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PRICING AND GREEN QUALITY DECISIONS IN TWO-STAGE GREEN SUPPLY CHAIN FOR SUBSTITUTABLE GREEN PRODUCTS: A GAME-THEORETIC APPROACH

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Abstract. Growing businesses are concerned with doing well both environmentally and economically. Pointing out this view, this paper explores the game theoretical approach (Stackelberg and Bertrand) for a two-echelon green supply chain where the duopolistic manufacturers produce two substitutable green products and sell their products through a common retailer. The demands for both green products are functions of the selling prices and green levels (GLs). The effects of power structures on optimal price and green level decisions and associated equilibrium decisions are examined in three scenarios. Firstly, trilateral competition manufacturer-led Stackelberg (MS); secondly, retailer-led Stackelberg (RS); and thirdly, vertical collaboration, and compares the optimal decisions analytically. Our investigations show that, in addition to increasing the product's greening level, vertical collaboration creates a win–win situation for collaboration members, whereas the manufacturer outside the collaboration experiences a decline in profits. Additionally, we find that the overall profit from vertical collaboration is greater than the sum of the individual profits corresponding to two participants in the trilateral competition models (MS and RS). Further, a selection criterion is developed for retailer to select the most suitable manufacturer for vertical collaboration. Finally, a numerical example and a sensitivity analysis are performed to determine the impact of parameters.

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1. Introduction

Environmental issues must be integrated into supply chain management in the ever-changing world of sustainable business practices, as human growth and survival are under considerable threat from the trend towards global warming. To achieve a competitive advantage in this circumstance, many manufacturing organizations are modifying their production methods by broadening the ideas of environmental protection and sustainability [1]. Consequently, incorporating green practices into supply chain management has become important with the increasing consumer awareness about green products. Consumer environmental awareness (CEA) is essential in achieving ecological sustainability because consumer purchasing behaviour and environmental consciousness are

Keywords. Green supply chain, substitutable products, pricing and green level decision, vertical collaboration model, game theory.

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strongly related to manufacturer and retailer greening activities throughout the supply chain [2,3]. Over 80% of consumers prefer green products, according to an Accenture worldwide survey, and 20% of consumers are prepared to pay more for eco-friendly solutions, according to a different Carbon Trust report [4]. As CEA grows, manufacturers and retailers face an escalating responsibility to provide more environment friendly products. For example, companies in the computer industry, such as Dell, Acer, and HP, have prioritized green computing to minimize energy usage, emissions, and e-waste. Adidas, a prominent sports garment maker, used Eco-Grip technology to remove hazardous compounds from raw materials [5]. As a result, professionals, researchers, and governments are increasingly interested in the concept of green supply chain management (GSCM). In the current competitive market, several companies have launched items that either fully or partially satisfy the needs of the same customers. Thus, they are referred to as substitute products because of their identical character. Many items, such as toothpaste, chocolates, and milk, are manufactured by multiple manufacturers and are considered as distinct yet substituted by retailers and consumers. The demand for substitutable products can be affected by factors like quality and price, leading to the use of substitute goods. The reason for selecting a substitute item is that consumers only want to go to other retail stores if their preferred item is not available due to a lack of time or a busy life. So, this study develops decision models for supply chains to attain profit-oriented as well as environmental goals; for this, we specifically focus on substitutable green products, with demands for both green products being a function of selling prices and green levels (GLs). Electric vehicles and LED light bulbs of two companies are some examples of substitutable green products. The cost for production of green products is more expensive over non-green products, resulting in higher selling prices. So, there is a need for some ideal coordination to reduce selling prices and increase GLs to increase profits. There are numerous contracts that can increase the performance of supply chain participants. Revenue sharing, cost sharing, wholesale price, two-part tariff and revenue-cost sharing contracts are some of the most popular trends for coordination between members of the supply chain and increases its profit [6]. In this article, we introduce a vertical collaboration model for coordinating supply chain members and increasing profitability by reducing selling prices and increasing GLs.

In this paper, we answer the following research questions:

- How does the availability of a substitute green product affect the market's demand?
- When retailers act as a Stackelberg leader, how does the performance of the supply chain change?
- What are the optimal sales price and green levels to get the highest supply chain profit?
- How could manufacturers and retailer work together to increase the alliance's overall profit by enhancing green quality and reducing the prices of both green products?

To answer the above research questions, an analytical model of a two-stage (two manufacturers and a single retailer) supply chain is established. The retailer orders and sells the two types of substitutable green products produced by duopoly manufacturers. The demands for both green products are a function of selling prices and green levels (GLs). Two manufacturers with different market sizes will have different product GLs since they cannot extract the same amount of investment to increase GLs. When selling products through a retailer, it is crucial to investigate the best decision-making features under different power structures and suggest a coordination contract mechanism that may promote increased profits for each participant involved and higher GLs. The profit functions of each member are developed and optimised in three scenarios, *viz.* trilateral competition manufacturer-led Stackelberg (MS), retailer-led Stackelberg (RS), and vertical collaboration. Firstly, a model of trilateral competition MS is built, in which each of the three competitors aims to maximise their own earnings, considering both manufacturers simultaneously work as Stackelberg leader and retailer as follower. Secondly, trilateral competition RS is discussed, in which retailer works as Stackelberg leader and both manufacturers simultaneously as follower. For these games, distinct equilibrium solutions are obtained. Thirdly, we define the vertical collaboration model between a manufacturer and retailer for making the best decisions. In the vertical collaboration model, a manufacturer and retailer collaborate to optimize pricing issues, GLs, and profits. The strength of the collaboration is assessed by taking the retailer's dominance into account. Finally, we look at the criteria for determining which manufacturer is a part of the collaboration for various potential market demands.

The novelties of this paper are as follows: Previous research in the SCM has mostly focused on price-dependent demand [7–11]. Recently, some researchers [12–14] investigated the pricing and GL decisions in a two-stage supply chain, considering a single manufacturer and a single retailer. This study examines the price and green level competition of two substitute green products in a two-tier supply chain where the two substitute items are manufactured by two different manufacturers. Optimal decisions are obtained in the MS game as well as the RS game. To the best of our knowledge, the literature that is currently available examines the optimal decisions using MS game only. Additionally, we introduce a vertical collaboration model to enhance the performance of the supply chain. Using this model, we can reduce selling prices while increasing the GLs of both products, resulting in increased profits.

The structure of the current study is as follows. Section 2 reviews the prior literature. In Section 3, notations and assumptions are introduced and develops mathematical models and examines the results. In Section 4, the stability of vertical collaboration is analysed. In Section 5, numerical examples, sensitivity analysis, and managerial insights are analysed. In Section 6, conclusions with future reference are discussed.

2. Literature review

In this section we categorised the literature review based on three main key parameters by the research namely: green supply chain and contracts, substitutable product, and price and green level (GL) decisions using game theory.

2.1. Green supply chain and contracts

Due to significant changes in the natural environment and the depletion of natural resources, many consumers are becoming interested in purchasing environmentally friendly items, even if they have to pay a high price for them. Thus, studying green supply chains (GSCs) becomes a fascinating field of study. Ghosh and Shah [15] examined the greening techniques using different GSC designs and discovered that channel members' collaboration helps to increase the greening level and maximise its profitability. Swami and Shah [16] investigated the problem of channel coordination in a GSC and discovered that the proportion of green elasticity and greening cost equals the proportion of the manufacturer's and retailer's ideal greening endeavours. In a dual channel GSC, Li *et al.* [17] investigated the pricing and greening strategies of the channel participants in both cooperative and noncooperative circumstances. They examined when to introduce dual-channel and found that it relies on the retail channel's degree of consumer reliability as well as the cost of greening. In any business, implementing a successful contract strategy among the business participants is the key to advancing sustainable development. In order to manage the whole channel, De Giovanni [18] developed a revenue-sharing contract in a CLSC (closed-loop supply chain). Ghosh and Shah [19] presented a cost-sharing agreement to encourage manufacturers to raise the degree of product greenness, but the suggested method fell short of the centralized approach. Zhang and Liu [20] demonstrated how a revenue-sharing contract can be utilized to successfully coordinate channels in a Stackelberg game. Johari and Hosseini-Motlagh [10] study how, in a competitive context, the corporate social responsibility (CSR) cost-sharing contract might improve market demand by considerably lowering selling prices while improving the amount of CSR activities. Mondal and Giri [14] addressed a two-tier sustainable supply chain that involves a single manufacturer and a single retailer. Their findings show that retailer-led revenue sharing can result in a win–win scenario for the manufacturer and the retailer, in addition to raising the product's greenness level. In the dual channel supply chain, Hosseini-Motlagh and Johari [7] analysed pricing, sales service strategies and warranty replacement in decentralised, semi-centralized, centralised, semi-coordinated, and coordinated scenarios. They showed that the suggested hybrid revenue-tariff contract results in a situation where all parties benefited. Lou *et al.* [21] examined the game-theoretical models of a two-stage supply chain considering two substitute products using optimal vertical pricing alliance.

According to the above literature review, previous research has investigated how various coordination contracts may coordinate GSC systems. However, the vertical collaboration model got less attention for coordinating GSC.

To fill this identified gap this article develops a vertical collaboration model for a manufacturer and retailer to enhance the total profit of the supply chain.

2.2. Substitutable products

This research is concerned with the topic of green channel coordination in the presence of environmentally conscious customers. More specifically, this study relates to supply chain models with two substitutable green products. Substitution is divided into three categories by Tang and Yin [22]: assortment, stock-out and pricing-based substitution. Products with the same features can be substituted using an assortment-based substitution strategy. When the preferred product is out of stock and a client can purchase another product as a replacement, this is known as stock-out-based substitution. When a shop uses differential pricing to make those goods replaceable, this is known as price-based substitution. The substitution was divided into two categories by Kim and Bell [23]: asymmetrical substitution and symmetrical substitution. The asymmetrical substitution is used when the unmet demand for one good is only partially covered by another good, while the symmetrical substitution is used when the entire unmet demand for one good is supplied by another good.

In the supply chain network, determining the appropriate valuation for substitutable goods has become a popular study area. Bish and Suwandechochai [24] investigated the effects of product replacement level and functional delay on ideal capacity and found that the firm's varied postponement processes had no effect on the link between ideal capacity and product replacement level. Zhao *et al.* [25] considered pricing prediction for substitutable items under various competing behaviours of producers and channel persons. Zhang *et al.* [26] investigated the pricing estimate and channel coordination mechanism of a green supply chain by adopting a hybrid production mode, and revealed that Rubinstein bargaining may increase the advantage of the noncooperative game by up to 33.3% and investigated the impact of purchasers' ecological consciousness on order volumes and coordination mechanisms by using a supply chain for two types of substitutable items: conventional and green items. Mondal and Giri [27] investigate strategies for a green closed-loop supply chain for substitutable products under government subsidy using cost sharing and revenue sharing contracts. Giri and Dey [28] developed a reverse supply chain to deal with stochastic demand and the uncertain quality of returned products. Singh and Mishra [11] developed a green inventory model for substitutable products under the concept of carbon emissions. In a two-stage supply chain with demand that is dependent on greening level, retail price, and marketing effort, Basiri and Heydari [12] examined the GSC coordination mechanism for green substitute products. They found that the collaboration model has the ability to increase the profit of the supply chain as compared to the cooperation model, but also increases the profitability of both the chain and its individual components. Ma *et al.* [29] investigated pricing estimates for substitutable items in a two-stage competing supply chain using game theory. Currently, Nielsen *et al.* [30] examined the strategic integration choices made by two competing GSCs, each having a manufacturer and a store (retailer), for handling replaceable goods. This study examines the trilateral pricing issues of green substitutable products by comparing the sales amounts of the two items while considering both future market needs and production costs.

According to the above literature review, previous research has mostly focused on traditional products in which demand is price-dependent [11, 21, 31]. This study aims to advance the promotion of eco-friendly products by analysing the availability of substitutable green products within the supply chain.

2.3. Price and GL decisions using game theory

Zhang and Liu [20] demonstrated how a revenue-sharing contract can be utilized to successfully coordinate channels in a Stackelberg game. In order to generate a stable profit distribution for a retailer-led supply chain while taking consideration of the retailer's negotiating power, Zhou *et al.* [32] proposed cooperative bargaining models. In a different research, Saha *et al.* [33] examined the impact of government subsidy programmes in situations where demand is dependent on price, greening level and green marketing effort. They looked at the supply chain under both green marketing effort sharing contracts and revenue sharing contracts and discovered that, under a policy of consumer subsidies, neither contract can enhance the performance of the supply chain.

However, they discovered that a manufacturer subsidy scheme encourages members to work together. Hosseini-Motlagh and Johari [8] offer fresh perspectives on CLSC management as well as theoretical and practical insights into the way CLSC managers apply it to advance economic and environmental sustainability. They examine how remanufacturing and energy-saving initiatives affect competitive sales prices and collection rates using game theory techniques. Mondal and Giri [34] considered the manufacturer's consistent pricing strategy and, based on the various competing behaviours of the retailers, established one centralised policy and three manufacturer-led decentralised scenarios, namely Collusion, Cournot (Nash), and Stackelberg. They discovered that retailers play the Stackelberg game and deal with the same fundamental market; being followers rather than leaders benefits the retailers. In the dual channel supply chain, Hosseini-Motlagh and Johari [9] used a game-theoretic technique to investigate equilibrium solutions for remanufactured product selling prices, acquisition prices, and transfer prices under decentralisation, centralisation, and coordination settings. Using a manufacturer-led Stackelberg game and a Bayesian game, Patare and Venkataraman [35] developed a dual-chain supply chain competition considering a single manufacturer and a single retailer. Recently, Pal *et al.* [36] examined the game-theoretical models of a closed-loop supply chain consisting of duopolistic manufacturers with two price- and product-quality-dependent demands for substitute products.

According to the above literature review, previous research has mostly focused on price-competition using game theory in supply chain system including single manufacturer and single retailer. Most of them used MS game. RS game got less attention in previous research [37]. To fill this identified gap, this article disused price and green level decisions using the MS game as well as the RS game.

2.4. Research gaps and our contributions

Table 1 summarises the literature review and compares our work to the most recent and relevant articles in five key parameters. As reviewed above and shown in Table 1 the substitutable green products have received less attention. We have identified the following research gap:

- The study the price and GL competition simultaneously in a two-stage supply chain with substitute green products is not addressed in the literature.
- The concept of a vertical collaboration model to increase the performance of supply chain participants has received less attention in the previous literature.
- There is less attention in a two-stage green supply chain involving duopolistic manufacturers and a monopoly retailer with two price-GL-sensitive demands. Most of the study focused on single manufacturer-single retailer.
- Most of the article on substitutable products in the two-stage supply chain considered a single manufacturer and single retailer [12, 13].
- There is no research paper to study the product price and GL competition in a two-stage supply chain in which retailer work as Stackelberg leader (RS) and manufacturer as follower. Most of the study work on the concept in which manufacturer work as Stackelberg leader.

This paper is closely related to the work of Lou *et al.* [21] and may be treated as extension of their work in following direction:

- Introduction of substitutable green products.
- Consideration of pricing as well as GLs issues.
- Discussion of model of trilateral competition MS as well as RS.
- Introduce a vertical collaboration model to enhance the supply chain performance.

3. Model descriptions

In this study, a two-stage GSC with two duopolistic manufacturers and one retailer is examined, and the estimated demand is modelled as a linear function of selling prices and GLs. The same demand function approximation is also used by Ghosh and Shah [19]. Retail prices are the decision variables under the control of the

TABLE 1. Comparison of this study with the relevant literature.

Author(s)	Game types		Demand Dependency		Nature of items		SC structure	Contract
	MS*	RS*	Price	Green level	Green	Substitute		
Ghosh and Shah [19]	✓		✓	✓	✓		1 manufacturer 1 retailer	Cost sharing
Basiri and Heydari [12]	✓		✓	✓	✓	✓	1 manufacturer 1 retailer	Collaboration
Mondal and Giri [34]	✓		✓	✓	✓		1 manufacturer 2 retailers	Cap-and-trade policy
Johari and Hosseini-Motlagh [10]	✓		✓				2 manufacturers 1 retailer	Cost sharing
Hosseini-Motlagh and Johari [8]	✓		✓				1 manufacturer 2 retailers	Cost-tariff contract
Lou *et al.* [21]	✓		✓			✓	2 manufacturers 1 retailer	Vertical collaboration
Heydari *et al.* [13]	✓		✓	✓	✓		1 manufacturer 1 retailer	Hybrid revenue and greening cost sharing
Mondal and Giri [27]	✓		✓	✓	✓	✓	2 manufacturers 1 retailer	Cost sharing and revenue sharing
Saha *et al.* [37]	✓	✓	✓	✓	✓	✓	2 manufacturers 1 retailer	Revenue sharing
Mondal and Giri [14]	✓		✓	✓	✓		1 manufacturer 1 retailer	Revenue sharing
Hosseini-Motlagh and Johari [7]	✓		✓				1 manufacturer 2 retailer	Hybrid revenue-tariff
Hosseini-Motlagh and Johari [9]	✓		✓				1 remanufacturer 1 collector	Two-part tariff contract
Pal *et al.* [36]	✓		✓	Quality level		✓	2 manufacturers	Game theory
Patare and Venkataraman [35]	✓		✓	Quality level		✓	2 manufacturers 2 retailers, dual channel	Horizontal and Vertical competition
This article	✓	✓	✓	✓	✓	✓	2 manufacturers 1 retailer	Vertical collaboration

Notes. *MS = manufacturer-led Stackelberg, RS = retailer-led Stackelberg.

TABLE 2. Definition of symbols.

Symbols	Description
a	Primary market potential of green product I
b	Primary market potential of green product II
c_i	Unit cost of green product i ($i = 1, 2$)
τ	Consumer environmental awareness (CEA) level
α	The marginal demand with respect to the sales price
θ	Sensitivity of switchover toward price difference
β	Degree of substitution between two green products, and $\beta\alpha$ represents the demand shift between the two green items ($0 < \beta < 1$)
η	Green investment efficiency of each manufacturer
D_i	Demand function of green product i
Π_{mi}	Profit of manufacturer i ($i = 1, 2$)
Π_r	Profit of the retailer
Π_{mir}	Total profit of collaboration when retailer collaborates with manufacturer i ($i = 1, 2$)
Decision variables	
w_i	Unit wholesale price of green product i to the retailer ($i = 1, 2$)
g_i	Green quality level of green product i ($i = 1, 2$)
p_i	Retail price of green product i ($i = 1, 2$)

retailer. The wholesale prices and GLs of substitutable green products are the decision variables for manufacturers. Table 2 contains the relevant notations for the following discussion.

In this paper, a and b represent the primary market potential for both products. Retail prices of both green products are $p_i(i = 1, 2)$ and α is price elasticity coefficient. Demand is assumed to be inversely proportional to price, hence α is using with negative sign in the demand function. β is degree of substitution ($0 < \beta < 1$) between two green products, so $\beta\alpha$ represents the demand shift between the two green items due to price effect [21]. Obviously, if $\beta = 0$ then customers are loyal to one particular product. The greening level (GLs) are $g_i(i = 1, 2)$, and the consumer environmental awareness (CEA) level is τ. More environmentally friendly items will be produced with higher g_i, while customers will be more receptive to green products with higher τ. Observe that if $\tau = 0$, customers have no curiosity in green products; as a result, this study parallels the SC models explored under price-sensitive demand [11,21,31], which we refer to as the situation in which manufacturers sell regular products. Parameter θ is defined as the sensitivity of switchovers toward price difference [12]. η is the green investment efficiency of the additional cost to increase the greenness of the products. So, manufacturer M_i requires an additional cost of $\eta g_i^2(i = 1, 2)$. Since, demand D_i must be non-negative, to avoid trivial situations, we must consider $a - \alpha c_1 > 0$ and $b - \alpha c_2 > 0$. This condition helps to obtain optimal decisions in this research.

Based on the above discussion the demand functions of both products are expressed as:

$$\begin{cases} D_1 = a - \alpha p_1 + \tau g_1 + \theta(p_2 - p_1) + \beta\alpha p_2 \\ D_2 = b - \alpha p_2 + \tau g_2 + \theta(p_1 - p_2) + \beta\alpha p_1. \end{cases} \tag{1}$$

We consider few assumptions in the suggested model; some of them help to make model more realistic, while others are used to simplification. However, each of these is consistent with earlier research in the subject. To construct a reliable mathematical model, it is assumed that:

(i) To make sure the non-negativity of the optimal decisions and to avoid trivial cases, it is assumed that $a - \alpha c_1 > 0$ and $b - \alpha c_2 > 0$ [14,21]. This situation always holds in almost all literature.
(ii) The CEA level of the customer's group is supposed to have been previously evaluated using survey procedures [12].
(iii) This study does not take stock-out problems into account because it is believed that each manufacturer's production capacity is adequate to fulfil market demand.

(iv) Despite the apparent disconnection between each product's potential market demand, they are mutually transformative under distinct sales prices. To put it another way, the two items' prospective customers are same. Therefore, the price coefficient (α) is same for both items in demand function.
(v) The degree of substitution of green product I with respect to green product II and *vice-versa* is same. Each product may have different degrees of substitution in actual use. In that scenario, the analysis procedure is the same, but there is a lot more computation involved.
(vi) Manufacturers must invest in innovative product research and development (R&D) to increase the greenness of their goods. Green product development costs are calculated as ηg_i^2, where η is the green investment efficiency [27,37].
(vii) Green investment cost coefficients should be sufficiently large such that $\beta\eta\alpha > \tau^2$ for $0 < \beta < 1$ in order to avoid the trivial analysis.

In the following subsection, we consider three different models: trilateral competition MS, trilateral competition RS, and the vertical collaboration model.

3.1. Trilateral competition manufacturer-led Stackelberg (MS)

In this model duopoly manufacturers and single retailer playing a Stackelberg game while the manufacturers are simultaneously playing a Bertrand game. A two-layer model is built to investigate the presence and originality of each solution, as well as the market share of each item.

First, examine Stackelberg game, in which decision order is as follows: Duopolistic manufacturers disclose their wholesale pricing and GLs first in the leader tier, followed by the retailer's choice. The profit functions of both manufacturers (1 & 2) and retailer are given as follows:

$$\Pi_{m1}(w_1, g_1) = (w_1 - c_1)D_1 - \eta g_1^2 \tag{2}$$
$$\Pi_{m2}(w_2, g_2) = (w_2 - c_2)D_2 - \eta g_2^2 \tag{3}$$
$$\Pi_r(p_1, p_2) = (p_1 - w_1)D_1 + (p_2 - w_2)D_2. \tag{4}$$

Now we will find the optimal selling prices for both green products. In this section we use "MS" to denote the optimal solution. For this, we proposed the following theorem.

Theorem 1. *$\Pi_r(p_1, p_2)$ is concave in both p_1 & p_2 and optimal decision variables p_1 & p_2 is given by equations (5) and (6)*

$$\begin{aligned} p_1^{\mathrm{MS}} = &\frac{1}{2(\beta-1)(\beta\alpha+\alpha+2\theta)\alpha}\big[(\beta^2-1)w_1^{\mathrm{MS}}\alpha^2 + \big(2w_1^{\mathrm{MS}}\beta\theta - 2w_1^{\mathrm{MS}}\theta \\ &- \tau g_2^{\mathrm{MS}}\beta - b\beta - \tau g_1^{\mathrm{MS}} - a\big)\alpha - \theta(g_1^{\mathrm{MS}}\tau + g_2^{\mathrm{MS}}\tau + a + b)\big] \end{aligned} \tag{5}$$

$$\begin{aligned} p_2^{\mathrm{MS}} = &\frac{1}{2(\beta-1)(\beta\alpha+\alpha+2\theta)\alpha}\big[(\beta^2-1)w_2^{\mathrm{MS}}\alpha^2 + \big(2w_2^{\mathrm{MS}}\beta\theta - 2w_2^{\mathrm{MS}}\theta \\ &- \tau g_1^{\mathrm{MS}}\beta - a\beta - \tau g_2^{\mathrm{MS}} - b\big)\alpha - \theta(g_1^{\mathrm{MS}}\tau + g_2^{\mathrm{MS}}\tau + a + b)\big] \end{aligned} \tag{6}$$

$$\begin{aligned} g_1^{\mathrm{MS}} = &\frac{\tau}{B_1}\big[4\eta\big((c_1\beta^2 + \beta c_2 - 2c_1\big)\alpha^2 + ((2c_1\beta + c_2\beta - 4c_1 + c_2)\theta + b\beta + 2a)\alpha \\ &+ (c_2\theta - c_1\theta + 2a + b)\theta) - \tau^2(c_2\alpha\beta - c_1\alpha + c_2\theta - c_1\theta + a)\big] \end{aligned} \tag{7}$$

$$\begin{aligned} g_2^{\mathrm{MS}} = &\frac{\tau}{B_1}\big[4\eta\big((c_2\beta^2 + \beta c_1 - 2c_2\big)\alpha^2 + ((2c_2\beta + c_1\beta - 4c_2 + c_1)\theta + a\beta + 2b)\alpha \\ &+ (c_1\theta - c_2\theta + 2b + a)\theta) - \tau^2(c_1\alpha\beta - c_2\alpha + c_1\theta - c_2\theta + b)\big] \end{aligned} \tag{8}$$

$$\begin{aligned} w_1^{\mathrm{MS}} = &\frac{1}{B_1}\big[((-16\beta c_2 - 32c_1)\alpha^2 + (16c_2\theta\beta + 64c_1\theta + 16c_2\theta + 16b\beta + 32a)\alpha \\ &- 16(2c_1\theta + c_2\theta + 2a + b)\theta)\eta^2 + 4\tau^2(c_2\alpha\beta + 3c_1\alpha + 3c_1\theta + c_2\theta + a)\eta - c_1\tau^4\big] \end{aligned} \tag{9}$$

$$w_2^{MS} = \frac{1}{B_1}\big[((-16\beta c_1 - 32c_2)\alpha^2 + (16c_1\theta\beta + 64c_2\theta + 16c_1\theta + 16a\beta + 32b)\alpha - 16(2c_1\theta + c_2\theta + 2a + b)\theta)\eta^2 + 4\tau^2(c_2\alpha\beta + 3c_1\alpha + 3c_1\theta + c_2\theta + a)\eta - c_1\tau^4\big] \quad (10)$$

where,

$$B_1 = (\tau^2 + 4\eta(\beta\alpha - 2\alpha - \theta))(\tau^2 - 4\eta(\beta\alpha + 2\alpha + 3\theta)).$$

Proof. See in Appendix A. □

Following that, the sales volume and profit of both manufacturers are examined, which are linked to a variety of characteristics.

Theorem 2. *In the trilateral competition MS, a manufacturer with a higher marginal profit also has a large sales volume, resulting in higher earnings considering assumptions* $a - \alpha c_1 > 0, b - \alpha c_2 > 0$ *and* $0 < \beta < 1$.

Proof. The difference in marginal profitability between the two manufacturers is given by

$$\left(w_1^{\text{MS}} - c_1\right) - \left(w_2^{\text{MS}} - c_2\right) = \frac{4\eta(a - b + (c_2 - c_1)(\alpha + \alpha\beta + 2\theta))}{4\eta(\beta + 2)\alpha + 12\eta\theta - \tau^2}\cdot \quad (11)$$

Using equations (5)–(8), the following optimal sales amounts of green products I and II are obtained-

$$\begin{cases} D_1^{\text{MS}} = a - \alpha p_1^{\text{MS}} + \tau g_1^{\text{MS}} + \theta\left(p_2^{\text{MS}} - p_1^{\text{MS}}\right) + \beta\alpha p_2^{\text{MS}} \\ D_2^{\text{MS}} = b - \alpha p_2^{\text{MS}} + \tau g_2^{\text{MS}} + \theta\left(p_1^{\text{MS}} - p_2^{\text{MS}}\right) + \beta\alpha p_1^{\text{MS}}. \end{cases} \quad (12)$$

The difference between the two amounts is then calculated-

$$D_1^{\text{MS}} - D_2^{\text{MS}} = \frac{2(\alpha + \theta)\eta(a - b + (c_2 - c_1)(\alpha + \alpha\beta + 2\theta))}{4\eta(\beta + 2)\alpha + 12\eta\theta - \tau^2}\cdot \quad (13)$$

Equation (13) demonstrates that which type of product has a bigger sales volume is determined by the difference in the value of basic market demands and the difference in the value of manufacturing costs, *i.e.*,

The difference in profitability between manufacturers 1 and 2 is determined by

$$\begin{cases} D_1^{\text{MS}} - D_2^{\text{MS}} > 0 \text{ if } a - b + (c_2 - c_1)(\alpha + \alpha\beta + 2\theta) > 0 \\ D_1^{\text{MS}} - D_2^{\text{MS}} < 0 \text{ if } a - b + (c_2 - c_1)(\alpha + \alpha\beta + 2\theta) < 0. \end{cases} \quad (14)$$

The difference in profitability between manufacturers 1 and 2 is determined by

$$\Pi_{m1}^{\text{MS}} - \Pi_{m2}^{\text{MS}} = \frac{((\beta - 1)(c_1 + c_2)\alpha + a + b)\eta(a - b + (c_2 - c_1)(\alpha + \alpha\beta + 2\theta))(\tau^2 - 8\eta(\alpha + \theta))}{(4\eta(\beta + 2)\alpha + 12\eta\theta - \tau^2)(4\eta(\beta - 2)\alpha - 4\eta\theta + \tau^2)}\cdot \quad (15)$$

This means that whether $\Pi_{m1}^{\text{MS}} - \Pi_{m2}^{\text{MS}}$ is positive or negative depends on the expression

$$a - b + (c_2 - c_1)(\alpha + \alpha\beta + 2\theta). \quad (16)$$

For the sake of clarity, we are using K to represent equation (16).

We conclude that

$$\text{If } K > 0, \text{ then } D_1^{\text{MS}} - D_2^{\text{MS}} > 0 \text{ which implies that } \Pi_{m1}^{\text{MS}} - \Pi_{m2}^{\text{MS}} > 0$$
$$\text{If } K < 0, \text{ then } D_1^{\text{MS}} - D_2^{\text{MS}} < 0 \text{ which implies that } \Pi_{m1}^{\text{MS}} - \Pi_{m2}^{\text{MS}} < 0.$$

This completes the proof. □

Using equations (7)–(10), and (12) we get,
The profit of manufacturer 1 is

$$\Pi_{m1}^{\mathrm{MS}} = \left(p_1^{\mathrm{MS}} - w_1^{\mathrm{MS}}\right) D_1^{\mathrm{MS}} - \eta\left(g_1^{\mathrm{MS}}\right)^2. \tag{17}$$

The profit of manufacturer 2 is

$$\Pi_{m2}^{\mathrm{MS}} = \left(p_2^{\mathrm{MS}} - w_2^{\mathrm{MS}}\right) D_2^{\mathrm{MS}} - \eta\left(g_2^{\mathrm{MS}}\right)^2. \tag{18}$$

And the retailer's profit is given by

$$\Pi_{r}^{\mathrm{MS}} = \left(p_1^{\mathrm{MS}} - w_1^{\mathrm{MS}}\right) D_1^{\mathrm{MS}} + \left(p_2^{\mathrm{MS}} - w_2^{\mathrm{MS}}\right) D_2^{\mathrm{MS}}. \tag{19}$$

3.2. Trilateral competition retailer-led Stackelberg (RS)

In this model, the retailer acts as a Stackelberg leader and both manufactures work as a Stackelberg follower. The retailer establishes profit margins ($m_i = p_i - w_i$) first, then the two upstream manufacturers set wholesale prices and product GLs. Backword induction is used to get the following optimal solutions.

Theorem 3. *Optimal profit margins, GLs and wholesale prices of both green products are given by:*

$$\begin{cases} m_1^{\mathrm{RS}} = \frac{\alpha^2 c_1(1-\beta^2)+(2c_1\theta(1-\beta)-b\beta-a)\alpha-\theta(a+b)}{2\alpha(\alpha\beta^2+2\beta\theta-\alpha-2\theta)} \\ m_2^{\mathrm{RS}} = \frac{\alpha^2 c_2(1-\beta^2)+(2c_2\theta(1-\beta)-a\beta-b)\alpha-\theta(a+b)}{2\alpha(\alpha\beta^2+2\beta\theta-\alpha-2\theta)} \end{cases} \tag{20}$$

$$\begin{aligned} g_1^{\mathrm{RS}} = \frac{\tau}{B_2}\big[&2\eta\big((c_1\beta^2+\beta c_2-2c_1)\alpha^2+((2c_1\beta+c_2\beta-4c_1+c_2)\theta+b\beta+2a)\alpha \\ &+(c_2\theta-c_1\theta+2a+b)\theta\big)-\tau^2(c_2\alpha\beta-c_1\alpha+c_2\theta-c_1\theta+a)\big] \end{aligned} \tag{21}$$

$$\begin{aligned} g_2^{\mathrm{RS}} = \frac{\tau}{B_2}\big[&2\eta\big((c_2\beta^2+\beta c_1-2c_2)\alpha^2+((2c_2\beta+c_1\beta-4c_2+c_1)\theta+a\beta+2b)\alpha \\ &+(c_1\theta-c_2\theta+2b+a)\theta\big)-\tau^2(c_1\alpha\beta-c_2\alpha+c_1\theta-c_2\theta+b)\big] \end{aligned} \tag{22}$$

$$\begin{aligned} w_1^{\mathrm{RS}} = \frac{1}{-B_2}\big[&\big((2\beta^2c_1-2\beta c_2-12c_1)\alpha^2+(4c_1\theta\beta-2c_2\beta\theta-2c_2\theta-24c_1\theta-2b\beta-4a)\alpha \\ &-2(5c_1\theta+c_2\theta+2a+b)\theta\big)\eta^2+\tau^2(c_2\alpha\beta+7c_1\alpha+7c_1\theta+c_2\theta+a)\eta-c_1\tau^4\big] \end{aligned} \tag{23}$$

$$\begin{aligned} w_2^{\mathrm{RS}} = \frac{1}{-B_2}\big[&\big((2\beta^2c_2-2\beta c_1-12c_2)\alpha^2+(4c_2\theta\beta-2c_1\beta\theta-2c_1\theta-24c_2\theta-2a\beta-4b)\alpha \\ &-2(5c_2\theta+c_1\theta+2b+a)\theta\big)\eta^2+\tau^2(c_1\alpha\beta+7c_2\alpha+7c_2\theta+c_1\theta+b)\eta-c_2\tau^4\big] \end{aligned} \tag{24}$$

where

$$B_2 = (\tau^2 + 2\eta(\beta\alpha - 2\alpha - \theta))(\tau^2 - 2\eta(\beta\alpha + 2\alpha + 3\theta)).$$

Proof. See in Appendix B. □

The profit of manufacturer 1 is

$$\Pi_{m1}^{\mathrm{RS}} = \left(p_1^{\mathrm{RS}} - w_1^{\mathrm{RS}}\right) D_1^{\mathrm{RS}} - \eta\left(g_1^{\mathrm{RS}}\right)^2. \tag{25}$$

The profit of manufacturer 2 is

$$\Pi_{m2}^{\mathrm{RS}} = \left(p_2^{\mathrm{RS}} - w_2^{\mathrm{RS}}\right) D_2^{\mathrm{RS}} - \eta\left(g_2^{\mathrm{RS}}\right)^2. \tag{26}$$

And the retailer's profit is given by

$$\Pi_{r}^{\mathrm{RS}} = \left(p_1^{\mathrm{RS}} - w_1^{\mathrm{RS}}\right) D_1^{\mathrm{RS}} + \left(p_2^{\mathrm{RS}} - w_2^{\mathrm{RS}}\right) D_2^{\mathrm{RS}} \tag{27}$$

where,

$$\begin{cases} D_1^{\mathrm{RS}} = a - \alpha p_1^{\mathrm{RS}} + \tau g_1^{\mathrm{RS}} + \theta\big(p_2^{\mathrm{RS}} - p_1^{\mathrm{RS}}\big) + \beta\alpha p_2^{\mathrm{RS}} \\ D_2^{\mathrm{RS}} = b - \alpha p_2^{\mathrm{RS}} + \tau g_2^{\mathrm{RS}} + \theta\big(p_1^{\mathrm{RS}} - p_2^{\mathrm{RS}}\big) + \beta\alpha p_1^{\mathrm{RS}}. \end{cases}$$

Next, we discuss a third model named the "Vertical Collaboration Model" in which a retailer collaborates with one of the manufacturers.

3.3. Vertical collaboration model

Multiple manufacturers may be unable to collaborate because they are all competing for market share with their own goods. In this model, the retailer has more decision-making authority, and he may select a manufacturer to build a vertical collaboration to increase his profit.

This section first describes the effects of the vertical collaboration on each participant's profit, and then compares the overall improved earnings to determine which vertical collaboration is best for the retailer. It is presumed that the complete profit is transferrable.

Case 1 (M1R). First, examine the vertical collaboration created by the retailer's and manufacturer 1. Wholesale price w_1 is irrelevant for this case. In this section we use "'" to denote the optimal solution. The profit function of this collaboration is defined as:

$$\Pi_{m1r}(p_1, p_2, g_1) = (p_1 - c_1)D_1 - \eta g_1^2 + (p_2 - w_2)D_2 \tag{28}$$

where p_1, p_2 and g_1 are decision variables.

The profit function of manufacturer 2 is still same as equation (3).

Solving $\left(\frac{\partial \Pi_{m1r}}{\partial p_1}\right) = 0$, $\left(\frac{\partial \Pi_{m1r}}{\partial p_2}\right) = 0$, and $\left(\frac{\partial \Pi_{m1r}}{\partial g_1}\right) = 0$ simultaneously for p_1, p_2, and g_1, we get,

$$\begin{aligned} p_1 &= \frac{1}{B_3}\big[\big((\beta^2-1)2c_1\alpha^2 + ((\beta-1)4c_1\theta - (\tau g_2 + b)2\beta - 2a)\alpha - (\tau g_2 + a + b)2\theta\big)\eta + c_1\tau^2(\alpha+\theta)\big] \\ p_2 &= \frac{1}{2B_3}\big[(\beta^2-1)4\eta w_2\alpha^2 + \big(((\beta-1)8w_2\theta - 4a\beta - 4\tau g_2 - 4b)\eta + \tau^2(\beta c_1 + w_2)\big)\alpha \\ &\quad - (\tau g_2 + a + b)4\theta\eta + \tau^2(c_1\theta + w_2\theta + \tau g_2 + b)\big] \\ g_1 &= \frac{-\tau}{B_3}\big[(\beta^2-1)c_1\alpha^2 + (2(\beta-1)c_1\theta + (\tau g_2 + b)\beta + a)\alpha + (\tau g_2 + a + b)\theta\big]. \end{aligned}$$

By putting the value of p_1, p_2, and g_1 from above into equation (3), we get

$$\Pi_{m2}(w_2, g_2) = \frac{1}{2}\big[-(\alpha+\theta)w_2^2 + (b + (\theta+\alpha\beta)c_1 + (\alpha+\theta)c_2 + \tau g_2)w_2 - (b + (\theta+\alpha\beta)c_1 + \tau g_2)c_2\big] - \eta g_2^2.$$

The optimum wholesale price (w_2') and GL (g_2') of manufacturer 2 are determined as:

$$\begin{cases} w_2' = \frac{((4\beta c_1 + 4c_2)\alpha + (4c_1 + 4c_2)\theta + 4b)\eta - c_2\tau^2}{8(\alpha+\theta)\eta - \tau^2}, \\ g_2' \; = \frac{(\beta c_1 - c_2)\alpha + (c_1 - c_2)\theta + b)\tau}{8(\alpha+\theta)\eta - \tau^2}. \end{cases} \tag{29}$$

Hence, profit of manufacturer 2 in this case is given by:

$$\Pi_{m2}' = \frac{\big([(\beta c_1 - c_2)\alpha + (c_1 - c_2)\theta + b]^2\eta\big)}{8(\alpha+\theta)\eta - \tau^2}. \tag{30}$$

Again, using the value of $w = w_2'$ and $g = g_2'$ from equation (29), we obtain optimal sales prices of two green products and GL of I product as:

$$p_1' = \frac{1}{B_3}\big[\big(2(\beta^2-1)c_1\alpha^2 + (4(\beta-1)c_1\theta - 2(\tau g_2' + b)\beta - 2b)\alpha - 2(\tau g_2' + a + b)\theta\big)\eta + c_1\tau^2(\alpha+\theta)\big] \tag{31}$$

$$p_2' = \frac{1}{2B_3}\big[(\beta^2-1)4\eta w_2'\alpha^2 + \big((8(\beta-1)w_2'\theta - 4a\beta - 4\tau g_2' - 4b)\eta + \tau^2(\beta c_1 + w_2')\big)\alpha - 4(\tau g_2' + a + b)\theta\eta + \tau^2(c_1\theta + w_2'\theta + \tau g_2' + b)\big] \tag{32}$$

$$g_1' = \frac{-\tau}{B_3}\big[(\beta^2-1)c_1\alpha^2 + ((\beta-1)c_1\theta + (\tau g_2' + b)\beta + a)\alpha + (\tau g_2' + a + b)\theta\big] \tag{33}$$

where,

$$B_3 = 4\alpha\eta((\beta+1)\alpha + 2\theta)(\beta-1) + \tau^2(\alpha+\theta).$$

The overall profit of the collaboration is calculated using equations (31)–(33), as follows:

$$(\Pi_{m1r})' = (p_1' - c_1)D_1' - \eta(g_1')^2 + (p_2' - w_2')D_2' \tag{34}$$

where,

$$\begin{cases} D_1' = a - \alpha p_1' + \tau g_1' + \theta(p_2' - p_1') + \beta\alpha p_2' \\ D_2' = b - \alpha p_2' + \tau g_2' + \theta(p_1' - p_2') + \beta\alpha p_1'. \end{cases}$$

Case 2 (M2R). Next, the scenario when the retailer selects manufacturer 2 to create a collaboration is considered. The wholesale price w_2 is irrelevant in this circumstance. In this section we use "$\circ$" to denote the optimal solution. The profit function of this collaboration is defined as:

$$\Pi_{m2r}(p_1, p_2, g_2) = (p_1 - w_1)D_1 - \eta g_2^2 + (p_2 - c_2)D_2. \tag{35}$$

In a similar way to case 1, by solving $\frac{\partial \Pi_{m2r}}{\partial p_1} = 0$, $\frac{\partial \Pi_{m2r}}{\partial p_2} = 0$ and $\frac{\partial \Pi_{m2r}}{\partial g_2} = 0$, due to the symmetry of the parameters as in case 1, we get the following optimal solutions:

$$\begin{cases} w_1^\circ = \frac{((4\beta c_2 + 4c_1)\alpha + (4c_1 + 4c_2)\theta + 4a)\eta - c_1\tau^2}{8(\alpha+\theta)\eta - \tau^2} \\ g_1^\circ = \frac{(\beta c_2 - c_1)\alpha + (c_2 - c_1)\theta + a)\tau}{8(\alpha+\theta)\eta - \tau^2}. \end{cases} \tag{36}$$

And optimal sales prices of two green products are as follows:

$$p_1^\circ = \frac{1}{2B_3}\big[4(\beta^2-1)\eta w_1^\circ\alpha^2 + \big((8(\beta-1)w_1^\circ\theta - 4b\beta - 4\tau g_1^\circ - 4a)\eta + \tau^2(\beta c_1 + w_1^\circ)\big)\alpha - 4(\tau g_1^\circ + a + b)\theta\eta + \tau^2(c_2\theta + w_1^\circ\theta + \tau g_1^\circ + a)\big] \tag{37}$$

$$p_2^\circ = \frac{1}{B_3}\big[\big(2(\beta^2-1)c_2\alpha^2 + (4(\beta-1)c_2\theta - 2(\tau g_1^\circ + a)\beta - 2b)\alpha - 2(\tau g_1^\circ + a + b)\theta\big)\eta + c_2\tau^2(\alpha+\theta)\big] \tag{38}$$

$$g_2^\circ = \frac{-\tau}{B_3}\big[(\beta^2-1)c_2\alpha^2 + (2(\beta-1)c_2\theta + (\tau g_1^\circ + a)\beta + b)\alpha + (\tau g_1^\circ + a + b)\theta\big]. \tag{39}$$

The overall profit of the collaboration is calculated using equations (37)–(39) as follows:

$$(\Pi_{m2r})^\circ = (p_1^\circ - w_1^\circ)D_1^\circ - \eta(g_2^\circ)^2 + (p_2^\circ - c_2)D_2^\circ \tag{40}$$

where,

$$\begin{cases} D_1^\circ = a - \alpha p_1^\circ + \tau g_1^\circ + \theta(p_2^\circ - p_1^\circ) + \beta\alpha p_2^\circ \\ D_2^\circ = b - \alpha p_2^\circ + \tau g_2^\circ + \theta(p_1^\circ - p_2^\circ) + \beta\alpha p_1^\circ. \end{cases}$$

And in this case the profit of manufacturer 1 is given by

$$\Pi_{m1}^\circ = \frac{[(\beta c_2 - c_1)\alpha + (c_2 - c_1)\theta + a]^2\eta}{8(\alpha+\theta)\eta - \tau^2}. \tag{41}$$

Next, we compare the selling prices and GLs for trilateral competition (MS) model and vertical collaboration model. For this, we propose the following theorem.

Theorem 4. *When a vertical collaboration develops, the sales prices of the two items are lower, and the GLs of the product i $(i = 1, 2)$ are greater only when the manufacturer i $(i = 1, 2)$ collaborates with the retailer respectively, otherwise, it will decrease compared to those in the trilateral competition (MS) model.*

Proof. Considering the assumptions $a - \alpha c_1 > 0$, $b - \alpha c_2 > 0$, and $0 < \beta < 1$, we can easily show that,

$$\begin{array}{llll}
\text{(i)} & p_1' - p_1^{\text{MS}} < 0 & \text{(ii)} & p_2' - p_2^{\text{MS}} < 0 \\
\text{(iii)} & p_1^{\circ} - p_1^{\text{MS}} < 0 & \text{(iv)} & p_2^{\circ} - p_2^{\text{MS}} < 0 \\
\text{(v)} & g_1' - g_1^{\text{MS}} > 0 & \text{(vi)} & g_2' - g_2^{\text{MS}} < 0 \\
\text{(vii)} & g_1^{\circ} - g_1^{\text{MS}} < 0 & \text{(viii)} & g_2^{\circ} - g_2^{\text{MS}} > 0.
\end{array}$$

This observation establishes a fact that, under a trilateral competition (MS) model, vertical collaboration will result in reduced sales prices and increased GLs for each product in order to produce bigger sales volumes. We may see these results numerically in Table 3. This condition can be summarised as "more greening activities at a lower price". □

Now, using the collaboration created by the retailer and manufacturer 1, we examine the change in profit on the collaboration and on the noncooperative manufacturer. In actuality, a comparable technique may be used to get the same result in a different scenario. Since $(\Pi_{m1}^{\text{MS}} + \Pi_r^{\text{MS}})$ and $(\Pi_{m1r})'$ have complex expressions, reducing fractions to a common denominator will result in additional items, making direct comparison difficult. As a result, we use an analytical approach to show how they are related

Theorem 5. *In the trilateral competition (MS) model, the overall profit of the collaboration exceeds the combined profit of the corresponding two competitors.*

i.e.

$$\left(\Pi_{m1}^{\text{MS}} + \Pi_r^{\text{MS}}\right) < (\Pi_{m1r})_{[(p_1',p_2',g_1'),w_2']}.$$

Proof. See in Appendix C. □

Here, left expression represents the combined profit earned by the manufacturer 1 and retailer in the trilateral competitive game (MS), while the right expression represents the overall profit of the collaboration. Same technique can be used to examine the case when the retailer and manufacturer 2 combine. We may see this result numerically in Table 3. In a similar way we can show for RS game

$$\left(\Pi_{m1}^{\text{RS}} + \Pi_r^{\text{RS}}\right) < (\Pi_{m1r})_{\left[(p_1',p_2',g_1'),w_2'\right]}.$$

Theorem 6. *When compared to the trilateral competitive (MS) game, the manufacturer who is not a member of the collaboration will see his or her profit decrease.*

Proof. Since the profit of manufacturer 2 in the trilateral competitive game is given by

$$\Pi_{m2}^{\text{MS}} = \left(p_2^{\text{MS}} - w_2^{\text{MS}}\right) D_2^{\text{MS}} - \eta\left(g_2^{\text{MS}}\right)^2.$$

And manufacturer 2's profit when retailer collaborate with manufacturer 1

$$\Pi_{m2}' = \frac{[(\beta c_1 - c_2)\alpha + (c_1 - c_2)\theta + b]^2 \eta}{8(\alpha + \theta)\eta - \tau^2}.$$

From these two expressions, using $a-\alpha c_1>0$, $b-\alpha c_2>0$, $\beta\eta\alpha>\tau^2$, and $0<\beta<1$, we can show that $\Pi_{m2}^{\mathrm{MS}}-\Pi_{m2}^{'}>0$. This result can be observed numerically in Table 3. The proof demonstrates that manufacturer 2's profit decreases when the retailer and manufacturer 1 create a vertical collaboration. A similar conclusion can be derived for manufacturer 1 if the retailer and manufacturer 2 create a vertical collaboration. Actually, the vertical collaboration reduces the non-cooperation manufacturer's negotiating strength; hence, Theorem 6 makes sense. □

In addition, we take into account the question of which manufacturer the retailer would select in order to optimise the overall improved earnings. The following expression is used to simplify the described issue.

$$\begin{aligned}&(\Pi_{m1r})^{'}-\Pi_{m1}^{\mathrm{MS}}-\Pi_{r}^{\mathrm{MS}}-\left[(\Pi_{m2r})^{\circ}-\Pi_{m2}^{\mathrm{MS}}-\Pi_{r}^{\mathrm{MS}}\right]\\&=(\Pi_{m1r})^{'}-(\Pi_{m2r})^{\circ}-\left(\Pi_{m1}^{\mathrm{MS}}-\Pi_{m2}^{\mathrm{MS}}\right)\\&=3((\beta-1)(c_1+c_2)\alpha+a+b)\eta^2KX\end{aligned}$$

where, $K=a-b+(c_2-c_1)(\alpha+\alpha\beta+2\theta)$

and,

$$X=\frac{\begin{array}{c}(256(\beta-1)((3\beta^2-4)\alpha^2+2\theta(3\beta-4)\alpha-\theta^2)\alpha(\beta\alpha+\alpha+2\theta)(\alpha+\theta)\eta^3\\+64\tau^2((3\beta^2-4)\alpha^2+2\theta(3\beta-4)\alpha-\theta^2)(\alpha+\theta)^2\eta^2\\+16\tau^4((2\beta^2+1)\alpha^2+\theta(2\beta+1)\alpha+3\theta^2)(\alpha+\theta)\eta\\-4\tau^6(\alpha\beta+\theta)^2)\end{array}}{\begin{array}{c}(4(\alpha\beta+2\alpha+3\theta)\eta-\tau^2)(8(\alpha+\theta)-\tau^2)^2\\(4(\beta-1)\alpha(\alpha\beta+\alpha+2\theta)\eta+\tau^2(\alpha+\theta))(4(\alpha\beta-2\alpha-\theta)+\tau^2)\end{array}}.$$

Here we observe that the expression

$$K=a-b+(c_2-c_1)(\alpha+\alpha\beta+2\theta)\tag{42}$$

decide which manufacturer is better co-operator for retailer. For this we propose the following theorem.

Theorem 7. *If $K>0$, the retailer prefers selecting manufacturer 1 for establishing a pricing collaboration. If $K<0$, cooperating with manufacturer 2 is preferable for the retailer. Where,*

$$K=a-b+(c_2-c_1)(\alpha+\alpha\beta+2\theta).$$

Proof. According to $a-\alpha c_1>0$, $b-\alpha c_2>0$, and $0<\beta<1$, we have

$$(\Pi_{m1r})^{'}-(\Pi_{m2r})^{\circ}-\left(\Pi_{m1}^{\mathrm{MS}}-\Pi_{m2}^{\mathrm{MS}}\right)=3((\beta-1)(c_1+c_2)\alpha+a+b)\eta^2KX$$

where, $K=a-b+(c_2-c_1)(\alpha+\alpha\beta+2\theta)$ and $X>0$ (Proof is given in Appendix D).

Hence,

- If $K>0$, then $(\Pi_{m1r})^{'}-(\Pi_{m2r})^{\circ}-(\Pi_{m1}^{\mathrm{MS}}-\Pi_{m2}^{\mathrm{MS}})>0$, and the retailer chooses manufacturer 1.
- If $K<0$, then $(\Pi_{m1r})^{'}-(\Pi_{m2r})^{\circ}-(\Pi_{m1}^{\mathrm{MS}}-\Pi_{m2}^{\mathrm{MS}})<0$, and the retailer chooses manufacturer 2.

In the explanation above, comparing the overall enhanced profits is the method for choosing the best vertical partnership. □

In the following part, we will explore the stability of the ideal collaboration and suggest ways to distribute the additional benefit. In reality, it appears that the degree of substitution, the difference in value of basic market demands and difference in value of production costs will determine which producer will be preferred for collaboration with the retailer. As a result of the debate above, we are aware that the expression $K=a-b+(c_2-c_1)(\alpha+\alpha\beta+2\theta)$, is eligible for determining which manufacturer generates the most sales and earnings and also for determining which manufacturer will cooperate best with the retailer.

Following that, we demonstrate the link between each provided by the manufacturer's sales volume and the retailer's optimal co-operator. For this, we propose the following theorem.

Theorem 8. *The optimal manufacturer to collaborate with the retailer is the one that generates the highest sales volumes.*

Proof. The expression $K = a - b + (c_2 - c_1)(\alpha + \alpha\beta + 2\theta)$, as given in equation (16), determines which producer has the highest sales volumes and earnings. Now, using the same formulation $K = a - b + (c_2 - c_1)(\alpha + \alpha\beta + 2\theta)$ from equation (42), we determine which producer is a better co-operator for the retailer:

– If $K > 0$, then $D_1^{\mathrm{MS}} - D_2^{\mathrm{MS}} > 0$, which implies that $(\Pi_{m1r})' - (\Pi_{m2r})^\circ - (\Pi_{m1}^{\mathrm{MS}} - \Pi_{m2}^{\mathrm{MS}}) > 0$. Therefore, the retailer collaborates with manufacturer 1.
– If $K < 0$, then $D_1^{\mathrm{MS}} - D_2^{\mathrm{MS}} < 0$, which implies that $(\Pi_{m1r})' - (\Pi_{m2r})^\circ - (\Pi_{m1}^{\mathrm{MS}} - \Pi_{m2}^{\mathrm{MS}}) < 0$. Therefore, the retailer collaborates with manufacturer 2.

□

4. Stability of the vertical collaboration

A collaboration is only stable when none of its members have any reason to leave. In this part, we look at the stability of the optimum collaboration while taking the retailer's dominance into account. Without loss of generality, we will consider that the retailer and manufacturer 1 form the optimal vertical collaboration in the discussion that follows. Similar analysis and conclusions may be applied to the other circumstance.

First, depending on retailer's dominance, the question of how much profit the retailer should share at most arises. We suppose that retailer presents the distribution plan, and the other member chooses whether or not to accept, based on retailer's dominance. The stability of the optimal partnership in this situation is then taken into account under the assumption that the two firms are conservative.

If manufacturer 1 refuses to work with the retailer, then the retailer will create a partnership with manufacturer 2. Manufacturer 1 here just receives Π_{m1}°. Similarly, if manufacturer 2 refuses to work with the retailer, then the retailer will collaborate with manufacturer 1. Manufacturer 2 here just receives Π'_{m2}.

So, in the collaboration between the retailer and manufacturer 1, the retailer might receive the greatest share

$$(\Pi_{m1r})' - \Pi_{m1}^\circ. \tag{43}$$

Similar to this, under the collaboration between the retailer and manufacturer 2, the retailer could only share

$$(\Pi_{m2r})^\circ - \Pi'_{m2}. \tag{44}$$

Now, we solve and simplify the following expressions

$$\begin{aligned}(\Pi_{m1r})' &- \Pi_{m1}^\circ - [(\Pi_{m2r})^\circ - \Pi'_{m2}] \\ &= \frac{((\beta-1)(c_1+c_2)\alpha+a+b)\eta^2 K\left(\eta(\beta^2-1)\alpha^3+\left(\theta(\beta+3)(\beta-1)\eta+\frac{\tau^2\beta^2}{4}\right)\alpha^2+2\theta\left((\beta-1)\theta\eta+\frac{\tau^2\beta}{4}\right)\alpha+\frac{\tau^2\theta^2}{4}\right)}{16(\alpha\eta+\eta\theta-\frac{1}{8}\tau^2)^2\left(\eta(\beta^2-1)\alpha^2+\left(2(\beta-1)\theta\eta+\frac{\tau^2}{4}\right)\alpha+\frac{\tau^2\theta}{4}\right)}\end{aligned}$$

and using Theorem 7, we have

$$(\Pi_{m1r})' - \Pi_{m1}^\circ - \left[(\Pi_{m2r})^\circ - \Pi'_{m2}\right] > 0. \tag{45}$$

This suggests that a collaboration between the retailer and manufacturer 1 might result in higher earnings. Given the foregoing, the optimal vertical collaboration is stable when the two manufacturers are risk-averse and unable to bear any risk.

Next, we examine the scenario in which the retailer is likewise conservative. When the retailer claims

$$(\Pi_{m1r})' - \Pi_{m1}^\circ$$

from the ideal collaboration, manufacturer 1 may decline this allocation, resulting in decreased retailer earnings. As a result, the retailer's strong optimal solution is to request for

$$(\Pi_{m2r})^{\circ} - \Pi'_{m2}. \tag{46}$$

In this case, the profit of manufacturer 1 is:

$$(\Pi_{m1r})' - [(\Pi_{m2r})^{\circ} - \Pi'_{m2}] = (\Pi_{m1r})' - (\Pi_{m2r})^{\circ} + \Pi'_{m2}. \tag{47}$$

From equation (45), we can see that:

$$(\Pi_{m1r})' - (\Pi_{m2r})^{\circ} + \Pi'_{m2} > \Pi^{\circ}_{m1}.$$

So, manufacturer 1 and the retailer have no reason to leave the partnership under the terms of this allocation. Hence, the optimal vertical collaboration is stable in this situation.

The retailer's profit as estimated by equation (44) appears fairer than the one provided by equation (43), since manufacturer 1 also has to increase his income. Therefore, we decide to use this rule to divide earnings between the two alliance partners.

By Theorem 5 and 6, we have:

$$(\Pi_{m2r})^{\circ} - \Pi'_{m2} > \Pi^{\mathrm{MS}}_{m2} + \Pi^{\mathrm{MS}}_{r} - \Pi'_{m2} > \Pi^{\mathrm{MS}}_{r}.$$

It indicates that the retailer's profit mentioned in equation (44) is greater than the profit in the trilateral competition (MS) game.

However, upon joining the price collaboration, the profit of manufacturer 1 may be smaller than in the trilateral competition game. This will be demonstrated in the numerical demonstration.

5. Numerical analysis and sensitivity analysis

5.1. Numerical analysis

A numerical experiment is conducted in this part to demonstrate the use of the suggested model and solution method. This numerical example is evaluated by using software *MAPLE*. Here, the value of numerical data are taken from [14,21] with some adjustments as: production cost per unit of green product I $c_1 = 140$, production cost per unit of green product II $c_2 = 100$, marginal demand $\alpha = 1.8$, degree of substitution between the two green products $\beta = 0.5$, sensitivity of switchover toward price difference $\theta = 0.3$, Consumer Environmental Awareness (CEA) level $\tau = 0.7$, Green investment efficiency $\eta = 20$, primary market potential of green product I $a = 500$, primary market potential of green product II $b = 350$. Table 3 shows the outputs of decision variables in different scenarios.

First, we analyse the profitability of manufacturers and retailers under a trilateral competition and a partial collaboration scenario. Given that the optimal co-operator of the retailer may vary depending on which pair of (a, b) is used, we will only look at the scenario where manufacturer 1 is a part of the optimal vertical collaboration in this instance. Given $a \in [500, 1200]$ and $b = 350$. It is obvious that equation (42) is true, ensuring that manufacturer 1 is the best partner for the retailer.

In a condition of trilateral competition model (MS and RS), the profit functions of the retailer are provided by equations (19) and (27) respectively, while in a situation of vertical collaboration, it is given by equation (44). The graph of the profit of the retailer for different a under different scenarios is given in Figure 1. The profit of the retailer in the RS game gradually increases more than in the vertical collaboration model when the difference between a and b increases.

Next, the profit functions of manufacturer 1 in the scenario of trilateral competition model (MS and RS) are provided by equations (17) and (25) respectively, while the profit function of manufacturer 1 in the scenario of partial collaboration is provided by equation (47). The graph of the profit of the manufacturer 1 for different

TABLE 3. Optimal outputs.

Optimal decisions	MS	RS	M1R	M2R
p_1	371.61	371.35	318.33	356.94
p_2	327.21	326.96	312.06	275.60
g_1	0.94	0.94	3.12	0.68
g_2	0.91	0.91	0.64	3.07
Π_{m1}	12 127.95	6079.80	–	6534.34
Π_{m2}	11 387.01	5708.69	5654.91	–
Π_r	27 469.24	39 244.06	–	–
$(\Pi_{m1r})'$	–	–	47 617.33	–
$(\Pi_{m2r})^\circ$	–	–	–	46 595.42

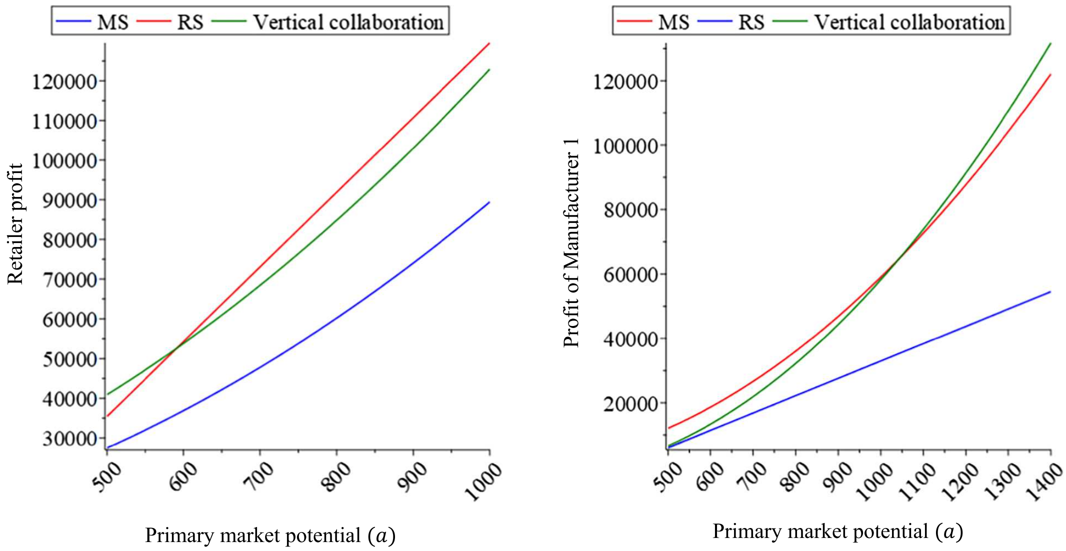

FIGURE 1. Profit of retailer and manufacturer 1 under different scenarios.

$a \in [500, 1400]$ under different scenarios is given by Figure 1. The profit of manufacturer 1 in RS game is lower than that of MS game and vertical collaboration model, but profit in the vertical collaboration model gradually increases more than in the MS game when the difference between a and b increases.

5.2. Sensitivity analysis

We evaluate the effects of changes in parameters. By changing the parameter value independently while keeping the remaining parameters fixed, the sensitivity analysis has been carried out numerically. We take the collaboration created by the retailer and manufacturer 1 to calculate the sensitivity analysis. We can use the same procedure to examine the case when the retailer and manufacturer 2 combine. Π_{m1}^{MS} and Π_r^{MS} are the profits of manufacturer 1 and retailer in the trilateral game (MS), respectively, while $(\Pi_{m1r})'$ is the overall profit of the collaboration. Table 4 demonstrates the numerical output of parameters. As the value of θ increases, we can see that the profit of manufacturer 1 decreases while the profits of the retailer and collaboration model increase. As the value of α increases, we can see that the profit of manufacturer 1 decreases as well as the profits of the retailer and collaboration model also decrease. As the value of τ increases, we can see that the profit of

Table 4. Sensitivity analysis of parameter variations.

Parameters	Value	% Change	Π_{m1}^{MS}	Π_{r}^{MS}	Π_{r}^{RS}	$(\Pi_{m1r})'$
	0.21	−30	12 503.03	26 751.35	39 254.96	47 458.97
	0.24	−20	12 376.11	26 994.38	39 251.33	47 513.09
θ	0.27	−10	12 251.10	27 233.65	39 247.69	47 565.86
	0.30	0	12 127.95	27 469.24	39 244.06	47 617.33
	0.33	+10	12 006.63	27 701.22	39 240.43	47 667.55
	0.36	+20	11 887.09	27 929.67	39 236.80	47 716.57
	0.39	+30	11 769.29	28 154.66	39 233.16	47 764.42
	1.26	−30	21 509.02	49 457.88	72 498.32	84 999.75
	1.44	−20	17 608.58	40 042.44	61 413.57	69 180.11
α	1.62	−10	14 561.08	32 960.89	50 328.81	57 099.10
	1.80	0	12 127.95	27 469.24	39 244.06	47 617.33
	1.98	+10	10 151.94	23 107.50	28 159.31	40 015.06
	2.16	+20	8525.22	19 576.56	17 074.56	33 815.46
	2.34	+30	7171.39	16 673.79	5989.80	28 690.24
	0.49	−30	12 111.87	27 412.01	39 175.38	47 490.65
	0.56	−20	12 116.60	27 428.82	39 198.28	47 527.85
τ	0.63	−10	12 121.96	27 447.90	39 221.17	47 570.07
	0.70	0	12 127.95	27 469.24	39 244.06	47 617.33
	0.77	+10	12 134.59	27 492.86	39 266.95	47 669.67
	0.84	+20	12 141.86	27 518.76	39 289.85	47 727.10
	0.91	+30	12 149.77	27 546.95	39 312.74	47 789.67

Table 5. Effect of parameter changes on different variables.

Parameters	Change in parameters	Π_{m1}^{MS}	Π_{r}^{MS}	Π_{r}^{RS}	$(\Pi_{m1r})'$
θ	Increases	Decrease	Increase	Decrease	Increase
α	Increases	Decrease	Decrease	Decrease	Decrease
τ	Increases	Increase	Increase	Increase	Increase

manufacturer 1 increases as well as the profits of the retailer and collaboration model also increase. All of these analyses can be seen in Figures 2 and 3 and Table 5.

In the Figures 2 and 3 we use some abbreviations as:

– M1(MS) = profit of manufacturer 1 in MS game.
– R (MS) = profit of retailer in MS game.
– M1R = overall profit of the collaboration when collaboration is created by the retailer and manufacturer 1.
– R (RS) = profit of retailer in RS game.

5.3. Managerial insights

This study has various management implications for supply chain managers and sustainability specialists, the most significant of which is balancing product pricing and green quality. Our study reveals that the SC managers can offer more green products with lower prices by using the proposed contract. Our research shows that the SC managers may use the suggested partnership to provide more green items at reduced rates. In fact, proper coordination amongst the SC decision-makers under the suggested system not only lowers the selling prices but also increases customer satisfaction among environmentally conscious consumers. A vertical collaboration

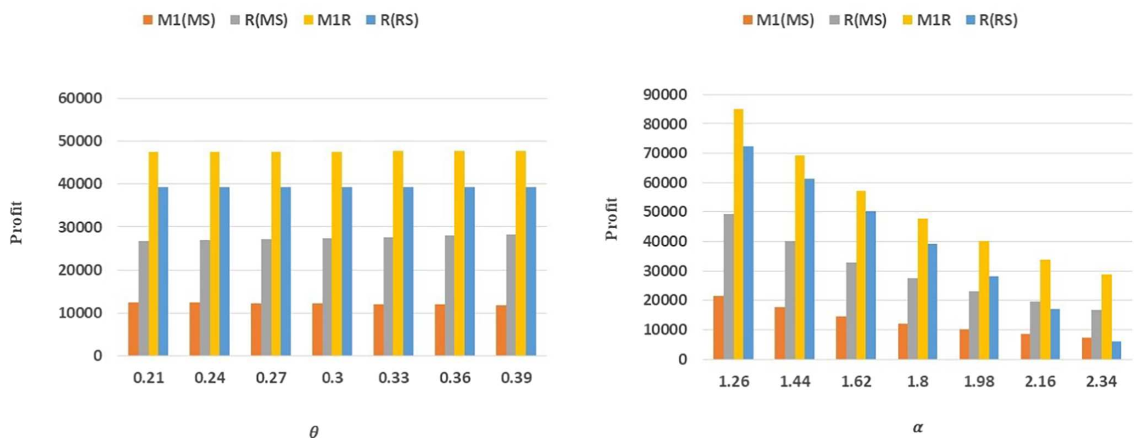

FIGURE 2. Sensitivity analysis when θ and α are changed.

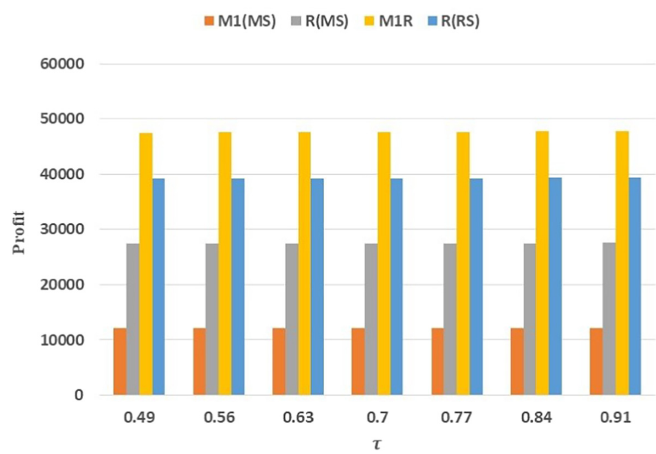

FIGURE 3. Sensitivity analysis when CEA level (τ) is changed.

strategy can increase profits for both alliance members while also stimulating demand by lowering the prices of green products. The manager should mainly concentrate on marginal demand because this parameter is more sensitive than other parameters.

6. Conclusions

This research examines the pricing and GL decisions of duopolistic competitive manufacturers who make substitutable green items and sell them through a common retailer. We investigate the optimal decisions in three scenarios namely; trilateral competition MS, RS, and vertical collaboration. First, in a trilateral competitive model (MS), both manufacturers work as Stackelberg leader and retailer work as follower and result shows that the manufacturer with a bigger sales volume achieves superior performance in terms of earnings. Second, in a trilateral competitive model (RS), retailer work as Stackelberg leader and both manufacturers work as follower and result shows that profits of both manufacturers are lower than MS model. Thirdly, a vertical collaboration model is created in which a manufacturer and retailer collaborate to optimise pricing issues, GLs, and profits. Our investigation reveals that vertical collaboration can achieve channel coordination and allows both alliance members to gain more profit than the lateral competition scenario. The following are some theoretic contributions made by this study:

(i) The equivalent relationship between sales volume and profit for each firm is displayed.
(ii) A selection criterion is developed for retailer to select the most suitable manufacturer for vertical collaboration.
(iii) The link between each manufacturer's sales volume and the retailer's suitable partner is examined.
(iv) When a vertical collaboration develops, the sales prices of the two items are lower and GLs are greater than those in the trilateral competition model (MS).
(v) The overall profit of the collaboration exceeds the combined profit of the corresponding two competitors in the trilateral competition models MS and RS.

An allocation mechanism for the vertical collaboration is provided by taking into account the retailer's dominance, which is the condition under which the collaboration is stable. The numerical example shows the valuable improvement in total profit between vertical collaboration and the trilateral competition model. For future research, the established model is limited with two items, which may be further extended by assuming multi-items. The degree of substitution for each product may vary in practice, which has not been considered in this work. In this study, it is assumed that both manufacturers produce green items; hence, another avenue of research might investigate scenarios in which one manufacturer produces green products and another produces non-green products [27]. We propose a vertical collaboration model in this paper. Therefore, one can also study the effect of revenue sharing, cost-sharing, or a two-part tariff contract [14, 37].

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Appendix A.

Proof of Theorem 1. In accordance with the decision order, the backward derivation is used. With w_1, g_1, w_2 and g_2 fixed, we deal with the retailer's goal function. Solving $\frac{\partial \Pi_r}{\partial p_1} = 0$ and $\frac{\partial \Pi_r}{\partial p_2} = 0$ simultaneously for p_1 and p_2, we get:

$$\begin{cases} p_1 = \frac{(\beta^2-1)w_1\alpha^2+(2w_1\beta\theta-2w_1\theta-\tau g_2\beta-b\beta-\tau g_1-a)\alpha-\theta(g_1\tau+g_2\tau+a+b)}{2(\beta-1)(\beta\alpha+\alpha+2\theta)\alpha} \\ p_2 = \frac{(\beta^2-1)w_2\alpha^2+(2w_2\beta\theta-2w_2\theta-\tau g_1\beta-a\beta-\tau g_2-b)\alpha-\theta(g_1\tau+g_2\tau+a+b)}{2(\beta-1)(\beta\alpha+\alpha+2\theta)\alpha}. \end{cases}$$

Now, the Hessian matrix of $\Pi_r(p_1, p_2)$ is:

$$H = \begin{pmatrix} -2(\beta+\theta) & 2(\theta+\beta\alpha) \\ 2(\theta+\beta\alpha) & -2(\beta+\theta) \end{pmatrix}.$$

For $0 < \beta < 1$, $\theta > 0$ and $\alpha > 0$, H is negative definite. As a result, the above solution is the optimal solution of $\max \Pi_r(p_1, p_2)$.

Further, using Bertrand game theory between manufacturers, by substituting (5) and (6) into (2) and (3), the profit functions of manufacturer 1 and 2 are converted into:

$$\Pi_{m1} = \frac{1}{2}\left[-(\alpha+\theta)w_1^2 + (\alpha c_1 + \theta c_1 + \alpha\beta w_2 + w_2\theta + \tau g_1 + a)w_1 - (\alpha\beta w_2 + \tau g_1 + w_2\theta + a)c_1\right] - \eta g_1^2$$
$$\Pi_{m2} = \frac{1}{2}\left[-(\alpha+\theta)w_2^2 + (\alpha c_2 + \theta c_2 + \alpha\beta w_1 + w_1\theta + \tau g_2 + b)w_2 - (\alpha\beta w_1 + \tau g_2 + w_1\theta + b)c_2\right] - \eta g_2^2.$$

Both Π_{m1} and Π_{m2} are viewed as two-variable functions for themselves, based on each participant's decision variable. Therefore, optimal wholesale prices and GLs for both manufacturers can be derived by solving simultaneously $\frac{\partial \Pi_{m1}}{\partial w_1} = 0$, $\frac{\partial \Pi_{m1}}{\partial g_1} = 0$, $\frac{\partial \Pi_{m2}}{\partial w_2} = 0$, and $\frac{\partial \Pi_{m2}}{\partial g_2} = 0$.

Now, the Hessian matrix of $\Pi_{mi}(w_i, g_i)$ is:

$$H_{mi} = \begin{vmatrix} \frac{\partial^2 \Pi_{mi}}{\partial w_i^2} & \frac{\partial^2 \Pi_{mi}}{\partial w_i \partial g_i} \\ \frac{\partial^2 \Pi_{mi}}{\partial w_i \partial g_i} & \frac{\partial^2 \Pi_{mi}}{\partial g_i^2} \end{vmatrix} = \begin{vmatrix} -(\alpha+\theta) & \frac{\tau}{2} \\ \frac{\tau}{2} & -2\eta \end{vmatrix} = \frac{8\eta(\alpha+\theta) - \tau^2}{4}.$$

According to our assumption $\beta\eta\alpha > \tau^2$, profit functions for both manufacturers are concave. So, $(w_1^{\mathrm{MS}}, g_1^{\mathrm{MS}})$ and $(w_2^{\mathrm{MS}}, g_2^{\mathrm{MS}})$ are the unique solutions of $\max \Pi_{m1}$ and $\max \Pi_{m2}$ respectively. By substituting these optimal values $(w_1^{\mathrm{MS}}, g_1^{\mathrm{MS}})$ and $(w_2^{\mathrm{MS}}, g_2^{\mathrm{MS}})$ in equations (5) and (6), we get the required optimal value of p_1^{MS} and p_2^{MS}. This completes the proof. □

Appendix B.

Proof of Theorem 2. First, the two manufacturers' decisions are calculated by substituting $m_i^{\mathrm{RS}} = p_i^{\mathrm{RS}} - w_i^{\mathrm{RS}}$ in equations (2) and (3), which indicate the per-unit profit margin for product i. Therefore, optimal wholesale prices and GLs for both manufacturers can be derived by solving simultaneously $\frac{\partial \Pi_{m1}}{\partial w_1} = 0$, $\frac{\partial \Pi_{m1}}{\partial g_1} = 0$, $\frac{\partial \Pi_{m2}}{\partial w_2} = 0$, and $\frac{\partial \Pi_{m2}}{\partial g_2} = 0$. After simplification, wholesale prices w_i^{RS} and GLs g_i^{RS} are obtained in terms of m_i^{RS}.

Using the values of w_i^{RS} and g_i^{RS} in the profit function of the retailer given by equation (4), we get Π_r^{RS} in terms of m_i^{RS}. Hence, profit margins are calculated by solving $\frac{\partial \Pi_r^{\mathrm{RS}}}{\partial m_i^{\mathrm{RS}}} = 0$, $\frac{\partial \Pi_r^{\mathrm{RS}}}{\partial m_i^{\mathrm{RS}}} = 0$ simultaneously. After simplification, profit margins m_i^{RS} are obtained in equation (20).

Concavity can be shown in a similar way as in the proof of Theorem 1. □

Appendix C.

Proof of Theorem 5. Consider the collaboration formed between the retailer and manufacturer 1. $\Pi_{m1r}(p_1, p_2, g_1)$ is a quadratic function with a deterministic expression, so equations (31)–(33) are optimal solutions for $\max \Pi_{m1r}(p_1, p_2, g_1)$, which indicates that the value of $\Pi_{m1r}(p_1, p_2, g_1)$ is lower for different values of p_1, p_2, and g_1. We know that $(p_1', p_2^{\mathrm{MS}}, g_1^{\mathrm{MS}})$ is the optimal solution for $\max \Pi_{m1r}(p_1, p_2, g_1)$ for $w_2 = w_2^{\mathrm{MS}}$ (which is calculated by equation (10)). As a result, the value of $\Pi_{m1r}(p_1, p_2, g_1)$ under $(p_1', p_2^{\mathrm{MS}}, g_1^{\mathrm{MS}})$ is greater than the one under $(p_1^{\mathrm{MS}}, p_2^{\mathrm{MS}}, g_1^{\mathrm{MS}})$. To describe this relationship, we utilize the following inequality:

$$\Pi_{m1r[(p_1^{\mathrm{MS}}, p_2^{\mathrm{MS}}, g_1^{\mathrm{MS}}), w_2^{\mathrm{MS}}]} < \Pi_{m1r[(p_1', p_2^{\mathrm{MS}}, g_1^{\mathrm{MS}}), w_2^{\mathrm{MS}}]}.$$

Comparing w_2^{MS} and w_2' using the assumption $a - \alpha c_1 > 0$, we get

$$w_2' - w_2^{\mathrm{MS}} < 0.$$

As a result, when $w_2 = w_2'$ (provided by Eq. (29)) and the sales prices of the two items stay constant (p_1', p_2^*), we can see from equation (28) that the profit is greater than

$$(\Pi_{m1r})_{[(p_1', p_2^{\mathrm{MS}}, g_1^{\mathrm{MS}}), w_2^{\mathrm{MS}}]}.$$

So, from the above discussion, we can write as:

$$(\Pi_{m1r})_{[(p_1', p_2^{\mathrm{MS}}, g_1^{\mathrm{MS}}), w_2^{\mathrm{MS}}]} < (\Pi_{m1r})_{[(p_1', p_2^{\mathrm{MS}}, g_1^{\mathrm{MS}}), w_2']}. \tag{C.1}$$

Additionally, for $w_2 = w_2'$, (p_1', p_2') is the unique solution to $\max \Pi_{m1r}(p_1, p_2)$. Hence, we can see that

$$(\Pi_{m1r})_{[(p_1', p_2^{\mathrm{MS}}, g_1^{\mathrm{MS}}), w_2']} < (\Pi_{m1r})_{[(p_1', p_2', g_1'), w_2']}. \tag{C.2}$$

So, from (C.1) and (C.2) we have

$$(\Pi_{m1r})_{[(p_1^{\mathrm{MS}},p_2^{\mathrm{MS}},g_1^{\mathrm{MS}}),w_2^{\mathrm{MS}}]} < (\Pi_{m1r})_{[(p'_1,p'_2,g'_1),w'_2]}.$$

□

Appendix D.

Proof of $X > 0$ of Theorem 7.

$$X = \frac{\begin{array}{c}256(\beta-1)((3\beta^2-4)\alpha^2+2\theta(3\beta-4)\alpha-\theta^2)\alpha(\beta\alpha+\alpha+2\theta)(\alpha+\theta)\eta^3\\ +64\tau^2((3\beta^2-4)\alpha^2+2\theta(3\beta-4)\alpha-\theta^2)(\alpha+\theta)^2\eta^2\\ +16\tau^4((2\beta^2+1)\alpha^2+\theta(2\beta+1)\alpha+3\theta^2)(\alpha+\theta)\eta\\ -4\tau^6(\alpha\beta+\theta)^2\end{array}}{\begin{array}{c}(4(\alpha\beta+2\alpha+3\theta)\eta-\tau^2)(8(\alpha+\theta)-\tau^2)^2\\ (4(\beta-1)\alpha(\alpha\beta+\alpha+2\theta)\eta+\tau^2(\alpha+\theta))(4(\alpha\beta-2\alpha-\theta)+\tau^2)\end{array}}.$$

Let

$$A_1 = 256(\beta-1)((3\beta^2-4)\alpha^2+2\theta(3\beta-4)\alpha-\theta^2)\alpha(\beta\alpha+\alpha+2\theta)(\alpha+\theta)\eta^3 > 0 \quad (\text{as } 0<\beta<1).$$

And

$$A_2 = 64\tau^2((3\beta^2-4)\alpha^2+2\theta(3\beta-4)\alpha-\theta^2)(\alpha+\theta)^2\eta^2 < 0.$$

So,

$$A_1 > A_2.$$

Again, let

$$A_3 = 16\tau^4((2\beta^2+1)\alpha^2+\theta(2\beta+1)\alpha+3\theta^2)(\alpha+\theta)\eta.$$

and

$$A_4 = 4\tau^6(\alpha\beta+\theta)^2.$$

Considering the assumption $\beta\eta\alpha > \tau^2$ for $0<\beta<1$, we see that

$$A_3 > A_4.$$

So, the numerator of expression X is positive.

In a similar way, considering the assumptions, we can see that the denominator of expression X is

$$(4(\alpha\beta+2\alpha+3\theta)\eta-\tau^2)(8(\alpha+\theta)-\tau^2)^2(4(\beta-1)\alpha(\alpha\beta+\alpha+2\theta)\eta+\tau^2(\alpha+\theta))(4(\alpha\beta-2\alpha-\theta)+\tau^2) > 0.$$

Hence, $X > 0$ under the conditions of the Theorem 7. □

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