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# SOCIAL WELFARE MAXIMIZATION OF TOURIST BUS SERVICE SYSTEM IN THE M/M/1 QUEUE WITH STRATEGIC TOURISTS

Nidhi and Sudeep Singh Sanga*

**Abstract.** Tourism is experiencing a transformative phase in its development, playing a pivotal role in the country's economic growth. The potential for further advancement lies in effective tourism management. Recognizing this vital aspect, the present study investigates the application of the M/M/1 queueing model, considering the strategic behavior of tourists in a tourist bus service (TBS) system. A TBS system involves the provision of bus services to tourists, ensuring convenient and efficient travel to various destinations. Understanding the phenomenon of balking as a strategic behavior of tourists is crucial, where tourists strategically decide whether to join or balk based on the queue length, thereby highlighting its impact on the dynamics of tourists' decision-making. In the proposed model, our attention is directed toward understanding the equilibrium and socially optimal strategies adopted by incoming tourists through a reward-cost structure in the observable queue. This structure allows us to analyze and identify the most effective and efficient strategies for tourists based on the balance between the rewards they receive and the costs they incur. Furthermore, government strategies involve interventions and policies by authorities to regulate and optimize the functioning of the TBS system. These strategies may include the implementation of subsidies or taxes on buses aimed at influencing bus arrival rates. We examine how governmental strategies are employed to identify the optimal bus buffer size objectively, ultimately seeking to maximize social welfare in the TBS system. Through a series of numerical experiments, we aim to unravel the dynamics of the optimal strategies while assessing the impact of varying information levels and TBS system parameters on the overall societal benefit. The ultimate objective is to pave the way for an optimized and more beneficial tourism landscape, fostering positive outcomes in the tourism sector.

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## 1. Introduction

In the midst of global advancements and economic progress, tourism stands as a cornerstone for both individual enjoyment and national development. As nations thrive and economies flourish, individuals seek ways to enrich their lives, often turning to travel as a means of experiencing new cultures and destinations. In India, a country renowned for its diverse heritage and scenic beauty, the travel and tourism sector plays an important role, contributing to the continuous growth of the nation's gross domestic product. This sector's prominence

*Keywords.* M/M/1 queue, tourist bus service system, observable queue, strategic behavior, government strategies, social welfare maximization.

Department of Mathematics, SV National Institute of Technology, Surat, Gujarat 395007, India.
*Corresponding author: ssanga@amhd.svnit.ac.in

highlights the growing importance of tourism in the country's economic landscape. Motivated by this trend, this paper focuses on the aspect of queueing management in the tourist bus service (TBS) system, seeking to explore its significance and impact within this thriving sector [10, 11, 14]. At tourism destinations, buses often serve as a primary transportation choice for tourists. However, the inherent randomness in demand and supply for this service can create scenarios where imbalances occur. There may be instances where numerous tourists wait for buses or where an excess of idle buses await tourists. In these situations, the travel operations (TO) manager incurs waiting costs due to idle vehicles, or tourists bear the waiting costs. To model and manage such dynamics, a matching queueing system is utilized, which allows the arrival of both tourists and buses from the opposite end. They are matched on a first-come, first-served (FCFS) basis, then services are provided immediately, and both exit the system together. Introduced by Kendall [9], this model provides insight into the matching of supply and demand in stochastic service systems. Matching (or double-ended) queueing problems have been studied by a few researchers [3,4,7]. Wang *et al.* [19] examined a matching queueing system featuring a gated policy. Within this framework, they not only explored the dynamics of how taxis respond to queue length thresholds but also delved into understanding the social welfare optimization problems. Wang *et al.* [20] researched a matching queue characterized by a two-point matching time. Their research delved into the analysis of passengers' equilibrium strategy and the socially optimal strategy in the system.

In tourist destinations, it becomes important to strike a balance between the availability of tourists and the demand for tourist buses for efficient and seamless travel experiences. This involves aligning the benefits for tourists seeking tourist buses and optimizing the service of tourist buses, both of which are key challenges within this matching queueing model. A critical concern arises when the bus queue becomes excessively long due to an ample waiting space, resulting in increased costs that the TO manager has to bear. This prolonged queue not only affects the TO manager's expenses but also diminishes the overall social welfare, which encompasses the benefits accrued by both the TO manager and tourists. To overcome this inefficiency and enhance overall social welfare within the system, government strategies, such as the provision of subsidies or the imposition of taxes, are implemented. Subsidies serve as a tool to motivate TO managers to improve services, whether by increasing frequency or enhancing operational efficiency. On the flip side, taxes are strategically employed to manage congestion and promote efficient scheduling by guiding the flow of bus services to align with broader societal goals. These measures actively influence and regulate bus arrival rates, thereby shifting the focus toward optimizing the queue length of the bus. By fine-tuning the bus queue length, the objective is to enhance the overall social welfare, aligning the interests of tourists and the TO manager. The objective of maximizing social welfare is to establish a more balanced and beneficial environment for the operational efficiency of the TBS system. Notable works on social welfare maximization have been conducted by several authors [5,18,19]. Nguyen and Phung-Duc [13] studied a matching queueing system where both the supply and demand sides exhibit strategic behaviors. They solved the social welfare optimization problem in both observable and unobservable cases to determine the socially optimal strategies for both sides.

In the bustling dynamics of tourist queues, the decision of tourists to refrain from joining due to overcrowding introduces a significant challenge. There is growing interest in understanding the strategic behavior of tourists in queueing systems. The initial exploration of queueing systems involving customer strategic behavior was pioneered by Naor [12] in the observable queues. The observable queue is used to describe a system in which tourists have the ability to observe the state of the system and the tourist queue before making their decisions. On the other hand, the unobservable queue states that tourists do not know the exact length of the queue, but they have some information about the estimated arrival rates of tourists and buses. With this information, they decide whether to join the system or not based on their utilities. In these queues, the terms equilibrium (selfish) and socially optimal strategies describe tourists' behavior in situations where they formulate their strategy based on individual utility or social welfare [8,15,17,21,22]. This strategic decision-making by tourists affects not only their waiting times but also influences the overall dynamics of the queues. Understanding these behaviors helps in designing better systems to manage queues, ensuring tourists have a seamless travel experience. For deeper insights into strategic queueing systems, valuable resources include the works of Hassin and Haviv [8], Burnetas and Economou [1] and Burnetas *et al.* [2]. Economou and Manou [6] investigated customers' strategic behavior in

TABLE 1. Notations.

| | |
|---|---|
| $K$ | Capacity for buses |
| $\lambda_0$ | Arrival rate of tourists |
| $\lambda_1$ | Arrival rate of buses |
| $\Lambda_0$ | Potential arrival rate of tourists |
| $\beta$ | Effective arrival rate of buses |
| $q$ | Joining probability |
| $p_0$ | Bus fare |
| $R$ | Reward |
| $C_0$ | Waiting cost for tourists |
| $C_1$ | Waiting cost for buses |
| $C_f$ | Fuel expenses |
| $C$ | Societal congestion cost |
| $p_1$ | Subsidy/tax rate |
| $t$ | Time |
| $\rho$ | Traffic intensity |
| $E[N_t]$ | Average queue length of tourists |
| $E[N_b]$ | Average queue length of buses |
| $W[N_t]$ | Average waiting time for tourists |
| $W[N_b]$ | Average waiting time for buses |
| $U_0$ | Utility function for tourists |
| $U_1$ | Utility function for buses |
| $k_0$ | Selfish optimal threshold for tourists |
| $k^{0'}$ | Socially optimal threshold for tourists |
| $S_w$ | Social welfare |
| $K^*$ | Optimal bus buffer size |
| $S_w(K^*)$ | Optimal social welfare |

the fluid queue, considering two levels of information upon arrival. In recent research, Tian *et al.* [16] conducted a study on a single server Markovian queue with strategic customers at different service rates. The focus of the investigation was to explore and understand the equilibrium strategies adopted by customers in the system.

The queueing literature has identified a significant research gap concerning optimal strategies for both tourists and buses within the tourist bus service system. This paper aims to fill that void by delving deeper into the strategic behavior of tourists in the tourist bus service system. Our study focuses on exploring how these strategies vary under different levels of available information with the overarching goal of maximizing social welfare. Emphasizing the importance of social welfare is a key aspect of our research, which aims to enhance the efficiency and effectiveness of the TBS system. We consider two types of optimal strategies for tourists in the observable system. Equilibrium strategies are those adopted by tourists to maximize their individual benefits, while socially optimal strategies are chosen with the aim of maximizing overall social welfare for the collective well-being of the society. Additionally, we implement government strategies to maximize social welfare by determining the optimal bus buffer size. The model is constructed using queueing notations outlined in Table 1. The remainder of the paper is structured as follows: Section 2 provides a precise description and analysis of the tourist bus service system. Section 3 concentrates on the observable queue case and deduces the threshold policy, considering both individual and social welfare perspectives. The discussion in Section 4 delves into government strategies aimed at maximizing social welfare. Section 5 presents numerical examples illustrating the influence of various information levels and parameters on the TBS system. Finally, Section 6 concludes the paper by summarizing the key findings.

## 2. Model description

In this study, a tourist bus service (TBS) system is designed as a matching queueing model. The TBS system involves tourists arriving at the bus stand prior to the tourist spot, where they disembark from their private or public transport vehicles. From there, they board buses to reach the tourist spot. It considers groups of one to ten tourists traveling together as a single entity, referred to as one tourist. The formulation of the model is based on certain assumptions outlined as follows:

- The arrivals of tourists and buses follow Poisson distribution with rates $\lambda_0$ and $\lambda_1$, respectively.
- Tourists possess the option to balk, with a probability $(1-q)$, upon observing congested queues.
- The waiting space for buses is limited to a maximum $K$ at the bus stand. Notably, there are no limitations on the number of tourists joining the bus stand, allowing an unlimited influx of tourists.
- The TBS system operates on a first-come, first-served basis for both tourists and buses. As soon as the transport bus becomes available, tourists board the buses immediately without any waiting time.
- Tourists facing queue-related waiting costs of $C_0$ and paying a bus fare of $p_0$ but gaining a reward $R$ upon taking a bus.
- Tourist buses incur waiting costs of $C_1$ and contribute to societal congestion cost $C$ per unit of time, with buses also facing fuel expenses of $C_f$ per trip.
- To enhance tourist service, the government can modify bus arrival rates by implementing either a subsidy/tax of $p_1$. A positive $p_1$ signifies a governmental subsidy to buses, while a negative $p_1$ implies a tax on each trip. The aim is to strike a balance between improving service quality for tourists and mitigating congestion in the TBS system.

Apart from the strengths of the developed model, there are a few limitations, which are outlined below:

- The model strictly adheres to a first-come, first-served approach without any provision for priority tourists.
- All buses are fully operational without taking into account the possibility of breakdowns or operational problems.

Let $|K(t)|$ denote the queue length of either tourists or buses at the bus stand at time $t$. When $K(t)<0$, it signifies that buses are awaiting new tourists. If $K(t)=0$, the TBS system is empty. In the scenario where $K(t)>0$, it indicates tourists are waiting for buses. This TBS system forms a one-dimensional continuous-time Markov chain characterized by a state space $Z=\{-K,-K+1,\ldots,-1,0,\ldots\}$. The pictorial representation of the TBS system is shown in Figure 1.
Now, we analyze the model in the steady-state environment. The steady-state probability of being in state $j$ in the system is represented by

$$\Pi_j=\lim_{t\to\infty}\Pi_j(t)=\begin{cases}\text{Probability of } |j| \text{ buses waiting for the arrival of new tourists,} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for } -K\leqslant j<0 \\ \text{Probability of an empty system,} \quad \text{for } j=0 \\ \text{Probability of } j \text{ tourists waiting for buses,} \quad \text{for } j>0.\end{cases}$$

The flow balance equations, integral to modeling the TBS system's dynamics, establish an equilibrium between the inflow and outflow rates. These equations are

$$\lambda_0 q\Pi_{-K}=\lambda_1\Pi_{-K+1} \tag{1}$$

$$(\lambda_0 q+\lambda_1)\,\Pi_j=\lambda_0 q\Pi_{j-1}+\lambda_1\Pi_{j+1}, \quad j=-K+1,-K+2,\ldots \tag{2}$$

Now, these equations are solved using a recursive approach. From equation (1), it follows

$$\Pi_{-K+1}=\frac{\lambda_0 q}{\lambda_1}\Pi_{-K}. \tag{3}$$

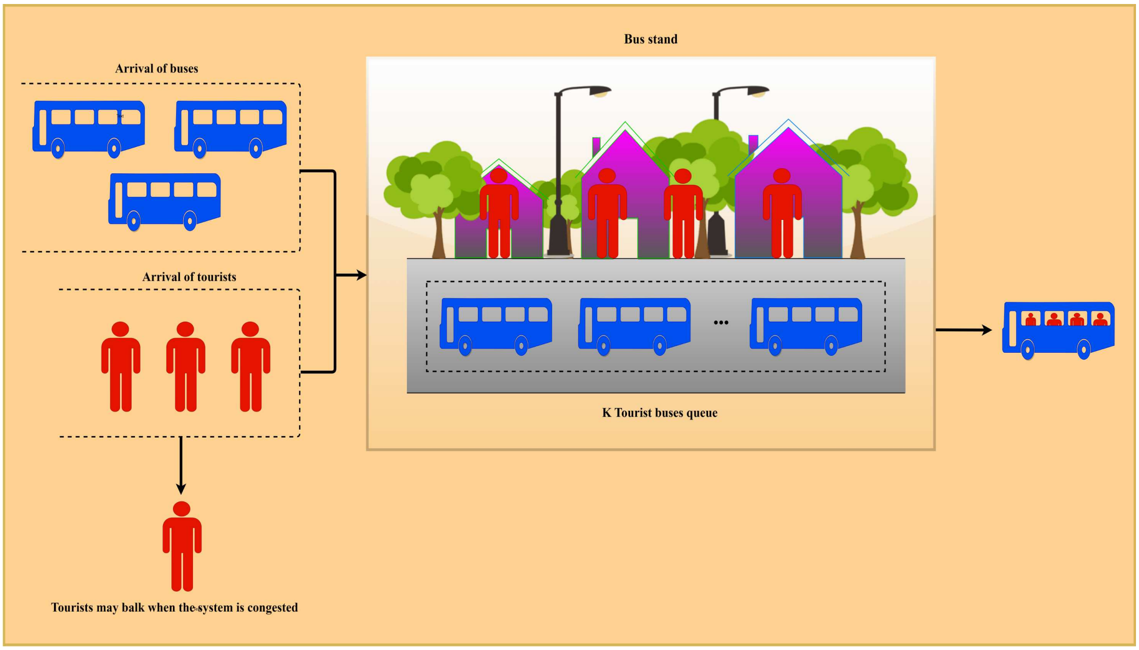



FIGURE 1. Pictorial representation of tourist bus service (TBS) system.

Put $n = -K + 1$ in equation (2), we get

$$(\lambda_0 q + \lambda_1)\,\Pi_{-K+1} = \lambda_0 q \Pi_{-K} + \lambda_1 \Pi_{-K+2} \tag{4}$$

$$\Pi_{-K+2} = \left(\frac{\lambda_0 q}{\lambda_1}\right)^2 \Pi_{-K}. \tag{5}$$

In general, we have

$$\Pi_j = \left(\frac{\lambda_0 q}{\lambda_1}\right)^{K+j} \Pi_{-K}, \quad j = -K+1, -K+2, \ldots \tag{6}$$

where $\Pi_{-K}$ is computed using the normalized condition.

$$\sum_{j=-K}^{\infty} \Pi_j = 1. \tag{7}$$

Now, we obtain the probability distribution for the TBS system, as given below:

$$\Pi_j = \begin{cases} 1-\rho, & j = -K \\ \rho^{K+j}(1-\rho), & j = -K+1, -K+2, \ldots \end{cases} \tag{8}$$

**Remark 2.1.** Stability criteria for the TBS system is $\rho = \frac{\lambda_0 q}{\lambda_1} < 1$.

### System metrics

– The average queue length of tourists and the expected queue length of buses are

$$E[N_t] = \sum_{j=0}^{\infty} j\Pi_j = \frac{\rho^{K+1}}{1-\rho} \tag{9}$$

$$E[N_b] = \sum_{j=-K}^{0} (-j)\Pi_j = K - \frac{\rho - \rho^{K+1}}{1-\rho}. \tag{10}$$

– The average waiting time for a tourist and the expected waiting time for a bus are

$$W[N_t] = \frac{E[N_t]}{\lambda_0} = \frac{\rho^{K+1}}{\lambda_0(1-\rho)} \tag{11}$$

$$W[N_b] = \frac{E[N_b]}{\beta} = \frac{K}{\lambda_0 q} - \frac{\rho - \rho^{K+1}}{\lambda_0 q(1-\rho)} \tag{12}$$

where the effective arrival rate of buses is $\beta = \lambda_1 (1 - \Pi_{-N}) = \lambda_1 \rho = \lambda_0 q$.

### Utility function

– The utility function for each tourist and bus joining the waiting queue are

$$U_0 = R - p_0 - C_0 W[N_t] \tag{13}$$

$$U_1 = p_0 + p_1 - C_f - C_1 W[N_b]. \tag{14}$$

## 3. Observable system

In this observable queue scenario related to tourism, tourists are well-informed about the queue dynamics. When a queue for the bus is present, arriving tourists promptly receive service and seamlessly integrate into the TBS system. For the tourist queue, tourists assess its length and adhere to a threshold policy: if it falls below a threshold, they willingly join the queue and wait. However, if the queue surpasses this threshold, tourists opt not to join, preferring to wait for a less crowded period or explore alternate tourist transportation options. This system empowers tourists to make well-informed decisions, optimize their choices based on observable queue information, and enhance their tourism experience.

### 3.1. Tourists' equilibrium strategies

In pursuit of maximizing individual benefit through a pure threshold strategy $k_0$ as an equilibrium joining approach, the tourist's decision to join or not relies on the current number of tourists in the TBS system. The threshold $k_0$ should meet these criteria based on the tourist's utility function, which are given below:

$$R - p_0 - k_0 \frac{C_0}{\lambda_1} \geqslant 0 \text{ and } R - p_0 - (k_0+1)\frac{C_0}{\lambda_1} < 0. \tag{15}$$

Therefore,

$$k_0 = \left\lfloor \frac{\lambda_1 (R - p_0)}{C_0} \right\rfloor. \tag{16}$$

### 3.2. Socially optimal strategies of tourists

In the exploration of the socially optimal strategy for an arriving tourist, the focus lies on a specific threshold known as the socially optimal threshold. This threshold represents a point at which the overall social benefit is maximized. The TBS system under consideration follows a one-dimensional continuous-time Markov chain, characterized by a state space $\{-K, -K+1, \ldots, -1, 0, \ldots, k^0\}$, where $k^0$ denotes the tourist buffer size. Now, we write the steady-state equations using flow balance as follows:

$$\lambda_0 q \Pi_{-K} = \lambda_1 \Pi_{-K+1} \tag{17}$$

$$(\lambda_0 q + \lambda_1)\, \Pi_j = \lambda_0 q \Pi_{j-1} + \lambda_1 \Pi_{j+1}, \quad j = -K+1, -K+2, \ldots, k^0 - 1 \tag{18}$$

$$\lambda_0 q \Pi_{k^0-1} = \lambda_1 \Pi_{k^0}. \tag{19}$$

By employing the recursive method, we derive the steady-state probabilities for the TBS system.

$$\Pi_j = \begin{cases} \frac{1-\rho_0}{1-\rho_0^{-j+k^0+1}}, & j = -K \\ \frac{(1-\rho_0)\rho_0^{K+j}}{1-\rho_0^{K+k^0+1}}, & j = -K+1, -K+2, \ldots, k^0 \end{cases} \tag{20}$$

where $\rho_0 = \frac{q\Lambda_0}{\lambda_1}$ and $\Lambda_0$ is the potential arrival rate of tourists.
Then, the effective arrival rate of tourists is

$$A_{eff} = \Lambda_0 \left(1 - \Pi_{k^0}\right) = \Lambda_0 \left( \frac{1 - \rho_0^{K+k^0}}{1 - \rho_0^{K+k^0+1}} \right). \tag{21}$$

The expected number of tourists and buses in the queue are

$$E[N_t'] = \sum_{j=0}^{k^0} j \Pi_j = \frac{\rho_0^{K+1}}{1 - \rho_0^{K+k^0+1}} \left( -k^0 \rho_0^{k^0} + \frac{1 - \rho_0^{k^0}}{1 - \rho_0} \right) \tag{22}$$

$$E[N_b'] = \sum_{j=-K}^{0} (-j) \Pi_j = \frac{1}{1 - \rho_0^{K+k^0+1}} \left( K - \frac{\rho_0 - \rho_0^{K+1}}{1 - \rho_0} \right). \tag{23}$$

The social welfare of the TBS system is

$$S_w^{'}\left(k^0\right) = A_{eff}\left(R + p_1 - C_f\right) - C_0 E[N_t'] - C_1 E[N_b'] - CK. \tag{24}$$

The optimal threshold $k^{0'}$ for social welfare must satisfy the following inequalities:

$$S_w^{'}\left(k^{0'}\right) - S_w^{'}\left(k^{0'} + 1\right) \geqslant 0 \tag{25}$$

$$S_w^{'}\left(k^{0'}\right) - S_w^{'}\left(k^{0'} - 1\right) \geqslant 0. \tag{26}$$

After some algebraic calculations, the equations (25) and (26) transform into:

$$- \Lambda_0 \left(R + p_1 - C_f\right) (1 - \rho_0)^2 + C_0 \rho_0 \left[ \left(k^{0'} + 1\right)(1 - \rho_0) - \rho_0^{K+1} \left(1 - \rho_0^{k^{0'}+1}\right) \right] - C_1 \rho_0 [K - K\rho_0 - \rho_0 + \rho_0^{K+1}] \geqslant 0 \tag{27}$$

$$\left(k^{0'} + 1\right)(1 - \rho_0) - \rho_0^{K+1} \left(1 - \rho_0^{k^{0'}+1}\right) \geqslant \frac{T}{C_0 \rho_0} \tag{28}$$

where $T=\Lambda_0\left(R+p_1-C_f\right)\left(1-\rho_0\right)^2+C_1\rho_0\left[K-K\rho_0-\rho_0+\rho_0^{K+1}\right]$ and

$$\Lambda_0\left(R+p_1-C_f\right)\left(1-\rho_0\right)^2+C_0\rho_0\left[-k^{0'}\left(1-\rho_0\right)+\rho_0^{K+1}\left(1-\rho_0^{k^{0'}}\right)\right]+C_1\rho_0[K-K\rho_0-\rho_0+\rho_0^{K+1}]\geqslant 0 \quad (29)$$

$$k^{0'}\left(1-\rho_0\right)-\rho_0^{K+1}\left(1-\rho_0^{k^{0'}}\right)\leqslant\frac{T}{C_0\rho_0}. \quad (30)$$

From equations (28) and (30), we have

$$k^{0'}\left(1-\rho_0\right)-\rho_0^{K+1}\left(1-\rho_0^{k^{0'}}\right)\leqslant\frac{T}{C_0\rho_0}\leqslant\left(k^{0'}+1\right)\left(1-\rho_0\right)-\rho_0^{K+1}\left(1-\rho_0^{k^{0'}+1}\right). \quad (31)$$

Now, our task is to determine whether there exists a unique value of $k^{0'}$ that fulfills

$$k^{0'}\left(1-\rho_0\right)-\rho_0^{K+1}\left(1-\rho_0^{k^{0'}}\right)=\frac{T}{C_0\rho_0}. \quad (32)$$

Let

$$f\left(y\right)=y\left(1-\rho_0\right)-\rho_0^{K+1}\left(1-{\rho_0}^{y}\right). \quad (33)$$

The first-order derivative of $f(y)$ is

$$g(\rho_0)=\frac{df(y)}{dy}=1-\rho_0+\rho_0^{K+1+y}\ln\rho_0. \quad (34)$$

The second-order derivative of equation (33) is

$$\frac{d^2f(y)}{dy^2}=\rho_0^{K+1+y(\ln\rho_0)^2}>0. \quad (35)$$

Thus, $f(y)$ is a concave function.
The first-order derivative of $g(\rho_0)$ is

$$\frac{dg\left(\rho_0\right)}{d\rho_0}=-1+\left(K+1+y\right)\rho_1^{K+y}\ln\rho_0+\rho_0^{K+y}. \quad (36)$$

The second-order derivative of $g(\rho_0)$ is

$$\frac{d^2g\left(\rho_0\right)}{d\rho_0^2}=\left[\left(K+1+y\right)\left(K+y\right)\ln\rho_0+\left(2K+2y+1\right)\right]\rho_0^{K+y-1}. \quad (37)$$

When $0<\rho_0<1$ so $\frac{df(y)}{dy}=g(\rho_0)>g(\rho_0=1)=0$, which implies that $f(y)$ is an increasing function in $y$ for $\rho_0\in(0,1)$. Thus, there exists a unique root of equation (32).
If $\rho_0=1$, there is not any positive number $k^{0'}$ that satisfies equation (32). As a result, there is no optimal threshold for social welfare.
When $\rho_0>1$, then $\frac{d^2g(\rho_0)}{d\rho_0^2}>0$, thus $\frac{dg(\rho_0)}{d\rho_0}$ is an increasing function with $\rho_0$. We find that $\frac{dg(\rho_0)}{d\rho_0}>\frac{dg(\rho_0)}{d\rho_0}|_{\rho_0=1}=0$ for $\rho_0>1$. Therefore, we need to check whether $y_0=1$ is the root of the equation (32). If $y_0=1$ is the root, then the optimal threshold for the maximum social welfare is $k^{0'}=1$. If not, there exists a unique root $y_0$ that satisfies equation (32), and the optimal threshold becomes $k^{0'}=\lfloor y_0\rfloor$.

## 4. Government strategies

Within the realm of tourism, government strategies serve as essential tools for effective planning to shape the operational dynamics of the TBS system. As part of this, the government endeavors to determine the optimal bus buffer size. This seeks to not only maximize social welfare but also ensure non-negative utility for tourists and buses. This multi-step process involves deriving the optimal bus buffer size $K^*$ exclusively to enhance societal benefit, followed by adjusting pricing $p_0$ to secure non-negative tourist utility. Subsequently, determining the $p_1$ value to tax or subsidize bus operators ensures a balanced bus utility. This systematic approach navigates the complexities of the tourism sector, aiming to optimize the bus buffer size while maintaining tourist satisfaction and equitable treatment for bus operators, contributing to the overall welfare of the tourism industry.
The expected social welfare per unit of time is

$$\begin{aligned} S_w(K) &= \lambda_0 U_0(K) + \beta U_1(K) - CK \\ &= \lambda_0 (R - p_0) + \beta (p_0 + p_1 - C_f) - C_0 E[N_t] - C_1 E[N_b] - CK \\ &= \lambda_0 (R + p_1 - C_f) - \frac{C_0 \rho^{K+1}}{1-\rho} - C_1 \left( K - \frac{\rho - \rho^{K+1}}{1-\rho} \right) - CK. \end{aligned} \tag{38}$$

The maximization problem of the social welfare function is

$$\begin{aligned} \max_K S_w(K) &= \lambda_0 U_0(K) + \alpha U_1(K) - CK \\ &= S_w(K^*) \end{aligned} \tag{39}$$

where $K^*$ is the optimal bus buffer size.
Now, the first derivative of the equation (38) is

$$\frac{dS_w(K)}{dK} = -\frac{C_0 \rho^{K+1} \ln \rho}{1-\rho} - C_1 \left( 1 + \frac{\rho^{K+1} \ln \rho}{1-\rho} \right) - C. \tag{40}$$

The second derivative of $S_w(K)$ is

$$\frac{d^2 S_w(K)}{dK^2} = -\rho^{K+1} (\ln \rho)^2 \left( \frac{C_0 + C_1}{1-\rho} \right) < 0. \tag{41}$$

Thus, $S_w(K)$ is a concave function, so there exists an optimal point where social welfare is maximum. Therefore, $\frac{dS_w(K)}{dK} = 0$ which implies

$$K^* = \frac{\ln \left( -\frac{(C_1 + C)(1-\rho)}{(C_0 + C_1) \ln \rho} \right)}{\ln \rho} - 1. \tag{42}$$

After determining the optimal bus buffer size, we can make adjustments to the values of $p_0$ and $p_1$ to ensure that both tourist and bus operator utilities remain non-negative. It is crucial to note that adjusting $p_0$ does not impact the optimal taxi buffer size. When adjusting $p_1$, a positive value indicates subsidizing the bus operator, while a negative value means imposing a tax on the bus operator with a magnitude of $-p_1$. This strategic adjustment is designed to guarantee that both tourists and bus operators experience positive utility, ensuring a balanced operational environment within the tourist bus service system.

**Remark 4.1.** Social welfare function is positive if

$$\min S_w(K) = \min \left( \lambda_0 (R + p_1 - C_f) - \frac{C_0 \rho^{K+1}}{1-\rho} \right) - C_1 \left( K - \frac{\rho - \rho^{K+1}}{1-\rho} \right) - CK) > 0. \tag{43}$$

Here, with $0 < \rho < 1$, this implies $\rho_{N+1} < \rho$, which gives

$$\lambda_0 \left(R + p_1 - C_f\right) - \frac{C_0 \rho^{K+1}}{1-\rho} - C_1 \left(K - \frac{\rho - \rho^{K+1}}{1-\rho}\right) - CK > \lambda_0 \left(R + p_1 - C_f\right) - \frac{C_0 \rho}{1-\rho} - \left(C_1 + C\right) K. \quad (44)$$

From equation (44), we get

$$K < \frac{\lambda_0 \left(R + p_1 - C_f\right)(1-\rho) - C_1(1-\rho)}{\left(C_1 + C\right)(1-\rho)}. \quad (45)$$

**Remark 4.2.** Optimal buffer is positive if

$$\frac{\rho \ln \rho}{1-\rho} < -\left(\frac{C_1 + C}{C_0 + C_1}\right). \quad (46)$$

## 5. Numerical results

In this section, we aim to demonstrate the influence of system parameters on tourists' strategies and social welfare in different scenarios. For the purpose of our analysis, we consider two sets ($S_1$ and $S_2$) of fixed system parameters:
$S_1 : \lambda_0 = 15, \Lambda_0 = 15, \lambda_1 = 18, p_0 = 15, p_1 = 20, R = 60, C_0 = 15, C_1 = 20, C_f = 30, C = 2, q = 0.75$ and
$S_2 : \lambda_0 = 20, \Lambda_0 = 20, \lambda_1 = 24, p_0 = 20, p_1 = 25, R = 60, C_0 = 20, C_1 = 25, C_f = 35, C = 3, q = 0.8$.

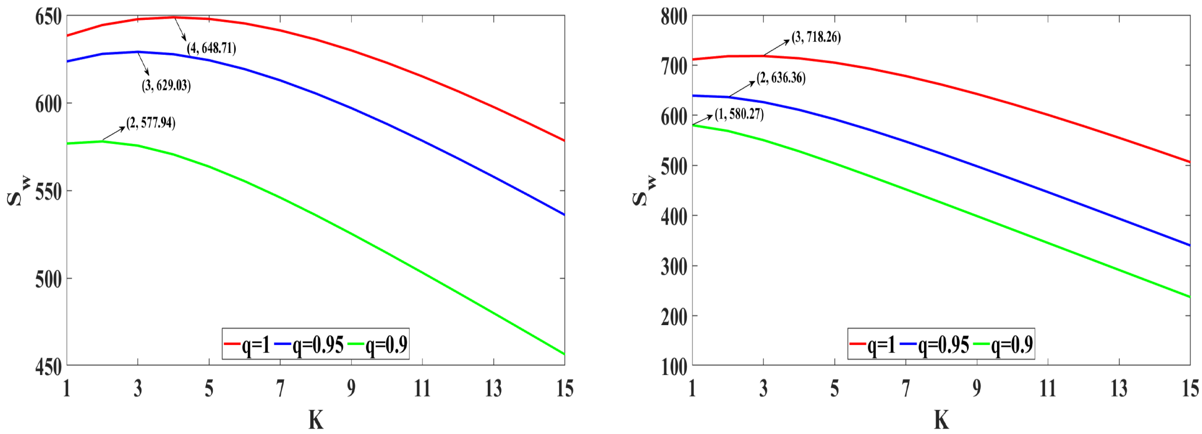
Figures 2 and 3 depict how social welfare changes concerning variations in parameters of joining probability $q$ and arrival rate of buses $\lambda_1$, aiming to find the optimal value for the capacity $K$ that maximizes social welfare. The observations indicate that as $q$ increases, social welfare also increases, suggesting a positive correlation. Conversely, an increase in $\lambda_1$ results in an increase in social welfare. The optimal value $K^*$ corresponds to the point where social welfare reaches its maximum value.

Moving to Figures 4 and 5, these visuals reveal a concave nature of the social welfare concerning the buffer $K$ under different parameter values of the arrival rate of tourists $\lambda_0$ and reward $R$. On comparing the two sets, it is evident that $S_2$ exhibits a higher value of social welfare function. This discrepancy is primarily attributed to the fact that all system parameters within $S_2$ possess higher values than those in $S_1$. This elevation in parameter values collectively contributes to a more favorable environment in $S_2$, thereby increasing overall social welfare in the TBS system.

Table 2 presents the optimal threshold for tourists solely concerned about their individual benefits, derived from the equation (16) for $S_1$. As the system parameter $\lambda_1$ increases, the optimal threshold $k_0$ for tourists also increases. Conversely, when $\lambda_1$ is fixed, higher bus fares $p_0$ lead to a reduction in the optimal threshold $k_0$. This implies that higher bus fares encourage tourists to adopt a more cautious approach, setting a lower threshold for prioritizing their individual benefits within the TBS system.

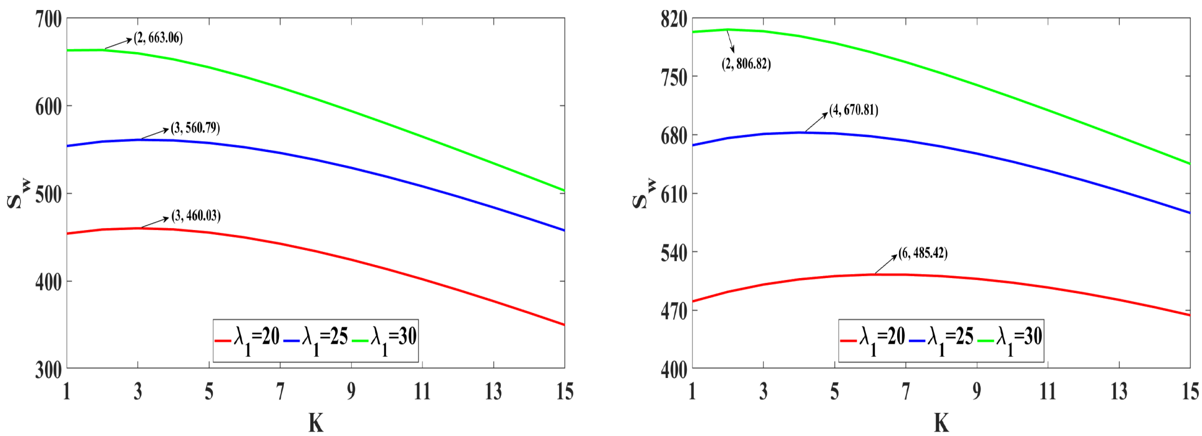
In Table 3, we determine the optimal threshold of socially concerned tourists with the goal of maximizing social welfare using equation (32) under two scenarios: when $\lambda_1 < \Lambda_0$ and $\lambda_1 > \Lambda_0$ for $S_1$. A comparison between these cases reveals that when $\lambda_1 > \Lambda_0$, the optimal social threshold $k^{0'}$ is consistently greater than $\lambda_1 < \Lambda_0$ case. Additionally, it shows that the optimal social threshold $k^{0'}$ increases with an increase in $K$ and decreases with an increase in $q$.

Additionally, Table 4 reveals insights into the optimal buffer and maximum social welfare under varying arrival rates for $S_1$. This shows that a simultaneous increase in $\lambda_0$ and $\lambda_1$ expands the buffer range that guarantees positive social welfare. Both the optimal buffer $K^*$ and maximum social welfare $S_w(K^*)$ also increase with these increments. Comparing scenarios with different $\lambda_0$ and $\lambda_1$ values, it is evident that an increased difference between these arrival rates results in an expanded range of buffer values and higher maximum social welfare. However, it results in a decrease in the optimal buffer. Understanding these trends helps in making decisions regarding parameter adjustments to optimize the TBS system's social welfare.




(a) (b)

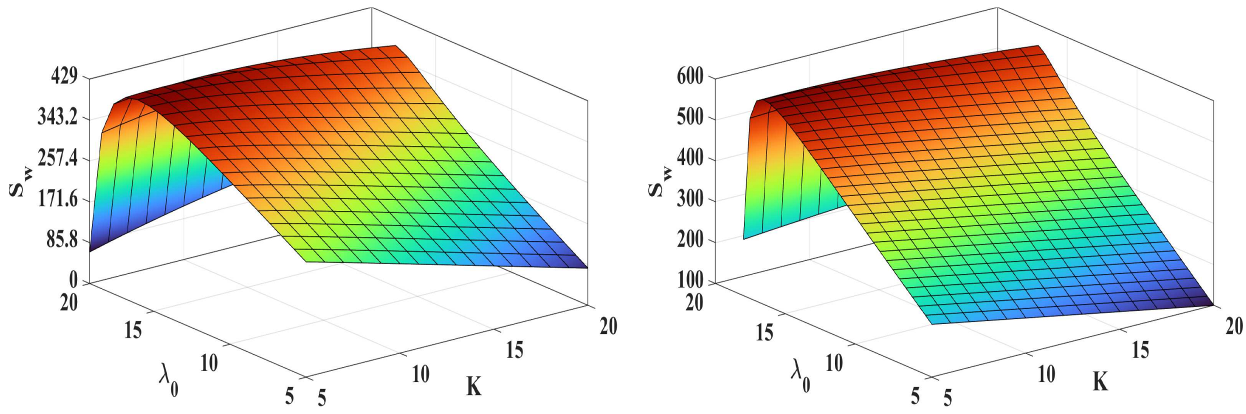
FIGURE 2. $K$ *vs.* $S_w$ on varying $q$. (a) For $S_1$. (b) For $S_2$.




(a) (b)

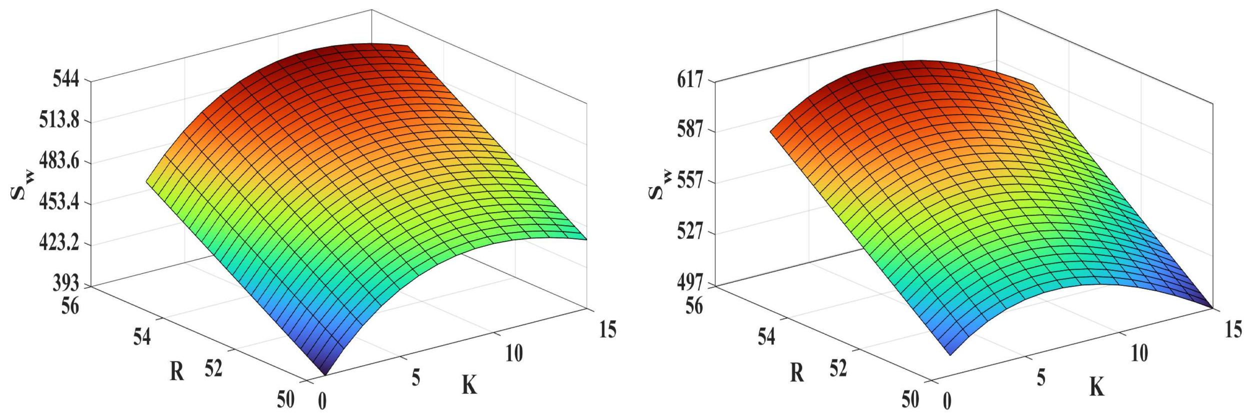
FIGURE 3. $K$ *vs.* $S_w$ on varying $\lambda_1$. (a) For $S_1$. (b) For $S_2$.

The analysis of numerical results highlights the significant influence of various parameters, such as the arrival rate of tourists and buses, waiting costs, and rewards, on the TBS system. These parameters are pivotal in shaping the social welfare of the system. For illustrative purposes, we have examined two sets of system parameters to evaluate their impact on the TBS system. For instance, in set $S_1$, the arrival rates for tourists and buses are 15 and 18, respectively, while in set $S_2$, they are 20 and 24. The results indicate that an increase in the arrival rates for both tourists and buses generally leads to a rise in social welfare. However, if these rates increase excessively, it may result in a decline in social welfare due to issues such as overcrowding and operational efficiencies. Therefore, achieving optimal arrival rates is critical to maintaining a balanced system that supports social welfare. Moreover, all other parameters in set $S_2$ have higher values compared to those in set $S_1$, leading to varying outcomes. Notably, set $S_2$ yields a higher value of the social welfare function, underscoring the importance of meticulous parameter management to achieve the desired level of social welfare. Higher social welfare in the system not only enhances the tourist experience but also fosters local economic growth, community well-being,




(a) (b)

FIGURE 4. $(K, \lambda_0)$ *vs.* $S_w$. (a) For $S_1$. (b) For $S_2$.




(a) (b)

FIGURE 5. $(K, R)$ *vs.* $S_w$. (a) For $S_1$. (b) For $S_2$.

and increased tourism activity, ultimately contributing to a more sustainable and thriving TBS system. Thus, by carefully managing system parameters, decision-makers can effectively improve the overall performance and efficiency of the TBS system.

## 6. Conclusion

In this paper, we investigated the tourists' equilibrium and socially optimal strategies in the observable queue for a tourist bus service system. The identified threshold-type equilibrium and socially optimal strategies provide valuable insights into the dynamics of the TBS system. These strategies not only contribute to a more efficient and satisfactory experience for tourists within the queue but also result in enhanced service efficiency and reduced waiting times. To strike a balance in benefits between tourists and buses, the government has either provided subsidies to buses or imposed taxes on them. The subsidies provided encourage more buses to participate in the TBS system, potentially improving service availability. On the other hand, the taxes

TABLE 2. Optimal equilibrium threshold of tourists in the observable system.

| | $p_0$ | | | |
|---|---|---|---|---|
| $\lambda_1$ | 12 | 16 | 20 | 24 |
| 16 | $k_0 = 35$ | $k_0 = 31$ | $k_0 = 27$ | $k_0 = 22$ |
| 19 | $k_0 = 42$ | $k_0 = 37$ | $k_0 = 32$ | $k_0 = 27$ |
| 22 | $k_0 = 48$ | $k_0 = 43$ | $k_0 = 37$ | $k_0 = 31$ |
| 25 | $k_0 = 55$ | $k_0 = 48$ | $k_0 = 42$ | $k_0 = 35$ |
| 28 | $k_0 = 62$ | $k_0 = 54$ | $k_0 = 47$ | $k_0 = 39$ |

TABLE 3. Socially optimal threshold of tourists in the observable system.

| | $(\Lambda_0, \lambda_1)$ | | | |
|---|---|---|---|---|
| $(K, q)$ | $(15, 18)$ | $(18, 23)$ | $(17, 15)$ | $(21, 18)$ |
| $(15, 0.7)$ | $k^{0'} = 25$ | $k^{0'} = 31$ | $k^{0'} = 13$ | $k^{0'} = 15$ |
| $(15, 0.8)$ | $k^{0'} = 21$ | $k^{0'} = 27$ | $k^{0'} = 12$ | $k^{0'} = 13$ |
| $(15, 0.9)$ | $k^{0'} = 18$ | $k^{0'} = 23$ | $k^{0'} = 11$ | $k^{0'} = 12$ |
| $(20, 0.7)$ | $k^{0'} = 28$ | $k^{0'} = 35$ | $k^{0'} = 17$ | $k^{0'} = 18$ |
| $(20, 0.8)$ | $k^{0'} = 25$ | $k^{0'} = 31$ | $k^{0'} = 15$ | $k^{0'} = 17$ |
| $(20, 0.9)$ | $k^{0'} = 21$ | $k^{0'} = 26$ | $k^{0'} = 14$ | $k^{0'} = 16$ |

TABLE 4. Optimal values of $(K^*, S_w(K^*))$ in the range of buffer.

| $(\lambda_0, \lambda_1)$ | $K^*$ | Range of buffer | $S_w(K^*)$ |
|---|---|---|---|
| $(15, 18)$ | 2 | 29.75 | 395.81 |
| $(18, 20)$ | 3 | 34.32 | 468.39 |
| $(22, 27)$ | 2 | 47.07 | 600.13 |
| $(23, 24)$ | 4 | 42.13 | 590.34 |
| $(25, 30)$ | 2 | 52.79 | 678.52 |

imposed act as a regulatory mechanism, ensuring responsible and efficient bus operations. The overarching goal of maximizing social welfare has been achieved by determining the optimal number of buses. The theoretical findings have been supported by numerical experiments, contributing to the practical applicability of the model. The model's practicality was evident in various aspects, especially in its positive impact on both tourists and buses. Tourists benefited from a more effective and responsive bus service, improving the overall experience. Simultaneously, buses experience improved operational conditions, underscoring the model's effectiveness in elevating the overall efficiency and functionality of the TBS system. In the study, we also observed that the effective functioning of the system depends on an adequate number of buses to meet tourist demands while avoiding overcrowding in the system. By striking this balance, we can effectively handle the flow of tourists and maximize their overall experience and satisfaction, thereby contributing to the maximization of social welfare for the TBS system. A potential direction for future research involves extending the analysis to include the prioritization of tourists and considering scenarios where the arrival processes for both tourists and buses vary over time. These expansions can provide a more nuanced understanding of optimizing queue strategies in the TBS system.

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## Data availability statement

No data was used for the research described in the article.

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