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PRICING STRATEGY AND ITS IMPACT ON THE EFFORT OF COMMUNITY LEADER AND PLATFORM: UNIFORM OR DIFFERENTIATED PRICING?

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Abstract. This paper is motivated by the rapid development of community group buying (CGB), where the CGB platform dramatically relies on the community leader to provide last-mile services and fulfill consumers' orders. Considering two types of community leaders, the friend role and seller role, this work adopts a game-theoretical model and investigates how the pricing strategy, uniform pricing strategy (N) or differentiated pricing strategy (Y), affects players' performance and decisions on effort level. This study shows that the commission rate is an essential factor in stimulating the role transformation of community leaders. A significantly large commission rate results in the friend role community leader with lower trust value changing into the seller role. Generally, the community leader works harder under the uniform pricing scenario except in situations with a significant commission rate and moderate sensitivity coefficient of trust value. However, the effort level of the platform is jointly influenced by the pricing strategy, commission rate, and the role of a community leader. Moreover, regardless of the commission rate, when the community leader is a friend role and the trust value is high, both the platform and community leader can gain higher profits under the uniform pricing scenario than the differentiated pricing case. It indicates that a win–win situation can be achieved.

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1. Introduction

The widespread use of social media platforms, such as WeChat, coupled with the continually rising demands for online shopping on e-commerce platforms, has engendered a new retail industry, Community Group Buying (CGB). It is a business model that operates within the framework of a community-based economy and pre-sale methodology. The community leader is a pivotal node in this mode, utilizing the WeChat communication platform to establish direct connections with both suppliers and consumers through the CGB platform [31]. The pre-sale mechanism of CGB aligns goods supplied with demands, mitigating inventory losses. Simultaneously, CGB integrates innovative features from both fresh retail and social e-commerce business models, which diminishes operational costs within the intermediate links of the retail industry supply chain [33]. Moreover,

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the target of next-day delivery satisfies consumers' requirements for swift transactions, then fostering lots of loyal users and attracting new market shares through multi-level proliferation, significantly reducing customer acquisition costs.

Along with CGB's advantages, the COVID-19 epidemic has imposed restrictions on offline interpersonal contact, and people have spontaneously organized CGB [31], which further accelerates its growth. It shows that the overall transaction size increased by 300% in 2019. Such a powerful market has attracted many Internet giants, such as Meituan, JD.com, and Alibaba, to join and operate as a CGB platform. In 2020, the growth rate of China's CGB market was as high as 111.8%[1]. It reports that the daily order volume of Duoduo Shopping is about 15 million and the daily order volume of Meituan Preferred has exceeded 20 million. The average daily transaction volume of Meituan Preferred is about 100 million yuan. Duoduo grocery shopping is about 165 million yuan[2].

The most significant difference between the CGB business mode and other online group buying ones is the community leader who links the platform-end and consumers. Generally, as the publisher of commodity information, the community leader first selects relatively high-quality commodities and then publishes the corresponding information to community members through WeChat or a small program, which can reduce the information collection time for users. Consumers can order through the link and pick up products from community leaders the next day. In other words, the community leader plays the role of traffic acquisition and monetization, community operation, and after-sales service, interacting throughout the entire online and offline purchasing process. Such a multiple-roles phenomenon may lead the community leader to switch the role in interacting with consumers. Heide and Wathne [10] classify the roles that merchants played in the interactions into seller and friend based on the role theory. Köhler *et al.* [19] divide it into task-oriented interaction and social-oriented according to the features of interactions.

In the CGB model, especially during the outbreak of COVID-19, people trusted the community leader and were able to buy goods mostly relying on him. It indicates that the community leader may play as a friend. On the other hand, an imbalance between supply and demand resulted in the platform losing the control of commodity price so that the community leader would price differently from that on the CGB platform, *i.e.*, a higher price. In this situation, the role of a retailer happens. It is noted that the form of interaction adopted by community leader not only has an influence on consumers' attitudes to purchase through CGB, but also the platform's operation. Specifically, a friend role means the community leader works harder, *i.e.*, a higher effort level, and publishes goods information with higher quality as well as lower prices, which is liked by consumers. Then, the customer acquisition cost will be lower, which is positive for the platform. Otherwise, if the community leader acts as a retailer, adopting a differentiated pricing strategy, some consumers may be lost which is a harmful side or directly snap up goods on platform.

Motivated by the rapid development of CGB and multiple roles, a friend or seller role may adopted by a community leader, we establish a community group buying supply chain including a CGB platform and a community leader. Before and during the outbreak of COVID-19, different pricing strategies, uniform or differentiated pricing, may be implemented. Additionally, the community leader presents multiple roles that affect consumers' attitudes to purchasing on the CGB platform. Therefore, this paper investigates the impacts of different pricing strategies on players' performance in a CGB supply chain, specifically,

(1) What is the equilibrium decision of the CGB platform and community leader under different scenarios? How does the commission rate offered by the CGB platform impact the role transformation of a community leader between a friend and a seller?
(2) What is the impact of different pricing strategies on players' effort levels? How do these effects interact with the trust value of community leaders and the commission rate?
(3) Which pricing strategy, the uniform pricing strategy or the differentiated one, is preferred to the CGB platform (or community leader)? Does the win–win situation exist, and under which conditions?

[1] https://ecoapp.qianzhan.com/detials/220422-6cf743e1.html?uid=ffffffff-fb4b-b515-ffff-ffffcb303f11.
[2] https://www.360doc.cn/document/74928380_1086981100.html.

To answer these questions, we establish a game-theoretic model including a CGB platform and multiple-roles community leader. Consumers can purchase through either the platform channel or the link launched by a community leader. The platform may implement different pricing strategies: (1) scenario N, referring to the uniform pricing strategy, under which the community leader has no power to set the price of launching goods information; (2) scenario Y, referring to the differentiated pricing strategy, under which the community leader can select which goods to publish, but also determine the corresponding price. Besides, the community leader presents multiple-roles characteristic, acting as a friend or seller role, which affects consumers' trust value. Based on this, we derive players' performance under different scenarios and then identify how pricing strategy, commission rate, and the sensitivity coefficient of trust value influence the optimal decision on price, effort level, and role transformation.

We obtain the following research findings. Firstly, we emphasize the impact of commission rate and sensitivity coefficient of trust value on the role transformation of community leader. Under the differentiated pricing strategy, role transformation consistently occurs for the community leader of a seller role. However, the case of a friend role community leader becomes complex. A low commission rate encourages him to maintain his current role, while a high commission rate and low trust value lead to transformation into a seller role.

Secondly, the implemented pricing strategy influences both the platform and community leader's effort level. Specifically, the community leader is more inclined to exert higher efforts under scenario N due to the lack of pricing power in contrast to scenario Y. Regarding the platform, the effort level is higher under the differentiated pricing case with a friend role community leader if the commission rate and sensitivity coefficient of trust value are small. Nevertheless, a seller role community leader has the opposite effect on the difference between the platform's effort levels under scenarios Y and N.

Thirdly, although the profits of the platform and community leader exhibit diverse discrepancies between the uniform and differentiated pricing strategies, further affected by the commission rate, sensitivity coefficient of trust value, and the role played by community leader, they are superior under scenario N when the community leader is a friend role and the sensitivity coefficient of trust value is large. Therefore, a win–win situation is achieved.

Our work is highlighted in the following contributions. First, we focus on the community group buying supply chain through game theory, which is less examined in the current literature that explores the evaluation of this mode [43] and the coordination contract design [21]. Besides, our work is motivated by the realistic phenomenon of the different pricing strategies, the uniform and differentiated pricing modes, implemented before and during the outbreak of COVID-19. Moreover, according to role theory and trust theory, we characterize consumer trust in the community leader with multiple roles, a friend or seller role.

The remainder of this paper is organized as follows. In Section 2, we exhibit a literature review for the related research content of this paper. In Section 3, the model and assumptions are established. In Section 4, we compare and analyze the uniform pricing model and the differentiated pricing model. In Section 5, numerical analysis is carried out to verify the relevant important conclusions. Section 6 summarizes the primary findings and discusses future research.

2. Literature review

This paper is closely related to four streams of literature: the group buying supply chain, pricing strategy, promotional effort, and the value of trust (social interaction).

First, our work is relevant to the research on group buying operations. Group buying is a unique selling strategy for a seller to benefit from facilitating consumer interaction. Jing and Xie [14] point out that implementing group buying is more profitable than the traditional individual-selling strategy when interpersonal information-sharing efficiency is high enough. Similarly, Ming and Tunca [26] establish a continuous-time dynamic game theoretical model to characterize consumer behavior and explore whether the group buying business can improve a firm's performance. It demonstrates that group buying discounts can efficiently boost market demands when the base demand is neither too low nor too high. In contrast to the above research under a monopolistic market, Yan *et al.* [39] extend it to competition and discuss how e-tailers with asymmetric demand information select selling modes between individual buying and group buying. Contrarily, given the selling mode, Sun *et al.* [34] investigate which supply chain structure is preferred, monopoly or competition. It indicates that the e-tailer benefits more under competition when the threshold on group size is very high and consumers are more impatient to the group buying. Zhang and Liu [41] study the impact of pricing and ordering in the group buying model and consider the joint decision of pricing and ordering for short-life-cycle products in a competitive market. In addition to the above research on the firm's performance of introducing group buying [7, 46], scholars also analyze consumer's intention to participate in group buying [18, 30], the pricing decision under different information sharing cases, either the seller or the informed buyer alone sharing, or both sharing together [6].

Though numerous scholars have developed research on group buying, they mainly focus on the traditional online group buying. There are few kinds of literature on community group buying, including the evaluation of this mode [43], the coordination contract design [21], and the operation strategy of nanostore [33]. Sun and Zhang [33] explore whether the nanostore should cooperate with the CGB platform, acting as the community leader. However, our work differs from it by given the CGB supply chain structure consisting of a CGB platform and a community leader. Then, we discuss how different pricing strategies, the uniform and differentiated pricing strategy, influence on players' effort level considering the multiple-roles characteristic of community leader.

Second, this research contributes to the literature on pricing strategy in a multi-channel supply chain. Recently, scholars have discussed price discrimination, dynamic pricing, pricing matching [4, 13], and profit-margin guarantee [45]. Additionally, some studies have concentrated on the different pricing strategies provided by different supply chain members. For instance, Hou *et al.* [12] explore how competition influences the optimal pricing strategy for content product producers, the traditional fixed-price or pay-as-you-want strategy. From the perspective of a supplier, Vakharia and Wang [35] consider the wholesale pricing strategy for multiple downstream retailers, and results show that the supplier and less efficient retailer prefer the retailer-specific wholesale price and the more efficient one likes the uniform strategy. Under different information structures (symmetric or asymmetric), Zhang and Zheng [42] find that the optimal price of main and bundled products are positively and negatively related to consumers' green preference, respectively, under the differentiated pricing strategy. Meanwhile the optimal price of both products presents stability under the uniform pricing strategy. Moreover, the e-commerce platform has flourished, which gives the manufacturer or retailer more than one selling channel option. Thus, Zhen and Xu [44] examine who should introduce the marketplace channel under different pricing strategies. Based on the literature of Zhen and Xu [44], Xi and Zhang [37] further discuss the best choice of marketplace channel addition from the viewpoints of three parties, including the manufacturer, the reseller, and the third-party platform. They find that the manufacturer and platform have different preferences regarding the combination of channel addition and pricing strategy.

In contrast to the above research, we consider a CGB supply chain where the CGB platform has the power to determine whether the community leader can control the selling price and analyze how the pricing strategy implemented by the CGB platform affects the community leader's effort level and role transformation.

Besides, this paper is also relevant to the research on promotional effort. Promotional effort is a primary measure to derive consumer demands regardless of online or offline channels, which includes advertising, sales promotion, and service enhancement. Generally, advertising can expand the potential market base and inform consumers a detailed understanding of goods. Li *et al.* [24] examine the online and offline joint advertising promotion strategy, and the channel demand is affected by the advertising level of sellers and the O2O platform. Targeted promotion efforts are also widely utilized by e-commerce platforms. Hao and Yang [9] investigate whether an online platform should conduct targeted promotion and in which mode, free or paid advertising. Besides, providing sales promoting, such as gifts [5, 17] and discounts [25, 28] can attract some price-sensitive consumers. Some specific sales promotion, such as green marketing by retailer, are efficient tools to stimulate consumers with green awareness to purchase. Then Hong and Guo [11] evaluate the impact of green product promotion efforts on firms' performance under three cooperation models. Moreover, a high-quality service, especially during the COVID-19 (*i.e.*, no-contract delivery), is one factor that determines consumer purchase in addition to price. Under the optimal service level, Kumar [20] proposes an optimization model to solve the pricing mechanism based on inventory. Considering different power structures, the centralized, manufacturer Stackelberg, and vertical Nash structure, in a competitive green dual-channel supply chain, Pal *et al.* [29] examine the optimal green innovation effort of the manufacturer and promotional effort of the retailer.

The above research mainly analyzes the influence of different supply chain members' promotion effort levels on the supply chain system and what kind of effort level can make the supply chain optimal. However, this paper considers the promotion effort level of the CGB platform and community leader, which presents different efficiencies in improving consumer utility. Specifically, we assume that the effort level coefficient of the community leader is greater than that of the CGB platform and then analyze whether the promotion effort advantage of community leader has impacts on platform's effort level. The reason is that the community leader maintains a more intimate relationship with consumers than the CGB platform. Then, given the same promotion level of CGB platform and community leader, the latter may generate more significant impacts on consumers.

The last stream of related research mainly focuses on the value of trust (social interaction). Numerous studies have shown that social information sharing has effects on consumer choice [8] and company performance, such as sales [3] and revenue [1]. Therefore, some marketing strategies to proactively incentivize and manage social interactions have been explored from the sellers' perspective. For example, Yang *et al.* [40] explore the impacts of online consumer reviews on firms' performance under both centralized and decentralized channels and point out that only when online consumer reviews can provide sufficiently favorable information is it wise to adopt such a strategy. Xia *et al.* [38] analyze how interactions between social groups, fashion leaders and fashion followers, influence the manufacturer to adjust the launch sequence of luxury fashion products, through which channels firstly, the retail or direct channel. In addition to the research of manipulating trust diffusion among consumers, scholars also investigate how to maintain consumer trust in transactions, such as online group buying auction and rewards-based crowdfunding, before which is accomplished [15, 16, 36]. Enhancing consumer's perceived value for a program, *i.e.*, time-based and volume-based discounts [22], is critical to sustaining their trust because they must deal with transactional uncertainty in the pioneering online group buying [30].

The extant literature has pointed out that consumer trust plays a significant role in the success of online group buying and discussed how to increase it through marketing methods. Similarly, in this paper, we think whether a consumer purchases a good depends on the price and the trust value for community leader in CGB supply chain. However, what differs from the mentioned research is that the consumer is heterogeneous in trusting one community leader. Moreover, according to the symbol of the sensitivity coefficient of trust, the community lead is divided into two types, the friend and seller one. Such a setting reflects the multiple-roles characteristic of community leader in a CGB supply chain.

Although CGB has achieved great success in our daily lives, especially during the outbreak of COVID-19, the relevant theoretical research on operation management needs to be further enriched. These studies on the impacts of e-commerce in a group buying supply chain mainly focus on the traditional online group buying, that is, regarding group buying platforms as the external environment for supply chain operation. In other words, the promotional effort of CGB platform is not considered. On the other hand, COVID-19 has greatly influenced the CGB supply chain, *i.e.*, customers' assumption, the price of goods, and community leader's role change. Therefore, considering different pricing strategies before and during COVID-19, the uniform and differentiated pricing strategy, we investigate its impacts on the decision regarding the effort level of the CGB platform and community leader with multiple-roles characteristic. Table 1 shows the differences between this paper and relevant studies.

3. Model description and hypotheses

This paper considers a community group buying supply chain consisting of a CGB platform, a community leader (community store manager), and consumers. In the CGB supply chain, according to the product information provided by suppliers, the platform releases pre-sale information to determine the product price p and the effort level of the platform e_P; the community leader first selects some products at the effort level e_L and then sends the product information through the WeChat group and QQ group. Consumers can order either

TABLE 1. Differences between this paper and relevant studies.

Theme	Literature	Research content	Research gap	This study
Group buying supply chain	Zhang and Liu [41]	Whether the retailer should launch a group buying	Focus on the traditional group buying supply chain	Given the framework of CGB supply chain, investigate how different pricing strategies affect the effort level of community leader and platform
	Sun and Zhang [33]	Whether the nanostore should cooperate with CGB platform	Do not consider the effort of community leader	
Pricing strategy	Xi and Zhang [37]; Zhen and Xu [44]	The introduce of marketplace channel and pricing strategy	The pricing strategy is implemented by the one who adds a new channel	The CGB platform determines the pricing strategy, and examine its impacts on community leader's optimal decision
Promotion effort	Pal *et al.* [29]	The decision on green innovation effort of manufacturer and promotional effort of retailer	Firms' effort directly affects market demands (Bertrand price competition)	Firms' effort can enhance consumer's utility then has influence on demands. (Utility function)
Value of trust	Xia *et al.* [38]	How interactions between social groups require sellers to adjust their launch sequence of product and pricing decision	Do not study the social interaction between community leaders and consumer	Analyzes the different trust values of social interactions for community leaders
	Yang *et al.* [40]	Online consumer review (social interaction between consumers) affects consumer's perceived quality of products		

through the information released by the community leader or directly purchase and pay on the CGB platform and then choose a community store as a self-pickup point for the product. Obviously, consumers can choose the community leader whom they think has a higher trust value to place an order. In other words, if consumers feel that the trust value of the community leader is not enough, they can choose to place an order directly on the platform. In the end, the platform counts the number of product orders on the day, uniformly purchases the ordered goods from suppliers the next day, and then sends them to the corresponding community store. Lastly, consumers generally can pick up the goods on the second day after placing the order, which indicates the consumption process is ending. The platform must share a certain percentage of incomes as a commission to the community leader. The CGB supply chain structure is shown in Figure 1.

3.1. The uniform pricing model (Scenario N)

The uniform pricing strategy indicates that platform strictly control the pricing power of the community leader. That is, the community leader cannot arbitrarily change the product price, resell, or maliciously promote the product. Consumers are heterogenous in the trust for community leader, x, and we assume that x is uniformly distributed in the interval $[0,1]$, *i.e.*, $x \in [0,1]$. The consumer utility from purchasing goods through the community leader channel is

$$U_L = v + \alpha x - p + \beta e_L, \tag{1}$$

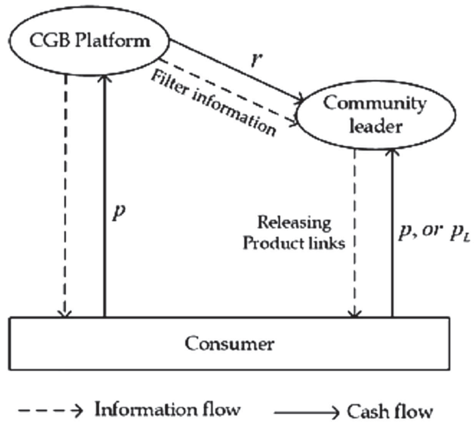

FIGURE 1. Structure of the CGB supply chain.

where v represents the average basic value of the product, αx is the trust value for the community leader, and α is the sensitivity coefficient of trust value. p is the product price determined by the CGB platform, and e_L means the effort level of community leader, which can be expressed as product inventory, effort to send product information, delivery service, customer acquisition cost, etc. β is the effort level sensitivity coefficient to consumer's utility.

In this paper, α may be positive or negative which is related to consumer's perception of the role played by community leader. Specifically, there are two roles of community leader: the friend and seller role. According to role theory, as for customer, when the community leader acts a friend role, he recommends high-quality and cost-effective products to friends in the community to maintain better social interactions; and when the community leader plays a seller role, he may be more inclined to recommend products with low-cost performance, focusing on selling products and completely self-interest. Besides, the trust theory considers that personal trust in a social context may play an important role in the decision-making process [23]. Su *et al.* [32] point out that the trust towards friend role positively influences social commerce engagement intention. Therefore, in this paper, we utilize $\alpha > 0$ to represent the case that consumer perceives the community leader as a friend role, and $\alpha < 0$ to reflect that consumer perceives the community leader as a seller role[3].

Besides, as for the parameter β, due to the intimate relationship between community leader and his neighbors, the customer acquisition cost of whom is lower than that of the CGB platform. It conveys that the same effort level provided by community leader and CGB platform, the former has a larger influence on consumer utility. Furthermore, we restrict β to be not too large to make the results comparable. Therefore, we assume $1 < \beta < 2$ in our work.

Similar to the assumptions in Niu *et al.* [27] and Lan and Yu [21], we utilize the cost function $C(e_L) = \frac{k{e_L}^2}{2}$, where $k(k > 0)$ is the effort cost coefficient, that is, the total amount of funds required to improve each unit service. Such a convex expression indicates that the increasing cost to improve per unit effort level.

The consumer utility from purchasing goods through the CGB platform channel is

$$U_P = v - p + e_P, \tag{2}$$

where e_P is the platform's effort level. It can be expressed as the optimization degree of platform interface, consumer service, payment service, after-sales service, etc. The corresponding cost is $C(e_P) = \frac{k{e_P}^2}{2}$. In order

[3]For the case of $\alpha = 0$, it implies that consumers' trust on community leader has no influence on consuming utility, which cannot reflect the research focus of our work, *i.e.*, how does the commission rate offered by CGB platform impact the role transformation of community leader between a friend and seller? Therefore, we don't consider the case of $\alpha = 0$.

to reflect the lower customer acquisition cost of community leader, the sensitivity coefficient of the platform's effort level to consumer utility is set to 1. The meanings of other characters are similar to those in formula (1). To ensure the validity of the model, it is assumed that $\alpha > \max\left\{\frac{1+\beta^2}{k(2+r)}, \frac{\beta^2}{k(2-r)}\right\}$ *or* $\alpha < -\frac{1+\beta^2}{2k}$.

Consumers can buy goods through either the community leader channel or the platform channel and will choose the one that generates larger utility. Thus, when $U_L \geq \max\{U_P, 0\}$, consumers purchase from the community leader channel, otherwise, when $U_P \geq \max\{U_L, 0\}$, the transaction happens on the platform channel. The demands for community leader channel and platform channel are D_L and D_P, respectively, expressed as

$$D_L = \begin{cases} \frac{\alpha+\beta e_L - e_P}{\alpha} & \alpha > 0 \\ \frac{-\beta e_L + e_P}{\alpha} & \alpha < 0 \end{cases}, \tag{3}$$

and

$$D_P = \begin{cases} \frac{-\beta e_L + e_P}{\alpha} & \alpha > 0 \\ \frac{\alpha+\beta e_L - e_P}{\alpha} & \alpha < 0 \end{cases}. \tag{4}$$

For the community leader, he can obtain r percentage of the sales sold through the launched product link. Thus, his profit is:

$$\pi_L = rpD_L - \frac{k}{2}{e_L}^2. \tag{5}$$

Generally, in practice, the revenue sharing fee r adopted by platform is less than 50%. Therefore, we assume $0 < r < 0.5$.

The profit of platform is:

$$\pi_P = (1-r)pD_L + pD_P - \frac{k}{2}{e_P}^2, \tag{6}$$

consisting of three parts: the revenue from the community leader channel (the first part) and the platform channel (the second part), and the cost of effort (the third part).

Decision-making sequence: (1) Firstly, the platform decides the effort level e_P and the commodity price p, (2) Then, the community leader decides the effort level e_L.

When $\alpha > 0$, we derive the optimal solutions by backward induction, ${e_L}^* = \frac{(1-r)\alpha\beta}{r(2\beta^2-1)}, {e_P}^* = \frac{(1-r)\alpha}{r(2\beta^2-1)}, p^* = \frac{k(1-r)\alpha^2}{r^2(2\beta^2-1)}, {\pi_L}^* = \frac{k(1-r)\alpha^2\left((1+3r)\beta^2-2\right)}{2r^2(2\beta^2-1)^2}$, ${\pi_P}^* = \frac{k(1-r)^2\alpha^2}{2r^2(2\beta^2-1)}$.

When $\alpha > 0$, similarly, the following results are obtained: ${e_L}^* = \frac{-\alpha\beta}{r(2\beta^2-1)}, {e_P}^* = \frac{-\alpha}{r(2\beta^2-1)}, p^* = \frac{k\alpha^2}{r^2(2\beta^2-1)}, {\pi_L}^* = \frac{k\alpha^2\left(\beta^2-2\right)}{2r^2(2\beta^2-1)^2}$, ${\pi_P}^* = \frac{k\alpha^2}{2r^2(2\beta^2-1)}$.

Lemma 3.1. (1) *When the community leader is a friend role,* $\frac{\partial p^*}{\partial\alpha} > 0, \frac{\partial {e_P}^*}{\partial\alpha} > 0, \frac{\partial {e_L}^*}{\partial\alpha} > 0, {e_L}^* > {e_P}^*, \frac{\partial({e_L}^*-{e_P}^*)}{\partial\alpha} > 0$. (2) *When the community leader is a seller role,* $\frac{\partial p^*}{\partial\alpha} < 0, \frac{\partial {e_P}^*}{\partial\alpha} < 0, \frac{\partial {e_L}^*}{\partial\alpha} < 0, {e_L}^* > {e_P}^*, \frac{\partial({e_L}^*-{e_P}^*)}{\partial\alpha} < 0$.

Lemma 3.1 indicates that different roles played by community leader have opposite influences on equilibrium results under the uniform pricing scenario. Specifically, the optimal price, the optimal effort level of platform, and community leader increase (decrease) with the sensitivity coefficient of trust value, α, when the community leader is a friend role (seller role), *i.e.*, $\alpha > 0$ ($\alpha < 0$). Additionally, though the optimal effort level of community leader is always greater than that of platform, the difference between them decreases with the sensitivity coefficient of trust value, *i.e.*, $\frac{\partial({e_L}^*-{e_P}^*)}{\partial\alpha} < 0$ if the community leader acts as a seller role; otherwise, it changes positively, *i.e.*, $\frac{\partial({e_L}^*-{e_P}^*)}{\partial\alpha} > 0$. The explanations can be summarized as follows. The case of $\alpha > 0$ is intuitive. When the community leader is a seller role, a larger sensitivity coefficient of trust value, α, means a less negative influence on consumer utility. This leads to the community leader with a smaller intention to improve the effort level. Thus, the difference between both players' effort levels shrinks.

3.2. The differentiated pricing model (Scenario Y)

Under this scenario, due to the impact of the epidemic, the primary target is to meet people's daily living materials. Therefore, the country and platform have relaxed control over the price power of community leader. In other words, the community leader can price and sell goods, either adopting the strategy of cashback selling at a lower price or being able to resell the product at a higher price to become a "new retailer". The consumer utility from purchasing goods through the community leader channel is

$$U_L = v + \alpha x - p_L + \beta e_L, \tag{7}$$

where p_L is the independent price of the goods sold by community leader, and the meanings of other characters are same as those in formula (1).

Similar with the uniform pricing scenario, consumers can purchase goods through one of two channels. Then the demand of community leader channel is

$$D_L = \begin{cases} \frac{\alpha+\beta e_L-e_P-p_L+p}{\alpha} & \alpha > 0 \\ \frac{e_P-\beta e_L+p_L-p}{\alpha} & \alpha < 0 \end{cases} \tag{8}$$

and the demand of the platform channel is:

$$D_P = \begin{cases} \frac{e_P-\beta e_L+p_L-p}{\alpha} & \alpha > 0 \\ \frac{\alpha+\beta e_L-e_P-p_L+p}{\alpha} & \alpha < 0 \end{cases}. \tag{9}$$

The community leader's profit is

$$\pi_L = rpD_L - (p - p_L)D_L - \frac{k}{2}{e_L}^2, \tag{10}$$

including three parts: the sales share of goods (the first part), the promotion expenses or the extra profit (the second part), and the effort cost (the third part). It should be noted that the price in the first part is determined by the platform and the community leader can only decide on whether enhance or reduce the actually selling price p_L.

Decision-making sequence: (1) Firstly, the platform decides the effort level e_P and the commodity price p, (2) Then, the community leader makes decision on the effort level e_L and the actual selling price p_L.

Similarly, when $\alpha > 0$, we can obtain: ${p_L}^* = \frac{k^2(4-4r+3r^2)\alpha^2+(1-r)\beta^4-k\alpha(r+4\beta^2-4r\beta^2+r^2\beta^2)}{kr^2(4k\alpha-1-2\beta^2)}$, $p^* = \frac{(2k\alpha-\beta^2)(k(2-r)\alpha-\beta^2)}{kr^2(4k\alpha-1-2\beta^2)}$, ${e_L}^* = \frac{k(2+r)\alpha\beta-\beta-\beta^3}{kr(4k\alpha-1-2\beta^2)}$, ${e_P}^* = \frac{k(2-r)\alpha-\beta^2}{kr(4k\alpha-1-2\beta^2)}$, ${\pi_L}^* = \frac{(2k\alpha-\beta^2)(1-k(2+r)\alpha+\beta^2)^2}{2kr^2(4k\alpha-1-2\beta^2)^2}$, ${\pi_P}^* = \frac{(k(2-r)\alpha-\beta^2)^2}{2kr^2(4k\alpha-1-2\beta^2)}$.

When $\alpha < 0$, the equilibrium solutions are: ${p_L}^* = \frac{2k^2(2-r)\alpha^2+(1-r)\beta^4+k\alpha(4\beta^2-r(3\beta^2-1))}{-kr^2(1+4k\alpha+2\beta^2)}$, $p^* = \frac{(2k\alpha+\beta^2)^2}{-kr^2(1+4k\alpha+2\beta^2)}$, ${e_L}^* = \frac{\beta(1+2k\alpha+\beta^2)}{kr(1+4k\alpha+2\beta^2)}$, ${e_P}^* = \frac{2k\alpha+\beta^2}{kr(1+4k\alpha+2\beta^2)}$, ${\pi_L}^* = \frac{-(2k\alpha+\beta^2)(1+2k\alpha+\beta^2)^2}{2kr^2(1+4k\alpha+2\beta^2)^2}$, ${\pi_P}^* = \frac{(2k\alpha+\beta^2)^2}{-2kr^2(1+4k\alpha+2\beta^2)}$.

Lemma 3.2. (1) *There exists α_1, when the community leader is a friend role and the commission rate $r < \frac{2}{1+2\beta^2}$, $\frac{\partial {e_L}^*}{\partial\alpha} > 0$, $\frac{\partial {e_P}^*}{\partial\alpha} < 0$, $\frac{\partial({e_L}^*-{e_P}^*)}{\partial\alpha} > 0$, $\frac{\partial p^*}{\partial\alpha} < 0$ if $\alpha < \alpha_1$, $\frac{\partial p^*}{\partial\alpha} > 0$ if $\alpha > \alpha_1$; There exists r_1 and α_2, when the community leader is a friend role and the commission rate $r < r_1$, $\frac{\partial {p_L}^*}{\partial\alpha} < 0$ if $\alpha < \alpha_2$, $\frac{\partial {p_L}^*}{\partial\alpha} > 0$ if $\alpha > \alpha_2$, when the commission rate $r_1 < r < \frac{2}{1+2\beta^2}$, $\frac{\partial {p_L}^*}{\partial\alpha} > 0$; When the community leader is a friend role and the commission rate $r > \frac{2}{1+2\beta^2}$, $\frac{\partial p^*}{\partial\alpha} > 0$, $\frac{\partial {e_L}^*}{\partial\alpha} < 0$, $\frac{\partial {e_P}^*}{\partial\alpha} > 0$, $\frac{\partial({e_L}^*-{e_P}^*)}{\partial\alpha} < 0$, $\frac{\partial {p_L}^*}{\partial\alpha} < 0$ if $\alpha < \alpha_2$, $\frac{\partial {p_L}^*}{\partial\alpha} > 0$ if $\alpha > \alpha_2$.* (2) *When the community leader is a seller role, $\frac{\partial p^*}{\partial\alpha} < 0$, $\frac{\partial {p_L}^*}{\partial\alpha} < 0$, $\frac{\partial {e_P}^*}{\partial\alpha} >$*

$0, \frac{\partial e_L{}^*}{\partial \alpha} < 0, \frac{\partial (e_L{}^* - e_P{}^*)}{\partial \alpha} < 0$; *where* $r_1 = \frac{5+4\beta^2}{2\beta^2} - \frac{1}{2}\sqrt{\frac{25+32\beta^2+16\beta^4}{\beta^4}}, \alpha_1 = \frac{1+2\beta^2}{4k} + \frac{\sqrt{k^2(2-r)(2-r-2r\beta^2)}}{4k^2(2-r)}$, *and* $\alpha_2 = \frac{1+2\beta^2}{4k} + \frac{\sqrt{k^2(4-4r+3r^2)(4-8r(1+\beta^2)+r^2(3+8\beta^2+4\beta^4))}}{4k^2(4-4r+3r^2)}$.

Lemma 3.2 shows that when the community leader is a friend role, the commission rate will affect the changing trend of the optimal decision variable, while such impacts do not appear if the community leader is a seller role. Specifically, when the community leader is a friend role and the commission rate is small, the platform's price p^* firstly decreases and then increases with the sensitivity coefficient of trust value. However, a larger r, *i.e.*, $r > \frac{2}{1+2\beta^2}$, makes p^* always be positively correlated with α. On the other hand, regardless of the commission rate r, the platform's price is negatively influenced by α when the community leader acts as the seller role. As for the community leader's price $p_L{}^*$, only when the commission rate r is moderate does it always increase with the sensitivity coefficient of trust value α if he plays the friend role. When r is large enough or relatively low, $p_L{}^*$ firstly decreases and then increases with α. If the community leader is a seller role, his optimal price continuously decreases with α.

When the community leader is a friend role and the commission rate is small, with the increase of the sensitivity coefficient of trust value, the community leader will work harder, *i.e.*, $\frac{\partial e_L{}^*}{\partial \alpha} > 0$, and the platform may reduce the effort level, *i.e.*, $\frac{\partial e_P{}^*}{\partial \alpha} < 0$. However, when the commission rate is larger than threshold $\frac{2}{1+2\beta^2}$, it will make the community leader more degenerate, then the platform can only work harder. Additionally, we find that the threshold $\frac{2}{1+2\beta^2}$ decreases with the sensitivity coefficient of effort level, β. It indicates that when the customer acquisition efficiency is higher, the increase in the sensitivity coefficient makes the community leader be more inclined to reduce his effort level. When the community leader is a seller role, the one with lower trust puts in more effort than the one with more considerable trust; however, the platform works harder with the increase of consumer trust.

Proposition 3.3. (1) *When the community leader is a friend role, and the commission rate* $r < r_2$, *he maintains the current role, i.e.,* $p_L{}^* < p^*$; *there exists* α_3 *and* α_4, *when the commission rate* $r_2 < r < \frac{2}{1+2\beta^2}$, *he changes into a seller role if* $\alpha_3 < \alpha < \alpha_4$, *i.e.,* $p_L{}^* > p^*$; *otherwise, he maintains the current role if* $\alpha > \alpha_4$ *or* $\alpha < \alpha_3$; *when the commission rate* $r > \frac{2}{1+2\beta^2}$, *he changes into a seller role if* $\alpha < \alpha_4$; *otherwise, he maintains the current role if* $\alpha > \alpha_4$; (2) *When the community leader is a seller role, he will always change into a friend role, i.e.,* $p_L{}^* < p^*$; *Where* $\alpha_3 = \frac{(3-r)\beta^2-1-\sqrt{1-2(3-r)\beta^2+(1+6r+r^2)\beta^4}}{2k(2-3r)}, \alpha_4 = \frac{(3-r)\beta^2-1+\sqrt{1-2(3-r)\beta^2+(1+6r+r^2)\beta^4}}{2k(2-3r)}$ *and* $r_2 = \frac{2\sqrt{\beta^6(3+2\beta^2)}}{\beta^4} - 3 - \frac{1}{\beta^2}$.

Proposition 3.3 characterizes the situation under which the role transformation of community leader happens. When the community leader is a seller role, the optimal price $p_L{}^*$ will not be higher than the platform's price p^*, implying the change of role always exists. At the beginning of the leader-member relationship establishment, the community leader will reduce selling price to gain consumer's trust, changing himself into a friend role. It is consistent with the literature Cai *et al.* [2], pointing out that the community leader starts by playing the role of "seller," and trust gradually accumulates and develops with the continuation of the transaction relationship. In practice, we can also find that merchants may play the role of "friends" and maintain good personal relationships with consumers.

Additionally, when the community leader is a friend role, as the commission rate changes, three different situations appear depending on the sensitivity coefficient of trust value α. Firstly, if the commission rate is small, regardless of α, the community leader will not set the selling price higher than the platform's price, *i.e.*, $p_L{}^* < p^*$. Therefore, the platform can reduce the possibility of high-priced sales by controlling the commission rate. Secondly, in the case of a moderate commission rate, only when the community residents trust the community leader very much or the trust value is small, the community leader will maintain the friend role by setting a lower price. However, the community leader with moderate trust value, *i.e.*, $\alpha_3 < \alpha < \alpha_4$, will sell goods at a higher price, which will deplete the interests of friends to obtain more profits. Thirdly, when the commission rate is large, the community leader with higher trust value will keep the price low and maintain a friend role, while the leader with less trust value will raise the price, changing into a seller role.

Lemma 3.4. (1) *There exists* α_5*, when the community leader is a friend role and the commission rate* $r < \frac{2}{1+2\beta^2}$*,* ${e_L}^* < {e_P}^*$ *if* $\alpha < \alpha_5$*,* ${e_L}^* > {e_P}^*$ *if* $\alpha > \alpha_5$*; when the commission rate* $r > \frac{2}{1+2\beta^2}$*,* ${e_L}^* > {e_P}^*$*.* (2) *There exists* α_6*, when the leader is a seller role,* ${e_L}^* > {e_P}^*$ *if* $\alpha < \alpha_6$*,* ${e_L}^* < {e_P}^*$ *if* $\alpha > \alpha_6$*; Where* $\alpha_5 = \frac{\beta\left(\beta^2-\beta+1\right)}{k(2\beta-2+r+r\beta)}, \alpha_6 = \frac{\beta\left(\beta^2-\beta+1\right)}{2k(\beta-1)}$.

What conveys in Lemma 3.4 is how the relationship between both players' effort levels is affected by the commission rate, r, and the sensitivity coefficient of trust value, α. Specifically, when the community leader is a seller role, only parameter α affects. That is, a smaller (larger) α makes the effort level of community leader higher (lower) than that of the platform, *i.e.*, ${e_L}^* > {e_P}^*$ (${e_L}^* < {e_P}^*$). It indicates that the community leader with a lower trust will work harder. However, the one who can obtain enough trust from consumers will reduce effort level below that of the platform.

When the community leader is a friend role, whether the effort level of community leader outweighs that of the platform hinges on both parameters r and α. If the commission rate is small, the community leader with a lower (higher) trust value pays less (more) effort than the platform. However, when the commission rate is significant, the effort level of the community leader is greater than that of the platform, whatever the sensitivity coefficient of trust value is. In other words, with the commission rate increasing, the community leader with a lower trust value will put in more effort, just like the one with a higher trust value. Therefore, the platform can improve the commission rate to induce the community leader with a lower trust value to make more efforts. Unfortunately, along with such stimulation is a higher price decided by the community leader (as shown in the first part of Prop. 3.3), and then the role transformation happens.

4. Comparative analysis

This section compares the platform's and community leader's optimal decision on price, effort level, and performance under different pricing strategies, respectively. We utilize the subscript "Y" to represent the differentiated pricing model and the subscript "N" to denote the uniform pricing model. Besides, the superscript "*" refers to the optimal decision.

Proposition 4.1. (1) *When the community leader is a friend role,* (i) *there exists* r_3*, when the commission rate* $r < r_3$*, the optimal price of platform in Scenario Y is higher than that in Scenario N if* $\alpha < \frac{\beta^2}{k}$*, i.e.,* ${p_Y}^* > {p_N}^*$*, otherwise,* ${p_Y}^* < {p_N}^*$ *if* $\alpha > \frac{\beta^2}{k}$*;* (ii) *there exists* α_7 *and* α_8*, when the commission rate* $r > \frac{2}{1+2\beta^2}$ *or* $\frac{1}{7} < r < \frac{2}{9}$ *and* $\sqrt{\frac{5+r}{1+2r}} < \beta < 2$ *or* $\frac{2}{9} < r < \frac{1}{4}$ *and* $\sqrt{\frac{5+r}{1+2r}} < \beta < \sqrt{\frac{2-r}{2r}}$*,* ${p_Y}^* < {p_N}^*$ *if* $\alpha < \alpha_8$ *or* $\alpha > \frac{\beta^2}{k}$*, otherwise,* ${p_Y}^* > {p_N}^*$ *if* $\alpha_8 < \alpha < \frac{\beta^2}{k}$*;* (iii) *when* $r_3 < r < \min\left\{\frac{2}{1+2\beta^2}, \frac{5-\beta^2}{-1+2\beta^2}\right\}$*,* ${p_Y}^* > {p_N}^*$ *if* $\alpha < \alpha_7$ *or* $\alpha_8 < \alpha < \frac{\beta^2}{k}$*,* ${p_Y}^* < {p_N}^*$ *if* $\alpha_7 < \alpha < \alpha_8$ *or* $\alpha > \frac{\beta^2}{k}$*.* (2) *When the community leader is seller role,* ${p_Y}^* < {p_N}^*$ *if* $\alpha < \frac{-\beta^2}{k}$*,* ${p_Y}^* > {p_N}^*$ *if* $\alpha > \frac{-\beta^2}{k}$*; Where* $r_3 = \frac{4\sqrt{2}\beta-2\beta^2-3}{2\beta^2-1}, \alpha_7 = \frac{6\beta^2-r\left(2\beta^2-1\right)-3-\sqrt{2\beta^2-1}\sqrt{2\beta^2+r^2(2\beta^2-1)+r(6+4\beta^2)-9}}{8k(1-r)}$*, and* $\alpha_8 = \frac{6\beta^2-r\left(2\beta^2-1\right)-3+\sqrt{2\beta^2-1}\sqrt{2\beta^2+r^2(2\beta^2-1)+r(6+4\beta^2)-9}}{8k(1-r)}$.

From Proposition 4.1, it can be analyzed that when the community leader is a friend role, and the commission rate is low, if the sensitivity coefficient of trust value α is small, the platform's price under the differentiated pricing model will be higher than that under the uniform pricing model. However, when α becomes larger, the platform will lower the price under scenario Y. When the community leader is a seller role, the opposite relationship happens.

Additionally, when the commission rate is small, the platform's decision on price under scenario Y varies with the role of community leader. Specifically, for the friend-role community leader who is relatively less influential, the platform will raise the price to make the community leader more competitive. If the community leader is relatively more influential, the platform will lower the price to form more fierce competition. However, for the seller role community leader who is relatively less influential, the platform will set a relatively lower price

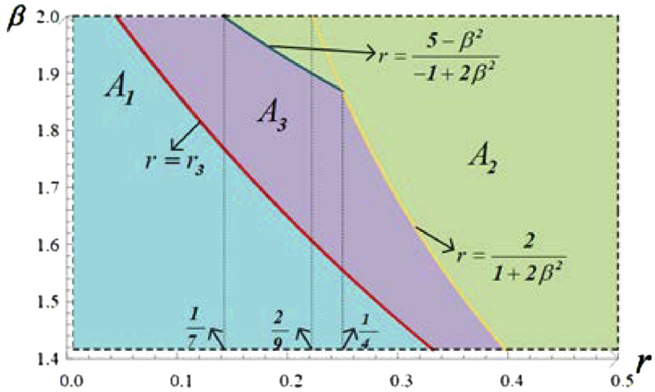

FIGURE 2. The impacts of β and r on the difference between $p_Y{}^*$ and $p_N{}^*$.

so that the community leader has more difficulty in surviving in the market; When the community leader is more influential, the platform will set a relatively high price to make the community leader easily survive in the market. Through the above different decisions on price, the platform can screen out more high-quality community leaders and cultivate these community leaders with middle trust value.

Figure 2 shows the influence of the commission rate r and the community leader's effort level sensitivity coefficient β on the relationship between the optimal price of platform under the two models. The main situations that will occur are divided into three cases, namely blue, green, and purple area. The region A_1 drawn blue is that the community leader plays a friend role and the commission rate is small (case i). The region A_2 drawn green characterizes the parameter range of case (ii). According to Proposition 4.1, we can find that with the increase of sensitivity coefficient of trust value α, the relationship between the optimal price of the platform in the two modes has changed three times $\left(\alpha < \alpha_8 \to \alpha_8 < \alpha < \frac{\beta^2}{k} \to \alpha > \frac{\beta^2}{k}\right)$. When α is small or large, the optimal price of the platform under scenario Y will be smaller than that under scenario N, and only when it falls into a moderate interval will the platform prices be higher under scenario Y. Similarly, we plot case (iii) as the region A_3 drawn purple. In this case, as the sensitivity coefficient of trust value α increases, the relationship between $p_Y{}^*$ and $p_N{}^*$ changes four times $\left(\alpha < \alpha_7 \to \alpha_7 < \alpha < \alpha_8 \to \alpha_8 < \alpha < \frac{\beta^2}{k} \to \alpha > \frac{\beta^2}{k}\right)$.

Proposition 4.2. (1) *When the community leader is a friend role and the commission rate* $r < \frac{2}{1+2\beta^2}$*, the optimal effort of platform in Scenario Y is higher than that in Scenario N if* $\alpha < \frac{\beta^2}{k}$*, i.e.,* $e_{PY}{}^* > e_{PN}{}^*$*, vice versa; When the commission rate* $r > \frac{2}{1+2\beta^2}$*, the optimal effort of platform in Scenario Y is lower than that in Scenario N if* $\alpha < \frac{2\beta^2-1}{4k(1-r)}$ *or* $\alpha > \frac{\beta^2}{k}$*, otherwise, the optimal effort of platform in Scenario Y is higher than that in Scenario N.* (2) *When the community leader is a seller role, the optimal effort of platform in Scenario Y is lower than that in Scenario N if* $\alpha < -\frac{\beta^2}{k}$*, vice versa.*

Proposition 4.2 indicates not only the commission rate r and the sensitivity coefficient of trust value α, but also the role played by community leader jointly affect the effort level of platform under different pricing models. Firstly, we discuss the situation of the community leader being a friend role. If the commission rate is small, a larger α, *i.e.*, $\alpha > \frac{\beta^2}{k}$, makes the platform put in more effort under the uniform pricing case than that under the differentiated pricing case. However, more effort could be paid under scenario Y if parameter α below the threshold $\frac{\beta^2}{k}$. If the commission rate is large, platform's optimal effort level will further change. Only when the sensitivity coefficient of trust value is moderate, it is higher in the differentiated pricing model than that in the uniform pricing model. Moreover, regardless of the commission rate, when the community leader is more influential, the platform will reduce the effort level and obtain more profits through the channel operated by community leader who has the advantage of acquiring consumer's trust.

Secondly, when the community leader is the seller role, platform's optimal effort level under different pricing situation will not be affected by the commission rate. In this case, the change of the platform's optimal effort level is just opposite to that when the community leader is a friend role and the commission rate is small. In other words, when the trust value of the community leader is large, the platform's effort level under uniform pricing model will be lower than that under the differentiated pricing model. For the seller role community leader whose trust value is small, it is not a favorable choice for the platform to improve the effort level. The platform prefers to make more efforts when the community leader can gain enough consumer trust.

Proposition 4.3. (1) *When the community leader is a friend role and the commission rate $r < r_4$, the community leader's effort level in Scenario Y is lower than that in Scenario N, i.e., ${e_{LY}}^* < {e_{LN}}^*$; there exists α_9 and α_{10}, when $r_4 < r < \frac{2}{1+2\beta^2}$, the community leader's effort level in Scenario Y is higher than that in Scenario N if $\alpha_9 < \alpha < \alpha_{10}$, otherwise, the community leader's effort level in Scenario Y is lower than that in Scenario N; when $r > \frac{2}{1+2\beta^2}$, the community leader's effort level in Scenario Y is higher than that in Scenario N if $\alpha < \alpha_{10}$, vice versa.* (2) *When the community leader is a seller role, the community leader's effort level in Scenario Y is always lower than that in Scenario N; Where $r_4 = \frac{3}{2} + \beta^2 - 4\beta^4 + \sqrt{2(1+\beta^2)(2\beta^2-1)^3}$, $\alpha_9 = \frac{6\beta^2-1-2r-\sqrt{17+4r^2-28\beta^2+4\beta^4+4r(8\beta^4-2\beta^2-3)}}{8k(1-r)}$, and $\alpha_{10} = \frac{6\beta^2-1-2r+\sqrt{17+4r^2-28\beta^2+4\beta^4+4r(8\beta^4-2\beta^2-3)}}{8k(1-r)}$.*

From Proposition 4.3, we can find that the effort level of community leader under the differentiated pricing case is lower than that under the uniform pricing model in most cases. Only when the commission rate is large and the sensitivity coefficient of trust value is moderate, the community leader will work harder under scenario Y. The reasons can be analyzed as follows. Under the uniform pricing case, the community leader has no power to price, resulting in paying more effort to gain revenue, especially when the commission rate is low. However, when it comes to the case of differentiated pricing, such a passive situation will be improved. Because the community leader has two ways to increase profit, flexibly balancing the relationship between selling price and effort level.

It indicates that when the community leader is a friend role, the platform can guide the community leader's effort level by controlling the commission rate and implementing different pricing strategies. Specifically, the platform can reduce commission rate so that the effort level of community leader is higher in the case of uniform pricing. Additionally, increasing the commission rate to stimulate the community leader work harder under the differentiated pricing model is also applicable.

Proposition 4.4. (1) *There exists r_5, when the community leader is a friend role and the commission rate $r < r_5$, the platform prefers the differentiated pricing strategy if $\alpha < \frac{\beta^2}{k}$, i.e., ${\pi_{PY}}^* > {\pi_{PN}}^*$, vice versa; There exists α_{11} and α_{12}, when the commission rate $r_5 \le r < \frac{5-\beta^2}{1+5\beta^2}$, the platform prefers the differentiated pricing strategy if $\alpha < \alpha_{11}$ or $\alpha_{12} < \alpha < \frac{\beta^2}{k}$, the platform prefers the uniform pricing strategy if $\alpha_{11} < \alpha < \alpha_{12}$ or $\alpha > \frac{\beta^2}{k}$; when the commission rate $r > \frac{5-\beta^2}{1+5\beta^2}$, the platform prefers the differentiated pricing strategy if $\alpha_{12} < \alpha < \frac{\beta^2}{k}$, otherwise, the platform prefers the uniform pricing strategy.* (2) *When the community leader is a seller role, the platform prefers the uniform pricing strategy if $\alpha < \frac{-\beta^2}{k}$, vice versa; where $\alpha_{11} = \frac{k(3-2r)\left(2\beta^2-1\right)-\sqrt{k^2(2\beta^2-1)(2\beta^2-9+4r(3+2\beta^2)-r^2(4+8\beta^2))}}{8k^2(1-r)^2}$, $\alpha_{12} = \frac{k(3-2r)\left(2\beta^2-1\right)+\sqrt{k^2(2\beta^2-1)(2\beta^2-9+4r(3+2\beta^2)-r^2(4+8\beta^2))}}{8k^2(1-r)^2}$, and $r_5 = \frac{3+2\beta^2-2\sqrt{\beta^2(2\beta^2-1)}}{2+4\beta^2}$.*

Proposition 4.4 has summarized how the platform adopts the optimal pricing strategy according to the community leader's characteristic, a friend role or a seller role, and consumer's trust level. Specifically, when the community leader is a friend role, various commission rate r may cause three kinds of results. The first one is a small r. If the sensitivity coefficient of trust value is also small, the optimal profit obtained by the platform under the differentiated pricing case will be higher than that under the uniform pricing case; otherwise, the platform is more profitable under a uniform pricing strategy. The second one is a moderate r, under which a not too small or a large enough sensitivity coefficient of trust value, *i.e.*, $\alpha_{11} < \alpha < \alpha_{12}$ or $\alpha > \frac{\beta^2}{k}$, can make the platform perform better under scenario N. When it comes to the third case, a larger r, if the parameter α is great as well, adopting strategy N may be advisable. Summarily, regardless of the commission rate, the platform can obtain more profits under the uniform pricing model as long as the sensitivity coefficient of trust value is large and the community leader plays a friend role. However, if consumers still greatly trust the community leader who is a seller role, *i.e.*, $\alpha > \frac{-\beta^2}{k}$, it will make the platform be worse under scenario N than Y.

5. Numerical analysis

In this section, we employ specific numerical examples to further explore players' performance under different pricing strategies and vividly verify the primary conclusions derived above as well. To better analyze the impacts of the sensitivity coefficient of trust value α, without loss of generality, we set $k = 1$ and $\beta = 1.5$.

5.1. Comparison of the optimal effort level

As can be seen from Figure 3, when the community leader is a friend role, under the uniform pricing model, the effort level of community leader will always be greater than that of the platform. Under the differentiated pricing model, when the commission rate is large, similar to the uniform pricing model, the effort level of the community leader is larger than that of the platform. However, when the commission rate is small, the effort level of the community leader may be lower than that of the platform if the sensitivity coefficient of trust value α is not large enough. Under the uniform pricing model, the community leader can only enhance benefits through paying more effort, leading to the one with higher trust value work harder, as shown in Figure 3a. In contrast, under the differentiated pricing model, the community leader has another way, manipulating price, besides controlling effort level, to improve revenue. This contributes to less effort paid by the community leader with higher trust value. On the other hand, a smaller commission rate further discourages the community leader's enthusiasm, resulting in a more dramatic decline in the effort level, as shown in Figure 3b1.

Figure 4 characterizes the situation under which the community leader is a seller role. Figure 4a shows that the community leader's effort level is greater than the platform's when implementing a uniform pricing strategy, which is similar to that in Figure 3a. However, under the differentiated pricing case, in contrast to players' behavior when the community leader is a friend role (see in Fig. 3b2), the optimal effort level of community leader dose not consistently surpass than that of the platform, as shown in Figure 4b. Under the uniform pricing scenario, the community leader acting as a seller role always works harder than the platform. In contrast, when it comes to the differentiated pricing model, if the sensitivity coefficient of trust value for a seller role community leader is large, the platform will pay more effort (see in Fig. 4b), which is similar to the feature when the commission rate is small and the community leader is a friend role, as shown in Figure 3b1.

Figures 5a–5b shows how the community leader's optimal effort level under different pricing strategies varies with the commission rate. Specifically, when the community leader is a friend role and the commission rate is small, his optimal effort level under the differentiated pricing model remains lower than that under the uniform pricing model. As the commission rate increases, the optimal effort level curve of the community leader under different pricing strategies exhibits two intersection points firstly and then into one point. That is, when the commission rate is moderate, *i.e.*, $r = 0.3$, compared with the uniform pricing case, the effort level of community leader with moderate trust value under the differentiated pricing model will be higher. However, if the commission rate is large, *i.e.*, $r = 0.4$, it just requires a lower trust value that can stimulate the community leader to exert more effort under the differentiated pricing model, as shown in the left part in Figure 5a3.

Additionally, what interesting in Figure 5a3 is when the commission received by the community leader is larger, the higher the trust value, the lower the effort level. Under the differentiated pricing model, increasing the commission rate does not stimulate the friend-role community leader to exert more effort. Therefore, when

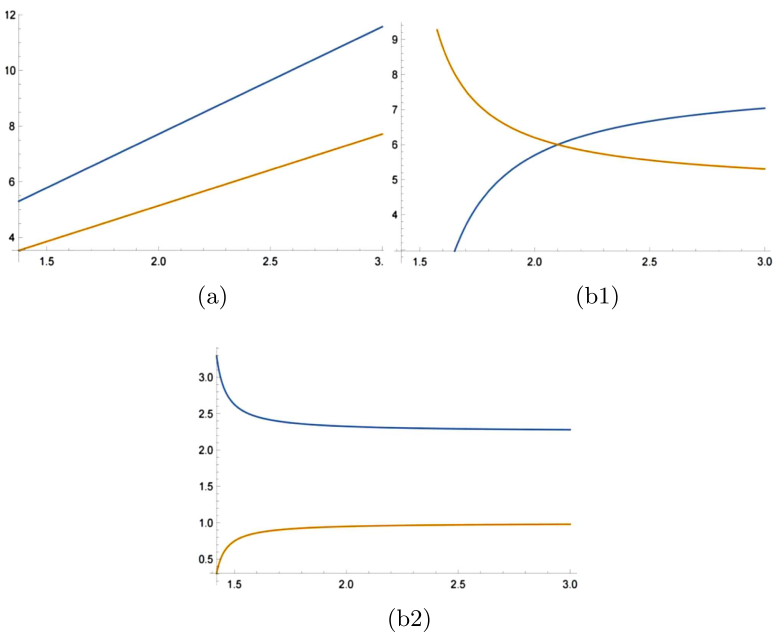

FIGURE 3. The optimal effort level (friend role). (a) Uniform pricing strategy. (b1) Differentiated pricing strategy $\left(r < \frac{2}{1+2\beta^2}\right)$. (b2) Differentiated pricing model $\left(r > \frac{2}{1+2\beta^2}\right)$.

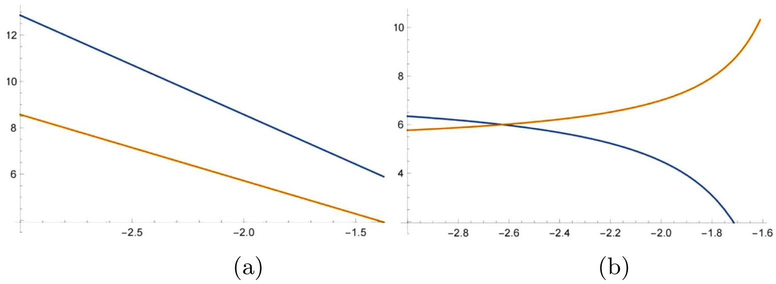

FIGURE 4. The optimal effort level (seller role). (a) Uniform pricing model. (b) Differentiated pricing model.

platform decides on the commission rate, it is necessary to consider whether it can promote the community leader.

Figure 5b shows the changes in the optimal effort level of community leader under the two models when the community leader is a seller role. The effort level under the differentiated pricing model is always lower than that under the uniform pricing model, which is the same as shown in Figure 5a1. However, the difference is that the impact of the sensitivity coefficient of trust value α on effort level is opposite if the role player by community leader changes. Specifically, the optimal effort level increases with α if the community leader acts as a friend role, while a seller role makes the effort level negatively influenced by α.

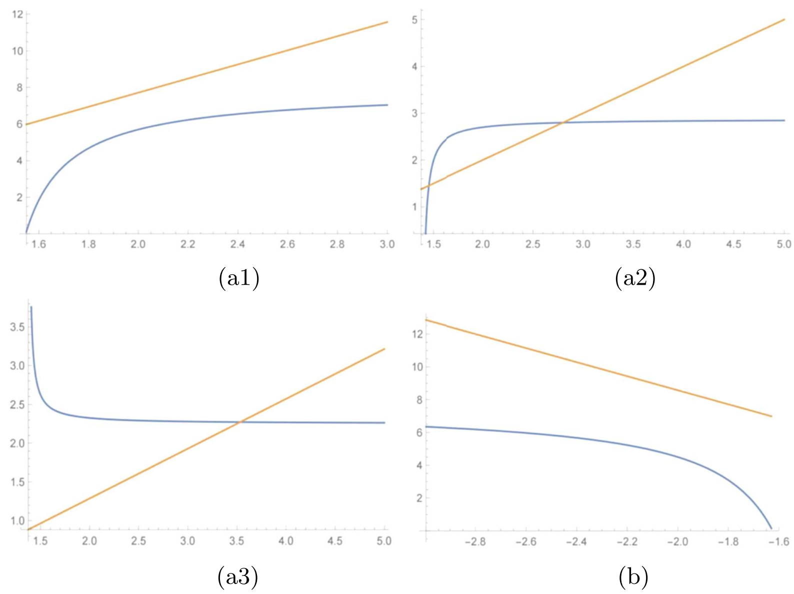

FIGURE 5. The optimal effort level of community leader. (a1) Friend role ($r = 0.1$). (a2) Friend role ($r = 0.3$). (a3) Friend role ($r = 0.4$). (b) Seller role ($r = 0.1$).

5.2. Comparison of the optimal price

We have found that players' decisions on price under different scenarios are influenced by commission rate in Propositions 3.3 and 4.1. In this section, we further plot these impacts through setting various values of r.

As can be seen from Figure 6, firstly, when the community leader is a friend role and the sensitivity coefficient of trust value α is small, if the commission rate is small, *i.e.*, Figure 6a1, compared with ${p_N}^*$ and ${p_L}^*$, the optimal price of the platform under the differentiated pricing model ${p_Y}^*$ is the highest, while it is the lowest when the commission rate is large (the left part in Fig. 6a4). Because when the commission rate increases, the community leader with low trust value will exert greater effort, which contributes to a higher price determined by community leader. Secondly, when the community leader is a friend role and α is high, regardless of the value of r, the optimal price of the platform is the highest under the uniform pricing model (the upper right part in Figs. 6a1–6a4). The platform relies on the community leader with a higher trust value to increase prices and earn more profits. This is why the platform hopes to recruit community leaders with high trust value as much as possible. Thirdly, however, when the community leader is a seller role, the opposite situation occurs. When the trust value is large, the platform's price under the uniform pricing model is the lowest (the lower right part in Fig. 6b).

Under the differentiated pricing model, in most cases, for example, a friend-role community leader and a small commission rate r (Fig. 6a1), and a seller role community leader (Fig. 6b), the price of the community leader is lower than that of the platform. Such a low-price strategy can attract more consumers to purchase from the community leader, then generating more profits. However, when the community leader is a friend role and the commission rate is large, the price set by a community leader with moderate trust value may surpass that determined by the platform (Figs. 6a3 and 6a4). This observation suggests that the commission rate is a crucial factor in stimulating the role transformation of community leader.

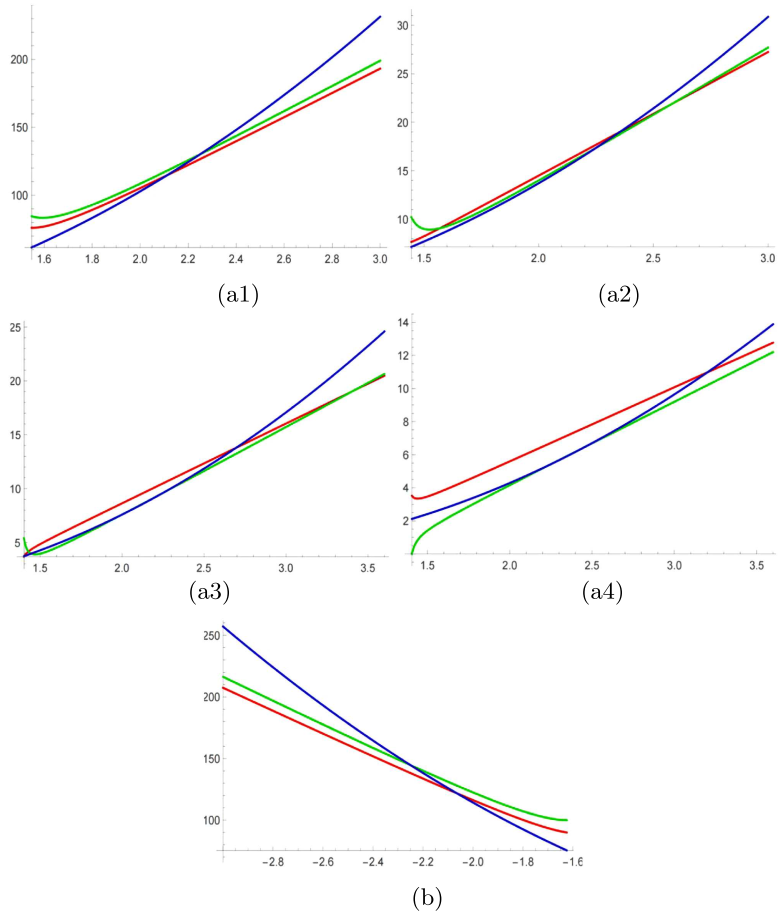

FIGURE 6. The optimal price of community leader and platform. (a1) Friend role ($r = 0.1$). (a2) Friend role ($r = 0.25$). (a3) Friend role ($r = 0.32$). (a4) Friend role ($r = 0.4$). (b) Seller role ($r = 0.1$).

5.3. Comparison of the optimal profit

Figure 7 describes the relationship between the platform's optimal profit and the sensitivity coefficient of trust value α under various pricing strategies. We can find that when the community leader is a friend role and the commission rate is small, it is better for the platform to adopt the differentiated pricing strategy if parameter α is small (Fig. 7a1). However, as r increases, strategy Y is not a favorable choice for the platform when the sensitivity coefficient of trust value is either too small or too large (Fig. 7a2). Therefore, as for a friend-role community leader with lower trust value, the platform should reduce (increase) the commission rate r under the differentiated (uniform) pricing strategy. It is profitable to restrict the pricing power of community leader, *i.e.*, scenario N, when the sensitivity coefficient of trust value is large. When the community leader is a seller role, it is better for the platform to adopt the uniform pricing strategy if the parameter α is small; otherwise, implementing the differentiated pricing strategy is preferred (Fig. 7b).

Furthermore, we plot how the community leader's profit varies with the sensitivity coefficient of trust value α in Figure 8. Surprisingly, though the community leader can decide on price under the differentiated pricing scenario, his profit is not consistently greater than the uniform pricing case, especially for the one with a high

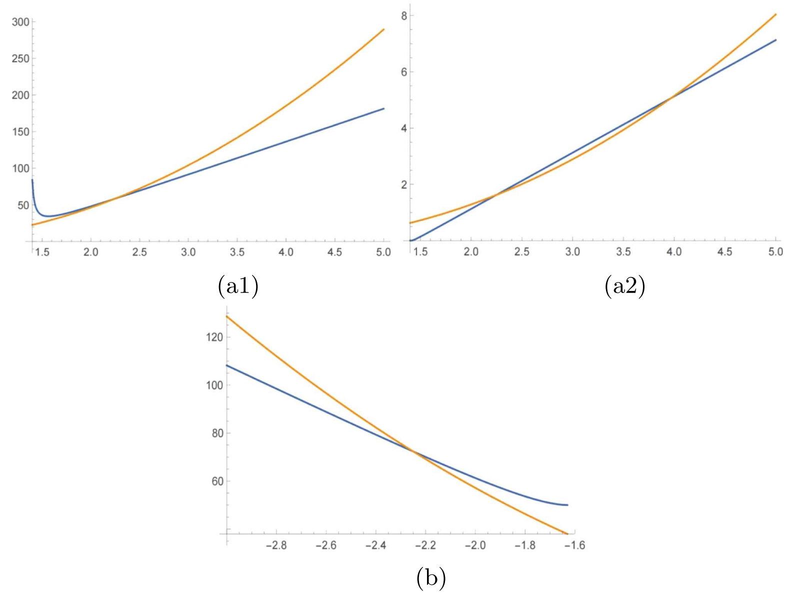

FIGURE 7. The optimal profit of the platform under different pricing scenarios. (a1) Friend role ($r = 0.1$). (a2) Friend role ($r = 0.4$). (b) Seller role ($r = 0.1$).

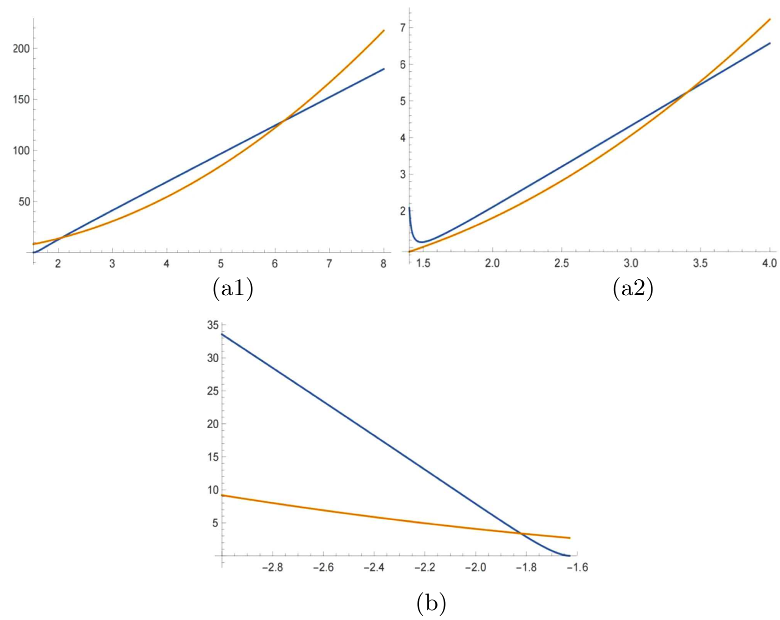

FIGURE 8. The optimal profit of community leader under different pricing scenarios. (a1) Friend role ($r = 0.1$). (a2) Friend role ($r = 0.4$). (b) Seller role ($r = 0.1$).

trust value. Moreover, the uniform pricing strategy seems disadvantageous to community leaders because of the lost in pricing power. It is also invalid for the friend-role community leader with low trust value if the commission rate is small (the lower left part in Fig. 8a1).

Combined with Figures 7a1, 7a2 and 8a1, 8a2, we can conclude that when the community leader is a friend role and the parameter α is large, regardless of the commission rate, the optimal profit of the platform and community leader under the uniform pricing strategy is higher than that under the differentiated pricing strategy. It indicates that both players can achieve a win–win situation under scenario N. However, such a balance is broken by the social role transformation from a friend to seller role. In other words, when the community leader is a seller role, a sizeable α makes the platform benefit more from scenario Y whereas the community leader still prefers scenario N.

6. Conclusions

Recently, community group buying has flourished, particularly during the outbreak of the COVID-19 epidemic. Aiming at the different pricing strategies before and during COVID-19, this paper explores the optimal effort level of the CGB platform and community leader who may play either the friend or seller role. Based on this, we further discuss the impacts of pricing strategy, the trust value of community leader, and the commission rate. The platform can implement a uniform pricing strategy under which the community leader has no pricing power or a differentiated pricing strategy under which players price independently. Besides, the community leader may act as a friend or seller role depending on setting a lower or higher price than platform. We summarize the following primary conclusions and management insights.

Firstly, the commission rate is an important factor that stimulates the role transformation of the friend-role community leader. When the commission rate is larger, the friend-role community leader becomes more likely to change his role, raising product price. Thus, the platform can reduce the commission rate to suppress the role-change of a friend role community leader. However, the seller role community leader will always change himself into a friend role, whatever the commission rate is. It is consistent with reality. During the epidemic, the community leader whom consumers trust sets a lower price than the platform's.

Secondly, the effort level of the community leader is higher under the uniform pricing model than under the differentiated pricing model in most cases. When the commission rate is small, the friend role community leader with higher trust value will work harder. Otherwise, he can gain considerable revenue from selling commissions by exerting less efforts. Therefore, the platform can strategically set a commission rate to motivate the community leader. For the friend role community leader with lower trust value, increasing the commission rate is advisable. However, the platform should limit the commission rate for the one that consumers trust extremely.

Lastly, our work shows that a win–win situation can be achieved. Under the uniform pricing scenario, regardless of the size of commission rate, when the community leader is a friend role and the trust value is high, both the platform and community leader gain higher profits.

This paper also has some limitations. First, we set the commission rate to be exogenous, which is motivated by the phenomenon that some prominent platforms, such as Amazon.com and JD.com, have given it for specific categories. Second, due to the complexity of calculation, this paper only considers the impacts of trust value and pricing strategy. Considering more factors will obtain other meaningful results. Besides, we assume that consumers can buy from either one community leader or CGB platform. However, in practice, multiple community leaders can provide service for consumers.

Based on the above discussion, some future research directions can be proposed. Though the setting of the commission rate is reasonable, it deserves to explore the impacts that a platform with great power can endogenously determine the commission rate. Moreover, incorporating other factors, for example, the reference effect that influences consumers' trust value for community leaders, may reveal diverse findings. Last, extending the study to a competitive scenario, where consumers can purchase from multiple community leaders, to examine how competition influences the effort level and profits may be more realistic.

Appendix A.

Proof of Lemma 3.1. When $\alpha > 0$, because $0 < r < 0.5$ and $1 < \beta < 2$, $\frac{\partial p^*}{\partial \alpha} = \frac{2k(1-r)\alpha}{r^2(2\beta^2-1)} > 0$, $\frac{\partial e_P{}^*}{\partial \alpha} = \frac{(1-r)}{r(2\beta^2-1)} > 0$, $\frac{\partial e_L{}^*}{\partial \alpha} = \frac{(1-r)\beta}{r(2\beta^2-1)} > 0$. Since $e_L{}^* - e_P{}^* = \frac{(1-r)\alpha(\beta-1)}{r(2\beta^2-1)} > 0$, $e_L{}^* > e_P{}^*$. $\frac{\partial(e_L{}^*-e_P{}^*)}{\partial \alpha} = \frac{(1-r)(\beta-1)}{r(2\beta^2-1)} > 0$.

When $\alpha < 0$, $\frac{\partial p^*}{\partial \alpha} = \frac{2k\alpha}{r^2(2\beta^2-1)} < 0$, $\frac{\partial e_P{}^*}{\partial \alpha} = \frac{-1}{r(2\beta^2-1)} < 0$, $\frac{\partial e_L{}^*}{\partial \alpha} = \frac{-\beta}{r(2\beta^2-1)} < 0$, Since $e_L{}^*-e_P{}^* = \frac{-\alpha(\beta-1)}{r(2\beta^2-1)} > 0$, $e_L{}^* > e_P{}^*$. $\frac{\partial(e_L{}^*-e_P{}^*)}{\partial \alpha} = \frac{-(\beta-1)}{r(2\beta^2-1)} < 0$. □

Proof of Lemma 3.2. (1) When the leader is friend role ($\alpha > 0$), $\frac{\partial e_L{}^*}{\partial \alpha} = \frac{\beta\left(2-r-2r\beta^2\right)}{r(4k\alpha-1-2\beta^2)^2}$, if $r < \frac{2}{1+2\beta^2}$, $\frac{\partial e_L{}^*}{\partial \alpha} > 0$, if $r > \frac{2}{1+2\beta^2}$, $\frac{\partial e_L{}^*}{\partial \alpha} < 0$.

$\frac{\partial e_P{}^*}{\partial \alpha} = \frac{-\left(2-r-2r\beta^2\right)}{r(4k\alpha-1-2\beta^2)^2}$, if $r < \frac{2}{1+2\beta^2}$, $\frac{\partial e_P{}^*}{\partial \alpha} < 0$, if $r > \frac{2}{1+2\beta^2}$, $\frac{\partial e_P{}^*}{\partial \alpha} > 0$.

$\frac{\partial p^*}{\partial \alpha} = \frac{8k^2(2-r)\alpha^2-4k(2-r)\alpha\left(1+2\beta^2\right)-\beta^2\left(r+2r\beta^2-4\left(1+\beta^2\right)\right)}{r^2(4k\alpha-1-2\beta^2)^2}$, it's easy to see that the denominator is positive. Thus, the sign of $\frac{\partial p^*}{\partial \alpha}$ depends on the numerator. If $r > \frac{2}{1+2\beta^2}$, the numerator is positive. If $r < \frac{2}{1+2\beta^2}$, setting $8k^2(2-r)\alpha^2 - 4k(2-r)\alpha\left(1+2\beta^2\right) - \beta^2\left(r+2r\beta^2-4\left(1+\beta^2\right)\right) = 0$, we can obtain two roots $\alpha_1 = \frac{1+2\beta^2}{4k} + \frac{\sqrt{k^2(2-r)(2-r-2r\beta^2)}}{4k^2(2-r)}$ and $\alpha_1{}' = \frac{1+2\beta^2}{4k} - \frac{\sqrt{k^2(2-r)(2-r-2r\beta^2)}}{4k^2(2-r)}$. The root $\alpha_1{}'$ is not satisfied with the condition $\alpha > \max\left\{\frac{1+\beta^2}{k(2+r)}, \frac{\beta^2}{k(2-r)}\right\}$. Thus, $\frac{\partial p^*}{\partial \alpha} < 0$ if $\alpha < \alpha_1$, $\frac{\partial p^*}{\partial \alpha} > 0$ if $\alpha > \alpha_1$.

$\frac{\partial p_L{}^*}{\partial \alpha} = \frac{r^2\left(12k^2\alpha^2+\beta^2+2\beta^4-6k\alpha\left(+2\beta^2\right)\right)+4\left(4k^2\alpha^2+\beta^2+\beta^4-2k\alpha\left(+2\beta^2\right)\right)+r\left(1-16k^2\alpha^2-2\beta^2-4\beta^4+8k\alpha\left(+2\beta^2\right)\right)}{r^2(4k\alpha-1-2\beta^2)^2} = \frac{N_1}{N_2}$, the sign of $\frac{\partial p_L{}^*}{\partial \alpha}$ depends on the numerator N_1. From $N_1 = 0$, when $\frac{2}{3+2\beta^2} < r < \frac{2}{1+2\beta^2}$, the equation has no root. Then when $r > \frac{2}{1+2\beta^2}$ or $r < \frac{2}{3+2\beta^2}$, we can obtain two roots $\alpha_2 = \frac{1+2\beta^2}{4k} + \frac{\sqrt{k^2(4-4r+3r^2)(4-8r(1+\beta^2)+r^2(3+8\beta^2+4\beta^4))}}{4k^2(4-4r+3r^2)}$ and $\alpha_2{}' = \frac{1+2\beta^2}{4k} - \frac{\sqrt{k^2(4-4r+3r^2)(4-8r(1+\beta^2)+r^2(3+8\beta^2+4\beta^4))}}{4k^2(4-4r+3r^2)}$. The root $\alpha_2{}'$ is not satisfied with the condition $\alpha > \max\left\{\frac{1+\beta^2}{k(2+r)}, \frac{\beta^2}{k(2-r)}\right\}$. But when $r_1 = \frac{5+4\beta^2}{2\beta^2} - \frac{1}{2}\sqrt{\frac{25+32\beta^2+16\beta^4}{\beta^4}} < r < \frac{2}{3+2\beta^2}$. The root α_2 is not also satisfied with the condition $\alpha > \max\left\{\frac{1+\beta^2}{k(2+r)}, \frac{\beta^2}{k(2-r)}\right\}$. So when $r < r_1$ or $r > \frac{2}{1+2\beta^2}$, $\frac{\partial p_L{}^*}{\partial \alpha} < 0$ if $\alpha < \alpha_2$, $\frac{\partial p_L{}^*}{\partial \alpha} > 0$ if $\alpha > \alpha_2$ and when the commission ratio $r_1 < r < \frac{2}{1+2\beta^2}$, $\frac{\partial p_L{}^*}{\partial \alpha} > 0$ can be derived.

From $\frac{\partial(e_L{}^*-e_P{}^*)}{\partial \alpha} = \frac{(1+\beta)\left(2-r-2r\beta^2\right)}{r(4k\alpha-1-2\beta^2)^2}$, $\frac{\partial(e_L{}^*-e_P{}^*)}{\partial \alpha} > 0$ if $r < \frac{2}{1+2\beta^2}$ and $\frac{\partial(e_L{}^*-e_P{}^*)}{\partial \alpha} < 0$ if $r > \frac{2}{1+2\beta^2}$ can be derived.

(2) When the leader is seller role ($\alpha < 0$), $\frac{\partial p^*}{\partial \alpha} = -\frac{4\left(2k\alpha+\beta^2\right)\left(2k\alpha+1+2\beta^2\right)}{r^2(4k\alpha+1+2\beta^2)^2}$. Because $\alpha < -\frac{1+\beta^2}{2k}$, $\frac{\partial p^*}{\partial \alpha} < 0$ can be proved. Similarly, $\frac{\partial p_L{}^*}{\partial \alpha} = -\frac{4\left(4k^2\alpha^2+\beta^2+\beta^4+2k\alpha\left(1+2\beta^2\right)\right)-r\left(8k^2\alpha^2+\beta^2+2\beta^4+4k\alpha\left(1+2\beta^2\right)-1\right)}{r^2(4k\alpha+1+2\beta^2)^2} = \frac{N_3}{N_4}$. The sign of $\frac{\partial p_L{}^*}{\partial \alpha}$ depends on the numerator N_3. It's easy to prove that N_3 decreases with r. When $r = 0.5$, $N_3 = 3\left(4k^2\alpha^2 + \beta^2 + \beta^4 + 2k\alpha\left(1+2\beta^2\right)\right) + \frac{1}{2}(\beta^2+1) > 0$. $\frac{\partial p_L{}^*}{\partial \alpha} < 0$ can be derived.

From $\frac{\partial e_P{}^*}{\partial \alpha} = \frac{2}{r(4k\alpha+1+2\beta^2)^2}$, $\frac{\partial e_P{}^*}{\partial \alpha} > 0$ can be proved.

From $\frac{\partial e_L{}^*}{\partial \alpha} = \frac{-2\beta}{r(4k\alpha+1+2\beta^2)^2}$, $\frac{\partial e_L{}^*}{\partial \alpha} < 0$ can be proved.

From $\frac{\partial(e_L{}^*-e_P{}^*)}{\partial \alpha} = \frac{-2(1+\beta)}{r(4k\alpha+1+2\beta^2)^2}$, $\frac{\partial(e_L{}^*-e_P{}^*)}{\partial \alpha} < 0$ can be proved.

□

Proof of Proposition 3.3. (1) When the leader is friend role ($\alpha > 0$), $p_L{}^* - p^* = -\frac{k^2\alpha^2(2-3r)+\beta^4+k\alpha\left(1-(3-r)\beta^2\right)}{kr(4k\alpha-1-2\beta^2)}$.

It's easy to see that the sign of $p_L{}^* - p^*$ depends on the numerator. When $r < \frac{2\sqrt{\beta^6(3+2\beta^2)}}{\beta^4} - 3 - \frac{1}{\beta^2}$, $p_L{}^*-p^* < 0$. Thus, $p_L{}^* < p^*$ if $r < \frac{2\sqrt{\beta^6(3+2\beta^2)}}{\beta^4}-3-\frac{1}{\beta^2}$ can be proved. When $r > \frac{2\sqrt{\beta^6(3+2\beta^2)}}{\beta^4}-3-\frac{1}{\beta^2}$, solving

$k^2\alpha^2(2-3r)+\beta^4+k\alpha\big(1-(3-r)\beta^2\big)=0$, we can obtain two roots $\alpha_3=\frac{(3-r)\beta^2-1-\sqrt{1-2(3-r)\beta^2+(1+6r+r^2)\beta^4}}{2k(2-3r)}$ and $\alpha_4=\frac{(3-r)\beta^2-1+\sqrt{1-2(3-r)\beta^2+(1+6r+r^2)\beta^4}}{2k(2-3r)}$. When $r>\frac{2}{1+2\beta^2}$, the root α_3 is not satisfied with the condition $\alpha>\max\Big\{\frac{1+\beta^2}{k(2+r)},\frac{\beta^2}{k(2-r)}\Big\}$. So when $\frac{2\sqrt{\beta^6(3+2\beta^2)}}{\beta^4}-3-\frac{1}{\beta^2}<r<\frac{2}{1+2\beta^2}$, $k^2\alpha^2(2-3r)+\beta^4+k\alpha\big(1-(3-r)\beta^2\big)<0$ if $\alpha_3<\alpha<\alpha_4$, *i.e.*, ${p_L}^*>p^*$; $k^2\alpha^2(2-3r)+\beta^4+k\alpha\big(1-(3-r)\beta^2\big)>0$ if $\alpha>\alpha_4$ *or* $\alpha<\alpha_3$, *i.e.*, ${p_L}^*<p^*$. Similarly, when $r>\frac{2}{1+2\beta^2}$, ${p_L}^*>p^*$ if $\alpha<\alpha_4$, ${p_L}^*>p^*$ if $\alpha>\alpha_4$.

(2) When the leader is seller role, ${p_L}^*-p^*=\frac{2k^2\alpha^2+k\alpha\big(3\beta^2-1\big)+\beta^4}{kr(4k\alpha+1+2\beta^2)}$, because $1<\beta<2$, ${p_L}^*-p^*<0$ can be derived.

□

Proof of Lemma 3.4. (1) When the leader is friend role ($\alpha>0$), ${e_L}^*-{e_P}^*=\frac{k\alpha(r+2\beta+r\beta-2)-\beta\big(\beta^2+1-\beta\big)}{kr(4k\alpha-1-2\beta^2)}$. It's easy to see that the sign of ${e_L}^*-{e_P}^*$ depends on the numerator. When $r<\frac{2}{1+2\beta^2}$, ${e_L}^*<{e_P}^*$ if $\alpha<\frac{\beta\big(\beta^2-\beta+1\big)}{k(2\beta-2+r+r\beta)}$, ${e_L}^*>{e_P}^*$ if $\alpha>\frac{\beta\big(\beta^2-\beta+1\big)}{k(2\beta-2+r+r\beta)}$. When $r>\frac{2}{1+2\beta^2}$, because $\alpha>\frac{\beta^2}{k(2-r)}$, $k\alpha(r+2\beta+r\beta-2)-\beta\big(\beta^2+1-\beta\big)>0$, *i.e.*, ${e_L}^*>{e_P}^*$.

(2) When the leader is seller role ($\alpha<0$), ${e_L}^*-{e_P}^*=\frac{2k\alpha(\beta-1)+\beta\big(1+\beta^2-\beta\big)}{kr(4k\alpha+1+2\beta^2)}$. Similarly, ${e_L}^*>{e_P}^*$ if $\alpha<\frac{\beta\big(\beta^2-\beta+1\big)}{2k(\beta-1)}$, ${e_L}^*<{e_P}^*$ if $\alpha>\frac{\beta\big(\beta^2-\beta+1\big)}{2k(\beta-1)}$.

□

Proof of Proposition 4.1. (1) When the leader is friend role ($\alpha>0$), ${p_Y}^*-{p_N}^*=\frac{-\big(k\alpha-\beta^2\big)\big(4k^2\alpha^2(1-r)-k\alpha(3-r)\big(2\beta^2-1\big)+2\beta^4-\beta^2\big)}{kr^2(4k\alpha-1-2\beta^2)(2\beta^2-1)}$. It's easy to see that the sign of ${p_Y}^*-{p_N}^*$ depends on the numerator. When $r<\frac{4\sqrt{2}\beta-2\beta^2-3}{2\beta^2-1}$, $4k^2\alpha^2(1-r)+2\beta^4-\beta^2-k\alpha(3-r)\big(2\beta^2-1\big)<0$, ${p_Y}^*-{p_N}^*>0$ if $\alpha<\frac{\beta^2}{k}$; ${p_Y}^*-{p_N}^*<0$ if $\alpha>\frac{\beta^2}{k}$. When $r>\frac{4\sqrt{2}\beta-2\beta^2-3}{2\beta^2-1}$, solving $(4k^2\alpha^2(1-r)+2\beta^4-\beta^2-k\alpha(3-r)\big(2\beta^2-1\big)=0$, we can obtain two roots $\alpha_7=\frac{6\beta^2-r\big(2\beta^2-1\big)-3-\sqrt{2\beta^2-1}\sqrt{2\beta^2+r^2(2\beta^2-1)+r(6+4\beta^2)-9}}{8k(1-r)}$ and $\alpha_8=\frac{6\beta^2-r\big(2\beta^2-1\big)-3+\sqrt{2\beta^2-1}\sqrt{2\beta^2+r^2(2\beta^2-1)+r(6+4\beta^2)-9}}{8k(1-r)}$. When $r>\frac{2}{1+2\beta^2}$, the root α_7 is not satisfied with the condition $\alpha>\max\Big\{\frac{1+\beta^2}{k(2+r)},\frac{\beta^2}{k(2-r)}\Big\}$. When $\frac{4\sqrt{2}\beta-2\beta^2-3}{2\beta^2-1}<r<\frac{2}{1+2\beta^2}$, the β will influence the existence of α_7. When $\frac{1}{7}<r<\frac{2}{9}$ and $\sqrt{\frac{5+r}{1+2r}}<\beta<2$ or $\frac{2}{9}<r<\frac{1}{4}$ and $\sqrt{\frac{5+r}{1+2r}}<\beta<\sqrt{\frac{2-r}{2r}}$, The root α_7 is not satisfied with the condition $\alpha>\frac{1+\beta^2}{k(2+r)}$. Thus, when the commission ratio $r>\frac{2}{1+2\beta^2}$ or $\frac{1}{7}<r<\frac{2}{9}$ and $\sqrt{\frac{5+r}{1+2r}}<\beta<2$ or $\frac{2}{9}<r<\frac{1}{4}$ and $\sqrt{\frac{5+r}{1+2r}}<\beta<\sqrt{\frac{2-r}{2r}}$, ${p_Y}^*<{p_N}^*$ if $\alpha<\alpha_8$ or $\alpha>\frac{\beta^2}{k}$, ${p_Y}^*>{p_N}^*$ if $\alpha_8<\alpha<\frac{\beta^2}{k}$; otherwise, ${p_Y}^*>{p_N}^*$ if $\alpha<\alpha_7$ *or* $\alpha_8<\alpha<\frac{\beta^2}{k}$, ${p_Y}^*<{p_N}^*$ if $\alpha_7<\alpha<\alpha_8$ or $\alpha>\frac{\beta^2}{k}$.

(2) When the leader is seller role ($\alpha<0$), ${p_Y}^*-{p_N}^*=\frac{-\big(k\alpha+\beta^2\big)\big(4k^2\alpha^2-3k\alpha\big(2\beta^2-1\big)+2\beta^4-\beta^2\big)}{kr^2(4k\alpha+1+2\beta^2)(2\beta^2-1)}$. It's easy to see that the sign of ${p_Y}^*-{p_N}^*$ depends on the numerator. Because $1<\beta<2$, $4k^2\alpha^2+2\beta^4-\beta^2-3k\alpha\big(2\beta^2-1\big)>0$. Thus, ${p_Y}^*<{p_N}^*$ if $\alpha<\frac{-\beta^2}{k}$, ${p_Y}^*>{p_N}^*$ if $\alpha>\frac{-\beta^2}{k}$.

□

Proof of Proposition 4.2. (1) When the leader is friend role ($\alpha>0$), ${e_{PY}}^*-{e_{PN}}^*=\frac{\big(k\alpha-\beta^2\big)\big(2\beta^2-1-4k(1-r)\alpha)\big)}{kr(4k\alpha-1-2\beta^2)(2\beta^2-1)}$. When $r<\frac{2}{1+2\beta^2}$, because $\alpha>\frac{1+\beta^2}{k(2+r)}$, $2\beta^2-1-4k(1-r)\alpha)<0$ can be proved. Thus, we can get ${e_{PY}}^*>{e_{PN}}^*$ if $\alpha<\frac{\beta^2}{k}$, ${e_{PY}}^*<{e_{PN}}^*$ if $\alpha>\frac{\beta^2}{k}$. When $r>\frac{2}{1+2\beta^2}$, there exists $\alpha=\frac{2\beta^2-1}{4k(1-r)}$, $2\beta^2-1-4k(1-r)\alpha=0$. Therefore, ${e_{PY}}^*<{e_{PN}}^*$ if $\alpha<\frac{2\beta^2-1}{4k(1-r)}$ or $\alpha>\frac{\beta^2}{k}$, ${e_{PY}}^*>{e_{PN}}^*$ if $\frac{2\beta^2-1}{4k(1-r)}<\alpha<\frac{\beta^2}{k}$.

(2) When the leader is seller role ($\alpha < 0$), ${e_{PY}}^* - {e_{PN}}^* = \frac{\left(k\alpha+\beta^2\right)\left(2\beta^2-1+4k\alpha\right)}{kr(4k\alpha+1+2\beta^2)(2\beta^2-1)}$. Because $\alpha < -\frac{1+\beta^2}{2k}$, $2\beta^2 - 1 + 4k\alpha < 0$ can be derived. Thus, ${e_{PY}}^* < {e_{PN}}^*$ if $\alpha < -\frac{\beta^2}{k}$, ${e_{PY}}^* > {e_{PN}}^*$ if $\alpha > -\frac{\beta^2}{k}$.

□

Proof of Proposition 4.3. (1) When the leader is friend role ($\alpha > 0$), ${e_{LY}}^* - {e_{LN}}^* = \frac{\beta\left(1-4k^2(1-r)\alpha^2-\beta^2-2\beta^4+k\alpha\left(6\beta^2-1-2r\right)\right)}{kr(4k\alpha-1-2\beta^2)(2\beta^2-1)}$. It's easy to see that the sign of ${e_{LY}}^* - {e_{LN}}^*$ depends on the numerator. When $r < r_4 = \frac{3}{2} + \beta^2 - 4\beta^4 + \sqrt{2(1+\beta^2)(2\beta^2-1)^3}$, $1 - 4k^2(1-r)\alpha^2 - \beta^2 - 2\beta^4 + k\alpha\left(6\beta^2 - 1 - 2r\right) < 0$ can be proved. So ${e_{LY}}^* < {e_{LN}}^*$ if $r < \frac{3}{2} + \beta^2 - 4\beta^4 + \sqrt{2(1+\beta^2)(2\beta^2-1)^3}$. When $r > \frac{3}{2} + \beta^2 - 4\beta^4 + \sqrt{2(1+\beta^2)(2\beta^2-1)^3}$, by solving $1 - 4k^2(1-r)\alpha^2 - \beta^2 - 2\beta^4 + k\alpha\left(6\beta^2 - 1 - 2r\right) = 0$, we can obtain two roots $\alpha_9 = \frac{6\beta^2-1-2r-\sqrt{17+4r^2-28\beta^2+4\beta^4+4r(8\beta^4-2\beta^2-3)}}{8k(1-r)}$ and $\alpha_{10} = \frac{6\beta^2-1-2r+\sqrt{17+4r^2-28\beta^2+4\beta^4+4r(\ 8\beta^4-2\beta^2-3)}}{8k(1-r)}$. But when $r > \frac{2}{1+2\beta^2}$, the root α_9 is not satisfied with the condition $\alpha > \max\left\{\frac{1+\beta^2}{k(2+r)}, \frac{\beta^2}{k(2-r)}\right\}$. Thus, when the commission ratio $r_4 < r < \frac{2}{1+2\beta^2}$, ${e_{LY}}^* < {e_{LN}}^*$ if $\alpha < \alpha_9$ or $\alpha > \alpha_{10}$, ${e_{LY}}^* > {e_{LN}}^*$ if $\alpha_9 < \alpha < \alpha_{10}$; when the commission ratio $r > \frac{2}{1+2\beta^2}$, ${e_{LY}}^* > {e_{LN}}^*$ if $\alpha < \alpha_{10}$, ${e_{LY}}^* < {e_{LN}}^*$ if $\alpha > \alpha_{10}$.

(2) When the leader is seller role ($\alpha < 0$), ${e_{LY}}^* - {e_{LN}}^* = \frac{\beta\left(4k^2\alpha^2+k\alpha\left(6\beta^2-1\right)-1+\beta^2+2\beta^4\right)}{kr(4k\alpha+1+2\beta^2)(2\beta^2-1)}$. Because $1 < \beta < 2$, $4k^2\alpha^2 - 1 + \beta^2 + 2\beta^4 + k\alpha\left(6\beta^2 - 1\right) < 0$ can be derived. Thus, ${e_{LY}}^* > {e_{LN}}^*$.

□

Proof of Proposition 4.4. (1) When the leader is friend role ($\alpha > 0$), ${\pi_{PY}}^* - {\pi_{PN}}^* = \frac{-\left(k\alpha-\beta^2\right)\left(\left(4k^2-8k^2r+4k^2r^2\right)\alpha^2+\left(3k-2kr-6k\beta^2+4kr\beta^2\right)\alpha+2\beta^4-\beta^2\right)}{(4k\alpha-1-2\beta^2)(2\beta^2-1)}$. It's easy to see that the sign of ${\pi_{PY}}^* - {\pi_{PN}}^*$ depends on the numerator. When $r < r_5 = \frac{3+2\beta^2-2\sqrt{\beta^2(2\beta^2-1)}}{2+4\beta^2}$, $(4k^2 - 8k^2r + 4k^2r^2)\alpha^2 + \left(3k - 2kr - 6k\beta^2 + 4kr\beta^2\right)\alpha + 2\beta^4 - \beta^2 > 0$ can be proved. So when $r < r_5$, ${\pi_{PY}}^* > {\pi_{PN}}^*$ if $\alpha < \frac{\beta^2}{k}$, ${\pi_{PY}}^* < {\pi_{PN}}^*$ if $\alpha > \frac{\beta^2}{k}$. When $r > \frac{3+2\beta^2-2\sqrt{\beta^2(2\beta^2-1)}}{2+4\beta^2}$, by solving $(4k^2 - 8k^2r + 4k^2r^2)\alpha^2 + \left(3k - 2kr - 6k\beta^2 + 4kr\beta^2\right)\alpha + 2\beta^4 - \beta^2 = 0$, we obtain two roots $\alpha_{11} = \frac{k(3-2r)\left(2\beta^2-1\right)-\sqrt{k^2(2\beta^2-1)(2\beta^2-9+4r(3+2\beta^2)-r^2(4+8\beta^2))}}{8k^2(1-r)^2}$, and $\alpha_{12} = \frac{k(3-2r)\left(2\beta^2-1\right)+\sqrt{k^2(2\beta^2-1)(2\beta^2-9+4r(3+2\beta^2)-r^2(4+8\beta^2))}}{8k^2(1-r)^2}$. But when $r > \frac{5-\beta^2}{1+5\beta^2}$, the root α_{11} is not satisfied with the condition $\alpha > \max\left\{\frac{1+\beta^2}{k(2+r)}, \frac{\beta^2}{k(2-r)}\right\}$. Thus, when the commission ratio $r_5 \le r < \frac{5-\beta^2}{1+5\beta^2}$, ${\pi_{PY}}^* > {\pi_{PN}}^*$ if $\alpha < \alpha_{11}$ or $\alpha_{12} < \alpha < \frac{\beta^2}{k}$, ${\pi_{PY}}^* < {\pi_{PN}}^*$ if $\alpha_{11} < \alpha < \alpha_{12}$ or $\alpha > \frac{\beta^2}{k}$; when the commission ratio $r > \frac{5-\beta^2}{1+5\beta^2}$, ${\pi_{PY}}^* > {\pi_{PN}}^*$ if $\alpha_{12} < \alpha < \frac{\beta^2}{k}$, ${\pi_{PY}}^* < {\pi_{PN}}^*$ if $\alpha < \alpha_{12}$ or $\alpha > \frac{\beta^2}{k}$.

(2) When the leader is seller role ($\alpha < 0$), ${\pi_{PY}}^* - {\pi_{PN}}^* = \frac{\left(k\alpha+\beta^2\right)\left(4k^2\alpha^2+\left(6\beta^2-3\right)k\alpha+2\beta^4-\beta^2\right)}{-(4k\alpha+1+2\beta^2)(2\beta^2-1)}$. Because $1 < \beta < 2$, $4k^2\alpha^2 + \left(6\beta^2 - 3\right)k\alpha + 2\beta^4 - \beta^2 > 0$ can be derived. Thus, ${\pi_{PY}}^* < {\pi_{PN}}^*$ if $\alpha < \frac{-\beta^2}{k}$, ${\pi_{PY}}^* > {\pi_{PN}}^*$ if $\alpha > \frac{-\beta^2}{k}$.

□

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