RAIRO-Oper. Res. 58 (2024) 4769–4790
https://doi.org/10.1051/ro/2024179

RAIRO Operations Research
www.rairo-ro.org

RESEARCH ON THE INFLUENCE OF SPILLOVER EFFECT ON DUAL-CHANNEL SUPPLY CHAIN UNDER THE BACKGROUND OF LIVE-STREAMING MARKETING

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Abstract. In the context of live streaming marketing, it is crucial for members of the supply chain to allocate resources not only to enhance product quality through quality improvement efforts but also towards marketing initiatives aimed at boosting sales. This paper focuses on a dual-channel supply chain comprising a brand vendor and a streamer and investigates the spillover effect of the live streaming channel on traditional channels and product goodwill. By analyzing the Stackelberg game and differential game, this study compares the optimal equilibrium strategy and equilibrium profit under two decentralized models: the brand vendor-dominated decentralized model and the streamer-dominated decentralized model, as well as the centralized decision-making model. Additionally, this paper presents the optimal strategy and profit function of the product goodwill trajectory for the brand vendor, streamer, and the entire supply chain, considering different spillover effects using numerical simulation. The analysis reveals that the level of quality improvement consistently increases over time, while the variation in product goodwill is more diverse. The optimal pricing of products is influenced by the market share of the channel with goods under the decision of the two decentralized models. Risk-averse brands tend to collaborate with waist and tail streamers to gain control over bargaining power, while risk-seeking brands often choose top streamers to distribute their goods when the market share of goods channels is significant. The overall profitability of the supply chain is influenced by consumers' preferences for quality and marketing.

Mathematics Subject Classification. 90B06, 90B60.

Received December 13, 2023. Accepted September 11, 2024.

1. Introduction

In the post-epidemic era, live broadcast e-commerce has witnessed significant growth worldwide [1,2]. According to the China Internet Network Information Center's report, as of December 2023, the number of online live broadcast users in China is projected to reach 816 million, marking an increase of 65.01 million compared to December 2022. This accounts for 74.7% of the total Internet users [3]. Coresight estimates that the live streaming market in the United States will reach 20 billion US dollars by 2022 and is expected to grow to 32 billion US dollars by 2023 [4]. Currently, Facebook and Instagram serve as the primary platforms for live streaming. The immense consumption potential and dividends offered by live streaming have led to a period of robust global

Keywords. Live-streaming marketing, spillover effect, goodwill, dual-channel supply chain.

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development in this industry. Major companies have ventured into the live streaming domain. For instance, in China, Taobao introduced a live streaming module in 2016, triggering an industry trend [5]. Subsequently, Douyin, JD.com, and Kuaishou joined the live streaming industry and intensified the competition. In the United States, TikTok has also expanded its live broadcast module to further capture the consumer market [6].

After recognizing the immense business opportunities presented by live streaming, brand owners have capitalized on this medium to attract consumers and expand their product live streaming channels for revenue generation [7,8]. In this regard, the selection of delivery streamers holds paramount importance for brand owners. A critical factor influencing live streaming delivery is the impact of the delivery streamers themselves [9]. The possession of private domain traffic by certain top streamers can significantly boost sales. However, in recent years, China's live streaming industry has witnessed the pronounced effects of the long-tail phenomenon and the Matthew effect. Simultaneously, instances of live streaming mishaps have persisted. Several celebrities have faced scrutiny regarding their product deliveries, and certain top streamers or key opinion leaders (KOLs) have occasionally violated regulations (*e.g.*, Wei Ya's high-profile tax evasion and Li Jiaqi's deceptive advertising) [10]. The spillover effect relies on the size of the follower base engaging with the streamer. When top streamers conduct live streams for product promotions, a negative spillover effect can occur, significantly impacting the brand's reputation associated with the live stream. In response, platforms and brand owners are actively supporting mid-tier and long-tail streamers. For instance, Taobao launched the "9 Major Support Plans" in 2021 to address the shortcomings of mid-tier and long-tail streamers, aiming to foster a diverse and vibrant live streaming ecosystem.

With the rapid growth of live streaming, the competition between traditional channels employed by brand owners and the emerging delivery channels has become increasingly pronounced. In the current stage, it is crucial to address the conflicts faced by brand owners and explore optimized and enhanced multi-channel sales strategies that account for the spillover effects of live streaming on traditional channels. For example, in a successful case, in Douyin, a well-known platform for live streaming in China, L'Oreal, on the one hand, selected talents to cooperate based on the target group, and finally reached a cooperation with 100 KOLs and KOCs to open up channels for live streaming; On the other hand, merchants opened up a full-day self-operated live broadcast room on the platform, and the GMV of new product payment hit a new high during the event. This paper aims to tackle this urgent issue by examining the spillover effect between channels and its impact on product goodwill. It establishes a dual-channel supply chain analysis model involving brand owners and streamers with goods, and analyzes the optimal equilibrium strategy and profit function under two decentralized models (one led by brand owners and the other dominated by streamers with goods) as well as a centralized decision-making model. The research focuses on scenarios where brand owners seek waist and tail streamers, head streamers, or key opinion leader (KOL) streamers for sales and brand self-broadcasting.

As a result, the innovation of this research and the main objective of the work is as follows:

(1) Incorporate the spillover effect as an important variable into the dual-channel model.
(2) Two decentralized decision-making models and centralized decision-making models for brand owners' live broadcast sales under different power organizations are proposed.
(3) Solving the optimal equilibrium decision of supply chain agent in three models and how to maximize the benefits.

Our analysis consists of seven sections. Section 1 describes research topics and background. Section 2 reviews relevant literature. Section 3 problem description and model building. Section 4 solve three decision-making models and compare them. Section 5 numerical simulation analysis of the model. Section 6 provides the conclusions, limitation, and future research of this study.

2. Literature review

Research in this field primarily focuses on examining the influence of live streaming on product goodwill, channel demand, and the spillover effect on pricing coordination within the supply chain.

Scholars in the field of live streaming predict that the industry will undergo a transformation into live streaming sales, which is increasingly becoming a popular marketing tool and is expected to be widely adopted worldwide [1, 2]. Wang *et al.* [11] proposed in their research that the influence of the delivery streamer plays a crucial role in the sales process within the live streaming sales model. Research conducted by Liu *et al.* [12] suggests that brand owners should identify the characteristics of streamers and consumers before selecting a live delivery strategy, and accordingly adjust the pricing and channel strategies for maximizing profits. Furthermore, consumers' willingness to purchase is a vital factor contributing to the success of live streaming. Lu *et al.* [13] employed signal theory and uncertainty theory to explore the factors influencing consumers' purchasing decisions, and proposed product routes and social routes as means to attract consumers to make purchases. Additionally, Tong *et al.* [14] discovered that factors affecting the live broadcast room environment, such as visual complexity and music rhythm, can enhance consumers' willingness to continue watching. These findings collectively underscore the substantial practical significance of research conducted in the field of live streaming.

When it comes to constructing a model for the issue of live streaming, scholars have identified goodwill as an important variable. The concept of goodwill was initially introduced by Nerlove [15], who suggested that advertising expenditure could establish goodwill, allowing it to function as a form of physical capital stock. Since then, the Nerlove–Arrow model has been expanded and applied to various fields. Nair [16] argues that the formation of goodwill is essentially a dynamic process, and the dynamic nature of quality-based competition can be examined by incorporating quality into the differential game model of goodwill formation. Pasin *et al.* [17] proposes that product quality is a significant factor influencing the accumulation of goodwill, with both quality enhancement and goodwill of duopoly companies playing a role in signaling market share. Consequently, the impact of brand goodwill has garnered increasing attention from major innovative companies, challenging the notion that the best price and quality level of innovative products are always positively correlated with the brand goodwill of innovative companies [18]. Subsequently, goodwill has gradually emerged as a key variable in the study of supply chain issues. Guan *et al.* [19] posits that the manufacturer's quality improvement and the retailer's advertising efforts positively affect consumer goodwill, and explores the optimal strategy for supply chain coordination.

Regarding the fundamental concept of spillover effects, Abhishek *et al.* [20] discovered that e-commerce channels can have either a positive or negative impact on traditional channels, indicating the existence of spillover effects in the dual-channel online and offline sales supply chain. Subsequently, numerous scholars have recognized the significance of spillover effects as a key variable and have incorporated it into inter-channel supply chain models. Initially, Perry *et al.* [21] examined service spillover effects and suggested that, under monopolistic competition, manufacturers incentivize homogeneous retailers to provide services by setting franchise fees and maintaining retail prices. As consumption channels continue to diversify, spillover effects influence the marketing strategy choices of manufacturers and retailers by affecting consumer acceptance [22]. In the current era of live streaming, which has gained considerable momentum, it has become an important sales channel with significant spillover effects on traditional channels. Li *et al.* [23] has investigated the impact of live streaming, considering spillover effects on consumer surplus and social welfare. Moreover, the existence of spillover effects not only affects the income of the platform itself but also has positive or negative implications for competitors. Zhao *et al.* [24] propose that when new streamers join a platform, they not only introduce competition but also bring their own regional traffic, resulting in direct spillover effects on the platform. In summary, this paper provides a theoretical foundation for considering spillover effects as a significant variable influencing the supply chain model. However, it is noteworthy that scholars have yet to explore how differences in the categories of streamers selected by brand owners can lead to variations in spillover effects.

The supply chain model has found extensive applications across various domains. In the context of carbon emission reduction in the fuel vehicle supply chain, Fu *et al.* [25] examined the pricing issue in a dual-channel supply chain consisting of traditional offline channels operated by retailers and online channels established by manufacturers to promote carbon reduction. Furthermore, in the realm of pricing decisions in live streaming, the pricing conflict between online and offline channels has become increasingly prominent due to the rapid growth of e-commerce. Additionally, it has been observed that promotional services have a significant impact

TABLE 1. Comparison of the related literature on e-commerce and this paper.

Literature	Multi-channel	Live-streamers' marketing	Spillover effect	Platform format	Core problem	Decision variables
Cui *et al.* [1]	×	×	×	√	Whether to adopt livestream selling	Selling price and platform commission
Hao *et al.* [2]	×	×	×	√	Probe the Influence of Resale and Agency sale	Level of live streaming selling efforts Wholesale price
Liu *et al.* [4]	√	√	×	√	Investigate the impact of live-streaming selling	The unit price of the e-commerce/live-streaming channel
Li *et al.* [7]	√	√	×	√	What conditions should the retailer introduce an influencer marketing channel	Unit product price in the TO channel and ML channel; Utility of Watching a live broadcast through the ML channel
Lu *et al.* [13]	×	×	×	√	How live streaming affects consumers' purchase intention (PI) in online markets.	Consumers' PPCS and PVS with broadcasters
Han *et al.* [18]	×	×	×	×	Considers the innovative product's pricing and quality level under different brand goodwill	The brand goodwill of the innovative enterprise and the consumers' quality expectation of the traditional product
Guan *et al.* [19]	×	×	×	×	What are the optimal results of these differential game models?	The level of quality improvement and the retailer commands the selling price
Abhishek *et al.* [20]	√	×	√	√	When they should use an agency selling format	Spillover effect into traditional channel
Yang *et al.* [22]	√	√	√	√	How do the two vital parameters impact on the decisions of the supply chain members	Consumer acceptance and spillover effects
Li *et al.* [23]	×	×	√	√	What are the impacts of the optimal decisions and spillover effects on their optimal retail prices and profits?	Probability of receiving a high-type quality signal; spillover utility generated by Retailer i's live streams
Fu *et al.* [25]	√	×	√	×	Carbon emission reduction in the production of ordinary supply chains	The green preference of consumers
Gu *et al.* [26]	√	×	×	×	Study the impact of in-sale service on pricing strategy and choose the appropriate pricing strategy	The variation of overall supply chain profit with in-sale service
Xiong *et al.* [27]	√	√	×	√	The impact of streamer traffic and streamer display on the composition of dual channels	Wholesale price of products and streamer display effect
Liang *et al.* [28]	×	√	×	√	Study the influence of product quality control level and consumer quality perception on manufacturer's price decision	Coefficient of quality cost and consumer sensitivity coefficient
Zhang *et al.* [29]	√	×	×	×	To investigate the impact of the warranty's quality signal on the pricing and warranty decisions	The proportion of direct-channel buyers and the Quality signal of online reviews
Li and Mizuno [30]	√	×	×	×	How to adjust the pricing and inventory decisions in every period	Inventory level
Barman *et al.* [31]	√	×	×	×	Maximise the supply chain profit by minimising the amount of carbon emissions	Carbon tax protocol, green technology, carbon cap-and-trade protocol
Barman *et al.* [31]	×	×	×	×	Developing a multi-objective supply chain inventory management by considering deteriorating products	Number of deliveries from the manufacturer to the retailer per cycle.
Barman *et al.* [33]	×	×	×	×	A multi-objective sustainable economic production quantity model is proposed with partially back-ordering shortages	Selling price per unit product, greening level of product
Paul *et al.* [34]	×	×	×	×	Find an optimal replenishment time and an optimal green concerning level by considering profit maximization	Market demand, Green sensitive parameter
This study	√	√	√	√	Under the influence of spillover effects, how to optimize the selection of merchants' live broadcast strategies	Spillover effect of live streaming channels and the market share of live broadcast channels

on consumer purchasing behavior [26]. In addition, the characteristics of streaming media have an important impact on supply chain pricing and coordination in live broadcast channels [27]. It is also necessary to be aware of another important subject of the supply chain, and the decision-making of brand owners also plays an important role, Liang *et al.* [28] primarily focused on the impact of the manufacturer's quality control level on the supply chain. Zhang *et al.* [29] constructed a dual-channel supply chain model involving manufacturers and retailers, investigating the influence of the proportion of pass-through buyers and the quality of online reviews on consumer sensitivity to warranties. Meanwhile, Li and Mizuno [30] considered three potential power structures – manufacturer-led, retailer-led, and equilibrium models – between manufacturers and retailers in dual-channel supply chains. Their findings revealed that in situations of low wholesale prices, manufacturers and retailers prefer an equilibrium model, whereas manufacturers tend to favor a manufacturer-led model when wholesale prices are high. It can be seen that existing studies on optimal pricing problems in the dual-channel supply chain model do not incorporate the live streaming delivery channel as a form of dual-channel supply chain. Furthermore, the analysis of real-world live streaming delivery issues under different power structures using spillover effects remains unexplored.

Through sorting out, the previous literature is sorted out to form the following Table 1. This paper offers a thorough examination of the influence of spillover effects and product goodwill on the decision-making and coordination among members of a supply chain. The study also examines the most effective pricing and decision-making strategies employed by brand owners, streamers, and the entire supply chain, while considering the influence of different power dynamics.

3. Problem description and model building

This study proposes that within the realm of live streaming, the dual-channel supply chain consists of two key components: a brand (B) and a streaming streamer (A). The categorization of product distribution by brand owners can be delineated into two primary channels: live streaming channels and traditional sales channels. Traditional sales refer to the functioning of physical brick-and-mortar stores, as well as online sales conducted through conventional e-commerce platforms. Live delivery involves the careful selection of a delivery streamer on the live delivery platform who possesses strong alignment with the brand. A specific proportion of commission is subsequently remunerated to the streamer for their role in promoting and selling the brand's products. Among the various factors, the brand's product quality control and service quality improvement are of paramount importance in determining the extent of quality enhancement, denoted as $Q(t)$. The marketing effort level of the streamer is categorized as $E(t)$, indicating a comprehensive product marketing strategy employed by the streamer. This strategy includes promoting the streamer's own private domain traffic and highlighting the unique characteristics of the products. Additionally, it involves providing comprehensive information about the products and leveraging platform public domain traffic for data push and big data recommendation purposes. The determination of commodity sales prices is contingent upon the influence wielded by the party with dominant authority in pricing negotiations. The parameters involved in the model are shown in Table 2.

The streamer, through their influence and private domain traffic consisting of fans, acquires increased bargaining power, which ultimately determines the final price of products. Figure 1 illustrates the supply chain structure of the two types of channels. The marketing effort level of the streamer is categorized as $E(t)$, representing a comprehensive product marketing strategy employed by the streamer. This strategy includes promoting the streamer's own private domain traffic and highlighting the unique characteristics of the products. Additionally, it involves providing comprehensive information about the products and leveraging platform public domain traffic for data push and big data recommendation purposes. The determination of commodity sales prices is contingent upon the influence wielded by the party with dominant authority in pricing negotiations. The streamer, through their influence and private domain traffic consisting of fans, acquires increased bargaining power, resulting in the final product price, denoted as P. The supply chain structure of the two types of channels is depicted in Figure 1. In order to clarify the relationship between each subject more intuitively, draw a vector diagram as shown in Figure 2. In addition, the variable relationships among the subjects are shown in Figure 3.

Assumption 1. *The level of quality improvement in a unit product is contingent upon the quality improvement endeavors undertaken by brand owners [35]. The quality improvement level is a dynamic variable that undergoes changes over time and is influenced by both quality improvement initiatives and the natural rate of decline. $Q(0)$ suggests that the initial efforts to improve product quality, $\tau > 0$, are driven by the cumulative diminishing level effect perceived by consumers. Analogous to previous studies [19]. The following equations are presented:*

$$Q'(t) = \omega R(t) - \tau Q(t). \tag{1}$$

Assumption 2. *The level of quality improvement and marketing effort can have a significant impact on the goodwill of a product, and the goodwill itself is subject to change over time [36]. According to the modified*

TABLE 2. Model notations.

Notation list		
Indices (sets)		
a	Set of streamers, $a \in A = \{1, 2, \cdots, \overline{a}\}$	
b	Set of brand owners, $b \in B = \left\{1, 2, \cdots, \overline{b}\right\}$	
s	Set of scenario led by brand owners, $s \in S = \{1, 2, \cdots, \overline{s}\}$	
x	Set of scenario led by streamers, $x \in X = \{1, 2, \cdots, \overline{x}\}$	
c	Set of scenario centralized decision-making, $c \in C = \{1, 2, \cdots, \overline{c}\}$	
t	Set of time period, $t \in T = \left\{1, 2, \cdots, \overline{t}\right\}$	
Parameters		
σ	The commission ratio of the streamer with goods is divided	$0 < \sigma < 1$
ω	Influence coefficient of quality improvement level of brand owner	$\omega > 0$
τ	Cumulative decreasing level effect of product quality	$\tau > 0$
γ	Marketing preference coefficient	$0 < \gamma < 1$
k_e	Coefficient of Marketing Effort Cost	$0 < k_e < 1$
η	Quality preference coefficient	$\eta > 0$
k_q	Quality improvement cost factor	$0 < k_q < 1$
α	Market capacity	$\alpha > 0$
β	Sensitivity coefficient of inter-channel price competition	$0 < \beta < 1$
μ	Impact of product goodwill on sales volume	$\mu > 0$
ρ	Discount factor	$0 < \rho < 1$
Decision variables		
r	Spillover effect of live streaming channels	$0 < r < 1$
λ	Market share of live broadcast channels	$0 < \lambda < 1$
$P_b(t)$	Product prices through traditional sales channels	
$P_s(t)$	Product prices of live streaming channels	
Auxiliary variables		
$R(t)$	Quality improvement efforts	
$E(t)$	Level of marketing effort	
$Q(t)$	Quality improvement level per unit product	
$D_b(t)$	Demand function of traditional sales channel	
$D_a(t)$	Demand function of live broadcast channel with goods	
π_B	Profit function of brand owner	
π_s	Profit function of the streamer with goods	

goodwill model proposed by, the differential equation model of product goodwill can be derived based on the quality improvement level ($Q(t)$) and marketing effort level ($E(t)$) at time t, as described in this paper [37, 38].

$$G'(t) = \eta Q(t) + \gamma E(t) - \delta G(t), G(0) = G_0. \tag{2}$$

Assumption 3. *Reference to previous research [39, 40], in line with Dockner and Meng's description of quality improvement cost and marketing effort cost, this study considers the quality improvement cost of merchants as $k_q Q^2(t)/2$, with k_q representing the quality improvement cost coefficient. Additionally, the marketing effort cost of streamers is denoted as $k_e E^2(t)/2$, with k_e representing the marketing effort cost coefficient.*

Assumption 4. *The augmentation of product goodwill will have a substantial impact on consumers' purchasing inclination, subsequently influencing market demand. According to prior research, the demand function of goods is frequently derived using the variables of goodwill and price, as demonstrated below:*

$$D(t) = f(G(t))h\left(p(t)\right) \tag{3}$$

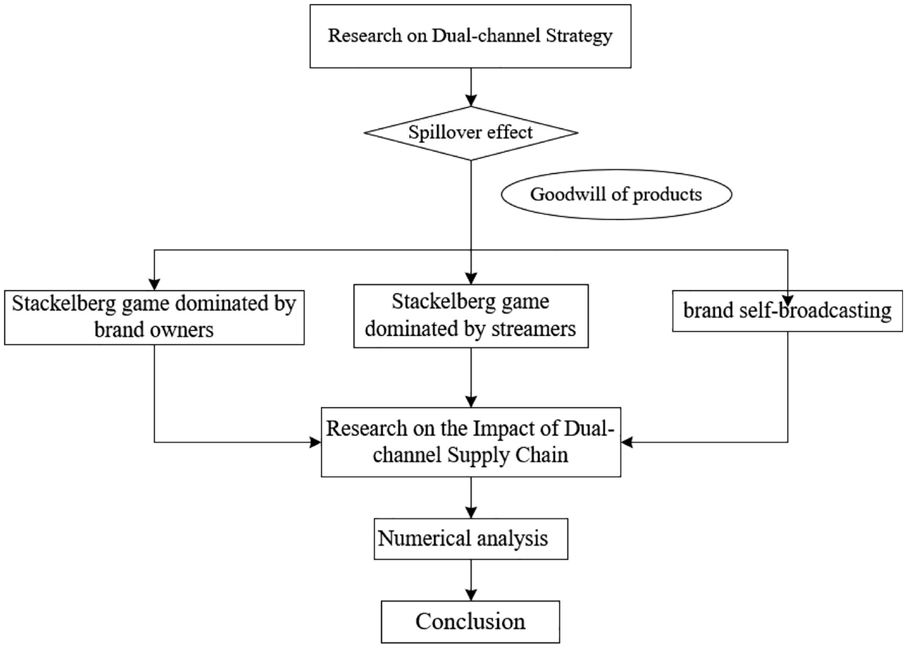

FIGURE 1. Chart of step of manuscript.

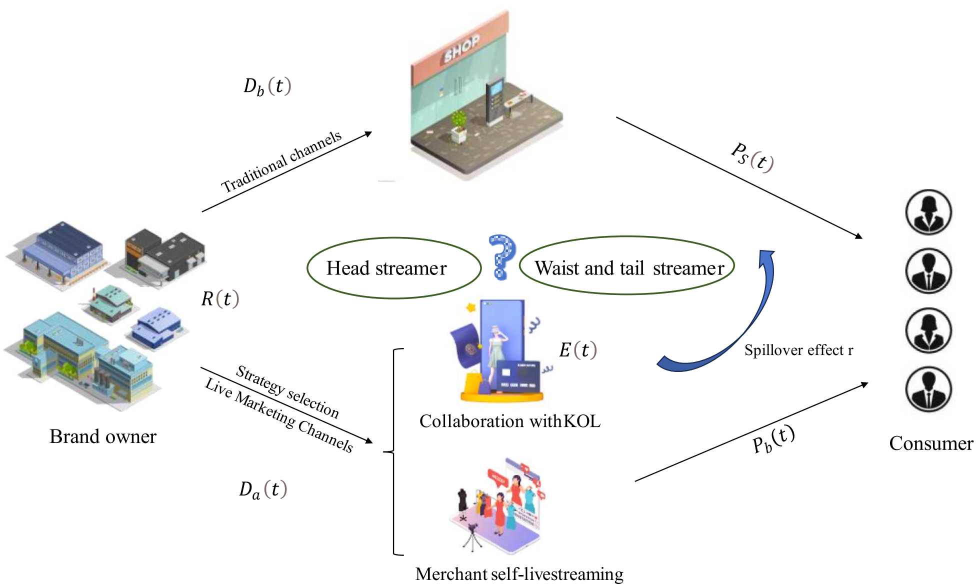

FIGURE 2. Dual-channel supply price structure chart of brand owner product sales.

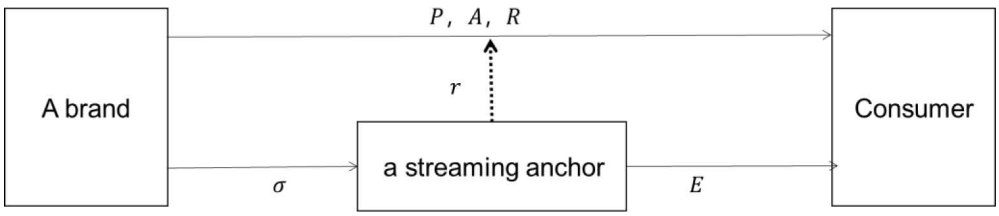

FIGURE 3. Diagram of a dual-channel supply chain considering spillover effects in the context of live streaming.

where $f(G(t)) = \mu G(t)$, μ is the influence degree of product goodwill on sales volume, $h(p(t)) = \alpha - \beta p(t)$, α is market capacity, and β is demand price elasticity coefficient.

$$D(t) = \mu G(t)\left[\alpha - \beta p(t)\right]. \tag{4}$$

In this paper, we examine the traditional sales channels and live broadcast with goods as two distinct types of channels. We construct the demand functions for each channel based on the aforementioned single channel demand function [29]. Considering the objective of mitigating price conflicts between two types of channels, it is assumed that manufacturers will adopt the same pricing strategy as retailers, denoted as strategy $P_a = P_b = p$. Referring to the simplification of the dual-channel model in the dual-channel supply chain [41], the demand function in this paper is presented as follows:

$$\begin{cases} D_a(t) = (\lambda\alpha - \theta\beta p(t))\,\mu G(t) \\ D_b(t) = \left[(1-\lambda)\,\alpha - (1-\theta)\,\beta p(t)\right]\mu G(t). \end{cases} \tag{5}$$

Among the variables mentioned, λ represents the market share of brand self-operated channels, $\beta = \beta_1 + \beta_2$, $\theta = \beta/\beta_1$, $1 - \theta = \beta/\beta_2$, while β_1, β_2 respectively denote the price demand elasticity of retailer channels and streamer live streaming channels.

Assumption 5. *It is assumed that retailers and streamers possess the same discount factor ρ, engage in similar quality improvement efforts $R(t)$, implement comparable marketing strategies $E(t)$, and set product prices $p(t)$ as independent variables. Additionally, the levels of quality improvement and product goodwill are considered dependent variables $Q(t)$ and $G(t)$. Additionally, there exists a spillover effect between the two channels, whereby the live streaming channel can have either a positive or negative spillover effect r on the traditional sales channel [20, 22, 23]. The sales volume of the live streaming channel is expected to have a positive impact on the sales volume of the traditional channels, as it encourages consumers to make purchases through these channels. In the two channels, the profit functions of brand owners, streamers, and the entire supply chain can be expressed as follows:*

$$\pi_B = (1 - \sigma + r)\,D_a(t)p\,(t) + D_b(t)p(t) - k_r R^2(t)/2 \tag{6}$$

$$\pi_A = \sigma D_a(t)p(t) - k_e E^2(t)/2 \tag{7}$$

$$\pi = \left[D_b(t) + D_a(t)\right]p(t) - k_r R^2(t)/22 - k_e E^2(t)/2. \tag{8}$$

4. ANALYSIS AND SOLUTION MODELING

4.1. Decentralized management model-Stackelberg game dominated by brand owners (DB model)

Currently, it is common for brand owners to showcase their collaboration with influential streamers, capitalizing on their strong brand influence. Brand owners and streamers engage in Stackelberg games, employing decentralized business models. The decision-making process involves several steps. Firstly, the brand assesses market conditions and product quality to determine the commodity price $P(t)$ and the necessary efforts $R(t)$ for quality improvement. Subsequently, the brand selects a streamer that is highly compatible with the product. The streamer then decides on their marketing effort $E(t)$ to conduct live streaming. This mode is denoted by the superscript S and is characterized by the simultaneous pursuit of long-term profit maximization. The decentralized operation mode, led by the brand, gives rise to decision-making problems for both the brand and the streamer.

$$\begin{cases} \max\limits_{p(t),R(t)} \left\{\pi_B^S = \int_0^\infty e^{-\rho t}\left\{(1-\sigma+r)\, D_a(t)p(t) + D_b(t)p(t) - k_q R^2(t)/2\right\} \mathrm{d}t\right\} \\ \max\limits_{E(t)} \left\{\pi_A^S = \int_0^\infty e^{-\rho t}\left\{\sigma D_a(t)p(t) - k_e E^2(t)/2\right\} \mathrm{d}t\right\} \\ \text{s.t. } G'^{(t)} = \eta R(t) + \gamma E(t) + \varphi A(t) - \delta G(t), \quad G(0) = G_0 > 0. \end{cases} \tag{9}$$

Theorem 1. *The optimal strategy and profit function for each subject in the decentralized management mode, dominated by brand owners, can be described as follows:*

(1) *The following are the components of balanced sales price, quality improvement efforts, and marketing efforts:*

$$p^S = \frac{\alpha\,(1-\sigma\lambda + r\lambda)}{2\beta\,(1-\sigma\theta + r\theta)}, \quad R^S = \frac{\omega L_1 L_3 L_4}{k_q}, \quad E^S = \frac{\sigma\gamma L_2 L_3}{k_e}.$$

Among them, $L_1 = \frac{\alpha^2(1-\sigma\lambda+r\lambda)^2}{(1-\sigma\theta+r\theta)}$, $L_2 = \frac{\alpha^2(1-\sigma\lambda+r\lambda)(2\lambda-\theta-\lambda\theta\sigma+\lambda\theta r)}{(1-\sigma\theta+r\theta)^2}$, $L_3 = \frac{\mu}{4\beta(\rho+\delta)}$, $L_4 = \frac{\eta}{(\rho+\tau)}$.

(2) *The optimal trajectory for enhancing brand quality is:*

$$Q^S(t) = e^{-\tau t}\left(Q_0^S - Q_\infty^S\right) + Q_\infty^S. \tag{10}$$

The variable $Q_\infty^S = \frac{\omega^2 L_1 L_3 L_4}{\tau k_q}$ *represents the asymptotic value of the level of product quality improvement over an infinite period of time.*

(3) *The ideal path for maximizing product goodwill is*

$$G^S(t) = e^{-\delta t}\left(G_0^S - G_\infty^S - \frac{\eta\left(Q_0^S - Q_\infty^S\right)}{\delta - \tau}\right) + G_\infty^S + \frac{\eta\left(Q_0^S - Q_\infty^S\right)}{\delta - \tau} e^{-\tau t}. \tag{11}$$

The variable $G_\infty^S = \frac{1}{\delta}\left(\frac{\eta\omega^2 L_1 L_3 L_4}{\tau k_q} + \frac{\gamma^2\sigma L_2 L_3}{k_e}\right)\frac{\omega^2 L_1 L_3 L_4}{\tau k_q}$ *represents the enduring value of product goodwill over an indefinite period of time.*

(4) *The decentralized operation led by retailers yields the following optimal profits for brand owners, streamers, and the entire supply chain.*

$$\pi_B^S = L_1 L_3\left(L_4 Q_0 + G_0\right) + \frac{L_1 L_3^2}{\rho}\left(\frac{\omega^2 L_1 L_4^2}{2k_q} + \frac{\gamma^2\sigma L_2}{k_e}\right) \tag{12}$$

$$\pi_A^S = \sigma L_2 L_3\left(L_4 Q_0 + G_0\right) + \frac{\sigma L_2 L_3^2}{\rho}\left(\frac{\omega^2 L_1}{k_q} + \frac{\gamma^2\sigma L_2}{2k_e}\right) \tag{13}$$

$$\pi^S = L_3\left(L_1 + \sigma L_2\right)\left(L_4 Q_0 + G_0\right) + \frac{L_3^2}{\rho}\left[\frac{\omega^2 L_1\left(L_1 L_4^2 + 2\sigma L_2\right)}{2k_q} + \frac{\sigma\gamma^2 L_2\left(2L_1 + \sigma L_2\right)}{k_e}\right]. \tag{14}$$

Proof. The inverse inductive method is utilized to resolve the master–slave game. In this particular approach, the formulation of the Hamilton–Jacobi–Bellman (HJB) equation for the streamer with is initially derived from the Berman theory. It has been established that the truth of statement $G(t) > 0$ is valid. Firstly, the Hamilton–Jacobi–Bellman (HJB) equation for the streamer with is presented as follows:

$$\rho V_A^S = \max_{E(t)}\left\{\sigma p^S\left(\lambda\alpha - \theta\beta p^S\right)\mu G^S - \frac{k_e\left(E^S\right)^2}{2} + \frac{\partial V_A^S}{\partial Q^S}\left(\omega R^S - \tau Q^S\right) + \frac{\partial V_A^S}{\partial G^S}\left(\eta Q^S + \gamma E^S - \delta G^S\right)\right\} \tag{15}$$

where V_B^S is the optimal value function of the streamer, and according to the first-order optimality condition, the optimal marketing effort level E^S can be obtained as follows:

$$E^S = \frac{\gamma}{k_e}\frac{\partial V_A^S}{\partial G^S}. \tag{16}$$

The HJB equation for brand vendors is:

$$\rho V_B^S = \max_{p(t)} \Big\{ (1-\sigma+r)\, p^S \left(\lambda\alpha - \theta\beta p^S\right) \mu G^S + p^S \left[(1-\lambda)\,\alpha - (1-\theta)\,\beta p^S\right] \mu G^S - \frac{k_q \left(R^S\right)^2}{2} + \frac{\partial V_B^S}{\partial Q^S}\left(\omega R^S - \tau Q^S\right) + \frac{\partial V_B^S}{\partial G^S}\left(\eta Q^S + \gamma E^S - \delta G^S\right) \Big\} \tag{17}$$

where V_B^S represent the optimal value function for the retailer. HJB equation is widely used in solving dynamic pricing optimal strategy [41,42]. By substituting the formula into the aforementioned Hamilton–Jacobi–Bellman (HJB) equation, and applying the first-order optimality condition, we can derive the optimal equilibrium sales price V_B^S and the corresponding effort R^S for quality improvement

$$p^S = \frac{\alpha\,(1-\sigma\lambda+r\lambda)}{2\beta\,(1-\sigma\theta+r\theta)} \tag{18}$$

$$R^S = \frac{\omega}{k_q}\frac{\partial V_B^S}{\partial Q^S}. \tag{19}$$

□

By substituting equations (16), (18), and (19) into equation (17), we can derive the equations that describe the optimal value function for small and medium-sized businesses and streamers.

$$\rho V_B^S = \left\{ \left(\eta\frac{\partial V_B^S}{\partial G^S} - \tau\frac{\partial V_B^S}{\partial Q^S}\right) Q^S + \left(\frac{\mu L_1}{4\beta} - \delta\frac{\partial V_B^S}{\partial G^S}\right) G^S + \frac{\omega^2}{2k_q}\left(\frac{\partial V_B^S}{\partial Q^S}\right)^2 + \frac{\gamma^2}{k_e}\frac{\partial V_A^S}{\partial G^S}\frac{\partial V_B^S}{\partial G^S} \right\} \tag{20}$$

$$\rho V_A^S = \left\{ \left(\eta\frac{\partial V_A^S}{\partial G^S} - \tau\frac{\partial V_A^S}{\partial Q^S}\right) Q^S + \left(\frac{\sigma\mu L_2}{4\beta} - \delta\frac{\partial V_A^S}{\partial G^S}\right) G^S + \frac{\gamma^2}{2k_e}\left(\frac{\partial V_A^S}{\partial G^S}\right)^2 + \frac{\omega^2}{k_q}\frac{\partial V_A^S}{\partial G^S}\frac{\partial V_B^S}{\partial G^S} \right\}. \tag{21}$$

According to the formula of the above-mentioned optimal function value, we can make $V_B^S = f_1 Q + f_2 G + f_3$, $V_A^S = c_1 Q + c_2 G + c_3$, where f_1, f_2, f_3, c_1, c_2, c_3 are all constants, and their expressions are:

$$f_1 = L_1 L_3 L_4,\ f_2 = L_1 L_3,\ f_3 = \frac{1}{\rho}\left(\frac{\omega^2}{2k_q}f_1^2 + \frac{\gamma^2}{k_e}c_2 f_2\right) \tag{22}$$

$$c_1 = \sigma L_2 L_3 L_4,\ c_2 = \sigma L_2 L_3,\ c_3 = \frac{1}{\rho}\left(\frac{\gamma^2}{2k_e}c_2^2 + \frac{\omega^2}{k_q}c_2 f_2\right). \tag{23}$$

Replace equations (22) and (23) with equations (16), (18), and (19) to derive the optimal values for quality improvement level, product price, and marketing effort level, respectively.

$$p^{S*} = \frac{\alpha\,(1-\sigma\lambda+r\lambda)}{2\beta\,(1-\sigma\theta+r\theta)}, \quad R^S = \frac{\omega L_1 L_3 L_4}{k_q}, \quad E^{S*} = \frac{\sigma\gamma L_2 L_3}{k_e}. \tag{24}$$

By substituting equilibrium strategy formula (24) into formula (11), the optimal evolution path of product goodwill is $G^S(t) = e^{-\delta t}\left(G_0^S - G_\infty^S - \frac{\eta\left(Q_0^S - Q_\infty^S\right)}{\delta-\omega}\right) + G_\infty^S + \frac{\eta\left(Q_0^S - Q_\infty^S\right)}{\delta-\omega}e^{-\omega t}$, where $G_\infty^S = \frac{1}{\delta}\left(\frac{\eta\omega^2 L_1 L_3 L_4}{\tau k_q} + \frac{\gamma^2\sigma L_2 L_3}{k_e}\right)$. By substituting the formula into the original equation, it is possible to derive the profit functions for brand owners, streamers, and the entire supply chain. After obtaining the certificate.

Corollary 1. *When the spillover effect* $\frac{\sigma\theta-1}{\theta} < r < 1$, *the market share of the live streaming channel under the decentralized business model* $\lambda \in \left[\frac{\theta}{2-\sigma\theta+r\theta}, \frac{1+\theta(2r-2\sigma+1)}{2+(r-\sigma)(1+\theta)}\right]$, *the decision-making supply chain under the leadership of brand owners will adopt dual-channel sales; When* $\lambda \in \left(0, \frac{\theta}{2-\sigma\theta+r\theta}\right)$, *brand owners choose to sell only in traditional channels;* $\lambda \in \left(\frac{1+\theta(2r-2\sigma+1)}{2+(r-\sigma)(1+\theta)}, 1\right)$, *the brand chooses to sell only in the live streaming channel. When the spillover effect* $-1 < r < \frac{\sigma\theta-1}{\theta}$, *the brand chooses to distribute only in the traditional sales channel.*

According to Corollary 1, the interplay between the spillover effect r and market share λ of live streaming is evident in the formulation of brand sales channel selection strategy. When the magnitude of the spillover effect surpasses a specific threshold $(\sigma - 1/\theta)$, and the market share of live streaming falls within a predetermined range, the brand will consider implementing the dual-channel sales strategy. Live streaming directly addresses the challenges faced by traditional sales channels by offering real-time interactivity and authenticity. Additionally, it may have unpredictable spillover effects on traditional sales channels. In the contemporary era, the selection of live streaming channels must align with the evolving trends, as these channels have the potential to foster customer loyalty and expand the market for brand owners. When the market share λ of live streaming is limited, brand owners should focus on establishing control over their products in traditional sales channels. However, when the market share λ of live streaming reaches a high level, brand owners can take advantage of the strong fan loyalty associated with live streaming and introduce products specifically designed for this platform. When the spillover effect of the live streaming channel on the traditional channel is negative, it can significantly disrupt the sales of the traditional channel. The negative spillover effect from the live streaming channel can squeeze the traditional sales channel, leading the brand to opt for selling through the traditional channel.

4.2. Decentralized management model-Stackelberg game dominated by streamer (DS model)

When a brand's products generate a strong response in the live broadcast room hosted by a top streamer, it results in a noticeable spillover effect on the sales of traditional retailers. Moreover, the streamers who promote these products hold the power to influence the pricing of the goods. To address potential conflicts arising from price discrepancies between the top streamer's live broadcast room and traditional sales channels, retailers align their prices with those set by the top streamer. The decision-making process follows a specific sequence: first, the brand determines the level of quality improvement $Q(t)$, and then it selects the streamer on the live broadcast platform who best aligns with the product. Once the streamer establishes a strong presence, they gain the authority to determine the price $P(t)$ of the commodity and the level of marketing effort $E(t)$ required for live streaming. In the superscript mode, the optimal strategy-solving model is as follows:

$$\begin{cases} \max\limits_{A(t),Q(t)} \left\{\pi_B^X = \int_0^\infty e^{-\rho t} \left\{(1-\sigma+r)\, D_a(t)p(t) + D_b(t)p(t) - k_q Q^2(t)/2\right\} \mathrm{d}t\right\} \\ \max\limits_{E(t),P(t)} \left\{\pi_A^X = \int_0^\infty e^{-\rho t} \left\{\sigma D_a(t)p(t) - k_e E^2(t)/2\right\} \mathrm{d}t\right\} \\ \text{s.t. } G'^{(t)} = \eta Q(t) + \gamma E(t) + \varphi A(t) - \delta G(t),\ G(0) = G_0 > 0. \end{cases} \tag{25}$$

Theorem 2. *The optimal strategy for each subject within the decentralized business model, led by the streamer, can be described as follows:*

(1) *The following are the components of balanced sales price, quality improvement efforts, and marketing efforts:*

$$p^X = \frac{\lambda\alpha}{2\theta\beta}, \quad R^X = \frac{\omega K_1 L_3 L_4}{k_q}, \quad E^X = \frac{\sigma\gamma K_2 L_3}{k_e}.$$

Among them, $K_1 = \frac{\lambda\alpha^2(2\theta+r\lambda\theta-\sigma\lambda\theta-\lambda)}{\theta^2}$, $K_2 = \frac{\lambda^2\alpha^2}{\theta}$.

(2) *The optimal trajectory for enhancing brand quality is:*

$$Q^X(t) = e^{-\tau t}\left(Q_0^X - Q_\infty^X\right) + Q_\infty^X. \tag{26}$$

The variable $Q_\infty^X = \frac{\omega^2 K_1 L_3 L_4}{\tau k_q}$ *represents the asymptotic value of the level of product quality improvement over an infinite period of time.*

(3) *The ideal path for maximizing product goodwill is*

$$G^X(t) = e^{-\delta t}\left(G_0^X - G_\infty^X - \frac{\eta\left(Q_0^X - Q_\infty^X\right)}{\delta - \omega}\right) + G_\infty^X + \frac{\eta\left(Q_0^X - Q_\infty^X\right)}{\delta - \omega}e^{-\omega t}. \tag{27}$$

The variable $G_\infty^X = \frac{1}{\delta}\left(\frac{\eta\omega^2 K_1 L_3 L_4}{\tau k_q} + \frac{\gamma^2 \sigma K_2 L_3}{k_e}\right)\frac{\omega^2 K_1 L_3 L_4}{\tau k_q}$ *represents the enduring value of product goodwill over an indefinite period of time.*

(4) *The decentralized operation led by livestream hosts yields the following optimal profits for brand owners, streamers, and the entire supply chain.*

$$\pi_B^X = K_1 L_3\left(L_4 Q_0 + G_0\right) + \frac{K_1 L_3^2}{\rho}\left(\frac{\omega^2 K_1 L_4^2}{2k_q} + \frac{\gamma^2 \sigma K_2}{k_e}\right) \tag{28}$$

$$\pi_A^X = \sigma K_2 L_3\left(L_4 Q_0 + G_0\right) + \frac{\sigma K_2 L_3^2}{\rho}\left(\frac{\omega^2 K_1}{k_q} + \frac{\gamma^2 \sigma K_2}{2k_e}\right) \tag{29}$$

$$\pi^X = L_3\left(K_1 + \sigma K_2\right)\left(L_4 Q_0 + G_0\right) + \frac{L_3^2}{\rho}\left(\frac{\omega^2 K_1\left(K_1 L_4^2 + 2\sigma K_2\right)}{2k_q} + \frac{\sigma\gamma^2 K_2\left(K_1 + 2\sigma K_2\right)}{k_e}\right). \tag{30}$$

The proof process for this proposition is identical to that of Theorem 1.

Inference 1. Under the leading mode of the streamer (that is, choose to cooperate with the top streamer), when the market share of the live channel is $\lambda \in (0, \frac{2\theta}{1+\theta}]$, the brand will adopt dual-channel sales; When the market share of the live streaming channel is $\lambda \in (\frac{2\theta}{1+\theta}, 1)$, the brand will only choose the live streaming channel for distribution, and the upper limit of the market share of the live streaming channel is less than the upper limit when the brand chooses to cooperate with the waist and tail streamers, that is, $\frac{2\theta}{1+\theta} < \frac{1+\theta(2r-2\sigma+1)}{2+(r-\sigma)(1+\theta)}$.

Based on the aforementioned Inference 1, it is apparent that the top streamer on the platform plays an active role in price negotiations with brands due to their substantial private domain traffic pool, significant influence, and widespread popularity among the public. Brand owners often seek to leverage the influence of the top streamer to endorse their products. However, establishing a brand's intellectual property (IP) and cultivating consumer loyalty towards the brand can be challenging. When the market share λ of live streaming is relatively low, brand owners frequently choose to collaborate with prominent streamers to endorse their products. This strategy aims to evaluate the spillover effect of these streamers on the traditional channels of the brand owners, facilitating the transition from the private domain traffic of streaming streamers to actual product users. However, when the market share λ of live streaming spans a wide range, it becomes crucial to consider the interdependence between Internet development and a robust supply chain system. In such circumstances, the live streaming channel has the potential to capture a significant portion of the traditional channel market and emerge as the primary platform for commodity sales. In this scenario, if the dominant entity in the market continues to grow and control private domain traffic and negotiation power, it will result in a reduction of the brand's market presence and have a significant impact on the traditional sales channels. In comparison to a scenario where the brand holds dominance, the brand is more inclined to adopt a dual-channel sales strategy in collaboration with waist and tail streamers. Unlike the top streamer, who incurs high pit fees and commission fees, the waist and tail streamers operate with more refinement and lower pit fees. This enables the brand to continue selling through dual channels even when the market share of the live streaming channel is high.

4.3. Centralized management model-brand self-broadcasting (CB model)

To optimize the profitability of the entire supply chain, this study presents a differential game model that considers the integrated scenario of a cooperative differential game between brand owners and streamers over an infinite time horizon. This model encompasses the determination of the optimal selling price, quality improvement level, and marketing effort level for the entire supply chain. Assuming a given discount level ρ and denoting the centralized management mode with a superscript C, the comprehensive decision model for optimizing the supply chain can be expressed as follows:

$$\begin{cases} \max\limits_{Q(t),P(t),E(t),A(t)} \left\{\pi^C = \int_0^\infty e^{-\rho t}\left\{\left(D_b(t)+D_a(t)\right)p(t) - k_q Q^2(t)/2 - k_e E^2(t)/2\right\}\,\mathrm{d}t\right\} \\ \text{s.t. } G'^{(t)} = \eta Q(t) + \gamma E(t) + \varphi A(t) - \delta G(t), \quad G(0) = G_0 > 0. \end{cases} \tag{31}$$

Theorem 3. *The optimal strategy in the context of centralized management mode can be described as follows:*

(1) *The following are the components of balanced sales price, quality improvement efforts, and marketing efforts:*

$$p^C = \frac{\alpha}{2\beta}, \quad R^C = \frac{\omega\alpha^2 L_3 L_4}{k_q}, \quad E^C = \frac{\gamma\alpha^2 L_3}{k_e}\cdot$$

Among them, $K_1 = \frac{\lambda\alpha^2(2\theta + r\lambda\theta - \sigma\lambda\theta - \lambda)}{\theta^2}$, $K_2 = \frac{\lambda^2\alpha^2}{\theta}$.

(2) *The optimal trajectory for enhancing brand quality is:*

$$Q^C(t) = e^{-\rho t}\left(Q_0^C - Q_\infty^C\right) + Q_\infty^C. \tag{32}$$

The variable $Q_\infty^C = \frac{\omega^2\alpha^2 L_3 L_4}{\tau k_q}$ *represents the asymptotic value of the level of product quality improvement over an infinite period of time.*

(3) *The ideal path for maximizing product goodwill is*

$$G^C(t) = e^{-\delta t}\left(G_0^C - G_\infty^C - \frac{\eta\left(Q_0^C - Q_\infty^C\right)}{\delta - \omega}\right) + G_\infty^C + \frac{\eta\left(Q_0^C - Q_\infty^C\right)}{\delta - \omega}e^{-\omega t}. \tag{33}$$

The variable $G_\infty^C = \frac{1}{\delta}\left(\frac{\eta\omega^2\alpha^2 L_3 L_4}{\tau k_q} + \frac{\gamma^2\alpha^2 L_3}{k_e}\right)$ *represents the enduring value of product goodwill over an indefinite period of time.*

(4) *The optimal profit of the whole supply chain under centralized operation is*

$$\pi^C = \alpha^2 L_3\left(L_4 Q_0 + G_0\right) + \frac{\alpha^4 L_3^2}{\rho}\left(\frac{\omega^2 L_4^2}{2k_q} + \frac{\gamma^2}{2k_e}\right)\cdot \tag{34}$$

The proof process for this proposition is identical to that of Theorem 1.

4.4. Comparison and discussion

After solving the equilibrium strategy values of the above three models, the optimal strategies of the three models are summarized and compared below, as shown in the Table 3.

In order to ensure the successful implementation of dual-channel sales under the two decentralized models, it is necessary to meet the preconditions of $\frac{\sigma\theta-1}{\theta} < r < 1$ and $\frac{\theta}{2-\sigma\theta+r\theta} < \lambda \leq \frac{2\theta}{1+\theta}$. All of the aforementioned conditions hold true in the subsequent inferences.

4.4.1. Comparative analysis of optimal strategies in two decentralized models

Sensitivity analysis 1. The relationship between the optimal product price and the optimal strategy in the dual-channel decentralized decision-making model, where brand owners and streamer lead, can be described as follows:

TABLE 3. Optimal policy equilibrium value under three kinds of models.

Equilibrium decisions	Model classification		
	DB model	DS model	CS model
Product prices p	$\frac{\alpha(1-\sigma\lambda+r\lambda)}{2\beta(1-\sigma\theta+r\theta)}$	$\frac{\lambda\alpha}{2\theta\beta}$	$\frac{\alpha}{2\beta}$
Quality improvement efforts R	$\frac{\omega L_1 L_3 L_4}{k_q}$	$\frac{\omega K_1 L_3 L_4}{k_q}$	$\frac{\omega\alpha^2 L_3 L_4}{k_q}$
Level of marketing effort E	$\frac{\sigma\gamma L_2 L_3}{k_e}$	$\frac{\sigma\gamma K_2 L_3}{k_e}$	$\frac{\gamma\alpha^2 L_3}{k_e}$
Stable value of product quality Q_∞	$\frac{\omega^2 L_1 L_3 L_4}{\tau k_q}$	$\frac{\omega^2 K_1 L_3 L_4}{\tau k_q}$	$\frac{\omega^2\alpha^2 L_3 L_4}{\tau k_q}$
Stable value of product goodwill G_∞	$\frac{1}{\delta}\left(\frac{\eta\omega^2 L_1 L_3 L_4}{\tau k_q}+\frac{\gamma^2\sigma L_2 L_3}{k_e}\right)$	$\frac{1}{\delta}\left(\frac{\eta\omega^2 K_1 L_3 L_4}{\tau k_q}+\frac{\gamma^2\sigma K_2 L_3}{k_e}\right)$	$\frac{1}{\delta}\left(\frac{\eta\omega^2\alpha^2 L_3 L_4}{\tau k_q}+\frac{\gamma^2\alpha^2 L_3}{k_e}\right)$
Profit Function of Brand Owner π_B	$L_1 L_3\left(L_4 Q_0+G_0\right)+\frac{L_1 L_3^2}{\rho}\left(\frac{\omega^2 L_1 L_4^2}{2k_q}+\frac{\gamma^2\sigma L_2}{k_e}\right)$	$K_1 L_3\left(L_4 Q_0+G_0\right)+\frac{K_1 L_3^2}{\rho}\left(\frac{\omega^2 K_1 L_4^2}{2k_q}+\frac{\gamma^2\sigma K_2}{k_e}\right)$	–
Profit function of the streamer with goods π_A	$\sigma L_2 L_3\left(L_4 Q_0+G_0\right)+\frac{\sigma L_2 L_3^2}{\rho}\left(\frac{\omega^2 L_1}{k_q}+\frac{\gamma^2\sigma L_2}{2k_e}\right)$	$\sigma K_2 L_3\left(L_4 Q_0+G_0\right)+\frac{\sigma K_2 L_3^2}{\rho}\left(\frac{\omega^2 K_1}{k_q}+\frac{\gamma^2\sigma K_2}{2k_e}\right)$	–
Overall profit of supply chain π	$L_3\left(L_1+\sigma L_2\right)\left(L_4 Q_0+G_0\right)+\frac{L_3^2}{\rho}\left[\frac{\omega^2 L_1\left(L_1 L_4^2+2\sigma L_2\right)}{2k_q}+\frac{\sigma\gamma^2 L_2(2L_1+\sigma L_2)}{k_e}\right]$	$L_3\left(K_1+\sigma K_2\right)\left(L_4 Q_0+G_0\right)+\frac{L_3^2}{\rho}\left(\frac{\omega^2 K_1\left(K_1 L_4^2+2\sigma K_2\right)}{2k_q}+\frac{\sigma\gamma^2 K_2(K_1+2\sigma K_2)}{k_e}\right)$	$\alpha^2 L_3\left(L_4 Q_0+G_0\right)+\frac{\alpha^4 L_3^2}{\rho}\left(\frac{\omega^2 L_4^2}{2k_q}+\frac{\gamma^2}{2k_e}\right)$

(1) if $\frac{\theta}{2-\sigma\theta+r\theta}<\lambda\leq\theta$, then $p^S\geq p^X$, and if $\theta<\lambda\leq\frac{2\theta}{1+\theta}$, then $p^S<p^X$;
(2) $R^S>R^X$, $E^S<E^X$, $Q_\infty^S>Q_\infty^X$, $G_\infty^S>G_\infty^X$.

Based on Sensitivity analysis 1 (1), it is evident that when the market share of the live streaming channel is relatively small, indicating the dominance of the traditional channel, the product price is lower when the brand selects the top streamer for live streaming compared to collaborating with middle and lower-tier streamers. This situation aligns more closely with the characteristics of the contemporary dual-channel supply chain market, which integrates live broadcasting and goods sales. The current market value of the top streamer's live broadcasts is comparatively lower than the historical sales channel utilized by brand owners. This phenomenon can be attributed to the top streamer's possession of a distinct and exclusive domain traffic pool. The primary objective of the leading live broadcast is to engage in price negotiations with merchants, securing competitive prices, in order to enhance fan retention and increase its own influence. Subsequently, the live broadcast platform focuses on enhancing its marketing strategies, resulting in a positive spillover effect and cumulative impact when combined with the price advantages. This approach aims to attract potential consumers from the general public domain traffic pool. As the market share of the live streaming channel reaches a significant level, the bargaining power of the top streamer experiences a substantial increase. However, without considering the horizontal competition within the live streaming industry, there is a likelihood that the product sales price will be raised to achieve its own marginal benefit. In order to maintain profitability in traditional sales channels, brand owners often employ price reduction strategies to mitigate the decline in demand by leveraging the competitive advantage of lower prices.

Based on Sensitivity analysis 1 (2), it is evident that when brand owners collaborate dominantly with waist-tail streamers, the quality improvement efforts of brand owners and the long-term stability of quality improvement are significantly superior compared to cooperation with top streamers. However, the marketing efforts of waist-tail streamers are inferior to those of head-tail streamers. Currently, the brand chamber of commerce prioritizes enhancing the intrinsic quality of the product to improve its reputation and goodwill. Due to their limited private domain traffic and inadequate team operation, the waist and tail streamers collaborating with the brand focus on efforts to enhance product marketing, increase sales volume, and strengthen their bargaining power with brand owners by augmenting their own influence. Brand owners choose to collaborate with prominent influencers, and top streamers currently hold a favorable position in the bargaining game with merchants. This enables them to prioritize enhancing their marketing capabilities to drive product sales. It is understandable that brands and retailers prioritize their bargaining power and self-interests. However, from the perspective of consumers, the quality of the product remains the fundamental factor in increasing the likelihood of repeat purchases. In the long term, while the marketing strategies employed by companies that prioritize short-term gains may lead to a temporary boost in sales, the decline in product quality will ultimately result in a decrease in repeat purchases.

4.4.2. A comparative analysis of optimal strategies between a decentralized decision-making model and a centralized decision-making model dominated by brand owners

Sensitivity analysis 2.

$$\text{When } \frac{\sigma\theta-1}{\theta}<r<\sigma, \text{if } \frac{\theta}{2-\sigma\theta+r\theta}<\lambda\leq\theta, \text{then } p^S\geq p^C, \text{if } \theta<\lambda\leq\frac{2\theta}{1+\theta}, \text{then } p^S<p^C;$$

$$\text{When } \sigma<r<1, \text{if } \frac{\theta}{2-\sigma\theta+r\theta}<\lambda\leq\theta, \text{then } p^S<p^C, \text{and if } \theta<\lambda\leq\frac{2\theta}{1+\theta}, \text{then } p^S>p^C.$$

According to Sensitivity analysis 2, the centralized decision-making model considers the demands of streamers with goods, in contrast to the brand owner-driven decision-making model. When the spillover effect of the delivery channel on the traditional channel is minimal, it indicates a low level of cooperation between the brand and waist-tail streamers. If traditional brands predominantly hold the current market share, the centralized decision-making model is expected to adopt a higher optimal pricing strategy compared to the brand dominant model. This phenomenon can be attributed to the minimal spillover effect and the dominant presence of the traditional market, which leads to the neglect of the influence of live streaming channels on traditional channels. Consequently, merchants continue to focus their efforts on traditional channels as their primary battleground. When selecting streamers for collaborative delivery, it is crucial to consider their individual income and commission rates, and appropriately set higher prices to achieve marginal profits. When brand owners engage in self-promotion of their brands, the price tends to be lower compared to when they opt for collaborative delivery with streamers, regardless of streamer commission and airtime allocation.

When the spillover effect of the delivery channel on the traditional channel is significant, a high market share of the live broadcasting channel has a substantial impact on the traditional channel. In a centralized decision-making scenario, the optimal price is lower compared to a brand-led decision-making scenario. This phenomenon occurs when the live streaming channel dominates the market share of brand owners. In such cases, brand owners rely on the live broadcast platform to build their own private domain traffic pools. Instead of collaborating with streamers for product promotion, brand owners who choose centralized decision-making, specifically self-broadcasting, do not need to consider dynamics with the streamers. They focus more on promoting and introducing the product itself. Currently, our intention is to implement a pricing strategy that attracts a larger volume of traffic to enhance consumer engagement. This approach involves conducting comprehensive publicity and promotional activities, contributing to building a stronger brand reputation and fostering greater consumer trust. Ultimately, our goal is to cultivate customer loyalty towards our brand.

5. Numerical analysis

According to the previous analysis, MATLAB numerical simulation can intuitively analyze the influence of each parameter on the optimal strategy. Referring to the existing numerical studies of live streaming marketing dual-channel supply chain [19, 22, 26]. The initial parameters can be set as: $\alpha = 5$, $\beta = 2$, $\theta = 0.5$, $\omega = 0.9$, $\gamma = 0.6$, $\mu = 0.85$, $\rho = 0.3$, $\tau = 0.3$, $\eta = 0.6$, $\sigma = 0.2$, $k_q = k_e = 1$, $Q_0 = 5$, $G_0 = 12$.

Figures 4 and 5 illustrate the trajectories of the quality effort level and product goodwill over time in two decentralized models and centralized decision models. In this context, we discuss the parameters $\lambda = 0.3$ and $r = 0.4$. The figures clearly show that regardless of the model, the level of improvement in product quality consistently follows a monotonic increasing trend, with $Q^S_\infty > Q^C_\infty > Q^X_\infty$. The overall goodwill of products initially exhibits a decline, followed by an improvement in product quality, resulting in a subsequent increase.

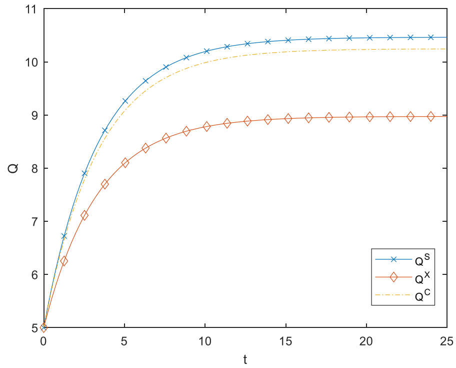

FIGURE 4. Quality improvement levels in various situations.

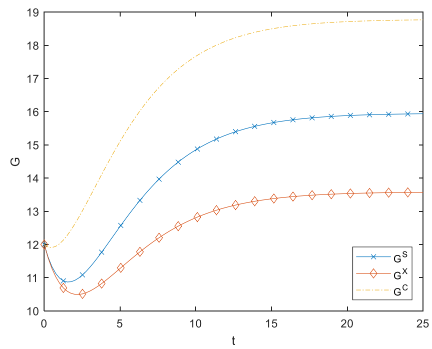

FIGURE 5. Goodwill of product value in various circumstances.

Eventually, the overall goodwill stabilizes at a consistent value, with $G^C_\infty > G^S_\infty > G^X_\infty$. This trend can be attributed to the influence of the natural decline rate and the lag in product quality enhancement. In the context of centralized decision-making, the stable value of product goodwill is highest in the brand-dominated model, followed by the streamer model. The impact on product goodwill is influenced by the level of quality improvement and marketing efforts. It is observed that brand owners tend to lead in a higher level of quality improvement compared to streamers. Despite the relatively lower marketing effort level of streamers, consumers exhibit a stronger preference for product quality over marketing preferences. Additionally, the product goodwill is higher when streamers are dominant.

In the context of live streaming commerce, brand owners often find themselves in a position of dominance when collaborating with top streamers, while they tend to have a dominant position when collaborating with supporting and trailing streamers. The choice of collaborating with top streamer results in a significant positive or negative spillover effect of the live streaming channel on the traditional channel. However, when a waist and tail streamer is chosen as the primary collaborator, the spillover effect of the live streaming channel on the brand's traditional channel is minimal. Real brands make decisions to employ streamers in three distinct situations. Collaborating with top streamers results in a larger positive spillover effect or a smaller negative spillover effect. On the other hand, collaborating with waist and tail streamers leads to a small but positive spillover effect. Here, we represent the spillover effect in the aforementioned three cases using $r = 0.6$, $r = -0.6$, and $r = 0.2$, respectively.

Figure 6 presents a comparison between two scenarios: one where the brand selects the top streamer, resulting in a significant positive spillover effect, and another where the brand chooses the waist and tail streamer. It

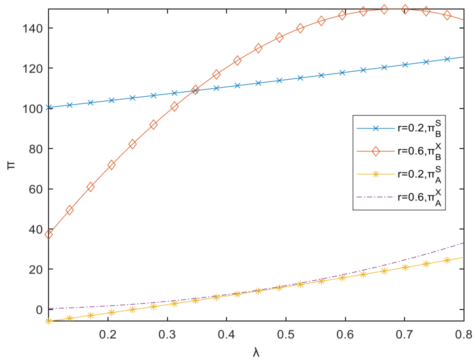

FIGURE 6. Revenue function of brand and streamer under two decentralized decision models with spillover effects $r = 0.2$ and $r = 0.6$.

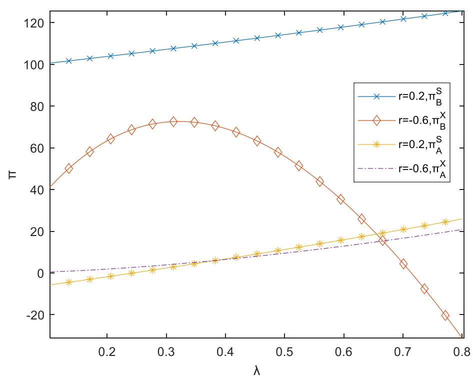

FIGURE 7. Revenue function of brand and streamer under two decentralized decision models with spillover effects $r = 0.2$ and $r = -0.6$.

is evident that in situations where the market share of the goods channel is limited and the market is not yet saturated, the brand chamber of commerce tends to opt for collaborating with waist and tail streamers in order to gain control over the bargaining process. Currently, despite the significant spillover effect, the sales volume of the goods channel remains relatively low. Therefore, brand owners should consider incorporating traditional distribution channels for their products. When the market share of a particular delivery channel reaches a significant scale, it becomes crucial for merchants to seek collaboration with influential figures who align with their product positioning. By leveraging the private domain traffic and fan base of these influencers, merchants can rapidly expand the reach of their live broadcast channels and enhance their marginal profit. This collaboration creates a mutually beneficial situation for both the merchants and the influencers, resulting in a win–win outcome.

When a brand selects the top streamer and experiences a negative spillover effect, as depicted in Figure 7, the brand demonstrates a pattern of initial growth followed by a diminishing effect with the increase of the variable being discussed. However, this effect is consistently smaller compared to the collaboration between the brand and the secondary and tertiary streamers. Although the current increase in profit for the merchants is not substantial, there is still an overall upward trend in profitability. During this period, the brand will seek to regain its bargaining power by strategically selecting waist and tail streamers for live broadcasts with goods. This approach aims to prevent any negative spillover from the live broadcast channel onto the traditional channel.

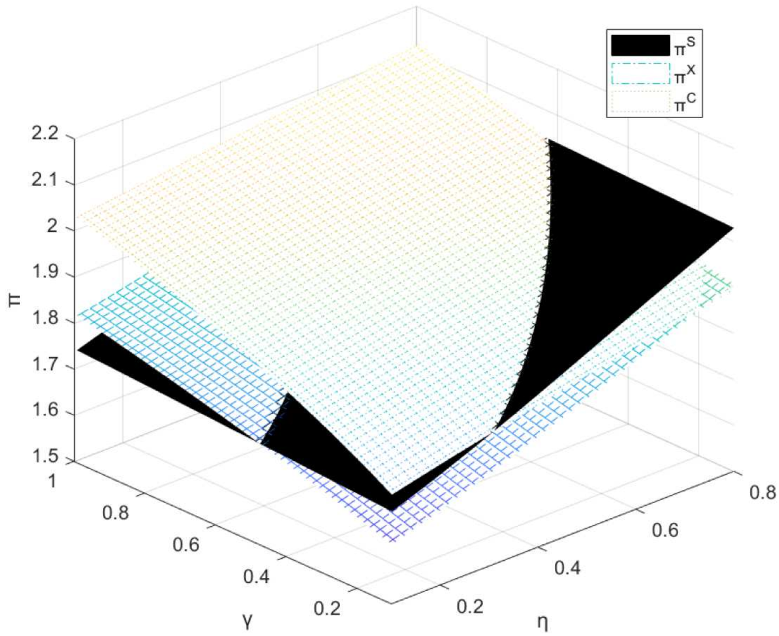

Figure 8. Changes of overall profit of supply chain by quality preference coefficient η and marketing preference coefficient γ under three modes.

This strategic decision highlights the strong support of the live broadcast platform for prominent influencers and the redirection of traffic towards them. The goal is to prevent significant losses to the platform and businesses caused by adverse spillover effects on their interests. To summarize, in situations where the market share of live broadcasts with goods remains relatively small, it is advisable for brand owners to opt for waist and tail streamers for their live broadcasts. In an era where the industry is experiencing heightened saturation, brand owners must exercise caution when selecting streamers for product sales. This strategic decision not only enables them to maintain a competitive edge but also empowers them to mitigate potential risks. Consequently, brand owners may choose to collaborate with waist and tail streamers in order to secure favorable bargaining power.

Figure 8 illustrates the influence of the quality preference coefficient (η) and marketing preference coefficient (γ) on the overall profit of the supply chain under three modes. It is evident that when consumers' preference coefficient η for quality and the preference coefficient γ for marketing efforts of streamers with goods exceed a certain value, the relationship between the overall profit of the supply chain follows the pattern $\pi^C > \pi^X > \pi^S$. This indicates that consumers are equally sensitive to both product quality and the marketing promotion ability of streamers with goods, and the brand self-broadcasting strategy best aligns with consumers' preferences for these two factors. Although top streamers have stronger marketing efforts compared to waist and tail streamers, due to consumers' preference for quality, they are more inclined to choose products with goods brought by waist and tail streamers. On the other hand, when consumers' preference coefficient γ for marketing efforts of streamers with goods exceeds a certain value, while their preference coefficient η for quality falls below a certain value, the relationship between the overall profit of the supply chain is $\pi^S > \pi^C > \pi^X$. These consumers are more influenced by the marketing level of streamers but are not highly sensitive to the quality of the products themselves. They are usually fans of online celebrity streamers and are easily swayed by sensational live broadcasts. For this type of consumer, brand owners should choose to collaborate with top streamers who have a private domain traffic and a large fan base to achieve the highest profit for the overall supply chain through their high marketing efforts. Furthermore, when consumers' preference coefficient η for product quality and marketing preference coefficient γ for streamers with goods are below a certain value, the relationship between the overall profit of the supply chain is $\pi^C > \pi^S > \pi^C$. During this time, product quality and marketing promotion have minimal impact on consumers, and consumers are more price-sensitive. It is observed that the centralized decision-making model, specifically the brand self-broadcasting strategy, incurs lower operating costs as there is no need to pay high commission fees or pit fees to streamers. Consequently, consumers often prioritize price preferences. Additionally, when the brand chooses to collaborate with top streamers, the top streamer's strong bargaining power leads to lower product prices. However, in many cases, despite the lower product prices in top streamer live broadcasts, there is often a risk of poor product quality.

6. Conclusions and outlook

In this paper, we present a dual-channel supply chain model that comprises brand owners and streamers. By integrating the quality improvement initiatives of brand owners, the marketing endeavors of streamers, and the spillover effect of live streaming on dual-channel demand, we consider the quality improvement level and product goodwill as crucial state variables. Differential equations are formulated to depict the trajectories of these variables. The primary objective of this study is to examine and compare three decision-making models for decentralized and centralized decision-making within the brand owner and streamer cooperation context. Specifically, the models encompass brand owner-led decentralized decision-making, streamer-led decentralized decision-making, and a centralized decision-making model. These models correspond to three distinct scenarios: collaboration between the brand owner and the top streamer, collaboration between the brand owner and waist and tail streamers, and brand self-broadcasting. Furthermore, this paper analyzes and compares the optimal pricing and equilibrium strategies for products in each of these situations. Through conducting a comprehensive comparative analysis and simulation, we have derived the following conclusions:

(1) Regardless of the mode, there is a progressive improvement in product quality, which eventually stabilizes at a fixed value. On the other hand, the product goodwill tends to fluctuate over time, initially decreasing, then increasing, and ultimately stabilizing at a fixed value. Notably, the decentralized model, primarily controlled by brand owners, exhibits the highest level of stability in terms of quality improvement.

(2) On the other hand, the decentralized model led by brand owners involves collaboration with waist and tail streamers, resulting in a relatively smaller spillover effect. When the market share of live streaming is low, brand owners tend to leverage their bargaining power and collaborate with less popular streamers. However, when the market share of live streaming is high, brand owners carefully select streamers to promote their products. Presently, collaborating with top streamers has the potential to generate favorable spillover effects and yield significant profits. However, it is crucial to acknowledge that they may also encounter adverse spillover effects, which can significantly undermine their own profitability. It will affect the spillover effect with the random risk that the streamer may have during the live streaming. Considering the current circumstances, risk-averse merchants are inclined to collaborate with waist and tail streamers. On the other hand, risk-seeking merchants are more likely to collaborate with top streamers, leveraging their market influence to promote their products.

(3) The profitability of the supply chain is significantly influenced by consumers' preferences for both product quality and marketing. A higher coefficient of influence for quality improvement level and marketing efforts on product goodwill can effectively facilitate collaboration between brand owners and merchants, leading to collective efforts in promoting product sales.

The above research provides a reference for the development of brand owners and streamers in the live broadcast ecology. On the one hand, brand owners should continuously improve the level of product quality improvement, and open up self-broadcast modules in live broadcast channels; on the other hand, streamers should continuously improve the level of marketing efforts while obtaining profits. In future developments, it would be valuable to integrate the platform into the supply chain system, thereby investigating the potential effects of big data marketing, including platform infrastructure and traffic push, on the resilience of the supply chain. However, there are still some deficiencies in this paper. Regarding the spillover effect of the online live broadcast channel mentioned in this article on the traditional channels of brand owners, here we only regard this spillover effect as one-way. In real life, there are often spillover effects of traditional channels on online live broadcasts due to the marketing and promotion of brand owners in traditional channels. In future research, the impact of two-way spillover effects can be considered in supply chain research.

Funding

This paper is a project funded by the National Social Science Fund, "Research on Governance Mechanism of Network Platform Enterprises from the Perspective of Quasi-public Goods" (ProjectNo.: 20BGL096), a project funded by Jiangsu Social Science Fund, "Game Analysis of Governance Evolution of Webcast Platform from the Perspective of Quasi-public Goods" (ProjectNo.: KYCX22_2982).

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