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USING THE HYBRID UNDESIRABLE NETWORK DATA ENVELOPMENT ANALYSIS MODEL TO EVALUATE THE EFFICIENCY OF TAIWAN'S SOCIAL CARE SYSTEM

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Abstract. In this study, a hybrid undesirable network data envelopment analysis model is developed for assessing the efficiency of social care systems. Studies investigating the performance of social care activities have mostly focused on a single social care type. To date, no study has proposed an integrated framework for assessing social care performance. To address this gap in the literature, the developed model incorporates four divisions for a comprehensive performance measurement: disability care, child and youth care, women care, and elderly care. In the model, the government's expenditure on social care is the initial input, and the efficiency of social care activities in the aforementioned four divisions is evaluated in the second stage. Social care facilities and social care workers serve as the nonradial input and radial input, respectively. Care recipients and those affected by violence serve as the desirable output and undesirable output, respectively, in the second stage. The empirical results have several practical implications for Taiwan's social care system.

Mathematics Subject Classification. 90C30.

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1. Introduction

Social care typically refers to the support that the public sector provides to impoverished, vulnerable, or disadvantaged individuals and households through social assistance and protection services [10,29]. The provision of shelter for those who are socially vulnerable or affected by violence as well as their families has also entered the scope of social care [16]. Despite the use of diverse terminology, such as *social service*, *social assistance*, *social welfare*, and *social security*, to represent the notion of social care, the main focus remains promoting the welfare of individuals and society as well as improving people's quality of life [35]. Social care can be regarded as a system that protects people from social risks and helps them to return to a normal standard of living, thus restoring their social status [7,23,37]. Such a system aims to meet the physical and emotional needs of dependent adults and children [18]. Other outcomes of the social care system, in addition to enhancing quality of life, include improving people's health status, contributing to positive changes in families, mitigating discrimination, and maximizing economic participation and benefits and personal dignity [54].

Keywords. Data envelopment analysis, social care system, undesirable, hybrid measure, network model.

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Social care systems are primarily aimed at offering service, assistance, and protection to vulnerable or disadvantaged groups in terms of health, economic status, or cultural or social circumstances [53]. The identification of vulnerable and disadvantaged groups is regarded as the key step in establishing a social care system. This information helps the authorities to understand the type of care and assistance that the target groups require. According to Sousa *et al.* [69], individuals who are unable to independently perform activities of daily living should be identified as primary recipients of social care; Children, youth, and disabled people in unfavorable socioeconomic situations are generally regarded as the most vulnerable groups, and they require the full support of the social care system [16]. Women affected by violence and substance abuse represent another major vulnerable group that must be supported by the social care system [21, 22]. Older adults who are unable to independently perform activities of daily living (*e.g.*, tasks at home, mobility, recreation, or safety procedures) are another group requiring assistance from this system [69]. Finally, those who are unemployed, those who had accidents at work, single mothers, immigrants, and those with chronic illness are also regarded as vulnerable and disadvantaged groups [48, 57].

With the development of Taiwan's economy, social welfare has received increasing attention from local authorities. The social welfare framework in Taiwan principally consists of social assistance, social protection, and national health insurance. The social care system aims to assist and protect the following four vulnerable and disadvantaged groups: children and youth, older adults, women, and people with disabilities. Social care policy is coordinated by the Ministry of Health and Welfare, which formulates policies and allocates budgets to local governments. The social welfare bureaus in each region are responsible for the actual implementation of policies, including the establishment of social care facilities, recruitment of social workers, and identification and investigation of vulnerable groups. In 1980, the Elderly Welfare Act and Disability Welfare Act were enacted; subsequently, the Children and Youth Welfare Act and Domestic Violence Prevention Act were enacted in 1989 and 1995, respectively. According to statistics from the Ministry of Health and Welfare for 2021, Taiwan had 1470 social care institutions and 40 000 social care workers, including professionals and volunteers; the public expenditure on the social care system in 2021 was US$6.63 billion. Despite relevant legislation and increased public expenditure to enhance social care and protection, vulnerable groups still experience unfavorable situations in communities. For instance, the number of disabled people experiencing domestic violence and number of abused children and youth were both over 12 000 in 2021. Therefore, improving the efficiency of the social care system should be a key priority of the authorities.

The main purpose of this study is to develop a hybrid undesirable network data envelopment analysis (DEA) model to assess the performance of the social care system. The DEA model has been applied to investigate the performance of social care services, but studies have mostly focused only on the efficiency evaluation of a single sector. For example, Ni Luasa *et al.* [55], Weatherall *et al.* [83], and Luan *et al.* [44] have assessed the efficiency of social care for people with disabilities; Vrabková and Vaňková [81] and Visic and Kordić [80] have considered the performance of long-term care for older adults; Achoki *et al.* [1] and Lo Storto [42] have assessed the performance of care and protection for children and youth. To date, no study has proposed an integrated framework for assessing social care performance. To address this gap in the literature, the model for performance measurement developed in this study incorporates four divisions: disability care, child and youth care, women care, and elderly care. In addition, because social care expansion depends on public expenditure [36, 61], an administrative efficiency measurement for the social care system is added in the initial stage of the model. The outcomes of social care have been demonstrated to improve quality of life [35, 54]; therefore, the security and health situation, which represents the performance of quality of life, is considered in the final stage of the developed model.

The principal methodological contribution of this study is the use of a hybrid undesirable network DEA model; that is, a hybrid measure is used to compute efficiency. Related studies, such as Kalhor and Matin [34], Maghbouli *et al.* [45], and Michali *et al.* [50], have applied the radial measure in the undesirable network DEA model. However, the radial measure might not be appropriate for unit invariance, translation invariance, and monotonicity [15, 43, 74]. Other studies, such as those of Chen *et al.* [14], Fukuyama and Weber [26], and Shi *et al.* [65], have adopted the nonradial measure to assess efficiency in the undesirable network DEA model.

The radial measure assumes that inputs and outputs change proportionally, whereas the nonradial measure assumes that inputs and outputs change nonproportionally. However, the difference in changeability between variable and semifixed factors has not been considered in the two measures [31]. That is, variable factors are subject to change proportionally while semifixed factors are subject to change disproportionably. For instance, Shao *et al.* [64] and Visic and Kordić [80] employed fixed assets and social workers as outputs to assess efficiency for senior care. In practice, the number of social workers can be adjusted according to the demand of social care recipients through the recruitment of volunteers; however, fixed assets cannot be rapidly adjusted as the scale of demand changes. These differences need to be defined in the DEA model ([75, 78]); and the hybrid undesirable network DEA model developed in this study addresses this inadequacy.

The remainder of this paper is organized as follows: in Section 2, the literature on using the DEA model for evaluating the efficiency of social care services is reviewed, and in Section 3, the hybrid undesirable network DEA model is presented. In Section 4, the result of an empirical evaluation applying the data of 20 Taiwanese regions is presented, and the conclusions are provided in Section 5.

2. Literature review

Studies have applied the DEA model to analyze social welfare and well-being and have added indicators related to the social care system. For example, Martín and Mendoza [47] investigated quality of life by using the DEA model and incorporated social care facilities in the efficiency measurement. Cuadrado-Ballesteros *et al.* [17] and González *et al.* [27] have regarded social care services as factors in the assessment of quality of life. Some studies have focused on the performance of social care organizations or social care policies. Medina-Borja and Triantis [49] established a radial multistage DEA model to measure the efficiency of social service organizations. Wichmann *et al.* [84] also used the DEA model to evaluate the efficiency of long-term care facilities. Shtals *et al.* [66] applied the radial DEA model to assess the technical and cost efficiencies of social care institutions. Hu *et al.* [30] defined minimum living allowance, employment rate, and hospital beds as the outputs and expenditure on social security as the input to evaluate social welfare development in China through the radial DEA model.

The DEA model has been extensively applied in efficiency measurements of social care services for disadvantaged and vulnerable groups. Social care and long-term care for older adults have become a popular topic because of rapidly aging populations worldwide. Björkgren *et al.* [8] applied the DEA model to evaluate the efficiency of long-term care services provided to frail older adults. Delellis and Ozcan [19] and Iparraguirre and Ma [33] employed nursing home data to assess the efficiency of social care provision for older people by using the DEA model. Ni Luasa *et al.* [55] estimated the technical and scale efficiencies of nursing homes providing long-term care. Weatherall *et al.* [83] used the radial DEA model to assess the efficiency of mental disability care systems in 32 countries, with care expenditure and years of disability being used as the input and output, respectively, in the assessment. Luan *et al.* [44] used the radial DEA model, with social care institutions and older people receiving care being defined as the input and output, respectively, and investigated the influence of countries' level of economic and industrial development on the efficiency of the social care system. Shao *et al.* [64] employed the nonradial DEA model to evaluate the performance of 186 senior care service centers. Their inputs were fixed assets and social workers, whereas the number of older people receiving support and their satisfaction were used as outputs.

Measuring the efficiency of disability care services has also attracted DEA researchers. Blank and Valdmanis [9] modified the radial three-stage DEA to assess the cost efficiency of the social care system for the disabled population. Agovino and Rapposelli [2] used the radial DEA model to analyze employment initiatives for disabled people, with the aim of promoting policies that support the social inclusion of disabled people; Agovino and Rapposelli [3] subsequently analyzed the influence of the efficiency of these initiatives by using a regression model. Wu *et al.* [86] focused on long-term care support for older adults and applied a nonradial intertemporal DEA model to assess the efficiency of the system over a 10-year period for 22 administrative regions; in that study, care receivers and caregivers represented the input and output, respectively. Mirmozaffari *et al.* [51] used

the radial DEA model, with length of stay and patients employed as the input and output, respectively, to evaluate the efficiency of disability care services. Vrabková and Vaňková [81] used employees and costs as the inputs and income as the output to examine the allocative efficiency of senior long-term care institutions by using the radial DEA model. In a similar study, Visic and Kordić [80] defined assets and staff costs as the inputs and revenue as the output to analyze the performance of nursing homes by using the intertemporal DEA model.

Social care and protection for children and youth have also been considered in some DEA studies. Song [67] evaluated the efficiency of childcare centers by using the radial DEA model, aiming to verify the quality of child services in the social care system. Vaz *et al.* [79] used the radial DEA model to evaluate the efficiency of nonprofit institutions that provide care to children and young people. They defined costs as an input and care recipients as an output, respectively, in their empirical evaluation. Kang and Elwang [35] also applied the radial DEA model to evaluate the efficiency of child and youth psychological support services, with government expenditure and number of social workers being used as the input and output, respectively. Achoki *et al.* [1] used the radial DEA model and employed medical personnel and funds as the inputs and child mortality rate as the undesirable output to assess the technical and scale efficiency of child health services. Lo Storto [42] investigated the performance of care for children and older people and proposed a radial two-stage DEA model to compare the efficiency of the social service system in different municipalities. The assumption of inputs and outputs of the previous studies has been briefly summarized in Table 1.

3. Methodology

The DEA model, developed by Charnes *et al.* [12] and Banker *et al.* [6], has been extensively used by subsequent researchers with various assumptions. Many studies focusing on assessing economic, energy, or environmental efficiency have incorporated the concept of negative output into relevant models and proposed different measures to address undesirable outputs. Seiford and Zhu [62,63] applied a monotone decreasing transformation to treat undesirable outputs which shows that the DEA model how improve the performance *via* increase of the desirable outputs and decrease of the undesirable outputs. Morais and Camanho [52] used a similar transformation to change undesirable outputs into positive factors. Liu *et al.* [41] handled undesirables by multiplying them by -1. Some studies, such as those of Korhonen and Luptacik [38] and Fukuyama and Weber [25], have measured undesirable outputs exactly as they would inputs in efficiency assessments, which simultaneously take into account the undesirable outputs and the desirable outputs. Samavati *et al.* [58], Tavana *et al.* [72], Sarkhosh-Sara *et al.* [59], and Michali *et al.* [50] transformed the relationship between reference sets and current values in mathematical constraints to evaluate the effect of an excessive number of undesirable outputs on efficiency. The assumption can address the undesirable outputs in the network or the dynamic structure.

Researchers are increasingly focusing on the internal structure of overall efficiency. Some studies have decomposed a unit under evaluation by using several stages or divisions, with each characterized by its own inputs and outputs, and added undesirables into the structure. Li *et al.* [39], Mahboubi *et al.* [46], Puri *et al.* [56], and Wu *et al.* [85] have used a two-stage framework, with undesirables being used as intermediates or final outputs, to assess stage and overall efficiencies. Kalhor and Matin [34], Maghbouli *et al.* [45], and Michali *et al.* [50] have extended the structure to obtain an undesirable network framework consisting of multiple stages and divisions; they then used pollution as the undesirable output to assess the efficiency of the system. The aforementioned studies have applied the radial measure to evaluate efficiency in their undesirable network framework. However, some researchers have suggested that the radial measure is not appropriate for models with unit invariance, translation invariance, and monotonicity [15,43,74]. Other researchers have assessed efficiency involving undesirables by using the nonradial measure. For instance, Chen and Liu [13], Hajaji *et al.* [28], Li *et al.* [40], Tavassoli *et al.* [73], and Xiao *et al.* [87] have used the nonradial measure to compute slacks for undesirable outputs and efficiency scores. An *et al.* [4], Fu [24], and Song *et al.* [68] have established nonradial two-stage DEA models to assess efficiency when undesirable outputs are involved; and Chen *et al.* [14], Fukuyama and Weber [26], and Shi *et al.* [65] have further extended the two-stage DEA model into a network structure.

TABLE 1. Efficiency in social care – literature review.

Dimension	Study	Input	Output
Social welfare	Hu *et al.* [30]	Expenditure on social security, Employment, population	Disposable income, Unemployment rate, GDP
	Medina-Borja and Triantis [49]	Expenses, Trained volunteers, Paid staff, People trained in health and safety	Income, Families assisted after a local emergency, Service quality, Outcome achievement index
	Shtals *et al.* [66]	Healthcare professionals, Caregivers, Nurses and social educators, Other employees of the institution	Number of bed-days
	Wichmann *et al.* [84]	Nursing, Beds, Health professionals	Comfort assessment of dying, Economic outcome
Elderly care	Björkgren *et al.* [8]	Nursing staff, Beds in the unit	Cost
	Iparraguirre and Ma [33]	Staff per client, Front-line staff ratio, Gross expenditure, Nursing/residential costs	Older recipients of the social care services
	Luan *et al.* [44]	Pension for retired personnel, Pension resource institutions	Population of the elderly receiving service
	Ni Luasa *et al.* [55]	Medical staff, Non-medical staff, Number of beds	Total patient days
	Shao *et al.* [64]	Capital source, Hardware facilities, Team composition	Elderly care recipients, Staff training, Financial performance
	Weatherall *et al.* [83]	Disease investment, Health care expenditure	Disability-adjusted life years per patient
Disability care	Agovino and Rapposelli [2]	Temporary, Layoff hours, Employed women, Foreign resident population	Number of disabled people employed
	Agovino and Rapposelli [3]	Temporary, Layoff hours, Employed women, Foreign resident population	Number of disabled people employed
	Blank and Valdmanis [9]	Nursing personnel, Medical personnel, Auxiliary personnel	Patient days
	Mirmozaffari *et al.* [51]	Average Length of Stay, Average occupational and physical therapy charges	Average severe patients, Average semi-severe patients, Average mild patients
	Visic and Kordić [80]	Tangible assets, Staff costs, Material costs	Total revenues
	Vrabková and Vaňková [81]	Employees, Costs per bed	Incomes
	Wu *et al.* [86]	Long-term care facilities, Caregivers	Long-term care receivers
Children and youth	Song [67]	Number of teachers, Care capacity for infants	Enrollment of infants
	Vaz *et al.* [79]	Staff cost, Operating costs	Social service users
	Kang and Elwang [35]	Budget, Workers	Social service users
	Achoki *et al.* [1]	Total funds, Medical personnel, Nursing personnel	Health intervention coverage
	Lo Storto [42]	Expenditure	Social infrastructure, Education infrastructure, Sport and culture infrastructure

The nonradial measure was reported to be superior to the radial measure in terms of reducing the limitations of variable selection, such as ratio value and zero value in input/output [20]; however, whether the nonradial measure is absolutely superior to the radial measure remains under debate. Avkiran *et al.* [5] highlighted the defects of radial and nonradial measures, and Sueyoshi and Sekitani [71] and Sueyoshi and Goto [70] revealed the strengths and weaknesses of the two measures. Tone [75] and Tone and Tsutsui [76] developed a model that integrates the radial and nonradial measures. In studies using undesirable factors, the radial measure assumes that inputs and outputs change proportionally, whereas the nonradial measure assumes that inputs and outputs change nonproportionally. Some researches measured efficiency by assuming adjustment to be fully proportional between various inputs [60, 82]. The assumption represents that variable inputs are increased or decreased proportionally with semifixed inputs. The actual operations may not adhere to the assumption, in which variable inputs and semifixed inputs would not change by equal magnitude [32]. However, conventional undesirable network DEA models have not considered the difference in changeability between variable and semifixed factors. To address this research gap, this study establishes a hybrid undesirable network DEA that measures variable factors and semifixed factors by using the radial and nonradial measures, respectively.

The framework of the social care system is presented in Figure 1. *Government expenditure on social care* is designated as the initial input, with reference to Cepparulo and Giuriato [11]. The administrative performance is measured in the first stage of the model. The efficiency of the social care activities is evaluated in the second stage. With reference to related studies, such as Luan *et al.* [44], Romenska *et al.* [57], Lo Storto [42], and Visic and Kordić [80], and the Taiwanese context, social care activities are classified into four divisions: child and youth care, elderly care, disability care, and women care. *Social care facilities*, which are characterized by semifixed input, are employed as the nonradial input in the efficiency measurement for the social care system, and *social care workers*, which are characterized by variable input, are used as the radial input. *Care recipients* and *those affected by violence* are used as the desirable output and undesirable output, respectively, in the second stage. *Public security* and *national health outcomes* are designated as the final output to measure quality of life, with reference to the work of Kang and Elwang [35] and Newman *et al.* [54].

In accordance with the framework of the social care system, the hybrid undesirable network DEA can be modeled as follows. The first stage is used to assess administrative performance, and an N-dimension set of decision-making units represents the sample under evaluation. The DMU_o represents a certain region under evaluation and is subject to $\mathrm{DMU}_o \in N$. The ith initial input is labeled as $x_i \in R_+^I$. The j_kth link factor between the first and second stages refers to the radial inputs for K divisions in the second stage and is expressed as $z_{j_k}^{Rk} \in R_+^{J_k}$. The unknown intensity variable λ_n can be solved by mathematical programming. The technology set for the first stage can be expressed as follows:

$$T^{1\mathrm{st}} = \left\{ \left(x_i, z_{j_k}^{Rk}\right) : x_i \geq \sum_{n=1}^{N} x_{ni} \cdot \lambda_n(\forall i), z_{j_k}^{Rk} \leq \sum_{n=1}^{N} z_{nj_k}^{Rk} \cdot \lambda_n(\forall j, \forall k), \sum_{n=1}^{N} \lambda_n = 1, \lambda_n \geq 0 \right\}. \tag{1}$$

The second stage consists of K-dimension divisions: child and youth care, elderly care, disabled people care, and woman care. The J_kth radial input $z_{j_k}^{Rk} \in R_+^{J_k}$ $(\forall j)$, which links the first stage and the second stage, serves as the radial input for the kth division of the second stage, and the m_kth nonradial input is labeled as $z_{m_k}^{NRk} \in R_+^{M_k}(\forall m)$. The unknown intensity variable is expressed as $\mu_n^k(\forall k)$ for the kth division. The p_kth desirable output and the q_kth undesirable output are represented by $w_{p_k}^{Dk} \in R_+^{P_k}(\forall p_k)$ and $w_{q_k}^{UDk} \in R_+^{Q_k}(\forall q_k)$, respectively. The technology set for the kth division in the second stage can be expressed as follows:

$$\begin{aligned} T_k^{2\mathrm{nd}} = \Bigg\{ & \left(z_{j_k}^{Rk}, z_{m_k}^{NRk}, w_{p_k}^{Dk}, w_{q_k}^{UDk}\right) : z_{j_k}^{Rk} \geq \sum_{n=1}^{N} z_{nj_k}^{Rk} \cdot \mu_n^k(\forall j_k), z_{m_k}^{NRk} \geq \sum_{n=1}^{N} z_{nm_k}^{NRk} \cdot \mu_n^k(\forall m_k), \\ & w_{p_k}^{Dk} \leq \sum_{n=1}^{N} w_{np_k}^{Dk} \cdot \mu_n^k(\forall p_k), w_{q_k}^{UDk} \geq \sum_{n=1}^{N} w_{nq_k}^{UDk} \cdot \mu_n^k(\forall q_k), \sum_{n=1}^{N} \mu_n^k = 1, \mu_n^k(\forall k) \geq 0 \Bigg\}. \ (\forall k). \end{aligned} \tag{2}$$

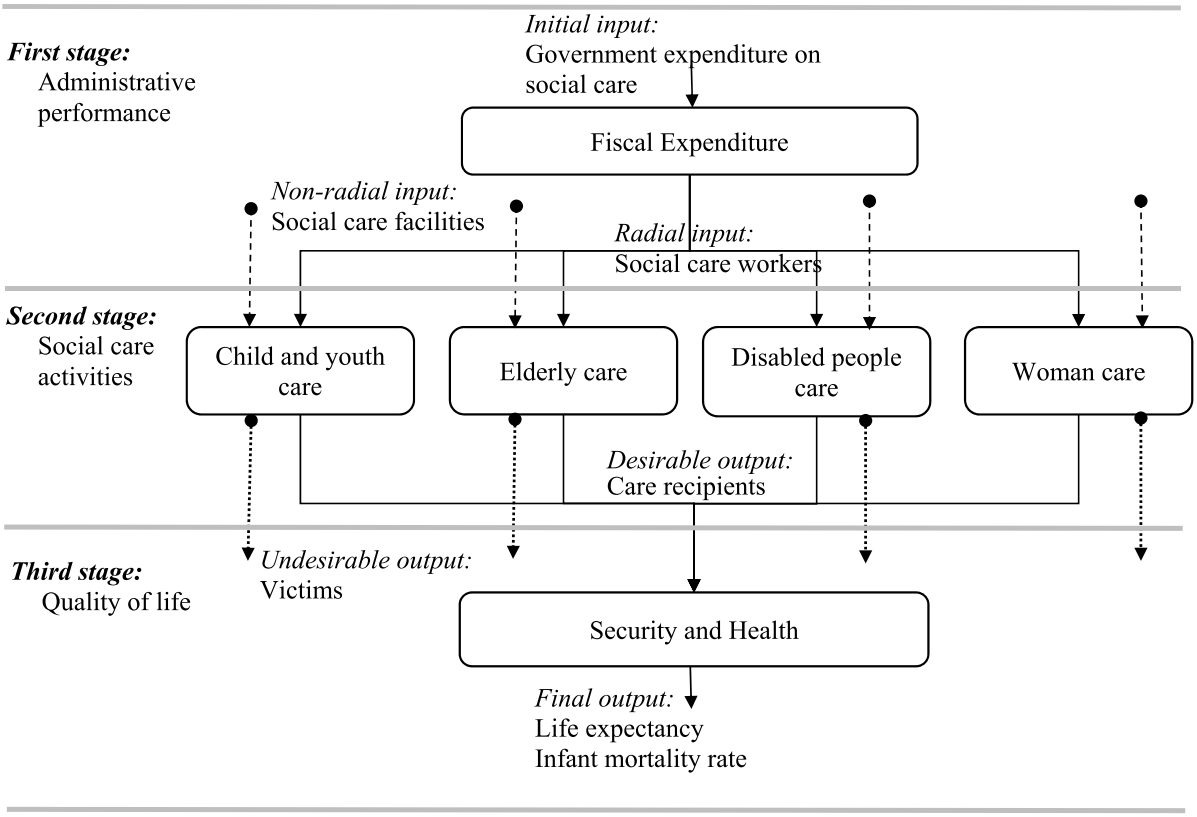

FIGURE 1. Framework of the social care system measured using the hybrid undesirable network DEA model.

The third stage uses social security and national health as indicators to reveal the quality of life in the social care system. The p_kth desirable output $w_{p_k}^{Dk} \in R_+^{P_k}(\forall p, \forall k)$, which links the second stage and the third stage, also serves as the input for the third stage. The final outputs consist of desirable and undesirable outputs, and the gth desirable output and the hth undesirable output are expressed as $y_g^D \in R_+^G$ and $y_h^{UD} \in R_+^H$, respectively. The unknown intensity variable is represented by ρ_n for the third stage. The technology set for the third stage can be expressed as follows:

$$T_k^{3\text{rd}} = \left\{ \left(w_{p_k}^{Dk}, y_g^D, y_h^{UD}\right) : w_{p_k}^{Dk} \geq \sum_{n=1}^{N} w_{np_k}^{Dk} \cdot \rho_n^k(\forall p_k), y_g^D \leq \sum_{n=1}^{N} y_{ng}^D \cdot \rho_n(\forall g), \right.$$
$$\left. y_h^{UD} \geq \sum_{n=1}^{N} y_{nh}^{UD} \cdot \rho_n(\forall h), \sum_{n=1}^{N} \rho_n = 1, \rho_n \geq 0 \right\}. \tag{3}$$

Thus, the hybrid undesirable DEA model in accordance with the framework in Figure 1 can be formulated as follows:

$$\min_{\theta, \lambda, \mu^k, s^\varepsilon}$$

s.t. : (the first stage)

$$
\begin{aligned}
& x_i = \sum_{n=1}^{N} x_{ni} \cdot \lambda_n + s_i^-(\forall i); && \text{initial input} \\
& z_{j_k}^{Rk} = \sum_{n=1}^{N} z_{nj_k}^{Rk} \cdot \lambda_n - s_{j_k}^{k+}(\forall j_k, \forall k); \\
& \text{(the second stage)} \\
& \theta^k \cdot z_{j_k}^{Rk} \geq \sum_{n=1}^{N} z_{nj_k}^{Rk} \cdot \mu_n^k(\forall j_k, \forall k); && \text{radial input} \\
& z_{m_k}^{NRk} = \sum_{n=1}^{N} z_{nm_k}^{NRk} \cdot \mu_n^k + s_{m_k}^{k-}(\forall m_k, \forall k); && \text{nonradial input} \\
& w_{p_k}^{Dk} = \sum_{n=1}^{N} w_{np_k}^{Dk} \cdot \mu_n^k - s_{p_k}^{k+}(\forall p_k, \forall k); && \text{desirable output} \\
& w_{q_k}^{UDk} = \sum_{n=1}^{N} w_{nq_k}^{UDk} \cdot \mu_n^k + s_{q_k}^{k+}(\forall q_k, \forall k); && \text{undesirable output} \\
& \text{(the third stage)} \\
& w_{p_k}^{Dk} = \sum_{n=1}^{N} w_{np_k}^{Dk} \cdot \rho_n^k + s_{p_k}^{k-}(\forall p_k, \forall k); \\
& y_g^D = \sum_{n=1}^{N} y_{ng}^D \cdot \rho_n - s_g^+(\forall g); && \text{desirable final output} \\
& y_h^{UD} = \sum_{n=1}^{N} y_{nh}^{UD} \cdot \rho_n + s_h^+(\forall h); && \text{undesirable final output} \\
& \sum_{n=1}^{N} \lambda_n = 1, \sum_{n=1}^{N} \mu_n^k = 1(\forall k), \sum_{n=1}^{N} \rho_n = 1; \\
& \lambda_n \geq 0, \mu_n^k(\forall k) \geq 0, \rho_n \geq 0; \\
& s_i^-, s_{j_k}^{k+}(\forall k), s_{m_k}^{k-}(\forall k), s_{p_k}^{k+}(\forall k), s_{q_k}^{k+}(\forall k), s_{p_k}^{k-}(\forall k), s_g^+, s_h^+ \geq 0; \\
& \theta^k \leq 1. && (4)
\end{aligned}
$$

The objective value ε, which combines the assumptions of Tone [75] and Tone and Tsutsui [77] and is used to measure the overall efficiency, is defined as follows:

$$
\varepsilon = \frac{\left[1 - \frac{J}{I+H+J+P+M+Q} \cdot \left(\sum_{k=1}^{K} \theta^k\right) - \frac{\alpha_1}{I+H+J+P+M+Q}\right]}{\left[1 + \frac{\alpha_2}{G+J+P}\right]}, \tag{5}
$$

where

$$
\begin{aligned}
\alpha_1 &= \sum_{i=1}^{I} \frac{s_i^-}{x_i} + \sum_{k=1}^{K} \sum_{m_k=1}^{M_k} \frac{s_{m_k}^{k-}}{z_{m_k}^{NRk}} + \sum_{k=1}^{K} \sum_{q_k=1}^{Q_k} \frac{s_{q_k}^{k+}}{w_{q_k}^{UDk}} + \sum_{p_k=1}^{P_k} \frac{s_{p_k}^{k-}}{w_{p_k}^{Dk}} + \sum_{h=1}^{H} \frac{s_h^+}{y_h^{UD}} \\
\alpha_2 &= \sum_{k=1}^{K} \sum_{j_k=1}^{J_k} \frac{s_{j_k}^{k+}}{z_{j_k}^{Rk}} + \sum_{k=1}^{K} \sum_{p_k=1}^{P_k} \frac{s_{p_k}^{k+}}{w_{p_k}^{Dk}} + \sum_{g=1}^{G} \frac{s_g^+}{y_g^D}.
\end{aligned}
$$

In function (4), the total number of radial inputs is defined as $J(=\sum_{k=1}^{K} j_k)$ in the second stage, the total number of nonradial inputs is defined as $M(=\sum_{k=1}^{K} m_k)$, the total number of desirable outputs is defined as $P(=\sum_{k=1}^{K} p_k)$, and the total number of undesirable outputs is defined as $Q(=\sum_{k=1}^{K} q_k)$. The symbol $\theta^k(\forall k)$ represents an unknown variable and can be solved through the radial measure. The symbol $s_i^-(\forall i)$ represents the initial input slack, and $s_{j_k}^{k+}(\forall j_k, \forall k)$ represents the output slack in the first stage. The input slack of the second stage is labeled as $s_{m_k}^{k-}(\forall m_k, \forall k)$. The slacks for the desirable and undesirable outputs of the second stage are labeled as $s_{p_k}^{k+}(\forall p_k, \forall k)$ and $s_{q_k}^{k+}(\forall q_k, \forall k)$, respectively. The symbol $s_{p_k}^{k-}(\forall p_k, \forall k)$ represents the input slack of the third stage. The desirable and undesirable outputs of the third stage are labeled as $s_g^+(\forall g)$ and $s_h^+(\forall h)$, respectively. These slack variables are unknown variables and can be solved through the nonradial measure.

The stage efficiencies can be measured using the following functions:

$$\text{First stage efficiency:}\quad \text{EFF}^{1\text{st}} = \frac{\left[1 - \frac{1}{I}\cdot\left(\sum_{i=1}^{I}\frac{s_i^-}{x_i}\right)\right]}{\left[1 + \frac{1}{J}\cdot\left(\sum_{k=1}^{K}\sum_{j_k=1}^{J_k}\frac{s_{j_k}^{k+}}{z_{j_k}^{Rk}}\right)\right]} \tag{6}$$

Second stage efficiency:

$$\text{EFF}^{2\text{nd}} = \frac{\left[1 - \frac{J}{J+M+Q}\cdot\left(\sum_{k=1}^{K}\theta^k\right) - \frac{1}{J+M+Q}\cdot\left(\sum_{k=1}^{K}\sum_{m_k=1}^{M_k}\frac{s_{m_k}^{k-}}{z_{m_k}^{NRk}} + \sum_{k=1}^{K}\sum_{q_k=1}^{Q_k}\frac{s_{q_k}^{k+}}{w_{q_k}^{UDk}}\right)\right]}{\left[1 + \frac{1}{P}\cdot\left(\sum_{k=1}^{K}\sum_{p_k=1}^{P_k}\frac{s_{p_k}^{k+}}{w_{p_k}^{Dk}}\right)\right]} \tag{7}$$

$$\text{Third stage efficiency:}\quad \text{EFF}^{3\text{rd}} = \frac{\left[1 - \frac{1}{P+H}\cdot\left(\sum_{p_k=1}^{P_k}\frac{s_{p_k}^{k-}}{w_{p_k}^{Dk}} + \sum_{h=1}^{H}\frac{s_h^+}{y_h^{UD}}\right)\right]}{\left[1 + \frac{1}{G}\cdot\left(\sum_{g=1}^{G}\frac{s_g^+}{y_g^D}\right)\right]}. \tag{8}$$

The divisional efficiencies in the second stage can be measured using the following function:

$$\text{EFF}_k^{2\text{nd}} = \frac{\left[1 - \frac{J_k}{J_k+M_k+Q_k}\cdot\left(\theta^k\right) - \frac{J_k}{J_k+M_k+Q_k}\cdot\left(\sum_{m_k=1}^{M_k}\frac{s_{m_k}^{k-}}{z_{m_k}^{NRk}} + \sum_{q_k=1}^{Q_k}\frac{s_{q_k}^{k+}}{w_{q_k}^{UDk}}\right)\right]}{\left[1 + \frac{1}{p_k}\cdot\left(\sum_{p_k=1}^{P_k}\frac{s_{p_k}^{k+}}{w_{p_k}^{Dk}}\right)\right]} \quad (\forall k) \tag{9}$$

To explore the source of the inefficiency, this study uses the optimal solution (s_i^{-*}, $s_{m_k}^{k-*}$, $s_{p_k}^{k+*}$, $s_{q_k}^{k+*}$, s_g^{+*}, s_h^{+*}, θ^{k*}) to assess factor inefficiency indices as follows:

$$\text{Initial input inefficiency:}\quad \text{FI}_i^I = \frac{s_i^{-*}}{x_i}(\forall i) \tag{10}$$

$$\text{Radial input inefficiency:}\quad \text{FI}_k^R = 1 - \theta^{k*}(\forall k) \tag{11}$$

$$\text{Nonradial input inefficiency:}\quad \text{FI}_{m_k}^{NR} = \frac{s_{m_k}^{k-*}}{z_{m_k}^{NRk}}(\forall m_k, \forall k) \tag{12}$$

$$\text{Desirable output inefficiency:}\quad \text{FI}_{p_k}^{D} = \frac{s_{p_k}^{k+*}}{w_{p_k}^{Dk}}(\forall p_k, \forall k) \tag{13}$$

$$\text{Undesirable output inefficiency:}\quad \text{FI}_{q_k}^{UD} = \frac{s_{q_k}^{k+*}}{w_{q_k}^{UDk}}(\forall q_k, \forall k) \tag{14}$$

$$\text{Desirable final output inefficiency:}\quad \text{FI}_{g}^{DF} = \frac{s_{g}^{+*}}{y_g^{D}}(\forall g) \tag{15}$$

$$\text{Undesirable final output inefficiency:}\quad \text{FI}_{h}^{UDF} = \frac{s_{h}^{+*}}{y_h^{UD}}(\forall h). \tag{16}$$

4. Empirical results

For the empirical evaluation, the data of 20 regions across Taiwan are applied to assess the efficiencies of the social care system. The data are collected from the databases of the National Statistics Bureau of Taiwan (https://www.stat.gov.tw) and Ministry of Health and Welfare (https://dep.mohw.gov.tw/) in 2021. Government expenditure on social care work serves as the initial input in the social care system. The numbers of social workers working with disabled people, children and youth, and older adults are used as the radial inputs. Because of the lack of relevant data, this study uses the number of volunteer working hours for women care instead of the number of workers. The numbers of social care facilities for the four vulnerable groups are used as the nonradial inputs. The official public data only provides the number of social care facilities and the unit of measurement is without classification adjustment for size and quality. The numbers of people who are served by, protected by, or settled in social care facilities serve as the desirable outputs in the four divisions and the numbers of people who are abandoned, abused, or affected by domestic violence serve as the undesirable outputs. Average life expectancy, offender rate, and infant mortality rate, representing the quality-of-life outcomes, serve as the final outputs in the social care system. The specific interpretation for all variables is shown as follows, and the descriptive statistics for the variables are presented in Table 2.

Initial input:

- Government expenditure on social care work: refers to the regional government's annual expenditure used on social care.

Radial input:

- Social workers for disabled people: refers to the number of employee in social care institutions for people with disabilities.
- Social workers for children and youth: refers to the number of employee in social care institutions for abused and abandoned children and adolescents.
- Social workers for long-term care and nursing houses for older adults: refers to the number of employee in long-term care institutions and nursing houses for older adults.
- Volunteer working hours for women care: refers to volunteer service hours in the protection and resettlement of abused women.

Nonradial input:

- Social care facilities for people with disabilities: refers to the number of institutions for people with disabilities.
- Social care facilities for children and youth: refers to the number of institutions for abused and abandoned children and adolescents.
- Long-term care and nursing homes for older adults: refers to number of long-term care institutions and nursing houses for older adults.
- Shelters for women: refers to the number of shelters for abused women.

Desirable output:

- Disabled people served by or settled in social care facilities: refers to the number of persons with disabilities who are placed or served in social care institutions.
- Children and youth served by or settled in social care facilities: refers to the number of abused and abandoned children and adolescents who are placed or served in social care institutions.
- Residents in long-term care and nursing homes for older adults: refers to the number of abandoned elderly people placed in long-term care institutions or nursing homes.
- Women protected by or settled in shelters: refers to the number of abused women protected by or settled in shelters.

TABLE 2. Descriptive statistics.

Variable	Mean	S.D.	Max	Min
Initial input:				
Government expenditure on social care work (million NT$)	9945.7	9733.6	29 677.1	1761.1
Radial input:				
Social workers for disabled people	394.9	314.6	1118.0	48.0
Social workers for children and youth	358.4	479.9	2019.3	37.5
Social workers for long-term care and nursing houses for older adults	1313.1	1338.0	5449.0	56.0
Volunteer working hours for women care	52 735.9	64 864.0	224 767.0	64.0
Nonradial input:				
Social care facilities for people with disabilities	12.5	10.8	42.0	2.0
Social care facilities for children and youth	5.4	4.5	17.0	1.0
Long-term care and nursing homes for older adults	52.7	54.1	211.0	3.0
Shelters for women	3.0	3.1	10.0	1.0
Desirable output:				
Disabled people served by or settled in social care facilities	729.8	627.1	2149.0	53.0
Children and youth served by or settled in social care facilities	96.1	82.9	283.0	5.0
Residents in long-term care and nursing homes for older adults	2415.0	2340.3	9138.0	133.0
Women protected by or settled in shelters	130.3	180.9	587.0	1.0
Undesirable output:				
Disabled people affected by domestic violence	634.9	652.4	2666.0	50.0
Abused children and youth	628.0	610.0	2110.0	36.0
Neglected, abandoned, or abused older people	79.1	89.1	388.0	13.0
Women affected by domestic violence	58 295.7	56 144.5	181 658.0	1758.0
Desirable final output:				
Average life expectancy	80.2	1.7	84.1	76.5
Undesirable final output:				
Offender rate	1280.4	290.0	1696.8	647.9
Infant mortality rate	4.1	1.9	9.6	1.5

Undesirable output:

- Disabled people affected by domestic violence: refers to the number of people with disabilities affected by domestic violence and sexual assault.
- Abused children and youth: refers to the number of children and adolescents exposed to domestic violence or abandonment.
- Neglected, abandoned, or abused older people: refers to the number of elderly people who have been neglected, abandoned, or abused.
- Women affected by domestic violence: refers to the number of women affected by domestic violence and sexual assault.

Desirable final output:

- Average life expectancy: refers to the average of years people can expect to live.

Undesirable final output:

- Offender rate: refers to the rate of criminal cases per 100 000 population.
- Infant mortality rate: refers to the rate of children who die before their first year of life per 1000 live births.

The overall efficiency (*i.e.*, ε) and stage efficiency (*i.e.*, EFF^{1st}, EFF^{2nd}, and EFF^{3rd}) have been evaluated in a single DEA implementation and are listed in Table 3. The results indicate that two regions, Taipei City (No. 2) and Penghu County (No. 17), have highly efficient social care systems; moreover, their stage efficiency scores

TABLE 3. Overall and stage efficiencies.

No	Region	Overall efficiency	Stage efficiency		
			First	Second	Third
1	New Taipei City	0.877	1	0.804	1
2	Taipei City	1	1	1	1
3	Taoyuan City	0.566	0.496	0.716	1
4	Taichung City	0.505	0.449	0.604	1
5	Tainan City	0.445	1	0.397	0.259
6	Kaohsiung City	0.462	0.423	0.716	0.325
7	Yilan County	0.615	1	0.657	0.203
8	Hsinchu County	0.522	1	0.427	0.461
9	Miaoli County	0.546	0.619	0.457	1
10	Changhua County	0.436	0.863	0.386	0.430
11	Nantou County	0.249	0.364	0.261	0.293
12	Yunlin County	0.255	0.138	0.936	1
13	Chiayi County	0.199	0.165	0.647	0.212
14	Pingtung County	0.313	0.488	0.434	0.294
15	Taitung County	0.114	0.122	0.135	0.451
16	Hualien County	0.270	0.186	0.812	0.245
17	Penghu County	1	1	1	1
18	Keelung City	0.579	0.407	0.829	1
19	Hsinchu City	0.681	1	0.490	1
20	Chiayi City	0.645	1	0.686	0.327
	Average	0.514	0.636	0.620	0.625

are equal to one. In terms of first-stage efficiency, eight regions (Nos. 1, 2, 5, 7, 8, 17, 19, and 20) are efficient, suggesting that these regions have the strongest administrative performance. Regarding second-stage efficiency, Taipei City and Penghu County are the most efficient, suggesting that these two regions have the most efficient social care services for disabled people, children and youth, older adults, and women. In terms of third-stage efficiency, nine regions (Nos. $1, 2, 3, 4, 9, 12, 17, 18$, and 19) are efficient, achieving the highest performance in quality of life.

Taitung County (No. 15) is the region with the lowest score (0.114) for overall efficiency, and its first- and second-stage efficiency scores (0.122 and 0.135, respectively) are also the lowest among all regions. Yilan County (No. 7) has the lowest score (0.203) for third-stage efficiency. The average overall efficiency score is 0.514, with the average scores for first-, second, and third-stage efficiency being 0.636, 0.620, and 0.625, respectively. The results reveal that the efficiencies of the three stages are relatively similar.

The efficiencies of the four divisions (*i.e.*, $\mathrm{EFF}_k^{\mathrm{2nd}}$) are listed in Table 4. The results indicate that eight regions (Nos. 1, 2, 5, 7, 8, 11, 17, and 18) have efficient disabled care, and the region with the lowest score (0.707) is Hualien County (No. 16). Nine regions (Nos. 2, 4, 6, 7, 11, 12, 13, 16, and 17) have efficient child and youth care, and Tainan City (No. 5) is the region with the lowest score (0.206). Thirteen regions (Nos. 1, 2, 3, 4, 5, 6, 7, 8, 11, 16, 17, 18, and 20) have efficient elderly care, and Hsinchu City (No. 19) is the region with the lowest score (0.665). Five regions (Nos. 1, 2, 12, 17, and 18) have efficient women care, and Taitung County (No. 15) is the region with the lowest score (0.032).

Of the four social care divisions, elderly care has the highest efficiency, with an average score of 0.932, followed by disabled care and child and youth care, with average scores of 0.876 and 0.716, respectively. Women care has by far the lowest efficiency of the four divisions in the social care system, with an average score of 0.497.

To explore differences by region type, the 20 regions are classified as either metropolitan (city) or county and compared using the average efficiency scores (Tab. 5). The overall efficiency of metropolitan regions (0.753) is

TABLE 4. Divisional efficiencies.

No	Region	Disabled people	Child and youth	Elderly	Woman
1	New Taipei City	1	0.359	1	1
2	Taipei City	1	1	1	1
3	Taoyuan City	0.839	0.562	1	0.649
4	Taichung City	0.797	1	1	0.212
5	Tainan City	1	0.206	1	0.164
6	Kaohsiung City	0.757	1	1	0.430
7	Yilan County	1	1	1	0.301
8	Hsinchu County	1	0.442	1	0.185
9	Miaoli County	0.885	0.432	0.836	0.247
10	Changhua County	0.845	0.270	0.814	0.212
11	Nantou County	1	1	1	0.066
12	Yunlin County	0.998	1	0.776	1
13	Chiayi County	0.639	1	0.787	0.437
14	Pingtung County	0.766	0.304	0.784	0.593
15	Taitung County	0.584	0.932	0.979	0.032
16	Hualien County	0.707	1	1	0.634
17	Penghu County	1	1	1	1
18	Keelung City	1	0.538	1	1
19	Hsinchu City	0.785	0.515	0.665	0.305
20	Chiayi City	0.921	0.758	1	0.480
	Average	0.876	0.716	0.932	0.497

TABLE 5. Differences in efficiency by regional classification.

	Overall efficiency	Stage efficiency			Divisional efficiency			
		First	Second	Third	Disabled people	Child and youth	Elderly	Woman
City	0.640	0.753	0.693	0.768	0.900	0.660	0.963	0.582
County	0.411	0.540	0.559	0.508	0.857	0.762	0.907	0.428

higher than that of county regions (0.411). The stage efficiencies of metropolitan regions are also higher than those of county regions. The efficiency scores of disabled care and elderly care in the metropolitan regions are 0.900 and 0.963, respectively, which are higher than the scores for the county regions, but the gap is small. The efficiency score of women care in the metropolitan regions is 0.582, which is higher than that in the county regions (0.428). Child and youth care is the only category for which the county regions outperform the metropolitan regions in efficiency (0.762 *vs.* 0.660).

This study uses the factor inefficiency index to identify sources of inefficiency in the social care system, with a higher score representing a greater influence on inefficiency. The average values of the radial input inefficiency (*i.e.*, FI_k^R) and nonradial input inefficiency (*i.e.*, $\mathrm{FI}_{m_k}^{NR}$) indices are reported in Table 6. For the nonradial input inefficiency, the value of women care (0.367) is the highest of all four divisions, followed by the value of child and youth care (0.277). The value of elderly care is 0.002, and only one region has a positive value for this indicator.

For the radial input inefficiency, the value of women care is the highest (0.071) of the four divisions, followed by the average value of disability care (0.039). The average value of elderly care is the lowest (0.008).

In terms of the different input types, the nonradial input inefficiency indices are higher than the radial input inefficiency indices in two divisions, namely child and youth care and women care. The gaps between the radial

Table 6. Nonradial and radial input inefficiency indices.

No	Region	Nonradial input				Radial input			
		Disabled people	Child and youth	Elderly	Woman	Disabled people	Child and youth	Elderly	Woman
1	New Taipei City	–	0.961	–	–	–	0.141	–	–
2	Taipei City	–	–	–	–	–	–	–	–
3	Taoyuan City	0.109	0.631	–	0.717	0.105	0.039	–	0.043
4	Taichung City	0.083	–	–	0.789	0.002	–	–	0.296
5	Tainan City	–	0.620	–	0.928	–	–	–	–
6	Kaohsiung City	0.051	–	–	0.829	0.098	–	–	0.225
7	Yilan County	–	–	–	–	–	–	–	–
8	Hsinchu County	–	0.913	–	0.314	–	–	–	–
9	Miaoli County	0.001	0.441	–	0.561	0.113	–	–	–
10	Changhua County	0.042	0.547	–	0.865	0.035	–	0.004	0.167
11	Nantou County	–	–	–	0.768	–	–	–	0.222
12	Yunlin County	–	–	–	–	–	–	0.032	–
13	Chiayi County	–	–	–	–	0.058	–	0.043	–
14	Pingtung County	0.137	0.477	–	0.691	0.121	0.125	0.045	0.292
15	Taitung County	0.108	0.149	0.047	–	–	–	–	–
16	Hualien County	–	–	–	–	0.012	–	–	–
17	Penghu County	–	–	–	–	–	–	–	–
18	Keelung City	–	–	–	–	–	–	–	–
19	Hsinchu City	–	0.638	–	0.880	0.126	–	0.035	–
20	Chiayi City	–	0.167	–	–	0.101	–	–	0.167
	Average	0.027	0.277	0.002	0.367	0.039	0.015	0.008	0.071

and nonradial indices are nonsignificant in disability care and elderly care (0.012 *vs.* 0.006, respectively). The results imply that excessive social care facilities in women care and child and youth care is the main factor leading to inefficiency, whereas most elderly care facilities are used efficiently. Notably, the radial input – the number of social care workers – is more efficient than the nonradial input.

The average values of the undesirable output inefficiency (*i.e.*, $\mathrm{FI}_{q_k}^{UD}$) and desirable output inefficiency (*i.e.*, $\mathrm{FI}_{p_k}^{D}$) indices are reported in Table 7. For the undesirable output inefficiency, the value of women care is the highest (0.274), followed by the values of child and youth care and elderly care (0.190 and 0.177, respectively). The lowest average value (0.126) is observed for disability care (0.126). Regarding the desirable output inefficiency, the value of women care is the highest (2.342), followed by child and youth care (0.352). The average values of disability care and elderly care are both lower than 0.1.

In terms of the different output types, the average values of the undesirable output are higher than those of the desirable output for disability care and elderly care, whereas the average scores of the desirable output are higher than those of the undesirable output for child and youth care and women care. This finding implies that the low number of care recipients is the factor causing low efficiency in child and youth care and women care, whereas the excessive number of people requiring care is the largest factor for disability care and elderly care.

Table 8 lists the initial input (*i.e.*, FI_i^I), desirable final output (*i.e.*, FI_g^{DF}), and undesirable final output (*i.e.*, FI_h^{UDF}) indices as measured using the developed model. The average value of the initial input is 0.024, with three regions (Nos. 6, 16, and 17) having a positive index. This result indicates that most regions achieve best practice for government expenditure utilization. The average value of the desirable final output – life expectancy – is 0.057. The average values of the undesirable outputs, namely offender rate and infant mortality rate, are 0.263 and 0.008, respectively. The results indicate that life expectancy and infant mortality rate have

TABLE 7. Undesirable and desirable output inefficiency indices.

No	Region	Undesirable output				Desirable output			
		Disabled people	Child and youth	Elderly	Woman	Disabled people	Child and youth	Elderly	Woman
1	New Taipei City	–	0.820	–	–	–	–	–	–
2	Taipei City	–	–	–	–	–	–	–	–
3	Taoyuan City	0.270	0.643	–	0.292	–	–	–	–
4	Taichung City	0.525	–	–	0.796	–	–	–	0.756
5	Tainan City	–	0.682	–	0.632	–	1.750	–	1.926
6	Kaohsiung City	0.579	–	–	0.657	–	–	–	–
7	Yilan County	–	–	–	0.397	–	–	–	1.887
8	Hsinchu County	–	0.198	–	–	–	0.424	–	3.834
9	Miaoli County	–	–	0.422	–	0.086	0.977	0.028	2.288
10	Changhua County	0.389	0.507	0.554	0.515	–	1.406	–	1.290
11	Nantou County	–	–	–	–	–	–	–	9.218
12	Yunlin County	0.002	–	0.640	–	–	–	–	–
13	Chiayi County	0.079	–	0.593	0.608	0.494	–	0.001	0.824
14	Pingtung County	0.404	0.441	0.603	0.106	0.017	1.143	–	0.075
15	Taitung County	0.026	0.057	–	0.734	0.636	–	0.005	22.307
16	Hualien County	0.236	–	–	0.580	0.298	–	–	0.272
17	Penghu County	–	–	–	–	–	–	–	–
18	Keelung City	–	0.158	–	–	–	0.762	–	–
19	Hsinchu City	0.017	0.294	0.722	–	0.213	0.339	0.124	1.313
20	Chiayi City	–	–	–	0.157	0.049	0.246	–	0.860
	Average	0.126	0.190	0.177	0.274	0.090	0.352	0.008	2.342

a negligible effect on the efficiency of the social care system, whereas the impact of offender rate on efficiency is relatively strong. The finding implies that, from the perspective of quality of life, the inadequate outcome in public security is the main factor causing low efficiency in the social care system. By contrast, the national health outcomes have only a minor impact on the efficiency.

5. Conclusion

This study develops a hybrid undesirable network DEA model to assess the efficiency of the social care system. Previous studies investigating the performance of social care activities have mostly focused on a single social care type. This study establishes an integrated framework, which incorporates four social care types, administrative performance, and quality of life measurement, for the social care system. The main methodological contribution of this study is the developed model's use of hybrid measurement; semifixed inputs and variable inputs are measured through nonradial and radial measures, respectively, in the efficiency measurement involving undesirable factors. The empirical evaluation uses the data of 20 regions in Taiwan to examine the hybrid undesirable network DEA model and assess the social care system's overall efficiency, stage efficiency, and divisional efficiency. The factor inefficiency indices are also calculated to explore the influence of various factors on inefficiency.

The empirical results have several practical implications for Taiwan's social care system. First, in terms of overall efficiency, only two regions are efficient, and only minor differences are noted in the efficiencies of the three stages. This finding implies that most local authorities should improve the efficiency of their social care systems. Second, in terms of divisional efficiencies, elderly care and disability care exhibit higher performance than the other care types in most regions. Women care has the lowest efficiency within the social care system.

TABLE 8. Initial input and final output inefficiency indices.

No	Region	Initial input	Final output		
		Government expenditure	Life expectancy	Offender rate	Infant mortality rate
1	New Taipei City	–	–	–	–
2	Taipei City	–	–	–	–
3	Taoyuan City	–	–	–	–
4	Taichung City	–	–	–	–
5	Tainan City	–	0.151	0.382	0.002
6	Kaohsiung City	0.313	–	0.375	0.006
7	Yilan County	–	0.261	0.462	0.005
8	Hsinchu County	–	0.094	0.461	–
9	Miaoli County	–	–	–	–
10	Changhua County	–	–	–	–
11	Nantou County	–	0.083	0.417	0.014
12	Yunlin County	–	–	–	–
13	Chiayi County	–	0.240	0.625	0.014
14	Pingtung County	–	–	0.634	0.034
15	Taitung County	–	0.022	0.781	0.054
16	Hualien County	0.144	0.146	0.553	0.041
17	Penghu County	–	–	–	–
18	Keelung City	0.019	–	–	–
19	Hsinchu City	–	–	–	–
20	Chiayi City	–	0.134	0.574	–
	Average	0.024	0.057	0.263	0.008

Third, the nonradial and radial input inefficiency indices reveal that excessive social care facilities established for women care and child and youth care are the main factors driving inefficiency. The use of a variable input (social care workers) in the model yields more favorable performance than the use of a semifixed input (social care facilities). Fourth, the undesirable and desirable output inefficiency indices reveal that the influence of the undesirable output on inefficiency is greater than that of the desirable output in disability care and elderly care. The influence of the desirable output on inefficiency is higher than that of the undesirable output in child and youth care and women care. Fifth, the final output inefficiency indices reveal that the inadequate outcome in public security is the main factor causing low efficiency in the social care system, with the national health outcomes having less of an effect. Sixth, except for child and youth care, the efficiencies in metropolitan regions are higher than those in county regions.

The results also offer a practical direction for individual regional government in social care efficiency improvement. For example, The Taichung City (No. 4) was assessed lower in the efficiencies of first stage and the divisions of disabled people and woman. The local government needs to increase social care facilities and workers for disabled care and woman care to improve the efficiencies. In addition, it also has to decrease the number of disabled people affected by domestic violence and women affected by domestic violence in the region to enhance the performance of quality of life.

The present empirical results reveal that the efficiency of elderly care is higher than that of the other social care types, with the efficiency of women care being the lowest. This result may be attributed to local governments' increased promotion of long-term care for older adults in recent years to address rapid population aging. However, protection of and assistance for vulnerable women should be improved in most Taiwanese regions, especially outside of the island's main cities. Utilization of social care workers in the model yields more favorable results than the use of social care facilities, because social care workers represent a variable input that can be adjusted rapidly with the scale of people in need of social care. To address the excess of social care facilities, authorities

may consider adopting a combined operation mode by providing unused social care facilities to residents as activity centers or preschools. Regarding the outcomes of social care, an excessive number of people in vulnerable groups needing care (*i.e.*, the undesirable outputs) is not the greatest factor leading to inefficiency in the child and youth care and women care divisions. The finding implies that the monitoring of domestic violence against and substance abuse among women and children has been effective as a result of the well-established Domestic Violence Prevention Notification Channel in Taiwan. However, the monitoring mechanism for abuse or abandonment of disabled or older people has room for improvement.

The factor inefficiency index of government expenditure was assessed as a positive value only in three regions. The result shows that there was no budget waste in most regions. The study found that the inefficiency indices of social care workers, who serve disabled people and women care, were higher than the divisions of child and youth care and elderly people care. The finding implies that the two divisions, disability care and women care, have not effectively translated financial resources into service capacity within the social care system. Therefore, this study suggests that local governments are supposed to allocate more budget to divisions with higher efficiency to enhance the capacity of social care services. In addition, the public sector should establish an exclusive notification channel, similar to that for domestic violence prevention, for disabled people and older adults who are affected by violence. Finally, public security and national health reflect the quality of life in the social care system. The relevant authorities should carefully consider how to provide assistance and protection to vulnerable groups and further reduce crime through the use of efficient social care services.

Although the study provides managerial insights and implications for policy in the social care system, it still has limitations. The main limitation lies in the incomplete data. For example, the empirical evaluation used the number of volunteer working hours for women's care instead of the number of social workers due to a lack of data. Another limitation is the range of efficiency values. The developed DEA model assessed efficiency values by using the slacks and the radial ratios, which only have unique values solved by using the present hybrid measure, but the range of values has not been discussed. These limitations may offer opportunities for future research.

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Data availability statement

The datasets generated during and/or analyzed during the current study are available from the online databases of the National Statistics Bureau of Taiwan (https://www.stat.gov.tw) and Ministry of Health and Welfare (https://dep.mohw.gov.tw/).

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