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COORDINATION OF A SOCIALLY RESPONSIBLE TWO-STAGE SUPPLY CHAIN UNDER RANDOM YIELD AND DEMAND

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Abstract. This study integrates corporate social responsibility (CSR) and channel coordination in a supply chain (SC) under random yield and random demand. In the SC, a supplier with random yield determines wholesale price and production input, and a producer with CSR decides order quantity and CSR investment facing CSR-related random demand. For centralized SCs with and without CSR, we prove the unimodality of the expected profits of the SCs for generic random yield and demand, and show the uniqueness of optimal ratio of order quantity to production input quantity. We disclose that the optimal expected profit of the SC with CSR is larger than that of the SC without CSR, and the profit difference increases with CSR effort and expected yield. We also find that the optimal CSR investment increases with the expected yield. Subsequently, the Nash equilibrium solutions of decentralized SCs under wholesale price and revenue-sharing contracts are analyzed. Next, we present a revenue and CSR sharing contract to realize channel coordination and win-win of SC members and related stakeholders. Lastly, we verify theoretical statements of centralized and decentralized SCs *versus* three numerical examples, and present managerial insights for the effect of yield uncertainty and CSR effort.

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1. Introduction

A supply chain (SC) comprises independent members usually with conflicting self-interested objectives. Supply chain coordination (SCC) is a way to make SC members of a decentralized setting behave coherently each other, so as to achieve higher profits, customer service improvements, etc. Contracting is a widely used coordination mechanism in a decentralized SC that motivates its members to behave nearly or exactly as well as what they would in a centralized setting. Indeed, SCC is an important part of SC management that has received considerable attention from researchers and practitioners in recent years.

The SC under consideration consists of a supplier and a producer, and provides a single agricultural product for customers. In this SC, the brand producer assumes its corporate social responsibility (CSR) which is considered as an enterprise's activities exhibiting ethical responsibilities, social and environmental to its stakeholders such as employees, customers and related communities. Recent studies suggest that consumers prefer buying agri-food products with CSR attributes [1]. The CSR activities and related investment by the producer in the

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SC of concern are conceived to bring reward demand [36,47]. Naturally, customers' demand is uncertain in the real world and often described as some kind of stochastic distribution. So, we take the CSR into account by adding a reward demand element derived from it to the stochastic demand function.

Likewise, the yield of the agricultural supplier is also uncertain owing to such factors as weather condition, diseases and natural disasters. In this paper, the widely used stochastically proportional yield [10] is adopted to model uncertain yield *via* multiplying the quantity of production input by a random factor. Notably, studies that integrate CSR into the SC coordination with random demand have only emerged in the last few years [46]. As a consequence, inquiry into coordination for socially responsible SCs under uncertain supply and demand remains scant.

The aim of this paper is to study how to design contract terms coordinating a decentralized SC to improve its performance and to benefit its members as well as other stakeholders. To our best knowledge, this paper represents one of the first attempts to examining how to coordinate an SC involving CSR together with random yield and demand. Here, we focus on the supplier's production planning of raw-material under random yield, the producer's decisions of order quantity and CSR investment under CSR-related random demand. Our study addresses the following questions: (1) Is the total profit of the socially responsible SC unimodal with respect to the supplier's production-planning decision, the producer's procurement decision and the CSR investment under random yield and random demand? (2) How do the uncertain yield and the CSR activities affect the coordination of typical SC contracts such as wholesale price (WP) and revenue-sharing (RS) contracts? (3) If typical contracts such as RS contract and modified RS contract fail to coordinate the SC, how to design a contract to coordinate the SC. (4) How do uncertain yield and CSR effort affect the profits of SC members and the welfare of stakeholders under channel coordination?

In this study, we first develop the centralized models with and without CSR as benchmark cases and then solve for the Nash equilibrium solutions of the non-coordinated models with and without CSR under WP and RS contracts. Finally, we consider designing a new RS-based contract to coordinate the SC.

The rest of this paper is organized as follows. Section 2 is literature review. Section 3 states the SC coordination problem and analyzes centralized SCs. In Section 4, the decentralized SCs under WP contract, RS contract and a proposed contract are investigated. In section 5, the validity of theoretical analysis is illustrated by numerical examples. In Section 6, we present conclusions and future research directions.

2. Literature review

Three relevant streams of existing literature, namely, coordination of SC under random demand, coordination of SC under random yield and coordination of SC with CSR, are synthesized.

2.1. Coordination of SC under random demand

As for SCC, most works focus on coordination of SCs with different contracts which rule the transactions between SC members. A contract is said to coordinate the SC if the set of SC optimal actions is a Nash equilibrium, and total profit of the SC is maximized.

The first SCC model considered a single supplier selling to a single retailer under random demand [4]. For that SC, the widely used WP contract does not coordinate the SC owing to the double marginalization effects [4]. Thus, different contracts such as buy-back, revenue sharing, quantity discount and quantity flexibility contracts, are proposed to coordinate the supplier-retailer SC under random demand. Cachon and Lariviere [5] showed that there always exist corresponding RS contracts equivalent to aforementioned other contracts for coordinating two-echelon SC under random demand. Since the RS contract is concerned in this study, the following review only focuses on the SCC by RS contracts in recent years under random demand. For generic review of SCC, one can refer to [3] and [5].

In the early studies of SCC, only the uncertainty of demand is concerned, and the uncertainty of yield is not considered. For the SCC under random demand, various SC settings have been investigated in recent years. Xie *et al.* [39] showed that the revenue and cost sharing contract can increase the profits of SC members in both the online and the offline channel by properly setting the RS ratio and cost-sharing ratio. Ma *et al.* [25] coped with the coordination of a three-echelon fresh agricultural SC involving freshness-keeping effort with asymmetric information, and proposed a coordination contract based on cost and revenue sharing. For the demand related to sales effort and the age of the product on shelf, Avinadav [2] studied marketing and operational decisions of a manufacturer-retailer SC selling a perishable product under the RS contract. Zhang *et al.* [44] investigated decisions and coordination of retailer-led fresh-product SC under two-period dynamic pricing and portfolio contracts, and found that the option contract and the cost-sharing contract coordinate the SC in the first and second periods, respectively. Yang *et al.* [42] coped with coordination problem of a suppler-retailer fresh-product SC with demand information updating, and proposed a bidirectional risk-sharing contract to realize channel coordination.

In summary, different contracts are employed to coordinate different SCs. Among many type contracts, the RS contract has been frequently used to coordinate different SCs under random demand. For the SCs with additional characteristics except for random demand, the standard RS contract may not coordinate those SCs, and therefore the mixed contract based on RS are devised to coordinate complicated SCs.

2.2. Coordination of SC under random yield

The issue of SC coordination under random yield has attracted much attention. Early studies considered the SC with random yield and deterministic demand [11, 19]; but later works were more concerned with random yield and random demand (RYRD). For instance, Güler and Bilgiç [7] extended Gurnani and Gerchak [11] to the case of RYRD and proposed two mixed types of contracts to coordinate the SC. Similarly, Zhao and Wu [45] investigated a supplier-retailer agri-food SC under RYRD, and showed how the RS contract coordinates the SC. He and Zhao [12] dealt with the coordination problem of a three-echelon SC with a supplier, a manufacturer and a retailer under RYRD, and proposed a return policy combined with the WP contract to coordinate the SC. Analyzing coordination in a supplier-retailer SC facing defective shipment products and random demand, Guler and Keskin [10] found that buy-back and RS contracts coordinate the SC. Hu *et al.* [18] presented a RS policy with an order penalty and rebate contract coordinating the SC with a flexible ordering policy under RYRD. Chen and Yang [6] studied a supplier-buyer SC with emergency backup sourcing under RYRD, and proposed a subsidy contract and a punishing/subsidizing contract to coordinate the buyer-Stackelberg game and the supplier-Stackelberg game, respectively.

Giri *et al.* [8], extending [12], considered random yields of both the supplier and the manufacturer in the three-echelon SC and proposed composite contracts with buyback, sales rebate, and penalty to coordinate the SC. Luo and Chen [24] examined the coordination of a supplier-retailer SC for short life-cycle product under RYRD, and proposed an improved RS scheme coupled with a surplus subsidy. The coordination problem of a supplier-buyer SC for a perishable product was investigated by Hu and Feng [17] under RYRD and service requirement, and the RS contract was found to achieve channel coordinate under suitable parameters. Anderson and Monjardino [1] studied contract design for the three-echelon agricultural SC comprising suppliers, growers and buyers when growers are risk averse, and designed a new discount contract achieving channel coordination. Ye *et al.* [43] explored coordination for agricultural SC consisting of multiple risk-averse farmers and an agribusiness firm under RYRD, and proposed a hybrid coordination contract to realize optimal decisions. Xie *et al.* [40] analyzed a seller-buyer SC where the buyer is subject to uncertain demand and yield, and they showed the coordination effects of buyback contract.

In summary, different mechanisms, such as RS, buyback and mixed contracts have been designed to coordinate the SCs with various settings. Recently, the configuration of the SC under RYRD has become more and more sophisticated. However, SC coordination under RYRD has rarely been integrated with CSR.

2.3. SC coordination with CSR

2.3.1. Deterministic demand

As for the quantitative models and methods on integrating CSR into SC coordination, most works focused on deterministic demand in the past. For a socially responsible SC with price-dependent demand, Hsueh and Chang [16] showed that sharing the CSR investment by transfer payment among SC members achieves channel coordination. For the SC with deterministic demand in retail price and CSR level, Ni and Li [30] found that a win-win performance under certain conditions.

For the SC under linear demand in selling price and green efforts, Goering [30] utilized a two-part tariff contract to coordinate the SC. Panda [31], Modak *et al.* [26], and Panda *et al.* [32] explored the SC coordination problems, in which CSR is counted by consumer surplus and the demand is linear in retail price. Probing into the CSR coordination between a manufacturer and retailer, Panda [31] showed that the RS contract achieves channel coordination. Similarly, Modak *et al.* [26] investigated the coordination for a dual-channel SC comprising a retailer and a manufacturer with CSR, and found that the combination of all unit quantity discounts and a franchise fee achieves channel coordination. In later, Modak *et al.* [27] studied the coordination problem of a socially responsible SC comprising a manufacturer and two competitive retailers. Studying coordination of a manufacturer–distributor–retailer SC, Panda *et al.* [32] presented a contract-bargaining process resolving channel conflict in a manufacturer's CSR setting. For a manufacturer-retailer SC both exhibiting CSR, Panda *et al.* [33] found that the quantity discount contract achieves channel coordination. Panda *et al.* [34] further coped with the coordination problem of a socially responsible manufacturer-retailer closed-loop SC. Using social work donation as a CSR practice, Modak [28] investigated the coordination of a manufacturer-retailer closed-loop SC under deterministic linear demand in price and CSR, and showed that a two-part tariff contract realizes channel coordination. Xin *et al.* [41] studied the coordination problem of a socially responsible logistics SC, and designed a two-part pricing contract ensuring implementation of CSR accounting by consumer surplus.

For the SC under exogenous price and deterministic demand, Hsueh [15] established a bi-level programming model for a three-stage SC and showed the realization of win-win for CSR and the profits of SC members by proper SC collaboration. Wu [38] dealt with the coordination of a SC with an upstream firm and a downstream one considering decisions of pricing and CSR efforts, and found that sharing their costs or benefits realize channel coordination. Liu *et al.* [22] considered channel coordination in a CSR SC composed of a dominant retailer and multiple homogeneous suppliers, and suggested a CSR cost sharing mechanism of coordination. Heydari and Rafiei [13] studied the coordination problem of a manufacturer-retailer SC facing consumers' environmental and social awareness under deterministic demand, and used the production cost and CSR cost sharing contract to coordinate the chain. Liu *et al.* [23] tackled the coordination of a SC comprising a retailer, a supplier with CSR, and a supplier without CSR under deterministic demand related to CSR and retailer price.

In short, previous review indicated that most works of SC coordination in the presence of CSR focus on deterministic demand. In the real world, however, the market demand is usually uncertain.

2.3.2. Random demand

For the SC with CSR under random demand, Hsueh [14] made an early attempt to handle the SC coordination problem under normal random demand and exogenous price. They adopted a RS contract to coordinate the manufacturer-supplier SC where the supplier or the manufacturer assumes CSR. More recently, Nematollahi *et al.* [29] extended the model Hsueh [14] to demand variance influenced by CSR investment. Jokar and Hosseinimotlagh [20] studied a similar SC coordination problem and proposed a hybrid of WP and buyback contract to coordinate the chain.

Different from previous studies concerned with normal random demand, Zhao and Yin [46] investigated the coordination problem of a SC comprising a manufacturer with CSR and a retailer under CSR-and-price dependent multiplicative generic random demand. They suggested a modified RS contract coordinating the SC. Raza [36] considered the similar SC coordination problem as in Zhao and Yin [46] under CSR-and-price dependent additive generic random demand so as to extend the problem to the case of partial information. Zhao *et al.*. [47] studied the coordination problem of a manufacturer-retailer SC, where both members exhibit CSR under CSR-related random demand, and utilized a modified RS contract coordinating the SC.

2.4. Research gaps and contributions

The above literature review has clearly revealed that most past works focused on the coordination problem of different SC settings with CSR under deterministic demand related to CSR and/or price. Studies on the socially responsible SC coordination under random demand remain rare; further, an inquiry into the channel coordination of SC integrating CSR under random yield and demand has not been done.

In this paper, inspired by the SC coordination problem in agricultural sector, we merge CSR and channel coordination of SC under random yield and random demand for the first time. This study differs from previous works [14, 20, 29] in simultaneously considering generic random demand and yield. Compared to [47] and [46], this paper has two distinguished differences. First, this paper considers the random yield of the supplier while previous works assume deterministic yield. Secondly, the salvage of unsold products and the cost of lost sales are concerned in this paper while those are ignored in previous works.

The contributions of this study are three folds. First, given generic random yield and random demand, we establish the unimodality of the expected profit (EP) of centralized SCs with CSR. The merits resulting from CSR in centralized SCs are revealed by comparing optimal decisions and profits of centralized SCs with CSR and centralized SCs without CSR. Second, we present the Nash equilibrium solutions of non-coordinated models for the decentralized SCs with CSR under WP and RS contracts. In addition, we show that WP, RS and modified RS contracts do not coordinate the decentralized SC with CSR. Third, we propose a new RS-based contract to coordinate the SC with CSR. We calibrate our theoretical results with case study and numerical examples, and shed light on the effect of random yield and CSR effort. As for practical implication, the managerial insights are illustrated by case study and numerical analysis.

3. Problem statement and centralized models

3.1. Problem statement

The socially responsible SC providing a single agricultural product for customers consists of one upstream raw-material supplier, *e.g.*, a fish farmer, and one downstream end-product producer, *e.g.*, a frozen fish producer. The characteristic of the SC is that the supplier has random yield and the socially responsible producer faces with CSR-related random demand. For the supplier such as a fish farmer, the yield Q is stochastic given certain inputs, *e.g.*, fish larvae and fish feed, because of such uncertain factors as weather and diseases [43, 45]. As did in the existing literature [18, 20, 24], the stochastically proportional yield (SPY) $Q = \theta X$ is adopted to model the random yield of the supplier, where X is a vector of production inputs, Q is the yield, and θ is the random output factor (ROF).

In the SC, the brand producer implements CSR activities by related investment, *e.g.*, investment for environmental protection. Unlike any marketing or sale effort, CSR effort is not directly related to product sales of the producer; but it is conducive to establishing and maintaining a reputation of the producer, which helps to indirectly increase market sales. The CSR activities of a producer perceived by the public usually can increase the goodwill of buying the products for the people with CSR awareness. Existing studies [35, 37] showed that the CSR activities of firms perceived by consumers can improve the consumer satisfaction which is an important content of consumer utility. Moreover, consumers' satisfaction has positive effects on consumer loyalty [35], and affects their purchasing intension in the future. To account for the positive effect of CSR on customers' demand, the CSR-related demand function $L(y) = K\sqrt{y}$ [36, 47] is adopted to model the reward demand from CSR, where y is the producer's CSR investment and K is the CSR effect factor (CSREF) of the producer. Clearly, $L(y)$ is concave with respect to y, and is a non-decreasing function of y. By nature, the customer demand is uncertain, and the portion of the demand independent of CSR is described by the stochastic demand d with probability density function (PDF) and cumulative distribution function (CDF). Here, as did in [36, 46], the

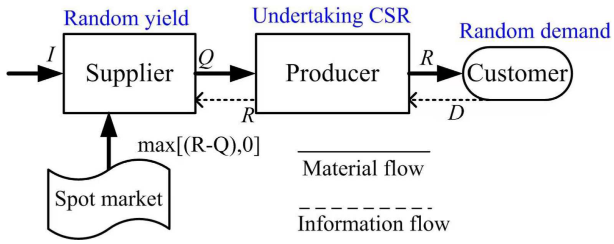

Figure 1. Expected profits and CSR investment changing with the CSREF K in NE 3.

total demand is assumed to be the additive-form and be the sum of random demand d independent of the producer's CSR, and reward demand $L(y)$ resulting from CSR activities of the producer, *i.e.*, $D = d + L(y)$. In this study, only a single selling period of SC is considered.

The sequence of events for the two-stage SC is as follows:

(1) The supplier declares wholesale price ω for raw materials;
(2) Based on the wholesale price ω, the producer decides the quantity of raw material R to be ordered and the CSR investment y;
(3) After knowing the ordered quantity of raw material R, the supplier determines the production input I;
(4) The supplier provides the producer with the realized yield Q. The supplier needs to purchase some raw materials in the spot market if Q is less than R because of the random yield of the supplier;
(5) The producer's output is R units of the final product after receiving R units of raw materials, then sells final product at an exogenous price p after the demand is realized during a single selling period;
(6) Finally, the producer makes payment to the supplier under some contract specifying the transfer payment.

In above event (4), it is reasonable to assume that the purchasing price c_p from the spot market is higher than the wholesale price ω of the supplier [12]. Without loss of generality, we assume that one unit output needs one unit input in events (4) and (5). Figure 1 depicts the material and information flows of the SC with the sequence of events listed above. The uniqueness of the coordination problem is predicated on both random yield and CSR-related random demand, and the presence of producer's CSR activity under random yield and random demand adds dramatically the complexity of the SC coordination problem. To better state the problem, the nomenclature and assumptions used to formulate the problem are stated below.

3.2. Nomenclature, assumptions, and profit functions

The variables, parameters and cost coefficients for the problem are listed as follows.

To simplify the SC coordination problem, we made the following assumptions:

(1) Symmetric information is assumed, so that both members know information on all cost coefficients and parameters describing random yield and demand, at the start of the game.
(2) The members of the SC are rational and risk neutral.
(3) For the supplier, unit salvage value is less than unit production cost, $h_s < c_s$ and $c_p > \omega$ (game setting).
(4) For the producer, let $(p - h_m + s_m)$, $h_m < c_m$ and $p > c_p + c_m$.

Assumptions 1 and 2 are basic assumptions for SC coordination, and have been widely adopted in the field of SC coordination, *e.g.*, in [14,20,29,46], so did in this paper. For the agricultural perishable product, such as the fish (the frozen fish), its salvage $h_s(h_m)$ approaches zero [2]. So, it is reasonable that unit salvage $h_s(h_m)$ is less than unit production cost $c_s(c_m)$ in assumption 3(4). The price in the spot market higher than the wholesale price for agricultural raw material is usually observed in the real world. In assumption 4, the price of final products p is obviously more than the unit salvage of unsold final product h_m, *i.e.*, $p > h_m$. Owing to $s_m > 0$, then $p - h_m + s_m > 0$. Naturally, the price of final product is greater than the sum of the price of unit raw material c_p and unit production cost c_m.

In a practical SC, each member of the SC usually makes decisions independently and tries to maximize its own profit based on some contract defining the transfer payment and regulations. For convenience, the contract between two members of the SC is denoted by $\Omega(\alpha_1 \cdots \alpha_i \cdots \alpha_n)$, where α_i is the ith parameter of the contract. Then, the transfer payment T can be written as the function of contract parameters $T = f(\alpha_1 \cdots \alpha_i \cdots \alpha_n)$. To solve the SC coordination problem, the profit of each member should be first accounted for a given transfer payment T.

Based on above assumptions and notations, the profit of the supplier is as follows:

$$\pi_s = T + h_s[uI - R]^+ - c_p[R - uI]^+ - c_S I. \tag{1}$$

On the right-hand side of equation (1), the first term is the transfer payment, the second term is the salvage revenue of unsold products, the third term is the replenishment cost from the spot market, and the last term is the production cost of the supplier.

The profit of the producer is as follows.

$$\pi_m(R, y) = p \cdot Min(R, D) + h_m[R - D]^+ - c_m R - T - s_m[D - R]^+ - y. \tag{2}$$

On the right-hand side of equation (2), the first term is the sale revenue of final products, and the second term is the salvage revenue of unsold final products. The third term is the production cost of the producer, the fourth term is the transfer payment paid by the producer, followed by the cost of lost sales, and the last term is the CSR investment.

3.3. Centralized model without CSR

The benchmark problems of centralized SC with CSR and centralized SC without CSR are first stated. The centralized SC without CSR ($D = d$ and $y = 0$) is to maximize the expected profit of the SC through concurrent selection of the supplier's production input I and the producer's order quantity R by a single decision maker. Note that order quantity is still a decision in the centralized SC owing to the presence of random yield.

Summing equations (1) and (2), the profit of the SC when y is zero is as follows:

$$\begin{aligned} \pi_c^o(I, R) = p \cdot Min(R, D) + h_m[R - D]^+ - s_m[D - R]^+ \\ - c_m R - c_p[R - uI]^+ - h_s[uI - R]^+ - c_s I. \end{aligned} \tag{3}$$

Then, the EP function of the SC is as follows (see Appendix A):

$$\begin{aligned} E(\pi_c^o) = (p - h_m + s_m)S(R) + (c_p - h_s)S(I, R) \\ - (c_m + c_p - h_m)R - (c_s - h_s u_1)I - s_m u_d \end{aligned} \tag{4}$$

where $S(I, R) = E[\min(uI, R)] = R - I \cdot \int_A^{R/I} G(u)du$, $S(R) = E[\min(R, d)] = R - \int_0^R F(x)dx$.

To describe the relationship between the supplier's decision I and the producer's decision R, the ratio of order quantity to production input quantity for raw material $z = R/I$ is introduced. Denoting $W(z) = \partial S(I, R)/\partial I$, we have the following proposition.

Proposition 1. *For any given K falling in the value domain of $W(z)$, there is a unique z satisfying $W(z) = K$.*

Proof. See Appendix B. □

The monotone of $W(z) = \partial S(I, R)/\partial I$ shown in Proposition 1 is a critical property for the SC with random yield, which ensures the uniqueness of z^*.

To solve the centralized model, we first study the concavity of the EP function $E(\pi_c^o)$. For the concavity of $E(\pi_c^o)$, we have the following Theorem 1.

Theorem 1. *For the SC without CSR, the EP function $E(\pi_c^o)(I,R)$ is concave with respect to I and R, and there exists a unique decision pair (I_c^o, R_c^o) maximizing $E(\pi_c^o)$.*

Proof. See Appendix D. □

Theorem 1 and its proof procedure indicate that there is a unique decision pair (I_c^o, R_c^o) maximizing $E(\pi_c^o)$ regardless of stochastic distribution forms of the ROF and the random demand. The unimodality of $E(\pi_c^o)$ is the necessary condition coordinating the SC. The following Theorem 2 further specifies the (I_c^o, R_c^o) maximizing the EP of the SC.

Theorem 2. *The centralized optimal decisions (I_c^o, R_c^o) are determined by the following simultaneous equations.*

$$\begin{aligned} W(z^*) &= (c_s - h_s\mu_1)/(c_p - h_s)(a) \\ F(R) &= [p + s_m - c_m - h_s - (c_p - h_s)G(z^*)]/(p - h_m + s_m)(b). \end{aligned} \tag{5}$$

Proof. See Appendix D. □

Note that $-\int_A^{z^*} G(u)du = (c_s - h_s\mu_1)/(c_p - h_s) - z^*G(z^*)$ by equation (5a). Replacing I by R/z and $S(I,R)$ by $R - I \cdot \int_A^{R/I} G(u)du = R/z(z - \int_A^z G(u)du)$ in equation (4), the optimal EP of the SC can be written as follows.

$$E(\pi_c^o) = (p - h_m + s_m)S(R) - [c_m - hm + ks]R - s_m\mu_d. \tag{6}$$

Hereinafter, let $ks = h_s + (c_p - h_s)G(z^*)$.

3.4. Centralized model with CSR

Based on equation (4), we have the following EP for the SC with CSR:

$$\begin{aligned} E(\pi_c) = (p - h_m + s_m)S(R) - (c_p - h_s)S(I,R) - (c_m + c_p - h_m)R \\ - (c_s - h_s\mu_1)I - s_m(\mu_d + K\sqrt{y}) - y \end{aligned} \tag{7}$$

where $S(R) = E[\min(R, d + L(y))] = R - \int_0^{R-L(y)} F(x)dx$, $S(I,R) = R - I \cdot \int_A^{R/I} G(u)du$

The difference between $E(\pi_c)$ and $E(\pi_c^o)$ lies in the dissimilarity between $E(D)$ and $E(d)$ along with extra cost of CSR investment y. Note that the decisions are for the centralized model with CSR. Since $E[\pi_c(I,R,y)]$ is concave with respect to I and R given the CSR investment y by Theorem 1, the following proposition is derived from $\partial E(\pi_c)/\partial I$ and $\partial E(\pi_c)/\partial R$ by equation (7).

Proposition 2. *Given CSR investment y, the centralized optimal decisions (I_c, R_c) are determined by the following simultaneous equations.*

$$\begin{aligned} W(z^*) &= (c_s - h_s\mu_1)/(c_p - h_s)(a) \\ F[R - L(y)] &= [p + s_m - c_m - h_s - (c_p - h_s)G(z^*)]/(p - h_m + s_m)(b). \end{aligned} \tag{8}$$

Note that the z^* in equation (8a) is identical with that in equation (5a). Then, the optimal I_c is R_c/z^*. Based on Theorem 2 and Propositions 1 and 2, we have Corollary 1.

Corollary 1. *For the centralized SC, the unique z^* is designated by equation (9), and is only relevant to the supplier's ROF and cost parameters along with exogenous c_p.*

$$W(z^*) = (c_s - h_s\mu_1)/(c_p - h_s). \tag{9}$$

Proof. See Appendix C. □

Corollary 1 means that the z^* is fixed in a centralized SC and is not concerned with the producer's parameters. Based on the optimal decisions (I_c, R_c), the optimal CSR investment of the producer is designated in the following Proposition 3.

Proposition 3. *For the centralized SC with CSR, the optimal decision y_c is fixed and as follows:*

$$y_c = \frac{K^2}{4}[p - c_m - h_s - (c_p - h_s)G(z^*)]^2. \tag{10}$$

Proof. See Appendix B. □

According to the optimal decision y_c in equation (10), we have the following Corollary 2 on the relationship between y_c and μ_1.

Corollary 2. *For the centralized SC with CSR, the optimal CSR investment y_c is related to the mean of supplier's ROF μ_1, and increases with μ_1.*

Proof. See Appendix C. □

According to Proposition 3, the optimal CSR investment of the producer for the centralized SC with CSR is unique given the cost parameters and CDF of ROF. Then, we have the following Theorem 3 on the unimodality of the EP of the SC with CSR.

Theorem 3. *For the SC with CSR, is unimodal with respect to I, R and y, and there exists a unique decision vector (I_c, R_c, y_c) maximizing $E(\pi_c)$.*

Proof. See Appendix D. □

The unimodality of $E(\pi_c)$ stated in Theorem 3 is the necessary condition coordinating the SC. Fortunately, the expected profit of most of SCs satisfy unimodality including the socially responsible SC in this study.

Based on Propositions 2 and 3, we have the following optimal EP of the SC with CSR through replacing I by R/z in equation (8) and simplification.

$$\begin{aligned} E(\pi_C) = (p - h_m + s_m)S(R) - [c_m - h_m + ks]R - s_m\mu_d \\ - s_m K^2[p - c_m - ks]/2 - K^2[p - c_m - ks]^2/4. \end{aligned} \tag{11}$$

3.5. Comparison of centralized models

To learn the effect of CSR on the centralized SCs, we compared the decisions of I and R along with the optimal EPs of the SCs between the centralized SC with CSR and the centralized SC without CSR. The relationship between (I_c, R_c) maximizing $E(\pi_c)$ and (I_c^o, R_c^o) maximizing $E(\pi_c^o)$ is stated in Proposition 4.

Proposition 4. *For the centralized SCs, the optimal decision $I_c(R_c)$ is larger than $I_c^o(R_c^o)$, and the relationships between them are shown in the following equation.*

$$\begin{aligned} I_c = I_c^o + K\sqrt{y_c}/z^*(a) \\ R_c = R_c^o + K\sqrt{y_c}(b). \end{aligned} \tag{12}$$

Proof. See Appendix B. □

Proposition 4 indicates that the CSR investment of the producer incentives the producer to increase the order quantity of raw materials in order to meet the reward demand in relation to CSR activities. Correspondingly, the supplier should increase the production input to match the increase of R. An interesting observation of the effect of CSR is that the optimal EP of the SC with CSR is more than that of the SC without CSR, as stated in below Theorem 4.

Theorem 4. *For the centralized SCs, $E(\pi_c)$ is larger than $E(\pi_c^o)$, and the difference between them is shown in the following equation.*

$$E(\pi_c) - E(\pi_c^o) = \frac{K^2}{4}[p - c_m - h_s - (c_p - h_s)G(z^*)]^2. \tag{13}$$

Proof. See Appendix D. □

Theorem 4 further supports the practical value of CSR for the SC. Owing to equations (10), (13) in Theorem 4 and Corollary 2, the below Corollary 3 is trivial.

Corollary 3. *For the centralized SCs, the difference between $E(\pi_c)$ and $E(\pi_c^o)$ equals y_c and increases with μ_1.*

Corollary 3 indicates that the mean of ROF affects the difference between $E(\pi_c)$ and $E(\pi_c^o)$. Moreover, the more the μ_1 is, the more the difference is.

4. Decentralized models

In practical SCs, the decisions of SC members are usually made separately. For the decentralized SC, the sequence of events under different contacts is the same with that in Section 3.1, but the transfer payments in event (6) are different for different contracts. The decentralized SCs under WP contract, RS contract and a proposed contract are studied in this section. For the purpose of comparison, we tackle the SC with CSR and the SC without CSR.

4.1. Wholesale price contract

Wholesale price contract has been widely used in practical SCs. Under the WP contract $\Omega = \omega$, the transfer payment is $T(\omega) = \omega \cdot R$. By substituting T into Eq. (1) and calculating its expectation, the EP of the supplier $E(\pi_s^\omega)$ is as follows.

$$E(\pi_s^w) = (\omega - c_p)R + (c_p - h_s)S(I, R) - (c_s - h_s\mu_1)I. \tag{14}$$

Note that $\omega \le c_p$, $c_s > h_s$, $c_s > h_s$. To avoid uninteresting cases of negative profit of the supplier, $S(I, R)$ needs to be positive under the WP contract.

Similarly, the EP of the producer $E(\pi_m^w)$ is as follows.

$$E(\pi_m^w) = (p - h_m + s_m)S(R) - (c_m - h_m + \omega)R - s_m(\mu_d + K\sqrt{y}) - y \tag{15}$$

where $S(R) = E[\min(R, d + K\sqrt{y})] = R - \int_0^{R-L(y)} F(x)dx$.

Notably, the WP contract cannot coordinate the two-stage SC under random demand. Here, we present the optimal decisions and the optimal EPs of SC members under the WP contract. Those EPs of SC members are as the baseline profits.

4.1.1. WP contract for SC without CSR

First, we investigate the decentralized SC without CSR under the WP contract. The related results are then extended to the SC with CSR. The supplier's EP of SC without CSR $E(\pi_{s,o}^w)$ is the same with $E(\pi_s^w)$. Let y be zero and D equal d. We have the following producer's EP of SC without CSR.

$$E(\pi_{m,o}^w) = (p - h_m + s_m)S(R) - (c_m - h_m + \omega)R - s_m\mu_d. \tag{16}$$

On the optimal decisions of the supplier and the producer, we have the following proposions.

Proposition 5. *Under the WP contract, there exists a unique $z_w{}^*$ shown in equation (17) maximizing $E(\pi_{s,o}^w)$ and $E(\pi_s^w)$.*

$$W(z_w^*) = \partial S(I_w^*, R)/\partial I = (c_s - h_s\mu_1)/(c_p - h_s). \tag{17}$$

Proof. See Appendix B. □

From Proposition 5, given the producer's order quantity R, the supplier always chooses $I_w^{o*} = z_w^* R$ to maximize its own EP. Note that $z^* = z_w^*$ by comparing equation (9) with equation (17)). As for the producer's optimal decision, it is prescribed in Proposition 6.

Proposition 6. *Under the WP contract with specified ω, there exists a unique R_w^{o*} maximizing $E(\pi_{m,o}^w)$, and the optimum R_w^{o*} satisfies equation (18).*

$$F(R) = (p + s_m - c_m - \omega)/(p - h_m + s_m). \tag{18}$$

Proof. See Appendix B. □

Equation (18) in Proposition 6 means that the producer's optimal decision R_w^{o*} is closely related to the WP ω specified by the supplier.

Since the supplier decides ω followed by the decision of producer's order quantity, the supplier should consider the optimal policy of the producer and makes its best decision. The best ω of the supplier can be derived by backward induction method.

Replacing R by $F^{-1}[(p + s_m - c_m - \omega)/(p - h_m + s_m)]$ and I by R/z^* in equation (14) and reducing $\partial E(\pi_s^w)/\partial\omega = 0$, the optimal ω is designated by the following equation.

$$F^{-1}[\frac{p + s_m - c_m - \omega}{p - h_m + s_m}] = \frac{[\omega - c_p + kt]}{(p - h_m + s_m)f(\frac{p+s_m-c_m-\omega}{p-h_m+s_m})}. \tag{19}$$

where $kt = [(c_p - h_s)(z^* - \int_A^{z^*})G(u)du - (c_s - h_s\mu_1)]/z^*$.

Owing to $F^{-1}[]$ is monotone and is larger than 0, there is usually a unique ω_w^{o*} as the solution of equation (19) for $\omega > cp - kt$. However, in some case the solution of equation (19) ω_w^{o*} may be larger than c_p (this is not practical), and then the optimal ω is the maximum value in the range $(c_p - kt, c_p)$ accepted by the producer.

Given the optimal WP, there is a unique Nash equilibrium for decentralized SC without CSR under the WP contract. Based on the ω_w^{o*} chosen by the supplier, the optimal R_w^{o*} of the producer is designated by equation (18). From Proposition 6, we know that R_w^{o*} satisfying equation (18) maximizes the producer's EP. After knowing the order quantity, the supplier always chooses the optimal I as R/z to maximize its EP.

4.1.2. WP contract for SC with CSR

As aforementioned, the z_w* for the decentralized socially responsible SC under WP contract is the same with the z^* under centralized SCs. So, the optimal I of the supplier is R_w^*/z^*, where R_w^* is the producer's optimal order quantity. According to the first-order condition, $\partial E(\pi)_m^w/\partial R = 0$ and $\partial E(\pi)_m^w/\partial y = 0$, we have the value of R_w^* in Proposition 7.

Proposition 7. *Under the WP contract with specified ω, there exist unique R_w^* and y_w^* maximizing $E(\pi_m^w)$ and the optimal R_w^* and y_w^* satisfy equation (20).*

$$\begin{aligned} R_d^* &= F^{-1}[\frac{p + s_m - c_m - \omega}{p - h_m + s_m}] + \frac{K^2}{2}(p - c_m - \omega)(a) \\ y_d^* &= \frac{K^2}{4}(p - c_{m-\omega})(b). \end{aligned} \tag{20}$$

From equation (20), we know that the producer's optimal decisions depend on the WP ω decided by the supplier. To maximize its EP, the rational supplier always chooses the ω satisfying $\partial E(\pi_s^w)/\partial\omega = 0$. Similar to the SC without CSR, the best ω of the supplier is derived by backward induction method.

Replacing R by the term of right-hand side in equation (20a) and I by R/z^* in equation (14) and reducing $\partial E(\pi_s^w)/\partial\omega = 0$, the optimal ω_w^* is designated as follows.

$$F^{-1}[\frac{p+s_m-c_m-\omega}{p-h_m+s_m}] - \frac{\omega - c_p + kt}{f(\frac{p+s_m-c_m-\omega}{p-h_m+s_m})(p-h_m+s_m)} + \frac{K^2}{2}(p-c_m-2\omega+c_p-kt) = 0. \tag{21}$$

Based on the ω_w^* chosen by the supplier and accepted by the producer, the optimal R_w^* of the producer is designated by equation (20a), which maximizes the producer's EP by proposition 7. After knowing the order quantity, the supplier chooses the optimal decision I as R_w^*/z^* to maximize its EP.

Note that the optimal CSR investment is $K^2[p-c_m-h_s-(c_p-h_sG(u*))]^2/4$ (see Eq. (10)) in the centralized SC, while the optimal one is $y_w^* = K^2(p-c_m-\omega_w^*)^2/4$ here. We can verify $\omega_w^* \neq h_s + (c_p-h_s)G(z^*)$. Thus, the WP contract does not coordinate the SC.

4.2. Revenue-sharing contract

RS contracts have been adopted to coordinate the two-stage SC with random yield and random demand [10] and certain two stage SCs with CSR under random demand [14, 31]. This motivates us to analyze the decentralized socially responsible SC under the RS contract.

Under standard RS contract $\Omega = \omega, \phi$ where $0 < \phi < 1$, the supplier declares the wholesale price ω (may be less than c_s). Then the producer makes decisions of the order quantity R and CSR investment y, followed by the supplier's decision of production input I. At the end of the selling season, the producer returns ϕ proportion of total revenue to the supplier in order to compensate the supplier's relatively low WP. For the RS contract, the transfer payment T is $T(\omega,\phi) = \phi(pMin(R,D) + h_m[R-D]^+) + \omega\cdot R$. Then, we can obtain the following EPs of the supplier and the producer by substituting T into equations (1) and (2), respectively.

$$E(\pi_s^r) = \phi(p-h_m)S(R) + (\omega - c_p + \phi h_m)R + (c_p-h_s)S(I,R) - (c_s - h_s\mu_1)I \tag{22}$$

$$E(\pi_m^r) = [(1-\phi)(p-h_m)+s_m]S(R) - [c_m - (1-\phi)h_m + \omega]R - s_m(\mu_d + K\sqrt{y}) - y. \tag{23}$$

4.2.1. RS contract for SC without CSR

The supplier's EP of SC without CSR $E(\pi_{s,o}^r)$ is the same with $E(\pi_s^r)$. Let y be zero and D equal d. We have the following producer's EP of SC without CSR.

$$E(\pi_{m,o}^r) = [(1-\phi)(p-h_m)+s_m]S(R) - [c_m - (1-\phi)h_m + \omega]R - s_m\mu_d. \tag{24}$$

According to the first-order condition of $\partial E(\pi_s^r)/\partial I = 0$, we know that the optimal z_r^* maximizing $E(\pi_{s,o}^r)$ or $E(\pi_s^r)$ satisfies $W(z_r^*) = \partial S(I_r^*, R)/\partial I = (c_s - h_s\mu_1)/(c_p - h_s)$. Note that $z_r^* = z_w^* = z^*$.

As for the producer's optimal decision, we have the following result by the first-order condition of $\partial E(\pi_{m,o}^r/\partial R = 0$.

$$F_{r,o}^* = [(1-\phi)p + s_m - c_m - \omega]/[(1-\phi)(p-h_m)+s_m]. \tag{25}$$

Theorem 5 states the effect of RS contract on coordinating the SC without CSR.

Theorem 5. *For the SC without CSR, the RS contract achieves channel coordination under the following contract parameters*

$$\omega = ks + \phi\left[\frac{(p-h_m(p+s_m-c_m-ks))}{p-h_m+s_m} - p\right] \tag{26}$$

and $\frac{(p-h_m+s_m)\max(0,ks-c_p)}{p(p-h_m+s_m)-(p-h_m)(p+s_m-c_m-ks)} < \phi < \frac{(p-h_m+s_m)ks}{p(p-h_m+s_m)-(p-h_m)(p+s_m-c_m-ks)}$, $E(\pi_m^s) = \lambda E(\pi_c^o - (1-\lambda)s_m\mu_d$ *where* $\lambda = [(1-\phi)(p-h_m+s_m)]/(p-h_m+s_m)$.

Proof. See Appendix D. □

Theorem 5 indicates that the supplier selects the ω shown in equation (26) achieving channel coordination of the SC without CSR, given a certain ϕ.

Theorem 5 shows that RS contract coordinates the SC without CSR under random yield and demand by sharing the uncertainties of yield and demand *via* the revenue.

4.2.2. RS contract for SC with CSR

For the SC with CSR, the optimal decision of the supplier is $I_r^* = R_r^*/z_r^*$, where R_r^* is the optimal order quantity of the producer and the WP contract and $z_r^* = z^*$. According to the first-order condition, $\partial E(\pi_m^r)/\partial R$ and $\partial E(\pi_m^r)/\partial y = 0$, we have the value of in Proposition 8.

Proposition 8. *Under the RS contract with specified ω, there exist unique R_r^* and y_r^* maximizing $E(\pi_m^r)$ and the optimal R_r^* and y_r^* satisfy equation (27).*

$$\begin{aligned} R_r^* &= F^{-1}\left[\frac{(1-\phi)p + s_m - c_m - \omega}{(1-\phi)(p-h_m)+s_m}\right] + \frac{K^2}{2}[(1-\phi)p - c_m - \omega](a) \\ y_r^* &= \frac{K^2}{4}[(1-\phi)p - c_m - \omega]^2(b). \end{aligned} \tag{27}$$

Proof. See Appendix B. □

Proposition 8 shows that the producer's optimal decisions depend on the WP ω and the RS ratio ϕ. Based on Proposition 8, the following Theorem 6 is deduced.

Theorem 6. *The RS contract does not coordinate the SC with CSR.*

Proof. See Appendix D. □

In the socially responsible SC, revenue sharing mechanism makes the two members share the risk of yield and demand uncertainty. But there is no mechanism making the supplier share the cost of CSR investment, so that the RS contract fails to coordinate the two-stage SC with CSR. Considering this point, we design a new contract named revenue and CSR sharing (RCSRS) contract to coordinate the SC.

4.3. Revenue and CSR sharing contract

The proposed RCSRS contract is an extension of existing RS contract. The basic idea is to add the producer's CSR cost sharing mechanism into standard RS contract. In the RCSRS contract, let $(1-\eta)y$ CSR cost of the producer is shared by the supplier directly. So, the CSR cost of the producer is reduced to η where $0 < \eta < 1$.

Based on previous description, the RCSRS contract is denoted by $\Omega = \omega, \phi, \eta$. Then, the EPs of the supplier and the producer are as follows.

$$E(\pi_s^n) = \phi(p-h_m)S(R) + (\omega - c_p + \phi h_m)R + (c_p - h_s)S(I,R) - (c_s - h_s\mu_1)I - (1-\eta)y \tag{28}$$

$$E(\pi_m^n) = [(1-\phi)(p-h_m)+s_m]S(R) - [c_m - (1-\phi)h_m + \omega]R - s_m(\mu_d + F\sqrt{y}) - \eta y. \tag{29}$$

From the first-order condition of $\partial E(\pi_s^n)/\partial I = 0$, the optimal z_n^* maximizing $E(\pi_n^r)$ satisfies $W(z_n^*) = \partial s(I_n^*, R)/\partial I = (c_s - h_s\mu_1)/(c_p - h_s)$. Based on $\partial E(\pi_m^n)/\partial R = 0$ and the fixed z_n^*, we have the following Proposition 9 on the optimal decisions of the producer.

Proposition 9. *Under the RCSRS contract with given parameters ω, ϕ, η, there exist unique R_n^* and y_n^* maximizing $E(\pi_m^n)$ and the optimal R_n^* and y_n^* satisfy equation (30).*

$$R_n^* = F^{-1}[\frac{(1-\phi)p + s_m - c_m - \omega}{(1-\phi)(p-h_m)+s_m}] + \frac{K^2}{2}[\frac{(1-\phi)p - c_m - \omega}{\eta}](a)$$
$$y_n^* = \frac{K^2}{4}[\frac{(1-\phi)p - c_m - \omega}{\eta}]^2(b). \tag{30}$$

Next, we investigate the effect of the RCSRS contract on coordinating the SC with CSR. If the RCSRS contract is able to coordinate the SC, we can find suitable contract parameters $< \omega, \phi, \eta >$ maximizing the EP of every member of the SC. The suitable parameters of $< \omega, \phi, \eta >$ coordinating the SC are designated by equation (31) in Theorem 7.

Theorem 7. *The RCSRS contract $\Omega(\omega, \phi, \eta)$ with the parameters shown in equation (31) is able to coordinate the socially responsible SC under random yield and random demand*

$$\omega = ks + \phi[\frac{(p - h_m(p + s_m - c_m - ks))}{p - h_m + s_m} - p](a)$$
$$\eta = (1-\phi) + \frac{\phi s_m(h_m - c_m - ks)}{(p - c_m - ks)(p - h_m + s_m)}(b) \tag{31}$$

where ϕ is the value specified by the following inequality.

$$\frac{(p - h_m + s_m)\max(0, ks - c_p)}{p(p - h_m + s_m) - (p - h_m)(p + s_m - c_m - ks)} < \phi < \frac{(p - h_m + s_m)ks}{p(p - h_m + s_m) - (p - h_m)(p + s_m - c_m - ks)}. \tag{32}$$

Proof. See Appendix D. □

Theorem 7 means that the RCSRS contract with the parameters designated in equation (31) coordinates the SC. But the revenue-sharing ratio ϕ is limited to the range prescribed by inequality equation (32). The specific value of ϕ is determined by relative bargaining powers of the two SC members. The terms and parameters of the RCSRS contract should be determined at the beginning of sequential games stated in Section 3.1.

Note that a modified RS contract [47] in which the revenue is defined as $pMin(R, D) + h_m[R - D]^+ - y$ has been proposed to coordinate a manufacturer-retailer SC with CSR under random demand. However, the modified RS contract does not coordinate the SC with CSR under random yield, as stated in below Corollary 4.

Corollary 4. *The modified RS contract is not able to coordinate the SC with CSR under random yield and demand.*

Proof. See Appendix C. □

Corollary 4 indicates that the introduction to random yield of SC in this paper makes the modified RS contract fail to coordinate the SC with CSR under random yield and demand.

5. Numerical examples

Numerical examples are used to illustrate some propositions and theorems in this section. The effects of yield uncertainty and CSREF on the EPs are analyzed. The $MATLAB^{TM}$ is adopted to compute all numerical examples.

5.1. Description of numerical example

Three numerical examples (NEs) with different random yields and demands are constructed by the assumptions listed in Section 3.2. For the three NEs, the cost parameters of the supplier and the producer are listed in Table 1. In NE 1 and NE 2, we assume that the ROF μ in the SPY model follows the same uniform distribution in the interval $[A, B]$. In NE 3, however, the ROF follows exponential distribution in the interval $[A, B]$, where $g(\mu) = exp(-x/\mu_1)/\mu_1$ and $\mu_1 = 1$. For the purpose of comparison, the ranges of decisions I and R in all NEs are the same and are within [50, 200]. In three NEs, the random demand d has the same mean $\mu d = 100$. In NE 1, d follows normal distribution with standard deviation $\theta = 20$. In NE 2, d is exponential distribution, while it follows uniform distribution within [50, 150] in NE 3. For the reward demand from the producer's CSR, the CSREF is set to be four, *i.e.*, $K = 4$.

In every NE, the values of cost and price parameters are set for easily solving related equations by MATLAB, and are set to make the EPs positive. The mean of ROF is usually less than one in SPY model. Here, the mean of ROF is set as 0.8 in NEs 1 and 2, accordingly the values of A and B are 0.6 and 1.0, respectively.

5.2. Computational results of numerical examples

Firstly, we check some properties of the EPs for centralized SCs. According to Proposition 1, $W(z) = zG(z) - \int_A^z G(u)du$ is monotone in the domain $z \in [A, B]$. The curves of $W(z)$ *versus* z are shown in Figure 2. The monotonicity of $W(z)$ shown in Figure 2 is consistent with the statement in Proposition 1.

Next, we verify some important conclusions for centralized SC with CSR and centralized SC without CSR. According to equation (4), the maximal EPs of the SC without CSR $E(\pi_c^o)$ for NEs are shown in Table 2. From equation (7), the maximal EPs of the SC with CSR $E(\pi_c)$ for three NEs are shown in Table 2. Taking NE 2 as an example, we find that $R_c = R_c^o + L(y_c) = 118.87 + 9.34 = 128.21$ and $I_c = I_c^o + L(y_c)/z^* = 164.84 + 9.34/0.7211 = 177.79$. Those observations agree with Proposition 4, *i.e.*, $I_c(R_c)$ is larger than $I_c^o(R_c^o)$. According to the EPs listed in Table 2, we can observe that $E(\pi_c)$ is larger than $E(\pi_c^o)$. Taking NE 3 as an example, we find $E(\pi_c) - E(\pi_c^o) = 116.28 - 109.89 = 6.39$ while the difference stated in Theorem 4 is $0.25K^2[p_s - c_m - h_s - (c_p - h_s)G(z^*)]^2 = 0.25 * 16 * [2.0 - 0.4 - 0.2 - (1.2 - 0.2) * 0.1359]_2 = 6.39$. That result agrees with Theorem 4.

Subsequently, we verify the conclusions for decentralized SCs and the coordination effect of the proposed RCSRS contract. For the SC without CSR and the SC with CSR under the WP contract, the corresponding computational results for NE 1 and NE 2 are listed in Table 3. As for the SC without CSR and the SC with CSR under the RS contract, the optimal decisions of every member in NE 1 and NE 2 are obtained for $\phi = 0.3$, as shown in Table 3. Under the RCSRS contract, the optimal decisions of every member for the SC with CSR in three NEs are presented in Table 3 where $\phi = 0.3$.

From the data shown in Table 3, we find that the optimal WPs of the supplier for NE 1 and NE 2 without CSR are 1.34, and 0.96, respectively, which are in the interval (c_s, c_p), *i.e.*, (0.2, 1.35) and (0.3,1.2), respectively. An interesting result observed from Table 3 is that the RS contract is able to coordinate the SC without CSR. This observation agrees with Theorem 5. From the data for SCs with CSR in Table 3, it can be seen that the RS contract does not coordinate the SC with CSR. This conclusion is consistent with Theorem 6. In addition, we find that the EPs of the socially responsible SC and its members are higher than those of the SCs without CSR under the WP contact and the RS contract in NE 1 and NE 2. Those results indicate that undertaking CSR is beneficial to the SC and its members even for uncoordinated decentralized SCs.

For the decentralized SC with CSR, the parameters of RCSRS are obtained by solving equation (31). Based on those parameters and equations (28) and (29), the EPs and variables for three NEs are listed in Table 3. Comparing the related data in Table 3 with the data in Table 2, it is clear that the RCSRS contract coordinates the SCs with CSR under normal-distributed, exponential-distributed and uniform-distributed demands. Note that the ROF is uniform distribution in NE 1 and NE 2, and is exponential distribution in NE 3. In conclusion, the proposed RCSRS is able to coordinate the supplier-producer SC with CSR under generic random yield and demand, which is in accordance with the statement in Theorem 7.

TABLE 1. Variable parameter description.

Notations for the supplier	Notations for the producer
I: the production input of the supplier (decision variable);	R: ordered quantity of raw materials of the producer(decision variable)
Q: the yield of supplier for input I, $Q = u \cdot I$	c_m: unit production cost of the producer
u: the supplier's ROF supporting on $[A, B]$, $0 < A < B \leq 1$)	h_m: unit salvage of unsold products of the producer
$g(u)$: the PDF of ROF u, $g(u) > 0$	s_m: unit cost of lost sales for the producer
$G(u)$: the CDF of ROF u	exogenous price of final products of the SC
μ_1:the mean of the supplier's ROF u	d: random demand of customers
c_s: unit production cost of the supplier	$f(x)$: PDF of d
h_s: unit salvage of unsold products of the supplier	$F(x)$: CDF of d
ω: unit wholesale price of raw materials	μ_d: the mean of random demand d;
c_d: exogenous price of raw material in the spot maket, $c_p > \omega$)	L: the reward demand function $L(y) = K\sqrt{y}$
Π_s: profit function of the supplier	D: the total demand $D = d + L$;
Other notations	K: the CSREF of the producer
T: transfer payment from producer to supplier	Π_m: profit function of the producer
$\Omega(\alpha_1 \cdots \alpha_i \cdots \alpha_n)$: transaction contract between two members of the SC	Superscripts and subscripts
, where is α_i the ith key parameter of Ω;	c: mark of the centralized SC;
Π_c: profit function of the centralized SC	o: mark of the SC without CSR
$E(\cdot)$: operator for mathematical expectation;	w: mark of the decentralized SC under wholesale price contract;
$[]_+$: operator for $[z]+ = Max(z, 0)$	r: mark of the decentralized SC under RS contract
z: the ratio of order quantity to production input quantity for raw material, $z = R/I$	n: mark of the decentralized SC under revenue and CSR sharing contract; *: optimal value mark of related variables

So, let the acceptable ω_d be $0.7c_p = 0.84$.

Further, we can find $\phi = 0.45$ within the range [0, 0.472] and related RCSRS contract ($\omega = 0.016, \eta = 0.541$) so that $E(\pi_s^n) > E(\pi_s^w)$, *i.e.*, 53.88.14 > 52.35, and $E(\pi_m^n) > E(\pi_m^w)$ *i.e.*, 62.40 > 55.85, for NE 3. The EP of every member under channel coordination with RCSRS contract is higher than that under the WP contract so that the "win-win" for two members of the SC with CSR is realized.

Clearly, using or devising a SC contract able to coordinate the SC such as the RCSRS contract proposed in this paper is beneficial to the SC members and related stakeholders (improving the profits and CSR investment).

5.3. Sensitivity analysis of yield uncertainty and CSRE

In the SC with CSR addressed here, the ROF of the supplier μ_1 and the exogenous price of raw material in the spot market c_p together prescribe the effect of uncertain yield. From equations (7) and (28), we have $\partial E(\pi_c)/\partial c_p = S(I, R) - R = -I \cdot \int_A^Z G(u)du < 0$. Clearly, $E(\pi_c)$ and $E(\pi_s^n)$ decrease with c_p. However, the

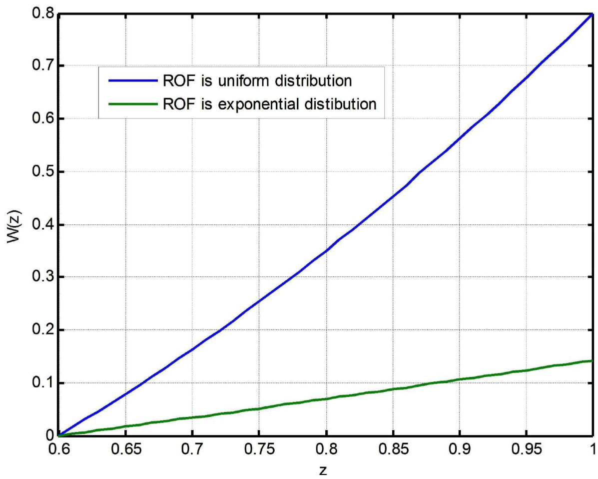

FIGURE 2. Expected profits and CSR investment changing with the mean of ROF.

TABLE 2. The parameters and demands of three numerical examples.

Parameters	c_s	h_s	c_p	μ_1	p	c_m	h_m	s_m	μ_d	Variance
NE1	0.2	0.1	1.35	0.8	1.8	0.4	0.35	0.1	100	20^2
NE2	0.3	0.1	1.2	0.8	2.0	0.4	0.3	0.05	100	100^2
NE3	0.3	0.2	1.2	1.0	2.0	0.4	0.3	0.1	100	$100^2/12$

TABLE 3. The EPs and decision variables for centralized SCs.

	NE	$E(\pi_c)$	I_c	R_c	y_c	$L(y)$	μ_d	z^*
1	No CSR	101.80	174.71	115.47	0	0	100	0.6609
	With CSR	106.72	188.15	124.35	4.93	8.88	108.88	0.6609
2	No CSR	53.33	164.84	118.87	0	0	100	0.7211
	With CSR	58.78	177.79	128.21	5.45	9.34	109.34	0.7211
3	No CSR	109.89	142.19	125.78	0	0	100	0.8846
	With CSR	116.28	153.62	135.89	6.39	10.11	109.38	0.8846

effect of yield uncertainty and CSREF on the EPs of every member are not so straightforward as c_p. Therefore, we perform sensitivity analysis and discuss the effect of yield uncertainty and CSREF on the EPs of every member and the SC in NEs 1 and 3 under channel oordination by RCSRS contract under $\phi = 0.3$.

Firstly, we keep the parameters of NE 1 and NE 3 unchanged except for the mean of supplier's ROF μ_1. The alteration of μ_1 is realized by changing the values of A from 0.1 to 0.9 and keeping $B = 1.0$ in NE 1. For NE 3, the alteration of μ_1 is realized by changing μ_1 from 0.85 to 1.2 directly while keeping A and B unchanged. The expected profits and CSR investment *versus* μ_1 in NE 1 and NE 3 are presented in Figures 3(a) and 3(b), respectively. It is found that the EPs of the supplier, the producer and the SC all increase with the mean of ROF in Figure 3. The reason may be that the EP of the SC increases with the mean of yield by equation (7). According to the y_c shown in Figure 3, we find that the optimal CSR investment y_c or the difference between $E(\pi_c)$ and $E(\pi_c^o)$ increases with μ_1. This observation is consistent with Corollaries 2 and 3.

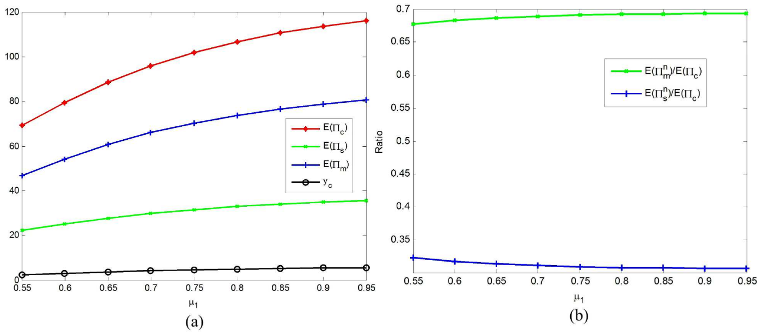

FIGURE 3. The function of W(z) *versus* z.

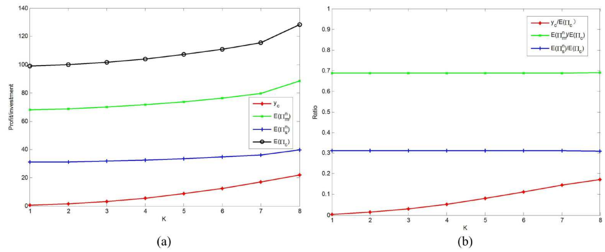

FIGURE 4. The structure of the two-stage SC with CS.

From previous observations from NE 1 and NE 3, the following managerial insights are revealed: (1) Increasing the expected yield of the supplier is beneficial to improve the EPs of the SC and all members under channel coordination by RCSRS contract; (2) Increasing the expected yield of the supplier is also beneficial to the stakeholders of the SC (the optimal CSR investment increases); (3) Compared with the EP of the SC without CSR, the EP of the socially responsible SC increases faster as μ_1 increases. Lastly, the effect of CSREF K on the EPs and CSR investment for NE 3 is investigated. From equation (10) in Proposition 4 or equation (13) in Theorem 4, it is clear that the difference between $E(\pi_c)$ and $E(\pi_c^o)$ or the optimal CSR investment increases with K. This can be observed in Figure 4(a). From the EPs shown in Figure 4(a), it can be seen that the EPs of the SC and every member all increase with K.

According to the ratio of CSR investment to $E(\pi_c)$ in Figure 4(b), it can be seen that y_c increases faster than $E(\pi_s^n)$ and $E(\pi_m^n)$. In addition, the ratio of $E(\pi_m^n)$ to slightly increases with K while the ratio of to slightly decreases with K. Clearly, increasing K is beneficial to the SC and related stakeholders. Observing the effect of changing K, we have the following managerial insights: (4) The producer is more beneficial than the supplier in the SC setting when increasing the CSREF K. (5) Increasing the producer's CSREF by the CSR effort such as careful selection and lean management of CSR activities is an effective way to enhance the SC members' expected profit and the stakeholders' welfare, *e.g.*, CSR investment, for the socially responsible SC.

6. CONCLUSION AND FUTURE WORK

In this paper, we tackle the coordination problem of a two-stage socially responsible SC comprising a supplier and a producer in a single selling season under random yield and CSR-related random demand. As did in existing papers, the SPY $Q = \epsilon X$ is used to model random yield of the supplier, and the total demand $D = d + L(y)$ is the additive-form that is the sum of reward demand from CSR and random demand unrelated to the CSR.

TABLE 4. The EPs and decision variables for decentralized SCs.

Scenario		Ω	I	R	y	z^*	$E(\pi_s)$	$E(\pi_m)$	$E(\pi_s+\pi_m)$
NE1 no CSR	WP	1.34	113.24	74.84	0	0.6609	78.45	0.54	78.99
	RS	0.09	174.71	115.47	0	0.6609	31.38	70.42	101.80
NE1 with CSR	WP	1.32	115.99	76.66	0.025	0.6609	79.03	1.87	80.90
	RS	0.09	184.04	115.47	2.37	0.6609	33.47	72.79	106.26
	RCSRS	0.023	188.15	124.35	4.93	0.6609	43.84	62.88	106.72
NE2 no CSR	WP	0.963	69.14	49.86	0	0.7211	26.42	10.71	37.13
	RS	0.032	164.84	118.87	0	0.7211	27.76	25.56	53.33
NE2 with CSR	WP	0.968	75.43	54.39	1.595	0.7211	29.12	13.88	43.00
	RS	0.032	171.36	123.57	1.38	0.7211	30.48	26.94	57.43
	RCSRS	0.032	177.79	128.21	5.45	0.7211	30.47	28.31	58.78
NE3 with CSR	WP	0.84	117.40	103.86	2.31	0.8846	52.35	55.85	108.20
	RS	0.016	149.19	131.97	2.40	0.8846	46.06	69.26	115.32
	RCSRS	0.016	153.62	135.89	6.39	0.8846	53.88	62.40	116.28

Note: For NE 3 under the WP contract, we have $\omega_d = 1.4679 > c_p = 1.2$ by solving equation (21).

For the SC under consideration, firstly the unimodality of the EP with respect to I and R for the centralized SC without CSR is proved, and then the unimodality of the EP with respect to I, R and y for the centralized SC with CSR is established. We find that the unimodality of the EP of the SC with CSR is regardless of stochastic distribution forms of the ROF and random demand. Moreover, the property of ratio of order quantity to production input quantity $z = R/I$ is disclosed. The value of optimal z is fixed and is not relevant to the producer's parameters for the centralized SCs. Through rigorous mathematical deduction and numerical verification, we have the following findings on the centralized SCs: (1) The optimal decisions of I and R for the SC with CSR are larger than those for the SC without CSR; (2) The optimal EP of the SC with CSR is larger than that of the SC without CSR, and the difference between $E(\pi_c)$ and $E(\pi_c^o)$ equals the optimal CSR investment y_c; (3) The optimal CSR investment y_c and the difference between $E(\pi_c)$ and $E(\pi_c^o)$ increases with the mean of supplier's ROF and the CSREF K. Those findings highlight the benefits of undertaking CSR.

Then, the Nash equilibrium solutions of the non-coordinated decentralized SCs under the WP contract and the RS contract are analyzed. We show that the WP, the RS and the modified RS contracts do not coordinate the SC with CSR. Those findings are different from the conclusions that the RS contracts [14, 29] coordinate the SC with CSR under random demand, and the modified RS contract coordinate the SC with CSR under random demand [36, 46, 47]. But we find that the RS contract coordinates the SC without CSR under random yield and demand, as observed in [10]. Lastly, we proposed a RCSRS contract to coordinate the SC. The RCSRS contract is the extension of standard RS contract by incorporating the sharing mechanism of the producer's CSR investment. The coordination effect of the RCSRS contract is proved and the relationship among its parameters is presented. The theoretical analysis is validated and illustrated by a real case and three numerical examples. Through theoretical and computational analysis, we have the following important observations for the SC on channel coordination: (1) increasing expected yield is an effective way to improve the profits of SC and its members, and the welfare of stakeholders; (2) increasing CSR effect factor K is beneficial to improve the EPs of the SC and all members, and is also beneficial to the stakeholders of the SC; moreover, the resulting benefit of the producer is more than that of the supplier; (3) using a SC contract able to coordinate the SC such as the RCSRS contract proposed in this paper is not only beneficial to improve the EPs of SC and its members, but also beneficial to improve the welfare of the stakeholders.

Based on those observations, the following managerial insights for managing the decentralized SCs with CSR under random yield and demand are identified. On one hand, the SC manager should find ways to incentive the supplier with random yield to improve the yield, e.g, providing the supplier with technique service or training, in order to reduce the yield uncertainty. On the other hand, the SC manager should implement lean management of CSR activities to improve the CSREF so that all of the stakeholders are benefited.

In this paper, we are concentrated on the coordination problem of a two-stage socially responsible SC facing random yield, and price-independent and CSR-dependent random demand. In practice, the demand of many products is price-dependent random demand. So, the producer's price decision should be considered in future. Furthermore, the SC coordination problem under generic random yields except for SPY needs to be addressed in future. In addition, other forms of CSR-related random demand except for $D = d + L(y)$ should be investigated. Considering the off-line and online channels for the SC with CSR is another interesting research direction.

Appendix A. The EP of the SC without CSR

Let $f_1 = p \cdot Min(R, D) + h_m[R-D]^+ - s_m[D-R]^+ - c_m R$ and $f_2 = h_s[uI-R]^+ - c_D[R-uI]^+ - c_s I$. Clearly, $\pi_c(I,R) = f_1 + f_2$. Note that $S(R) = E[\min(R,D)] = R - \int_0^R F(x)dx$, $E([R-D]^+) = \int_0^R (R-x)f(x)dx = \int_0^R F(x)dx$. Then, we have the following equations.

$$E([R-D]^+) = R - E[\min(R,D)] \tag{A.1}$$

$$E([D-R]^+) = \mu_1 - E[\min(R,D)]. \tag{A.2}$$

Therefore, we have the following equation.

$$E[f_1] = (p - h_m + s_m)S(R) - (c_m - h_m)R - s_m\mu_d \tag{A.3}$$

Note that $S(I,R) = E[\min(uI,R)] = E[I\min(u,R/I)] = I(R/I - \int_A^{R/I} G(u)du) = R - I\int_A^{R/I} G(u)du$. Then, we have the following equations.

$$E([uI-R]^+) = E(uI) - S(I,R) = \mu I - S(I,R) \tag{A.4}$$

$$E([R-uI]^+) = E(R) - S(I,R) = R - S(I,R). \tag{A.5}$$

Therefore, $E(f_2) = h_s[\mu_1 I - S(I,R)] - c_s I - c_p[R - S(I,R)]$. $E(f_2)$ can be rewritten as follows.

$$E(f_2) = (c_p - h_s)S(I,R) - (c_s - h_s\mu_1)I - c_p R. \tag{A.6}$$

Adding (A.3) to (A.6), we have equation (4) in Section 3.3.

Appendix B. Proof of Propositions

B.1. Proof of Proposition 1

Note that $S(I,R) = R - I \cdot \int_A^{R/I} G(u)du$ and $W(z) = \partial S(I,R)/\partial I$. Then, we have $W(z) = zG(z) - \int_A^z G(u)du$. Because of $z = R/I > 0$ and $g(z) > 0$, we know $\partial W(z)/\partial z = zg(z) >=$. Owing to $z > 0$ and, $W(z)$ is an increasing function and monotone in the domain $z \in [A,B]$. Therefore, there is only one $z \in [A,B]$ satisfying $W(z) = K$, where K is a constant and equals $\partial S(I,R)/\partial I$.

B.2. Proof of Proposition 2

B.3. Proof of Proposition 3

From the first-order condition, the optimal decision y is the solution of simultaneous equations $\partial E(\pi_c)/\partial y = 0$, $\partial E(\pi_c)/\partial I = 0$ and $\partial E(\pi_c)/\partial R = 0$. From equation (7), we have $\partial E(\pi_c)/\partial y = 0.5K[(p-h_m+s_m)F(R-L(y)) - s_m]/\sqrt{y} - 1$. Based on $\partial E(\pi_c)/\partial y = 0$, we have $y = K^2[(p - h_m + s_m)F(R - L(y)) - s_m]^2/4$. Substituting $F(R - L(y))$ in equation (8b) into the right-hand side of y, we have equation (10). Note that z^* is unique, as stated in Corollary 1.

B.4. Proof of Proposition 4

First, we know that the values of z^* in equations (5a) and (8a) are the same. Comparing equation (8b) with equation (5a), we know that $R_c = R_c^o + L(y_c) = R_c^o + K\sqrt{y}$ because $F(x)$ is monotone. Because of $I = R/z^*$, we have $I_c = R_c/z^* = (R_c^o + L(y_c))/z^*$ and $I_c^o = R_c^o/z^*$. Therefore, we have equation (12a). Due to $y \geq 0$, it is clear the optimal decision $I_c(R_c)$ is larger than $I_c^o(R_c^o)$.

B.5. Proof of Proposition 5

According to equation (14), we have $\partial E(\pi_s^w)/\partial I = (c_p - h_s)\partial S(I,R)/\partial I - (c_s - h_s\mu_1)$ and $\partial^2 E(\pi_s^w)/\partial I^2 = -(c_p - h_s)g(R/I)R^2/I^3$. Note that $c_p - h_s > 0$ and $g(R/I) > 0$. So, we have $\partial^2 E(\pi_s^w)/\partial I^2 < 0$. Therefore, $E(\pi_s^w)$ is concave with respect to I. Thus, given R, there exists sole maximizing. Note that $z = R/I$ and $W(z) = \partial S(I,R)/\partial I = (c_s - h_s\mu_1)/(c_p - h_s)$. From first-order condition $\partial E(\pi_s^w)/\partial I = 0$, we know that the optimal z_w^* maximizing $E(\pi_s^w)$ designated by equation (17) is unique. Note that the formulation of $E(\pi_{s,o}^w)$ is identical with $E(\pi_s^d)$. So, equation (17) holds for $E(\pi_{s,o}^w)$.

B.6. Proof of Proposition 6

Find the first and second derivatives of equation (16) with respect to R, and then we have $\partial E(\pi_{m,o}^w)/\partial R = (p - h_m + s_m)[1 - F(R)] - (c_m - h_m + \omega)$ and $\partial^2 E(\pi_{m,o}^w)/\partial R^2 = -(p - h_m + s_m)f(R) < 0$. Thus, $E(\pi_{m,o}^w)$ is concave with respect to R. Given ω, there exists a sole R_w^{o*} maximizing $E(\pi_m^w)$. From the first order condition $\partial E(\pi_{m,o}^w)/\partial R = 0$, we have equation (18).

Appendix C. Proof of Corollaries

C.1. Proof of Corollary 1

From equations (5a) and (8a), we have equation (9) for the centralized SC. Since c_p is exogenous price of raw material in the spot market and the cost parameters of supplier are fixed, we know that z^* is unique in light of Proposition 1. Note that $W(z) = zG(z) - \int_Z^z G(u)du$. From equation (9), it is clear that the z^* is only related to the supplier's ROF $(\mu_1, G(z))$ and cost parameters c_s and h_s, and c_p.

C.2. Proof of Corollary 2

From equation (10), we know that y_c is related to $G(z^*)$. Because $W(z^*) = (c_s - h_s\mu_1)/(c_p - h_s)$ in equation (9) and $W(z)$ is an increasing function for $z \in [A, B]$, we know that z^* decreases with μ_1. Since $G(z^*)$ increases with z^*, $G(z^*)$ decreases with μ_1. Because of, due to and $G(z^*)$ decreasing with μ_1, increases with μ_1. Therefore, y_c increases with μ_1 by equation (10).

C.3. Proof of Corollary 3

C.4. Proof of Corollary 4

Under the modified RS contract, the EP of the manufacturer $(E(\pi_m'))$ can be written as $E(\pi_m') = [(1-\phi)(p - h_m) + s_m]S(R) - [c_m - (1-\phi)h_m + \omega]R - s_m(\mu_d + K\sqrt{y}) - (1-\phi)y$. Comparing this EP with that in equation (29), we know that $(1-\phi)$ should equal η to coordinate the SC. However, we know $\eta = (1-\phi) + \frac{\phi s_m(h_m - c_m - ks)}{(p - c_m - ks)(p - h_m + s_m)} \neq 1 - \phi$ from equation (31b) in Theorem 6.

Appendix D. Proof of Theorems

D.1. Proof of Theorem 1

From equation (4), we have the following derivatives: $\frac{\partial E(\pi_c^o}{\partial I} = (c_p - h_s)\frac{\partial S(I,R)}{\partial I} - (c_s - h_s\mu_1)$, $\frac{\partial E(\pi_c^o)}{\partial R} = (p - h_m + s_m)\frac{\partial S(R)}{\partial R} + (c_p - h_s)\frac{\partial S(I,R)}{\partial R} - (c_m + c_p - h_m)$, $\frac{\partial^2 E(\pi_c^o)}{\partial I^2} = -(c_p - h_s)g(\frac{R}{I})\frac{R^2}{I^3} < 0$, $\frac{\partial^2 E(\pi_c^o)}{\partial I \partial R} = -(c_p - h_s)g(\frac{R}{I})\frac{R}{I^2}$,

$\frac{\partial^2 E(\pi_c^o)}{\partial R^2} = \frac{\partial^2[(p-h_m+s_m)S(R)]}{\partial R^2} + \frac{\partial^2(c_p-h_s)S(I,R)}{\partial R^2}$. The first term of $\frac{\partial^2 E(\pi_c^o)}{\partial R^2}$ is $(p-h_m+s_m)\frac{\partial^2 S(R)}{\partial R^2} = -(p-h_m+s_m)f(R) < 0$ because of $(p-h_m+s_m) > 0$ (refer to assumption). Due to $S(I,R) = R - I\int_A^{R/I} G(u)du$, the second term of $\frac{\partial^2 E(\pi_c^o)}{\partial R^2}$ can be written as $\frac{\partial^2(c_p-h_s)S(I,R)}{\partial R^2} = -(c_p-h_s)\frac{1}{I}g(\frac{R}{I}) < 0$. Thus, we have $\frac{\partial^2 E(\pi_c^o)}{\partial R^2} = -(p-h_m+s_m)f(R) - (c_p-h_s)g(\frac{R}{I})/I < 0$.

Then, we have the following Hessian matrix $H(I,R) = \begin{pmatrix} \frac{\partial^2 E(\pi_c^o)}{\partial I^2} & \frac{\partial^2 E(\pi_c^o)}{\partial I \partial R} \\ \frac{\partial^2 E(\pi_c^o)}{\partial R \partial I} & \frac{\partial^2 E(\pi_c^o)}{\partial R^2} \end{pmatrix}$, where $H_1(I,R) = \frac{\partial^2 E(\pi_c^o)}{\partial I^2} < 0$, $H_3(I,R) = \frac{\partial^2 E(\pi_c^o)}{\partial R^2} < 0$, $H_2(I,R) = \frac{\partial^2 E(\pi_c^o)}{\partial I^2}\frac{\partial^2 E(\pi_c^o)}{\partial R^2} - (\frac{\partial^2 E(\pi_c^o)}{\partial I \partial R}^2 = (p-h_m+s_m)(c_p-h_s)\frac{R^2}{I^3}f(R)G(\frac{R}{I})/I < 0$. So, the Hessian matrix is negative.

Then, we know that $E(\pi_c^o)$ is concave for I and R. Since $E(\pi_c^o)$ is concave for I and R, there must exist only one decision pair (I_c^o, R_c^o) maximizing $E(\pi_c^o(I,R))$.

D.2. Proof of Theorem 2

From the first-order condition, the optimal decisions (I_c^o, R_c^o) are the solutions of $\partial E(\pi_c^o/\partial I = 0$ and $\partial E(\pi_c^o/\partial R = 0$. From equation (4), we have $\partial E(\pi_c^o/\partial I = (c_n - h_s)\partial S(I,R)/\partial I - (c_s - h_s\mu_1)$ and $\partial E(\pi_c^o/\partial R = (p-h_m+s_m)\partial S(R)/\partial R + (c_n - h_s)\partial S(I,R)/\partial R - (c_m + c_p - h_m)$. From Proposition 1, we know that $W(z^*) = \partial S(I_c, R_c)/\partial I$, where $z^* = R_c^o/I_c^o$. Because of $S(I,R) = R - I\cdot\int_A^{R/I} G(u)du$, we have $\partial S(I_c, R_c)/\partial R = 1 - G(z^*)$ and then equation (5a). Note that $\partial S(R)/\partial R = 1 - F(R)$ and $\partial S(I,R)/\partial R = 1 - G(z)$. We have $(p-h_m+s_m)(1-F(R)) + (c_p-h_s)[1-G(z^*)] = (c_m + c_p - h_m)$, which can be reduced to equation (5b). Then, the optimal production input $I_c^o = R_c^o/z^*$.

D.3. Proof of Theorem 3

From equation (8a) in Proposition 2 and Corollary 1, we know the optimal z^* is unique. According to Proposition 3, the optimal y is unique. Note that the CDF of random demand $dF(x)$ is an increasing function and monotone. Since the term of right-hand side in equation (8b) is fixed, $R - L(y)$ is unique. Owing to the uniqueness of y, the optimal order quantity R is also unique. Because z^* is unique and the optimal I equals R/z^*, the optimal production input I_c of the supplier is unique.

D.4. Proof of Theorem 4

Note that equations (6) and (11). Then, we have the difference $E(\pi_c) - E(\pi_c^o) = (p-h_m+s_m)[S(R_c) - S(R_c^o)] - [c_m - h_m + ks](R_c - R_c^o) - s_m K^2[p - c_m - ks]/2 - K^2[p - c_m - ks]^2/4$. Note that $S(R_c) = R_c - \int_0^{R_c - L(y)} F(x)dx = R_c - \int_0^{R_c^o} F(x)dx$ and $S(R_c^o) = R_c^o - \int_0^{R_c^o} F(x)dx$. So, $S(R_c) - S(R_c^o) = R_c - R_c^o$. Because of $R_c = R_c^o + K\sqrt{y}$ in equation (12b), we have $R_c - R_c^o = K^2[p - c_m - ks]/2$. Then, $E(\pi_c) - E(\pi_c^o) = (s_m + p - c_m - ks)K^2[p - c_m - ks]/2 - s_m K^2[p - c_m - ks]/2 - K^2[p - c_m - ks]^2/4$. Therefore, we have equation (13) by reduction.

D.5. Proof of Theorem 5

Replacing the ω in equation (25) by the ω shown in equation (26), we have $F(R_{r,o}^*) = [p + s_m - c_m - h_s - (c_p - h_s)G(z^*)]/(p - h_m + s_m)$. Clearly, $R_{r,o}^*$ is identical with the R_c^o shown in equation (5b) for centralized SC without CSR. Note that $z_r^* = z^*$ and $I_{r,o}^* = R_{r,o}^*/z_r^*$. Therefore, $R_{r,o}^*$ and $I_{r,o}^*$ equal R_c^o and I_c^o, respectively. So, the RS contract with the parameters shown in equation (26) can coordinate the SC. Owing to $\omega > 0$ and $\omega < c_p$, we have. Replacing the ω in equation (24) by the ω shown in equation (26), we have. Note that in equation (7) and. Then, we have $\frac{(p-h_m+s_m)\max(0, ks-c_p)}{p(p-h_m+s_m)-(p-h_m)(p+s_m-c_m-ks)} < \phi < \frac{(p-h_m+s_m)ks}{p(p-h_m+s_m)-(p-h_m)(p+s_m-c_m-ks)}$. Replacing the ω in equation (24) by the ω shown in equation (26), we have $E(\pi_{m,o}^r) = [(1-\phi)(p-h_m) + s_m]S(R) - [c_m - (1-\phi)h_m + \omega]R - s_m\mu_d$. Note that $E(\pi_c^o) = (p - h_m + s_m)S(R) - [c_m - hm + ks]R - s_m\mu_d$ in equation (7) and $\lambda = \frac{(1-\phi)(p-h_m)+s_m}{p-h_m+s_m}$. Then, we have $E(\pi_m^s) = \lambda E(\pi_c^o) - (1-\lambda)s_m\mu_d$.

D.6. Proof of Theorem 6

Replacing the ω in equation (27b) by the ω shown in equation (26), we have the following optimal CSR investment $y_r^* = \frac{K^2}{4}[p - c_m - h_s - (c_p - h_s)G(z^*)\frac{(1-\phi)(p-h_m)+s_m}{p-h_m+s_m}]^2$. Only if $\phi = 0$, $y_r^* = \frac{K^2}{4}[p - c_m - h_s - (c_p - h_s)G(z^*)]^2$ and $y_r^* = y_c$ (see Eq. (10)). However, we know $\phi = 0$ from Theorem 4, which contradicts with $\phi = 0$.

D.7. Proof of Theorem 7

Replacing the ω and η in equation (30b) by those shown in equation (31), we have $y_n^* = \frac{K^2}{4}[p - c_m - h_s - (c_p - h_s)G(z^*)]^2$, which is the same with the y_c shown in equation (10). Replacing the ω in equation (30a) by that shown in equation (31a), we have $R_n^* = F^{-1}[(p + s_m - c_m - ks)/(p - h_m + s_m)] + K\sqrt{y_n^*} = F^{-1}[\frac{p+s_m-c_m-ks}{p-h_m+s_m}] + \frac{K^2}{2}(p - c_m - ks)$. From equations (8b) and (10), we have $R_c = F^{-1}[\frac{p+s_m-c_m-ks}{p-h_m+s_m}] + \frac{K^2}{2}(p - c_m - ks)$ for centralized SC with CSR. Note that $z_n^* = z^*$ and $I_n^* = R_n^*/z^*$. Thus, R_n^* and I_n^* equal R_c and I_c, respectively, and $y_n^* = y_c$. Therefore, the proposed contract with the parameters shown in equation (31) can coordinate the SC. Owing to $\omega > 0$ and $\omega < c_p$, we have $\frac{(p-h_m+s_m)\max(0,ks-c_p)}{p(p-h_m+s_m)-(p-h_m)(p+s_m-c_m-ks)} < \phi < \frac{(p-h_m+s_m)ks}{p(p-h_m+s_m)-(p-h_m)(p+s_m-c_m-ks)}$.

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References

[1] E. Anderson and M. Monjardino, Contract design in agriculture supply chains with random yield. *Eur. J. Oper. Res.* **277** (2019) 1072–1082.

[2] T. Avinadav, The effect of decision rights allocation on a supply chain of perishable products under a revenue-sharing contract. *Int. J. Prod. Econ.* **225** (2020) 107587.

[3] N. Bart, T. Chernonog and T. Avinadav, Revenue-sharing contracts in supply chains: A comprehensive literature review. *Int. J. Prod. Res.* **59** (2021) 6633–6658.

[4] G.P. Cachon, Supply chain coordination with contracts. In Vol. 11 *Handbooks in operations research and management science*. Elsevier (2003) 227–339.

[5] G.P. Cachon and M.A. Lariviere, Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Science* **51** (2005) 30–44.

[6] K. Chen and L. Yang, Random yield and coordination mechanisms of a supply chain with emergency backup sourcing. *Int. J. Prod. Research* **52** (2014) 4747–4767.

[7] M.G. Güler and T. Bilgiç, On coordinating an assembly system under random yield and random demand. *Eur. J. Oper. Res.* **196** (2009) 342–350.

[8] B.C. Giri, S. Bardhan and T. Maiti, Coordinating a three-layer supply chain with uncertain demand and random yield. *Int. J. Prod. Res.* **54** (2016) 2499–2518.

[9] G.E. Goering, Corporate social responsibility and marketing channel coordination. *Res. Econ.* **66** (2012) 142–148.

[10] M.G. Guler and M.E. Keskin, On coordination under random yield and random demand. *Expert Syst. Appl.* **40** (2013) 3688–3695.

[11] H. Gurnani and Y. Gerchak, Coordination in decentralized assembly systems with uncertain component yields. *Eur. J. Oper. Res.* **176** (2007) 1559–1576.

[12] Y. He and X. Zhao, Coordination in multi-echelon supply chain under supply and demand uncertainty. *Int. J. Prod. Econ.* **139** (2012) 106–115.

[13] J. Heydari and P. Rafiei, Integration of environmental and social responsibilities in managing supply chains: A mathematical modeling approach. *Comput. Ind. Eng.* **145** (2020) 106495.

[14] C.F. Hsueh, Improving corporate social responsibility in a supply chain through a new revenue sharing contract. *Int. J. Prod. Econ.* **151** (2014) 214–222.

[15] C.F. Hsueh, A bilevel programming model for corporate social responsibility collaboration in sustainable supply chain management. *Transp. Res. Part E: Logist. Transp. Rev.* **73** (2015) 84–95.

[16] C.F. Hsueh and M.-S. Chang, Equilibrium analysis and corporate social responsibility for supply chain integration. *Eur. J. Oper. Res.* **190** (2008) 116–129.

[17] B. Hu and Y. Feng, Optimization and coordination of supply chain with revenue sharing contracts and service requirement under supply and demand uncertainty. *Int. J. Prod. Econ.* **183** (2017) 185–193.

[18] F. Hu, C. Lim and Z. Lu, Coordination of supply chains with a flexible ordering policy under yield and demand uncertainty. *Int. J. Prod. Econ.* **146** (2013) 686–693.

[19] K. Inderfurth and J. Clemens, Supply chain coordination by risk sharing contracts under random production yield and deterministic demand. *OR Spectr.* **36** (2014) 525–556.

[20] A. Jokar and S. Hosseinimotlagh, Simultaneous coordination of order quantity and corporate social responsibility in a two-echelon supply chain: A combined contract approach. *J. Oper. Res. Soc.* **71** (2020) 69–84.

[21] M. Lerro, F. Caracciolo, R. Vecchio and L. Cembalo, Consumer's side of corporate social responsibility: A nonhypothetical study. *J. Consum. Aff.* **52** (2018) 689–710.

[22] Y. Liu, B.T. Quan, J. Li and J.Y.-L. Forrest, A supply chain coordination mechanism with cost sharing of corporate social responsibility. *Sustainability* **10** (2018) 1227–1252.

[23] Y. Liu, Q. Xu and Z.y. Liu, A coordination mechanism through value-added profit distribution in a supply chain considering corporate social responsibility. *Manage. Decis. Econ.* **41** (2020) 586–598.

[24] J. Luo and X. Chen, Coordination of random yield supply chains with improved revenue sharing contracts. *Eur. J. Ind. Eng.* **10** (2016) 81–102.

[25] X. Ma, S. Wang, S.M. Islam and X. Liu, Coordinating a three-echelon fresh agricultural products supply chain considering freshness-keeping effort with asymmetric information – sciencedirect. *Appl. Math. Model.* **67** (2019) 337–356.

[26] N.M. Modak, S. Panda, S.S. Sana and M. Basu, Corporate social responsibility, coordination and profit distribution in a dual-channel supply chain. *Pac. Sci. Rev.* **16** (2014) 235–249.

[27] N.M. Modak, S. Panda and S.S. Sana, Pricing policy and coordination for a two-layer supply chain of duopolistic retailers and socially responsible manufacturer. *Int. J. Logist. Res. Appl.* (2015) 1–22.

[28] N.M. Modak, N. Kazemi and L.E. Cárdenas-Barrón, Investigating structure of a two-echelon closed-loop supply chain using social work donation as a corporate social responsibility practice. *Int. J. Prod. Econ.* **207** (2019) 19–33.

[29] M. Nematollahi, S.M. Hosseini-Motlagh and J. Heydari, Coordination of social responsibility and order quantity in a two-echelon supply chain: A collaborative decision-making perspective. *Int. J. Prod. Econ.* **184** (2017) 107–121.

[30] D. Ni and K.W. Li, A game-theoretic analysis of social responsibility conduct in two-echelon supply chains. *Int. J. Prod. Econ.* **138** (2012) 303–313.

[31] S. Panda, Coordination of a socially responsible supply chain using revenue sharing contract. *Transp. Res. Part E Logist. Transp. Rev.* **67** (2014) 92–104.

[32] S. Panda, N. Modak, M. Basu and S. Goyal, Channel coordination and profit distribution in a social responsible three-layer supply chain. *Int. J. Prod. Econ.* **168** (2015) 224–233.

[33] S. Panda, N. Modak and D. Pradhan, Corporate social responsibility, channel coordination and profit division in a two-echelon supply chain. *Int. J. Manag. Sci. Eng. Manag.* **11** (2016) 22–33.

[34] S. Panda, N.M. Modak and L.E. Cárdenas-Barrón, Coordinating a socially responsible closed-loop supply chain with product recycling. *Int. J. Prod. Econ.* **188** (2017) 11–21.

[35] E. Park, K.J. Kim and J.K. Sang, Corporate social responsibility as a determinant of consumer loyalty: An examination of ethical standard, satisfaction, and trust. *J. Bus. Res.* **76** (2017) 8–13.

[36] S.A. Raza, Supply chain coordination under a revenue-sharing contract with corporate social responsibility and partial demand information. *Int. J. Prod. Econ.* **205** (2018) 1–14.

[37] S. Wang, Y.K. Liao, W.Y. Wu and K.B.H. Le, The role of corporate social responsibility perceptions in brand equity, brand credibility, brand reputation, and purchase intentions. *Sustainability* **13** (2021) 11975.

[38] C.H. Wu, Collaboration and sharing mechanisms in improving corporate social responsibility. *Cent. Eur. J. Oper. Res.* **24** (2016) 681–707.

[39] J. Xie, W. Zhang, L. Liang, Y. Xia, J. Yin and G. Yang, The revenue and cost sharing contract of pricing and servicing policies in a dual-channel closed-loop supply chain. *J. Clean. Prod.* **191** (2018) 361–383.

[40] L. Xie, J. Ma and M. Goh, Supply chain coordination in the presence of uncertain yield and demand. *Int. J. Prod. Res.* (2020) 1–17.

[41] B. Xin, L. Zhang and L. Xie, Pricing decision of a dual-channel supply chain with different payment, corporate social responsibility and service level. *RAIRO:RO* **56** (2022) 49–75.

[42] H. Yang and J. Peng, Coordinating a fresh-product supply chain with demand information updating: Hema fresh o2o platform. *RAIRO:RO* **55** (2021) 285–318.
[43] F. Ye, Q. Lin and Y. Li, Coordination for contract farming supply chain with stochastic yield and demand under cvar criterion. *Oper. Res.* **20** (2020) 369–397.
[44] C. Zhang, Y. Wang and L. Zhang, Risk-averse preferences in a dual-channel supply chain with trade credit and demand uncertainty. *RAIRO:RO* **55** (2020).
[45] X. Zhao and F. Wu, Coordination of agri-food chain with revenue-sharing contract under stochastic output and stochastic demand. *Asia-Pac. J. Oper. Res.* **28** (2011) 487–510.
[46] X. Zhao and R. Yin, Coordination of a socially responsible two-stage supply chain under price-dependent random demand. *4OR Q. J. Oper. Res.* **16** (2018) 379–400.
[47] X. Zhao, N. Li and L. Song, Coordination of a socially responsible two-stage supply chain under random demand. *Asia-Pac. J. Oper. Res.* **36** (2019) 1950029.