RAIRO-Oper. Res. 58 (2024) 3985–4012
https://doi.org/10.1051/ro/2024158

RAIRO Operations Research
www.rairo-ro.org

PRICING POLICIES FOR A COMPETITIVE DUAL-CHANNEL SUPPLY CHAIN WITH GREEN INVESTMENT AND SALES EFFORT UNDER REVENUE-SHARING CONTRACT

Abhijit Barman[1], Rubi Das[1], Pijus Kanti De[1] and Shib Sankar Sana[2,*]

Abstract. In view of the mushroom development of web-based business and green production, a few organizations have moved to produce ecological friendly items and utilized the online channel to build up the competition. This paper deals with a two-echelon sustainable dual-channel supply chain containing two manufacturers and a single retailer. One manufacturer of the investigated supply chain produces a traditional non-green item, and another one releases a substitutable green item. The demand of both the item at the online and offline channel is a function of the sales price, greenness level, sales effort of the retailer. Furthermore, we have discussed the criteria for optimal pricing and greening strategy of supply chain members and the sales effort level of the retailer under a centralized and decentralized model. To enhance the supply chain performance, a revenue-sharing contract has been proposed, subject to the pareto improvement of each channel member. The numerical result demonstrates that the proposed revenue-sharing contract can enhance the manufacturer's profit as well as system profit compared to the centralized and decentralized model and ensure higher profit for all the members than the decentralized model. Moreover, the item in the centralized model enjoys the highest green level than all other scenarios. Sensitivity analysis exhibits that a higher value of greening cost reduces the manufacturer's green level, decreasing the sales price.

Mathematics Subject Classification. 90B06, 90B30, 90B50.

Received July 20, 2022. Accepted July 31, 2024.

1. Introduction

Supply chain sustainability has been considered as a significant aspect to achieve growth in the economy. Manufacturing industries implemented concepts and issues of sustainability in their supply chains [18]. Production of green products has a positive impact on the environment and health; on the other hand, green technology investment increases the competitive advantage [34]. Green GPS units, solar speakers, organic food are extensively in high demand in the market. In response to increasing public consciousness of green products, companies prefer producing this type of product. Tesla, an automobile company, concentrates on only developing green cars and sells electric cars. But still, the traditional non-green products are the option of purchase for some

Keywords. Dual-channel supply chain, sales effort, greening decision, sales price, centralized, decentralized, revenue-sharing contract, game theory.

[1] Department of Mathematics, National Institute of Technology Silchar, Silchar 788010, Assam.

[2] Department of Mathematics, Kishore Bharati Bhagini Nivedita College, Ramkrishna Sarani, Kolkata 700060, India.

*Corresponding author: shib_sankar@yahoo.com

consumers. For example, Toyota and BMW sell both petrol cars and electric cars. The question arises of which car (electronic or petrol) customers should purchase depends on price, sales effort, quality, social awareness, etc. The co-existence of green and non-green manufactured products at retailer warehouses is the concern of this paper.

In the past few decades, the fast advancement of E-business had a significant influence on the purchasing behavior of consumers, and that encouraged the supply chain members to redesign and change the strategy of distribution [24]. The manufacturer can sell their products through direct online channels, retail channel, or by the combination of these two channels. This paper considers the combination selling strategy. Companies are used this dual-distribution channel in recent competitive markets. The co-existence of green or non-green products, different levels of environmental responsibility, dual-distribution strategy lead to various optimum pricing decisions. The manufacturer could acquire different measures to maximize the benefit by some sales effort (such as advertising, giving discounts, etc.). Customers would be convenient through sales effort by the manufacturer and retailer, and so this phenomenon affects the dual-channel framework of the supply chain. Developing pricing strategies within a dual-channel supply chain framework entails making decisions on how to price products that are distributed through both direct and indirect avenues. This situation commonly arises when manufacturers market their products directly as well as through intermediaries such as retailers or distributors. The primary challenge revolves around optimizing pricing choices to strike a balance between the interests of both channels and to maximize overall profitability.

Setting price strategies within a green supply chain framework involves the methods that businesses adopt to determine their product or service prices while keeping in mind environmental responsibility and societal concerns. Green supply chain management integrates environmentally friendly practices into different supply chain stages, including material sourcing, production, distribution, and disposal. The pricing approaches in this context aim to find a balance between economic viability and ecological and social considerations. Here are several aspects to think about when considering pricing policies in a green supply chain model:

(1) Environmental Expenses: include the expenses linked to environmental aspects, such as resource usage, pollution, emissions, and waste, into the final product cost. This process involves estimating externalises and incorporating them into the overall product price.
(2) Carbon Price Structure: introduce a method that accounts for the cost of carbon emissions, effectively internalizing these costs. This might involve setting a rate per ton of CO2 emitted during the product's life-cycle, which then becomes part of the product's price.
(3) Green Investment: gauge customers' potential willingness to pay a premium for eco-friendly products. By conducting market research, you can pinpoint an acceptable range for sustainable products and set prices accordingly.

While implementing eco-friendly pricing strategies demonstrates a commitment to sustainability and attracts environmentally conscious customers, it's also important for businesses to gauge the market's response to price adjustments. Striking the right balance between sustainability and affordability is pivotal for the success of these strategies. Moreover, educating customers about the merits of environmentally friendly products can be a key factor in justifying slightly higher prices.

The relationship between channel members of a supply chain for two complementary products in a duopoly market is either cooperative or non-cooperative. Coordination and cooperation between channel members increase the capability of supply chain operation. The supply chain can get a more economical and ecological advantage by integrating green and non-green item production. In the non-cooperative scenario, the member who has more power is considered as a leader, and the other party is the follower.

In view of the above explanation, motivation is occurred to study the arrival of green items into the present business sectors and look at how makers of green items compete with non-green item makers. Past most of the literature coexistence of green and non-green items mainly often investigated in general multi-layer supply chain system. This paper studies the dual-channel supply chain system in addition to the competitive market of the green and non-green items in the duopoly market. The following research questions are aimed to answer in this article.

(a) How should the supply chain establish the optimal pricing decision for green and non-green products in the online channel and retail channel in the duopoly market?
(b) What are the contrasts between the optimum decisions of different scenarios under fluctuation of greening effort and sales effort?
(c) What kind of supply chain contract is beneficial for members of the supply chain, and can a supply chain coordination contract improve the profit of the supply chain system?
(d) What impacts of system parameters such as price-elasticity, cross-price elasticity, green investment cost coefficient, sales effort cost coefficient, etc., on system performance and decisions?

To answer the above questions, in this research, we have developed a duopoly supply chain model serving an environmentally conscious consumer. The novelty of the research is that it not only ensure the production process of green manufacturing, it concludes the consumer demand and production system of non-green manufacturer also fulfill the requirement of divergence of the consumer. For this, firstly, we have designed the framework considering green and non green manufacturer where two manufacturers separately produce different products for the preference of the consumer; one of them manufacture a green item and another one makes traditional non-green item. Manufacturers sell the item or product through a common retailer, and they also have their own direct online channel. Secondly, the competition in market to sale the green and non-green items in a two-layer supply chain with a dual-distribution channel has been studied under centralized decision making model, decentralized decision making model, and revenue-sharing contract model. To show the coordination performance and the supply chain profitability, we have established the revenue sharing contract with better optimal decisions. The demand for each item is depend on sales price, product greening level, and sales effort of the retailer. At last, the model is demonstrated through some numerical illustration to study the impacts of major parameters which are affects on optimal pricing, greening decisions and the optimal profitability. Also, some marketing decisions are pointed out. To the best of our knowledge, this problem has not previously been studied with this competitive market environments.

2. Literature review

The relevant literature pertained to this study is essentially classified into the following sections: supply chain model with product greening and retailer's marketing effort, supply chain model with the competitive manufacturer, and supply chain model with revenue-sharing contract.

2.1. Supply chain model with product greening decision and retailer's marketing effort

Numerous studies have delved into the integration of environmentally friendly practices within supply chains and promotional strategies. Ghosh and Shah [20] analyzed supply chain performance in relation to demand influenced by pricing and green initiatives, shedding light on both manufacturer and retailer actions. Gao *et al.* [19] explored how sales efforts, pricing, and assortment decisions impact channel structure and member choices, ultimately affecting profitability. Madani and Rasti-Barzoki [32] investigated the factors guiding administrators' decisions in organizations within an ecological framework. Zhu and He [53] examined the interplay between sales price and green competition, revealing a nuanced impact on product greening due to double marginalization effects. Basiri and Heydari [10] investigated coordination in a two-stage green supply chain considering factors like selling price, sales effort, and greenness-sensitive demand. Dey and Saha [17] proposed a game model to assess pricing and greening decisions among manufacturers and purchasers, revealing insights through different acquisition scenarios. Zerang *et al.* [49] delved into closed-loop supply chains, exploring how retail prices and sales efforts influence demand and network success through diverse game approaches. Alamdar *et al.* [2] discussed a two-stage closed-loop supply chain model considering fuzzy sales prices and demand sensitivity to marketing efforts, employing various game models. Kuiti *et al.* [27] presented a supply chain system in terms of transportation cost and pack-size reduction considering environmental waste and pollution and uniquely combine transportation, product design, and retail decisions. Raza and Govindaluri [39] formulated a supply chain model offering two items, *i.e.*, regular and green items in the presence of partial stochastic demand and cannibalization. In an effort to assess the impact of incorporating environmental considerations and advertising efforts, Parsaeifar *et al.* [35] employed a non-cooperative game model that examined various aspects spanning economic and environmental realms. Xin *et al.* [46] examined a two-tier supply chain system for environmentally conscious buyers facing demand uncertainty, focusing on green product attributes. Shen *et al.* [41] used a game-theoretic approach to illustrate quality distinctions between green and non-green items within a two-tier supply chain setup. Investigating the consequences of greening levels and promotional endeavors in a closed-loop supply chain, Mondal and Giri [33] explored how these factors influence the performance of supply chain constituents, ultimately leading to enhanced outcomes. Green technology investment in three layer supply chain [9], product deterioration with backlogging [6], deterioration with two layer inventory [5], Stackelberg game in decentralized model [13], multi item deterioration [4], price sensitivity demand in multi-item inventory system [3], price discount scheme in non instantaneous inventory [7], return refund scheme [15] have been explored by different researcher under different scenarios of supply chain.

2.2. Dual channel supply chain

Given the advancement of e-channels, a significant portion of earlier research on dual-channel supply chains has primarily concentrated on optimizing ordering and pricing strategies. According to the findings by Dan *et al.* [12], organizations that focus on a select few direct channels tend to achieve superior economic performance compared to less adept entities with broader business involvement. Taleizadeh *et al.* [42] investigated the effects of marketing endeavors within dual-channel supply chains and introduced a pricing approach involving a two-part tariff arrangement. In a separate study, Li *et al.* [28] examined a scenario where a dual-channel supply chain's initiation was contingent on the cost of environmentally-friendly practices and customer loyalty, ultimately proposing conditions under which an online channel wouldn't be launched if greening costs surpassed a specific threshold. Zand *et al.* [48] recently outlined a dual-channel supply chain with environmental considerations, exploring cases where governmental restrictions on eco-friendly practices impact decision-making and member profitability. Heydari *et al.* [23] delved into a pricing and environmental decision framework within a three-layer supply chain, incorporating dual-channel supply chain considerations and multi-tier dynamics. Building upon this, Barman *et al.* [4] extended the work of Heydari *et al.* [23] by introducing a two-tier supply chain with eco-friendly decisions. Other scholars explored the dual-channel challenge in the context of green products and promotional strategies. For instance, Ranjan and Jha [38] investigated the influence of adopting a dual-channel setup on the performance of a two-layer supply chain. Their analysis encompassed variables like selling prices, eco-friendliness levels, and promotional efforts, particularly focusing on a scenario where standard products are distributed through retail channels and green items through online channels. Additionally, Wang and Song [44] concentrated on a two-stage supply chain involving uncertain demand influenced by green spending and marketing endeavors. This study considered the sale of both green and non-green items through offline and online channels. Notably, the domain of green dual-channel supply chains that simultaneously addresses eco-friendly and conventional products with marketing strategies has limited literature. The present paper tackles a supply chain problem involving dual-channel dynamics, wherein decisions concerning product pricing, eco-friendly practices, and retailer sales efforts are treated as variables, encompassing both green and non-green products.

2.3. Supply chain model with competitive manufacturer

This section focuses on the literature where different channel members of a supply chain achieve optimum decisions and have disparate capability in the market-place. Tsay and Agrawal [43] analyzes a supply chain framework with a manufacturer and two competiting retailers under retail price and service level dependent demand. Adida and DeMiguel [1] analyzed a supply chain model where multiple manufacturer supply the product to the multiple retailer [51] examined a two-layer supply chain with a single retailer and two competitive manufacturer; also discuss the pricing strategies of a substitutable item under fuzzy environment. That model was further extended by Zhao *et al.* [52] using game theoretic approach to evaluate centralized and decentralized case. Sana *et al.* [40] presented a three-stage supply chain model with multiple supplier, multiple manufacturer, multiple retailer inspecting both perfect and imperfect featured items. Two different manufacturer compete with each other to sell the product to a retailer under uncertain theory and game-theory approaches [25]. Jamali and Rasti-Barzoki [26] analyzed competitiveness between manufacturer and retailer in a dual-channel supply chain dealing with both green and non-green items. Giri *et al.* [22] established a duopoly market situation considering two manufacturer and one retailer supply chain framework with sales price dependent demand. Different researcher have implemented the application of NSGA-II in competitive supply chain to perform the optimal decision of supply chain [8].

2.4. Supply chain model with revenue-sharing contract

We have explored the literature review concerning revenue-sharing contracts, focusing primarily on their application within the context of green supply chain models. Our research concentrated on these specific aspects. Zhang and Liu [50] developed into achieving effective cooperation through revenue-sharing contracts in a three-stage green supply chain system, especially when facing uncertain demand. De Giovanni [16] conducted a comprehensive analysis of how revenue-sharing contracts can be employed to coordinate closed-loop green supply chain systems.

Giri *et al.* [21] introduced a closed-loop supply chain model that considered factors such as sales price, guarantee session, and greenness-sensitive demand to foster coordination among members using revenue-sharing mechanisms. Li *et al.* [29] introduced stochastic demand and fairness concerns within green supply chain models, examining the impact of key parameters on revenue-sharing models. Raj *et al.* [37] scrutinized revenue-sharing cooperation within the supply chain context, accounting for factors like greening, sales price, corporate social responsibility, and order quantity's influence on demand. To achieve coordination within a dual-channel supply chain, Xu *et al.* [47] established revenue-sharing contracts that not only harmonized the system but also ensured a mutually beneficial outcome for supply chain participants. Xie *et al.* [45] favored the adoption of revenue-sharing contracts to align interests between offline retailers and online manufacturers, incorporating service efforts. Qu *et al.* [36] investigated shipping policies and default risk within a green supply chain framework under revenue-sharing contracts. Li *et al.* [30] explored frugal innovation in an online supply chain system, considering revenue-sharing contracts, and analyzing their implications. Profit and cost sharing contract in dual channel scenario has been explored in Das *et al.* [14].

3. Problem definition and notations

This paper focuses on a green supply chain with dual distribution channels consisting of two manufacturers and single retailer shown in Figure 2. To prefer the customer's choice in the current competitive market, one manufacturer produces green items, the other manufacturer produces non-green items with a minimum green level and sells both green, non-green products with different sales price through direct online channel together with a common retailer in traditional channel. To promote and motivate customers to purchase products from retail channel, retailer puts sales effort. Retailer recruits proficient deals staff, builds item publicizing endeavors, improves deals staff business preparing, improves item show space and other techniques. The green manufacturer commodity is green degree which is determined by itself. In this work, we aims at how the individuals of the supply chain take their optimal decisions, and how the purchasers' green preference influences the profitability of supply chain under dual-channel structure. The retailer sets two different sales price for both green and non-green item. Also, at the online channel, both green and non-green item sales price is different from the retail channel; therefore the proposed model try to balance a predictable pricing policy tend to maximize their profit.

The presumptions made to demonstrate the model are addressed as follows:

Assumption 1. *The cost of greening paid by the manufacturer influenced by green level of the item and rises exponentially. The higher the item green level, the higher the greening cost of the manufacturer. The manufacturers investment cost to construct green item is* $\frac{1}{2}e_i g_i^2$*,* $i = \{1, 2\}$ *for green manufacturer and non-green manufacturer respectively. Here, It is assumed that the non-green item also has a minimum green level, which is fixed initially [19, 28].*

Assumption 2. *The cost of sales effort invested by the retailer is influenced by degree of sales effort and increases exponentially. We assume that the cost of sales effort is* $\frac{1}{2}\tau_i s_i^2$*,* $i = \{1, 2\}$ *for green products and non-green products simultaneously [31].*

Assumption 3. *The demand depends on the selling price, green level and additionally retail channel demand also depends on it's sales effort. It is presumed to linearly decreases with self price of the member, but linearly increases with competitors sales price.*

The demand faced by the green manufacturer from retail channel is D_1 *and at online channel* D_1'*. Furthermore this demand is positively correlated with the greening level. Also, retail channel demand is taken as upward sloping of sales effort level.*

$$\begin{cases} D_1 = \alpha_1 a_1 - b_1 p_1 + c_1' p_1' + c_2 p_2 + c_2' p_2' + \gamma_1 g_1 + \eta_1 s_1 & (1) \\ D_1' = (1-\alpha_1) a_1 - b_1' p_1' + c_1 p_1 + c_2 p_2 + c_2' p_2' + \gamma_1' g_1. & (2) \end{cases}$$

In equation (1), the term $\alpha_1 a_1$ *indicates the potential market demand. The subtraction* $b_1 p_1$ *signifies the demand reduction due to self-price elasticity. In addition, the terms* $c_1' p_1'$*,* $c_2 p_2$*,* $c_2' p_2'$ *are included for cross price elasticity. The higher green level and retailer's effort positively improve the potential demand for this* $\gamma_1 g_1$*,* $\eta_1 s_1$ *are included with* D_1*. Similarly, the demand for green items has been formulated in the online channel in equation (2) with customer loyalty degree* $1 - \alpha_1$*.*

The demand faced by the non-green manufacturer from retail channel is D_2 *and at online channel is* D_2'*. Traditional non-green items are long-settled item in the market and in this manner the green quality level of these items are fixed.*

$$\begin{cases} D_2 = \alpha_2 a_2 - b_2 p_2 + c_1 p_1 + c_1' p_1' + c_2' p_2' + \gamma_2 g_2 + \eta_2 s_2 & (3) \\ D_2' = (1-\alpha_2) a_2 - b_2' p_2' + c_1 p_1 + c_1' p_1' + c_2 p_2 + \gamma_2' g_2. & (4) \end{cases}$$

In equation (3), $\alpha_2 a_2$ *denotes the initial market potential of non green item with retail consumer loyalty* α_2*. The term* $b_2 p_2$ *is subtracted due to self price elasticity. In addition, the other term are included due to cross price, green and retail effort elasticity parameters. Here,* a_1 *and* a_2 *denotes the basic market demand for the green and non-green item,* α_1 *and* α_2 *where* $0 < \alpha_1, \alpha_2 < 1$ *are consumers loyalty degree at the retail channel for green and non-green manufacturer,* $(1 - \alpha_1)$ *and* $(1 - \alpha_2)$ *consumers loyalty degree at the online channel [53]. Somewhat,* α_1 *and* α_2 *reflects the offer in the competitive market of retailer for both green and non-green item,* $\alpha_1, \alpha_2 > \frac{1}{2}$ *retail channel have a larger portion of market share than her competitor online channel.*

Assumption 4. $b_i > c_i$ *as well as* $b_i' > c_i'$ *where* $i = \{1, 2\}$*.*

Here, parameter $\{b_1, b_2\}$ *are self-price elasticity of demand through retail channel and* $\{b_1', b_2'\}$ *through online channel,* $\{c_1, c_2, c_1', c_2'\}$ *are cross-price sensitivity, which implies the demand shift between the two channels for both the products as for the retail cost. Because of expansions of the unit price, the number of consumers who surrender a channel is more noteworthy than the number of another channel's consumers who change to this channel because of expansions in the unit price of the other channel. This assumptions is used in the literature [12, 26].*

Assumption 5. $b_i > \gamma_i > \eta_i > 0$ *as well as* $b_i' > \gamma_i' > 0$ *where* $i = \{1, 2\}$*.*

It is expected that the self-price coefficient is more significant than the green level and sales effort level, which is additionally a typical supposition in the literature [11, 28].

Assumption 6. *$\gamma_1 > \gamma_2$ and $\gamma_1' > \gamma_2'$.*

γ_1 and γ_2 indicates the effect of green level at the retail channel for the first commodity green product and second commodity non-green product respectively. γ_1' and γ_2' indicates the same at online channel. Green level is an important factor for the first commodity green item. But there is also a slight fixed cost function for the second manufacturer who produces non-green item. This assumption implies that buyers would prefer to purchase green items. If green items aren't accessible, then, at that point they change their mind to non-green item. The proportion of customer interest in purchasing a non-green item is γ_2 at retail channel and γ_2' at online channel. The difference between γ_1 and γ_2 as well as γ_1' and γ_2' decides the quantity of customers who quit purchasing the non-green item at retail channel and online channel simultaneously [28].

Assumption 7. *$\eta_1 < \eta_2$.*

Parameter η_1 and η_2 indicates the sales effort co-efficient of green and non-green products. Retailer gives more effort to sell the non-green item because these items are not attractive enough for the buyers.

Assumption 8. *$p_i > w_i > M_i$ as well as $p_i' > M_i$ where $i = \{1, 2\}$.*

Retail costs should be more than wholesale costs due to the prerequisite for the positive benefit of retailer. In order to have the positive benefit of the manufacturer wholesale price for retail channel and online retail price have to be more than the manufacturing cost of the item.

Assumption 9. *The basic market potential is a significant factor which is positively corresponded with the selling prices, green degree and benefit, both under consistent or inconsistent pricing strategies.*

Consistent selling price is taken as an example, Figures 1a and 1b gives a more instinctive presentation. There are four values of basic market potential $\{a_1, a_2\}$ and $\{a_1', a_2'\}$, $\{a_1 < a_1', a_2 < a_2'\}$ where $\Delta a = (a_1' - a_1) + (a_2' - a_2)$. Two market demand functions with the values a_1 to a_1' and a_2 to a_2' are D' and D''. Now, $P\{p_1, p_1', p_2, p_2'\}$ and D are the equilibrium selling price and market demand when the basic market demand is a_1, a_2. P' and $\{D', D''\}$ equilibrium selling price and corresponding market demand when the basic market demand is $\{a_1', a_2'\}$. In Figures 1a and 1b, the area of region 1 represents the income PD when the basic market demand is $\{a_1, a_2\}$ and it's value increases or decreases in sync with the value of the profit. In Figure 1a, the area of region 3 represents the increment of profit when the basic market potential increases from a_1 to a_1' and a_2 to a_2' and price is not changed. In Figure 1b, the area of region 3 represents the increment of profit when the selling price increases from P to P'' and demand is not changed. In Figures 1a and 1b, the area of region 2 refers the increment of profit due to both price and demand increases.

3.1. Notations

The following notations in Table 1 are used to establish the model.

4. Decision making model and discussion

Based on the above model, the profit function of the manufactures and retailers is formulated first. Next, we examine the model under the centralized and decentralized policy. In the case of decentralized policy, we have used the manufacturer-Stackelberg game to solve the models. What's more, a revenue-sharing contract is provided to achieve the higher benefit compared to the centralized and decentralized policy. The various profit expressions will be well behaved and possess a unique optimum, which is convenient to the analysis and discussion in this section. We first study the equilibrium decisions under three scenarios, respectively, and then examine the impacts of channel leadership on the equilibrium outcomes.

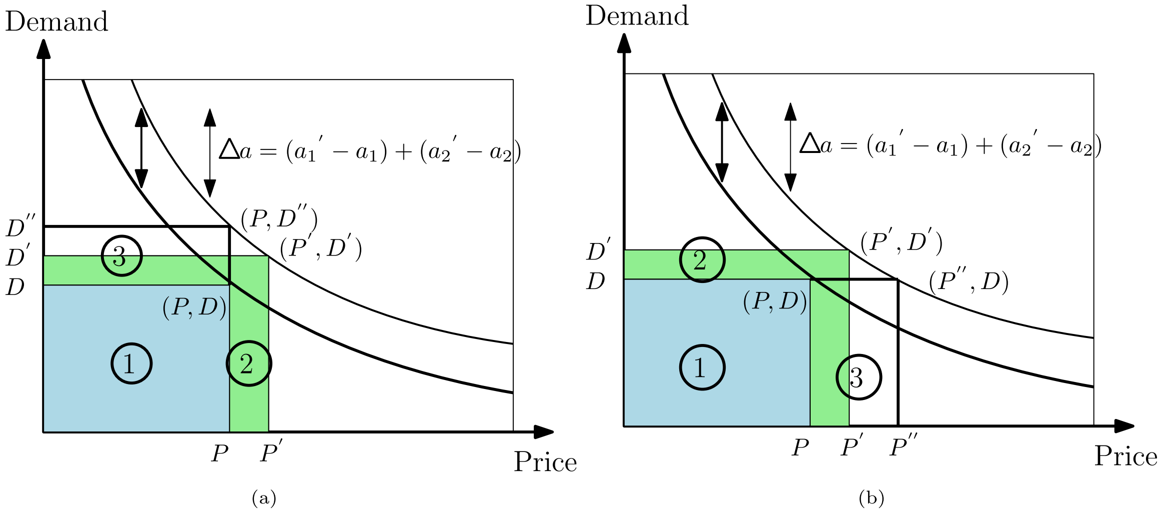


FIGURE 1. (a) Analysis of profit curve in supply chain. (b) e_1 *vs.* $D_2{}'$.

TABLE 1. Notations and descriptions.

Indices	
i	Index of manufacturer
$i = 1$	For green item manufacturer
$i = 2$	For non-green item manufacturer
Parameter	
M_i	Manufacturing cost per unit ($i = 1, 2$)
w_i	Wholesale price per unit ($i = 1, 2$)
g_2	Green quality level for non-green item
e_i	Green quality level cost coefficient per unit ($i = 1, 2$)
τ_i	Sales effort level cost coefficient per unit ($i = 1, 2$)
Dependent variable	
D_i	Demand of retail channel ($i = 1, 2$)
D_i'	Demand at online channel ($i = 1, 2$)
TP_{M_g}	Green manufacturer profit
TP_{M_n}	Non-green manufacturer profit
TP_R	Retailer profit
TP	Total profit
Superscript	
C	Centralized dual-channel supply chain
DC	Decentralized dual-channel supply chain
RS	Collaboration dual-channel supply chain
Subscript	
SC	Supply chain
Decision variables	
p_i	Sales price through retail channel ($i = 1, 2$)
p_i'	Sales price through online channel ($i = 1, 2$)
s_i	Sales effort level ($i = 1, 2$)
g_1	Green quality level of green item

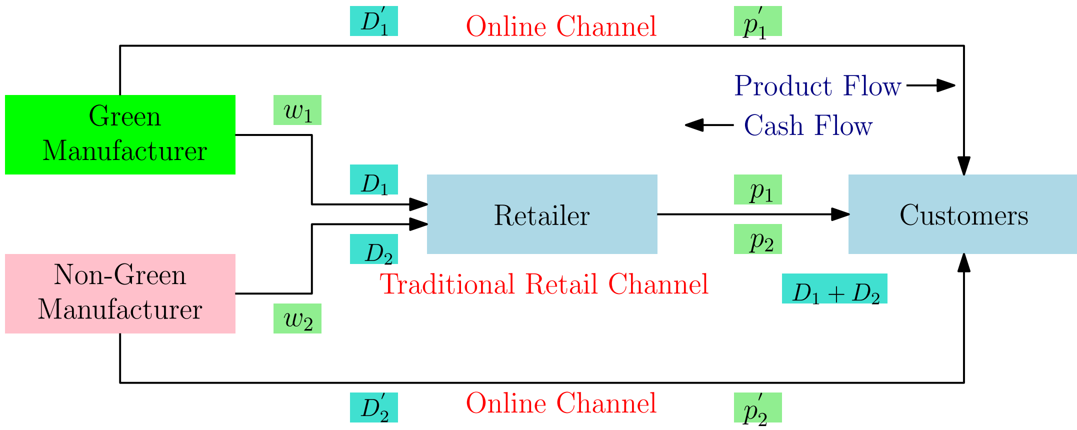


FIGURE 2. Logistic diagram of supply chain.

4.1. The profit function of each member

Under the suppositions given in the above section, the profit function of the supply chain members are given by equations (5)–(7).

$$\begin{cases} \mathrm{TP}_{M_g} = (w_1 - M_1)\, D_1 + (p_1' - M_1)\, D_1' - \dfrac{e_1 {g_1}^2}{2} & (5) \\ \mathrm{TP}_{M_n} = (w_2 - M_2)\, D_2 + (p_2' - M_2)\, D_2' - \dfrac{e_2 {g_2}^2}{2} & (6) \\ \mathrm{TP}_{R} = (p_1 - w_1)\, D_1 + (p_2 - w_2)\, D_2 - \dfrac{\tau_1 s_1^2}{2} - \dfrac{\tau_2 s_2^2}{2}. & (7) \end{cases}$$

In equations (5) and (6), the first term denotes the profit of manufacturer earned by selling the green and non-green items to the retail channel; the second term denotes the manufacturer's profit in the online channel. In equation (7), the first and second term denotes the profit earned by the retailer selling the green and non-green item to customers. The cost of generating green item is deduced from the green manufacturer profit in equation (5), this cost is $\frac{e_1 {g_1}^2}{2}$. Also, there is a slight fixed cost for the non-green item to maintain the minimum green level g_2, *i.e.*, $\frac{e_2 {g_2}^2}{2}$ which is deduced from the non-green manufacturer profit in equation (6) [10]. The retailer's sales effort cost for both green and non-green products is deducted from retailer profit in equation (7).

4.2. Centralized policy

In this scenario, a joint decision structure has been considered. The supply chain member build a grand coalition with joint channel coordination. Both the manufacturers and a common retailer are considered a whole supply chain system; they are vertically integrated and make their choices cooperatively and simultaneously to maximize the overall profit of the supply chain. The joint decision maker is involved to find the optimal decision of retail price (p_1, p_2), online price (p_1', p_2'), green level of green item (g_1), and sales efforts (s_1, s_2). The profit function of this scenario is stated as follows:

$$\begin{cases} \text{Max} \quad \mathrm{TP}^{C}_{\mathrm{SC}}\,(p_1, p_1', p_2, p_2', g_1, s_1, s_2) \\ \text{subject to constraints} \\ p_1, p_1' > M_1 > 0, \;\; p_2, p_2' > M_2 > 0, \;\; g_1 > 0 \;\; s_1, s_2 > 0. \end{cases} \qquad (8)$$

Using the equations (5)–(7), the joint profit function of the supply chain in the centralized scenario can be formulated as:

$$\mathrm{TP}_{\mathrm{SC}}^{C}(p_1, p_1', p_2, p_2', g_1, s_1, s_2) = (p_1 - M_1)\,D_1 + (p_1' - M_1)\,D_1' + (p_2 - M_2)\,D_2 + (p_2' - M_2)\,D_2' - \frac{e_1 g_1^2}{2} - \frac{e_2 g_2^2}{2} - \frac{\tau_1 s_1^2}{2} - \frac{\tau_2 s_2^2}{2}. \tag{9}$$

From the first to fourth terms of this expression indicates sales revenue of green and non-green item through the retail and online channel. The fifth to last term denotes the cost of green technology and cost of sales effort. Therefore, the joint profit $\mathrm{TP}_{\mathrm{SC}}^{C}$ is a function of seven decision variables $p_1, p_1', p_2, p_2', g_1, s_1, s_2$.

Proposition 1. *In the centralized scenario, the unique optimum solutions can be obtained from solving the following equations:*

$$\begin{aligned}
&(1)\ \ -2b_1p_1 + \alpha_1 a_1 + c_1' p_1' + c_2 p_2 + c_2' p_2' + \gamma_1 g_1 + \eta_1 s_1 + b_1 M_1 + c_1({p_1}' + p_2 + {p_2}' - M_1 - 2M_2) = 0\\
&(2)\ \ -2b_2p_2 + \alpha_2 a_2 + c_1 p_1 + c_1' p_1' + c_2' p_2' + \gamma_2 g_2 + \eta_2 s_2 + b_2 M_2 + c_2(p_1 + {p_1}' + {p_2}' - 2M_1 - M_2) = 0\\
&(3)\ \ -2{b_1}'{p_1}' a_1 - \alpha_1 a_1 + c_1 p_1 + c_2 p_2 + c_2' p_2' + \gamma_1' g_1 + {b_1}' M_1 + {c_2}'(p_1 + p_2 + {p_2}' - M_1 - 2M_2) = 0\\
&(4)\ \ -2{b_2}'{p_2}' + a_2 - \alpha_2 a_2 + c_1 p_1 + c_1' p_1' + c_2 p_2 + \gamma_2' g_2 + {b_2}' M_2 + {c_2}'(p_1 + p_2 + {p_1}' - 2M_1 - M_2) = 0\\
&(5)\ \ (p_1 - M_1)\gamma_1' + ({p_1}' - M_1)\gamma_2' - e_1 g_1 = 0\\
&(6)\ \ (p_1 - M_1)\eta_1 - \tau_1 s_1 = 0\\
&(7)\ \ (p_2 - M_2)\eta_2 - \tau_2 s_2 = 0.
\end{aligned} \tag{10}$$

Proof. The necessary condition to find the optimum solution is setting the first order partial derivative with respect to $(p_1, p_2, p_1', p_2', g_1, s_1, s_2)$ equals to zero. The first order differentiation gives the above linear equations under all the decision variables.

Appendix A shows that the solution obtained from the above equation are uniquely exist. The sufficient condition of optimally has been addressed in the appendix and the numerical section respectively. □

4.3. Decentralized policy

In a decentralized policy, both the manufacturer and retailer are part of a sequential game model. Each supply chain member has optimized their decision from the perspectives of their own choices. In this section, the Stackelberg game is used to analyze the decentralized model to acquire the optimum solutions for the chain members.

In this game, expecting the followers' reaction, the leader has optimized his/her own choices. Channel initiative that relies on the players' capacity to control the dynamic decision-making succession significantly affects channel execution [19].

In this model, the retailer, as a Stackelberg leader, first determines the sales price of each item and the sales effort level. Then both the green and non-green manufacturers decide their online sales price and greening level of the item. The problem has been solved in the reverse induction process, *i.e.*, the manufacturers first optimize their decisions for online selling price and greening level. Based on the manufacturer's findings, the retailer optimizes his decisions to maximize the supply chain profit. Therefore, the optimum results are obtained, which are given in the following propositions.

Proposition 2. *In decentralized model, under equilibrium situation, the optimum online selling price of the green and non-green manufacturer and greening level of the item are given by*

$$p_1' = \frac{2b_2'e_1B + 2b_2'\gamma_1'C - c_2'\gamma_2'e_1g_2 - c_2'e_1A}{(c_1'c_2'e_1 - 2b_2'E)} \tag{11}$$

$$g_1 = \frac{(\gamma_1 w_1 - \gamma_1 M_1 - \gamma_1' M_1)(c_1'c_2'e_1 - 2b_2'E) + \gamma_1' K_1}{e_1(c_1'c_2'e_1 - 2b_2'E)} \tag{12}$$

and $$p_2' = \frac{c_1'e_1B + c_1'\gamma_1'C - (\gamma_2'g_2 + A)E}{(c_1'c_2'e_1 - 2b_2'E)} \tag{13}$$

where, $K_1 = (2b_2'e_1B + 2b_2'\gamma_1'C - c_2'\gamma_2'e_1g_2 - c_2'e_1A)$, $A = a_2 - \alpha_2 a_2 + c_1p_1 + c_2p_2 + c_2'w_2 - c_2'M_2 + b_2'M_2$, $B = \alpha_1 a_1 - a_1c_1p_1 - c_2p_2c_1'w_1 + c_1'M_1 - b_1'M_1$, $C = \gamma_1'M_1 - \gamma_1 w_1 + \gamma_1 M_1$, $E = 2b_1'e_1 - {\gamma_1'}^2$.

Proof. Both green and non-green manufacturer first optimizes their online channel selling price. Also, the green manufacturer optimizes his greening level. Differentiating partially the green manufacturer profit $\left(\mathrm{TP}_{M_g}\right)$ in (5) with respect to p_1' and g_1 we get,

$$\begin{cases} \dfrac{\partial \mathrm{TP}_{M_g}}{\partial p_1'} = -2b_1'p_1' + \gamma_1'g_1 + a_1 - \alpha_1 a_1 + c_1p_1 + c_2p_2 + c_2'p_2' + c_1'w_1 - c_1'M_1 + b_1'M_1 & (14)\\ \dfrac{\partial \mathrm{TP}_{M_g}}{\partial g_1} = \gamma_1(w_1 - M_1) + \gamma_1'(p_1' - M_1) - e_1g_1. & (15)\end{cases}$$

The necessary condition for optimum online sales price (p_1') of green manufacturer and green level (g_1) are $\frac{\partial \mathrm{TP}_{M_g}}{\partial p_1'} = 0$ and $\frac{\partial \mathrm{TP}_{M_g}}{\partial g_1} = 0$.

Again, differentiating partially the non-green manufacturer profit (TP_{M_n}) in (6) with respect to p_2'

$$\frac{\partial \mathrm{TP}_{M_n}}{\partial p_2'} = c_2'(w_2 - M_2) + a_2 - \alpha_2 a_2 - b_2'p_2' + c_1p_1 + c_1'p_1' + c_2p_2 + \gamma_2'g_2 - b_2'(p_2' - M_2). \tag{16}$$

The necessary condition for optimum online sales price (p_2') of non-green manufacturer is $\frac{\partial \mathrm{TP}_{M_n}}{\partial p_2'} = 0$.

Solving the system of equation of p_1', g_1 and p_2' from (14)–(16), we get the simplest form of the decision variable as mentioned in (11)–(13). Therefore, the sufficient condition $\frac{\partial^2 \mathrm{TP}_{M_g}}{\partial {p_1'}^2} = -2b_1' < 0$ since $b_1' > 0$, $\frac{\partial^2 \mathrm{TP}_{M_g}}{\partial g_1^2} = -e_1 < 0$ since $e_1 > 0$ and $\frac{\partial^2 \mathrm{TP}_{M_g}}{\partial {p_1'}^2} * \frac{\partial^2 \mathrm{TP}_{M_g}}{\partial g_1^2} - \left\{\frac{\partial^2 \mathrm{TP}_{M_g}}{\partial p_1' g_1}\right\}^2 = 2b_1'e_1 - {\gamma_1'}^2 > 0$ when $2b_1'e_1 > {\gamma_1'}^2$ holds. On the other hands, $\frac{\partial^2 \mathrm{TP}_{M_n}}{\partial {p_2'}^2} = -2b_2' < 0$ since $b_2' > 0$. □

Proposition 3. *Under the decentralized policy,* TP_R *in* (7) *is concave with respect to* p_1, p_2, s_1 *and* s_2.

Proof. After the manufacturer's decision on online sales price, green level of product, the retailer decides the optimal retail sales price and sales effort. Substituting ${p_1'}^*$, ${p_2'}^*$, g_1^* in retailer's profit function (7), we can get the optimal profit of the retailer. The necessary condition for the first-order derivative of retailer profit function TP_R are shown in Appendix B. □

Proposition 4. *The impacts of the parameters* a_1, a_2 *on the optimal online sales price and product green label are as follows:*

(i) $\frac{\partial p_1'}{\partial a_1} > 0$, $\frac{\partial p_2'}{\partial a_1} > 0$ *and* $\frac{\partial g_1}{\partial a_1} > 0$ *when* $2b_2'E > c_1'c_2'e_1$ *holds, since* $0 < \alpha_1 < 1$.
(ii) $\frac{\partial p_1'}{\partial a_2} > 0$, $\frac{\partial p_2'}{\partial a_2} > 0$ *and* $\frac{\partial g_1}{\partial a_2} > 0$ *when* $2b_2'E > c_1'c_2'e_1$ *holds, since* $0 < \alpha_2 < 1$.

Proof. The online sales price of green manufacturer (p_1'), non-green manufacturer (p_2') and product green level (g_1) all are increasing function of market demand parameter a_1 and a_2. When $2b_2'E > c_1'c_2'e_1$ holds, then differentiating p_1' with respect to a_1 exhibit that $\frac{\partial p_1'}{\partial a_1} > 0$ and so on. It is obvious that, when the potential market demand of the product increase then manufacturers can increase their sales price with higher green level of products. Therefore, the higher values of a_1, a_2 will increase the overall profit of the supply chain. In Table 4, the behaviour of this parameter are shown in numerically. □

Proposition 5. *The impacts of the green elasticity cost parameters and wholesale price (e_1, w_1, w_2) on the optimal online sales price and product green level are as follows:*

(i) $\frac{\partial p_1'}{\partial e_1} < 0$ *when* $(4b_1'b_2' - c_1'c_2')(2b_2'e_1B + 2b_2'\gamma_1'C - e_1c_2'\gamma_2'g_2 - e_1c_2'A) < (2b_2'E - c_1'c_2'e_1)(2b_2'B - c_2'\gamma_2'g_2 - c_2'A)$ *holds.*
(ii) $\frac{\partial p_2'}{\partial e_1} < 0$ *when* $c_1'B < 2b_1'(c_1'c_2'e_1 - 2b_2'E)(\gamma_2'g_2 + A)$ *holds.*
(iii) $\frac{\partial g_1}{\partial e_1} < 0$ *when* $(\gamma_1M_1 + \gamma_1'M_1)(c_1'c_2'e_1 - 2b_2'E)^2 + e_1^2\gamma_1'(c_1'{c_2'}^2\gamma_2'g_2 + c_1'{c_2'}^2 + 8b_1'{b_2'}^2B) + 16b_1'{b_2'}^2{\gamma_1'}^2e_{1C} < \gamma_1w_1(c_1'c_2'e_1 - 2b_2'E)^2 + \gamma_1'{e_1}^2(2b_2'c_1'c_2'B + 4b_1'b_2'c_2'\gamma_2'g_2 + 4b_1'b_2'c_2'A) + 2b_2'{\gamma_1'}^2C(2c_1'c_2' + 2b_2'{\gamma_1'}^2)$.
(iv) $\frac{\partial p_1'}{\partial w_1} > 0$ *and* $\frac{\partial p_2'}{\partial w_2} > 0$ *when* $2b_2'E > c_1'c_2'e_1$ *and* $E > 0$ *holds.*

Proof. The decision variable p_1', p_2' and g_1 are decreasing function of green cost elasticity parameter e_1 whereas p_1' is increasing function of w_1 and p_2' is increasing function of w_2 under the certain conditions. The greenness level of the item reduces with the inducing cost coefficient related to green investment. The green-responsive purchasers will not accepting a lower green item at a more higher cost, which powers the leader to lessen the sales price of the item. □

4.4. Revenue-sharing contract

To expand the performance of the supply chain under decentralized scenario, this section presents a revenue-sharing contract to reach the higher profit of supply chain members. To be more specific, retailer wish to perform a revenue-sharing proportion δ_1, δ_2 $(0 < \delta_1, \delta_2 < 1)$ to the manufacturers. Due to the asymmetry of information, downstream retailer find out about market demand than upstream manufacturers and enjoy clear benefits in the game. We then discuss how this revenue-sharing mechanism affects optimum decisions whether the channel members will be profitable with the proposed mechanism. Based on these two points, we establish the retailer-led revenue-sharing contract model, in which the order and rules of the game are as follows: the green (and non-green) manufacturer first determines his optimal online sales price with optimal green level of green product, then retailer decides her optimal retail price with sales effort level in traditional channel. With the assumption above, the profit function of the supply chain members are as follows:

$$\begin{cases} \mathrm{TP}_{M_g} = (w_1 - M_1)\,D_1 + (p_1' - M_1)\,D_1' - \dfrac{e_1{g_1}^2}{2} + \delta_1(p_1 - w_1)D_1 & (17)\\ \mathrm{TP}_{M_n} = (w_2 - M_2)\,D_2 + (p_2' - M_2)\,D_2' - \dfrac{e_2{g_2}^2}{2} + \delta_2(p_2 - w_2)D_2 & (18)\\ \mathrm{TP}_R \;\; = (1-\delta_1)\,(p_1 - w_1)\,D_1 + (1-\delta_2)\,(p_2 - w_2)\,D_2 - \dfrac{\tau_1 s_1^2}{2} - \dfrac{\tau_2 s_2^2}{2}. & (19)\end{cases}$$

Proposition 6. *Under revenue-sharing contract, using Stackelberg game the optimum online sales price of both the manufacturers and greening level can be obtained as:*

$$p_1' = \frac{2b_2'\,(e_1Y + \gamma_1'Z) - c_2'e_1\,(\gamma_2'g_2 + X)}{(c_1'c_2'e_1 - 2b_2'E)} \tag{20}$$

$$g_1 = \frac{\{\gamma_1\,(w_1 + \delta_1p_1 - M_1 - \delta_1w_1) - \gamma_1'M_1\}\,(c_1'c_2'e_1 - 2b_2'E) + \gamma_1'K_2}{e_1\,(c_1'c_2'e_1 - 2b_2'E)} \tag{21}$$

$$p_2' = \frac{c_1'e_1Y + c_1'\gamma_1'Z - (\gamma_2'g_2 + X)\,E}{(c_1'c_2'e_1 - 2b_2'E)} \tag{22}$$

where $K_2 = (2b_2'e_1Y + 2b_2'\gamma_1'R - c_2'\gamma_2'e_1g_2 - c_2'e_1X)$, $X = a_2 - \alpha_2a_2 + c_1p_1 + c_2p_2 + c_2'w_2 - c_2'M_2 + c_2'\delta_2p_2 - c_2'\delta_2w_2 + b_2'M_2$, $Y = \alpha_1a_1 - a_1 - c_1p_1 - c_2p_2 - c_1'w_1 - c_1'\delta_1p_1 + c_1'M_1 + c_1'\delta_1w_1 - b_1'M_1$ *and* $Z = \gamma_1M_1 + \gamma_1\delta_1w_1 - \gamma_1w_1 - \gamma_1\delta_1p_1 + \gamma_1'M_1$.

Proof. Similar with the previous scenario, in revenue-sharing contract, green manufacturer determines first his online sales price (p_1'), product green level (g_1) and non-green manufacturer determines his online sales price (p_2') of non-green product. For the necessary condition of concavity of profit function TP_{M_g} in equation (17), We differentiate TP_{M_g} with respect to p_1' and g_1,

$$\begin{cases} \dfrac{\partial \text{TP}_{M_g}}{\partial p_1'} = -2b_1'p_1' + \gamma_1'g_1 + a_1 - \alpha_1a_1 + c_1p_1 + c_2p_2 + c_2'p_2' + c_1'w_1 + c_1'\delta_1p_1 - c_1'M_1 - c_1'\delta_1w_1 + b_1'M_1 & (23) \\ \dfrac{\partial \text{TP}_{M_g}}{\partial g_1} = \gamma_1(w_1 - M_1) + \gamma_1\delta_1(p_1 - w_1) + \gamma_1'(p_1' - M_1) - e_1g_1. & (24) \end{cases}$$

Again, differentiating TP_{M_n} in equation (18) with the decision variable p_2' we get,

$$\frac{\partial \text{TP}_{M_n}}{\partial p_2'} = -2b_2'p_2' + a_2 - \alpha_2a_2 + c_1p_1 + c_1'p_1' + c_2p_2 + c_2'w_2 + c_2'\delta_2p_2 - c_2'M_2 - c_2'\delta_2w_2 + b_2'M_2. \quad (25)$$

Solving the system of linear equations $\frac{\partial \text{TP}_{M_g}}{\partial p_1'} = 0$, $\frac{\partial \text{TP}_{M_g}}{\partial g_1} = 0$ and $\frac{\partial \text{TP}_{M_n}}{\partial p_2'} = 0$ from equations (23)–(25), we get the simplest form of optimal p_1', g_1 and p_2'. Therefore, the sufficient condition for optimality are $\frac{\partial^2 \text{TP}_{M_g}}{\partial {p_1'}^2} = -2b_1' < 0$ since $b_1' > 0$, $\frac{\partial^2 \text{TP}_{M_g}}{\partial g_1^2} = -e_1 < 0$ since $e_1 > 0$ and $\frac{\partial^2 \text{TP}_{M_g}}{\partial {p_1'}^2} * \frac{\partial^2 \text{TP}_{M_g}}{\partial g_1^2} - \left\{\frac{\partial^2 \text{TP}_{M_g}}{\partial p_1' g_1}\right\}^2 = 2b_1'e_1 - {\gamma_1'}^2 > 0$ when $2b_1'e_1 > {\gamma_1'}^2$ holds. On the other hands, $\frac{\partial^2 \text{TP}_{M_n}}{\partial {p_2'}^2} = -2b_2' < 0$ since $b_2' > 0$. □

Proposition 7. *Under the revenue-sharing contract,* TP_R *in* (19) *is concave with respect to* p_1, p_2, s_1 *and* s_2.

Proof. Substituting $p_1^{'*}$, $p_2^{'*}$, g_1^* in retailer profit function, we can get the optimal profit of the retailer. Differentiating TP_R from (19) w.r.to (p_1, p_2, s_1, s_2) and solving the system of equation $\frac{\partial \text{TP}_R}{\partial p_1} = 0$, $\frac{\partial \text{TP}_R}{\partial p_2} = 0$, $\frac{\partial \text{TP}_R}{\partial s_1} = 0$, $\frac{\partial \text{TP}_R}{\partial s_2} = 0$ mentioned in Appendix C, we easily reached the optimal decisions. Figure 4d signify the concavity of the retailer profit function with respect to decision variable p_1 and p_2. □

Proposition 8. *In the revenue sharing contract, the impacts of the parameters* a_1, a_2,δ_1, δ_2 *on the optimal online sales price and product green level are as follows:*

(i) $\frac{\partial p_1'}{\partial a_1} > 0$, $\frac{\partial p_2'}{\partial a_1} > 0$ *and* $\frac{\partial g_1}{\partial a_1} > 0$ *when* $2b_2'E > c_1'c_2'e_1$ *holds, since* $0 < \alpha_1 < 1$.
(ii) $\frac{\partial p_1'}{\partial a_2} > 0$, $\frac{\partial p_2'}{\partial a_2} > 0$ *and* $\frac{\partial g_1}{\partial a_2} > 0$ *when* $2b_2'E > c_1'c_2'e_1$ *holds, since* $0 < \alpha_2 < 1$.
(iii) $\frac{\partial p_1'}{\partial \delta_1} > 0$, *since* $p_1 > w_1$ *and* $\frac{\partial p_2'}{\partial \delta_2} > 0$, *since* $p_1 > w_1$ *when* $2b_2'E > c_1'c_2'e_1$ *holds.*

Proof. The optimum online sales price of both the green and non-green manufacturers under revenue-sharing contract likewise pursue a similar trend as the decentralized system described in Proposition 4. When the proportion of the revenue-sharing increases, the respective players also increase their online sales price. All together, the profit of all the members of the supply chain and the whole supply chain increases. Due to higher greening effort, the benefit of each supply chain members increases (see Figs. 6a–6j). □

Proposition 9. *In the revenue-sharing contract, the impact of the cost coefficient of green elasticity parameters and wholesale price* e_1, w_1, w_2 *on the optimal online sales price and product green level are as follows:*

(i) $\frac{\partial p_1'}{\partial e_1} < 0$ *when* $(4b_1'b_2' - c_1'c_2')\{2b_2'(e_1Y + \gamma_1'Z) - e_1c_2'(\gamma_2'g_2 + X)\} < (2b_2'E - c_1'c_2'e_1)(2b_2'Y - c_2'\gamma_2'g_2 - c_2'X)$ *holds.*
(ii) $\frac{\partial p_2'}{\partial e_1} < 0$ *when* $(4b_1'b_2' - c_1'c_2')\{c_1'e_1Y + c_1'\gamma_1'Z - E(\gamma_2'g_2 + X)\} < 2b_1'(e_1c_1'c_2' - 2b_2'E)(\gamma_2'g_2 + X)$ *holds.*

Table 2. Numerical datasets.

Datasets	α_1	α_2	a_1	a_2	b_1	b_1'	b_2	b_2'	c_1	c_1'	c_2	c_2'	γ_1	γ_1'	γ_2	γ_2'	η_1	η_2	M_1	M_2	e_1	e_2	τ_1	τ_2	δ_1	δ_2
	0.6	0.6	700	600	2.4	1.6	2.2	1.4	0.28	0.14	0.16	0.12	1.2	1.3	1.0	1.1	0.75	0.90	\$20	\$15	5	4	7.5	6.5	0.4	0.3

(iii) $\frac{\partial g_1}{\partial e_1} < 0$ *when* $e_1\left(e_1c_1'c_2' - 2b_2'E\right)\left(c_1'c_2' - 4b_1'b_2'\right)K_3 < \left(2c_1'c_2' - 8b_1'b_2' + 2b_2'{\gamma_1'}^2\right)\left\{K_3(e_1c_1'c_2' - 2b_2'E) + \gamma_1'K_2\right\}$ *holds, where* $K_3 = \left\{\gamma_1\left(w_1 + \delta_1 p_1 - M_1 - \delta_1 w_1\right) - \gamma_1' M_1\right\}$.

(iv) $\frac{\partial p_1'}{\partial w_1} > 0$ *and* $\frac{\partial p_2'}{\partial w_2} > 0$ *when* $2b_2'E > c_1'c_2'e_1$ *and* $E > 0$ *holds.*

Proof. Again, optimum online sales price of the manufacturer and greening level under revenue-sharing contract with the changes of green elasticity parameters and wholesale price has the same trend as decentralized model presented in Proposition 5. □

5. Numerical investigation

As indicated by the theoretical investigation, a numerical example will be provided to the situation examined in the above section for giving profoundly managerial experiences, and better show the above-mentioned hypothetical deduction for simple and instinctive arrangement. The numerical investigation will be performed based on three models: centralized, decentralized, and revenue-sharing contract.

In view of existing literature, we set the key parameter values, and sensitivity analysis is conducted. The values of the key parameters value are given in Table 2. In Section 5.4, we show how some significant parameters impact the selling price, greenness level, sales effort level, and optimum profit.

5.1. Optimal results of the profit function

To guarantee the possibility of the optimum outcomes, we actually look at all the necessary conditions. With the key parameters value of Table 2, the principal minors for the centralized model are $\Delta_1 = -4.8 < 0$, $\Delta_2 = 20.93 > 0$, $\Delta_3 = -65.65 < 0$, $\Delta_4 = 177.97 > 0$, $\Delta_5 = -171.41 < 0$, $\Delta_6 = 11.64 > 0$, $\Delta_7 = -2.16 < 0$. This satisfies the negative definite property of the Hessian matrix and the centralized system profit is joint concave function corresponding to $(p_1, p_2, g_1, p_1', p_2', s_1, s_2)$. To perform the mathematical model, the initial green level of the non-green item has been considered as a minimum level $g_2 = 5$ for packaging, procurement, or processing the item.

Figures 4a and 4c shows the concavity graph of total profit of the green manufacturer with respect to p_1', g_1 in decentralized case and revenue-sharing contract. In decentralized case, total profit of the green manufacturer ($\mathrm{TP}_{M_g} = \$23859$) when $p_1' = \$141.66$ and $g_1 = \$43.63$ and in revenue-sharing contract, ($\mathrm{TP}_{M_g} = \$28343$) when $p_1' = \$148.90$ and $g_1 = 54.04$. Figures 4b and 4d shows the concavity of the retailer with respect to (p_1, p_2). In decentralized case $\mathrm{TP}_R = \$25027$ when $p_1 = 152.07$ and $p_2 = \$137.31$ and in revenue-sharing contract $\mathrm{TP}_R = \$26517$ when $p_1 = \$158.78$ and $p_2 = \$137.73$.

5.2. Results analysis under centralized *vs.* decentralized policy

From given data, the computational outcomes got from the various models are summed up in Table 3. As expected, Table 3 shows that centralized decision model can accomplish a higher benefit compared with the decentralized decision model.

In the decentralized model, to avoid higher green item production costs, the manufacturer is hesitant to induce investment in greenness level. As a result, the product greenness level is relatively lower. The retailer also reduces his investment in promoting the product. Therefore, the sales price of green item reduces but simultaneously non-green manufacturer increases his selling price at the retail channel due to comparatively higher sales effort than that of green item, and the customer demand for the non-green item also decline. Except for the green

TABLE 3. Optimal results.

Datasets	${p_1}^*$	${p'_1}^*$	${p_2}^*$	${p'_2}^*$	${g_1}^*$	${s_1}^*$	${s_2}^*$	${\mathrm{TP}_{M_g}}^*$	${\mathrm{TP}_{M_n}}^*$	${\mathrm{TP}_R}^*$	${\mathrm{TP}_{\mathrm{SC}}}^*$
Centralized policy	152.80	167.78	128.76	145.07	70.29	13.28	15.75	25 252	25 635	28 539	79 426
Decentralized policy	152.07	141.66	137.31	108.85	43.63	8.20	10.70	26 494	23 859	25 027	75 380
Revenue sharing contract	158.78	148.90	137.73	131.26	54.04	5.33	7.53	31 968	28 348	26 517	86 834

TABLE 4. Optimal result under the major demand elasticity parameters.

	Parameters	Policy	${p_1}^*$	${p'_1}^*$	${p_2}^*$	${p'_2}^*$	${g_1}^*$	${s_1}^*$	${s_2}^*$	${\mathrm{TP}_{M_g}}^*$	${\mathrm{TP}_{M_n}}^*$	${\mathrm{TP}_R}^*$	${\mathrm{TP}_{\mathrm{SC}}}^*$
$a_1 = 860,\ a_2 = 750$	$b_1 = 2.6,\ b'_1 = 1.8,$ $b_2 = 2.4,\ b'_2 = 1.6$	Centralized	162.68	171.54	139.05	149.06	73.64	14.26	17.17	32 086	32 141	40 144	104 373
		Decentralized	163.21	146.85	148.71	114.08	44.98	9.32	12.28	33 550	30 185	36 038	99 773
		Revenue Sharing	169.87	153.79	148.93	136.41	56.37	5.99	8.62	41 316	36 375	37 829	115 522
	$b_1 = 2.4,\ b'_1 = 1.6,$ $b_2 = 2.2,\ b'_2 = 1.4$	Centralized	186.38	204.76	158.00	177.65	87.96	16.63	19.80	36 161	37 651	50 952	124 765
		Decentralized	178.58	169.20	163.05	135.78	50.78	10.85	14.26	39 248	35 638	44 096	118 984
		Revenue Sharing	187.23	178.42	163.38	158.81	64.44	7.03	10.02	48 888	43 104	46 513	138 506
	$b_1 = 2.2,\ b'_1 = 1.4,$ $b_2 = 2.0,\ b'_2 = 1.2$	Centralized	219.84	254.01	184.20	219.98	108.80	19.98	23.42	41 732	46 015	66 411	154 159
		Decentralized	196.85	198.81	167.11	164.78	58.49	12.68	14.83	46 806	42 573	51 067	140 448
		Revenue Sharing	210.32	212.87	182.22	190.26	75.62	8.42	11.84	59 554	52 728	58 464	170 748
$a_1 = 540,\ a_2 = 450$	$b_1 = 2.6,\ b'_1 = 1.8,$ $b_2 = 2.4,\ b'_2 = 1.6$	Centralized	104.57	110.23	87.90	94.84	43.75	8.45	10.10	14 462	13 666	8394	36 524
		Decentralized	115.87	99.60	102.73	68.07	32.70	4.58	5.92	13 372	11 689	8563	33 626
		Revenue Sharing	119.48	103.46	103.16	89.50	38.45	2.96	4.18	15 247	13 658	9122	38 029
	$b_1 = 2.4,\ b'_1 = 1.6,$ $b_2 = 2.2,\ b'_2 = 1.4$	Centralized	119.23	130.81	99.53	112.50	52.62	9.92	11.70	16 168	15 814	12 280	44 263
		Decentralized	125.56	114.14	111.57	81.92	36.48	5.55	7.14	15 908	14 110	11 323	41 342
		Revenue Sharing	130.34	119.36	112.08	103.72	43.62	3.62	5.04	18 411	16 455	12 103	46 970
	$b_1 = 2.2,\ b'_1 = 1.4,$ $b_2 = 2.0,\ b'_2 = 1.2$	Centralized	139.93	161.32	115.63	138.62	65.52	12.00	13.93	18 489	19 056	17 893	55 439
		Decentralized	138.20	134.19	123.05	101.24	41.68	6.82	8.73	19 396	17 462	15 175	52 033
		Revenue Sharing	144.78	141.56	123.74	123.68	50.78	4.48	6.18	22 811	20 403	16 312	59 527

manufacturer, consequently, the profit of non-green manufacturer, retailer, and overall supply chain will also reduce.

In the centralized scenario, the green item demand decreases, and non-green item demand increases. Although the centralized system gives higher benefits for the whole supply chain system, this model does not essentially make a higher benefit for both the channel members. The green manufacturer does not benefit from centralized model compare to the decentralized model. In searching for a better situation for the whole supply chain, the manufacturer sets higher green quality and level of sales effort, but the non-green manufacturer and retailer acquire better profitability compared to others. So, the centralized model is not an admissible technique for the green manufacturer. To assure the green manufacturer, the retailer compromises his benefit and needs another decision model that maximizes the whole supply chain benefit subject to an ensured acquire for the non-green manufacturer along with the retailer compared to the decentralized model.

5.3. Results analysis under revenue-sharing contract

Under the revenue-sharing contract, all decisions and total profit are coordinated to attain an optimum state compared to centralized and decentralized model. According to a practical viewpoint considering revenue-share for the retailer causes production costly however with a higher greening level compare to a decentralized model and leads to lower sales effort level compared to all the models. Under this contract, the retail price is higher among all the models, but the online sales price is higher compare to the decentralized model. This contract allows green and non-green manufacturers to gain more than they earn in a centralized and decentralized model. Also, allow the retailer to gain more compared to the decentralized scenario.

5.4. Sensitivity analysis

To examine the behaviour of the proposed models, sensitivity analysis is conducted of some key parameters.

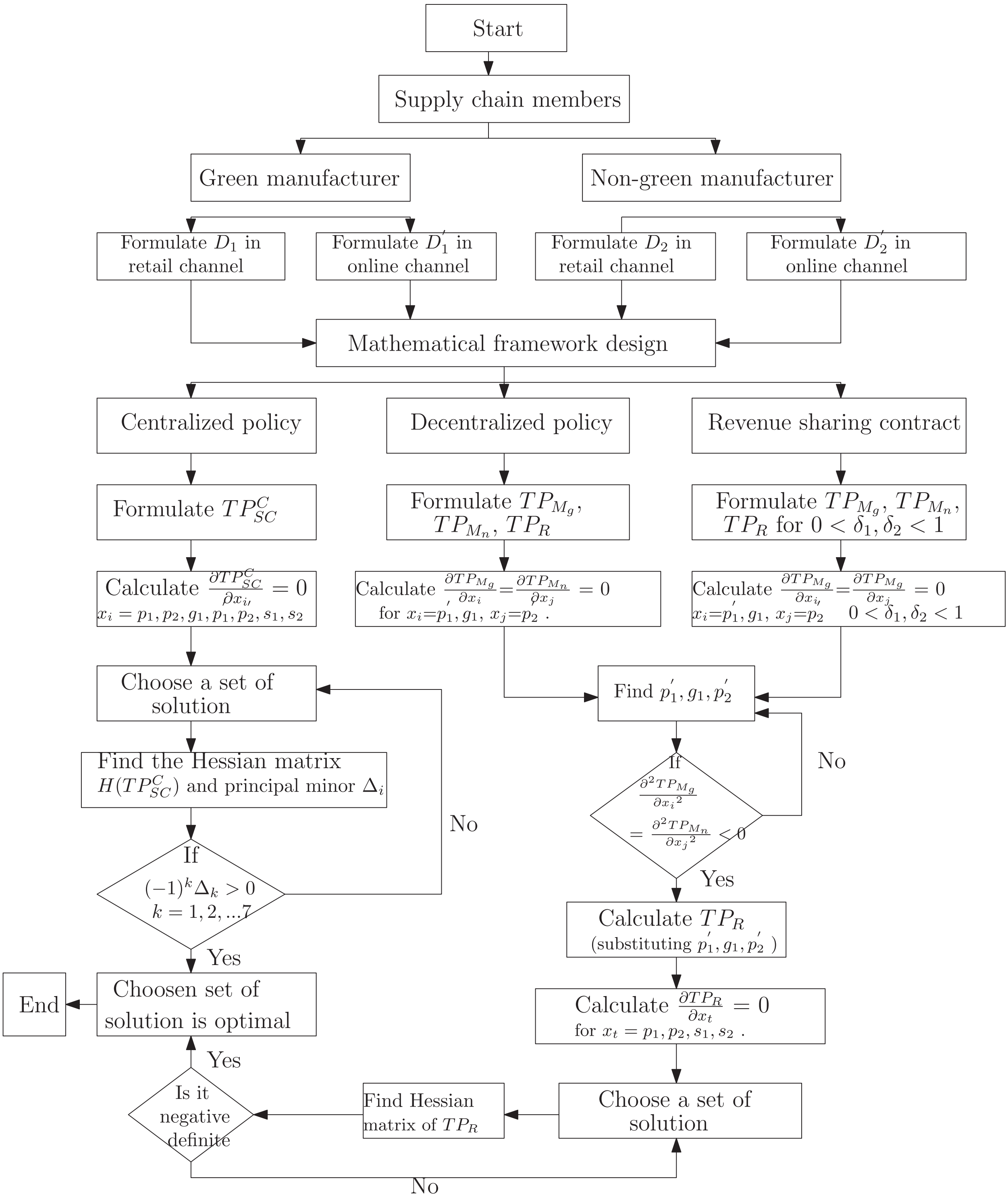


FIGURE 3. Flowchart of the optimal procedure.

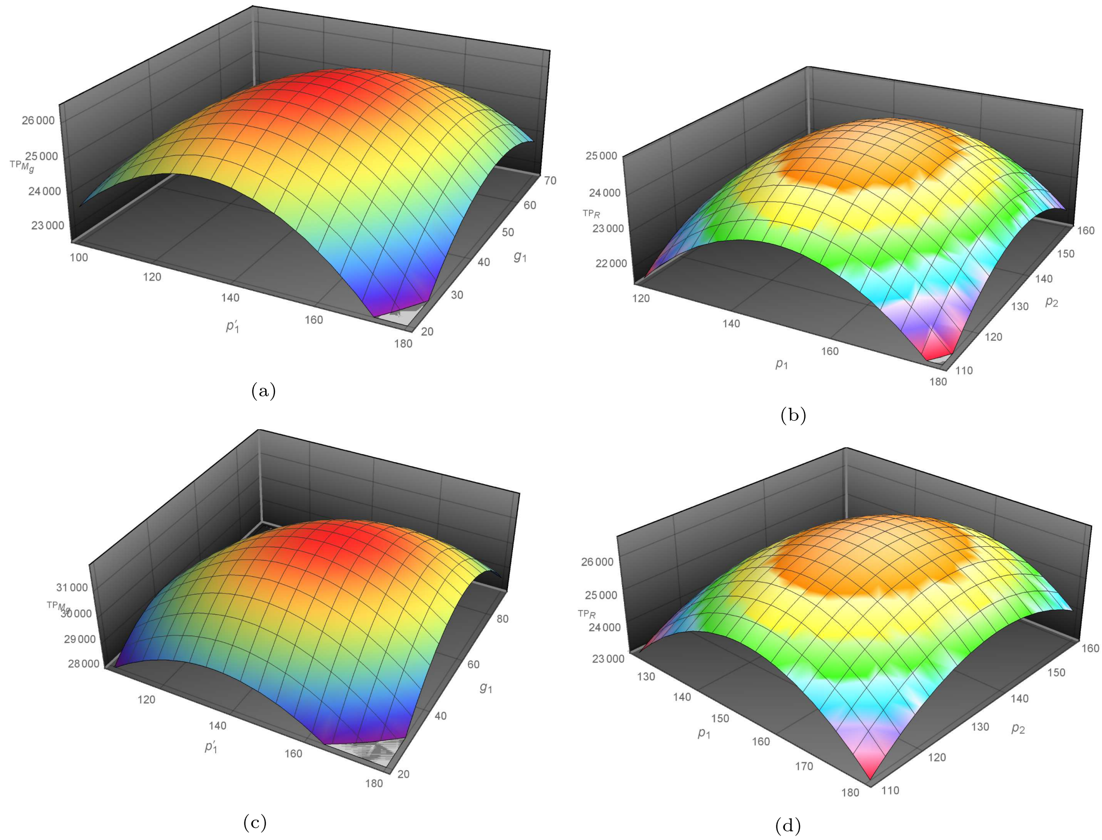


FIGURE 4. Concavity curve of TP_{M_g} and TP_R under decentralized and revenue sharing contract. (a) $\mathrm{TP}_{M}g$ with respect to p_1' and g_1 (Decentralized Scenario). (b) TP_R with respect to p_1 and p_2 (Decentralized Scenario). (c) $\mathrm{TP}_{M}g$ with respect to p_1' and g_1 (Revenue Sharing Contract). (b) TP_R with respect to p_1 and p_2 (Revenue Sharing Contract).

TABLE 5. Optimal result under different manufacturing cost.

Manuf. cost	Centralized Policy				Decentralized				Revenue Sharing Contract			
	TP_{M_g}	TP_{M_n}	TP_R	$\mathrm{TP}_{\mathrm{SC}}$	TP_{M_g}	TP_{M_n}	TP_R	$\mathrm{TP}_{\mathrm{SC}}$	TP_{M_g}	TP_{M_n}	TP_R	$\mathrm{TP}_{\mathrm{SC}}$
$M_1 = 28$, $M_2 = 21$	22 228	23 609	28 615	74 453	23 516	20 779	24 473	68 769	29 263	26 516	26 273	82 052
$M_1 = 24$, $M_2 = 18$	23 717	24 613	28 587	76 918	25 003	22 446	24 760	72 210	30 608	27 429	26 394	84 432
$M_1 = 20$, $M_2 = 15$	25 252	25 635	28 539	79 426	26 494	23 859	25 027	75 380	31 968	28 348	26 517	86 834
$M_1 = 16$, $M_2 = 12$	26 831	26 678	28 467	81 977	27 988	25 086	25 272	78 346	33 343	29 273	26 639	89 256
$M_1 = 12$, $M_2 = 09$	28 454	27 738	28 376	84 570	29 482	26 180	25 496	81 159	34 733	30 202	26 763	91 699

5.4.1. Effect of demand elasticity parameters $(a, b_1, b_1', b_2, b_2')$

Table 4 shows the effect of $(a, b_1, b_1', b_2, b_2')$ on optimal solution and supply chain members profit.

With increasing the value of a, both online and offline selling price, greening level, sale effort, manufacturer profit, retailer profit, whole supply chain profit increases. Increasing market potential (a) indicates the growth in market demand which motivates to bring up the sales price by the channel members together with green level and sales effort. Moreover, a rise in selling price is beneficial for the supply chain system in all the models.

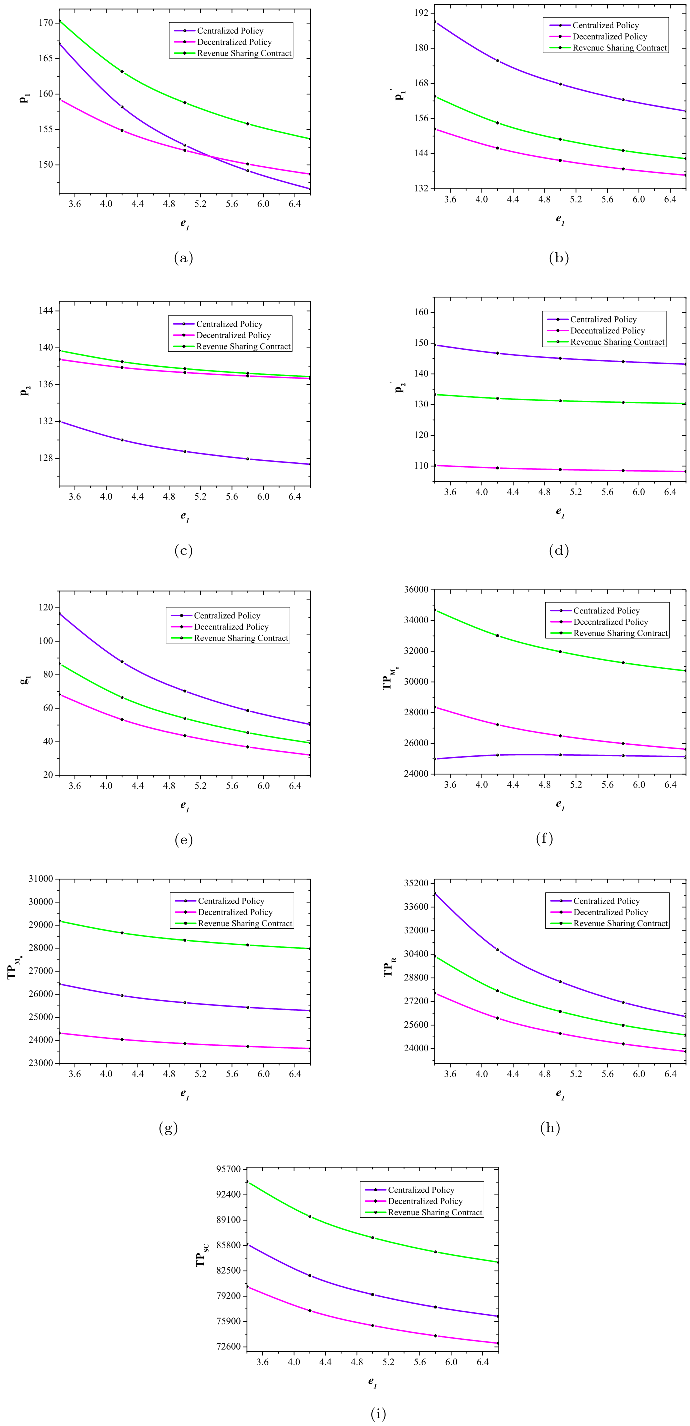


FIGURE 5. Changes of green elasticity parameter on optimal decision. (a) e_1 *vs.* p_1. (b) e_1 *vs.* p_1'. (c) e_1 *vs.* p_2. (d) e_1 *vs.* p_2'. (e) e_1 *vs.* g_1. (f) e_1 *vs.* TP_{M_g}. (g) e_1 *vs.* TP_{M_n}. (h) e_1 *vs.* TP_R. (i) e_1 *vs.* $\mathrm{TP}_{\mathrm{SC}}$.

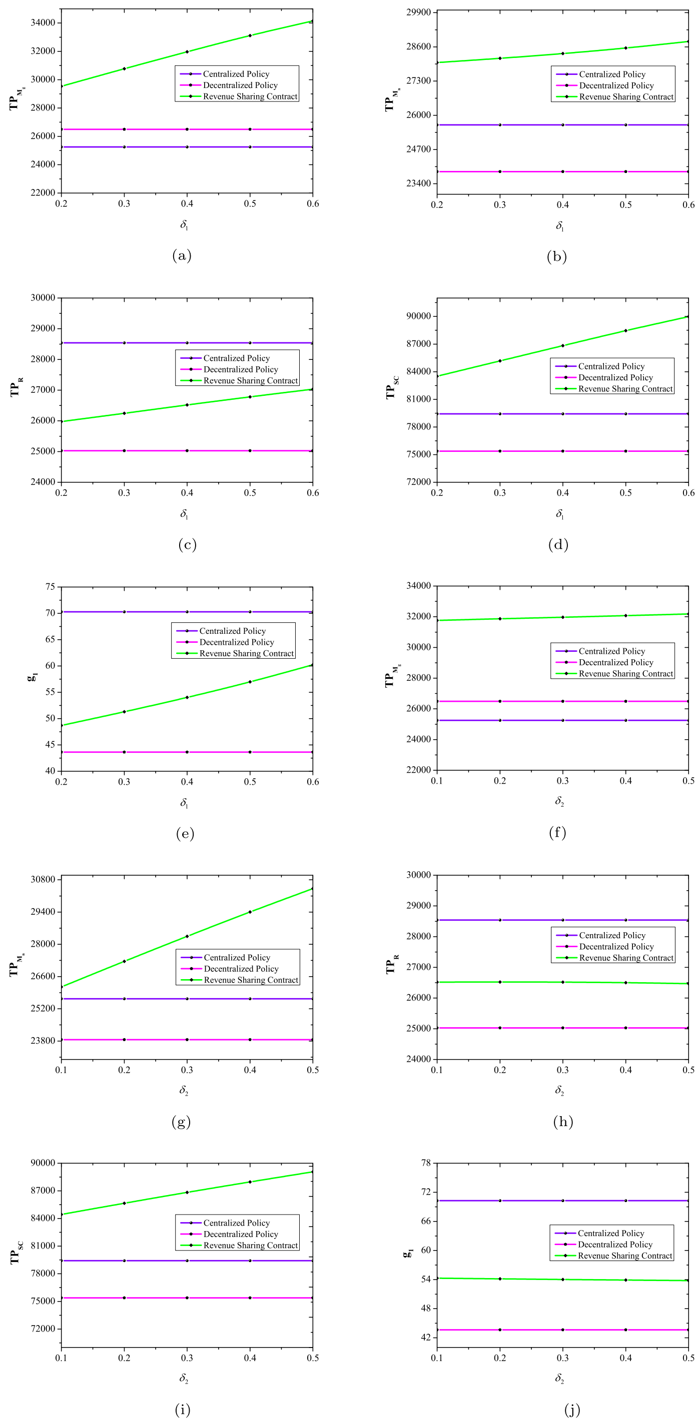


FIGURE 6. Changes of revenue-sharing proportion parameter on optimal decision. (a) δ_1 *vs.* TP_{M_g}. (b) δ_1 *vs.* TP_{M_n}. (c) δ_1 *vs.* TP_R. (d) δ_1 *vs.* $\mathrm{TP}_{\mathrm{SC}}$. (e) δ_1 *vs.* TP_{g_1}. (f) δ_2 *vs.* TP_{M_g}. (g) δ_2 *vs.* TP_{M_n}. (h) δ_2 *vs.* TP_R. (i) δ_2 *vs.* $\mathrm{TP}_{\mathrm{SC}}$. (j) δ_2 *vs.* g_1.

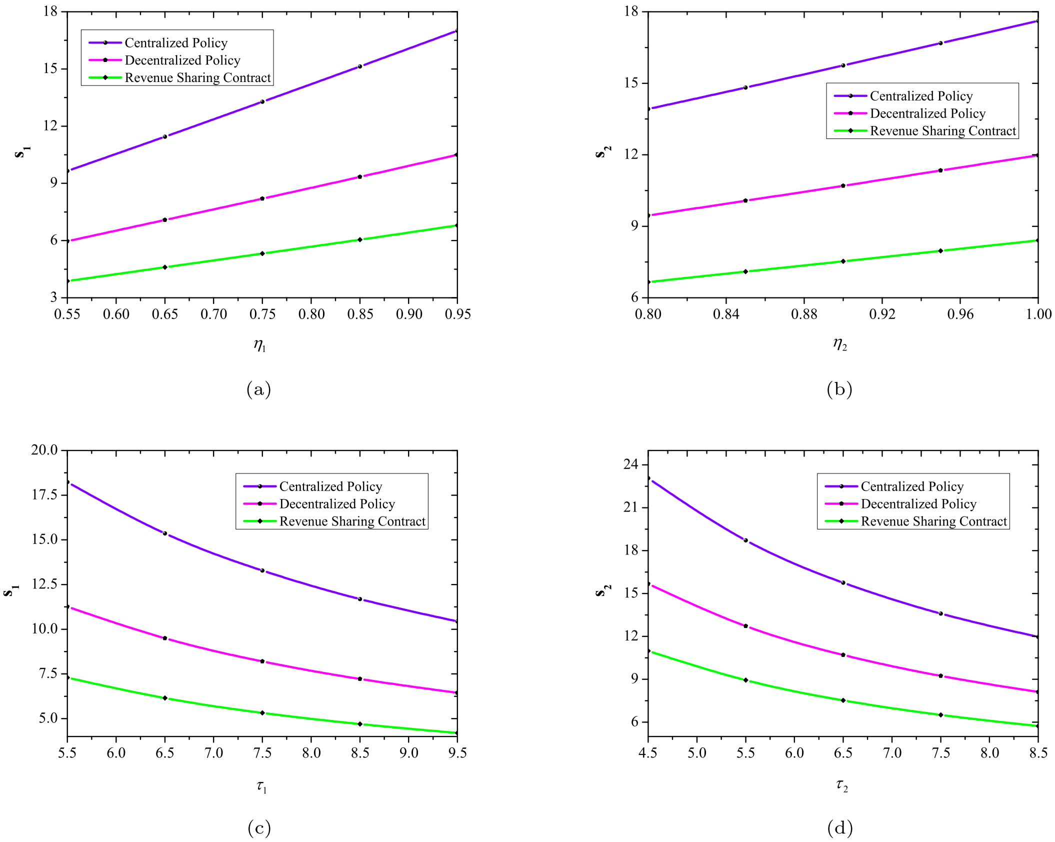


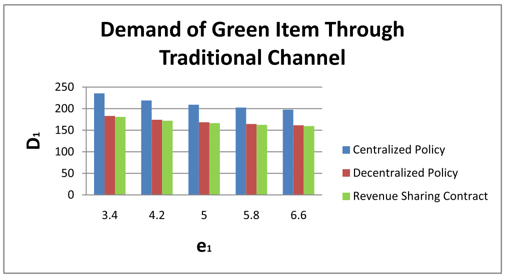
Figure 7. Effect on sales efforts s_1, s_2 with respect to η_1, η_2 and τ_1, τ_2. (a) η_1 *vs.* s_1. (b) η_2 *vs.* s_2. (c) τ_1 *vs.* s_1. (d) τ_2 *vs.* s_2.

From Table 4, increasing self-price elasticity (b_1, b_2, b_1', b_2'), reduces the selling price for both green and non-green manufacturer, greening level of the green item and sales effort of retailer in all the models. Table 4 shows that when (b_1, b_2, b_1', b_2') are placed in the lowest value the greenness level of item is highest. Also, retailer puts more sales effort on non-green items sale. At first, the relation $\mathrm{TP}_R > \mathrm{TP}_{M_n} > \mathrm{TP}_{M_g}$ for centralized model profit, $\mathrm{TP}_R > \mathrm{TP}_{M_g} > \mathrm{TP}_{M_n}$ for decentralized model profit and $\mathrm{TP}_{M_g} > \mathrm{TP}_{M_n} > \mathrm{TP}_R$ in case of revenue-sharing contract. But decreases in self-price sensitivity factor the relation changes to $\mathrm{TP}_{M_g} > \mathrm{TP}_{M_n} > \mathrm{TP}_R$, $\mathrm{TP}_{M_g} > \mathrm{TP}_R > \mathrm{TP}_{M_n} >$ and $\mathrm{TP}_{M_g} > \mathrm{TP}_R > \mathrm{TP}_{Mn}$.

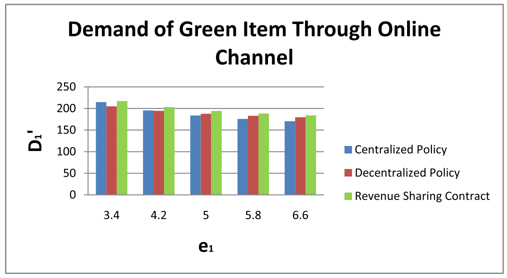
5.4.2. Effect of greening cost e_1

Perhaps the main elements in the suggested model is the greening cost e_1 which can influence the quality of solution. It is presumed that expanding e_1 in excess of an threshold cause a few inconveniences in the model. Expanding e_1 implies that more expenditure is needed for manufacturing a green item which thusly originates an exorbitant cost of green items for the consumers.

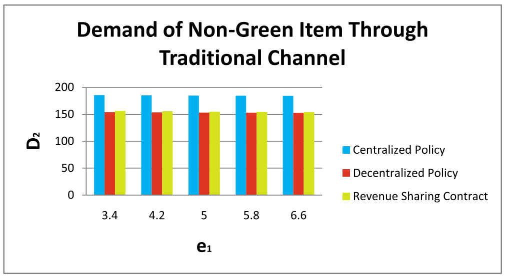
Figures 8a–8d illustrates the green and non-green item demand at both online and offline channel against e_1. According to Figures 8a and 8c demand of both green and non-green item through traditional retail channel is always highest in centralized model. Figures 8b and 8d shows that demand of green items highest in revenue-sharing contract and non-green item demand highest in decentralized model through online channel. Non-green item demand is not so much sensitive with the increasing value of e_1. We have found from the Figures 8a–8d



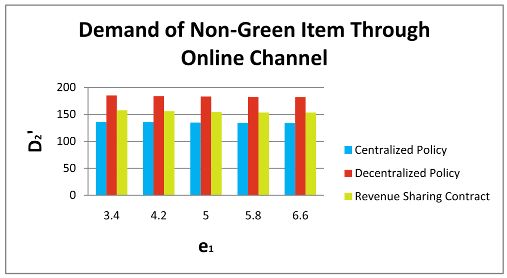
(a)



(b)



(c)



(d)

FIGURE 8. Changes of demand in supply chain with changes of parameter e_1. (a) e_1 *vs.* D_1. (b) e_1 *vs.* D_1'. (c) e_1 *vs.* D_2. (d) e_1 *vs.* D_2'.

that a higher green level reduces the item demand because by increasing e_1, the products will be less green in all the models (seen in Fig. 5e) which is an immediate after effect of high greening costs. From Figure 5e,the measure of ecological quality in the revenue-sharing contract is higher than the decentralized model. From the experimental perspective, this conduct exhibits that the suggested revenue-sharing contract is proficient for upgrading greenness level of the channel as for decentralized model.

From Figures 5a–5d, the manufacturer and retailer reduce the online and offline sales price of both the products. For explanation if we consider Figure 5a, it is observed that for higher values of e_1, sales price p_1 decreases simultaneously and the rate of decrees is high in centralized case compared to decentralized and revenue sharing contract. For $e_1 = 5.34$, the sales price is same in centralized and decentralized scenario. For all values of e_1, the revenue-sharing contract gives higher sales price (Figs. 5a–5d) and low sales effort and simultaneously higher ecological quality (Fig. 5e) compared to the decentralized model which suggests that the channel provides more affordable with high natural quality items to the consumers. This conduct is an attractive component of the suggested revenue-sharing model, which brings about more fulfilled consumers simultaneously with more environmental activities.

Figures 5f–5g shows the green and non-green manufacturer profit against e_1. For all estimations of e_1, the manufacturer's profit in the revenue-sharing contract is higher than the centralized and decentralized model, which indicates that the revenue-sharing contract is allowable from the manufacturer's viewpoint. Green manufacture profit under centralized increases with the increasing value of e_1; on the other hand, both the green and non-green manufacturer profit decreases in all the models. The manufacturer sets the greening cost e_1 in lower esteems to prevent extra expenses.

As it is evident from Figure 5h that the revenue-sharing contract is more profitable for the retailer compares to the decentralized model. Hence, the model is admissible to the retailer's perspective for different upsides of e_1. As seen in Figure 5i, the total supply chain profit reduces by enlarging e_1, which is rational conduct. Simultaneously, from Figure 5i, the suggested revenue-sharing contract brings about more benefit for the entire channel than the centralized and decentralized model.

5.4.3. *Effect of sales effort elasticity (η_1 and η_2) and sales effort cost (τ_1 and τ_2)*

Figures 7a and 7b follows the manner of retailer's choices about sales effort level by increasing (η_1 and η_2). Changes in sales effort elasticity coefficient extraordinarily affect the retailer's decision, it irrelevantly affects the manufacture's decision. With the increasing (η_1 and η_2) retailer increases his sales price to make more profit. Figures 7a and 7b exhibits that retailer under centralized scenario makes more sales effort. Moreover, it is worth focusing on that, in the real-life marketing scenario, the consumers are generally more delicate to cost and less to promotion effort. Furthermore, it is normal that the suggested revenue-sharing model prompts low retail cost than the decentralized situation in the practical marketing scenario.

Sales effort cost (τ_1, τ_2) is a basic expense value that can influence the level of sales effort and thusly impacts demand and order amount of green item in the suggested models. To prevent extra expenses by enlarging (τ_1, τ_2), the retailer decreases (s_1, s_2) (see Figs. 7c and 7d). This action neutralizes the effect of decreased (s_1, s_2) on-demand; also, keeping up demand volume, the retailer decreases its sales price. From the Figures 7c and 7d, under revenue-sharing contract, the retailer places the sales effort level lower/higher than centralized and decentralized model.Such tune-up of choice factors in the revenue-sharing contract sources an expanded demand for the green item and builds the member's order amount.

5.4.4. *Effect of δ_1 and δ_2*

When revenue-sharing co-efficient δ_1 and δ_2 is in a certain range, both the green and non-green manufacturer profit, retailer profit, and whole supply chain profit increase as this parameter increases (Figs. 6a–6d). The rate of increase of profit of the channel members is high. Sharing profit among the retailer and the manufacturer could assist the manufacturer in contributing more to green advancement.

Collaboration among individuals from the supply chain network is a viable methodology to develop the greening level of items further. Especially when purchasers are more worried about the natural qualities of items, the collaboration between members becomes especially significant. Moreover, collaboration doesn't guarantee the financial attentiveness of all individuals. For a revenue-sharing contract, the profit of the retailer will be reduced. Members of the supply chain ought to arrange a sensible offer rate to achieve a mutually advantageous result. From the controller's level, the foundation of natural projects is a fundamental setup to initiate undertaking collaboration and enrich social welfare.

5.4.5. *Effect of manufacturing cost*

Manufacturing costs have a significant influence on the effective design and manufacture of an item. With expanding the value of manufacturing cost, the benefit of the supply chain reduces in all the models in Table 5. The highest profit has been acquired in the revenue-sharing contract. Increasing manufacturing costs reduces the greenness level of the item. But an exception for the profit function of the retailer in centralized model is that this channel member profit is improved. Ultimately, the cost of manufacturing wields a substantial and direct impact on the overall profitability of a supply chain. Employing tactics to curtail manufacturing expenses by enhancing operational efficiency, adeptly managing suppliers, embracing technological advancements, and upholding product quality can all play a pivotal role in bolstering the fiscal health of a supply chain. Nevertheless, it remains imperative to find equilibrium between the pursuit of cost reduction and the preservation of product quality and customer contentment, as this equilibrium is paramount for achieving enduring prosperity.

6. Conclusion

By rising e-market preference and ecological concerns of customers throughout the world, offering dual-channel system with green along with non-green items to consumers turns into a competitive benefit for supply chains. Therefore, in the present day, the supply chain manager intent to introduce online marketing with their traditional retail channel and deals green item besides with non-green item to draw the considerations of naturally well-informed customers.

In this study, a two-layer dual-channel supply chain under the ecological concern is developed with two manufacturers and a single retailer. At present, the examined supply chain provides a regular non-green item to the consumers and targets for initiating a compatible green item. Demand for both items is an explicit term of the sales price, the items green level, and retailer sales effort level. The non-green item is a familiar item with a specified greening level. We construct a mathematical model and compute optimal decisions regarding sales prices for the two channels,sales effort level of the retailer and greening level of manufacturer for both the item.

Three decision models are investigated (a) centralized (b) decentralized, and (c) revenue-sharing scenario for upgrade the performance in ecological and economic phenomena of supply chain. In view of the analytical results of the centralized, decentralized, and revenue-sharing contract, customer demand, channel members benefits, and environmental advantages are compared, and we acquire a few exciting outcomes. All the scenarios are estimated utilizing numerical example. Numerical inspections reveal that the suggested revenue-sharing scenario is restoring demand and generating more profit for the manufacturers and overall system by suitable adjustment of all the decision variables, *i.e.*, sales price, green quality of product, sales effort level. Also, the result confirms that the adoption of the revenue-sharing contract makes a greener channel and simultaneously reduces the sales effort level. Accordingly, initiation of the suggested cooperation model outcomes in more fulfilled consumers as more sustainable activities simultaneously.

Managerial intuition of the given model can be summed up as: (i) Contributing a green item adjacent to the non-green item can prompt a synergic circumstance where all the members exploit more sales volume and more benefit. (ii) Cooperation in starting an online channel together with the offline channel prompts a more competitive price and simultaneously builds channel greenness. (iii) Although the decisions about sales effort level can impact their shares (iv) the proposed revenue-sharing model doesn't require for requesting a side installment program and can make a mutually beneficial arrangement for the channel members. For future research explores, considering the issue under uncertain demand is another option, the model could be inspecting as having probability functions following fuzzy set hypothesis. It very well may be expect to be that demand isn't linear and has non-linear functions. Moreover, as significant individuals from supply chains, suppliers could be considered in their character as contributors of protected and recyclable materials. Governments can assume a vital and significant part in progress of sustainability in supply chains and in the community.Another direction for additional exploration is to add government to the issue and explore the impact of its essence.

Appendix A.

Differentiating $\mathrm{TP}_{\mathrm{SC}}^{C}$ partially w.r.to $(p_1, p_2, g_1, p_1', p_2', s_1, s_2)$; the corresponding Hessian matrix in centralized scenario is derived as follows

$$H(\mathrm{TP}_{\mathrm{SC}}^{C}) = \begin{bmatrix} -2b_1 & c_1+c_2 & c_1+c_1' & c_1+c_2' & \gamma_1 & \eta_1 & 0 \\ c_1+c_2 & -2b_2 & c_2+c_1' & c_2+c_2' & 0 & 0 & \gamma_2 \\ c_1+c_1' & c_2+c_1' & -2b_1' & c_1+c_2' & \gamma_1' & 0 & 0 \\ c_1+c_2' & c_2+c_2' & c_1'+c_2' & -2b_2' & 0 & 0 & 0 \\ \gamma_1 & 0 & \gamma_1' & 0 & 0 & 0 & 0 \\ \eta_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Negative definite property of the above Hessian matrix is shown numerically due to complexity. Iterative solving the equation $\frac{\partial \mathrm{TP}_{\mathrm{SC}}^{C}}{\partial p_1}=0$, $\frac{\partial \mathrm{TP}_{\mathrm{SC}}^{C}}{\partial p_2}=0$, $\frac{\partial \mathrm{TP}_{\mathrm{SC}}^{C}}{\partial p_1'}=0$, $\frac{\partial \mathrm{TP}_{\mathrm{SC}}^{C}}{\partial p_2'}=0$, $\frac{\partial \mathrm{TP}_{\mathrm{SC}}^{C}}{\partial g_1}=0$, $\frac{\partial \mathrm{TP}_{\mathrm{SC}}^{C}}{\partial s_1}=0$, $\frac{\partial \mathrm{TP}_{\mathrm{SC}}^{C}}{\partial s_2}=0$ we simply get the solution set described in equation (10). Clearly, it has been observed that $\Delta_1=-2b_1<0$, $\Delta_2=4b_1b_2-(c_1+c_2)^2>0$ when $4b_1b_2>(c_1+c_2)^2$ holds and so on. To avoid the complexity, the illustration has been addressed in numerical in Section 5.

Appendix B.

In the decentralized scenario, by equating $\frac{\partial \mathrm{TP}_R}{\partial p_1}=0$, $\frac{\partial \mathrm{TP}_R}{\partial p_2}=0$, $\frac{\partial \mathrm{TP}_R}{\partial s_1}=0$, $\frac{\partial \mathrm{TP}_R}{\partial s_2}=0$, the retail sales price$(p_1,p_2)$ and sales effort level (s_1,s_2) obtained from solving the following:

(1)
$$\begin{aligned}&(\alpha_1a_1-b_1p_1+c_2p_2+\eta_1s_1)-b_1(p_1-w_1)+c_1(p_2-w_2)+\frac{\gamma_1}{e_1}(\gamma_1w_1-\gamma_1M_1-\gamma_1'M_1)\\&+\frac{1}{D}\Bigg[\left(c_1'+\frac{\gamma_1\gamma_1'}{e_1}\right)(2b_2'e_1B+2b_2'\gamma_1'C-c_2'\gamma_2'e_1g_2-c_2'e_1A)+c_2'\{-(\gamma_2'g_2+A)E+c_1'e_1B+c_1'\gamma_1'C\}\\&+(p_1+p_2-w_1-w_2)\{c_1'(-2b_2'c_1e_1-c_1c_2'e_1)+c_2'(-Ec_1-c_1c_1'e_1)\}\\&+(p_1-w_1)\gamma_1\gamma_1'(-2b_2'c_1-c_1c_2')\Bigg]=0\end{aligned}\tag{B.1}$$

(2)
$$\begin{aligned}&(\alpha_2a_2-b_2p_2+c_1p_1+\eta_2s_2)+c_2(p_1-w_1)-b_2(p_2-w_2)+\gamma_2g_2\\&+\frac{1}{D}[c_1'(2b_2'e_1B+2b_2'\gamma_1'C-c_2'\gamma_2'e_1g_2-c_2'e_1A)+c_2'\{-(\gamma_2'g_2+A)E+c_1'e_1B+c_1'\gamma_1'C\}\\&+(p_1+p_2-w_1-w_2)\{c_1'(-2b_2'c_2e_1-c_2c_2'e_1)+c_2'(-Ec_2-c_1'c_2e_1)\}\\&+(p_1-w_1)\gamma_1\gamma_1'(-2b_2'c_2-c_2c_2')]=0\end{aligned}\tag{B.2}$$

(3) $$(p_1-w_1)\eta_1-\tau_1s_1=0\tag{B.3}$$

(4) $$(p_2-w_2)\eta_2-\tau_2s_2=0.\tag{B.4}$$

The solutions obtained from equations (B.1)–(B.4) are uniquely exist if the Hessian matrix related to the retailer's profit function is negative definite. The Hessian matrix related to the profit function of the retailer is acquired as

$$\begin{bmatrix}\frac{\partial^2\mathrm{TP}_R}{\partial {p_1}^2}&\frac{\partial^2\mathrm{TP}_R}{\partial p_1\partial p_2}&\frac{\partial^2\mathrm{TP}_R}{\partial p_1\partial s_1}&\frac{\partial^2\mathrm{TP}_R}{\partial p_1\partial s_2}\\\frac{\partial^2\mathrm{TP}_R}{\partial p_2\partial p_1}&\frac{\partial^2\mathrm{TP}_R}{\partial {p_2}^2}&\frac{\partial^2\mathrm{TP}_R}{\partial p_2\partial s_1}&\frac{\partial^2\mathrm{TP}_R}{\partial p_1\partial s_2}\\\frac{\partial^2\mathrm{TP}_R}{\partial s_1\partial p_1}&\frac{\partial^2\mathrm{TP}_R}{\partial s_1\partial p_2}&\frac{\partial^2\mathrm{TP}_R}{\partial {s_1}^2}&\frac{\partial^2\mathrm{TP}_R}{\partial s_1\partial s_2}\\\frac{\partial^2\mathrm{TP}_R}{\partial s_2\partial p_1}&\frac{\partial^2\mathrm{TP}_R}{\partial s_2\partial p_2}&\frac{\partial^2\mathrm{TP}_R}{\partial s_2\partial s_1}&\frac{\partial^2\mathrm{TP}_R}{\partial {s_2}^2}\end{bmatrix}.$$

To exhibit the Hessian matrix is negative definite, the 2nd order partial derivatives of TP_R are shown below

$$\frac{\partial^2\mathrm{TP}_R^{\mathrm{DC}}}{\partial p_1^2}=-2b_1-\frac{2}{D}\left[\left(c_1'+\frac{\gamma_1\gamma_1'}{e_1}\right)(2b_2'c_1e_1+c_1c_2'e_1)+2c_2'(c_1c_1'e_1+Ec_1)\right]<0\tag{B.5}$$

$$\frac{\partial^2\mathrm{TP}_R^{\mathrm{DC}}}{\partial p_2^2}=-2b_2-\frac{2}{D}[c_1'(2b_2'c_2e_1+c_2c_2'e_1)+c_2'(c_2c_1'e_1+Ec_2)]<0\tag{B.6}$$

$$\frac{\partial^2\mathrm{TP}_R^{\mathrm{DC}}}{\partial s_1^2}=-\tau_1<0\tag{B.7}$$

$$\frac{\partial^2\mathrm{TP}_R^{\mathrm{DC}}}{\partial s_2^2}=-\tau_2<0.\tag{B.8}$$

Negative definite property of the above Hessian matrix is also shown numerically due to complexity.

Appendix C.

In the revenue-sharing contract, following manufacturer's decision retailer optimum retail sales price and sales effort level are as follows:

(1)
$$
\begin{aligned}
&(1-\delta_1)(\alpha_1 a_1 - b_1 p_1 + c_2 p_2 + \eta_1 s_1) - b_1(1-\delta_1)(p_1 - w_1) + c_1(1-\delta_2)(p_2 - w_2)\\
&+ (1-\delta_1)\left(\frac{\gamma_1^2}{e_1}\right)(w_1 + \delta_1 p_1 - M_1 - \delta_1 w_1) - (1-\delta_1)\left(\frac{\gamma_1\gamma_1' M_1}{e_1}\right) + (1-\delta_1)(p_1 - w_1)\left(\frac{\gamma_1^2\delta_1}{e_1}\right)\\
&+ \frac{1}{D}\Bigg[(1-\delta_1)\left(c_1' + \frac{\gamma_1\gamma_1'}{e_1}\right)(2b_2' e_1 Y + 2b_2'\gamma_1' Z - c_2'\gamma_2' e_1 g_2 - c_2' e_1 X) + (1-\delta_1)c_2'\{c_1' e_1 Y + c_1'\gamma_1' Z\\
&- (\gamma_2' g_2 + X)E\} + \{(1-\delta_1)(p_1 - w_1)c_1' + (1-\delta_2)(p_2 - w_2)c_1'\}\{2b_2'(-c_1 e_1 - c_1' e_1\delta_1 - \gamma_1\gamma_1'\delta_1)\\
&- c_1 c_2' e_1\} + \{(1-\delta_1)(p_1 - w_1)c_2' + (1-\delta_2)(p_2 - w_2)c_2'\}\left\{-Ec_1 - c_1 c_1' e_1 - {c_1'}^2 e_1\delta_1 - c_1'\gamma_1\gamma_1'\delta_1\right\}\\
&+ (1-\delta_1)(p_1 - w_1)\left(\frac{\gamma_1\gamma_1'}{e_1}\right)(-2b_2' c_1 e_1 - 2b_2' c_1' e_1\delta_1 - 2b_2'\gamma_1\gamma_1'\delta_1 - c_1 c_2' e_1)\Bigg] = 0 \qquad \text{(C.1)}
\end{aligned}
$$

(2)
$$
\begin{aligned}
&(1-\delta_1)(p_1 - w_1)c_2 + (1-\delta_2)(\alpha_2 a_2 - b_2 p_2 + c_1 p_1 + \eta_2 s_2) - (1-\delta_2)(p_2 - w_2)b_2 + (1-\delta_2)\gamma_2 g_2\\
&+ \frac{1}{D}\Big[(1-\delta_2)c_1'(2b_2' e_1 Y + 2b_2'\gamma_1' Z - c_2'\gamma_2' e_1 g_2 - c_2' e_1 X) + (1-\delta_2)c_2'\{-(\gamma_2' g_2 + X)E + c_1' e_1 Y\\
&+ c_1'\gamma_1' Z\} + \{(1-\delta_1)(p_1 - w_1)c_1' + (1-\delta_2)(p_2 - w_2)c_1'\}\left\{-2b_2' c_2 e_1 - c_2 c_2' e_1 - {c_2'}^2 e_1\delta_2\right\}\\
&+ \{(1-\delta_1)(p_1 - w_1)c_2' + (1-\delta_2)(p_2 - w_2)c_2'\}\{-(c_2 + c_2'\delta_2)E - c_2 c_1' e_1\}\\
&+ (1-\delta_1)(p_1 - w_1)\gamma_1\gamma_1'\left(-2b_2' c_2 - c_2 c_2' - {c_2'}^2\delta_2\right)\Big] = 0 \qquad \text{(C.2)}
\end{aligned}
$$

(3) $$(1-\delta_1)(p_1 - w_1)\eta_1 - \tau_1 s_1 = 0 \qquad \text{(C.3)}$$

(4) $$(1-\delta_2)(p_2 - w_2)\eta_2 - \tau_2 s_2 = 0. \qquad \text{(C.4)}$$

To exhibit the Hessian matrix is negative definite, the 2nd order partial derivatives of TP_R are given below

$$
\begin{cases}
\dfrac{\partial^2 \mathrm{TP}_R^{\mathrm{RS}}}{\partial p_1^2} = -2(1-\delta_1)\left(b_1 + \dfrac{\gamma_1^2\delta_1}{e_1}\right) - \dfrac{2(1-\delta_1)}{D}\Bigg[\left(c_1' + \dfrac{\gamma_1\gamma_1'}{e_1}\right)(2b_2' c_1 e_1 + 2b_2' c_1' e_1\delta_1 + 2b_2'\gamma_1\gamma_1'\delta_1 + c_1 c_2' e_1)\\
\qquad + c_2'\left(c_1 c_1' e_1 + {c_1'}^2 e_1\delta_1 + c_1'\gamma_1\gamma_1'\delta_1 - Ec_1\right)\Bigg] < 0 \qquad \text{(C.5)}\\
\dfrac{\partial^2 \mathrm{TP}_R^{\mathrm{RS}}}{\partial p_2^2} = -2(1-\delta_2)b_2 - \dfrac{2(1-\delta_1)}{D}\left[c_1'(2b_2' c_2 e_1 + c_2 c_2' e_1 + c_2' e_1\delta_2) + c_2'(Ec_2 + Ec_2'\delta_2 - c_1' c_2 e_1)\right] < 0 \qquad \text{(C.6)}\\
\dfrac{\partial^2 \mathrm{TP}_R^{\mathrm{RS}}}{\partial s_1^2} = -\tau_1 < 0 \qquad \text{(C.7)}\\
\dfrac{\partial^2 \mathrm{TP}_R^{\mathrm{RS}}}{\partial s_2^2} = -\tau_2 < 0. \qquad \text{(C.8)}
\end{cases}
$$

Equations (C.5)–(C.8) exists when $c_1 c_1' e_1 + {c_1'}^2 e_1\delta_1 + c_1'\gamma_1\gamma_1'\delta_1 > Ec_1$ and $Ec_2 + Ec_2'\delta_2 > c_1' c_2 e_1$ holds.

Acknowledgements

We extend our heartfelt gratitude to the esteemed reviewers and editor for their insightful suggestions that enhance the quality and impact of this research contribution.

Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

References

- [1] E. Adida and V. DeMiguel, Supply chain competition with multiple manufacturers and retailers. *Oper. Res.* **59** (2011) 156–172.
- [2] S.F. Alamdar, M. Rabbani and J. Heydari, Pricing, collection, and effort decisions with coordination contracts in a fuzzy, three-level closed-loop supply chain. *Expert Syst. Appl.* **104** (2018) 261–276.
- [3] A. Barman and P. De, A multi-item deteriorating inventory model under stock level-dependent, time-varying, and price-sensitive demand, in Recent Trends in Applied Mathematics: Select Proceedings of AMSE 2019. Springer, Singapore (2021) 1–12.
- [4] A. Barman, R. Das and P.K. De, Optimal pricing and greening decision in a manufacturer retailer dual-channel supply chain. *Mater. Today Proc.* **42** (2021) 870–875.
- [5] A. Barman, R. Das and P.K. De, An analysis of optimal pricing strategy and inventory scheduling policy for a non-instantaneous deteriorating item in a two-layer supply chain. *Appl. Intell.* **52** (2022) 4626–4650.
- [6] A. Barman, R. Das, P.K. De and J.K. Dash, Optimal pricing, ordering, and replenishment policies in a multi-item inventory system for deteriorating items under time-varying backlogging rate. *J. Ind. Integr. Manag.* **7** (2022) 235–256.
- [7] A. Barman, K. Maity, O. Banik, T. Datta and N. Majumder, A price discount scheme for non-instantaneous inventory system with shortage, in 2022 10th International Conference on Reliability, Infocom Technologies and Optimization (Trends and Future Directions) (ICRITO). IEEE (2022) 1–5.
- [8] A. Barman, A.K. Chakraborty, A. Goswami, P. Banerjee and P.K. De, Pricing and inventory decision in a two-layer supply chain under the weibull distribution product deterioration: an application of NSGA-II. *RAIRO-OR* **57** (2023) 2279–2300.
- [9] A. Barman, P.K. De, A.K. Chakraborty, C.P. Lim and R. Das, Optimal pricing policy in a three-layer dual-channel supply chain under government subsidy in green manufacturing. *Math. Comput. Simul.* **204** (2023) 401–429.
- [10] Z. Basiri and J. Heydari, A mathematical model for green supply chain coordination with substitutable products. *J. clean. prod.* **145** (2017) 232–249.
- [11] J. Chen, H. Zhang and Y. Sun, Implementing coordination contracts in a manufacturer Stackelberg dual-channel supply chain. *Omega* **40** (2012) 571–583.
- [12] B. Dan, G. Xu and C. Liu, Pricing policies in a dual-channel supply chain with retail services. *Int. J. Prod. Econ.* **139** (2012) 312–320.
- [13] R. Das, A. Barman and P.K. De, Integration of pricing and inventory decisions of deteriorating item in a decentralized supply chain: a Stackelberg-game approach. *Int. J. Syst. Assur. Eng. Manag.* **13** (2022) 479–493.
- [14] R. Das, A. Barman, B. Roy and P.K. De, Pricing and greening strategies in a dual-channel supply chain with cost and profit sharing contracts. *Environ. Dev. Sustain.* **25** (2022) 1–34.
- [15] P.K. De, A.K. Chakraborty, A. Barman and R. Das, Strategic analysis of a dual channel green supply chain with return-refund facility, in Applications of Operational Research in Business and Industries: Proceedings of 54th Annual Conference of ORSI. Springer, Singapore (2023) 295–309.
- [16] P. De Giovanni, Environmental collaboration in a closed-loop supply chain with a reverse revenue sharing contract. *Ann. Oper. Res.* **220** (2014) 135–157.
- [17] K. Dey and S. Saha, Influence of procurement decisions in two-period green supply chain. *J. Clean. Prod.* **190** (2018) 388–402.
- [18] M. Frostenson and F. Prenkert, Sustainable supply chain management when focal firms are complex: a network perspective. *J. Clean. Prod.* **107** (2015) 85–94.
- [19] J. Gao, H. Han, L. Hou and H. Wang. Pricing and effort decisions in a closed-loop supply chain under different channel power structures. *J. Clean. Prod.* **112** (2016) 2043–2057.
- [20] D. Ghosh and J. Shah, Supply chain analysis under green sensitive consumer demand and cost sharing contract. *Int. J. Prod. Econ.* **164** (2015) 319–329.
- [21] B. Giri, C. Mondal and T. Maiti, Analysing a closed-loop supply chain with selling price, warranty period and green sensitive consumer demand under revenue sharing contract. *J. Clean. Prod.* **190** (2018) 822–837.
- [22] R.N. Giri, S.K. Mondal and M. Maiti, Bundle pricing strategies for two complementary products with different channel powers. *Ann. Oper. Res.* **287** (2020) 701–725.
- [23] J. Heydari, K. Govindan and A. Aslani, Pricing and greening decisions in a three-tier dual channel supply chain. *Int. J. Prod. Econ.* **217** (2019) 185–196.
- [24] G. Hua, S. Wang and T.E. Cheng, Price and lead time decisions in dual-channel supply chains. *Eur. J. Oper. Res.* **205** (2010) 113–126.

[25] H. Huang and H. Ke, Pricing decision problem for substitutable products based on uncertainty theory. *J. Int. Manuf.* **28** (2017) 503–514.

[26] M.-B. Jamali and M. Rasti-Barzoki, A game theoretic approach for green and non-green product pricing in chain-to-chain competitive sustainable and regular dual-channel supply chains. *J. Clean. Prod.* **170** (2018) 1029–1043.

[27] M.R. Kuiti, D. Ghosh, S. Gouda, S. Swami and R. Shankar, Integrated product design, shelf-space allocation and transportation decisions in green supply chains. *Int. J. Prod. Res.* **57** (2019) 6181–6201.

[28] B. Li, M. Zhu, Y. Jiang and Z. Li, Pricing policies of a competitive dual-channel green supply chain. *J. Clean. Prod.* **112** (2016) 2029–2042.

[29] Q. Li, T. Xiao and Y. Qiu, Price and carbon emission reduction decisions and revenue-sharing contract considering fairness concerns. *J. Clean. Prod.* **190** (2018) 303–314.

[30] Z. Li, H. Zhang and R. Gao, Frugal innovation in supply chain cooperation considering e-retailer's platform value. *Soft Comput.* **24** (2020) 15373–15387.

[31] P. Ma, K. W. Li and Z.-J. Wang, Pricing decisions in closed-loop supply chains with marketing effort and fairness concerns. *Int. J. Prod. Res.* **55** (2017) 6710–6731.

[32] S.R. Madani and M. Rasti-Barzoki, Sustainable supply chain management with pricing, greening and governmental tariffs determining strategies: a game-theoretic approach. *Comput. Ind. Eng.* **105** (2017) 287–298.

[33] C. Mondal and B.C. Giri, Pricing and used product collection strategies in a two-period closed-loop supply chain under greening level and effort dependent demand. *J. Clean. Prod.* **265** (2020) 121335.

[34] R. Palevich, Lean Sustainable Supply Chain The: How to Create a Green Infrastructure with Lean Technologies. Ft Press (2011).

[35] S. Parsaeifar, A. Bozorgi-Amiri, A. Naimi-Sadigh and M.S. Sangari, A game theoretical for coordination of pricing, recycling, and green product decisions in the supply chain. *J. Clean. Prod.* **226** (2019) 37–49.

[36] S. Qu, Y. Zhou, Y. Zhang, M. Wahab, G. Zhang and Y. Ye, Optimal strategy for a green supply chain considering shipping policy and default risk. *Comput. Ind. Eng.* **131** (2019) 172–186.

[37] A. Raj, I. Biswas and S.K. Srivastava, Designing supply contracts for the sustainable supply chain using game theory. *J. Clean. Prod.* **185** (2018) 275–284.

[38] A. Ranjan and J. Jha, Pricing and coordination strategies of a dual-channel supply chain considering green quality and sales effort. *J. Clean. Prod.* **218** (2019) 409–424.

[39] S.A. Raza and S.M. Govindaluri, Greening and price differentiation coordination in a supply chain with partial demand information and cannibalization. *J. Clean. Prod.* **229** (2019) 706–726.

[40] S.S. Sana, J.A. Chedid and K.S. Navarro, A three layer supply chain model with multiple suppliers, manufacturers and retailers for multiple items. *Appl. Math. Comput.* **229** (2014) 139–150.

[41] B. Shen, Y. Cao and X. Xu, Product line design and quality differentiation for green and non-green products in a supply chain. *Int. J. Prod. Res.* **58** (2020) 148–164.

[42] A.A. Taleizadeh, E. Sane-Zerang and T.-M. Choi, The effect of marketing effort on dual-channel closed-loop supply chain systems. *IEEE Trans. Syst. Man Cybern. Syst.* **48** (2016) 265–276.

[43] A.A. Tsay and N. Agrawal, Channel dynamics under price and service competition. *Manuf. Serv. Oper. Manag.* **2** (2000) 372–391.

[44] L. Wang and Q. Song, Pricing policies for dual-channel supply chain with green investment and sales effort under uncertain demand. *Math. Comput. Simul.* **171** (2020) 79–93.

[45] J. Xie, W. Zhang, L. Liang, Y. Xia, J. Yin and G. Yang, The revenue and cost sharing contract of pricing and servicing policies in a dual-channel closed-loop supply chain. *J. Clean. Prod.* **191** (2018) 361–383.

[46] C. Xin, X. Chen, H. Chen, S. Chen and M. Zhang, Green product supply chain coordination under demand uncertainty. *IEEE Access* **8** (2020) 25877–25891.

[47] G. Xu, B. Dan, X. Zhang and C. Liu, Coordinating a dual-channel supply chain with risk-averse under a two-way revenue sharing contract. *Int. J. Prod. Econ.* **147** (2014) 171–179.

[48] F. Zand, S. Yaghoubi and S.J. Sadjadi, Impacts of government direct limitation on pricing, greening activities and recycling management in an online to offline closed loop supply chain. *J. Clean. Prod.* **215** (2019) 1327–1340.

[49] E.S. Zerang, A.A. Taleizadeh and J. Razmi, Analytical comparisons in a three-echelon closed-loop supply chain with price and marketing effort-dependent demand: game theory approaches. *Environ. Dev. Sustain.* **20** (2018) 451–478.

[50] C.-T. Zhang and L.-P. Liu, Research on coordination mechanism in three-level green supply chain under non-cooperative game. *Appl. Math. Model.* **37** (2013) 3369–3379.

[51] J. Zhao, W. Tang, R. Zhao and J. Wei, Pricing decisions for substitutable products with a common retailer in fuzzy environments. *Eur. J. Oper. Res.* **216** (2012) 409–419.

[52] J. Zhao, J. Wei and Y. Li, Pricing decisions for substitutable products in a two-echelon supply chain with firms' different channel powers. *Int. J. Prod. Econ.* **153** (2014) 243–252.

[53] W. Zhu and Y. He, Green product design in supply chains under competition. *Eur. J. Oper. Res.* **258** (2017) 165–180.