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OPTIMIZING SUBSTITUTION OF TWO PRODUCTS MODEL FOR EXPONENTIALLY INCREASING DEMAND UNDER INFLATION AND SHORTAGES

Gurudatt Rao Ambedkar[1], Chaman Singh[2] and Biswajit Sarkar[3,4,5*]

Abstract. This study addresses the limited attention given to product substitution in inventory models. Incorporating product substitution is crucial for determining reorder points, and safety stock, enabling businesses to optimize inventory levels, reduce costs, and maintain customer satisfaction. This study introduces an economic order quantity model tailored to an inflationary environment with shortages and one-way substitution between two deteriorating product types. Through comprehensive testing, this study evaluates the model under various substitution scenarios, including partial substitution. Findings highlight the significance of product substitution in inventory management, allowing businesses to optimize inventory levels, manage costs, and ensure customer satisfaction in dynamic environments with inflation and fluctuating product availability. This model provides the firm with the necessary information to determine the optimal ordering quantity of both products to optimize total benefit and enhance supply chain efficiency. The model demonstrates substantial cost advantages, with partial substitution resulting in an average cost reduction of approximately 9% compared to no substitution and about 45% compared to full substitution. Numerical experiments validate the applicability of the proposed model.

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1. Introduction

Inventory has a pivotal significance in all sectors of business. It is essential at every stage, from manufacturing to distribution and reaching the final customer. In today's rapidly changing environment, customer preferences evolve swiftly. When a desired product is unavailable, it can lead to customer frustration, which can have a detrimental impact on future revenue and the reputation of the product and business [57]. Understanding the customer's mindset becomes particularly challenging in cases of stock-out. Consumers may leave their preferred store and purchase the item from a competitor, wait for the product to become available, or settle for a similar alternative from their preferred retailer. This last option is commonly referred to as product substitution. In

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[1] Department of Mathematics, Hansraj College, University of Delhi, Delhi 110 007, India.

[2] Department of Mathematics, Acharya Narendra Dev College, University of Delhi, New Delhi 110019, India.

[3] Department of Industrial Engineering, Yonsei University, 50 Yonsei-ro, Sinchon-dong, Seodaemun-gu, Seoul 03722, South Korea.

[4] Center for Global Health Research, Saveetha Medical College, Saveetha Institute of Medical and Technical Sciences, Saveetha University, Chennai, Tamil Nadu 600077, India.

[5] Department of Mathematics, Lovely Professional University, Phagwara, Punjab 144411, India.

*Corresponding author: bsbiswajitsarkar@gmail.com

today's fast-paced world, customers often opt for a comparable product from the same store instead of going elsewhere, saving them time and effort. This choice may be influenced by the store's reputation, the brand, or the shopkeeper's recommendation of a suitable substitute. It is evident that the availability of one item in the inventory can impact the demand for other items as well. This situation is observed in various markets, including food and fruit products [41], clothing, electronics, fashion and beauty products, medicines (different brands with the same composition), and dairy products. Therefore, product substitution decisions should be analyzed before finalizing orders.

In scenarios involving product substitution, customers are provided with a range of options to choose from if the original version is not in stock. Product substitution occurs when a different item satisfies the customer's requirements in place of the desired product. This can happen when alternative products, either from the same manufacturer or similar products from different suppliers or organizations, are made available as substitutes. Traditional deterministic inventory models often do not consider product substitution but neglecting it can affect the efficiency of the inventory system in terms of client satisfaction and cost. However, there are numerous advantages to incorporating product substitution in an inventory system. Firstly, stockouts can be managed effectively by using substitutes, reducing overall holding costs by maintaining fewer inventories of the same type of product in multiple varieties. Ordering smaller lots of substitute products in larger quantities can decrease overall ordering costs and wait times. Cost-wise, cheaper substitutions can be attractive in terms of purchasing expenses. Substitutions can help reduce the amount of perishable goods in stock, such as consuming substitute stock first if it has an earlier expiration date. Additionally, product substitution can facilitate revenue-sharing arrangements among multiple systems, allowing inventory from one system to be utilized by another. This pooling of inventory helps mitigate demand uncertainties and reduce the need for safety stockpiles. Such an approach can serve as a fallback scheme using a common product as a reserve for a standard product.

1.1. Research questions

For a successful business, it is necessary to understand the problems and challenges related to product substitution. Some major questions arise while using product substitution which are as follows:

(1) How can a manager choose substitutable products and rapidly changing market requirements?
(2) What are the long-term effects of substitution on a brand and its reputation and associated profit strategy?
(3) How does a manager finalize the ordering quantity of an original product and substitutive product to gain the maximum benefit along with satisfying consumer demand?

1.2. Contributions of this research

Keeping all these questions and challenges in mind, this study focuses on two distinct types of items: major products and minor products. The major product is only used to fulfill demand when the minor product is unavailable, even if the major product could initially meet the demand. In certain cases, different generations (batch numbers, lots, or product versions) of the same product may exist, each with varying consumer preferences. These two types of items can be considered to have different demands. Product substitution is highly practical, and its effects have to be taken into account when developing traditional inventory frameworks. To provide a noble perception of the subject, a literature review relevant to our work has been conducted.

1.3. Orientation of the paper

The rest of the paper is organized as follows: Section 2 provides a literature review, Section 3 provides preliminaries about the paper such as notation and assumptions, Section 4 elaborates the mathematical model, Section 5 gives a numerical example with discussions, Section 6 gives insights of the study, and Section 7 provides concluding remarks and limitations of this study.

2. Literature review

The optimal inventory strategy is determined using traditional economic order quantity type models. Utilizing these models, researchers may determine the appropriate inventory lot size to obtain maximum system efficiency while minimizing inventory storage expenses. There are only a few irrational presumptions in these models, even though they have been successfully applied in the management area since the early 20th century. Harris [16] built the foundation of the economic order quantity (EOQ) model on the supposition that all commodities were of the highest caliber and that there were no storage facilities while in real-world business circumstances, none of these hypotheses can be applied. In many businesses sector deterioration of products plays an important role in deciding the optimal lot size and calculating the optimal cost. Deterioration can be increased or decreased based on storage conditions and type of products. There has been a lot of study on economic production quantity (EPQ) and EOQ inventory models with deteriorating products.

Drezner *et al.* [10] described the effectiveness of partial substitution over other methods of substitution. A joint replenishment problem (JRP) with the shortfall and partial demand substitution was considered by Chen *et al.* [5] and Eksler *et al.* [13]. The work of Drezner *et al.* [10] was expanded upon by Gernani and Drezner [15] using multiple product substitution. Zhu *et al.* [61] examined a customer-centric product management system without the consideration of deteriorating substitute products. Saxena *et al.* [50] considered deteriorating products to have a shelf-life, where the market demand followed a probability distribution function. Jaggi *et al.* [20] investigated a demand-dependent selling price inventory model for deteriorating goods. To explain the dependence among the items, Jaggi *et al.* [21] introduced a correlation between one and another. Mcgillivray *et al.* [32] examined how stocking control rules were affected by substitutable demand as well as the costs of excess inventory and shortages. A similar model with substitution was considered by Salameh *et al.* [45]. Wee [58] discussed a model for decaying products where demand decreases exponentially over a fixed time interval and the shortage was considered. An optimal inventory policy was created by Wee *et al.* [59].

To determine the best product price and replenishment cycle time, Edalatpour *et al.* [12] investigated an inventory model with substitution for price-sensitive demand and nonlinear holding cost [26, 51]. Khakbaz and Tirkolaee [23] studied hybrid manufacturing/remanufacturing systems (HMRSs) with waste electric and electronic equipment (WEEE) directive and two-way substitution under six different conditions. A mathematical model was developed to analyze the interactive effect of economic factors such as demand growth rate, interest rate, and inflation rate on the behaviors of retailers in Iran country [24]. Khakbaz *et al.* [25] extended their previous work [24] through a comparative study across different countries. According to Salameh and Jaber [44], the defective products might be retiled at a lower cost after a 100% screening procedure was complete. Mridha *et al.* [39] created a profitable policy for the supply chain but not for deteriorating substitutable products. Researchers have investigated a coordination model with an unreliable production process [60]. Sepehri *et al.* [49] examined a deteriorated product with substitutability. Li *et al.* [28] considered sustainable development goals (SDG) through inflation reduction but not for deteriorating products.

Arve and Zwart [1] developed a policy for optimizing investment strategy for new technology with asymmetric information about stochastic variables. Duong *et al.* [11] look at the effect of short-selling movement on cost unpredictability in the corporate security market and lay out the way that security short undercutting is certainly not a substitute for value selling. The demand for alternative products was thought to be linearly negatively impacted by complementary products, while the opposite was thought to be true [29]. Maity and Maiti [30] extended their previous work of Maity and Maiti [29]. An emissions-dependent model with substitutable deteriorated products was researched by Yadav *et al.* [22]. They considered a non-linear demand with cross-price elasticity. An inventory model was developed for manufacturing using substitution by Mokhtari *et al.* [35]. They presented the issue as a two-stage recourse-based integer stochastic program. They contrasted the effectiveness of their heuristics with the optimum match from a large-scale mixed integer linear algorithm. Asymmetric information about market demand and other factors was discussed by Sarkar and Guchhait [46] but they did not discuss deteriorating products.

Table A. Contribution of different authors.

Authors	Product substitution	Deterioration	Exponential increasing time-dependent demand	Inflation	Shortage
Chen *et al.* [5]	✓	✓	×	×	✓
Drezner *et al.* [10]	✓	×	×	×	✓
Edalatpour *et al.* [12]	✓	✓	×	×	×
Eksler *et al.* [13]	✓	✓	×	×	×
Gurnani *et al.* [15]	✓	×	×	×	×
Jaggi *et al.* [20]	✓	✓	×	✓	×
Jaggi *et al.* [21]	×	✓	×	✓	✓
Garai and Sarkar *et al.* [53]	×	×	×	×	✓
Maity and Maiti [29]	✓	✓	×	×	×
Maity and Maiti [30]	✓	✓	×	×	×
Mcgillivray *et al.* [32]	✓	×	×	×	✓
Mokhtari *et al.* [35]	✓	×	×	×	×
Salameh *et al.* [44]	✓	✓	×	×	×
Salameh *et al.* [45]	✓	×	×	×	✓
Singh *et al.* [51]	×	✓	✓	✓	×
Wee [58]	✓	✓	×	×	✓
Wee *et al.* [59]	✓	✓	×	×	×
Yu *et al.* [60]	×	✓	×	×	×
Singh *et al.* [52]	×	✓	×	×	×
Ghosh *et al.* [14]	×	✓	✓	×	✓
Hsu and Hsu [18]	×	✓	×	×	×
Rao *et al.* [42]	✓	×	×	×	×
Chakraborty *et al.* [3]	×	✓	×	×	×
Majumder *et al.* [31]	✓	✓	×	×	✓
Swierczek [55]	✓	×	×	×	×
This paper	✓	✓	✓	✓	✓

In the literature, it is found that several researchers have conducted different studies and developed different inventory models for various demand patterns and substitution options but as per knowledge, the articles did not take into account the substitution of products with exponentially increasing demand under shortages and inflationary environment. This work fills the research gap in this area. Table A in this article presents a comparison of the literature pertinent to the suggested model.

This research presents an enhanced inventory model that incorporates exponentially increasing demand, shortages, an inflationary environment, and product substitution while considering the impact of product deterioration. The objective is to determine the optimal order quantity and maximize overall profitability considering these complex factors. This study mathematically solves the model for determining the best possible production cycle length, which is crucial in achieving the maximum benefit. Furthermore, a sensitivity analysis is conducted.

3. Preliminaries

This section provides the related notation and assumptions of this study.

3.1. Notation and assumptions

The notation and assumptions utilized in the model are as follows:

Notation

Parameters

a_i	Base demand for the product, $a > 0$, $i = 1, 2$ (unit/time unit)
b_i	Mark up for the product, $b > 0$, $i = 1, 2$ ($/unit)
ch_i	Holding cost for the product, $i = 1, 2$ ($/unit/unit time)
C_0	Ordering cost for both products ($/order)
C_t	Transfer cost for product 1 into product 2 ($/unit)
θ_i	Deterioration rate for the product, $i = 1, 2$
r	Inflation rate
y_i	Order quantity of the product, $i = 1, 2$ (unit/cycle)

Decision variables

τ	Time interval during which no substitution is required (time unit)
T	Cycle time (time unit)

Functions

$I(t)$	Inventory level at the time t where $t \in [0, T]$
TAC	Total average cost ($/cycle)

3.2. Assumptions

The following assumptions are used to develop this model:

(1) Exponential demand is used to forecast and make plans for future growth. Products experience an increase in demand due to factors such as population growth, market expansion, or new product launches in a particular season. Businesses can anticipate their future inventory requirements and ensure that they have sufficient stock to meet the rising demand by incorporating exponential demand patterns into inventory models.

(2) Substitution of the standard product only when it runs out of stock (stockout-based substitution). By offering alternative products or components, businesses meet customer demand without completely depleting stock or experiencing stockouts.

(3) Shortage never occurs for substitutable products. Deterioration is constant for both products. Incorporating constant deterioration into an inventory model is relevant in industries where the quality or usability of inventory items deteriorates steadily over time.

(4) Inflation is considered to evaluate the time value of money. To ensure accurate valuation, maintain profitability, and effectively mitigate the effect of rising costs on the company's economy, inventory management must take inflation into account.

(5) The products have negligible lead time and replenishment is instantaneous. Once the standard product is not in stock, the product demand can be immediately fulfilled by a substituted product.

4. Model formulation

The inventory system initially consists of both a major and a minor product. However, when the minor product becomes unavailable due to shortages (*i.e.*, after the complete depletion of the minor product's stock), the major product's demand is affected. Consequently, customers begin considering alternative products and substitutive products fulfill the demand that was originally intended for the main product. This highlights the interdependence between the availability of the minor product and the customer's purchasing behavior, ultimately impacting the demand and substitution patterns for the major product.

The proposed model is developed under two scenarios.

(a) $\tau = t$ and (b) $t \leq \tau$.

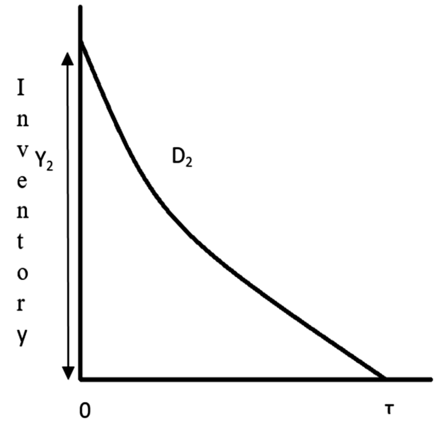

FIGURE 1. Inventory-time relationship of minor products.

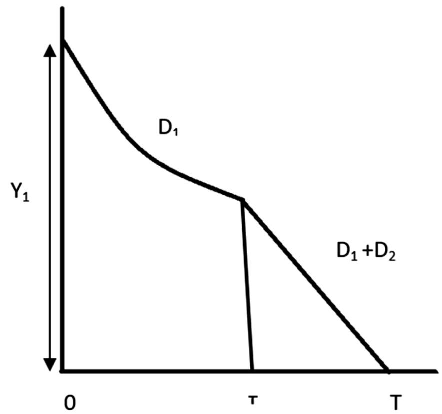

FIGURE 2. Inventory-time relationship of major products.

In both cases, the number of major items remains the same. The demand for both types of products is assumed to exponentially increasing (Figs. 1 and 2) based on the Heaviside function. The Heaviside function allows for straightforward modelling of demand patterns, helping to determine when to place an order based on sudden changes in demand. In this model, the demand is increasing exponentially when $t < \tau$ and constant when $t \geq \tau$.

$$D_1 = a_1 e^{b_1(t - t.H(t-\tau))}$$
$$D_2 = a_2 e^{b_2(t - t.H(t-\tau))}$$

where H is the Heaviside function defined as $H(t-\tau) = \begin{cases} 1 & t \geq \tau \\ 0 & t < \tau \end{cases}$.

4.1. Partial substitution

The differential equations describing the inventory level with constant deterioration rate θ_i, $i = 1, 2$ at the time $t \in [0, \tau]$ is

$$\frac{\mathrm{d}I_{21}(t)}{\mathrm{d}t} + \theta_2 I_{21}(t) = -D_2 \qquad 0 \leq t \leq \tau. \tag{1}$$

Solving the equation (1) using boundary condition $I_{21}(0) = y_2$, $I_{21}(\tau) = 0$, one can get

$$I_{21}(t) = \frac{a_2}{b_2+\theta_2}\left(e^{-\theta_2 t} - e^{b_2 t}\right) + y_2 e^{-\theta_2 t}.$$

The ordering quantity of minor products at the time $t = \tau$ is

$$y_2 = \frac{a_2}{b_2+\theta_2}\left(e^{(b_2+\theta_2)\tau} - 1\right). \tag{a}$$

$$\frac{\mathrm{d}I_{11}(t)}{\mathrm{d}t} + \theta_1 I_{11}(t) = -D_1 \qquad 0 \le t \le T. \tag{2}$$

The solution of equation (2) using the boundary condition $I_{11}(0) = y_1$, $I_{11}(T) = 0$ is given as

$$I_{11}(t) = \frac{a_1}{b_1+\theta_1}\left(e^{-\theta_1 t} - e^{b_1 t}\right) + y_1 e^{-\theta_1 t}$$

$$y_1 = \frac{a_1}{b_1+\theta_1}\left(e^{(b_1+\theta_1)T} - 1\right). \tag{b}$$

The differential equations describing the inventory level with constant deterioration rate θ_i, $i = 1, 2$ at the time $t \in [\tau, T]$ is

$$\frac{\mathrm{d}I_{12}(t)}{\mathrm{d}t} + \theta_1 I_{12}(t) = -(D_1 + D_2) \qquad \tau \le t \le T. \tag{3}$$

Solving equation (3) using boundary condition $I_{12}(T) = 0$, one can find

$$I_{12}(t) = \frac{a_1+a_2}{\theta_1}\left(e^{\theta_1(T-t)} - 1\right).$$

Holding cost. The expenses associated with storing and maintaining inventory over a fixed time period are known as holding costs or carrying costs. Holding cost includes various expenses incurred by a company for storing inventory, such as warehousing expenses, insurance, obsolescence, depreciation, opportunity cost, and costs related to handling and managing inventory. It is important to consider the impact of inflation on various components of the holding cost.

$$\begin{aligned}
\mathrm{HC} &= ch_2 \int_0^\tau e^{-rt} I_{21}(t)\,\mathrm{d}t + ch_1\left(\int_0^\tau e^{-rt} I_{11}(t)\,\mathrm{d}t + \int_\tau^T e^{-rt} I_{12}(t)\,\mathrm{d}t\right) \\
&= ch_2\left(\frac{1}{r+\theta_2}\left(1 - e^{-(r+\theta_2)\tau}\right)\left(\frac{a_2}{b_2+\theta_2} + y_2\right) + \frac{a_2}{(b_2+\theta_2)}\frac{1}{(b_2-r)}\left(1 - e^{(b_2-r)\tau}\right)\right) \\
&\quad + ch_1\left(\frac{1}{r+\theta_1}\left(1 - e^{-(r+\theta_1)\tau}\right)\left(\frac{a_1}{b_1+\theta_1} + y_1\right) + \frac{a_1}{(b_1+\theta_1)}\frac{1}{(b_1-r)}\left(1 - e^{(b_1-r)\tau}\right)\right) \\
&\quad + ch_1\left(\frac{a_1+a_2}{\theta_1}\right)\left(\frac{1}{r+\theta_1}\left(e^{\theta_1 T-(r+\theta_1)\tau} - e^{-rT}\right) + \frac{1}{r}\left(e^{-rT} - e^{-r\tau}\right)\right).
\end{aligned} \tag{4}$$

Deterioration cost. Deterioration in inventory refers to the decline in the quality or value of items held in stock over time. It commonly occurs in industries where inventory consists of perishable goods, such as food products, pharmaceuticals, or chemicals. Deterioration in inventory with inflation refers to the decline in the purchasing power or value of inventory items over time due to the effects of inflation.

$$\begin{aligned}
\mathrm{DC} &= \theta_2 \int_0^\tau e^{-rt} I_{21}(t)\,\mathrm{d}t + \theta_1\left(\int_0^\tau e^{-rt} I_{11}(t)\,\mathrm{d}t + \int_\tau^T e^{-rt} I_{12}(t)\,\mathrm{d}t\right) \\
&= \theta_2\left(\frac{1}{r+\theta_2}\left(1 - e^{-(r+\theta_2)\tau}\right)\left(\frac{a_2}{b_2+\theta_2} + y_2\right) + \frac{a_2}{(b_2+\theta_2)}\frac{1}{(b_2-r)}\left(1 - e^{(b_2-r)\tau}\right)\right)
\end{aligned}$$

$$+\theta_1\left(\frac{1}{r+\theta_1}\left(1-e^{-(r+\theta_1)\tau}\right)\left(\frac{a_1}{b_1+\theta_1}+y_1\right)+\frac{a_1}{(b_1+\theta_1)}\frac{1}{(b_1-r)}\left(1-e^{(b_1-r)\tau}\right)\right)$$
$$+\theta_1\left(\frac{a_1+a_2}{\theta_1}\right)\left(\frac{1}{r+\theta_1}\left(e^{\theta_1 T-(r+\theta_1)\tau}-e^{-rT}\right)+\frac{1}{r}\left(e^{-rT}-e^{-r\tau}\right)\right). \tag{5}$$

Transformation cost. Transformation cost in an inventory model refers to the expenses incurred when substituting one product with another or converting or transforming raw materials or components into finished goods or intermediate products.

$$\mathrm{CT}=c_t\int_{\tau}^{T}a_2e^{-rt}\,\mathrm{d}t=\frac{c_ta_2}{r}\left(e^{-r\tau}-e^{-rT}\right). \tag{6}$$

Ordering cost. The expenses associated with placing and receiving an order for inventory items are referred to as ordering cost. It is one of the major components of the total inventory costs and plays a significant role in inventory management decisions.

$$\mathrm{OC}=C_0. \tag{7}$$

Average total cost. The average total cost refers to the average cost per unit of inventory, taking into account various costs associated with inventory management such as holding inventory (carrying costs) and ordering costs, transformation costs, and deterioration costs. It represents the total cost incurred by a business to maintain and replenish its inventory over a specific period.

$$\mathrm{TAC}=\frac{1}{T}(\mathrm{OC}+\mathrm{CT}+\mathrm{HC}+\mathrm{DC})$$
$$\mathrm{TAC}=\frac{1}{T}\Bigg\{C_0+\frac{c_ta_2}{r}\left(e^{-r\tau}-e^{-rT}\right)+(ch_2+\theta_2)\left(\frac{1}{r+\theta_2}\left(1-e^{-(r+\theta_2)\tau}\right)\left(\frac{a_2}{b_2+\theta_2}+y_2\right)\right.$$
$$\left.+\frac{a_2}{(b_2+\theta_2)}\frac{1}{(b_2-r)}\left(1-e^{(b_2-r)\tau}\right)\right)+(ch_1+\theta_1)\left(\frac{1}{r+\theta_1}\left(1-e^{-(r+\theta_1)\tau}\right)\right.$$
$$\left.\times\left(\frac{a_1}{b_1+\theta_1}+y_1\right)+\frac{a_1}{(b_1+\theta_1)}\frac{1}{(b_1-r)}\left(1-e^{(b_1-r)\tau}\right)\right)+(ch_1+\theta_1)$$
$$\times\left(\frac{a_1+a_2}{\theta_1}\right)\left(\frac{1}{r+\theta_1}\left(e^{\theta_1T-(r+\theta_1)\tau}-e^{-rT}\right)+\frac{1}{r}\left(e^{-rT}-e^{-r\tau}\right)\right)\Bigg\}. \tag{8}$$

Solution procedure

The gradient method is used for optimization, *i.e.*,

$$\frac{\partial\mathrm{TAC}}{\partial T}=0,\ \frac{\partial\mathrm{TAC}}{\partial\tau}=0. \tag{A}$$

Equation (A) is solved using MATHEMATICA 11.0 to obtain the values of T and τ. Substitute the values of T and τ and other parameters in equations (a), (b) and (8) to get ordering quantity and average total cost, respectively. To check the convexity of the average total cost function, one can use the Hessian matrix.

$$H=\begin{bmatrix}\frac{\partial^2\mathrm{TAC}}{\partial^2T} & \frac{\partial^2\mathrm{TAC}}{\partial T\partial\tau}\\ \frac{\partial^2\mathrm{TAC}}{\partial\tau\partial T} & \frac{\partial^2\mathrm{TAC}}{\partial^2\tau}\end{bmatrix}$$

Here $H_1=\frac{\partial^2\mathrm{TAC}}{\partial^2T}>0$, $H_2=\frac{\partial^2\mathrm{TAC}}{\partial^2\tau}>0$, $H_3=\frac{\partial^2\mathrm{TAC}}{\partial^2T}\frac{\partial^2\mathrm{TAC}}{\partial^2\tau}-\left(\frac{\partial^2\mathrm{TAC}}{\partial T\partial\tau}\right)^2>0$.

This proves the convexity of the average total cost function.

4.2. Full substitution

In this case, $y_2 = 0$, $\tau = 0$. The differential equation governing the inventory level with constant deterioration is defined as

$$\frac{\mathrm{d}I_{12}(t)}{\mathrm{d}t} + \theta_1 I_{12}(t) = -(D_1 + D_2). \tag{9}$$

Solving equation (9) using boundary conditions $I_{12}(0) = y_1$ and $I_{12}(T) = 0$, one can have

$$I_{12}(t) = \frac{a_1}{b_1+\theta_1}\left(e^{-\theta_1 t} - e^{b_1 t}\right) + \frac{a_2}{b_2+\theta_2}\left(e^{-\theta_1 t} - e^{(b_2+\theta_2-\theta_1)t}\right) + y_1 e^{-\theta_1 t}.$$

The ordering quantity of major product at time $t = T$ is

$$y_1 = \frac{a_1}{b_1+\theta_1}\left(e^{(b_1+\theta_1)T} - 1\right) + \frac{a_2}{b_2+\theta_2}\left(e^{(b_2+\theta_2)T} - 1\right).$$

Holding cost. The present worth of the holding cost is given by

$$\begin{aligned}
\mathrm{HC} &= ch_1 \int_0^T e^{-rt} I_{12}(t)\,\mathrm{d}t \\
&= ch_1\left\{\left(\frac{1}{r+\theta_1} - \frac{e^{-(r+\theta_1)T}}{r+\theta_1}\right)\left(\frac{a_1}{b_1+\theta_1} + \frac{a_2}{b_2+\theta_2} + y_1\right) + \frac{a_1}{(b_1+\theta_2)(b_1-r)}\left(1 - e^{(b_1-r)T}\right)\right. \\
&\quad \left. + \frac{a_2}{(b_2+\theta_2)(b_2+\theta_2-\theta_1-r)}\left(1 - e^{(b_2+\theta_2-\theta_1-r)}\right)\right\}.
\end{aligned} \tag{10}$$

Deterioration cost. The present worth of deterioration cost for full substitution of product is governed by the equation

$$\begin{aligned}
\mathrm{DC} &= \theta_1 \int_0^T e^{-rt} I_{12}(t)\,\mathrm{d}t \\
&= \theta_1\left\{\left(\frac{1}{r+\theta_1} - \frac{e^{-(r+\theta_1)T}}{r+\theta_1}\right)\left(\frac{a_1}{b_1+\theta_1} + \frac{a_2}{b_2+\theta_2} + y_1\right) + \frac{a_1}{(b_1+\theta_2)(b_1-r)}\left(1 - e^{(b_1-r)T}\right)\right. \\
&\quad \left. + \frac{a_2}{(b_2+\theta_2)(b_2+\theta_2-\theta_1-r)}\left(1 - e^{(b_2+\theta_2-\theta_1-r)}\right)\right\}.
\end{aligned} \tag{11}$$

Transformation and ordering costs. The cost associated with the substitution of a product is given by

$$\mathrm{CT} = c_t \int_0^T a_2 e^{-rt}\,\mathrm{d}t = \frac{c_t a_2}{r}\left(1 - e^{-rT}\right). \tag{12}$$

The ordering cost is

$$\mathrm{OC} = C_0. \tag{13}$$

The average total cost is

$$\begin{aligned}
\mathrm{TAC} &= \frac{1}{T}(\mathrm{OC} + \mathrm{CT} + \mathrm{HC} + \mathrm{DC}) \\
\mathrm{TAC} &= \frac{1}{T}\left\{C_0 + \frac{c_t a_2}{r}\left(1 - e^{-rT}\right) + (ch_1 + \theta_1)\left\{\left(\frac{1}{r+\theta_1} - \frac{e^{-(r+\theta_1)T}}{r+\theta_1}\right)\left(\frac{a_1}{b_1+\theta_1} + \frac{a_2}{b_2+\theta_2} + y_1\right)\right.\right. \\
&\quad \left.\left. + \frac{a_1}{(b_1+\theta_2)(b_1-r)}\left(1 - e^{(b_1-r)T}\right) + \frac{a_2}{(b_2+\theta_2)(b_2+\theta_2-\theta_1-r)}\left(1 - e^{(b_2+\theta_2-\theta_1-r)}\right)\right\}\right\}.
\end{aligned} \tag{14}$$

4.3. No substitution

There is no impact of demand for one product on other products. In this case $\tau = T$. In no substitution case, the inventory level at a time is given by the following differential equations.

$$\frac{\mathrm{d}I_{11}(t)}{\mathrm{d}t} + \theta_1 I_{11}(t) = -D_1 \qquad 0 \le t \le \tau$$
$$I_{11}(0) = y_1, \quad I_{11}(\tau) = 0$$
$$I_{11}(t) = \frac{a_1}{b_1 + \theta_1}\left(e^{-\theta_1 t} - e^{b_1 t}\right) + y_1 e^{-\theta_1 t} \tag{15}$$

The ordering quantity for the major product at the time $t = T$ is

$$y_1 = \frac{a_1}{b_1 + \theta_1}\left(e^{(b_1+\theta_1)T} - 1\right)$$
$$\frac{\mathrm{d}I_{21}(t)}{\mathrm{d}t} + \theta_2 I_{21}(t) = -D_2 \qquad 0 \le t \le \tau$$
$$I_{21}(0) = y_2, \quad I_{21}(T) = 0$$
$$I_{21}(t) = \frac{a_2}{b_2 + \theta_2}\left(e^{-\theta_2 t} - e^{b_2 t}\right) + y_2 e^{-\theta_2 t}. \tag{16}$$

The ordering quantity for minor product at the time $t = T$ is

$$y_2 = \frac{a_2}{b_2 + \theta_2}\left(e^{(b_2+\theta_2)T} - 1\right).$$

Holding cost. The present worth of holding cost $t \in [0, T]$ is defined by the equation

$$\mathrm{HC} = ch_2 \int_0^T e^{-rt} I_{21}(t)\,\mathrm{d}t + ch_1 \int_0^\tau e^{-rt} I_{11}(t)\,\mathrm{d}t$$
$$\mathrm{HC} = ch_1\left\{\left(1 - e^{-(r+\theta_1)T}\right)\left(\frac{a_1}{(b_1+\theta_1)(r+\theta_1)} + \frac{y_2}{(r+\theta_1)}\right) + \frac{a_1}{(b_1+\theta_1)(b_1-r)}\left(1 - e^{(b_1-r)T}\right)\right\}$$
$$+ ch_2\left\{\left(1 - e^{-(r+\theta_2)T}\right)\left(\frac{a_2}{(b_2+\theta_2)(r+\theta_2)} + \frac{y_2}{(r+\theta_2)}\right) + \frac{a_2}{(b_2+\theta_2)(b_2-r)}\left(1 - e^{(b_2-r)T}\right)\right\}. \tag{17}$$

Deteriorating cost. The deterioration cost associated with no substitution is given by

$$\mathrm{DC} = \theta_2 \int_0^T e^{-rt} I_{21}(t)\,\mathrm{d}t + \theta_1 \int_0^\tau e^{-rt} I_{11}(t)\,\mathrm{d}t$$
$$\mathrm{DC} = \theta_1\left\{\left(1 - e^{-(r+\theta_1)T}\right)\left(\frac{a_1}{(b_1+\theta_1)(r+\theta_1)} + \frac{y_2}{(r+\theta_1)}\right) + \frac{a_1}{(b_1+\theta_1)(b_1-r)}\left(1 - e^{(b_1-r)T}\right)\right\}$$
$$+ \theta_2\left\{\left(1 - e^{-(r+\theta_2)T}\right)\left(\frac{a_2}{(b_2+\theta_2)(r+\theta_2)} + \frac{y_2}{(r+\theta_2)}\right) + \frac{a_2}{(b_2+\theta_2)(b_2-r)}\left(1 - e^{(b_2-r)T}\right)\right\}. \tag{18}$$

Transformation and ordering costs. In no substitution case, transformation cost is zero, i.e.,

$$\mathrm{CT} = 0. \tag{19}$$

The associative ordering cost is

$$\mathrm{OC} = C_0. \tag{20}$$

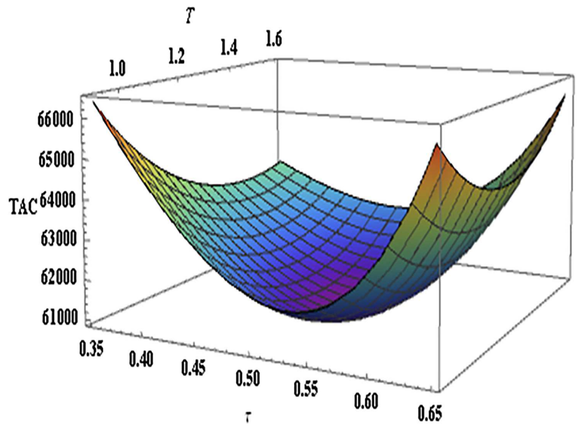

FIGURE 3. Convexity of total cost with respect to cycle time and partial substitution time.

The average total cost is

$$\mathrm{TAC} = \frac{1}{T}(\mathrm{OC} + \mathrm{CT} + \mathrm{HC} + \mathrm{DC})$$

$$\begin{aligned}\mathrm{TAC} = \frac{1}{T}\Bigg(C_0 + (ch_1+\theta_1)\Bigg\{\left(1 - e^{-(r+\theta_1)T}\right)\left(\frac{a_1}{(b_1+\theta_1)(r+\theta_1)} + \frac{y_2}{(r+\theta_1)}\right) \\ + \frac{a_1}{(b_1+\theta_1)(b_1-r)}\left(1 - e^{(b_1-r)T}\right)\Bigg\} + (ch_2+\theta_2)\Bigg\{\left(1 - e^{-(r+\theta_2)T}\right)\left(\frac{a_2}{(b_2+\theta_2)(r+\theta_2)} + \frac{y_2}{(r+\theta_2)}\right) \\ + \frac{a_2}{(b_2+\theta_2)(b_2-r)}\left(1 - e^{(b_2-r)T}\right)\Bigg\}\Bigg). \end{aligned} \quad (21)$$

5. Numerical example with sensitivity analysis

Decision-makers can evaluate the outcomes of various inventory policies using numerical examples. By inputting specific values for parameters like demand, holding costs, and ordering costs, decision-makers can observe the resulting inventory and costs. This makes it easier to make better decisions about order quantities, reorder points, safety stocks, and other aspects of inventory management.

5.1. Numerical example

The data are collected and utilized for validation with the MATHEMATICA 11.0 software in order to demonstrate and verify the developed model. Software is used to run the model and compare its predictions and results to the ones provided to determine their accuracy and dependability. The data taken for various used parameters are as follows [4, 18]: $a_1 = 200$ unit/day; $a_2 = 250$ unit/days; $b_1 = \$2$/unit; $b_2 = \$6$/unit; $\theta_1 = 0.01$; $\theta_2 = 0.02$; $ch_1 = \$10$/unit/unit time; $ch_2 = \$15$/unit/unit time; $r = 0.06$; $C_0 = \$30\,000$/setup; $C_t = \$200$/unit. Data is modified based on the convergence of the algorithm.

For partial substitution, the minimum total cost is \$60 949.8/cycle with cycle time $T = 1.20483$ days, and the time for product substitution is $\tau = 0.518877$ days. For full substitution, the minimum total cost is \$111 498/cycle with cycle time $T = 0.61094$ days. For no substitution, the minimum total cost is \$67 475/cycle and the cycle time T is 0.56856 days. Figure 3 shows that the obtained result is global minimum at the obtained value in the partial substitution case.

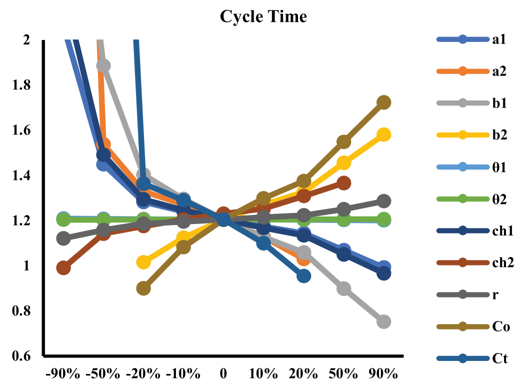

FIGURE 4. Changes in cycle time with respect to changes in various parameters for partial substitution.

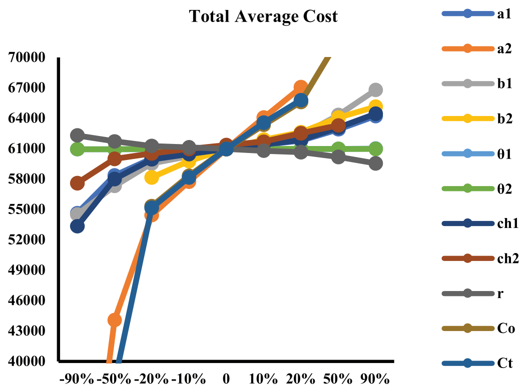

FIGURE 5. Changes in average total cost with respect to changes in various parameters for partial substitution.

5.2. Sensitivity analysis

The following tables provide valuable insights into the effects of different substitution strategies and parameter variations on the overall system performance. Graphical representations of sensitivity analysis from these tables are given in Figures 4–9.

5.3. Observations

The following observations from Tables 1–11 are given below.

– In the case of partial substitution, the following observations are made.
  (1) Table 7 demonstrates that the order quantity of major items rises as the holding cost of minor items increases.
  (2) According to Tables 5 and 6, the total average cost gradually increases as the deterioration rate of both products increases since it is needed to increase the order quantity of both products in this scenario.
  (3) Table 9 exhibits that the total cycle time increases with a high inflation rate while the total average cost decreases.

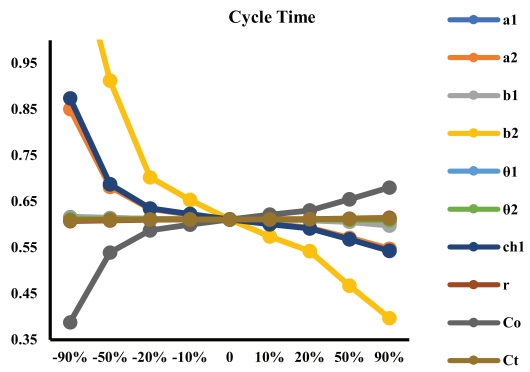

FIGURE 6. Changes in cycle time with respect to changes in various parameters for full substitution.

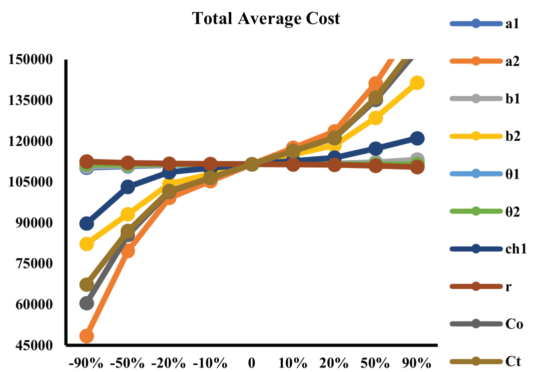

FIGURE 7. Changes in average total cost with respect to changes in various parameters for full substitution.

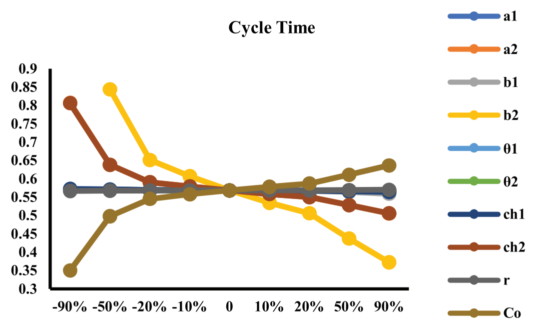

FIGURE 8. Changes in cycle time with respect to changes in various parameters for no substitution.

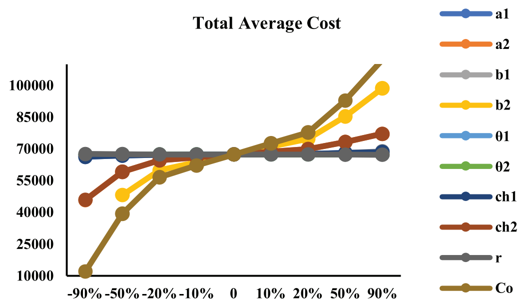

FIGURE 9. Changes in average total cost with respect to changes in various parameters for no substitution.

TABLE 1. Sensitivity analysis for the base demand (a_1) for the major product.

	Partial substitution					Full substitution			No substitution			
a_1	τ	T	TAC	y_1	y_2	T	TAC	y_1	T	TAC	y_1	y_2
−90%	0.528242	2.06236	54 603.4	618.305	957.11	0.616154	110 210	1678.16	0.572574	66 348.7	21.502	1262.58
−50%	0.521135	1.44895	58 341.4	865.704	915.283	0.613829	110 785	1751.35	0.570785	66 851.2	106.946	1248.6
−20%	0.519487	1.28185	60 020	967.367	905.839	0.612094	111 214	1805.66	0.569448	67 226.2	170.44	1238.26
−10%	0.519148	1.241	60 499.1	995.34	903.906	0.611516	111 356	1823.67	0.569004	67 350.9	191.494	1234.84
0	0.518877	1.20483	60 949.8	1021.41	902.365	0.61094	111 498	1841.63	0.56856	67 475.3	212.493	1231.44
10%	0.518662	1.17242	61 376.4	1045.79	901.142	0.610364	111 640	1859.54	0.568116	67 599.6	233.436	1228.04
20%	0.518492	1.1431	61 782.2	1068.73	900.181	0.60979	111 782	1877.4	0.567673	67 723.8	254.325	1224.66
50%	0.518195	1.0691	62 898.8	1130.65	898.5	0.60807	112 205	1930.67	0.566347	68 095.1	316.663	1214.59
90%	0.518142	0.992671	64 210.5	1201.29	898.197	0.605787	112 766	2001.01	0.564587	68 587.7	399.021	1201.35

TABLE 2. Sensitivity analysis for the base demand (a_2) for the minor product.

	Partial substitution					Full substitution			No substitution			
a_2	τ	T	TAC	y_1	y_2	T	TAC	y_1	T	TAC	y_1	y_2
−90%	0	6.08967	15 337.6	2 059 210	0	0.85034	48 418.2	1140.29	0.807485	45 891.2	404.826	532.229
−50%	0.409804	1.53744	44 056.7	2087.82	223.988	0.681566	79 644.6	1528	0.638667	59 162	259.706	949.923
−20%	0.5036	1.33037	54 440	1343.12	655.542	0.633439	99 131.2	1727.71	0.590872	64 582.5	226.804	1131.55
−10%	0.512453	1.27108	57 736.3	1181.06	779.905	0.621532	105 360	1786.16	0.579062	66 081.9	219.149	1183.07
0	0.518877	1.20483	60 949.8	1021.41	902.365	0.61094	111 498	1841.63	0.56856	67 475.3	212.493	1231.44
10%	0.523747	1.12705	64 061.5	859.173	1023.49	0.601409	117 559	1894.55	0.559112	68 780.2	206.624	1277.16
20%	0.527531	1.03025	67 034.3	689.675	1143.41	0.592751	123 556	1945.24	0.550532	70 009.5	201.39	1320.62
50%	NA	NA	NA	NA	NA	0.570749	141 235	2086.35	0.528736	73 337.9	188.492	1440.12
90%	NA	NA	NA	NA	NA	0.547778	164 279	2255.04	0.505988	77 160.6	175.62	1580.6

(4) According to Tables 1 and 2, the base demand of both products shows a positive effect on the ordering quantity of the corresponding product and a negative effect on the other product, while the total average cost increases in both cases.
(5) Table 11 illustrates that the cycle time of a major product exhibits reversible effects in relation to the transfer cost.

– For full substitution, one can make the following observations:
  (1) According to Table 9, cycle time increases while total average cost decreases with an increasing inflation rate.

TABLE 3. Sensitivity analysis for mark-up parameter (b_1) of demand for major product.

	Partial substitution					Full substitution			No substitution			
b_1	τ	T	TAC	y_1	y_2	T	TAC	y_1	T	TAC	y_1	y_2
−90%	0.543261	3.34836	54 475.9	971.542	1051.61	0.615213	110 728	1775.55	0.571742	66 835.4	121.496	1256.05
−50%	0.523104	1.88564	57 308.4	1131.89	926.691	0.613911	110 993	1801.14	0.570744	67 059.9	154.398	1248.28
−20%	0.519584	1.40363	59 525.4	1066.03	906.391	0.612362	111 268	1824.25	0.569594	67 287.3	186.57	1239.39
−10%	0.519129	1.29601	60 243.5	1043.25	903.796	0.611697	111 378	1832.74	0.569108	67 377.3	199.039	1235.64
0	0.518877	1.20483	60 949.8	1021.41	902.365	0.61094	111 498	1841.63	0.56856	67 475.3	212.493	1231.44
10%	0.518783	1.12654	61 644	1000.65	901.831	0.610081	111 631	1850.91	0.567943	67 582.3	227.002	1226.72
20%	0.518815	1.05854	62 326.3	981.004	902.013	0.609108	111 776	1860.58	0.567249	67 699	242.642	1221.43
50%	0.51949	0.898983	64 303.6	928.132	905.854	0.605365	112 300	1891.7	0.564616	68 115.5	297.088	1201.57
90%	0.521458	0.751923	66 781.8	868.558	917.147	0.597865	113 255	1936.66	0.559436	68 862.9	389.874	1163.4

TABLE 4. Sensitivity analysis for mark-up parameter (b_2) of demand for minor product.

	Partial substitution					Full substitution			No substitution			
b_2	τ	T	TAC	y_1	y_2	T	TAC	y_1	T	TAC	y_1	y_2
−90%	NA	NA	NA	NA	NA	1.2482	82 287.8	1594.53	–	–	–	–
−50%	NA	NA	NA	NA	NA	0.912545	93 224.7	1743.19	0.844195	48 191.5	443.447	976.848
−20%	0.622636	1.01607	58 135.5	667.489	991.065	0.702558	104 279	1790.12	0.651787	59 913.8	269.305	1148.4
−10%	0.565797	1.12332	59 755.4	852.013	944.15	0.653187	107 916	1814.46	0.606952	63 726.7	237.522	1191.62
0	0.518877	1.20483	6 0949.8	1021.41	902.365	0.61094	111 498	1841.63	0.56856	67 475.3	212.493	1231.44
10%	0.479389	1.27087	61 867.2	1180.52	864.514	0.574403	115 024	1870.83	0.535312	71 160.3	192.324	1268.77
20%	0.445624	1.32655	62 592	1332.09	829.751	0.542489	118 493	1901.48	0.506225	74 784.1	175.752	1304.2
50%	0.368024	1.45581	64 053.8	1756.85	738.6	0.46699	128 587	1998.23	0.437198	85 321.9	140.092	1402.44
90%	0.297989	1.58019	65 116.3	2284.08	636.055	0.396711	141 424	2130.99	0.37262	98 715.9	110.927	1521

TABLE 5. Sensitivity analysis for deterioration (θ_1) of the major product.

	Partial substitution					Full substitution			No substitution			
θ_1	τ	T	TAC	y_1	y_2	T	TAC	y_1	T	TAC	y_1	y_2
−90%	0.518759	1.20866	60 921.1	1022.43	901.697	0.610764	111 510	1838.97	0.568577	67 471.9	211.86	1231.57
−50%	0.518812	1.20696	60 933.8	1021.97	901.995	0.610842	111 505	1840.15	0.568569	67 473.4	212.141	1231.51
−20%	0.518851	1.20568	60 943.4	1021.63	902.218	0.610901	111 501	1841.04	0.568564	67 474.6	212.352	1231.47
−10%	0.518864	1.20526	60 946.6	1021.52	902.292	0.61092	111 499	1841.33	0.568562	67 475	212.422	1231.45
0	0.518877	1.20483	60 949.8	1021.41	902.365	0.61094	111 498	1841.63	0.56856	67 475.3	212.493	1231.44
10%	0.51889	1.20441	60 952.9	1021.3	902.439	0.61096	111 497	1841.93	0.568558	67 475.7	212.563	1231.42
20%	0.518903	1.20399	60 956.1	1021.18	902.513	0.610979	111 496	1842.22	0.568556	67 476.1	212.634	1231.41
50%	0.518942	1.20272	60 965.6	1020.85	902.734	0.611038	111 492	1843.11	0.56855	67 477.3	212.845	1231.36
90%	0.518993	1.20104	60 978.3	1020.41	903.027	0.611116	111 487	1844.29	0.568543	67 478.8	213.128	1231.3

(2) Based on Table 7, holding cost exhibits reversible effects on the ordering quantity of the product.
(3) Table 5 shows that the total average cost is reduced slowly with increments in the major product's deterioration rate.
(4) Tables 1 and 2 demonstrate that the cycle time decreases, while the total average cost and ordering quantity increase with an increasing base demand for the product.

– In the case of no substitution, one can make the following observations:

TABLE 6. Sensitivity analysis for deterioration (θ_2) of the minor product.

	Partial substitution					Full substitution			No substitution			
θ_2	τ	T	TAC	y_1	y_2	T	TAC	y_1	T	TAC	y_1	y_2
−90%	0.519692	1.20341	60 929.1	1018.21	900.871	0.612118	111 392	1840.78	0.569218	67 403.1	212.906	1227.13
−50%	0.51933	1.20404	60 938.3	1019.63	901.537	0.611594	111 439	1841.16	0.568925	67 435.2	212.722	1229.05
−20%	0.519058	1.20452	60 945.2	1020.7	902.034	0.611201	111 474	1841.44	0.568706	67 459.3	212.584	1230.48
−10%	0.518968	1.20467	60 947.5	1021.05	902.2	0.611071	111 486	1841.54	0.568633	67 467.3	212.539	1230.96
0	0.518877	1.20483	60 949.8	1021.41	902.365	0.61094	111 498	1841.63	0.56856	67 475.3	212.493	1231.44
10%	0.518787	1.20499	60 952.1	1021.76	902.531	0.610809	111 510	1841.72	0.568487	67 483.4	212.447	1231.92
20%	0.518696	1.20515	60 954.3	1022.12	902.696	0.610679	111 522	1841.82	0.568414	67 491.4	212.401	1232.39
50%	0.518425	1.20562	60 961.2	1023.18	903.191	0.610288	111 557	1842.1	0.568195	67 515.5	212.264	1233.83
90%	0.518063	1.20625	60 970.4	1024.6	903.85	0.609767	111 605	1842.48	0.567902	67 547.6	212.081	1235.74

TABLE 7. Sensitivity analysis for holding cost (ch_1) for the major product.

	Partial substitution					Full substitution			No substitution			
ch_1	τ	T	TAC	y_1	y_2	T	TAC	y_1	T	TAC	y_1	y_2
−90%	0.512029	2.22155	53 316.6	8552.06	864.246	0.874078	89 726	8444.53	0.57257	66 349.9	215.018	1262.54
−50%	0.515806	1.49157	57 977.4	1895.16	885.074	0.687183	103 264	2854.84	0.570783	66 851.9	213.89	1248.59
−20%	0.517786	1.29432	59 911	1242.29	896.188	0.635163	108 641	2116.51	0.569448	67 226.5	213.05	1238.26
−10%	0.51835	1.24671	60 448.9	1119.84	899.378	0.622338	110 123	1966.19	0.569003	67 351	212.771	1234.84
0	0.518877	1.20483	60 949.8	1021.41	902.365	0.61094	111 498	1841.63	0.56856	67 475.3	212.493	1231.44
10%	0.519371	1.16757	61 419.4	940.516	905.174	0.600691	112 785	1736.4	0.568117	67 599.5	212.215	1228.05
20%	0.519835	1.13409	61 862.2	872.829	907.824	0.591387	113 997	1646.1	0.567674	67 723.5	211.938	1224.67
50%	0.521081	1.05074	63 062.4	722.847	914.975	0.56776	117 275	1437.31	0.56635	68 094.4	211.11	1214.61
90%	0.52248	0.96672	64 440.6	595.066	923.061	0.543108	121 039	1247.53	0.564591	68 586.6	210.014	1201.38

TABLE 8. Sensitivity analysis for holding cost (ch_2) for the minor product.

	Partial substitution					Full substitution*			No substitution			
ch_2	τ	T	TAC	y_1	y_2	T	TAC	y_1	T	TAC	y_1	y_2
−90%	NA	NA	NA	NA	NA	NA	NA	NA	0.806224	45 965.6	403.549	5281.73
−50%	0.616076	0.991063	57 575.3	629.896	1652.99	NA	NA	NA	0.638531	59 176.2	259.608	1898.25
−20%	0.550023	1.14284	59 975.3	890.078	1097.02	NA	NA	NA	0.590839	64 586.7	226.782	1414.15
−10%	0.533566	1.17613	60 501.3	958.562	989.634	NA	NA	NA	0.579047	66 083.8	219.139	1314.4
0	0.518877	1.20483	60 949.8	1021.41	902.365	NA	NA	NA	0.56856	67 475.3	212.493	1231.44
10%	0.505614	1.23007	61 338.5	1079.73	829.934	NA	NA	NA	0.559124	68 778.5	206.631	1161.14
20%	0.493527	1.25258	61 679.8	1134.33	768.772	NA	NA	NA	0.550554	70 006.4	201.403	1100.67
50%	0.462606	1.30855	62 499.2	1281.22	631.147	NA	NA	NA	0.528779	73 331	188.517	960.34
90%	0.429945	1.3659	63 283.6	1449.91	511.077	NA	NA	NA	0.506048	77 150	175.654	832.211

Notes. (*)In the case of full substitution, there is no minor product.

TABLE 9. Sensitivity analysis for inflation rate (r).

	Partial substitution					Full substitution			No substitution			
r	τ	T	TAC	y_1	y_2	T	TAC	y_1	T	TAC	y_1	y_2
−90%	0.524419	1.12105	62 299.4	847.681	934.385	0.607363	112 485	1804.19	0.566892	67 655.6	211.449	1218.72
−50%	0.522059	1.1584	61 709.8	921.522	920.618	0.608958	112 045	1820.8	0.567633	67 575.4	211.912	1224.35
−20%	0.520181	1.18632	61 256.6	980.467	909.803	0.610149	111 716	1833.29	0.568189	67 515.3	212.26	1228.6
−10%	0.519534	1.19559	61 103.7	1000.78	906.107	0.610545	111 607	1837.46	0.568374	67 495.3	212.377	1230.02
0	0.518877	1.20483	60 949.8	1021.41	902.365	0.61094	111 498	1841.63	0.56856	67 475.3	212.493	1231.44
10%	0.51821	1.21404	60 795	1042.36	898.581	0.611335	111 389	1845.81	0.568745	67 455.4	212.609	1232.86
20%	0.517532	1.22322	60 639.3	1063.62	894.752	0.611729	111 281	1849.99	0.568931	67 435.4	212.725	1234.28
50%	0.515437	1.25051	60 167.7	1129.2	883.017	0.612907	110 955	1862.55	0.569487	67 375.5	213.075	1238.56
90%	0.512501	1.28621	59 528.5	1220.6	866.82	0.61447	110 523	1879.34	0.57023	67 295.7	213.542	1244.3

TABLE 10. Sensitivity analysis for ordering cost (C_0).

	Partial substitution					Full substitution			No substitution			
C_0	τ	T	TAC	y_1	y_2	T	TAC	y_1	T	TAC	y_1	y_2
−90%	NA	NA	NA	NA	NA	0.387965	60 420	505.191	0.350362	12 093.2	101.72	300.726
−50%	NA	NA	NA	NA	NA	0.539197	85 553.6	1219.8	0.498328	39 512.2	171.417	792.533
−20%	0.528994	0.899743	55 318.6	507.581	961.64	0.587459	101 490	1609.39	0.545567	56 710.7	198.402	1066.88
−10%	0.524069	1.08231	58 331.2	776.73	932.33	0.599809	106 543	1727.63	0.557659	62 148.5	205.731	1150.58
0	0.518877	1.20483	60 949.8	1021.41	902.365	0.61094	111 498	1841.63	0.56856	67 475.3	212.493	1231.44
10%	0.51346	1.29845	63 344.3	1253.48	872.081	0.621076	116 368	1951.99	0.578487	72 705.6	218.781	1309.83
20%	0.507793	1.37497	65 587.4	1478.44	841.44	0.630384	121 162	2059.18	0.587605	77 850.6	224.668	1386.08
50%	0.488767	1.54912	71 728.7	2139.77	745.882	0.654485	135 163	2365.11	0.611221	92 856.9	240.426	1604.17
90%	0.454572	1.7228	79 057.2	3075.35	599.387	0.680369	153 132	2744.96	0.636593	112 079	258.212	1875.75

TABLE 11. Sensitivity analysis for transfer cost (C_t).

	Partial substitution					Full substitution			No substitution*			
C_t	τ	T	TAC	y_1	y_2	T	TAC	y_1	T	TAC	y_1	y_2
−90%	0	4.02855	20 388.8	326 851	0	0.609199	67 311.8	1823.31	NA	NA	NA	NA
−50%	0	4.78683	37 978.8	1 501 010	0	0.609974	86 950.5	1831.44	NA	NA	NA	NA
−20%	0.474212	1.36371	55 143.7	1443.13	679.827	0.610554	101 679	1837.55	NA	NA	NA	NA
−10%	0.498587	1.28953	58 141.5	1229.43	793.837	0.610747	106 589	1839.59	NA	NA	NA	NA
0	0.518877	1.20483	60 949.8	1021.41	902.365	0.61094	111 498	1841.63	NA	NA	NA	NA
10%	0.536379	1.10002	63 528.6	808.475	1007.24	0.611133	116 408	1843.67	NA	NA	NA	NA
20%	0.551833	0.954449	65 777.9	578.143	1109.49	0.611326	121 317	1845.72	NA	NA	NA	NA
50%	NA	NA	NA	NA	NA	0.611904	136 045	1851.85	NA	NA	NA	NA
90%	NA	NA	NA	NA	NA	0.612674	155 682	1860.06	NA	NA	NA	NA

Notes. (*)Transfer cost is not applicable in the case of no substitution and there is no effect of transfer cost in the case of full substitution.

(1) Each product has its own independent inventory policies due to different demands. The occurrence of shortages for each type of product is considered as their corresponding cycle time.
(2) It is observed from Tables 7 and 8 that the order quantity of major and minor items increases as the holding cost of any item decreases.
(3) Table 6 reflects that the cycle time decreases, and how the total average cost of a major product varies with the product's increasing deterioration rate.
(4) Table 9 shows that the cycle time and ordering quantity increase gradually, while the total average cost decreases with an increasing inflation rate.
(5) It is observed from Tables 5 and 6 that, the deterioration rate of one product has the opposite impact on the ordering quantity of other products.

6. Managerial insights

Results show that partial substitution is the best other than two policies: full substitution and no substitution. Managers can decide which policy they want to implement for their industry based on cost or cycle time even though the partial substitution provides the minimum cost with a substitution time of 0.518877 days and a cycle time of 1.20483 days. Cycle times are less in the full substitution and no substitution cases rather than the partial substitution of deteriorating products even though the total costs are more. Thus, managers can choose either minimum cost or minimum cycle time. The scenario may differ for a multi-echelon model [7].

7. Conclusions and limitations

This study presented a model that examines the impact of product substitution on a deterministic inventory system with two products, taking into account the presence of deterioration and shortage in the major product under an inflationary environment. The decision variable was the cycle time. The model was mathematically and numerically analyzed to compare different scenarios, including partial substitution, full substitution, and no substitution. Various ordering quantity patterns were explored and the corresponding optimal costs were used in the model. To validate and strengthen the model, sensitivity analysis was performed using a numerical example, which provided important managerial insights. Findings indicated that partial substitution offers more significant benefits compared to full substitution and no substitution. Specifically, partial substitution resulted in an average cost that is approximately 9% lower than no substitution and 45% lower than complete substitution. Furthermore, the analysis demonstrated that no substitution was approximately 40% better than the full substitution. By understanding the impact of different parameters within different substitution methods, managers can maximize their benefits. Numerous types of products such as fruit items, clothing, house construction materials, and dairy products can be considered. Business managers can use this study to prepare a more profitable strategy because this study is highly practical and can be used in many business sectors under certain conditions [17]. Incorporating the exponentially increasing demand for the deteriorating product under an inflationary environment helps to identify customer preferences [33] and increase their profit using substitution strategy while contributing to achieving SDG [38].

While this model focused on a two-product inventory system, it can be widened with multi-item inventory systems. Additionally, this study considered a similar demand pattern for both products, but this can be extended to accommodate different demand patterns. This study was in a deterministic and crisp environment, which can be extended in a dynamic model and uncertain conditions with metaheuristic algorithms [43]. Waste of deteriorated products was not considered in this study [19]. Future research can explore multi-level substitution instead of single-level substitution, incorporating shortage, partial backlogging, and delivery policy [47]. Furthermore, the model can be analyzed under specific conditions [6], such as products with expiry dates [9], complex demand patterns [8], credit periods [56], returnable package [37], or products with imperfect quality [54]. Game policy can be used to solve the competitive game if more than one player is considered for the model [27,34]. The paper can be extended with fuzzy scenarios [2,36] and robust optimization analysis [40,48].

Data availability statement

Data has been provided in the model.

Author contribution statement

All authors contributed to the study conception, design, data preparation, and analysis.

Ethical approval

The submitted work is original and not published elsewhere in any form or language (partially or in full). All authors made substantial contributions to the conception or design of the work; or the acquisition, analysis, or interpretation of data.

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