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BIG DATA SERVICE OUTSOURCING AND COST-SHARING CHOICES FOR THE MANUFACTURER

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Abstract. The proliferation of digital technologies has revolutionized various industries, prompting enterprises to prioritize investment in big data analytics. Despite the associated value, enterprises must carefully assess the cost proposition of such investment. This study models a supply chain with a manufacturer and a retailer, investigating big data investment decisions and strategies of manufacturer as leader across varying scenarios. The results show that: if the manufacturer focus only on the big data service level, it will choose not to outsource. In the case of non-outsourcing, the pre-production big data service level, the pre-sale big data service level and the retailer's profit are higher, however, the manufacturer's profit depends on fixed cost. Moreover, the manufacturer has three options: it chooses non-outsourcing if the profits of supply chain members are decreased, it chooses outsourcing without coordination mechanism if only considers maximizing own profit, it chooses outsourcing with coordination mechanism if considers the profits of other members. If outsourcing is considered, the manufacturer can decide its cost-sharing rate according to different situations. When consumers need products with high satisfaction, they will improve the big data service level. And, the increased price is also acceptable to consumers. When consumers pay more attention to low price, appropriately reducing the big data service level can also satisfy consumers. In addition, this paper provides some management inspirations for decision-making and operation of supply chain.

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1. Introduction

The development of science and technology is leading the transformation of the industry, and the use of big data brings unlimited potential to enterprises[1]. Therefore, the major concern of the supply chain are not only production and sale of product like traditional supply chain but also the big data processing service. The development of Internet has penetrated every aspect of daily life and one of the original purposes of Internet is to communicate, resulting in applying big data technology. Big data technology is used to collect and process consumer preference related data in a more effective way than ever before by online shopping platform. For example, in the retail industry, Walmart and Procter & Gamble use big data to analyze consumers' product preference, achieve precise promotion, and provide consumers with better service. In the manufacturing industry,

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companies such as H&M and ZARA use big data to analyze consumer preference and predict demand at the same time to provide personalized service for consumers. In the online shopping platform, enterprises such as Taobao.com and JD.com are also using big data to analyze consumer preferences and provide targeted advertising. We can see that big data is used in all kinds of industries, which shows its importance. Hence, decision-makers can understand what consumers want well [15]. It will make the production and efficiency of enterprises well.

However, the cost is an issue that has to be considered when big data technology is applied [10,16]. The cost will be generated regardless of whether it is simple production, sale, or the application of new technology in the supply chain. And the cost can be divided into fixed cost and variable cost. Fixed cost including basic facilities, equipment, or housing, etc. Variable cost happens when target customer groups are serviced at some kind of level and it will vary with the service level [23]. Minimizing cost and maximizing profit have to be considered by decision-makers. This study considers fixed cost and variable cost of the manufacturer's big data service. All decision-makers have significant considerations when they face cost issue. For example, they either undertake the relevant cost by themselves or outsource the relevant operation to the service provider and pay the outsourcing service charge. In reality, these two models have different influences on the supply chain. Outsourcing or not depends on the relationship evaluation of decision-makers for profit and cost. Any model should be analyzed in the light of the specific situation [34]. Based on the above analysis, the purpose of this study is to explore what conditions the manufacturer will choose outsourcing and what impact it will have. The main issues discussed in this study are as follows:

(1) For big data service, the manufacturer can choose non-outsourcing or outsourcing. What are the conditions that force the manufacturer to make a different choice?
(2) When the manufacturer makes different choices, how does it affect the profits of supply chain members? How will big data service level be affected?
(3) When does the manufacturer use cost-sharing contract? What impact does this contract have on the profits of each member of the supply chain? When do supply chain members agree to use cost-sharing contract?

Based on the above problems, this paper constructs a supply chain composed of a manufacturer, a retailer and a big data service provider. This paper studies the change of supply chain members' profits and the change of big data service level when the manufacturer undertakes big data service by themselves or outsource big data service. The difference between the retailer and the manufacturer is that the retailer invests in big data service for product marketing, while the manufacturer invests in big data service for product production. The results show that there is a threshold for the manufacturer to choose to outsource. The retailer's profit will be damaged when the manufacturer chooses outsourcing, and the manufacturer uses cost-sharing contract as an incentive mechanism to compensate the retailer's profit loss. To a certain extent, cost-sharing mechanism can also increase the big data service level after outsourcing. Finally, numerical examples are used to illustrate the conclusion of this study, and the management enlightenment is given.

The main contributions of this study are as follows. Firstly, we explore the situation of supply chain members investing in big data service at the same time, and the different big data service decisions of supply chain members. Secondly, we study whether or not to outsource and how outsourcing can make supply chain decision-making and profit increase for manufacturer as leader in different situations, which is not deeply studied in previous literature. Finally, the findings of this research offer actionable insights and serve as a theoretical foundation for supply chain enterprises, equipping them with valuable guidance to inform their strategic decisions related to big data utilization and outsourcing practices. These contributions aim to bridge the gap between theoretical constructs and practical application in contemporary supply chain management.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature for this study. Section 3 describes and assumes the basic model. Section 4 analysis outsourcing model and non-outsourcing model. Section 5 analysis cost-sharing model. Section 6 provides numerical analysis. Section 7 provides the conclusions and management enlightenment. The proofs are presented in the Appendix A. The description of some notations is shown in Appendix B.

2. Literature review

The literature review of this study focuses on big data service, service outsourcing and cost-sharing contract in previous literature. Above several streams of literature provide a good direction and method for the research of this study.

2.1. Big data service

With the gradual increase of research on big data technology, some scholars consider that big data technology is very important for any enterprise, it can affect all the operation processes of enterprises [19] and the big data technology has brought more benefits to some enterprises [1, 4]. The big data technology tracks the preferences of consumers and provides products or services that can satisfy the needs of consumers well [21, 28]. Moreover, big data also has applications in supply chain. Based on big data and blockchain, Liu *et al.* [22] studies the incentive measures for information input service in the agricultural field, and the research results provided theoretical guidance for the government and enterprises to formulate and implement relevant strategies. Wu and Lang [29] uses differential game to study how big data marketing and reference promotion effects affect supply chain coordination. Song *et al.* [27] studies the problem of enterprises investing in big data technology to improve recovery rate and information sharing under closed-loop supply chain. Research shows that as competition increases, the negative impact of big data technology cost diminishes, and the optimal decision for manufacturer is to share information only with retailer. The application of big data technology not only has positive effects on traditional supply chains but also plays a significant role in low-carbon supply chains [9]. The significance of big data service cannot be overstated, as it has revolutionized the efficiency we analyze and interpret vast amounts of information. By harnessing the power of big data, enterprises can uncover patterns, trends, and correlations that helpful. This capability has profound implications for decision-making processes, strategic planning, and innovation across various industries. Consequently, exploring the big data service is not just beneficial but imperative for this study, as it provides a comprehensive understanding of big data-driven insights and applications.

2.2. Outsourcing

A number of literatures focus on outsourcing or non-outsourcing [5,23]. The technology or service is introduced to management by enterprises, the costs of each enterprise are different. So, enterprises choose outsourcing or not either because the cost issue [6] or because the service level needs to reach a more professional level, but this study considers the cause of cost. In terms of outsourcing model, the choices for each enterprise are different and the platform's choice for outsourcing model is also different when it plays different roles [25]. For enterprises, they should decide which outsourcing model to choose according to the constraints of each outsourcing model [35, 36]. Liu [20] studies the SLK/DIF expiration window assignment problem on a single machine with a general job-dependent location effect, where staff can decide to outsource by paying the corresponding fee, and demonstrates the advantages of outsourcing decisions and the difference between two different objectives. Farghadani-Chaharsooghi and Karimi [7] studies outsourcing planning and lateral transfer problems, proposed a mixed integer linear programming model, including outsourcing and other factors into the production route problem, and then solved the impact of demand uncertainty through robust optimization. However, the specific implementation of outsourcing should be considered clearly, otherwise not only the cost can't be reduced but also other issues may be introduced [34]. Outsourcing peripheral activities is the right choice for some enterprises, the benefits of outsourcing are to reduce operational costs and to increase operational flexibility for some enterprises. It will make enterprises focus on their own core business [5,24]. Faced with the changing market environment and the integration of partners, enterprises can achieve a win–win–win situation through outsourcing strategy [13]. Therefore, the consideration of outsourcing is indispensable in the decision-making degree of enterprises, and it will affect the interests of enterprises in some businesses directly [18]. Although existing research has extensively analyzed the environmental factors and implementation process of enterprises' outsourcing decisions, few studies have delved into outsourcing strategies under diversified scenarios. Therefore, this study explores outsourcing strategies in multiple situations, aiming to provide enterprises with more targeted and effective guidance.

2.3. Coordination contract

Traditional study on contract coordination focuses on the supply chain field. The contract coordinating mechanism can also be called incentive mechanism including cost-sharing contract, revenue-sharing contract, two-part tariffs contract, etc. [2]. In this study, we discuss the cost-sharing contract to coordinate the supply chain. A cost-sharing contract includes the following features: it improves not only the benefits and enthusiasm of the supply chain members but also the interests and efficiencies of the whole supply chain [3, 26]. And cost-sharing can also be divided into fixed cost-sharing and variable cost-sharing [17]. For its application, the service level of supply chain members can be improved by undertaking cost-sharing contracts [30]. The cost-sharing contract and revenue-sharing contract have influences for the service level and interests of low-carbon supply chain [11, 32]. Wu and Lang [29] studies the impact of big data marketing and reference promotion effect on consumers' conversion rate through differential game, and designed a two-part electricity price scheme and promotion cost-sharing scheme to coordinate the supply chain. Feng *et al.* [8] studies a dual-channel hazardous waste supply chain consisting of treatment facility and contractor, and discussed the impact of cost-sharing rate on the supply chain. The results show that cost-sharing contract can improve the profit of dual-channel supply chain. Based on the carbon cap-and-trade policy, Yang and Yao [33] constructs an evolutionary game model of the supply chain of fresh agricultural products and discusses the information cooperation strategies of each subject. The research results show that the information cooperation strategies of each subject are related to factors such as cost sharing mechanism. Moreover, we should consider a model or hypothetical with some appropriate analysis methods and compare the ratio of cost-sharing in different models [12]. Some conclusions can be drawn which the cost-sharing rate is not the same for different models [28]. The above research provides theoretical support for the coordination contract in this paper, but most of the literature does not discuss the cost-sharing strategy in different situations in depth, which also provides an exploratory direction for this study.

2.4. Summary

In general, existing literature shows the importance of big data and provides a good research target for this study. There is also a dearth of studies focusing on the outsourcing of big data service within the supply chain. And, there is limited exploration into the simultaneous investment in big data by manufacturer and retailer, as well as the subsequent impact on supply chain member profits and the level of retailer big data service. Specifically, while retailer often handle their own big data service independently, manufacturer may engage various supply chain players to manage their big data service. There is also little literature considering cost-sharing contract under different circumstances. This study addresses these gaps by examining the effects of big data service provided by manufacturer on the profitability of supply chain members and the level of retailer's big data service when managed by different supply chain players. Furthermore, it proposes using cost-sharing contract to encourage retailer experiencing losses and to enhance the level of manufacturing's big data service when a manufacturer opts for outsourcing. The key contributions of this study include enriching the field of big data service outsourcing and supply chain coordination contract, providing theoretical guidance for managing big data service cost, and offering managerial insights and strategies.

3. Problem descriptions

In this paper, we consider a supply chain with one manufacturer, one retailer and one big data service provider. This consideration helps us focus on the impact of the manufacturer's choice about outsourcing and cost-sharing, while maintaining the interpretability of the problem. The manufacturer's investment in big data service (BDS) is known as pre-production BDS. The difference between this and other influential factors such as logistics service mentioned in other literature lies in that this is what the manufacturer does before it makes a product, not during or after it is made. In addition, the manufacturer's investment in BDS will have an impact on the retailer's investment in BDS, which is different from other influential factors in literature. The retailer's investment in big data service is known as pre-sale BDS. When the manufacturer outsources pre-production BDS, the manufacturer pays service charge to the BDS provider, the retailer still maintains its original role. This setting is to compare choosing outsourcing with choosing non-outsourcing for the manufacturer and how to outsource. The outsourcing model is commonly used in past research [23,35]. In this era, big data has become more related to our lives. The impact of big data also becomes more important. It may be better than processing by the manufacturer itself if there is a professional BDS organization for the big data processing. However, many enterprises will choose to outsource pre-production BDS to reduce the cost of this service as same as most actual situations, more fund and attention can be put on their core business.

Similarly, referring to some outsourcing literature [35], we assume that the demand is sensitive to both the retail price p and BDS level. The BDS level includes the retailer's pre-sale BDS level e_1 and the manufacturer's pre-production BDS level e_2. BDS can improve the efficiency of enterprises, but also improve the needs of enterprises. For the manufacturer, BDS can help enterprises dig deeper into consumers' preferences and produce more satisfying products. This shows that the higher the BDS level is, the more the manufacturer can produce products to meet consumer demand, and consumers will increase the purchase quantity, which means that the higher the BDS level is, the greater the demand will be. For the retailer, doing some moderate publicity before the product sale is more conducive to consumers knowing the product, so as to achieve the purpose of increasing the purchase of products. The demand function is given by $d = a - bp + \lambda_1 e_1 + \lambda_2 e_2$. This study takes big data service as an influential factor to increase demand, the role of big data service is to make the products produced by the manufacturer to meet the needs of consumers further. A higher level of big data service means a higher level of consumers satisfaction, and consumers will increase the purchase of product. And this study considers the positive impact of big data services. If big data service brings negative effect, then supply chain members will not use big data service. Meanwhile, the big data service increases the demand based on the original demand. When the big data service level is zero, it is only the part that has not been increased, but it will not affect the original basic demand.

To simplify the calculation, it is assumed that the cost sensitivity (*i.e.*, μ_1 and μ_2) and demand sensitivity (*i.e.*, λ_1 and λ_2) are the same when the manufacturer undertakes the big data service by itself and when the big data service is outsourced, but this will not change the conclusion of this study. We use the backward induction approach to get the optimal solutions in different scenarios. To show the meaning of each parameter clearly, the meaning is shown in Table 1. In order to make the inquiry questions traceable and more in line with the actual situation, explaining the problem well, some parameters need to set restrictions. The specific restrictions are as follows: $2\mu_1 b - \lambda_1^2$, $A_0 > 0$. Refer to Ji *et al.* [14] and Xu *et al.* [31], the reason we set this qualification is that it is necessary to ensure that the Hessian Matrix is negative definite. In a practical sense, it is necessary to ensure that profit, price and demand are non-negative, it means that in the process of the game between the manufacturer and the retailer, the price of both sides is reasonable and the profit of both sides is greater than zero, otherwise the manufacturer and the retailer will not play the game. Therefore, we explain why these parameter limits should be set accordingly in the manuscript. Moreover, the description of some notations (*i.e.*, $A_i, i = 0 \sim 12$) is shown in Appendix B.

4. Outsourcing choices

4.1. Non-outsourcing – Model NO

In Model NO, the manufacturer conducts pre-production BDS by itself. The decision-making order is that the manufacturer decides the pre-production BDS level and the wholesale price first, then the retailer decides the pre-sale BDS level and the retail price. Refer to Appendix A for proof procedures and Appendix B for partial notations (*i.e.*, $A_i, i = 0 \sim 12$). Based on the analysis above, we can obtain the profit functions of the retailer and the manufacturer, respectively:

$$\pi_m^{\mathrm{NO}} = (w - c)d - \frac{1}{2}\mu_2 e_2^2 - f_2 \tag{4.1}$$

TABLE 1. The description of notations.

Notations	Description
Parameters	
π_m^i	The manufacturer's profit, $i = \text{NO, O, CS}$
π_r^i	The retailer's profit, $i = \text{NO, O, CS}$
π_b^i	The BDS provider's profit, $i = \text{NO, O, CS}$
d	Market demand
a	Market basic demand
b	Demand sensitivity coefficient of the retail price
f_1	Fixed cost of pre-sale BDS
f_2	Fixed cost of pre-production BDS
λ_1	Demand sensitivity coefficient of pre-sale BDS level
λ_2	Demand sensitivity coefficient of pre-production BDS level
μ_1	Cost sensitivity coefficient of pre-sale BDS level
μ_2	Cost sensitivity coefficient of pre-production BDS level
c	The manufacturer's unit production cost
Decision variables	
e_1^i	Pre-sale BDS level of the retailer, $i = \text{NO, O, CS}$
e_2^i	Pre-production BDS level of the manufacturer, $i = \text{NO, O, CS}$
p^i	Retail price of the product, $i = \text{NO, O, CS}$
w^i	Wholesale price of the product, $i = \text{NO, O, CS}$
m	Outsourcing service charge

$$\pi_r^{\text{NO}} = (p - w)d - \frac{1}{2}\mu_1 e_1^2 - f_1. \tag{4.2}$$

Lemma 1. *When the manufacturer undertakes pre-production BDS, the optimal pricing and service decisions are respectively as follows:*

$$p^{\text{NO}} = \frac{bcA_2 + 3\mu_1\mu_2 ab - \mu_2 a\lambda_1^2}{A_0 b} \tag{4.3}$$

$$w^{\text{NO}} = \frac{bcA_2 + 2\mu_1\mu_2 ab - \mu_2 a\lambda_1^2 + \mu_1\mu_2 b^2 c}{A_0 b} \tag{4.4}$$

$$e_1^{\text{NO}} = \frac{\lambda_1\mu_2(a - bc)}{A_0} \tag{4.5}$$

$$e_2^{\text{NO}} = \frac{\lambda_2\mu_1(a - bc)}{A_0}. \tag{4.6}$$

Consequently, the corresponding profits of the retailer and the manufacturer are as follows:

$$\pi_m^{\text{NO}} = \frac{\mu_1\mu_2(a - bc)^2}{2A_0} - f_2 \tag{4.7}$$

$$\pi_r^{\text{NO}} = \frac{\mu_1\mu_2^2(a - bc)^2(2b\mu_1 - \lambda_1^2)}{2A_0^2} - f_1. \tag{4.8}$$

Based on Lemma 1, we can obtain the results below which describe how the key parameters (*i.e.*, λ_1, λ_2, μ_1 and μ_2) affect the optimal decisions of the retailer and the manufacturer in Model NO.

Proposition 1. *When the manufacturer undertakes the pre-production BDS, we can get:*

(1) $\frac{\partial^2 p^{\mathrm{NO}}}{\partial\lambda_1\partial\lambda_2} > 0$, $\frac{\partial^2 w^{\mathrm{NO}}}{\partial\lambda_2\partial\lambda_1} > 0$, $\frac{\partial^2 e_1^{\mathrm{NO}}}{\partial\lambda_1\partial\lambda_2} > 0$, $\frac{\partial^2 e_2^{\mathrm{NO}}}{\partial\lambda_2\partial\lambda_1} > 0$;
(2) $\frac{\partial^2 p^{\mathrm{NO}}}{\partial\mu_1\partial\mu_2} > 0$, $\frac{\partial^2 w^{\mathrm{NO}}}{\partial\mu_2\partial\mu_1} > 0$, $\frac{\partial^2 e_1^{\mathrm{NO}}}{\partial\mu_1\partial\mu_2} > 0$, $\frac{\partial^2 e_2^{\mathrm{NO}}}{\partial\mu_2\partial\mu_1} > 0$.

Proposition 1 shows that one of BDS level demand (cost) sensitivity increase will promote an impact increase in another BDS level demand (cost) sensitivity to retail price, wholesale price and BDS level. For cost sensitivity, it indicates that when the BDS level cost sensitivity increases, it will further increase the cost of the manufacturer and the retailer, so they will further reduce the BDS level to cut cost. Due to the decline in the BDS level, consumers will reduce purchase of product, so they will further reduce price to ensure earning. However, for demand sensitivity, it indicates that when the BDS level demand sensitivity increases, the manufacturer and the retailer will further increase their BDS level in order to gain more revenue. As the BDS level goes up, its cost also increases, so the manufacturer and the retailer will further increase wholesale price and retail price to offset the increased cost.

Proposition 2. *When the manufacturer undertakes pre-production BDS, we can get:*

(1) $\frac{\partial^2 \pi_r^{\mathrm{NO}}}{\partial\lambda_1\partial\lambda_2} > 0$, $\frac{\partial^2 \pi_m^{\mathrm{NO}}}{\partial\lambda_2\partial\lambda_1} > 0$;
(2) $\frac{\partial^2 \pi_r^{\mathrm{NO}}}{\partial\mu_1\partial\mu_2} > 0$, $\frac{\partial^2 \pi_m^{\mathrm{NO}}}{\partial\mu_2\partial\mu_1} > 0$.

Proposition 2 shows that one of BDS level demand (cost) sensitivity increase will promote an impact increase in another BDS level demand (cost) sensitivity to the manufacturer's profit and the retailer's profit. It shows that when cost sensitivity increases, the cost of BDS will increase, and the revenue from selling product will not be enough to offset the increase in cost, resulting in further profit declines for the manufacturer and the retailer. When demand sensitivity increases, the manufacturer and the retailer will further increase their profits by raising price or increasing the BDS level.

4.2. Outsourcing – Model O

In Model O, the manufacturer hires the independent BDS provider to provide pre-production BDS. In this case, the role of the manufacturer in Model NO is played by the BDS provider. But the BDS provider only process the data, then reports the results to the manufacturer. Refer to Appendix A for proof procedures and Appendix B for partial notations (*i.e.*, $A_i, i = 0\sim12$).

The decision-making order in Model O is that the BDS provider decides the pre-production BDS level and the service charge first, then the manufacturer determines the wholesale price. Finally, the retailer decides the pre-sale BDS level and the retail price. Based on the analysis above, the profit functions of the retailer, the manufacturer and the BDS provider are as follows, respectively:

$$\pi_b^{\mathrm{O}} = md - \frac{1}{2}\mu_2 e_2^2 - f_2 \tag{4.9}$$

$$\pi_m^{\mathrm{O}} = (w - c)d - md \tag{4.10}$$

$$\pi_r^{\mathrm{O}} = (p - w)d - \frac{1}{2}\mu_1 e_1^2 - f_1. \tag{4.11}$$

Lemma 2. *When the service is outsourced to the independent BDS provider, the optimal pricing and service decisions are as follows, respectively:*

$$p^{\mathrm{O}} = \frac{bcA_2 + 7\mu_1\mu_2 ab - 3\mu_2 a\lambda_1^2}{A_1 b} \tag{4.12}$$

$$w^{\mathrm{O}} = \frac{bcA_2 + 6\mu_1\mu_2 ab - 3\mu_2 a\lambda_1^2 + \mu_1\mu_2 cb^2}{A_1 b} \tag{4.13}$$

$$e_1^{\mathrm{O}} = \frac{\lambda_1\mu_2(a - bc)}{A_1} \tag{4.14}$$

$$e_2^{\mathrm{O}} = \frac{\lambda_2\mu_1(a-bc)}{A_1} \tag{4.15}$$

$$m^{\mathrm{O}} = \frac{2\mu_2\left(-2\mu_1 b^2 c + bc\lambda_1^2 + 2\mu_1 ab - a\lambda_1^2\right)}{A_1 b}. \tag{4.16}$$

Consequently, the corresponding profits of the BDS provider, the manufacturer and the retailer are as follows:

$$\pi_b^{\mathrm{O}} = \frac{\mu_1\mu_2(a-bc)^2}{2A_1} - f_2 \tag{4.17}$$

$$\pi_m^{\mathrm{O}} = \frac{\mu_1\mu_2^2(a-bc)^2\left(2b\mu_1 - \lambda_1^2\right)}{A_1^2} \tag{4.18}$$

$$\pi_r^{\mathrm{O}} = \frac{\mu_1\mu_2^2(a-bc)^2\left(2b\mu_1 - \lambda_1^2\right)}{2A_1^2} - f_1. \tag{4.19}$$

Similarly, we acquire the following results which describe the effects of the parameters (*i.e.*, λ_1, λ_2, μ_1 and μ_2) on the optimal decisions respectively.

Proposition 3. *When the pre-production BDS is outsourced to the independent BDS provider, we can get:*

(1) $\frac{\partial^2 p^{\mathrm{O}}}{\partial\lambda_1\partial\lambda_2} > 0,\ \frac{\partial^2 w^{\mathrm{O}}}{\partial\lambda_2\partial\lambda_1} > 0,\ \frac{\partial^2 e_1^{\mathrm{O}}}{\partial\lambda_1\partial\lambda_2} > 0,\ \frac{\partial^2 e_2^{\mathrm{O}}}{\partial\lambda_2\partial\lambda_1} > 0,\ \frac{\partial^2 m^{\mathrm{O}}}{\partial\lambda_1\partial\lambda_2} > 0;$
(2) $\frac{\partial^2 p^{\mathrm{O}}}{\partial\mu_1\partial\mu_2} > 0,\ \frac{\partial^2 w^{\mathrm{O}}}{\partial\mu_2\partial\mu_1} > 0,\ \frac{\partial^2 e_1^{\mathrm{O}}}{\partial\mu_1\partial\mu_2} > 0,\ \frac{\partial^2 e_2^{\mathrm{O}}}{\partial\mu_2\partial\mu_1} > 0,\ \frac{\partial^2 m^{\mathrm{O}}}{\partial\mu_1\partial\mu_2} > 0.$

Proposition 3 shows that even though the BDS provider is hired to provide pre-production BDS. The change of the retail price, the wholesale price, the BDS level with respect to the demand sensitivity and cost sensitivity are similar to Model NO (Prop. 1). The impact of demand sensitivity and cost sensitivity on service charge is the same as the impact on wholesale price, retail price, and BDS level. It also shows that one of BDS level demand (cost) sensitivity increase will promote an impact increase in another BDS level demand (cost) sensitivity to retail price, wholesale price and BDS level.

Proposition 4. *When the service is outsourced to the independent BDS provider, we can get:*

(1) $\frac{\partial^2 \pi_r^{\mathrm{O}}}{\partial\lambda_1\partial\lambda_2} > 0,\ \frac{\partial^2 \pi_m^{\mathrm{O}}}{\partial\lambda_2\partial\lambda_1} > 0,\ \frac{\partial^2 \pi_b^{\mathrm{O}}}{\partial\lambda_1\partial\lambda_2} > 0;$
(2) $\frac{\partial^2 \pi_r^{\mathrm{O}}}{\partial\mu_1\partial\mu_2} > 0,\ \frac{\partial^2 \pi_m^{\mathrm{O}}}{\partial\mu_2\partial\mu_1} > 0,\ \frac{\partial^2 \pi_b^{\mathrm{O}}}{\partial\mu_1\partial\mu_2} > 0.$

Proposition 4 shows that the change in profits of the manufacturer and the retailer is similar to that in Proposition 2. And the BDS provider's profit changes are similar to that of the retailer and the manufacturer. The reason is that the role of the BDS provider in Model O is similar to that of the manufacturer in Model NO. Although the BDS provider appears in the supply chain as a third party, its decision-making behavior will also have an impact on demand, which is closely related to each member of the supply chain.

4.3. Comparison between Model NO and Model O

In order to find the difference between each model, some comparisons are used between Model NO and Model O. Furthermore, it can help us to get some extraordinary results. Refer to Appendix A for proof procedures and Appendix B for partial notations (*i.e.*, $A_i, i = 0{\sim}12$).

Proposition 5. *We can get from the comparison:*

(1) $e_1^{\mathrm{O}} < e_1^{\mathrm{NO}}$, $e_2^{\mathrm{O}} < e_2^{\mathrm{NO}}$;
(2) $w^{\mathrm{O}} < w^{\mathrm{NO}}$, $p^{\mathrm{O}} < p^{\mathrm{NO}}$ *if* $A_2 < 0$; $w^{\mathrm{O}} > w^{\mathrm{NO}}$, $p^{\mathrm{O}} > p^{\mathrm{NO}}$ *if* $A_2 > 0$.

Proposition 5 demonstrates that the pre-sale BDS level and the pre-production BDS level when the manufacturer chooses outsourcing are lower than that when the manufacturer chooses non-outsourcing. A reasonable explanation is that the manufacturer only considers reducing its own cost when it chooses outsourcing, but the change of the BDS level and the profit of other members of the supply chain are not considered. In addition, the manufacturer chooses to outsource BDS in order to reduce cost, which will lead to a decrease in the pre-production BDS level. For the manufacturer, although it outsources BDS, it needs to pay for the service charge. If the manufacturer lowers the wholesale price in the case of outsourcing, the retailer will definitely lower the retail price. The reason is that the increase in retail price by the retailer will lead to the decline of demand. However, the negative impact of falling demand on profit will be greater than the positive impact of rising retail price on profit. If the manufacturer raises wholesale price, the retailer has two ways to ensure profit do not fall. One is to raise price to ensure revenue, and the other is to lower price to ensure demand. However, whichever choice the retailer makes, it's profit will be hurt.

We can obtain the difference of the manufacturer's profit in different models: $\pi_m^{\mathrm{O}} - \pi_m^{\mathrm{NO}} = \frac{(2\mu_1 b-\lambda_1^2)\mu_1(a-bc)^2\mu_2^2}{A_1^2} - \frac{\mu_2\mu_1(a-bc)^2}{2A_0} + f_2$ (*i.e.*, $\pi_m^{\mathrm{O}} - \pi_m^{\mathrm{NO}} = A_3 + f_2$). While, the retailer prefers Model NO (*i.e.*, $\pi_r^{\mathrm{O}} - \pi_r^{\mathrm{NO}} < 0$). For the BDS provider, $\pi_b^{\mathrm{O}} = \frac{\mu_1(a-bc)^2\mu_2}{2A_1} - f_2 > 0$ (*i.e.*, $\pi_b^{\mathrm{O}} = A_4 - f_2$) and $A_4 > f_2$. If the profit of BDS provider is less than zero, then BDS provider will not reach an agreement with the manufacturer to undertake BDS. Therefore, this study only considers the profit changes of the manufacturer and the retailer under the condition that BDS provider can make profit. The difference between the total profits of the manufacturer and the retailer in Model O and the total profits of the manufacturer and the retailer in Model NO also can be obtained. The π_{m+r}^{O} denotes the total profits of the manufacturer and the retailer in Model O, the π_{m+r}^{NO} denotes the total profits of the manufacturer and the retailer in Model NO: $\pi_{m+r}^{\mathrm{O}} - \pi_{m+r}^{\mathrm{NO}} = \frac{3\mu_1\mu_2^2(a-bc)^2(2\mu_1 b-\lambda_1^2)}{2A_1^2} - \frac{\mu_1\mu_2(a-bc)^2(6\mu_1\mu_2 b-\mu_1\lambda_2^2-3\mu_2\lambda_1^2)}{2A_0^2} + f_2$ (*i.e.*, $\pi_{m+r}^{\mathrm{O}} - \pi_{m+r}^{\mathrm{NO}} = A_5 + f_2$). In addition, the above analysis shows that the change in the manufacturer's profit and the change in the total profits of the manufacturer and the retailer are related to pre-production BDS fixed cost f_2. From the perspective of the manufacturer, whether choosing outsourcing also depends on the pre-production BDS fixed cost f_2. The threshold principle exists when the manufacturer determines outsourcing or non-outsourcing. Therefore, the above analysis shows that $|A_3| < |A_5| < A_4$.

When $f_2 < |A_3|$, the manufacturer's profit, the retailer's profit and the total profits of them will decrease when the manufacturer chooses outsourcing, it is the best choice for the manufacturer to choose non-outsourcing. When $|A_3| < f_2 < |A_5|$, the manufacturer's profit will increase, the retailer's profit and the total profits of them will decrease when the manufacturer chooses outsourcing. The reason is that the level of the manufacturer's profit increase is lower than the level of the retailer's profit decrease. It is a good choice for the manufacturer, but it leads to disharmony in the operation of the supply chain due to the decrease of the retailer's profit and the total profits of them. When $f_2 > |A_5|$, the manufacturer's profit and the total profit of them will increase, the retailer's profit will decrease when the manufacturer chooses outsourcing. The reason is that the level of the manufacturer's profit increase is better than the level of the retailer's profit decrease. In this situation, the manufacturer should choose outsourcing and consider a good coordination mechanism or compensation mechanism to persuade the retailer to agree to outsourcing.

From the analysis in the above sections, we know that the manufacturer has three options. Firstly, the manufacturer chooses non-outsourcing if the profits of all supply chain members are decreased. Secondly, the manufacturer chooses outsourcing without coordination mechanism if it only considers maximizing its own profits. Thirdly, the manufacturer chooses outsourcing with coordination mechanism if it considers the profits of other members. When $|A_3| < f_2 < |A_5|$ or $f_2 > |A_5|$, the profit of the manufacturer in Model O is greater than that in Model NO, so the manufacturer can implement cost-sharing in both cases, and the results will be different.

5. Cost-sharing choices

5.1. Cost-sharing – Model CS

We discuss how the cost-sharing contract affects the optimal decisions of each member. According to the optimal conclusions from Lemma 1 to Proposition 5, we can obtain the results below. The level of the pre-production BDS and the pre-sale BDS will decrease, the profit of the manufacturer will increase, the profit of the retailer will decrease when the manufacturer chooses outsourcing. Therefore, a cost-sharing contract is introduced to coordinate the supply chain. Compared with Model O and Model NO, the profits of the manufacturer and the retailer, the level of BDS will be changed in Model CS. The discussion in this section is based on the manufacturer choosing outsourcing. The manufacturer as the subject of decision-making, has the right to choose outsourcing or non-outsourcing to maximize its benefits without considering other conditions. If the manufacturer considers the interests of other members, cost-sharing can be a way for the manufacturer to coordinate. Refer to Appendix A for proof procedures and Appendix B for partial notations (*i.e.*, $A_i, i = 0\sim 12$).

We assume that a cost-sharing contract is established between the manufacturer and the retailer. This cost-sharing contract is designed to solve the decrease of the retailer's profit. Similarly, the decision order in Model CS is the same as that in Model O. The difference with some previous models is that the cost-sharing model has two sharing rates [23], one of them is the sharing ratio of variable cost α and the other is the sharing ratio of fixed cost β. We can obtain the profit functions of the manufacturer, the BDS provider and the retailer, which are respectively expressed by:

$$\pi_b^{\text{CS}} = md - \frac{1}{2}\mu_2 e_2^2 - f_2 \tag{5.1}$$

$$\pi_m^{\text{CS}} = (w-c)d - md - \alpha\frac{1}{2}\mu_1 e_1^2 - \beta f_1 \tag{5.2}$$

$$\pi_r^{\text{CS}} = (p-w)d - (1-\alpha)\frac{1}{2}\mu_1 e_1^2 - (1-\beta) f_1. \tag{5.3}$$

Lemma 3. *When a cost-sharing contract is reached between the manufacturer and the retailer, the optimal price, the service charge and the service level are respectively as follows:*

$$w^{\text{CS}} = \frac{6\big(\big(\frac{1}{3}bc+a\big)\mu_2 - \frac{1}{6}c\lambda_2^2\big)(1-\alpha)^2 b\mu_1 + 5\big(\frac{1}{5}c(\alpha-1)b + a\big(\alpha-\frac{3}{5}\big)\big)\mu_2\lambda_1^2}{8b\big((1-\alpha)^2\big(b\mu_2 - \frac{1}{8}\lambda_2^2\big)\mu_1 + \frac{3}{4}\big(\alpha-\frac{2}{3}\big)\mu_2\lambda_1^2\big)} \tag{5.4}$$

$$p^{\text{CS}} = \frac{7\big(\big(\frac{1}{7}bc+a\big)\mu_2 - \frac{1}{7}c\lambda_2^2\big)(1-\alpha)^2 b\mu_1 + 5\big(\frac{1}{5}c(\alpha-1)b + a\big(\alpha-\frac{3}{5}\big)\big)\mu_2\lambda_1^2}{8b((1-\alpha)^2\big(b\mu_2 - \frac{1}{8}\lambda_2^2\big)\mu_1 + \frac{3}{4}\big(\alpha-\frac{2}{3}\big)\mu_2\lambda_1^2\big)} \tag{5.5}$$

$$e_1^{\text{CS}} = \frac{(a-bc)\lambda_1\mu_2(1-\alpha)}{8(1-\alpha)^2\big(b\mu_2 - \frac{1}{8}\lambda_2^2\big)\mu_1 + 6\big(\alpha-\frac{2}{3}\big)\mu_2\lambda_1^2} \tag{5.6}$$

$$e_2^{\text{CS}} = \frac{(a-bc)\lambda_2\mu_1(1-\alpha)^2}{8(1-\alpha)^2\big(b\mu_2 - \frac{1}{8}\lambda_2^2\big)\mu_1 + 6\big(\alpha-\frac{2}{3}\big)\mu_2\lambda_1^2} \tag{5.7}$$

$$m^{\text{CS}} = \frac{\big(\mu_1(1-\alpha)^2 b + \frac{3}{4}\lambda_1^2\big(\alpha-\frac{2}{3}\big)\big)\mu_2(a-bc)}{2b\big((1-\alpha)^2\big(b\mu_2 - \frac{1}{8}\lambda_2^2\big)\mu_1 + \frac{3}{4}\big(\alpha-\frac{2}{3}\big)\mu_2\lambda_1^2\big)}. \tag{5.8}$$

Consequently, the corresponding profits of the BDS provider, the manufacturer and the retailer are as follows:

$$\pi_b^{\text{CS}} = \frac{(a-bc)^2\mu_2(1-\alpha)^2\mu_1}{16(1-\alpha)^2\big(\mu_2 b - \frac{1}{8}\lambda_2^2\big)\mu_1 + 12\big(\alpha-\frac{2}{3}\big)\mu_2\lambda_1^2} - f_2 \tag{5.9}$$

$$\pi_m^{\text{CS}} = \frac{\mu_1\mu_2^2(1-\alpha)^2\big(\mu_1 b(1-\alpha)^2 + \frac{3}{4}\lambda_1^2\big(\alpha-\frac{2}{3}\big)\big)(a-bc)^2}{32\big((1-\alpha)^2\big(\mu_2 b - \frac{1}{8}\lambda_2^2\big)\mu_1 + \frac{3}{4}\big(\alpha-\frac{2}{3}\big)\mu_2\lambda_1^2\big)^2} - \beta f_1 \tag{5.10}$$

TABLE 2. The summary of each variable in the three models.

	Model NO	Model O	Model CS
π_m	$\frac{\mu_1\mu_2(a-bc)^2}{2A_0}-f_2$	$\frac{\mu_1\mu_2^2(a-bc)^2(2b\mu_1-\lambda_1^2)}{A_1^2}$	$\frac{\mu_1\mu_2^2(1-\alpha)^2\left(\mu_1 b(1-\alpha)^2+\frac{3}{4}\lambda_1^2\left(\alpha-\frac{2}{3}\right)\right)(a-bc)^2}{32\left((1-\alpha)^2\left(\mu_2 b-\frac{1}{8}\lambda_2^2\right)\mu_1+\frac{3}{4}\left(\alpha-\frac{2}{3}\right)\mu_2\lambda_1^2\right)^2}-\beta f_1$
π_r	$\frac{\mu_1\mu_2^2(a-bc)^2(2b\mu_1-\lambda_1^2)}{2A_0^2}-f_1$	$\frac{\mu_1\mu_2^2(a-bc)^2\left(2b\mu_1-\lambda_1^2\right)}{2A_1^2}-f_1$	$\frac{\mu_2^2(1-\alpha)^3\left(\mu_1 b(\alpha-1)+\frac{1}{2}\lambda_1^2\right)\mu_1(a-bc)^2}{64\left((1-\alpha)^2\left(b\mu_2-\frac{1}{8}\lambda_2^2\right)\mu_1+\frac{3}{4}\left(\alpha-\frac{2}{3}\right)\mu_2\lambda_1^2\right)^2}-(1-\beta)f_1$
π_b		$\frac{\mu_1\mu_2(a-bc)^2}{2A_1}-f_2$	$\frac{(a-bc)^2\mu_2(1-\alpha)^2\mu_1}{16(1-\alpha)^2\left(\mu_2 b-\frac{1}{8}\lambda_2^2\right)\mu_1+12\left(\alpha-\frac{2}{3}\right)\mu_2\lambda_1^2}-f_2$
e_1	$\frac{\lambda_1\mu_2(a-bc)}{A_0}$	$\frac{\lambda_1\mu_2(a-bc)}{A_1}$	$\frac{(a-bc)\lambda_1\mu_2(1-\alpha)}{8(1-\alpha)^2\left(b\mu_2-\frac{1}{8}\lambda_2^2\right)\mu_1+6\left(\alpha-\frac{2}{3}\right)\mu_2\lambda_1^2}$
e_2	$\frac{\lambda_2\mu_1(a-bc)}{A_0}$	$\frac{\lambda_2\mu_1(a-bc)}{A_1}$	$\frac{(a-bc)\lambda_2\mu_1(1-\alpha)^2}{8(1-\alpha)^2\left(b\mu_2-\frac{1}{8}\lambda_2^2\right)\mu_1+6\left(\alpha-\frac{2}{3}\right)\mu_2\lambda_1^2}$
w	$\frac{bcA_2+2\mu_1\mu_2 ab-\mu_2 a\lambda_1^2+\mu_1\mu_2 b^2 c}{A_0 b}$	$\frac{bcA_2+6\mu_1\mu_2 ab-3\mu_2 a\lambda_1^2+\mu_1\mu_2 cb^2}{A_1 b}$	$\frac{6\left(\left(\frac{1}{3}bc+a\right)\mu_2-\frac{1}{6}c\lambda_2^2\right)(1-\alpha)^2 b\mu_1+5\left(\frac{1}{5}c(\alpha-1)b+a\left(\alpha-\frac{3}{5}\right)\right)\mu_2\lambda_1^2}{8b\left((1-\alpha)^2\left(b\mu_2-\frac{1}{8}\lambda_2^2\right)\mu_1+\frac{3}{4}\left(\alpha-\frac{2}{3}\right)\mu_2\lambda_1^2\right)}$
p	$\frac{bcA_2+3\mu_1\mu_2 ab-\mu_2 a\lambda_1^2}{A_0 b}$	$\frac{bcA_2+7\mu_1\mu_2 ab-3\mu_2 a\lambda_1^2}{A_1 b}$	$\frac{7\left(\left(\frac{1}{7}bc+a\right)\mu_2-\frac{1}{7}c\lambda_2^2\right)(1-\alpha)^2 b\mu_1+5\left(\frac{1}{5}c(\alpha-1)b+a\left(\alpha-\frac{3}{5}\right)\right)\mu_2\lambda_1^2}{8b\left((1-\alpha)^2\left(b\mu_2-\frac{1}{8}\lambda_2^2\right)\mu_1+\frac{3}{4}\left(\alpha-\frac{2}{3}\right)\mu_2\lambda_1^2\right)}$
m		$\frac{2\mu_2\left(-2\mu_1 b^2 c+bc\lambda_1^2+2\mu_1 ab-a\lambda_1^2\right)}{A_1 b}$	$\frac{\left(\mu_1(1-\alpha)^2 b+\frac{3}{4}\lambda_1^2\left(\alpha-\frac{2}{3}\right)\right)\mu_2(a-bc)}{2b\left((1-\alpha)^2\left(b\mu_2-\frac{1}{8}\lambda_2^2\right)\mu_1+\frac{3}{4}\left(\alpha-\frac{2}{3}\right)\mu_2\lambda_1^2\right)}$

$$\pi_r^{\text{CS}}=\frac{\mu_2^2(1-\alpha)^3\left(\mu_1 b(\alpha-1)+\frac{1}{2}\lambda_1^2\right)\mu_1(a-bc)^2}{64\left((1-\alpha)^2\left(b\mu_2-\frac{1}{8}\lambda_2^2\right)\mu_1+\frac{3}{4}\left(\alpha-\frac{2}{3}\right)\mu_2\lambda_1^2\right)^2}-(1-\beta)f_1. \tag{5.11}$$

Lemma 3 depicts the optimal decisions of each member in the supply chain when a cost-sharing contract is reached between the manufacturer and the retailer. The summary of each variable in the three models is shown in Table 2.

When $|A_3|<f_2<|A_5|$ or $f_2>|A_5|$, the manufacturer has seven different purposes for cost-sharing, respectively: Scenario 1 represents the manufacturer considering the profit of itself. Scenario 2 represents the manufacturer considering the profit of the retailer. Scenario 3 represents the manufacturer considering the total profits of itself and the retailer. Scenario 4 represents the manufacturer considering the retailer's profit and the total profits of itself and the retailer. Scenario 5 represents the manufacturer considering its own profit and the retailer's profit. Scenario 6 represents the manufacturer considering the profit of itself and the total profits of itself and the retailer. Scenario 7 represents the manufacturer considering its own profit, the retailer's profit and the total profits of itself and the retailer. These seven different scenarios have different effects on fixed cost for the retailer and the manufacturer.

5.2. Profit analysis

5.2.1. The first cost-sharing case – Case 1

In this case, when $|A_3|<f_2<|A_5|$, the manufacturer's profit in Model O is greater than that in Model NO, the retailer's profit in Model O is less than that in Model NO, the total profits of the manufacturer and the retailer in Model O is less than that in Model NO. The description of scenarios refers to Section 5.1. Refer to Appendix B for partial notations (*i.e.*, $A_i, i=0\sim12$).

(1) Scenario 1: $\pi_m^{\text{CS}}-\pi_m^{\text{O}}=A_6-\beta f_1>0$ (*i.e.*, $f_1<\frac{|A_6|}{\beta}$);
(2) Scenario 2: $\pi_r^{\text{CS}}-\pi_r^{\text{NO}}=A_7+\beta f_1>0$ (*i.e.*, $f_1<\frac{|A_7|}{\beta}$);
(3) Scenario 3: $\pi_{m+r}^{\text{CS}}-\pi_{m+r}^{\text{NO}}=A_8+f_2>0$ (*i.e.*, $f_2>|A_8|$), that is, $|A_8|<f_2<|A_5|$;
(4) Scenario 4: $\pi_r^{\text{CS}}-\pi_r^{\text{NO}}=A_7+\beta f_1>0$ (*i.e.*, $f_1>\frac{|A_7|}{\beta}$) and $\pi_{m+r}^{\text{CS}}-\pi_{m+r}^{\text{NO}}=A_8+f_2>0$, (*i.e.*, $f_2>|A_8|$), that is, $|A_8|<f_2<|A_5|$;

Table 3. The value region for the fixed cost when $|A_3| < f_2 < |A_5|$.

Scenario	The value region
Scenario 1	$f_1 < \frac{|A_6|}{\beta}$
Scenario 2	$f_1 > \frac{|A_7|}{\beta}$
Scenario 3	$|A_8| < f_2 < |A_5|$
Scenario 4	$f_1 > \frac{|A_7|}{\beta}$ and $|A_8| < f_2 < |A_5|$
Scenario 5	$\frac{|A_7|}{\beta} < f_1 < \frac{|A_6|}{\beta}$
Scenario 6	$f_1 < \frac{|A_6|}{\beta}$ and $|A_8| < f_2 < |A_5|$
Scenario 7	$\frac{|A_7|}{\beta} < f_1 < \frac{|A_6|}{\beta}$ and $|A_8| < f_2 < |A_5|$

(5) Scenario 5: $\pi_m^{\mathrm{CS}} - \pi_m^{\mathrm{O}} = A_6 - \beta f_1 > 0$ ($i.e., f_1 < \frac{|A_6|}{\beta}$) and $\pi_r^{\mathrm{CS}} - \pi_r^{\mathrm{NO}} = A_7 + \beta f_1 > 0$, ($i.e., f_1 > \frac{|A_7|}{\beta}$), that is, $\frac{|A_7|}{\beta} < f_1 < \frac{|A_6|}{\beta}$;

(6) Scenario 6: $\pi_m^{\mathrm{CS}} - \pi_m^{\mathrm{O}} = A_6 - \beta f_1 > 0$ ($i.e., f_1 < \frac{|A_6|}{\beta}$) and $\pi_{m+r}^{\mathrm{CS}} - \pi_{m+r}^{\mathrm{NO}} = A_8 + f_2 > 0$, ($i.e., f_2 > |A_8|$), that is, $|A_8| < f_2 < |A_5|$;

(7) Scenario 7: $\pi_m^{\mathrm{CS}} - \pi_m^{\mathrm{O}} = A_6 - \beta f_1 > 0$ ($i.e., f_1 < \frac{|A_6|}{\beta}$), $\pi_r^{\mathrm{CS}} - \pi_r^{\mathrm{NO}} = A_7 + \beta f_1 > 0$ ($i.e., f_1 > \frac{|A_7|}{\beta}$) and $\pi_{m+r}^{\mathrm{CS}} - \pi_{m+r}^{\mathrm{NO}} = A_8 + f_2 > 0$ ($i.e., f_2 > |A_8|$), that is, $|A_8| < f_2 < |A_5|$ and $\frac{|A_7|}{\beta} < f_1 < \frac{|A_6|}{\beta}$.

However, Scenario 3, Scenario 4, Scenario 6 and Scenario 7 must satisfy the following condition: $|A_3| < |A_8| < |A_5|$. Scenario 5 and Scenario 7 must satisfy the following condition: $|A_7| < |A_6|$. In particular, Scenario 5 and Scenario 7 have the same meaning in the actual operation of supply chain. That is, when the profits of the manufacturer and the retailer increase, the sum of their profits also increases. However, according to theoretical analysis, in addition to the result of Scenario 5 ($i.e., |A_3| < f_2 < |A_5|$), the value region of fixed cost of the pre-production big data service can be further obtained in Scenario 7 ($i.e., |A_8| < f_2 < |A_5|$). The summary of each value region is shown in Table 3.

5.2.2. *The second cost-sharing case – Case 2*

In this case, when $f_2 > |A_5|$, the manufacturer's profit in Model O is greater than that in Model NO, the retailer's profit in Model O is less than that in Model NO, the total profits of the manufacturer and the retailer in Model O is greater than that in Model NO. The description of scenarios refers to Section 5.1. Refer to Appendix B for partial notations ($i.e., A_i, i = 0\sim12$). In different scenarios, similar to Case 1, the different conditions must be satisfied:

(1) Scenario 1: $\pi_m^{\mathrm{CS}} - \pi_m^{\mathrm{O}} = A_6 - \beta f_1 > 0$ ($i.e., f_1 < \frac{|A_6|}{\beta}$);

(2) Scenario 2: $\pi_r^{\mathrm{CS}} - \pi_r^{\mathrm{NO}} = A_7 + \beta f_1 > 0$ ($i.e., f_1 > \frac{|A_7|}{\beta}$);

(3) Scenario 3: $\pi_{m+r}^{\mathrm{CS}} - \pi_{m+r}^{\mathrm{O}} = A_9 - A_{10} > 0$ ($i.e., A_9 > A_{10}$);

(4) Scenario 4: $\pi_r^{\mathrm{CS}} - \pi_r^{\mathrm{NO}} = A_7 + \beta f_1 > 0$ ($i.e., f_1 > \frac{|A_7|}{\beta}$) and $\pi_{m+r}^{\mathrm{CS}} - \pi_{m+r}^{\mathrm{O}} = A_9 - A_{10} > 0$ ($i.e., A_9 > A_{10}$);

(5) Scenario 5: $\pi_m^{\mathrm{CS}} - \pi_m^{\mathrm{O}} = A_6 - \beta f_1 > 0$ ($i.e., f_1 < \frac{|A_6|}{\beta}$) and $\pi_r^{\mathrm{CS}} - \pi_r^{\mathrm{NO}} = A_7 + \beta f_1 > 0$ ($i.e., f_1 > \frac{|A_7|}{\beta}$), that is, $\frac{|A_7|}{\beta} < f_1 < \frac{|A_6|}{\beta}$;

(6) Scenario 6: $\pi_m^{\mathrm{CS}} - \pi_m^{\mathrm{O}} = A_6 - \beta f_1 > 0$ ($i.e., f_1 < \frac{|A_6|}{\beta}$) and $\pi_{m+r}^{\mathrm{CS}} - \pi_{m+r}^{\mathrm{O}} = A_9 - A_{10} > 0$ ($i.e., A_9 > A_{10}$);

TABLE 4. The value region for the fixed cost when $f_2 > |A_5|$.

Scenario	The value region
Scenario 1	$f_1 < \frac{|A_6|}{\beta}$
Scenario 2	$f_1 > \frac{|A_7|}{\beta}$
Scenario 3	$A_9 > A_{10}$
Scenario 4	$f_1 > \frac{|A_7|}{\beta}$ and $A_9 > A_{10}$
Scenario 5	$\frac{|A_7|}{\beta} < f_1 < \frac{|A_6|}{\beta}$
Scenario 6	$f_1 < \frac{|A_6|}{\beta}$ and $A_9 > A_{10}$
Scenario 7	$\frac{|A_7|}{\beta} < f_1 < \frac{|A_6|}{\beta}$ and $A_9 > A_{10}$

(7) Scenario 7: $\pi_m^{\mathrm{CS}} - \pi_m^{\mathrm{O}} = A_6 - \beta f_1 > 0$ ($i.e.$, $f_1 < \frac{|A_6|}{\beta}$), $\pi_r^{\mathrm{CS}} - \pi_r^{\mathrm{NO}} = A_7 + \beta f_1 > 0$ ($i.e.$, $f_1 > \frac{|A_7|}{\beta}$) and $\pi_{m+r}^{\mathrm{CS}} - \pi_{m+r}^{\mathrm{O}} = A_9 - A_{10} > 0$ ($i.e.$, $A_9 > A_{10}$);

However, Scenario 5 and Scenario 7 must satisfy the following condition: $|A_7| < |A_6|$. Scenario 3 does not give the result about the fixed cost of pre-sale BDS, but limits the value region of some parameters. Several results from other scenarios for the fixed cost of pre-sale BDS, with limitations on some parameters. The summary of each value region is shown in Table 4.

5.2.3. *Comparison between Case 1 and Case 2*

The regions of fixed cost of pre-sale BDS in Scenario 1, Scenario 2, and Scenario 5 of Case 1 are the same as that in Scenario 1, Scenario 2, and Scenario 5 of Case 2, respectively. It shows that if the manufacturer sees itself and the retailer as separate entities rather than as an integral whole, the fixed cost of pre-sale BDS in different cases has the same value region no matter what the fixed cost of pre-production BDS is. However, it shows the opposite result in the other scenarios of these two cases. That is, when the fixed cost of pre-production BDS changes, the fixed cost of pre-sale BDS will also take a different value region. It shows that when the manufacturer considers the total profits of itself and the retailer, the fixed cost of pre-sale BDS will change with the change of the fixed cost of pre-production BDS. In Scenario 3 of Case 1, there is no effect on the fixed cost of pre-sale BDS, but the fixed cost of pre-production BDS changes. It shows that as the decision-making body, the manufacturer's fixed cost will have great influence on the total profits of itself and the retailer. In Scenario 3 of Case 2, there is no effect on the fixed cost of pre-sale BDS and the fixed cost of pre-production BDS, which suggests that when the fixed cost of pre-production BDS exceeds the threshold, the change of the fixed cost of two cases has little impact on total profits of the manufacturer and the retailer. When the fixed cost of the manufacturer is large enough, no matter what cost-sharing choice the manufacturer makes, it will not change its own fixed cost, but will only affect the fixed cost of the retailer. The manufacturer and the retailer have to find ways to increase revenue if they want to increase profits. In addition, Scenario 1, Scenario 5, Scenario 6 and Scenario 7 of these two cases illustrate a particular situation: the manufacturer's profit after implementing cost-sharing is greater than that before implementing cost-sharing $\left(i.e., \pi_m^{\mathrm{CS}} > \pi_m^{\mathrm{O}}\right)$. Because the manufacturer helps the retailer share the cost, which makes the retailer work harder to sell, which leads to an increase in demand; And the manufacturer transfers part of the cost to the BDS provider, but the profit of BDS provider is still greater than zero, that is, $\pi_b^{\mathrm{CS}} = A_{11} - f_2 > 0$ ($i.e.$, $A_{11} > f_2$).

From what has been discussed above, the retailer will favor the cost-sharing model chosen by the manufacturer only in Scenario 3, Scenario 4, Scenario 5 and Scenario 7 of these two cases. The reason is that in these scenarios,

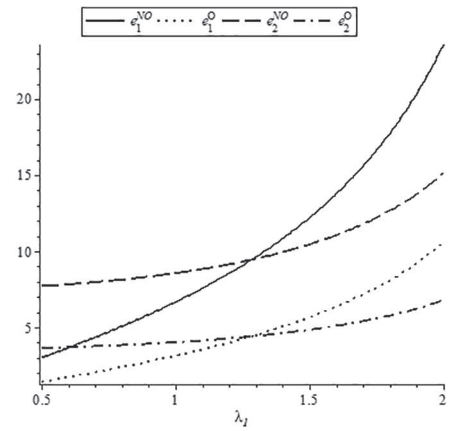

FIGURE 1. Change of e_1^{NO}, e_1^{O}, e_2^{NO} and e_2^{O} w.r.t λ_1.

the manufacturer considers either the retailer's profit or the total profits of itself and the retailer. When the retailer's profit increases, the retailer will certainly favor the manufacturer to choose cost-sharing. And when the total profits of the manufacturer and the retailer increase, either one's profit increases and the other's profit decrease or both of their profits increase. In the former situation, the manufacturer and the retailer negotiate that whose profit increases will compensate for whose profit decreases. However, Scenario 1, Scenario 2 and Scenario 6 do not consider the profit of each member clearly. The retailer will not favor the manufacturer's choice of cost-sharing in these three scenarios. In addition, the manufacturer, after the implementation of cost sharing incentive mechanism when the profits of the manufacturer and the retailer to increase, regardless of the BDS level increase or decrease, the profit will not be affected, because when the BDS level increasing cost, the manufacturer and the retailer to raise price, although this can lead to reduced demand. However, the negative impact of reduced demand on profits is less than the positive impact of increased BDS level. When the reduction of BDS level leads to reduced demand, the manufacturer and the retailer can lower price, and the positive impact of low prices on demand is greater than the negative impact of reduced BDS level on profits. In the first case, consumers pay more attention to product satisfaction, and in the second case, consumers pay more attention to low price.

6. Numerical examples

In the previous discussion, we analyzed the pre-production BDS outsourcing choice of the manufacturer and the results of the cost-sharing contract. However, these theoretical results are mainly based on the analysis of mathematical expressions. In order to better express these results, we will use some specific numerical values to show these results in this section. According to the definition of each parameter and their respective constraint conditions, refer to Lou *et al.* [23], we set the values of relevant parameters as: $a = 100$, $b = 4$, $c = 2$, $f_1 = 150$, $f_2 = 300$, $\lambda_1 = 1.3$, $\lambda_2 = 1.4$, $\mu_1 = 1.1$, $\mu_2 = 1.2$.

From Figures 1 to 4, it can be seen that the influence of cost sensitive μ_1 on BDS level is greater than that of cost sensitive μ_2 on BDS level, which indicates that no matter how well the product quality is, it still needs good publicity or public praise to sell the product, and the retailer is an important part of this link. It shows that when the manufacturer chooses non-outsourcing, the manufacturer and the retailer will concentrate on doing their own BDS. In addition, when the demand sensitivity and cost sensitivity change, the level of BDS in different models is not in strict order, but in different ranges with different orders, which indicates that the manufacturer and the retailer will adjust the BDS level according to the change of market demand and profit, so as to achieve the profit level satisfactory to both sides.

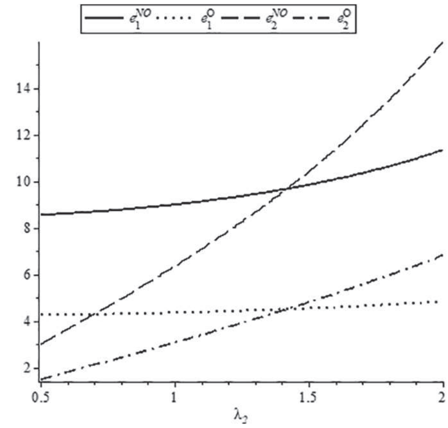

FIGURE 2. Change of e_1^{NO}, e_1^{O}, e_2^{NO} and e_2^{O} w.r.t λ_2.

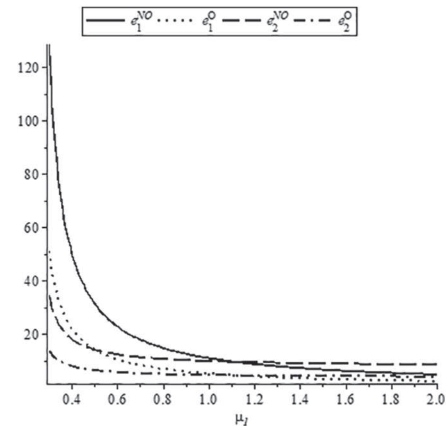

FIGURE 3. Change of e_1^{NO}, e_1^{O}, e_2^{NO} and e_2^{O} w.r.t μ_1.

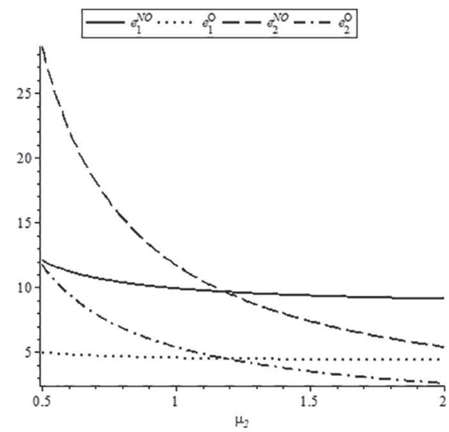

FIGURE 4. Change of e_1^{NO}, e_1^{O}, e_2^{NO} and e_2^{O} w.r.t μ_2.

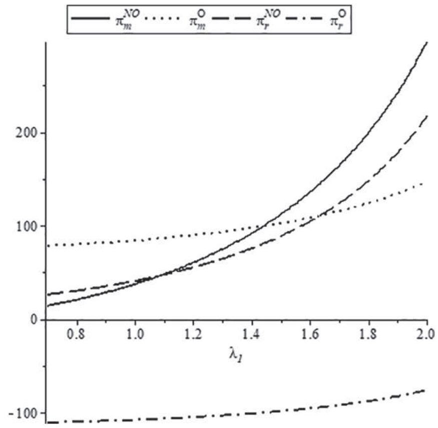

FIGURE 5. Change of π_m^{NO}, π_m^{O}, π_r^{NO} and π_r^{O} w.r.t λ_1.

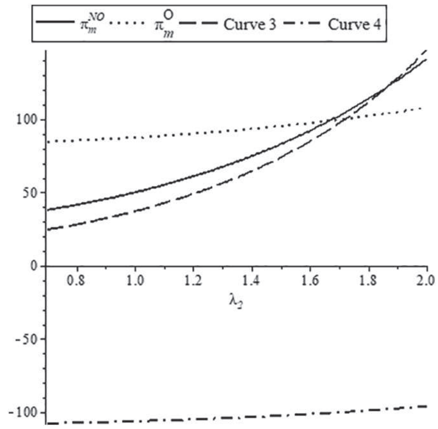

FIGURE 6. Change of π_m^{NO}, π_m^{O}, π_r^{NO} and π_r^{O} w.r.t λ_2.

From Figures 5 to 8, it can be seen that when the manufacturer chooses to outsource pre-production BDS, the pre-production BDS level will decline, it makes consumers feel dissatisfied with the product. This situation will lead to a decline in demand and reduce the retailer's profit further. The retailer should not consent to the manufacturer choosing outsourcing. In addition, the change of demand sensitivity has less impact on Model O, but more impact on Model NO. The reason is that in the Model NO, the manufacturer undertakes BDS themselves, which is directly related to the change of their profits. However, in the Model O, third-party BDS provider is introduced, which weakens the direct impact on the manufacturer's profit to a certain extent, and thus leads to the change of the retailer's profit. And cost sensitivity has a great impact on the retailer's profit in the Model O. The reason is that outsourcing reduces the retailer's profit. When the cost of third-party BDS provider increase, the profit of leading the manufacturer will not change significantly, so the retailer's profit margins can only be further compressed.

From Figures 9 to 16, it can be seen that when these factors have a positive influence, the profit gradient of both the manufacturer and the retailer will increase. That is, the higher the cost sensitivity, the lower the profit and the BDS level. When the cost sensitivity reaches a certain level, the service level and profit of the manufacturer and the retailer are both less than zero. Therefore, they will not implement BDS, so as to reduce

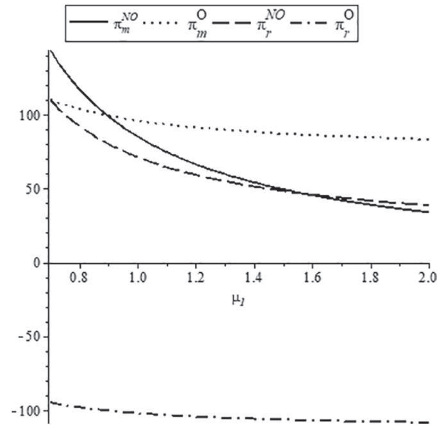

FIGURE 7. Change of π_m^{NO}, π_m^{O}, π_r^{NO} and π_r^{O} w.r.t μ_1.

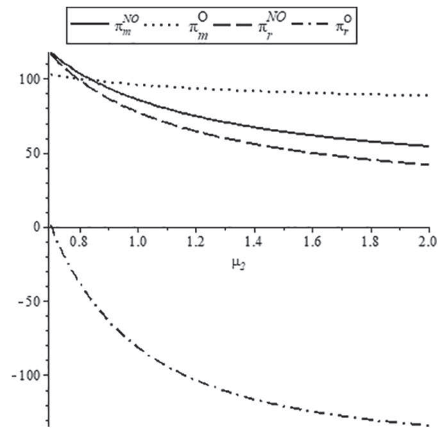

FIGURE 8. Change of π_m^{NO}, π_m^{O}, π_r^{NO} and π_r^{O} w.r.t μ_2.

cost and ensure that profits are greater than zero. In addition, it can be known from the partial derivative that the service level of the manufacturer and the retailer will affect each other, because the service level of both of them jointly affects the demand. The higher the cost of BDS for the manufacturer, the better the product, and the greater the effort of the retailer to attract consumers to buy, the higher the cost of BDS for the retailer. Similarly, if the retailer advertises their products well, their cost of BDS will increase, while the manufacturer will also increase their BDS in order to produce product that conform to the product advertised by the retailer, resulting in increased cost. This is in accordance with the conclusions in Propositions 1–4.

From Figures 1 to 16, it can be seen that compared with cost sensitivity, demand sensitivity has a greater impact on BDS level, profit and price. This shows that the manufacturer and the retailer are still focused on consumer demand. When consumers need products with high satisfaction, they will improve the BDS level. And, the increased price is also acceptable to consumers. When consumers pay more attention to low price, appropriately reducing the BDS level can also satisfy consumers.

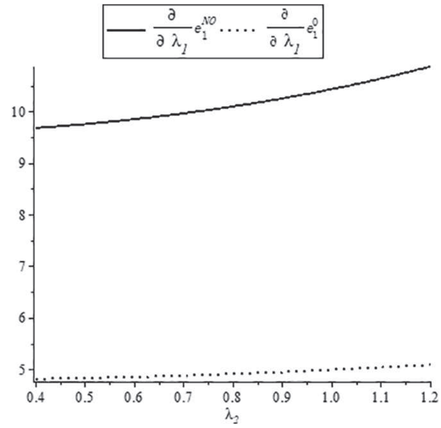

FIGURE 9. Change of $\frac{\partial e_1^{\mathrm{NO}}}{\partial \lambda_1}$ and $\frac{\partial e_1^{\mathrm{O}}}{\partial \lambda_1}$ w.r.t λ_2.

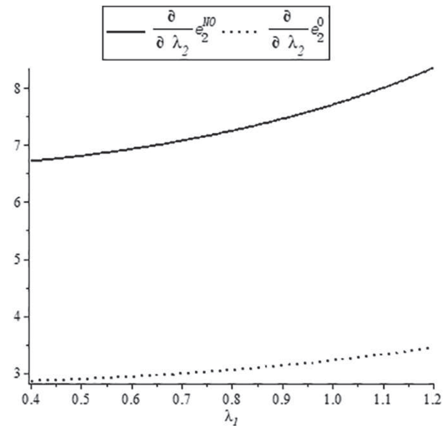

FIGURE 10. Change of $\frac{\partial e_2^{\mathrm{NO}}}{\partial \lambda_2}$ and $\frac{\partial e_2^{\mathrm{O}}}{\partial \lambda_2}$ w.r.t λ_1.

7. Conclusion and enlightenment

Conclusion

The development of big data has caused changes in the industry and brought considerable benefits to enterprises. However, its cost is also an important factor for enterprises to make trade-offs. In addition, the manufacturer, as the leader, is considered to make decisions in different situations. Therefore, this paper takes the supply chain composed of a manufacturer and a retailer as the main body of research, introduces a third-party service provider, and adopts the Stackelberg Game model to study the impact of whether the manufacturer chooses to outsource the pre-production big data service on the wholesale price, retail price, pre-production big data service level, pre-sale big data service level and the profits of the supply chain members. In addition, we also study the cost-sharing strategy of the manufacturer in different situations to achieve the effect of supply chain coordination.

The conclusions are as follows: (1) the pre-production big data service level, the pre-sale big data service level and the retailer's profit are higher in the case of non-outsourcing. For the manufacturer, its profit depends on the fixed cost of pre-production big data service. However, the relationship between wholesale price and retail

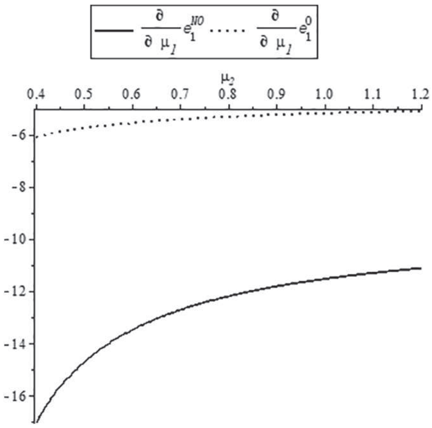

FIGURE 11. Change of $\frac{\partial e_1^{\mathrm{NO}}}{\partial \mu_1}$ and $\frac{\partial e_1^{\mathrm{O}}}{\partial \mu_1}$ w.r.t μ_2.

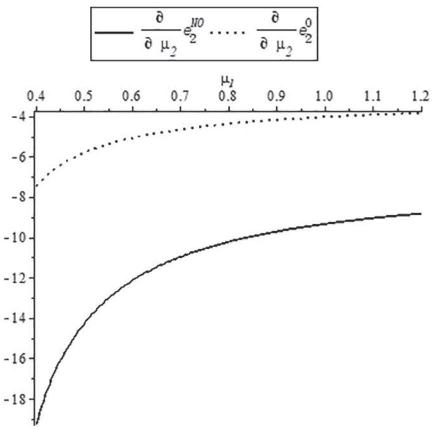

FIGURE 12. Change of $\frac{\partial e_2^{\mathrm{NO}}}{\partial \mu_2}$ and $\frac{\partial e_2^{\mathrm{O}}}{\partial \mu_2}$ w.r.t μ_1.

price depends on the change of sensitivity coefficient, respectively. (2) When the fixed cost of pre-production big data service is lower (higher), the manufacturer will choose non-outsource (outsource). However, from the perspective of big data service level, the manufacturer should choose not to outsource. From the perspective of profit, the manufacturer should decide whether to outsource based on the threshold of fixed cost of pre-production big data service. We know that the manufacturer has three options: the manufacturer chooses non-outsourcing if the profits of all supply chain members are decreased, the manufacturer chooses outsourcing without coordination mechanism if it only considers maximizing its own profits, the manufacturer chooses outsourcing with coordination mechanism if it considers the profits of other members. When fixed cost of pre-production big data service is change, the profit of the manufacturer in outsource model is greater than that in non-outsource model, but the manufacturer can implement cost-sharing in both cases, and the results will be different. (3) The manufacturer can adopt cost-sharing to coordinate the supply chain under different fixed cost of pre-production big data service, and can obtain different cost-sharing rate according to different coordination purposes. When consumers need product with high satisfaction, they will improve the BDS level. And, the

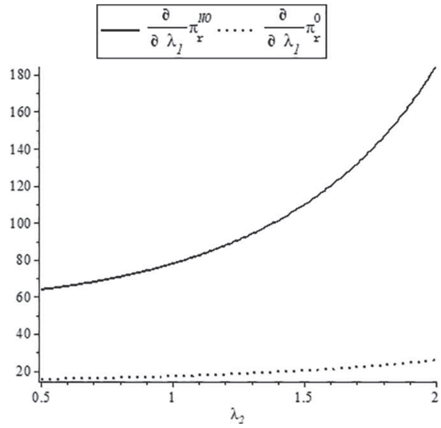

FIGURE 13. Change of $\frac{\partial \pi_r^{\mathrm{NO}}}{\partial \lambda_1}$ and $\frac{\partial \pi_r^{\mathrm{O}}}{\partial \lambda_1}$ w.r.t λ_2.

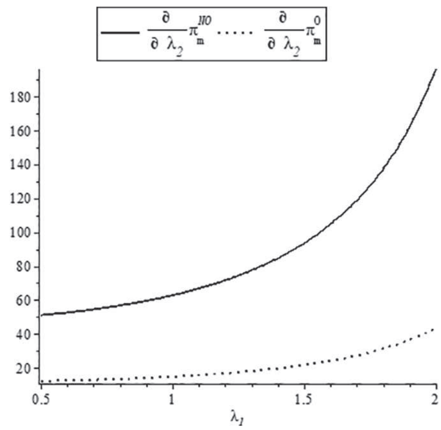

FIGURE 14. Change of $\frac{\partial \pi_m^{\mathrm{NO}}}{\partial \lambda_2}$ and $\frac{\partial \pi_m^{\mathrm{O}}}{\partial \lambda_2}$ w.r.t λ_1.

increased price is also acceptable to consumers. When consumers pay more attention to low price, appropriately reducing the BDS level can also satisfy consumers.

Managerial insights and implications

This paper provides some management implications and practical suggestions: (1) The decision to outsource operation within an organization necessitates a comprehensive analysis from various vantage points or core elements of enterprises. For instance, if enterprise's primary concern lies with the extent of outsourced activities, it must critically examine the depth and breadth of such engagements to determine the necessity of outsourcing. Conversely, if profitability is the paramount consideration without regard to the scale of outsourcing, then the emphasis ought to be placed on enhancing overall revenues. (2) In terms of financial performance, outsourcing may not be imperative when operational expenses are modest. However, when cost is substantial, a multi-faceted evaluation of potential outsourcing strategies is warranted to augment the collective benefits accruing to members of the supply chain. (3) We now reside in an epoch where collaborative success is paramount, and organizations cannot afford to solely prioritize their individual gains in strategic decision-making. Instead, they must take into account the shared interests of the entire supply network or partners, lest the integrity of

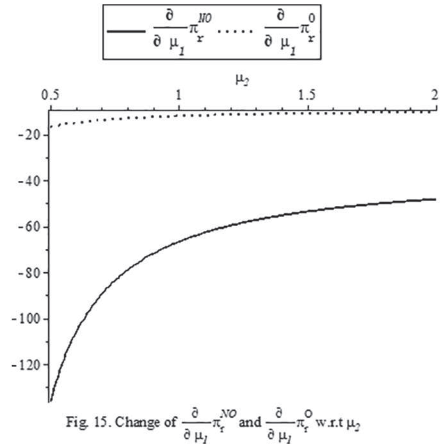

FIGURE 15. Change of $\frac{\partial \pi_r^{\mathrm{NO}}}{\partial \mu_1}$ and $\frac{\partial \pi_r^{\mathrm{O}}}{\partial \mu_1}$ w.r.t μ_2.

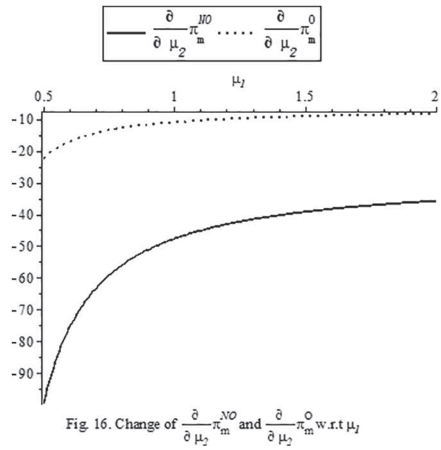

FIGURE 16. Change of $\frac{\partial \pi_m^{\mathrm{NO}}}{\partial \mu_2}$ and $\frac{\partial \pi_m^{\mathrm{O}}}{\partial \mu_2}$ w.r.t μ_1.

the entire supply chain be compromised. (4) Enterprises would be prudent to focus on discerning and meeting consumer demands, rather than pursuing a myopic strategy of indiscriminate cost reduction. Such short-sighted tactics ultimately result in diminished profitability, as they overlook the importance of delivering value that aligns with market expectations.

Future research

The future research of this study can be considered from the following aspects. (1) This study is constrained by several factors, foremost among which is its exclusive focus on the vertical dynamics within a single-channel supply chain framework. Subsequent study could expand upon this by incorporating analyses of multi-channel supply networks, the interplay of multiple stakeholders in competitive scenarios, and the integration of both horizontal and vertical relational complexities inherent in supply chain management. (2) A further limitation of the current research is its assumption of deterministic demand patterns, which contrasts sharply with the prevalent reality of market uncertainties. Future studies could address this gap by exploring the intricate web of competitive and cooperative interactions that arise in response to fluctuating and unpredictable consumer needs. (3) The present study employs a deterministic time game theoretic model, thereby overlooking potential temporal influences on the strategic choices of supply chain participants. Prospective research could delve into the realm of time-dependent differential games to ascertain the extent to which temporal variables might shape the decision-making processes within supply chains. By doing so, we can contribute to a more nuanced understanding of supply chain optimization under varying temporal conditions.

Appendix A.

Proof of Lemma 1. We solve the game of the manufacturer and the retailer using backward induction. By the first derivative of the retailer's profit function, we have:

$$\frac{\partial \pi_r^{\mathrm{NO}}}{\partial e_1^{\mathrm{NO}}} = (p-w)\lambda_1 - \mu_1 e_1 \tag{A.1}$$

$$\frac{\partial \pi_r^{\mathrm{NO}}}{\partial p^{\mathrm{NO}}} = -bp + e_1\lambda_1 + e_2\lambda_2 + a - (p-w)b. \tag{A.2}$$

Let (A.1) and (A.2) are equal zero, the retailer's optimal pre-sale BDS level and retail price is obtained as:

$$e_1^{\mathrm{NO}} = \frac{\lambda_1(-bw + e_2\lambda_2 + a)}{2\mu_1 b - \lambda_1^2} \tag{A.3}$$

$$p^{\mathrm{NO}} = \frac{\mu_1 bw + \mu_1 e_2\lambda_2 - w\lambda_1^2 + \mu_1 a}{2\mu_1 b - \lambda_1^2}. \tag{A.4}$$

Plug (A.3) and (A.4) into the manufacturer's profit function, we have:

$$\frac{\partial \pi_m^{\mathrm{NO}}}{\partial e_2^{\mathrm{NO}}} = \frac{-((-w+c)\lambda_2 + 2\mu_2 e_2)b\mu_1 + e_2\mu_2\lambda_1^2}{2\mu_1 b - \lambda_1^2} \tag{A.5}$$

$$\frac{\partial \pi_m^{\mathrm{NO}}}{\partial w^{\mathrm{NO}}} = \frac{\mu_1 b + ((c-2w)b + e_2\lambda_2 + a)}{2\mu_1 b - \lambda_1^2}. \tag{A.6}$$

Let (A.5) and (A.6) are equal zero, the manufacturer's optimal pre-production BDS level and wholesale price is obtained as:

$$e_2^{\mathrm{NO}} = \frac{\lambda_2\mu_1(a-bc)}{A_0} \tag{A.7}$$

$$w^{\mathrm{NO}} = \frac{bcA_2 + 2\mu_1\mu_2 ab - \mu_2 a\lambda_1^2}{A_0 b}. \tag{A.8}$$

We're going to get all the optimal solutions:

$$p^{\mathrm{NO}} = \frac{bcA_2 + 3\mu_1\mu_2 ab - \mu_2 a\lambda_1^2}{A_0 b} \tag{A.9}$$

$$w^{\mathrm{NO}} = \frac{bcA_2 + 2\mu_1\mu_2 ab - \mu_2 a\lambda_1^2 + \mu_1\mu_2 b^2 c}{A_0 b} \tag{A.10}$$

$$e_1^{\mathrm{NO}} = \frac{\lambda_1\mu_2(a-bc)}{A_0} \tag{A.11}$$

$$e_2^{\mathrm{NO}} = \frac{\lambda_2\mu_1(a-bc)}{A_0} \tag{A.12}$$

$$\pi_m^{\mathrm{NO}} = \frac{\mu_1\mu_2(a-bc)^2}{2A_0} - f_2 \tag{A.13}$$

$$\pi_r^{\mathrm{NO}} = \frac{\mu_1\mu_2^2(a-bc)^2(2b\mu_1-\lambda_1^2)}{2A_0^2} - f_1. \tag{A.14}$$

Hessian matrix of $\pi_m^{\mathrm{NO}}(w^{\mathrm{NO}}, e_2^{\mathrm{NO}})$ is

$$\begin{pmatrix} \frac{\partial^2\pi_m^{\mathrm{NO}}}{\partial w^{\mathrm{NO}2}} & \frac{\partial^2\pi_m^{\mathrm{NO}}}{\partial w^{\mathrm{NO}}\partial e_2^{\mathrm{NO}}} \\ \frac{\partial^2\pi_m^{\mathrm{NO}}}{\partial e_2^{\mathrm{NO}}\partial w^{\mathrm{NO}}} & \frac{\partial^2\pi_m^{\mathrm{NO}}}{\partial e_2^{\mathrm{NO}2}} \end{pmatrix} = \begin{pmatrix} -\frac{2b^2\mu_1}{2b\mu_1-\lambda_1^2} & \frac{b\mu_1\lambda_2}{2b\mu_1-\lambda_1^2} \\ \frac{b\mu_1\lambda_2}{2b\mu_1-\lambda_1^2} & -\mu_2 \end{pmatrix}. \tag{A.15}$$

$H1 = -\frac{2b^2\mu_1}{2b\mu_1-\lambda_1^2} < 0$, $H2 = \frac{b^2\mu_1 A_0}{(2b\mu_1-\lambda_1^2)^2} > 0$, constraints are given in the text: $A_0 > 0$.

The proof process of Hessian matrix of $\pi_r^{\mathrm{NO}}(p^{\mathrm{NO}}, e_1^{\mathrm{NO}})$ is similar to that of Hessian matrix of $\pi_m^{\mathrm{NO}}(w^{\mathrm{NO}}, e_2^{\mathrm{NO}})$. Hessian matrix of the manufacturer and the retailer are negative definite. Thus, we have got the solution. ☐

Proof of Proposition 1. According to Lemma 1, by taking the second partial derivative, we have:

$$\frac{\partial^2 p^{\mathrm{NO}}}{\partial\lambda_1\partial\lambda_2} = \frac{4\lambda_2(-bc+a)\mu_2\left(8\mu_1\mu_2 b+\mu_1\lambda_2^2-2\mu_2\lambda_1^2\right)\mu_1\lambda_1}{A_0^3 b} > 0 \tag{A.16}$$

$$\frac{\partial^2 w^{\mathrm{NO}}}{\partial\lambda_2\partial\lambda_1} = \frac{4\lambda_2(-bc+a)\mu_2\left(4\mu_1\mu_2 b+\mu_1\lambda_2^2-2\mu_2\lambda_1^2\right)\mu_1\lambda_1}{A_0^3 b} > 0 \tag{A.17}$$

$$\frac{\partial^2 e_1^{\mathrm{NO}}}{\partial\lambda_1\partial\lambda_2} = \frac{2\lambda_2(-bc+a)\mu_2\left(4\mu_1\mu_2 b+6\mu_1\lambda_2^2-\mu_2\lambda_1^2\right)\mu_1}{A_0^3} > 0 \tag{A.18}$$

$$\frac{\partial^2 e_2^{\mathrm{NO}}}{\partial\lambda_2\partial\lambda_1} = \frac{4\lambda_1(-bc+a)\mu_2\left(4\mu_1\mu_2 b+3\mu_1\lambda_2^2-2\mu_2\lambda_1^2\right)\mu_1}{A_0^3} > 0 \tag{A.19}$$

$$\frac{\partial^2 p^{\mathrm{NO}}}{\partial\mu_1\partial\mu_2} = \frac{\lambda_1^2\lambda_2^2(-bc+a)\left(8\mu_1\mu_2 b+\mu_1\lambda_2^2-2\mu_2\lambda_1^2\right)}{A_0^3 b} > 0 \tag{A.20}$$

$$\frac{\partial^2 w^{\mathrm{NO}}}{\partial\mu_2\partial\mu_1} = \frac{\lambda_1^2\lambda_2^2(-bc+a)\left(4\mu_1\mu_2 b+\mu_1\lambda_2^2-2\mu_2\lambda_1^2\right)}{A_0^3 b} > 0 \tag{A.21}$$

$$\frac{\partial^2 e_1^{\mathrm{NO}}}{\partial\mu_1\partial\mu_2} = \frac{\lambda_1\lambda_2^2(-bc+a)\left(4\mu_1\mu_2 b-\mu_1\lambda_2^2+2\mu_2\lambda_1^2\right)}{A_0^3} > 0 \tag{A.22}$$

$$\frac{\partial^2 e_2^{\mathrm{NO}}}{\partial\mu_2\partial\mu_1} = \frac{2\lambda_2\lambda_1^2(-bc+a)\left(4\mu_1\mu_2 b+\mu_1\lambda_2^2-2\mu_2\lambda_1^2\right)}{A_0^3} > 0. \tag{A.23}$$

Due to $A_0 > 0$, we get $8\mu_1\mu_2 b+\mu_1\lambda_2^2-2\mu_2\lambda_1^2-A_0 = 4\mu_1\mu_2 b+2\mu_1\lambda_2^2 > 0$, so $8\mu_1\mu_2 b+\mu_1\lambda_2^2-2\mu_2\lambda_1^2 > 0$. In the same way, we can prove $4\mu_1\mu_2 b+\mu_1\lambda_2^2-2\mu_2\lambda_1^2 > 0$, $4\mu_1\mu_2 b-\mu_1\lambda_2^2+2\mu_2\lambda_1^2 > 0$. ☐

Proof of Proposition 2. Similar to the proof of Proposition 1. ☐

Proof of Lemma 2. Similar to the proof of Lemma 1. ☐

Proof of Proposition 3. Similar to the proof of Proposition 1. ☐

Proof of Proposition 4. Similar to the proof of Proposition 1. ☐

Proof of Proposition 5.

$$e_1^{\mathrm{O}} - e_1^{\mathrm{NO}} = -\frac{4(-bc+a)\mu_2^2\lambda_1\left(\mu_1 b-\frac{1}{2}\lambda_1^2\right)}{(8\mu_1\mu_2 b-\mu_1\lambda_2^2-4\mu_2\lambda_1^2)(4\mu_1\mu_2 b-\mu_1\lambda_2^2-2\mu_2\lambda_1^2)} \tag{A.24}$$

$$e_2^{\mathrm{O}} - e_2^{\mathrm{NO}} = -\frac{4(-bc+a)\mu_1\mu_2\lambda_2\big(\mu_1 b - \frac{1}{2}\lambda_1^2\big)}{(8\mu_1\mu_2 b - \mu_1\lambda_2^2 - 4\mu_2\lambda_1^2)(4\mu_1\mu_2 b - \mu_1\lambda_2^2 - 2\mu_2\lambda_1^2)} \tag{A.25}$$

$$w^{\mathrm{O}} - w^{\mathrm{NO}} = \frac{2(-bc+a)\mu_2\big(2\mu_1\mu_2 b - \mu_1\lambda_2^2 - \mu_2\lambda_1^2\big)\lambda_2\big(2\mu_1 b - \lambda_1^2\big)}{b(8\mu_1\mu_2 b - \mu_1\lambda_2^2 - 4\mu_2\lambda_1^2)(4\mu_1\mu_2 b - \mu_1\lambda_2^2 - 2\mu_2\lambda_1^2)} \tag{A.26}$$

$$p^{\mathrm{O}} - p^{\mathrm{NO}} = \frac{4(-bc+a)\mu_2\big(\mu_1\mu_2 b - \mu_1\lambda_2^2 - \mu_2\lambda_1^2\big)\big(\mu_1 b - \frac{1}{2}\lambda_1^2\big)}{b(8\mu_1\mu_2 b - \mu_1\lambda_2^2 - 4\mu_2\lambda_1^2)(4\mu_1\mu_2 b - \mu_1\lambda_2^2 - 2\mu_2\lambda_1^2)} \tag{A.27}$$

$$\pi_m^{\mathrm{O}} - \pi_m^{\mathrm{NO}} = \frac{(2\mu_1 b - \lambda_1^2)\mu_1(a-bc)^2\mu_2^2}{A_1^2} - \frac{\mu_2\mu_1(a-bc)^2}{2A_0} + f_2 \tag{A.28}$$

$$\pi_{m+r}^{\mathrm{O}} - \pi_{m+r}^{\mathrm{NO}} = \frac{3\mu_1\mu_2^2(a-bc)^2\big(2\mu_1 b - \lambda_1^2\big)}{2A_1^2} - \frac{\mu_1\mu_2(a-bc)^2\big(6\mu_1\mu_2 b - \mu_1\lambda_2^2 - 3\mu_2\lambda_1^2\big)}{2A_0^2} + f_2. \tag{A.29}$$

□

Appendix B.

$$A_0 = 4\mu_1\mu_2 b - \mu_1\lambda_2^2 - 2\mu_2\lambda_1^2 \tag{B.1}$$

$$A_1 = 8\mu_1\mu_2 b - \mu_1\lambda_2^2 - 4\mu_2\lambda_1^2 \tag{B.2}$$

$$A_2 = \mu_1\mu_2 b - \mu_1\lambda_2^2 - \mu_2\lambda_1^2 \tag{B.3}$$

$$A_3 = \frac{\big(2\mu_1 b - \lambda_1^2\big)\mu_1(a-bc)^2\mu_2^2}{A_1^2} - \frac{\mu_1\mu_2(a-bc)^2}{2A_0} \tag{B.4}$$

$$A_4 = \frac{\mu_1\mu_2(a-bc)^2}{2A_1} \tag{B.5}$$

$$A_5 = \frac{3\mu_1\mu_2^2(a-bc)^2\big(2\mu_1 b - \lambda_1^2\big)}{2A_1^2} - \frac{\mu_1\mu_2(a-bc)^2\big(6\mu_1\mu_2 b - \mu_1\lambda_2^2 - 3\mu_2\lambda_1^2\big)}{2A_0^2} \tag{B.6}$$

$$A_6 = \frac{\mu_1\mu_2^2(1-\alpha)^2\big(\mu_1 b(1-\alpha)^2 + \frac{3}{4}\lambda_1^2\big(\alpha - \frac{2}{3}\big)\big)(a-bc)^2}{32\big((1-\alpha)^2\big(\mu_2 b - \frac{1}{8}\lambda_2^2\big)\mu_1 + \frac{3}{4}\big(\alpha - \frac{2}{3}\big)\mu_2\lambda_1^2\big)^2} - \frac{\mu_1\mu_2^2(a-bc)^2\big(2\mu_1 b - \lambda_1^2\big)}{A_1^2} \tag{B.7}$$

$$A_7 = \frac{\mu_1\mu_2^2(1-\alpha)^3\big(\mu_1 b(1-\alpha) - \frac{1}{2}\lambda_1^2\big)(a-bc)^2}{64\big((1-\alpha)^2\big(\mu_2 b - \frac{1}{8}\lambda_2^2\big)\mu_1 + \frac{3}{4}\big(\alpha - \frac{2}{3}\big)\mu_2\lambda_1^2\big)^2} - \frac{\mu_1\mu_2^2(a-bc)^2(2\mu_1 b - \lambda_1^2)}{2A_0^2} \tag{B.8}$$

$$A_8 = \frac{3\mu_2^2\big(\mu_1(\alpha-1)^2 b + \frac{2}{3}\lambda_1^2\big(\alpha - \frac{3}{4}\big)\big)(-bc+a)^2(\alpha-1)^2\mu_1}{64\big((\alpha-1)^2(b\mu_2 - \frac{1}{8}\lambda_2^2)\mu_1 + \frac{3}{4}\big(\alpha - \frac{2}{3}\big)\mu_2\lambda_1^2\big)^2} - \frac{\mu_2\mu_1(-bc+a)^2\big(6\mu_1\mu_2 b - \mu_1\lambda_2^2 - 3\mu_2\lambda_1^2\big)}{2A_0^2} \tag{B.9}$$

$$A_9 = \frac{3\mu_2^2\big(\mu_1(\alpha-1)^2 b + \frac{2}{3}\lambda_1^2\big(\alpha - \frac{3}{4}\big)\big)(-bc+a)^2(\alpha-1)^2\mu_1}{64\big((\alpha-1)^2\big(b\mu_2 - \frac{1}{8}\lambda_2^2\big)\mu_1 + \frac{3}{4}(\alpha - \frac{2}{3})\mu_2\lambda_1^2\big)^2} \tag{B.10}$$

$$A_{10} = \frac{3\mu_2^2\mu_1(-bc+a)^2\big(2\mu_1 b - \lambda_1^2\big)}{2A_1^2} \tag{B.11}$$

$$A_{11} = \frac{(\alpha-1)^2\mu_2(-bc+a)^2\mu_1}{16(\alpha-1)^2\big(b\mu_2 - \frac{1}{8}\lambda_2^2\big)\mu_1 + 12\big(\alpha - \frac{2}{3}\big)\mu_2\lambda_1^2} \tag{B.12}$$

$$A_{12} = 2\mu_1\mu_2 b - \mu_1\lambda_2^2 - \mu_2\lambda_1^2. \tag{B.13}$$

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