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MANUFACTURER ENCROACHMENT AND EXTENDED WARRANTY PROVISION

Yue Qi[1], Taofeng Ye[1,*] and Guoqing Zhang[2]

Abstract. This study focused on the interactions between manufacturer encroachment and extended warranty (EW) provision by examining the manufacturer's optimal encroachment decision with or without the extended warranty and the optimal EW provision decision under encroachment or no encroachment. Based on the combination of the two strategies, this study discussed four different models in which the manufacturer and retailer act as the Stackelberg leader and follower, respectively. It was shown that the manufacturer always finds it optimal to offer EW without encroachment. However, under encroachment, the manufacturer's motivation to offer EW weakens. Furthermore, when EW is not offered, the manufacturer can benefit from encroachment if the selling cost of the product is not sufficiently high. When the selling cost of EW is low, the manufacturer's motivation for encroachment strengthens. As the selling cost becomes moderate to high, offering EW weakens the manufacturer's motivation for encroachment. Our analysis reveals that for different values of the co-payment rate and the manufacturer's selling costs of products and EW, encroachment and EW provision may reinforce or impair each other. Therefore, manufacturer has to pay attention to the influence of one decison on another decision.

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1. Introduction

Extended warranty (EW) is a service that can protect consumers from the risks associated with product failure. Under this warranty, providers are responsible for repairing and replacing consumer products over a specified period. Unlike the base warranty bundled with the product, EW is an optional insurance service that requires consumers to pay an upfront premium, providing supplementary coverage beyond the expiration of the base warranty [21]. EW not only provides additional after-sales protection for consumers beyond the expiration of the base warranty but also represents a new source of profit for manufacturers [29, 33, 41]. In addition to improving revenue, providing extended warranties (EWs) can help firms promote brand image and consumer loyalty [10]. That is, offering EWs may help the firms form and improve the competitive advantage. Nowadays, an increasing number of manufacturers, such as Apple, Samsung, HP, and so on, offer their respective EWs to consumers. However, offering EWs is not without its drawbacks for manufacturers. Establishing a dedicated

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[1] School of Economics and Management/Jiangsu University of Science and Technology, Zhenjiang 212003, P.R. China.

[2] Department of Industrial and Manufacturing Systems Engineering/University of Windsor, Windsor, Canada.

*Corresponding author: ytf_jj@163.com

department to handle warranty-related issues incurs additional costs. Additionally, repair expenses during the warranty period can be substantial, potentially reducing the overall profitability of EWs.

Furthermore, an increasing number of manufacturers are expanding their market presence by developing direct sales channels or utilizing third-party marketplace platforms. This trend is largely driven by advancements in information technology and logistics. Companies like Huawei, Nike, Nikon, and HP have embraced this strategy. While such encroachment enables manufacturers to obtain more accurate demand data and potentially improve profit margins through direct sales [7], it also involves higher logistics costs, including inventory management and shipping. Thus, like EWs, market encroachment also carries its own set of challenges for manufacturers.

Manufacturers face decisions about whether to offer EWs and whether to expand their market presence through direct sales. Different firms across various industries adopt unique combinations of these strategies. For example, in the United States, Electrolux and Bosch exclusively distribute their products through authorized retailers but offer EWs directly through their websites or by phone. These companies choose to avoid direct market encroachment while still providing EWs. Conversely, some firms opt for market encroachment without offering EWs. Sony, for instance, initially relied on independent retailers for product distribution but offered EWs directly to consumers [8]. Recently, Sony has transitioned to a dual-channel strategy, distributing products through both retail and independent direct channels. Additionally, Sony has integrated the EW into the base warranty, effectively extending the base warranty period and eliminating the separate EW option. This shift represents a move from offering EWs without market encroachment to encroaching on the market without providing EWs. There are also companies that both encroach on the market and offer EWs. In the smartphone industry, for example, Huawei and Samsung have established direct sales channels and simultaneously provide their own EWs.

Given these varying cases, a natural question arises: why do firms choose different strategies? This study aims to explore this by examining the interplay between offering EW and manufacturer encroachment. Our primary research questions are: (1) How does the introduction of an EW influence a manufacturer's motivation for market encroachment? (2) How does market encroachment affect a manufacturer's decision to offer EWs? (3) What is the nature of the interaction between encroachment strategy and EW provision?

Through a detailed examination of these dynamics, we seek to provide new insights to guide the manufacturer in its decisions regarding offering EW and market encroachment. To achieve this, we analyze a two-echelon supply chain consisting of one manufacturer and one retailer. We develop four distinct models to explore scenarios based on whether manufacturer engages in market encroachment and offering the EW. These models, framed as manufacturer-Stackelberg games, includes: one without encroachment and EW, one with encroachment but without EW, one with EW but without encroachment, and one with both encroachment and EW. By deriving the equilibrium in these models, we investigate how different combinations of EW provision and market encroachment affect the manufacturer's profitability.

Our analysis shows that when EW is not offered, the manufacturer can benefit from encroachment if the selling cost of the product is not sufficiently high. Furthermore, when the manufacturer sells EW at a low cost, there exists a threshold for the selling cost of the product below which the manufacturer is better off from encroachment. The retailer can benefit from manufacturer encroachment only when the manufacturer's selling cost is moderate, irrespective of whether EW is offered. Furthermore, we find that when the selling cost of EW is low, the manufacturer's motivation for encroachment can be strengthened by offering the EW. As the selling cost of EW becomes moderate to high, the manufacturer's motivation for encroachment is weakened by offering the EW. We also find that depending on the values of the co-payment rate and the manufacturer's selling costs of the product and EW, these two strategies may be either complementary or substitutable. Specifically, it is shown that, from the manufacturer's perspective, encroachment and offering EW cannot reinforce each other in most cases.

This study makes several contributions to existing literature. First, while there is ample literature on manufacturer encroachment, none explores the impact of introducing the manufacturer's EW on the encroachment decision. Second, existing studies on EW distribution assume fixed product sales channel structures, whereas our study investigates manufacturer's introduction of an EW under different channel structures. Finally, manufacturer encroachment and EW provision are typically studied separately. To the best of our knowledge, our study is among the first to integrate these two streams of research and focus on the interactions between encroachment and EW provision.

The remainder of this paper is organized as follows. In the next section, we review related literature. After introducing the basic model in Section 3, we provide the equilibrium results for different models in Section 4. Given these results, we examine whether the manufacturer's encroachment decision varies regarding the provision of EW and how encroachment affects the manufacturer's decision to introduce EW in Section 5. Section 6 explores the interaction between EW provision and encroachment. Finally, we summarize our results and discuss future research directions in the last section. The equilibriums results and proofs are provided in the Appendices A and B, respectively.

2. Literature review

Research related to our work can be classified into two main streams: manufacturer encroachment and EW provision. Below, we review the pertinent studies within each of these areas separately.

2.1. Manufacturer encroachment

The first stream of related research focuses on manufacturer/supplier encroachment. Many studies have concentrated on whether and when manufacturers and retailers can benefit from encroachment. Chiang *et al.* [7] demonstrate that direct marketing can enhance a manufacturer's profitability by reducing double marginalization. Under certain conditions, encroachment benefits the retailer as well, due to the manufacturer lowering the wholesale price to mitigate the channel conflict. Arya *et al.* [1] show that both manufacturers and retailers can benefit from encroachment when the manufacturer is an ineffective retail competitor. Cattani *et al.* [6] propose a specific equal-pricing strategy that simultaneously optimizes the manufacturer's profit and benefits the retailer when the manufacturer encroaches on the market. Balasubramanian and Ponnachiyur Maruthasalam [2] argue that the coexistence of manufacturer encroachment and the retailer store brands can generate a win–win situation for both parties.

Some studies explore the quality decisions and information sharing involved in manufacturer encroachment. Zhang *et al.* [37] analyze the impact of manufacturer encroachment with endogenous quality decisions and information structure on quality and supply chain members, revealing that encroachment is always advantageous for the manufacturer, however, it only benefits the retailer when the manufacturer's direct selling cost is moderate. Wang and Zhuo [33] focus on different encroachment and information-sharing scenarios to investigate the conditions under which the supplier prefers to encroach and the retailer prefers to share demand information. They find that encroachment benefits both parties when the supplier's selling cost is either excessively low or high. Ha *et al.* [12] investigate the interaction between the retail platform information sharing decision and manufacturer encroachment decision which suggests these two decisions are complementary. Wang *et al.* [34] investigate the manufacturer encroachment with endogenous quality investment decision in a two-period supply chain. They find that encroachmentment is always optimal for the manufacturer and not always detrimental to retailer. Xue and Zhang [36] examine the manufacturer's encroachment decision by considering product quality as an endogenous variable under different power structures within the supply chain. They prove that if the manufacturer acts the leader in supply chain, encroachment could benefit both the manufacturer and the retailer, particularly with high efficiency of quality investment. Unlike previous studies that examined manufacturers' encroachment through their own channels, Fu *et al.* [9] investigates manufacturer's encroachment decision through online platform and analyzes manufacturer's encroachment decision in scenarios where the platform retains or forgoes its resell option. They found that, beyong the traditional "win–win" outcome between manufacturer and traditional retailer, manufacturer's encroachment on online platforms can also lead to "win–win" or "lose-lose" outcomes.

Among studies on manufacturer encroachment considering competition, Ponnachiyur Maruthasalam and Balasubramanian [30] examine the supplier's encroachment strategy within asymmetric retail competition. Their analysis reveals that asymmetric retail competition might lead the supplier to engage in encroachment and the efficient retailer can benefit from supplier encroachment, regardless of the quantity of competing retailers. There are literature focusing on the impact of encroachment in situation of coordination. Based on four supply chain structures with and without coordination, Cai [5] studies two Pareto zones concepts, the channel-adding Pareto zone and the contract-implementing Pareto zone. He demonstrates that when the retail channel has more cost advantage, both manufacturer and retailer can benefit from encroachment. Under a green supply chain framework, Hou *et al.* [16] examine the impact of collaborative promotion and the retailer's risk aversion behavior on manufacturer encroachment. The findings indicate that a cost-sharing contract can facilitate Pareto improvement for both the manufacturer and the retailer when the manufacturer encroaches.

Existing studies have primarily focused on the impact of encroachment on the profits of supply chain members, often neglecting the role of EW. Our study makes contributions by addressing these gaps in the literature. First, we integrate EW provision into the analysis of encroachment strategy, examining conditions under which either the manufacturer or retailer might benefit from encroachment, considering both scenarios with and without EW provision. Second, our research introduces a novel perspective by investigating how EW provision affects a manufacturer's decision to pursue encroachment. Third, we explore how the encroachment decision influences EW pricing. By addressing these overlooked aspects, our study offers a distinct context and raises new questions that extend beyond the current literature.

2.2. EW provision

The literature on EW primarily focuses on EW pricing decisions [13, 26, 29, 39, 41, 42] and their design [3,10,18,32]. Additionally, some studies have examined supply chain coordination involving EWs [20,27,35] and strategies for EW distribution [8,22,38].

The existing studies on EW provision remain relatively sparse. Some research examines EW provision within traditional supply chain frameworks. He *et al.* [14] investigate which of the manufacturers and retailers are better at providing EW and the effect of offering EW on supply chain profits. They find that the optimal EW provider is determined by customers' channel preferences. Liu *et al.* [43] examine the competition between the manufacturer and the retailer in offering EW, suggesting that introducing the retailer's EW does not always harm the manufacturer's profitability. Jiang and Zhang [19] analyze the effect of a retailer's EW on the manufacturer's base warranty and show that the manufacturer prefers to offer a base warranty only when it possesses a cost advantage in providing a warranty relative to the retailer. Other studies explore EW provision from the perspective of platform supply chains. Li *et al.* [23] focus on the conditions under which the manufacturer and platform provide the EW. They find that when the potential market is sufficiently large, both parties prefer providing EW; when the potential market is quite small, neither provides EW; and when the potential market is moderate, only one provides EW. Liu *et al.* [25] investigate optimal pricing and strategy selection for EW on the retail platform when one competing manufacturer offers bundled EW products while another manufacturer offers products without EW. The retail platform offering EW would harm the profit of the manufacturer who bundles EW with the product, and benefit the manufacturer who sells the product without EW. Zhang *et al.* [40] explore the optimal EW provision strategy in the platform supply chain under two online channel modes-agency mode and reselling mode-where both the platform and manufacturer can underwrite EW.

These studies mentioned focus on EW offerings within traditional or platform supply chains. In contrast, our research examines how a manufacturer's decision to offer EW is influenced by the presence of encroachment through a direct channel. Specifically, we investigate how encroachment impacts EW offering strategy and how EW provision interacts with manufacturer encroachment. These aspects are not addressed in the existing literature, highlighting the significant distinction between our study and previous research. In this direction, the study most relevant to ours is Chai *et al.* [21]. They discuss the retailer's decisions regarding whether to sell its own EW or resell the manufacturer's EW, given that the manufacturer encroaches on the market. Their findings suggest that selling the retailer's EW is profitable for both parties, regardless of whether it is offered by the manufacturer or the retailer. Different from their study, our research investigates how manufacturer encroachment affect its EW provision strategy, as well as how EW provision influences the manufacturer's encroachment decision. Our research context and focus are considerably different from those in Chai *et al.* [21].

Generally, the issues of both encroachment decision and EW provision have been well studied in the literature, nevertheless, none has analyzed their interactions. There is a lack of comprehensive studies on how offering EW influences the incentives for manufacturer encroachment, or how encroachment affects manufacturer's motivations to provide EW. Our research address these gaps by examining both strategies together to further shed light on the mutual influence and interactions between them, revealing insights that diverge from current findings. Additionally, our study provides valuable insights for manufacturers, indicating that under certain conditions, the combination of encroachment and EW provision can yield super-additive benefits, offering guidance for the manufacturer's strategic decision-making.

3. The model

Consider a two-echelon supply chain consisting of a manufacturer and a retailer, wherein the manufacturer distributes the repairable product through the retailer. The product is subject to a certain failure probability, $\alpha \in (0, 1)$, which is known to all but does not change over time. As in Mai *et al.* [27], the unit cost of the product is assumed to be $(1-\alpha)c$, where c is the unit cost to ensure that the product has zero failure. The manufacturer decides whether to directly open its own channel to customers. If the manufacturer encroaches on the market, it incurs a cost S when selling one product unit.

Moreover, the manufacturer may offer an EW that protects consumers against losses from failure. If a customer buys an EW, the manufacturer is responsible for repairing or replacing it once the product fails during the EW period. Moreover, we assumed that the product failed once at most during the EW period [27]. This assumption is realistic because the EW period is generally shorter than the lifetime of the product. Similar to Desai and Padmanabhan [8], the manufacturer's selling cost for each EW unit is assumed to be s. In this study, we assume $s < S$, which is based on the empirical reality that the sales process for small durable products, inclusive of advertising and distribution, is more intricate than that of EW.

According to the manufacturer's encroachment and EW provision decision, the following four models (see Fig. 1) must be discussed.

- Model NN. The manufacturer sells the product only through the retailer and does not offer EW.
- Model NM. The manufacturer distributes the product exclusively through the retailer and simultaneously sells EW directly to customers.
- Model EN. The product is sold in both the retail and direct channels, and the manufacturer does not offer EW.
- Model EM. The manufacturer distributes the product through both retail and direct channels and sells the EW directly to customers.

In each model, the retailer wholesales the product from the manufacturer at the unit cost w and then resells it to customers. To focus on the effect of the selling cost on the manufacturer's decision and to retain analytical tractability, we assume that the retailer's selling cost is zero. This result implies that the retailer has selling advantage over the manufacturer. This assumption is natural because the retailer specializes in marketing and possesses a more efficient sales network than the manufacturer. We assume that a market with a normalized size of one. First, consumers decide whether to buy a product. Once a customer purchases a product, he/she also needs to decide whether to buy EW. Each consumer buys at most one unit of the product and one unit of EW. The product offers consumer a monetary benefit v if effective. If the product breaks, consumer bears all repair cost r without EW. We use a monetary transfer to model EW compensation, $(1-\beta)r$, which is borne by the manufacturer, where $\beta \in (0, 1)$ is the customer's co-payment rate. Therefore, when the customer purchases EW, he/she only needs to pay βr when the product fails, which represents the net customer liability. In this study, to ensure that the quantity decisions of the product are positive, we assume that the parameter satisfies $(1-\alpha)(v-c) - \alpha r - s - S > 0$.

The customers are heterogeneous in risk aversion. Similar to Bian *et al.* [3] and Jiang and Zhang [19], we assume that the consumer utility function exhibits constant absolute risk aversion (CARA) and is denoted by

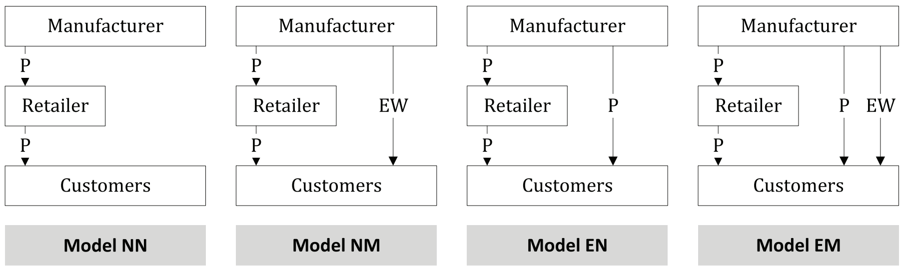

FIGURE 1. Manufacturer's distribution strategies, where P and EW correspond to the product and the extended warranty, respectively.

$\theta \in (0,1)$ the degree of risk aversion. Accordingly, the utility of a consumer who only purchases the product can be written as

$$u_P = (1-\alpha)v - \alpha r - p_P - \theta\alpha(1-\alpha)A, \tag{1}$$

where $A = (v+r)^2$ and p_P is the selling price of the product. In (1), the term $(1-\alpha)v - \alpha r - p_P$ is the risk-neutral customer's expected utility from purchasing the product at the retail price p_P, and $\theta\alpha(1-\alpha)A$ represents the disutility stemming from the risk of product failure, where $\alpha(1-\alpha)A$ denotes the variance of the "lottery" [3,8,27]. By contrast, the utility of the customer who purchases both the product and the EW is

$$u_{PS} = (1-\alpha)v - \alpha\beta r - p_P - p_S - \theta\alpha(1-\alpha)B, \tag{2}$$

where $B = (v+\beta r)^2$ and p_S is the sales price of EW. It's important to highlight that A and B themselves do not hold any real-world significance which are introduced solely for expressional and computational convenience. In particular, B increases as β increases, indicating that the cost borne by consumers for product breakdowns after purchasing the EW increases, while the responsibility undertaken by the manufacturer decreases. Therefore, in the subsequent analysis, we use the parameter B to represent the magnitude of β.

Given (1) and (2), the indifference point between purchasing the product alone and purchasing both the product and the EW can be obtained by setting $u_P = u_{PS}$, which yields $\theta_1 = \frac{p_S - \alpha r(1-\beta)}{\alpha(1-\alpha)(A-B)}$. Consumers with $\theta < \theta_1$ will buy the product alone. Moreover, the indifference point between purchasing both the product and the EW and buying nothing is $\theta_2 = \frac{(1-\alpha)v - \alpha r\beta - p_P - p_S}{\alpha(1-\alpha)B}$, which is obtained by setting $u_{PS} = 0$. Consumers with $\theta \in (\theta_1, \theta_2]$ will buy both the product and the EW, while consumers with $\theta \in (\theta_2, 1]$ are risk-averse because they buy nothing at all. Accordingly, the EW demand is in the $q_S = \theta_2 - \theta_1 = \frac{(A-B)(v(1-\alpha)-p_P)+(B-A\beta)r\alpha - Ap_S}{B(A-B)(1-\alpha)\alpha}$. The demand for the product is $q_P = \theta_2 = \frac{(1-\alpha)v - \alpha\beta r - p_P - p_S}{\alpha(1-\alpha)B}$, which includes all consumers who buy the product alone and those who buy both the product and the EW. Specifically, $q_P = q_{pr} + q_{pm}$ in Models EN and EM, where q_{pr} and q_{pm} are the retail and direct quantities of the product, respectively. Accordingly, we have the following inverse demand function of the product and the demand function of the EW,

$$p_P = (1-\alpha)v - \alpha r\beta - p_S - \alpha(1-\alpha)Bq_P, \tag{3}$$

$$q_S = \frac{\alpha r(1-\beta) - p_S + \alpha(1-\alpha)(A-B)q_P}{\alpha(1-\alpha)(A-B)}. \tag{4}$$

The list of the notations are summarized in Table 1.

4. Equilibrium results

In this section, we derive the equilibrium decisions of both the manufacturer and the retailer in different models. The equilibrium profits of the two players were used in subsequent analyses. In each model, we model

Table 1. Description of notations.

Notation	Description
α	Failure probability
c	The unit cost to ensure that the product has zero failure
S	Product selling cost in direct channel
s	EW selling cost
v	The value of usage when product works well
r	Repair cost of the product
β	After purchasing EW, the customer's co-payment rate
θ	Customer risk aversion coefficient
p_P	The selling price of the product
p_S	The sales price of the EW
q_{pr}	Retail quantity of the product
q_{pm}	Direct quantity of the product
π_r	The retailer's profit function
π_m	The manufacturer's profit function

the interactions between the manufacturer and retailer as a Stackelberg game, in which the manufacturer acts as the leader and the retailer as the follower.

4.1. Model NN

In model NN, the product was offered only through the retail channel. Moreover, there was no EW in the market. The game between the manufacturer and the retailer is conducted in two stages. In the second stage, after observing the wholesale price, the retailer determines the order quantity of the product to maximize

$$\pi_r(q_p) = (p_P - w)q_p.$$

From the first-order condition, we obtain $q_p^{\mathrm{NN}}(w) = \frac{(1-\alpha)v-\alpha r-w}{2\alpha(1-\alpha)A}$. In the first stage, anticipating the retailer's best response, the manufacturer sets the wholesale price of the product as

$$\begin{aligned} &\max_w \pi_m(w) = (w-(1-\alpha)c)\frac{(1-\alpha)v-\alpha r-w}{2\alpha(1-\alpha)A} \\ &\text{s.t. } w \leqslant (1-\alpha)v - \alpha r, \end{aligned}$$

where the constraint is used to ensure that demand in the retail channel remains non-negative. By solving the manufacturer's optimization problem, we obtain the following results.

Lemma 1. *In model NN, the manufacturer and retailer obtain*

$$\pi_m^{\mathrm{NN}} = \frac{\Phi^2}{8\alpha(1-\alpha)A} \quad \text{and} \quad \pi_r^{\mathrm{NN}} = \frac{\Phi^2}{16\alpha(1-\alpha)A}$$

respectively, in equilibrium, where $\Phi = (1-\alpha)(v-c) - \alpha r$.

4.2. Model EN

In model EN, the product is offered through both the direct and retail channels. Therefore, the total product supply in the market is $q_P = q_{pr}+q_{pm}$. In this model, the game between the manufacturer and the retailer consists of three stages. In the final stage, after observing the retailer's ordering decision, the manufacturer determines the direct quantity to maximize $\pi_m(q_{pm}) = (w-(1-\alpha)c)q_{pr} + (p_P-(1-\alpha)c-S)q_{pm}$. Differentiating $\pi_m(q_{pm})$

from q_{pm} and setting it to zero yields $q_{pm}^{\text{EN}}(q_{pr}) = \Phi - S - \alpha(1-\alpha)Aq_{pr}/2\alpha(1-\alpha)A$. In the second stage, anticipating the manufacturer's quantity decision and observing the wholesale price, the retailer decides its order quantity to maximize

$$\pi_r(q_{pr}) = \left(\frac{(1-\alpha)(v+c) - \alpha r + S - \alpha(1-\alpha)Aq_{pr}}{2} - w\right)q_{pr}.$$

From the first-order condition, we obtain $q_{pr}^{\text{EN}}(w) = \frac{(1-\alpha)(v+c)-\alpha r+S-2w}{2\alpha(1-\alpha)A}$. In the first stage, anticipating the retailer's best response, the manufacturer sets the wholesale price of the product as

$$\max_w \pi_m(w) = (w-(1-\alpha)c)\frac{(1-\alpha)(v+c)-\alpha r+S-2w}{2\alpha(1-\alpha)A} + \frac{((1-\alpha)(v-3c)-\alpha r-3S+2w)^2}{16\alpha(1-\alpha)A}$$
$$\text{s.t.} \quad \begin{cases} w \leqslant \frac{(1-\alpha)(v+c)-\alpha r+S}{2} \\ w \geqslant \frac{\alpha r+3S-(1-\alpha)(v-3c)}{2} \end{cases}$$

where the constraints are used to guarantee non-negative available quantities in both channels. By solving the manufacturer's constrained optimization problem, we obtain the following results.

Lemma 2. *In model EN, the equilibrium profits of the manufacturer and retailer are*

$$\pi_m^{\text{EN}} = \begin{cases} \frac{7S^2+3\Phi(\Phi-2S)}{12\alpha(1-\alpha)A}, & \text{if } S \leqslant \frac{3\Phi}{5} \\ \frac{(S-\Phi)(\Phi-3S)}{2\alpha(1-\alpha)A}, & \text{if } S > \frac{3\Phi}{5} \end{cases} \quad \text{and} \quad \pi_r^{\text{EN}} = \begin{cases} \frac{2S^2}{9\alpha(1-\alpha)A}, & \text{if } S \leqslant \frac{3\Phi}{5} \\ \frac{(\Phi-S)^2}{2\alpha(1-\alpha)A}, & \text{if } S > \frac{3\Phi}{5} \end{cases},$$

respectively.

4.3. Model NM

In model NM, the product is exclusively sold through the retail channel. Following the purchase, consumers can register online or *via* phone to acquire an EW directly from the manufacturer. This practice mirrors that of companies such as Electrolux and Bosch, where products are retailed but EW is exclusively available from the manufacturer *via* their official website or customer service hotline. The game between the manufacturer and the retailer involves three stages. In the final stage, after observing the retailer's order quantity of the product, the manufacturer decides on the selling price of EW to maximize $\pi_m(p_S) = (w-(1-\alpha)c)q_p + (p_S - \alpha(1-\beta)r - s)\frac{\alpha(1-\beta)r - p_S + \alpha(1-\alpha)(A-B)q_p}{\alpha(1-\alpha)(A-B)}$. Differentiating $\pi_m(p_S)$ from p_S and setting it to zero yields $p_S^{\text{NM}}(q_p) = \frac{s+2r\alpha(1-\beta)+\alpha(1-\alpha)(A-B)q_p}{2}$. In the second stage, by anticipating the selling price of EW and observing the wholesale price of the product, the retailer determines the order quantity of the product to maximize

$$\pi_r(q_p) = \left(\frac{2((1-\alpha)v-\alpha r) - s - \alpha(1-\alpha)(A+B)q_p}{2} - w\right)q_p.$$

From the first-order condition, we obtain $q_p^{\text{NM}}(w) = \frac{2((1-\alpha)v-\alpha r-w)-s}{2\alpha(1-\alpha)(A+B)}$. In the first stage, in anticipation of the retailer's best response, the manufacturer sets the wholesale price of the product as

$$\max_w \pi_m(w) = (w-(1-\alpha)c)\frac{2((1-\alpha)v-\alpha r-w)-s}{2\alpha(1-\alpha)(A+B)} + \frac{(2(A-B)((1-\alpha)v-\alpha r-w)-(3A+B)s)^2}{16\alpha(1-\alpha)(A+B)^2(A-B)}$$
$$\text{s.t.} \quad w \leqslant (1-\alpha)v - \alpha r$$

where constraint is used to ensure that the retail quantity is non-negative. By solving the manufacturer's constrained optimization problem, we obtain the following results.

Lemma 3. *In model NM, the manufacturer's and retailer's equilibrium profits are*

$$\pi_m^{\mathrm{NM}} = \frac{(7A+B)s^2 - 8(A-B)s\Phi + 4(A-B)\Phi^2}{4\alpha(1-\alpha)(A-B)(3A+5B)} \quad \text{and} \quad \pi_r^{\mathrm{NM}} = \frac{2(A+B)(\Phi-s)^2}{\alpha(1-\alpha)(3A+5B)^2},$$

respectively.

4.4. Model EM

In model EM, the manufacturer encroaches on the market and simultaneously provides EW to consumers directly. In this model, the total product supply in the market is $q_P = q_{pr} + q_{pm}$. The game between the manufacturer and the retailer takes place in three stages. In the last stage, after observing the retailer's ordering decision, the manufacturer simultaneously decides on the direct quantity of the product and the selling price of the EW to maximize

$$\begin{aligned}\pi_m(q_{pm}, p_S) &= (w-(1-\alpha)c)q_{pr} + ((1-\alpha)(v-c) - \alpha\beta r - p_S - \alpha(1-\alpha)B(q_{pr}+q_{pm}) - S)q_{pm} \\ &\quad + (p_S - \alpha(1-\beta)r - s)\frac{(\alpha(1-\beta)r - p_S + \alpha(1-\alpha)(A-B)(q_{pr}+q_{pm}))}{\alpha(1-\alpha)(A-B)}.\end{aligned}$$

Differentiating $\pi_m(q_{pm}, p_S)$ from q_{pm} and p_S and setting them to zero yields $q_{pm}^{\mathrm{EM}}(q_{pr}) = \frac{\Phi - s - S - B\alpha(1-\alpha)q_{pr}}{2\alpha(1-\alpha)B}$ and $p_S^{\mathrm{EM}}(q_{pr}) = \frac{s + 2\alpha r(1-\beta) + \alpha(1-\alpha)(A-B)q_{pr}}{2}$. In the second stage, by anticipating the manufacturer's decisions on the direct quantity of the product and the selling price of EW and observing the wholesale price of the product, the retailer decides the order quantity of the product to maximize

$$\pi_r(q_{pr}) = \left(\frac{(1-\alpha)(v+c) - \alpha r + S - A(1-\alpha)\alpha q_{pr}}{2} - w\right)q_{pr}.$$

From the first-order condition, we obtain $q_{pr}^{\mathrm{EM}}(w) = \frac{(1-\alpha)(v+c) - \alpha r + S - 2w}{2\alpha(1-\alpha)A}$. Finally, in the first stage, anticipating the retailer's best response, the manufacturer sets the wholesale price of the product as

$$\begin{aligned}\max_w \pi_m(w) &= (w-(1-\alpha)c)\frac{(1-\alpha)(v+c) - \alpha r + S - 2w}{2\alpha(1-\alpha)A} \\ &\quad + \frac{(1-\alpha)(v-3c) - \alpha r - 3S + 2w}{4} \cdot \frac{(2A(\Phi - s - S) - (B)(S-2w) - B((1-\alpha)(v+c) - \alpha r))}{4\alpha(1-\alpha)AB} \\ &\quad + \frac{((A-B)((1-\alpha)(v+c) - \alpha r - 2w + S) - 2As)}{4A} \cdot \frac{(A-B)(\Phi - S) - As}{2\alpha(1-\alpha)B(A-B)} \\ \text{s.t.} &\quad \begin{cases} w \leqslant \dfrac{(1-\alpha)(v+c) - \alpha r + S}{2} \\ w \geqslant \dfrac{B((1-\alpha)(v+c) - \alpha r) - 2A\Phi + 2As + (2A+B)S}{2B} \end{cases}\end{aligned}$$

where the constraints are used to ensure non-negative direct and retail quantities. Solving the manufacturer's constrained optimization problem yields the following results.

Lemma 4. *In model EM, the manufacturer's and retailer's equilibrium profits are*

$$\pi_m^{\mathrm{EM}} = \begin{cases} \dfrac{\begin{pmatrix} 3A^2s^2 - 6A(A-B)(\Phi-S)s + (A-B)(3A+4B)S^2 \\ +3A(A-B)\Phi^2 - 6A(A-B)\Phi S \end{pmatrix}}{12A(A-B)B(1-\alpha)\alpha}, \\ \text{if } \frac{\Phi}{2} \leqslant S < \frac{3A\Phi}{3A+2B} \quad \text{and} \quad s \leqslant \frac{3A\Phi-(3A+2B)S}{3A} \\ \dfrac{\begin{pmatrix} 2(A-B)(((3A-B)\Phi-(3A+B)S)s + (3A+B)\Phi S) \\ -A(3A-4B)s^2 - (A-B)(3(A+B)S^2 + (3A-B)\Phi^2) \end{pmatrix}}{4(A-B)B^2(1-\alpha)\alpha}, \\ \text{if } \frac{\Phi}{2} \leqslant S < \frac{3A\Phi}{3A+2B} \quad \text{and} \quad \frac{3A\Phi-(3A+2B)S}{3A} < s < \Phi - S \end{cases}$$

and

$$\pi_r^{\mathrm{EM}} = \begin{cases} \frac{2s^2}{9A(1-\alpha)\alpha}, & \text{if } \frac{\Phi}{2} \leqslant S < \frac{3A\Phi}{3A+2B} \quad \text{and} \quad s \leqslant \frac{3A\Phi-(3A+2B)S}{3A} \\ \frac{A(\Phi-s-S)^2}{2B^2(1-\alpha)\alpha}, & \text{if } \frac{\Phi}{2} \leqslant S < \frac{3A\Phi}{3A+2B} \quad \text{and} \quad \frac{3A\Phi-(3A+2B)S}{3A} < s < \Phi - S \end{cases},$$

respectively.

5. Analysis

Based on the above results, we now discuss the manufacturer's decisions regarding EW provision and encroachment.

5.1. EW provision

In the following section, we divide the analysis of the manufacturer's decisions on EW provision into two cases: with and without encroachment. The results without manufacturer encroachment are provided in the following proposition.

Proposition 1. *Without encroachment, when the selling cost satisfies $s < \Phi$, the manufacturer always finds it optimal to offer the EW. Moreover, when $s \leqslant \frac{\left(8A(A+B)-\sqrt{2A(A+B)(3A+5B)}\right)\Phi}{8A(A+B)}$, the retailer can benefit from the manufacturer's EW provision only.*

Proposition 1 states that a manufacturer can always benefit from offering EW when it chooses not to encroach on the market. When the product is distributed only through the retail channel, it can be found from Lemmas 1 and 3 that the manufacturer charges a lower wholesale price for the product in model NM than in model NN. Note that the retailer determines the order quantity of the product, which in turn determines the potential demand for EW in model NM. Consequently, a lower wholesale price encourages the retailer to decrease the selling price, attracting more customers to purchase the product. This, in turn facilitates EW demand. Consequently, manufacturers can enjoy the large sales of the EW.

When the selling cost of EW is not sufficiently high, the manufacturer may charge a lower EW price to make it more attractive to customers. This leads to customers with high risk aversion buying both the product and EW. In other words, a low price of the EW can facilitate the sales of the EW and indirectly boosts product sales. Consequently, the retailer can benefit from increased product demand.

Next, we discuss the manufacturer's EW provision decision under encroachment. The results are presented in the following proposition.

Proposition 2. *Denote $s_1 = \frac{3(A-B)((3A-B)\Phi-(3A+B)S)+B\sqrt{3(A-B)B}(3\Phi-5S)}{3A(3A-4B)}$, and suppose $B < \frac{3A}{10}$. Under encroachment, the manufacturer finds it optimal to offer the EW if $\frac{\Phi}{2} \leqslant S \leqslant \frac{5\Phi}{6}$ and $s \leqslant \min\left\{s_1, \frac{(A-B)(\Phi-S)}{A}\right\}$. Moreover, the retailer can benefit from the manufacturer's EW provision only when $\frac{3\Phi}{5} < S < \frac{5\Phi}{6}$ and $s \leqslant \frac{(A-B)(\Phi-S)}{A}$.*

Proposition 2 indicates that the manufacturer can benefit from offering EW only when the selling cost of the product is medium to high, and the selling cost of EW is not sufficiently high. When the selling cost of EW is low, the manufacturer charges a low EW price. This can stimulate the demand for EW and indirectly facilitate product sales. In other words, the manufacturer can now enjoy high product demand and high EW demand. Another finding presented in Proposition 2 is that when the manufacturer's selling cost of the product is relatively high, offering EW can lead to a win–win outcome for both the manufacturer and the retailer. When a manufacturer's selling efficiency is low, most product sales occur in the retail channel. In this situation, the retailer receives most of the additional product demand because of the manufacturer's EW provision. Therefore, offering EW is certainly a warm welcome for the retailer.

Proposition 1 states that the manufacturer always prefers to offer EW under no encroachment, whereas we find from Proposition 2 that under encroachment, the manufacturer offers EW only when the selling costs of the product and EW are not too high. This implies the following finding.

Remark 1. The manufacturer's motivation to offer the EW is weakened by encroachment.

The results of Remark 1 can be interpreted as follows. Under no encroachment, the manufacturer relies solely on the retail channel to sell its products. In this scenario, offering EW can simultaneously bring the manufacturer additional profits and facilitate retail demand. Given the latter effect, the manufacturer charges a higher wholesale price. That is, offering the EW gives the manufacturer additional profits from selling the EW, the large product demand, and the high profit margin of the product. As a result, the manufacturer always finds it optimal to offer EW.

By contrast, under encroachment, the manufacturer must lower the wholesale price to alleviate channel conflicts. In other words, the retailer can obtain the product at a lower cost relative to no encroachment. Moreover, offering the EW always leads to an increase in product demand. When the manufacturer's selling cost of the product is high, most of the product demand is satisfied by the retailer. In this situation, the retailer may charge a high selling price for the product owing to its dominant position in the market. This indicates that the manufacturer lowers the wholesale price, which does not necessarily lead to high product demand. Specifically, when the selling cost of EW is high, the manufacturer's profit from selling EW is very less and cannot cover the loss owing to the lowered wholesale price of the product. Therefore, the manufacturer does not offer EW.

There is a real-world example that illustrates the above conclusion. Previously, Sony exclusively distributed its products through independent retailers while directly marketing EW to consumers [8]. However, Sony has since transitioned to encroach on the market to sell products through dual channel and no longer offers the option to purchase EW separately on its official website. This strategic adjustment demonstrates how changes in product market structure can influence a manufacturer's decision regarding EW offerings.

5.2. Manufacturer encroachment

In this subsection, we investigate the manufacturer's encroachment decisions with and without EW and discuss whether and how offering the EW affects them.

Proposition 3. *When the EW is not offered, the manufacturer encroaches on the market if and only if* $S \leqslant \frac{5\Phi}{6}$*. Moreover, the retailer can benefit from manufacturer encroachment when* $\frac{3\Phi}{4\sqrt{2}} \leqslant S \leqslant \frac{(4-\sqrt{2})\Phi}{4}$.

Proposition 3 provides the conditions under which the manufacturer prefers encroachment to no encroachment. This indicates that the manufacturer can benefit from encroachment if the manufacturer's selling cost of the product is not sufficiently high. When the manufacturer encroaches on the market, as the manufacturer's selling cost of the product gradually increases from low to moderate, the wholesale price may be lowered to mitigate the channel conflicts between the direct and retail channels. There is a trade-off between margin benefits and scale benefits. Under these circumstances, encroachment brings the manufacturer more benefits from economies of scale because of the increased total quantity of the product, which can offset the loss from the reduced wholesale price. However, as the selling cost increases, the manufacturer may raise the wholesale price in pursuit of greater profitability from the retailer, which may exacerbate double marginalization and prompt the retailer to reduce the order quantity of the product, potentially leading to a decrease in overall product demand. Nonetheless, the manufacturer can still benefit from encroachment because of the marginal advantages it offers.

Specifically, opening the direct channel has little effect on retail demand only when the manufacturer's selling cost is relatively moderate and the difference between the retailer's advantage in marketing and the manufacturer's is moderate. As a result, the retailer would welcomes manufacturer encroachment because it can gain more from the lowered wholesale price and increased retail demand.

In the next proposition, we provide the manufacturer's encroachment decisions, given that EW is offered and the conditions under which the retailer prefers encroachment to no encroachment.

Proposition 4. *Denote* $S_1 = \frac{\left((3A+5B)(3A+B)+2B\sqrt{2B(3A+5B)}\right)(\Phi-s)}{3(A+B)(3A+5B)}$, $S_2 = \frac{\left(A(3A+5B)-2B\sqrt{A(A+B)}\right)(\Phi-s)}{A(3A+5B)}$, *and suppose* $s < \frac{3A\Phi}{6A+2B}$. *When the EW is offered, the manufacturer prefers to encroach on the market if and only if* $S < S_1$. *Moreover, the retailer can benefit from manufacturer encroachment if* $\max\left\{s, \frac{3\sqrt{A(A+B)}(\Phi-s)}{3A+5B}\right\} \leqslant S \leqslant S_2$.

Proposition 4 states that when EW is provided and the manufacturer can sell it at a low cost, there is a threshold for the selling cost of the product below which the manufacturer is better off from encroachment. Owing to the low selling cost of EW, the manufacturer charges a low EW price, which makes it very attractive to consumers. Because the selling cost of the product is also low, the manufacturer will make full use of the complementary effect between the product and EW to place a large quantity of the product in the direct channel. At this time, encroachment can reduce the manufacturer's reliance on the retailer to sell products efficiently. This leads to that the manufacturer can charge a high wholesale price to increase profits from the retail channel. In summary, offering EW gives the manufacturer an incentive to encroach on the market.

When the selling cost of the product is sufficiently high, the manufacturer would decrease the available quantity of the product in the direct channel. In this situation, the manufacturer's profit from selling the product directly may be negligible, and most product sales occur in the retail channel. Now, the manufacturer must lower the wholesale price to alleviate channel conflict due to encroachment and induce the retailer to order more. In this situation, the negative effect of encroachment on wholesale prices is evident. Although the manufacturer can sell EW at a low price and generate a positive effect on the wholesale price, the positive effect is weaker than the negative effect due to encroachment. In other words, encroachment lowers the manufacturer's profit; thus, the manufacturer would distributes the product only through the retail channel.

These results can be illustrated by the following industrial practices. For instance, Huawei encroaches on the market by selling products through both direct and retail channels, while offering its EW, Huawei Care+, through the direct sales channel. In contrast, Electrolux offers its EW through direct sales channel but sells its products exclusively through retail channels. The selling cost of home appliances, like those offered by Electrolux, is typically higher than that of electronic products. This variation in selling costs explains the differing market strategies, aligning with the research findings that manufacturers are less likely to encroach when the selling cost of products in the direct sales channel is high.

Moreover, Proposition 4 reveals that given the low selling cost of EW, the retailer can benefit from encroachment when the manufacturer's selling cost of the product is moderate. In this situation, the direct channel is active in equilibrium but is unable to threaten the retailer's market position. The manufacturer must turn to the retailer to sell the product, thereby lowering the wholesale price. On the other hand, the selling price of EW is also low because the manufacturer can sell it at a low cost. In summary, the retailer can simultaneously enjoy a low wholesale price and large product demand. For example, JD.com does not directly sell Huawei Care+, but it promotes this service on its product detail pages. This promotion increases the perceived value of Huawei's products, thereby boosting product demand.

Comparing Proposition 4 with Proposition 3, we have the following findings.

Remark 2. When the selling cost of EW is low, offering it strengthens the manufacturer's motivation for encroachment. As the selling cost becomes moderate-to-high, offering the EW weakens the manufacturer's motivation for encroachment.

When EW is unavailable, the motivation for manufacturer encroachment arises from the high profit margin in the direct channel as well as compelling the retailer to lower the selling price to induce more customers to buy the product. By contrast, when EW is offered, the manufacturer still needs to consider the complementary effect between the product and EW. When the manufacturer can sell EW at a low cost, it is highly appealing to customers with high risk aversion. This increases product demand, and thus, the manufacturer prefers to encroach on the market to have a finger in the pie. That is, the possibility of manufacturer encroachment is large, given that EW is offered. When the selling cost of EW is moderate to high, the high selling price deters some price-sensitive customers from purchasing and thus weakens the complementary effect between the product and EW. This potentially reduces the manufacturer's incentive to encroach, particularly when the product's selling cost is high.

Furthermore, Propositions 3 and 4 show that the manufacturer's encroachment decisions depend on whether it offers EW. An interesting question is whether and how the selling price of EW varies with manufacturer encroachment. We address this issue in the following proposition.

Proposition 5. *$p_S^{\mathrm{EM}} \geqslant p_S^{\mathrm{NM}}$ if (i) $s < \frac{3A\Phi}{6A+5B}$ and $\frac{3A(\Phi-s)}{3A+5B} \leqslant S \leqslant \frac{3(A+B)(\Phi-s)}{3A+5B}$, or (ii) $\frac{3A\Phi}{6A+5B} \leqslant s < \frac{3A\Phi}{6A+2B}$ and $S \leqslant \frac{3(A+B)(\Phi-s)}{3A+5B}$, and $p_S^{\mathrm{EM}} < p_S^{\mathrm{NM}}$ otherwise.*

Proposition 5 presents the conditions under which the selling price of EW is greater or lower under encroachment than under no encroachment. When the manufacturer's selling cost is low and the selling cost of the product is moderate, or when the selling cost of the EW is moderate and the selling cost of the product is not sufficiently high, the manufacturer's selling efficiency is obviously lower than that of the retailer, such that the retail channel contributes to most product sales. Moreover, the manufacturer must lower the wholesale price of the product to alleviate channel conflicts due to encroachment. This leads to that the retailer orders more products under encroachment than under no encroachment. Because the customers who purchase the product have the potential to buy EW, the manufacturer can charge a high price for EW to gain more profitability.

Furthermore, we see from Proposition 5 that when the manufacturer's selling cost of the product is sufficiently low or high, the EW price is greater under no encroachment than under encroachment. The manufacturer can enjoy significant profits and, accordingly, gain more from the direct channel when its selling efficiency is high. However, manufacturer encroachment negatively affects retail demand. Because the retailer's selling efficiency is higher than that of the manufacturer, the decreased retail demand covers the increased direct demand. That is, manufacturer encroachment leads to a decrease in total demand for the product. In this situation, the manufacturer charges a low price for EW to attract customers who tend to buy both the product and the EW. In contrast, when the selling cost is high, the manufacturer's profit from the product is almost fully from the retail channel. In this case, the manufacturer may charge a higher wholesale price. This, in turn, results in high retail prices and low total demand for the product. Because only customers who purchased the product would buy EW, the manufacturer charges a low EW price to take advantage of the facilitation of offering EW on product sales. As the selling cost of a product increases, its selling price also increases. Consequently, the potential demand for EW has decreased. The manufacturer must reduce the selling price of EW to cope with the decrease in demand.

5.3. Interactions between encroachment and EW provision

As discussed above, both encroachment and EW offerings may stimulate product demand and, in turn, improve the manufacturer's profit. A key question to investigate is how these two strategies interact. The following section discusses whether and how one of the two strategies affects the value of the other from the manufacturer's perspective.

Similar to Cachon and Swinney [4], here we discuss the interactions between encroachment and EW provision by figuring out whether these two strategies are complements or substitutes. If they are complements, then combining encroachment and EW provision may lead to a super-additive benefit in the sense that the incremental profit of the combination (the difference in profit over the base case without encroachment and EW) is more than the combined incremental profits of encroachment and providing EW separately, that is, $\pi_m^{\mathrm{EM}} - \pi_m^{\mathrm{NN}} \geqslant \left(\pi_m^{\mathrm{EM}} - \pi_m^{\mathrm{EN}}\right) + \left(\pi_m^{\mathrm{NM}} - \pi_m^{\mathrm{NN}}\right)$, which can be simplifies as $\pi_m^{\mathrm{EM}} - \pi_m^{\mathrm{EN}} - \left(\pi_m^{\mathrm{NM}} - \pi_m^{\mathrm{NN}}\right) \geqslant 0$. On the contrary, the two strategies are substitutes if $\pi_m^{\mathrm{EM}} - \pi_m^{\mathrm{EN}} - \left(\pi_m^{\mathrm{NM}} - \pi_m^{\mathrm{NN}}\right) < 0$.

Proposition 6. *Denote* $s_2 = \frac{2A((3A+B)\Phi-(3A+5B)S)-\sqrt{2AB(3A+5B)(2(7A+B)S^2-4(3A+B)\Phi S+(3A+B)\Phi^2)}}{2A(3A+B)}$ *and suppose* $\frac{\Phi}{2} < S \leqslant \frac{3\Phi}{5}$, $s \leqslant \frac{3A\Phi-(3A+2B)S}{3A}$ *and* $B \leqslant \frac{3A}{4}$.

- *Encroachment and offering EW are complements if (i)* $B \leqslant \frac{24A}{85}$ *and* $s \leqslant s_2$, *or (ii)* $\frac{24A}{85} < B \leqslant \frac{3A}{5}$, $S \leqslant \frac{\left(2(3A+5B)-\sqrt{10B(3A+5B)}\right)\Phi}{2(3A+5B)}$, *and* $s \leqslant s_2$.
- *Encroachment and offering EW are substitutes if (i)* $B \leqslant \frac{24A}{85}$ *and* $s > s_2$, *(ii)* $\frac{24A}{85} < B \leqslant \frac{3A}{5}$, $S \leqslant \frac{\left(2(3A+5B)-\sqrt{10B(3A+5B)}\right)\Phi}{2(3A+5B)}$ *and* $s > s_2$, *(iii)* $\frac{24A}{85} < B \leqslant \frac{3A}{5}$ *and* $S > \frac{\left(2(3A+5B)-\sqrt{10B(3A+5B)}\right)\Phi}{2(3A+5B)}$, *or (iv)* $B > \frac{3A}{5}$.

Proposition 6 indicates that given different values of the co-payment rate and the manufacturer's selling costs of the product and EW, manufacturer encroachment and EW provision may be either complementary or substitutable. Specifically, it can be found that encroachment and the offering of EW cannot reinforce each other in most cases. Because manufacturers are more concerned about when the two strategies are complementary, we thus only discuss this phenomenon.

Proposition 6 states that when the selling cost of the EW is low, the two strategies are complementary when either the selling cost of the product or the co-payment rate is also low. That is, the incremental profit from simultaneously implementing the two strategies over the base case surpasses the combined incremental profit from either encroaching on or offering EW in isolation. First, we examine a scenario in which the coy-payment rate and selling cost of EW are both low. Given the low co-payment rate, it is obvious that the attractiveness of EW is high. When the selling cost of EW is low, the manufacturer is incentivized to charge lower prices to stimulate demand for EW. Furthermore, when a manufacturer encroaches on the market and decides to offer EW, the high attractiveness of EW can effectively stimulate demand for the product in both direct and retail channels because higher risk-averse consumers prefer to purchase product together with EW. In other words, simultaneously implementing the two strategies resulted in greater product demand than implementing each strategy individually. Similarly, when the selling costs of the product and EW are low, the incremental increase in profit from simultaneously implementing the two strategies is greater than that from solely offering EW. This is because the manufacturer now makes more products available through the direct channel and enjoys a high profit margin.

6. Conclusions

6.1. Key findings

This study focuses on the manufacturer's optimal decisions regarding EW provision and market encroachment. To this end, we analyzed a two-echelon supply chain involving one manufacturer and one retailer, in which the interaction between the manufacturer and retailer is modeled as the manufacturer-Stackelberg game. Four distinct models are discussed depending on whether the manufacturer engages in market encroachment and offers EW. The focal point of this study is the interplay between these two strategies and how they mutually influence each other. The following highlights the main findings of this study.

First, we examined the impact of encroachment on the manufacturer's decision regarding EW provision. We found that the manufacturer consistently benefits from offering EW when not engaging in encroachment. In contrast, under encroachment, the manufacturer prefers to offer EW only if the selling cost of the product is medium to high, and the selling cost of EW is not excessively high. In both cases, we also identify the conditions under which the retailer can benefit from the manufacturer's EW provision. By analyzing how the manufacturer's EW strategy evolves in response to market encroachment, we reveal that the motivation for offering EW weakens as the manufacturer engages in encroachment.

Second, we investigated the impact of offering EW on the manufacturer's encroachment decision. Our results show that without EW, the manufacturer can benefit from encroachment if the selling cost of the product is not sufficiently high. Furthermore, when the manufacturer can offer EW at a low cost, there exists a threshold for the product's selling cost below which the manufacturer is better off from encroachment. The retailer can benefit from manufacturer encroachment only when the manufacturer's selling cost is moderate, irrespective of whether EW is offered. Furthermore, by evaluating the modifications in the manufacturer's encroachment decisions following the introduction of EW, we find that when the selling cost of EW is low, offering the EW can enhance the manufacturer's motivation to encroach. However, as the selling cost of EW becomes moderate to high, the manufacturer's motivation for encroachment weakens with the provision of EW.

Third, we investigate the interplay between manufacturer encroachment and EW provision. Our analysis reveals that, depending on the values of the co-payment rate and the manufacturer's selling costs for both the product and EW, these two strategies may either complement each other or serve as substitutes. Specifically, our findings indicate that, from the manufacturer's perspective, encroachment and offering EW cannot reinforce each other in most cases.

6.2. Theoretical and practical implications

This study makes significant contributions to the existing literature in several ways. First, we propose a new analytical framework that integrates the manufacturer's decisions interaction regarding market encroachment and EW provision. This framework expands the literature on manufacturer encroachment and EW strategies. Second, unlike previous studies on EW strategies that assume fixed product sales channel structures, our model allows for flexible decision-making under different market conditions, providing new insights into manufacturer's strategic decisions regarding EW provision in both encroachment and no encroachment scenarios. Third, we delve into the impact of market encroachment on the manufacturer's EW pricing decisions, further enriching the understanding of strategic EW pricing in complex market environments. Finally, our findings indicate that the complementarity and substitutability between manufacturer encroachment and EW provision strategies depend on the selling costs of both the product and EW, as well as the co-payment rate. This provides theoretical guidance for manufacturer's decision-making.

Our research findings have significant implications for both manufacturers and retailers in practical management. For manufacturers, understanding the interaction between market encroachment and EW provision strategies can help them formulate more effective market encroachment and EW introduction strategies. Our study suggests that manufacturers should carefully evaluate the selling costs of both the product and EW, as well as the co-payment rate, before deciding whether to implement these two strategies simultaneously. Additionally, manufacturers can enhance the potential benefits of concurrently implementing both strategies by reducing the selling costs of the product and EW. For retailers, our results indicate that they can improve their competitiveness and profitability by collaborating with manufacturers and leveraging the manufacturers' EW provision strategies.

6.3. Future research

We think that this study can be extended in several ways. First, we considered only one manufacturer and one retailer in this research. In the current competitive context, it would be natural to extend this study by introducing competitive manufacturers and/or competitive retailers. Such an extension can lead to a research setting that is more consistent with reality. Moreover, we assumed in this research that only the manufacturer could offer EW. Retailers in various industries offer EWs. Extending the current research by admitting that the retailer offering its EW would be an interesting topic and may provide some novel findings.

Appendix A. Equilibrium results

A.1. Results in model NN

In the second stage, upon observing the wholesale price of the product, the retailer determines the order quantity to maximize profit, *i.e.*, $\pi_r(q_{pr}) = (p_P - w_P)q_{pr}$, where $p_p = v(1-\alpha) - \alpha r - \alpha(1-\alpha)Aq_p$. From the first-order condition, we get $q_{pr}^{\mathrm{NN}}(w_P) = \frac{v(1-\alpha)-\alpha r - w_P}{2\alpha(1-\alpha)A}$.

In the first stage, anticipating the retailer's best response, the manufacturer sets w_P to maximize $\pi_m^{\mathrm{NN}}(w_P) = (w_P - (1-\alpha)c)\frac{v(1-\alpha)-\alpha r - w_P}{2\alpha(1-\alpha)A}$, by considering the constraint $w_P \leqslant v(1-\alpha) - \alpha r$, where the condition aims to guarantee non-negative demand in retail channel. Differentiating $\pi_m^{\mathrm{NN}}(w_P)$ with regard to w_P and letting it equal to zero gives $w_p = \frac{(v+c)(1-\alpha)-\alpha r}{2}$, and the constraint always satisfies. Consequently, the equilibrium quantity of the product that are offered in the retail channel is $q_{pr}^{\mathrm{NN}} = \frac{\Phi}{4\alpha(1-\alpha)A}$. The equilibrium profits of the manufacturer and the retailer are $\pi_m^{\mathrm{NN}} = \frac{\Phi^2}{8A(1-\alpha)\alpha}$, and $\pi_r^{\mathrm{NN}} = \frac{\Phi^2}{16A(1-\alpha)\alpha}$, where $\Phi = (v-c)(1-\alpha) - r\alpha$.

A.2. Results in model EN

In the last stage, the manufacturer has to decide the available quantity of the product in the direct channel after observing the retailer's order quantity decision, so the manufacturer determines q_{pm} to maximize $\pi_m^{\mathrm{EN}}(q_{pm}) = (w_P - (1-\alpha)c)q_{pr} + (p_P - (1-\alpha)c - S)q_{pm}$ where $p_P = v(1-\alpha) - \alpha r - \alpha(1-\alpha)A(q_{pr} + q_{pm})$. Differentiating $\pi_m^{\mathrm{EN}}(q_{pm})$ with regard to q_{pm} and making it equal to zero gives $q_{pm}^{\mathrm{EN}}(q_{pr}) = \frac{\Phi - S - A\alpha(1-\alpha)q_{pr}}{2A(1-\alpha)\alpha}$. In the second stage, by anticipating the manufacturer's quantity decision and observing the manufacturer's wholesale price, the retailer aims to maximize $\pi_r^{\mathrm{EN}}(q_{pr}) = \left(\frac{(v+c)(1-\alpha) - r\alpha + S - A(1-\alpha)\alpha q_{pr}}{2} - w_P\right)q_{pr}$ over q_{pr}. We obtain the retailer's best response $q_{pr}^{\mathrm{EN}}(w_P) = \frac{S+(v+c)(1-\alpha)-r\alpha-2w_P}{2A(1-\alpha)\alpha}$ from the first-order condition. Finally, the manufacturer's objective is to maximize $\pi_m^{\mathrm{EN}}(w_P) = (w_P - (1-\alpha)c)\frac{S+(v+c)(1-\alpha)-r\alpha-2w_P}{2A(1-\alpha)\alpha} + \frac{((v-3c)(1-\alpha)-r\alpha-3S+2w_P)^2}{16\alpha(1-\alpha)A}$ over w_P by considering the constraints $\frac{r\alpha+3S-(v-3c)(1-\alpha)}{2} \leqslant w_P \leqslant \frac{S+(v+c)(1-\alpha)-r\alpha}{2}$, which aims to guarantee non-negative available quantities in retail and direct channels. In order to make sure $\frac{r\alpha+3S-(v-3c)(1-\alpha)}{2} < \frac{S+(v+c)(1-\alpha)-r\alpha}{2}$, there must be $S < \Phi$. For the first constraint, it satisfies only when $S \leqslant \frac{3\Phi}{5}$. It's easy to verify that the latter constraint always satisfies. Since $\pi_m^{\mathrm{EN}}(w_P)$ is concave in w_P, it follows that the equilibrium wholesale price of the product can be written as

$$w_p^{\mathrm{EN}} = \begin{cases} w_p^{\mathrm{EN}-1} = \frac{3(v+c)(1-\alpha)-3r\alpha-S}{6}, & \text{if } S \leqslant \frac{3\Phi}{5} \\ w_p^{\mathrm{EN}-2} = \frac{r\alpha+3S-(v-3c)(1-\alpha)}{2}, & \text{otherwise} \end{cases}.$$

The other equilibrium decisions can be determined accordingly. The equilibrium quantities of the product that are offered by manufacturer and retailer are

$$q_{pm}^{\mathrm{EN}} = \begin{cases} q_{pm}^{\mathrm{EN}-1} = \frac{3\Phi-5S}{6\alpha(1-\alpha)A}, & \text{if } S \leqslant \frac{3\Phi}{5} \\ q_{pm}^{\mathrm{EN}-2} = 0, & \text{otherwise} \end{cases} \quad \text{and}$$

$$q_{pr}^{\mathrm{EN}} = \begin{cases} q_{pr}^{\mathrm{EN}-1} = \frac{2S}{3\alpha(1-\alpha)A}, & \text{if } S \leqslant \frac{3\Phi}{5} \\ q_{pr}^{\mathrm{EN}-2} = \frac{\Phi-S}{\alpha(1-\alpha)A}, & \text{otherwise} \end{cases}.$$

Moreover, the manufacturer's and the retailer's equilibrium profits are

$$\pi_m^{\mathrm{EN}} = \begin{cases} \pi_m^{\mathrm{EN}-1} = \frac{7S^2+3\Phi(\Phi-2S)}{12A(1-\alpha)\alpha}, & \text{if } S \leqslant \frac{3\Phi}{5} \\ \pi_m^{\mathrm{EN}-2} = \frac{(S-\Phi)(\Phi-3S)}{2A(1-\alpha)\alpha}, & \text{otherwise} \end{cases}, \quad \text{and}$$

$$\pi_r^{\mathrm{EN}} = \begin{cases} \pi_r^{\mathrm{EN}-1} = \frac{2S^2}{9A(1-\alpha)\alpha}, & \text{if } S \leqslant \frac{3\Phi}{5} \\ \pi_r^{\mathrm{EN}-2} = \frac{(\Phi-S)^2}{2A(1-\alpha)\alpha}, & \text{otherwise} \end{cases}, \quad \text{respectively.}$$

A.3. Results in model NM

In last stage, the manufacturer has to decide the available quantity of the EW after observing the retailer's order quantity decision, so the manufacturer determines p_S to maximize $\pi_m^{\mathrm{NM}}(p_S) = (w_P - (1-\alpha)c)q_{pr} + (p_S - \alpha(1-\beta)r - s)q_{sm}$, where $p_P = v(1-\alpha) - r\alpha\beta - p_S - B\alpha(1-\alpha)q_{pr}$ and $q_{sm} = \frac{r\alpha(1-\beta)-p_S+(A-B)\alpha(1-\alpha)q_{pr}}{(A-B)(1-\alpha)\alpha}$. Differentiating $\pi_m^{\mathrm{NM}}(q_{sm})$ with regard to p_S and making it equal to zero gives $p_S^{\mathrm{NM}}(q_{pr}) = \frac{s+2r\alpha(1-\beta)+(A-B)(1-\alpha)\alpha q_{pr}}{2}$. In the second stage, by anticipating the manufacturer's quantity decision and observing the manufacturer's wholesale price, the retailer aims to maximize $\pi_r^{\mathrm{NM}}(q_{pr}) = \left(\frac{2(v(1-\alpha)-r\alpha)-s-(A+B)(1-\alpha)\alpha q_{pr}}{2} - w_P\right)q_{pr}$ over q_{pr}. We obtain the retailer's best response $q_{pr}^{\mathrm{NM}}(w_P) = \frac{2(v(1-\alpha)-r\alpha-w_P)-s}{2(A+B)(1-\alpha)\alpha}$ from the first-order condition. Finally, by anticipating the retailer's ordering quantity decision, the manufacturer's objective is to maximize

$$\pi_m^{\mathrm{NM}}(w_P) = (p_S - \alpha(1-\beta)r - s)\frac{2(A-B)(v(1-\alpha) - r\alpha - w_P) - (3A+B)s}{4(A-B)(A+B)(1-\alpha)\alpha} + (w_P - (1-\alpha)c)\frac{2(v(1-\alpha) - r\alpha - w_P) - s}{2(A+B)(1-\alpha)\alpha}$$

over w_P by considering the constraint $w_P \leq \frac{2(v(1-\alpha)-r\alpha)-s}{2}$, which aims to guarantee non-negative available quantities in both retail and direct channels. Differentiating $\pi_m^{\mathrm{NM}}(w_P)$ with regard to w_P and making it equal to zero gives $w_P^{\mathrm{NM}} = \frac{(A-B)s+2(2(A+B)c+(A+3B)v)(1-\alpha)-2(A+3B)r\alpha}{6A+10B}$. It's easy to verify the inequality condition always satisfies under the condition that $s < \Phi$. Consequently, the equalibrium price of the EW and the quantity of the product that is offered by retailer are $p_S^{\mathrm{NM}} = \frac{(A+7B)s+2(A-B)\Phi+2r\alpha(3A+5B)(1-\beta)}{6A+10B}$, and $q_{pr}^{\mathrm{NM}} = \frac{2(\Phi-s)}{(3A+5B)(1-\alpha)\alpha}$. The manufacturer and the retailer's profits are

$$\pi_m^{\mathrm{NM}} = \frac{(7A+B)s^2 + 4(A-B)\Phi^2 - 8(A-B)\Phi s}{4(A-B)(3A+5B)(1-\alpha)\alpha}, \quad \text{and}$$

$$\pi_r^{\mathrm{NM}} = \frac{2(A+B)(\Phi-s)^2}{(3A+5B)^2(1-\alpha)\alpha},$$

respectively.

A.4. Results in model EM

In the last stage, the manufacturer has to decide the available quantity of the product and the price of EW after observing the retailer's order quantity decision, so the manufacturer determines q_{pm} and p_S to maximize $\pi_m^{\mathrm{EM}}(q_{pm}, p_S) = (w_P - (1-\alpha)c)q_{pr} + (p_P - (1-\alpha)c - S)q_{pm} + (p_S - \alpha(1-\beta)r - s)q_{sm}$ where $p_P = v(1-\alpha) - \alpha r + \alpha(1-\alpha)(A-B)q_{sm} - \alpha(1-\alpha)A(q_{pr}+q_{pm})$ and $q_{sm} = (r\alpha(1-\beta) - p_S + \alpha(1-\alpha)(A-B)(q_{pr}+q_{pm})/(\alpha(1-\alpha)(A-B))$. Differentiating $\pi_m^{\mathrm{EM}}(q_{pm}, p_S)$ with regard to q_{pm} and p_S respectively and making them equal to zero gives $q_{pm}^{\mathrm{EM}}(q_{pr}) = \frac{\Phi-s-S-B\alpha(1-\alpha)q_{pr}}{2B(1-\alpha)\alpha}$ and $p_S^{\mathrm{EM}}(q_{pr}) = ((A-B)(1-\alpha)\alpha q_{pr} + 2r\alpha(1-\beta) + s)/2$. In the second stage, by anticipating the manufacturer's decision and observing the manufacturer's wholesale price, the retailer aims to maximize $\pi_r^{\mathrm{EM}}(q_{pr}) = \frac{(1-\alpha)(v+c)-\alpha r-\alpha(1-\alpha)Aq_{pr}-2w_p+S}{2}q_{pr}$ over q_{pr}. We obtain the retailer's best response $q_{pr}^{\mathrm{EM}}(w_p) = \frac{(1-\alpha)(v+c)-\alpha r+S-2w_p}{2\alpha(1-\alpha)A}$ from the first-order condition. Finally, the manufacturer's objective is to maximize

$$\begin{aligned}\pi_m^{\mathrm{EM}}(w_P) = (w_P-(1-\alpha)c)&\frac{(v+c)(1-\alpha)-r\alpha+S-2w_P}{2A(1-\alpha)\alpha}\\ &+\frac{\Phi-2c(1-\alpha)+2w_P-3S}{4}\frac{(2A(\Phi-s-S)-B(\Phi+2c(1-\alpha)+S-2w_P))}{4AB(1-\alpha)\alpha}\\ &+\frac{(A-B)((c+v)(1-\alpha)+S-r\alpha-2w_P)-2As}{4A}\frac{(A-B)(\Phi-S)-As}{2(A-B)B(1-\alpha)\alpha}\end{aligned}$$

over w_P by considering the constraints $\frac{B((v+c)(1-\alpha)-r\alpha+S)-2A(\Phi-s-S)}{2B} \leq w_P \leq \frac{(v+c)(1-\alpha)-r\alpha+S}{2}$, which aim to guarantee non-negative available quantities in both retail and direct channels. In order to make sure $\frac{B((v+c)(1-\alpha)-r\alpha+S)-2A(\Phi-s-S)}{2B} < \frac{(v+c)(1-\alpha)-r\alpha+S}{2}$, there must be $s < \Phi - S$. Differentiating $\pi_m^{\mathrm{EM}}(w_P)$ with regard to w_P making it equal to zero gives $w_P^{\mathrm{EM}} = \frac{3((1-\alpha)(v+c)-\alpha r)-S}{6}$ that maximizes $\pi_m^{\mathrm{EM}}(w_P)$. For the former constraint, it satisfies only when $s \leq \frac{3A\Phi-(3A+2B)S}{3A}$ and $\frac{\Phi}{2} \leq S < \frac{3A\Phi}{3A+2B}$. It's easy to verify the latter constraint always satisfy. Since $\pi_m^{\mathrm{EM}}(w_P)$ is concave in w_P, it follows that the equilibrium wholesale price can be written as

$$w_P^{\mathrm{EM}} = \begin{cases} w_P^{\mathrm{EM}-1} = \frac{3((1-\alpha)(v+c)-\alpha r)-S}{6}, & \text{if } s \leq \frac{3A\Phi-(3A+2B)S}{3A} \\ w_P^{\mathrm{EM}-2} = \frac{B((v+c)(1-\alpha)-r\alpha+S)-2A(\Phi-s-S)}{2B}, & \text{otherwise} \end{cases}$$

which is under the condition that $\frac{\Phi}{2} \leqslant S < \frac{3A\Phi}{3A+2B}$. Consequently, the equilibrium price of the EW and the quantities of the product that are offered by manufacturer and retailer are

$$p_S^{\mathrm{EM}} = \begin{cases} p_S^{\mathrm{EM}-1} = \frac{3As+2(A-B)S+6Ar\alpha(1-\beta)}{6A}, & \text{if } s \leqslant \frac{3A\Phi-(3A+2B)S}{3A} \\ p_S^{\mathrm{EM}-2} = \frac{(A-B)(\Phi-S)-(A-2B)s+2r\alpha B(1-\beta)}{2B}, & \text{otherwise} \end{cases},$$

$$q_{pm}^{\mathrm{EM}} = \begin{cases} q_{pm}^{\mathrm{EM}-1} = \frac{3A\Phi-3As-(3A+2B)S}{6AB(1-\alpha)\alpha}, & \text{if } s \leqslant \frac{3A\Phi-(3A+2B)S}{3A} \\ q_{pm}^{\mathrm{EM}-2} = 0, & \text{otherwise} \end{cases}, \quad \text{and}$$

$$q_{pr}^{\mathrm{EM}} = \begin{cases} q_{pr}^{\mathrm{EM}-1} = \frac{2S}{3A\alpha(1-\alpha)}, & \text{if } s \leqslant \frac{3A\Phi-(3A+2B)S}{3A} \\ q_{pr}^{\mathrm{EM}-2} = \frac{\Phi-s-S}{B(1-\alpha)\alpha}, & \text{otherwise} \end{cases}.$$

The other equilibrium decisions can be determined accordingly. Moreover, the manufacturer's equilibrium profit is

$$\pi_m^{\mathrm{EM}} = \begin{cases} \pi_m^{\mathrm{EM}-1} = \frac{\begin{pmatrix} 3A^2s^2-6A(A-B)(\Phi-S)s \\ +3A(A-B)\Phi^2+(A-B)(3A+4B)S^2 \\ -6A(A-B)\Phi S \end{pmatrix}}{12A(A-B)B(1-\alpha)\alpha}, \\ \text{if } s \leq \frac{3A\Phi-(3A+2B)S}{3A} \\ \pi_m^{\mathrm{EM}-2} = \frac{\begin{pmatrix} 2(A-B)(((3A-B)\Phi-(3A+B)S)s+ \\ (3A+B)\Phi S)-A(3A-4B)s^2 \\ -(A-B)(3(A+B)S^2+(3A-B)\Phi^2) \end{pmatrix}}{4(A-B)B^2(1-\alpha)\alpha}, \\ \text{otherwise} \end{cases}$$

and the retailer's equilibrium profit is

$$\pi_r^{\mathrm{EM}} = \begin{cases} \pi_r^{\mathrm{EM}-1} = \frac{2s^2}{9A(1-\alpha)\alpha}, & \text{if } s \leq \frac{3A\Phi-(3A+2B)S}{3A} \\ \pi_r^{\mathrm{EM}-2} = \frac{A(\Phi-s-S)^2}{2B^2(1-\alpha)\alpha}, & \text{otherwise} \end{cases}.$$

Appendix B. Proofs

Proof of Proposition 1. The difference between π_m^{NN} and π_m^{NM} can be expressed as $\pi_m^{NM} - \pi_m^{NN} = \frac{2A(7A+B)s^2-16A(A-B)\Phi s+5(A-B)^2\Phi^2}{8A(A-B)(3A+5B)(1-\alpha)\alpha}$, in which the value under the condition of $s < \Phi$ is positive and always holds true. That's we have $\pi_m^{NM} > \pi_m^{NN}$.

The difference between π_r^{NN} and π_r^{NM} can be expressed as $\pi_r^{NM} - \pi_r^{NN} = \frac{f_1(s)}{16A(3A+5B)^2(1-\alpha)\alpha}$, where $f_1(s) = 32A(A+B)s^2 - 64A(A+B)\Phi s + (A-B)(23A+25B)(\Phi^2)$, and $f_1(s)$ is strictly convex in s. Since $f_1(s=0) = (A-B)(23A+25B)\Phi^2 > 0$ and $f_1(s=(v-c)(1-\alpha)-r\alpha) = -(3A+5B)^2\Phi^2 < 0$, there exists $s = \frac{\left(8A(A+B)-(3A+5B)\sqrt{2A(A+B)}\right)\Phi}{8A(A+B)}$ making $f_1(s) = 0$. It follows that $f_1(s) \geq 0$, that is $\pi_r^{NM} \geq \pi_r^{NN}$, for all $s \leq \frac{\left(8A(A+B)-(3A+5B)\sqrt{2A(A+B)}\right)\Phi}{8A(A+B)}$, and $f_1(s) < 0$, that is $\pi_r^{NM} < \pi_r^{NN}$, for all $\frac{\left(8A(A+B)-(3A+5B)\sqrt{2A(A+B)}\right)\Phi}{8A(A+B)} < s < \Phi$. □

Proof of Proposition 2. Under manufacturer encroachment, we separate the analysis of the diference between π_m^{EN} and π_m^{EM}, into four cases according to the value of S and s. For the results' tractablility, we suppose $B < \frac{3A}{10}$.

- $\frac{\Phi}{2} \leqslant S \leqslant \frac{3\Phi}{5}$.
  - (i) $s \leqslant \frac{3A\Phi-(3A+2B)S}{3A}$. Subtracting π_m^{EN-1} from π_m^{EM-1} gives $\pi_m^{EM-1} - \pi_m^{EN-1} = \frac{((A-B)\Phi-As-(A-B)S)^2}{4A(A-B)B(1-\alpha)\alpha}$ which remains positive and consistently holds under these conditions.
    Subtracting π_r^{EN-1} from π_r^{EM-1} gives $\pi_r^{EM-1} - \pi_r^{EN-1} = 0$.
  - (ii) $\frac{3A\Phi-(3A+2B)S}{3A} < s < \Phi - S$. Subtracting π_m^{EN-1} from π_m^{EM-2} gives $\pi_m^{EM-2} - \pi_m^{EN-1} = \frac{f_2(s)}{12A(A-B)B^2(1-\alpha)\alpha}$, where $f_2(s) = 3(A^2)(4B-3A)s^2 + 6A(A-B)((3A-B)-(3A+B)S)s - (A-B)(9A^2+9AB+7B^2)S^2 + 6(A-B)(3A^2+AB+B^2)\Phi S - 3(A-B)(3A^2-AB+B^2)\Phi^2$, is strictly concave in s, because of $B < \frac{3A}{10}$. Since $f_2\left(s = \frac{3A\Phi-(3A+2B)S}{3A}\right) = \frac{B^3(5S-3\Phi)^2}{3} > 0$ and $f_2(s = \Phi - S) = B^2\left((7B-4A)S^2 - 6B\Phi S + 3B\Phi^2\right) < 0$, there exists $s_1 = 3(A-B)((3A-B)\Phi-(3A+B)S) + B\sqrt{3(A-B)B}(3\Phi-5S)/3A(3A-4B)$ such that $f_2(s_1) = 0$. It follows that $f_2(s) \geqslant 0$, that is $\pi_m^{EM-2} \geqslant \pi_m^{EN-1}$, for all $\frac{3A\Phi-(3A+2B)S}{3A} < s \leqslant s_1$, and $f_2(s) < 0$, that is $\pi_m^{EM-2} < \pi_m^{EN-1}$ for all $s_1 < s < \Phi - S$.
    Subtracting π_r^{EN-1} from π_r^{EM-2}, gives $\pi_r^{EM-2} - \pi_r^{EN-1} = ((3A-2B)(3A+2B)S^2 - 18A^2(\Phi-s)S + 9A^2(\Phi-s)^2))/(18AB^2(1-\alpha)\alpha) < 0$, which always holds true.
- $\frac{3\Phi}{5} < S < \frac{5\Phi}{6}$.
  - (i) $s \leqslant \frac{3A\Phi-(3A+2B)S}{3A}$. Subtracting π_m^{EN-2} from π_m^{EM-1}, gives $\pi_m^{EM-1} - \pi_m^{EN-2} = \frac{\begin{pmatrix}(A-B)(3A+22B)S^2+6(A-B)(As-(A+4B)\Phi)S \\ +3A^2s^2-6A(A-B)\Phi s+3(A-B)(A+2B)\Phi^2\end{pmatrix}}{12A(A-B)B(1-\alpha)\alpha} > 0$, which always holds true.
    Subtracting π_r^{EN-2} from π_r^{EM-1}, gives $\pi_r^{EM-1} - \pi_r^{EN-2} = \frac{18\Phi S-5S^2-9\Phi^2}{18A(1-\alpha)\alpha} > 0$, which always holds true under these conditions.
  - (ii) $\frac{3A\Phi-(3A+2B)S}{3A} < s < \Phi - S$. Subtracting π_m^{EN-2} from π_m^{EM-2}, gives $\pi_m^{EM-2} - \pi_m^{EN-2} = \frac{f_3(s)}{4A(A-B)B^2(1-\alpha)\alpha}$, where $f_3(s) = A^2(4B-3A)s^2 + 2A(A-B)((3A-B)\Phi-(3A+B)S)s - (A-B)^2(3(A+2B)S^2 - 2(3A+4B)\Phi S + (3A+2B)\Phi^2)$, and $f_3(s)$ is strictly concave in s. Since $f_3\left(s = \frac{3A\Phi-(3A+2B)S}{3A}\right) = \frac{(3A-2B)B^2(5S-3\Phi)^2}{9} > 0$ and $f_3(s = \Phi - S) = B^2(\Phi-S)((3A-2B)\Phi + (6B-7A)S) < 0$, there exists $s = \frac{(A-B)(\Phi-S)}{A}$, such that $f_3\left(s = \frac{(A-B)(\Phi-S)}{A}\right) = 0$. It follows that $f_3(s) \geqslant 0$, that is $\pi_m^{EM-2} \geqslant \pi_m^{EN-2}$, for all $\frac{3A\Phi-(3A+2B)S}{3A} < s \leqslant \frac{(A-B)(\Phi-S)}{A}$, and $f_3(s) < 0$, that is $\pi_m^{EM-2} < \pi_m^{EN-2}$, for all $\frac{(A-B)(\Phi-S)}{A} < s < \Phi - S$.

Subtracting $\pi_r^{\text{EN}-2}$ from $\pi_r^{\text{EM}-2}$, gives $\pi_r^{\text{EM}-2} - \pi_r^{\text{EN}-2} = \frac{f_3(s)}{2AB^2(1-\alpha)\alpha}$, where $f_3(s) = A^2s^2 - 2A^2(\Phi - S)s + (A-B)(A+B)(\Phi - S)^2$, and $f_3(s)$ is strictly convex in s. Since $f_3\left(s = \frac{3A\Phi-(3A+2B)S}{3A}\right) = \frac{B^2\left(18\Phi S - 5S^2 - 9\Phi^2\right)}{9} > 0$ and $f_3(s = \Phi - S) = -B^2(\Phi - S)^2 < 0$, there exists $s = \frac{(A-B)(\Phi-S)}{A}$, such that $f_3\left(s = \frac{(A-B)(\Phi-S)}{A}\right) = 0$. It follows that $f_3(s) \geqslant 0$, that is $\pi_r^{\text{EM}-2} \geqslant \pi_r^{\text{EN}-2}$, for all $\frac{3A\Phi-(3A+2B)S}{3A} < s \leqslant \frac{(A-B)(\Phi-S)}{A}$, and $f_3(s) < 0$, that is $\pi_r^{\text{EM}-2} < \pi_r^{\text{EN}-2}$, for $\frac{(A-B)(\Phi-S)}{A} < s < \Phi - S$.

□

Proof of Proposition 3. We separate the analysises of the differences between π_m^{NN} and $\pi_m^{\text{EN}-1}$, and between π_r^{NN} and π_r^{EN}, into two cases according to the value of S.

- $S \leqslant \frac{3\Phi}{5}$.
  Subtracting π_m^{NN} from $\pi_m^{\text{EN}-1}$ gives $\pi_m^{\text{EN}-1} - \pi_m^{\text{NN}} = \frac{14S^2-12\Phi S+3\Phi^2}{24A(1-\alpha)\alpha}$, which is positive and always holds true under this condition.
  Subtracting π_r^{NN} from $\pi_r^{\text{EN}-1}$ gives $\pi_r^{\text{EN}-1} - \pi_r^{\text{NN}} = \frac{32S^2-9\Phi^2}{144A(1-\alpha)\alpha}$, where $\pi_r^{\text{EN}-1} < \pi_r^{\text{NN}}$, for all $S < \frac{3\Phi}{4\sqrt{2}}$, and $\pi_r^{\text{EN}-1} \geqslant \pi_r^{\text{NN}}$, for all $\frac{3\Phi}{4\sqrt{2}} \leqslant S \leqslant \frac{3\Phi}{5}$.
- $\frac{3\Phi}{5} < S < \Phi$.
  Subtracting π_m^{NN} from $\pi_m^{\text{EN}-2}$ gives $\pi_m^{\text{EN}-2} - \pi_m^{\text{NN}} = \frac{16\Phi S-12S^2-5\Phi^2}{8A(1-\alpha)\alpha}$, where $\pi_m^{\text{EN}-2} \geqslant \pi_m^{\text{NN}}$, for all $\frac{3\Phi}{5} < S \leqslant \frac{5\Phi}{6}$, and $\pi_m^{\text{EN}-2} < \pi_m^{\text{NN}}$, for all $\frac{5\Phi}{6} < S < \Phi$.
  Subtracting π_r^{NN} from $\pi_r^{\text{EN}-2}$ gives $\pi_r^{\text{EN}-2} - \pi_r^{\text{NN}} = \frac{8S^2-16\Phi S+7\Phi^2}{16A(1-\alpha)\alpha}$, where $\pi_r^{\text{EN}-2} \geqslant \pi_r^{\text{NN}}$, for all $\frac{3\Phi}{5} < S \leqslant \frac{(4-\sqrt{2})\Phi}{4}$, and $\pi_r^{\text{EN}-2} < \pi_r^{\text{NN}}$, for all $\frac{(4-\sqrt{2})\Phi}{4} < S < \Phi$.

□

Proof of Proposition 4. We separate the analysis of the differences between π_m^{NM} and $\pi_m^{\text{EM}-1}$, and between π_r^{NM} and $\pi_r^{\text{EM}-1}$, into two cases according to the value of S. For the results' tractability, we suppose $s < \frac{3A\Phi}{6A+2B}$, and the following analysis is based on this premise.

- $s < S \leqslant \frac{3A(\Phi-s)}{3A+2B}$.
  Subtracting π_m^{NM} from $\pi_m^{\text{EM}-1}$, gives $\pi_m^{\text{EM}-1} - \pi_m^{\text{NM}} = \frac{G_1(S)}{12AB(3A+5B)(1-\alpha)\alpha}$, where $G_1(S) = (3A+4B)(3A+5B)S^2 - 6A(3A+5B)(\Phi - s)S + 3A(3A+B)s^2 - 6A(3A+B)\Phi s + 3A(3A+B)\Phi^2$ is strictly convex in S. Differentiating $G_1(S)$ with regard to S and setting it equals zero gives $S = \frac{3A(\Phi-s)}{3A+4B}$ that minimizes $G_1(S)$. The minimum of $G_1(S)$ can be written as $G_1\left(S = \frac{3A(\Phi-s)}{3A+4B}\right) = \frac{12AB^2(\Phi-s)^2}{3A+4B}$, which is positive. That gives $\pi_m^{\text{EM}-1} > \pi_m^{\text{NM}}$.
  Subtracting π_r^{NM} from $\pi_r^{\text{EM}-1}$, gives $\pi_r^{\text{EM}-1} - \pi_r^{\text{NM}} = \frac{2G_2(S)}{9A(3A+5B)^2(1-\alpha)\alpha}$, where $G_2(S) = (3A+5B)^2S^2 - 9A(A+B)s^2 + 18A(A+B)\Phi s - 9A(A+B)\Phi^2$ is strictly convex in S. There is $G_2\left(S = \frac{3A(\Phi-s)}{3A+2B}\right) = \frac{9AB(3A-B)(3A+4B)(\Phi-s)^2}{(3A+2B)^2} > 0$ and $G_2(S = s) = B(21A + 25B)s^2 + 18A(A+B)\Phi s - 9A(A+B)\Phi^2$. Let $f_4(s) = B(21A + 25B)s^2 + 18A(A+B)\Phi s - 9A(A+B)\Phi$, which is strictly convex in s. Since $f_4(s = 0) = -9A(A+B)\Phi^2 < 0$ and $f_4(s = \frac{3A\Phi}{6A+2B}) = \frac{9AB(3A-B)(3A+4B)\Phi^2}{4(3A+B)^2} > 0$, there exists $s = \frac{\left(3(3A+5B)\sqrt{A(A+B)}-9A(A+B)\right)\Phi}{B(21A+25B)}$, giving $f_4(s = (3(3A+5B)\sqrt{A(A+B)} - 9A(A+B))\Phi/B(21A+25B)) = 0$. That is $G_2(S = s) < 0$ for all $s < \frac{\left(3(3A+5B)\sqrt{A(A+B)}-9A(A+B)\right)\Phi}{B(21A+25B)}$, and $G_2(S = s) \geqslant 0$ for all $\frac{\left(3(3A+5B)\sqrt{A(A+B)}-9A(A+B)\right)\Phi}{B(21A+25B)} \leqslant s < \frac{3A\Phi}{6A+2B}$.

When $s < \frac{\left(3(3A+5B)\sqrt{A(A+B)}-9A(A+B)\right)\Phi}{B(21A+25B)}$, there exist $S = \frac{3\sqrt{A(A+B)}(\Phi-s)}{3A+5B}$ giving $G_2\left(S = \frac{3\sqrt{A(A+B)}(\Phi-s)}{3A+5B}\right) = 0$. Such that $G_2(S) < 0$, that is $\pi_r^{\mathrm{EM}-1} < \pi_r^{\mathrm{NM}}$, for all $s < S < \frac{3\sqrt{A(A+B)}(\Phi-s)}{3A+5B}$, and $G_2(S) \geqslant 0$, that is $\pi_r^{\mathrm{EM}-1} \geqslant \pi_r^{\mathrm{NM}}$ for all $\frac{3\sqrt{A(A+B)}(\Phi-s)}{3A+5B} \leqslant S \leqslant \frac{3A(\Phi-s)}{3A+2B}$. When $\frac{\left(3(3A+5B)\sqrt{A(A+B)}-9A(A+B)\right)\Phi}{B(21A+25B)} \leqslant s < \frac{3A\Phi}{6A+2B}$, we have $G_2(S=s) \geqslant 0$, that is $\pi_r^{\mathrm{EM}-1} \geqslant \pi_r^{\mathrm{NM}}$.

– $\frac{3A(\Phi-s)}{3A+2B} < S < \Phi - s$.
Subtracting π_m^{NM} from $\pi_m^{\mathrm{EM}-2}$, gives $\pi_m^{\mathrm{EM}-2} - \pi_m^{\mathrm{NM}} = \frac{G_3(S)}{4B^2(3A+5B)(1-\alpha)\alpha}$, where $G_3(S) = -3(A+B)(3A+5B)S^2 + 2(3A+B)(3A+5B)(\Phi-s)S - (9A^2+12AB-B^2)(\Phi-s)^2$ and strictly concave in S. Since $G_3\left(S = \frac{3A(\Phi-s)}{3A+2B}\right) = \frac{4B^3(6A+B)(\Phi-s)^2}{(3A+2B)^2} > 0$ and $G_3(S=\Phi-s) = -4B^2(\Phi-s)^2 < 0$, there exists $S_1 = \frac{\left((3A+5B)(3A+B)+2B\sqrt{2B(3A+5B)}\right)(\Phi-s)}{3(A+B)(3A+5B)}$ giving $G_3(S=S_1) = 0$. Such that $G_3(S) \geqslant 0$, that is $\pi_m^{\mathrm{EM}-2} \geqslant \pi_m^{\mathrm{NM}}$, for all $\frac{3A(\Phi-s)}{3A+2B} < S \leqslant S_1$, and $G_3(S) < 0$, that is $S_1 < S < \Phi - s$ for all $\pi_m^{\mathrm{EM}-2} < \pi_m^{\mathrm{NM}}$.
Subtracting π_r^{NM} from $\pi_r^{\mathrm{EM}-2}$, gives $\pi_r^{\mathrm{EM}-2} - \pi_r^{\mathrm{NM}} = \frac{G_4(S)}{2B^2(3A+5B)^2(1-\alpha)\alpha}$, where $G_4(S) = A(3A+5B)^2S^2 - 2A(3A+5B)^2(\Phi-s)S + (3A+4B)(3A^2+6AB-B^2)(\Phi-s)^2$ is strictly concave in S. Since $G_4\left(S = \frac{3A(\Phi-s)}{3A+2B}\right) = \frac{4B^3(3A-B)(3A+4B)(\Phi-s)^2}{(3A+2B)^2} > 0$ and $G_4(S=\Phi-s) = -4B^2(A+B)(\Phi-s)^2 < 0$, there exists $S_2 = \frac{\left(A(3A+5B)-2B\sqrt{A(A+B)}\right)(\Phi-s)}{A(3A+5B)}$ giving $G_4(S_2) = 0$. Such that $G_4(S) \geqslant 0$, that is $\pi_r^{\mathrm{EM}-2} \geqslant \pi_r^{\mathrm{NM}}$, for all $\frac{3A(\Phi-s)}{3A+2B} < S \leqslant S_2$, and $G_4(S) < 0$, that is $\pi_r^{\mathrm{EM}-2} < \pi_r^{\mathrm{NM}}$, for all $S_2 < S < \Phi - s$.

□

Proof of Remark 2. As is shown in Propositions 3 and 4, when $S \leqslant \frac{5\Phi}{6}$ there is $\pi_m^{\mathrm{EN}} \geqslant \pi_m^{\mathrm{NN}}$ and when $s < S \leqslant \frac{\left((3A+5B)(3A+B)+2B\sqrt{2B(3A+5B)}\right)(\Phi-s)}{3(A+B)(3A+5B)}$ and $s < \frac{3A\Phi}{6A+2B}$, there is $\pi_m^{\mathrm{EM}} \geqslant \pi_m^{\mathrm{NM}}$.
We get that when $s < \frac{\left(9A^2-18AB-31B^2+10B\sqrt{2B(3A+5B)}\right)\Phi}{6(9A^2+12AB-B^2)}$, there is $\frac{5\Phi}{6} < \frac{\left((3A+5B)(3A+B)+2B\sqrt{2B(3A+5B)}\right)(\Phi-s)}{3(A+B)(3A+5B)}$. When $\frac{\left(9A^2-18AB-31B^2+10B\sqrt{2B(3A+5B)}\right)\Phi}{6(9A^2+12AB-B^2)} \leqslant s < \frac{3A\Phi}{6A+2B}$, there is $\frac{\left((3A+5B)(3A+B)+2B\sqrt{2B(3A+5B)}\right)(\Phi-s)}{3(A+B)(3A+5B)} < \frac{5\Phi}{6}$. □

Proof of Proposition 5. We separate the analysis of the difference between and into two cases according to the value of S.

– $s < S \leqslant \frac{3A(\Phi-s)}{3A+2B}$. Subtracting p_S^{NM} from $p_S^{\mathrm{EM}-1}$, we get $p_S^{\mathrm{EM}-1} - p_S^{\mathrm{NM}} = \frac{(A-B)((3A+5B)S-3A(\Phi-s))}{3A(3A+5B)}$. There is $(3A+5B)S - 3A(\Phi-s) \geqslant 0$, when $s < \frac{3A\Phi}{6A+5B}$ and $\frac{3A(\Phi-s)}{3A+5B} \leqslant S \leqslant \frac{3A(\Phi-s)}{3A+2B}$, or $\frac{3A\Phi}{6A+5B} \leqslant s < \frac{3A\Phi}{6A+2B}$ and $s < S \leqslant \frac{3A(\Phi-s)}{3A+2B}$. When $s < \frac{3A\Phi}{6A+5B}$ and $s < S < \frac{3A(\Phi-s)}{3A+5B}$, there is $(3A+5B)S - 3A(\Phi-s) < 0$.
– $\frac{3A(\Phi-s)}{3A+2B} < S < \Phi - s$. Subtracting p_S^{NM} from $p_S^{\mathrm{EM}-2}$, gives $p_S^{\mathrm{EM}-2} - p_S^{\mathrm{NM}} = \frac{(A-B)(3(A+B)(\Phi-s)-(3A+5B)S)}{2B(3A+5B)}$. Such that $p_S^{\mathrm{EM}-2} \geqslant p_S^{\mathrm{NM}}$ for all $\frac{3A(\Phi-s)}{3A+2B} < S \leqslant \frac{3(A+B)(\Phi-s)}{3A+5B}$, and $p_S^{\mathrm{EM}-2} < p_S^{\mathrm{NM}}$ for all $\frac{3(A+B)(\Phi-s)}{3A+5B} < S < \Phi - s$.

□

Proof of Proposition 6. For simplifying calculation, we suppose $\frac{\Phi}{2} < S \leqslant \frac{3\Phi}{5}$, $s \leqslant \frac{3A\Phi-(3A+2B)S}{3A}$ and $B \leq \frac{3A}{4}$. $\pi_m^{\mathrm{EM}} - \pi_m^{\mathrm{NN}} \geqslant \left(\pi_m^{\mathrm{EM}} - \pi_m^{\mathrm{EN}}\right) + \left(\pi_m^{\mathrm{NM}} - \pi_m^{\mathrm{NN}}\right)$ can be simplified as $\pi_m^{\mathrm{EM}} - \pi_m^{\mathrm{EN}} - \left(\pi_m^{\mathrm{NM}} - \pi_m^{\mathrm{NN}}\right) \geqslant 0$, we have $\pi_m^{\mathrm{EM}-1} - \pi_m^{\mathrm{NM}} - \left(\pi_m^{\mathrm{EN}-1} - \pi_m^{\mathrm{NN}}\right) = \frac{f_5(s)}{8AB(3A+5B)(1-\alpha)\alpha}$, where $f_5(s) = 2A(3A+B)s^2 - 4A((3A+B)\Phi - (3A+5B)S)s + 2(A-B)(3A+5B)S^2 - 4(A-B)(3A+5B)\Phi S + (A-B)(6A+5B)\Phi^2$ and it's strictly convex in s. Since $f_5(s=0) = 2(A-B)(3A+5B)S^2 - 4(A-B)(3A+5B)S\Phi + (A-B)(6A+5B)\Phi^2$, we can easily get that when $B \leqslant \frac{24A}{85}$, or $\frac{24A}{85} < B < \frac{3A}{5}$ and $\frac{\Phi}{2} < S \leqslant \frac{\Phi\left(2(3A+5B)-\sqrt{10B(3A+5B)}\right)}{2(3A+5B)}$, there is $f_5(s=0) \geqslant 0$, and when $\frac{24A}{85} < B \leqslant \frac{3A}{5}$ and $\frac{\Phi\left(2(3A+5B)-\sqrt{10B(3A+5B)}\right)}{2(3A+5B)} < S \leqslant \frac{3\Phi}{5}$, or $\frac{3A}{5} < B \leqslant \frac{3A}{4}$, there is $f_5(s=0) < 0$. Furthermore, there is $f_5\left(s = \frac{3A\Phi-(3A+2B)S}{3A}\right) = \frac{B\left(-2(21A-B)(3A+4B)S^2+36A(3A+5B)\Phi S-9A(3A+5B)\Phi^2\right)}{9A} < 0$.

Differentiating $f_5(s)$ with regard to s and setting it equal to zero gives $s = \frac{(3A+B)\Phi-(3A+5B)S}{3A+B}$ that minimizes $f_5(s)$ and there is $\frac{(3A+B)\Phi-(3A+5B)S}{3A+B} < \frac{3A\Phi-(3A+2B)S}{3A}$. We get that $f_5\left(s = \frac{(3A+B)\Phi-(3A+5B)S}{3A+B}\right) = \frac{B(3A+5B)\left(-2(7A+B)S^2+4(3A+B)\Phi S-(3A+B)\Phi^2\right)}{3A+B} < 0$. Giving $f_5(s) = 0$, we get $s_2 = \frac{\left(2A((3A+B)\Phi-(3A+5B)S)-\sqrt{2AB(3A+5B)((14A+2B)S^2-4(3A+B)\Phi S+(3A+B)\Phi^2)}\right)}{2A(3A+B)}$ and $s_3 = \frac{\left(2A((3A+B)\Phi-(3A+5B)S)+\sqrt{2AB(3A+5B)((14A+2B)S^2-4(3A+B)\Phi S+(3A+B)\Phi^2)}\right)}{2A(3A+B)}$. We conclude that

- $B \leqslant \frac{24A}{85}$, or $\frac{24A}{85} < B < \frac{3A}{5}$ and $\frac{\Phi}{2} < S \leqslant \frac{\Phi\left(2(3A+5B)-\sqrt{10B(3A+5B)}\right)}{2(3A+5B)}$. When $s \leqslant s_2$, there is $f_5(s) \geqslant 0$, that is $\pi_m^{\text{EM}-1} - \pi_m^{\text{NM}} - \left(\pi_m^{\text{EN}-1} - \pi_m^{\text{NN}}\right) \geqslant 0$, and when $s_2 < s \leqslant \frac{3A\Phi-(3A+2B)S}{3A}$, there is $f_5(s) < 0$, that is $\pi_m^{\text{EM}-1} - \pi_m^{\text{NM}} - \left(\pi_m^{\text{EN}-1} - \pi_m^{\text{NN}}\right) < 0$.
- $\frac{24A}{85} < B \leqslant \frac{3A}{5}$ and $\frac{\Phi\left(2(3A+5B)-\sqrt{10B(3A+5B)}\right)}{2(3A+5B)} < S \leqslant \frac{3\Phi}{5}$, or $\frac{3A}{5} < B \leqslant \frac{3A}{4}$. There is $f_5(s) < 0$, that is $\pi_m^{\text{EM}-1} - \pi_m^{\text{NM}} - \left(\pi_m^{\text{EN}-1} - \pi_m^{\text{NN}}\right) < 0$.

□

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