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TARGET SETTING AND BENCHMARKING BY THE POSSIBILITY OF SELECTING THE CLOSEST ALTERNATIVE BENCHMARK

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Abstract. Data envelopment analysis (DEA) stands out from other performance evaluation methods because it focuses on providing benchmarks. Benchmarking plays a vital role in improvement the performance of inefficient units, especially when classical models are combined with managerial perspectives, potentially resulting in a more feasible benchmark or target. Sometimes, choosing certain efficient units in the system as a benchmark and target isn't feasible from a management standpoint for inefficient units. This happens for various reasons, including the exclusivity of an efficient unit, the outlier status of an efficient unit, insufficient access to resources, or discrepancies with reality. Such units shouldn't be chosen as benchmark and target for inefficient units within the system. Therefore, disregarding such units in the benchmarking process based on the manager's preference and opinion can contribute to better alignment with reality. Implementing this strategy reduces the disparity between actual and efficient performance. This paper presents a model designed to address these considerations. The model takes into consideration managerial opinions, enabling managers to identify and ignore specific efficient units during benchmarking. It then proposes alternative benchmarks closest in similarity to improve efficiency for the inefficient units.

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1. Introduction

Benchmarking is an attitude prior to being acknowledged as a management technique. The main reason institutions and companies use benchmarking is because human beings naturally desire to learn better methods and strive for perfection and superiority. This inherent drive for improvement and growth motivates them to continuously seek the best situations and reach higher levels of performance.

DEA utilizes observations and technological assumptions to establish an empirical production possibility set (PPS) which helps determine the efficient frontier, which consists of efficient units, and is used as a reference for evaluating the remaining DMUs.

Keywords. Benchmarking, target setting, DEA, closest targets, DMU, managerial opinions.

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Nevertheless, Cook *et al.* [6] claim that "In the circumstance of benchmarking, the efficient DMUs, as defined by DEA, may not necessarily form a production frontier, but rather lead to a best-practice frontier". Specifically, the points on the best-practice frontier serve as potential benchmarks for the inefficient units. The targets are the coordinates of these benchmarks. These coordinates represent levels of operation for the inefficient DMUs, which would make them perform efficiently. For further reading on DEA and benchmarking, consider the studies by Adler *et al.* [1], Dai and Kuosmanen [8], Zanella *et al.* [23], Ruiz *et al.* [18], Ramón *et al.* [14], and Ruiz and Sirvent [16].

One of the most important issues in target setting and benchmarking is the combination of preference information and management opinion in the process of determining targets or analysis of efficiency. There are various alternative approaches to integrate preference information in efficiency analyses and target setting with DEA. These approaches include the use of hypothetical DMUs by Golany [9], DEA-benchmarking models by Cook *et al.* [7], specification of preference weights in weighted non-radial models by Zhu [24], the combination of DEA and interactive multiple objective linear programming (MOLP) techniques by Yang *et al.* [22], and benchmarking and target setting with expert preferences by Ruiz *et al.* [18]. The latter method involves using weight constraints obtained from analysts' views on the relative importance of inputs and outputs along with the AHP method to set targets. Furthermore, in a more recent study, Toloo *et al.* [20] utilized user subjective opinions to select efficient information system (IS) projects, proposing a validated DEA approach through a real-world case study involving 41 IS projects at a financial institution and 18 artificial projects.

Based on the mentioned articles, researchers aim to present new and valid methods in various fields to make their research more realistic by incorporating preference information.

In this paper, the main questions are, if certain efficient units do not meet the criteria for selection as a benchmark from a managerial perspective, how can the manager's opinion be incorporated into the process of determining the target and alternative pattern? What units can the manager disregard in benchmarking processes?

This article explores how managerial opinions can be applied in the process of determining the closest alternative benchmark or target under such circumstances. Additionally, the article examines which units can be disregarded by the manager during the benchmarking process. This article has two main contributions:

(1) By combining the manager's information and preferences in the benchmarking process, real and achievable targets for inefficient units can be determined.
(2) If some benchmarks in the system are deemed inappropriate by applying the manager's opinion, the closest alternative benchmark will be considered for it.

The standard DEA models yield targets that are usually the "furthest" efficient projection to the assessed unit. However, the distance to this efficient projection should be minimized, so that the resulting targets are as similar as possible to the inputs and outputs of the assessed unit. The rationale behind this is that when targets are closer, they suggest potential ways to improve the efficiency of inefficient units, leading to enhanced effectiveness with less effort. The problem of finding closest targets with DEA has been addressed in various studies by Aparicio *et al.* [3], Tone [21] and Aparicio *et al.* [2].

The proposed approach in this article is very useful in terms of the practical aspect of target setting and benchmarking. Integrating the opinion and preferences of the system manager in the benchmarking process by determining valid benchmark and ignoring unjustified benchmark is a new and valuable topic in DEA. Existing models for determining the closest targets and benchmarks fail to bridge the gap between the real and effective benchmarks and neglect various aspects that should be considered in establishing the actual targets and benchmarks. Therefore, addressing this issue in benchmarking and setting targets for inefficient units is valuable because the objectives need not only be achievable technically but also from a managerial perspective. In the real world, it is very common that some units are technically efficient, but for various reasons, may not be accepted as a suitable benchmark by some inefficient units. One of the major reasons making it not practical is the managerial perspective which make them not appropriate to be benchmark. Hence, applying management opinion in benchmarking process is essential part to achieve more realistic and achievable targets in the real world. The article is organized as follows: In Section 2, preliminary concepts of this topic are mentioned. In Section 3, a method for determining the allowed benchmarks for ignoring and then a model for determining alternative targets and benchmarks for efficiency units as inappropriate by management are provided. A numerical example to implement the approach is provided in Section 4. The last section concludes.

2. Bedding, background and analysis of the literature

In this section, some of the basic concepts used to present the original model are briefly explained. It should be noted that methodologically, in this article, we explain the approach used by Aparicio *et al.* [3] to find the closest targets. Suppose we have n of DMUs that use m inputs to generates outputs, which we represent as $(X_j, Y_j), j = 1, \ldots, n$:

$$X_j = (x_{1j}, \ldots, x_{mj})' \geq 0,\ \ X_j \neq 0,\ j = 1, \ldots, n,\ \text{and}\ Y_j = (y_{1j}, \ldots, y_{sj})' \geq 0,\ \ Y_j \neq 0,\ j = 1, \ldots, n.$$

The relative efficiency of each DMU is determined by referring to a set called the production possibility set, which can be considered with a non-parametric structure of observations and with specific assumptions and principles. The production possibility set in DEA under Constant Return to Scale (CRS) and Variable Returns to Scale (VRS) $T = \{(X, Y)/X \text{ can produce } Y\}$ is considered as follows:

$$T_{\text{CRS}} = \left\{ (x, y) \in R_+^m \times R_+^s : x \geq \sum_{j=1}^{n} \lambda_j x_j,\ y \leq \sum_{j=1}^{n} \lambda_j y_j, \lambda_j \geq 0,\ j = 1, \ldots, n \right\}$$
$$T_{\text{VRS}} = \left\{ (x, y) \in R_+^m \times R_+^s : x \geq \sum_{j=1}^{n} \lambda_j x_j,\ y \leq \sum_{j=1}^{n} \lambda_j y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0,\ j = 1, \ldots, n \right\}.$$

In DEA, the production possibility set (PPS) frontier can be found with other categories, for example, weak $\partial^w(T)$ and strong $\partial^s(T)$ efficiency frontiers are defined as follows:

$$\partial^w(T) := \{(x, y) \in T : \hat{x} < x, \hat{y} > y \Rightarrow (\hat{x}, \hat{y}) \notin T\}$$
$$\partial^s(T) := \{(x, y) \in T : \hat{x} \leq x, \hat{y} \geq y, (\hat{x}, \hat{y}') \neq (x, y) \Rightarrow (\hat{x}, \hat{y}) \notin T\}.$$

The efficient frontier of the PPS, $\partial(T)$, consists of the nondominated points of T, that is $\partial(T) = \{(x, y) \in T | (x', y') \in T, x' \leq x, y' \geq y \Rightarrow (x', y') = (x, y)\}$. $\partial(T)$ can also be expressed in terms of input and output weights as: $\partial(T) = \{(x, y) \in T | -vx + uy + u_0 = 0, -vx_j + uy_j + u_0 \leq 0, j = 1, \ldots, n, v > 0_m, u > 0_s, u_0 \in R\}$. Actually, in this latter set, it suffices to consider the conditions $-vx_j + uy_j + u_0 \leq 0$ only for the units in the set of extreme efficient points of the PPS, E[1].

Definition 1. DMU_0 with input–output vector (x_o, y_o) is said to be technically efficient if, and only if, $(x_o, y_o) \in \partial(T)$.

In order to measure technical efficiency, there are a lot of models in DEA. One of them is the weighted additive model by Lovell and Pastor [13], can be formulated under Variable Returns to Scale (VRS) as follows:

$$\max\ \ z = \sum_{i=1}^{m} w_i s_{io}^- + \sum_{r=1}^{s} w_r s_{ro}^+$$

s.t.

[1]E The extreme efficient units are the DMUs spanning the efficient faces of the frontier that cannot be expressed as a linear combination of the other DMUs. How to determine them is mentioned on page 15 of this article.

$$
\begin{aligned}
&\sum_{j\in E_{\mathrm{VRS}}} \lambda_{jo}x_{ij} = x_{io} - s_{io}^{-} && i=1,\ldots,m && (1.1)\\
&\sum_{j\in E_{\mathrm{VRS}}} \lambda_{jo}y_{rj} = y_{ro} + s_{ro}^{+} && r=1,\ldots,s && (1.2)\\
&\sum_{j\in E_{\mathrm{VRS}}} \lambda_{jo} = 1 && && (1.3)\\
&s_{io}^{-} \geq 0 && i=1,\ldots,m && (1.4)\\
&s_{ro}^{+} \geq 0 && r=1,\ldots,s && (1.5)\\
&\lambda_{jo} \geq 0 && j\in E_{\mathrm{VRS}} && (1.6)
\end{aligned}
\tag{1}
$$

where w_i, w_r and are weights representing the relative importance of unit inputs and unit outputs. The linear dual of model (1) can be written as follows:

$$
\begin{aligned}
z = \min\ & \sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r y_{ro} - u_0 \\
\text{s.t.}\quad & -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + u_0 \leq 0, && j\in E_{\mathrm{VRS}} && (2.1)\\
& v_i \geq w_i && i=1,\ldots,m && (2.2)\\
& u_r \geq w_r && r=1,\ldots,s. && (2.3)
\end{aligned}
\tag{2}
$$

Model (1) "maximizes" a weighted L_1 distance from the DMU_0 to the frontier of the production possibility set, and leading to a concurrent improvement in both output increase and input reduction. In contrast to the conventional weighted additive model, which establishes targets based on the farthest efficient projection to the evaluated DMU, it is widely recognized that the projection points produced by the weighted additive model consistently fall onto the strongly efficient frontier $\partial^s(T)$.

Unlike models that set the farthest targets, DEA literature has an approach that focuses on establishing the closest targets for inefficient units. But, applying this approach is not as simple as replacing "Max" with "Min" in model (1), determining the minimum distance and closest targets poses a computational challenge. This challenge arises from the intricacy of calculating the minimum distance to the frontier of DEA technology from an interior point, as this problem is equivalent to minimizing a convex function on the complement of a convex set.

Nowadays, there are principally two paths for determining closest targets in the DEA literature. The first one is based on identifying all the faces of the efficient frontier of the polyhedral DEA technology in a first stage, determining the minimum distance as the minimum of the distances to each of the faces in a multi-stage process. In this way, this first path is related to a combinatorial NP-hard problem and will not be explored in this paper. The second path corresponds to the approach proposed by Aparicio *et al.* [3], where the strongly efficient frontier is characterized by linear constraints and binary variables, which consequently allows the closest targets to be determined without calculating explicitly all the efficient faces by resorting to Mixed Integer Linear Programming. Next, we show the main result of Aparicio *et al.* [3].

The following theorem provides a useful characterization of $\partial(T)$, which will be used in the formulation of the benchmarking models we develop here, and to prove it, refer to Ruiz *et al.* [18]

Theorem 1.

$$
\partial(T) = \left\{ (x,y)\in R_{+}^{m+s} \,\middle|\, X = \sum_{j\in E}\lambda_j x_j,\ Y = \sum_{j\in E}\lambda_j y_j,\ \sum_{j\in E}\lambda_j = 1 \right.
$$

$$
-vx_j + uy_j + u_0 + d_j = 0 \qquad\qquad j\in E
$$

$$
\left.\begin{array}{ll}
v \geq 1, \quad u \geq 1, & \\
d_j \leq M b_j, & j \in E \\
\lambda_j \leq M(1-b_j), & j \in E \\
d_j, \lambda_j \geq 0 & \\
b_j \in \{0,1\}, & j \in E \\
u_0 \in R &
\end{array}\right\} \tag{3}
$$

where M is a big positive quantity.

This is an important result when we are interested in minimizing, instead of maximizing, the distance to the Pareto-efficient frontier of the PPS. Because it provides a characterization of (T) in terms of a set of linear constraints that can be added to the efficiency models.

Now, using Theorem 1, we can formulate a new version of the weighted additive model that minimizes the distance to $\|.\|_1$ (the closest targets) based on the integer linear programming in its brief form as follows[2]:

$$
\begin{array}{llll}
\text{Min } z_0 = \sum_{i=1}^{m} w_i s_{io}^{-} + \sum_{r=1}^{s} w_r s_{ro}^{+} & & & \\
\text{s.t.} \quad \sum_{j\in E} \lambda_j x_{ij} = x_{io} - s_{io}^{-} & & (4.1) & \\
\sum_{j\in E} \lambda_j y_{rj} = y_{ro} + s_{ro}^{+} & & (4.2) & \\
\sum_{j\in E} \lambda_j = 1 & & (4.3) & \\
-\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + u_0 + d_j = 0, & j \in E & (4.4) & (4) \\
v \geq 1, u \geq 1, & & (4.5) & \\
d_j, \lambda_j, w_i, w_r \geq 0 & & (4.6) & \\
d_j \leq M b_j, & j \in E & (4.7) & \\
\lambda_j \leq M(1-b_j), & j \in E & (4.8) & \\
b_j \in \{0,1\}, & j \in E & (4.9) & \\
u_0 \text{ free.} & & (4.10) &
\end{array}
$$

Note that the constraints of model (4) are obtained from the combination of constraints of models (1) and (2).

Indeed, (4.1)–(4.6) coincide with (1.1)–(1.6) and (2.1)–(2.3). The new constraints, (4.7)–(4.9), are the key to suitably mixing all the aforementioned restrictions, resorting to a set of card E_{VRS} binary variables b_j.

Proposition 1. DMU$_o$ *is technically efficient if and only if* $z_o^* = 0$.

Proof. Trivial. □

Definition 2. Suppose $p \in [1, \infty]$ p-norm is defined as follows:

$$
\|x\|_p = \begin{cases} \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}}, & \text{if } p \in [1, \infty) \\ \max\{|x_i|\} & \text{if } p = \infty. \end{cases} \tag{5}
$$

[2]In Aparicio *et al.* [3] $w_i = 1\ \forall i, w_r = 1, \forall r$, and w_r.

Note that for $p = 2$, we have the usual Euclidean distance. If $p = \infty$, then it is also called infinity or Chebyshev norm. Over R^n, the most commonly used norms are $\|.\|_p, p = 1, 2, \infty$.

Model (4) is a weighted non-oriented additive model that calculates input excesses and output shortage. For inefficient units, this model can be used for target setting and benchmarking. It should only be noted that unlike classical additive models, s^+, s^- instead of maximizing, these have been minimized because we are looking for the closest targets. As mentioned in the introduction, PPS efficient frontier represents the best performance frontier that Parato efficient units are on the frontier, enabling it to set efficient targets for different DMUs. As said in the introduction, the efficient frontier of the PPS represents a best practice frontier whose points enable the setting of efficient targets for the different DMUs. However, in spite of being technically efficient points of the PPS, some of them might not reflect the prior knowledge or accepted views of management. In this condition the selection of benchmarks that is made and the subsequent setting of targets are sometimes inconsistent with the preferences of management. To deal with this issue so we must interfere with the manager's opinion by ignoring inappropriate benchmarks in the target setting process.

It should be noted that the purpose of the presented algorithm is to determine the closest alternative model for the units under evaluation with the opinion of the management. Our proposed model is not an improvement of Aparicio *et al.*'s method. Because the basis of Aparicio *et al.*'s method is based on the concept of "dominance". The method presented by Aparicio *et al.* is able to determine benchmark or target only in the dominant area of the unit under the evaluation. If there is no justified target or benchmark in the dominant area, it cannot introduce the alternative benchmark or target in the non-dominant area. The main advantage of the proposed method over the Aparicio method is that the Aparicio method determines the benchmark and the close target within the dominant area of the DMU, but our proposed method determines the benchmark and the close target in both dominant or non-dominant areas (in the whole production possibility set). So it can be said that it determines the closest target and benchmark in the real sense for the unit under evaluation. Because sometimes the closest targets do not exist, with different norm, inside the dominant area. So, it can be safely said that the method of Aparicio *et al.* does not determine the closest target and benchmark, but rather it determines the close target. Or, more correctly, it determines the closest target and benchmark inside the dominant area. It can be said that the proposed method is an extension of the method of Aparicio *et al.*

3. Proposed method

According to what was mentioned, it is possible that sometimes these extreme efficiency points (or reference units) are not acceptable from the manager's point of view. It means that the manager does not accept them as a model for other ineffective units. In the real world, some units may not be placed based on their actual performance or may have reached this level of performance with special support and special privileges. Such units, because they are not placed in healthy competition with other units, should not be considered as benchmark for other units. because they will be the source of error and deviation from reality. Sometimes, the manager may not intend to benchmark some units for other inefficient units due to various other reasons such as lack of input resources or low cost efficiency. In this case, management by ignoring this type of unit should consider other extreme efficient units as fixed benchmarks and measure the inefficient units using these new benchmarks, and among them, determine the model units and targets for the inefficient DMUs. To achieve this goal, we must answer this question that: Which unit or units can the manager ignore in the benchmarking process if necessary? Is there a feature in all units that can be ignored if determined by the manager in the system? In order to answer, in mode of variable scale returns, we assume: $\alpha = \{$DMU, DMUs that can be ignored in the benchmarking process$\}$ and $\alpha \subset E_{\text{VRS}}$ *i.e.* α is a subset of the extreme efficient units.

3.1. Determination members of set α

It is clear that in benchmarking process, not all units can be ignored, because then the production possibility set frontier will not have a benchmark unit to introduce to the inefficient unit. On the other hand, $\alpha = \phi$ is also possible. In order to determine which units can be ignored by management, we will use a similar method

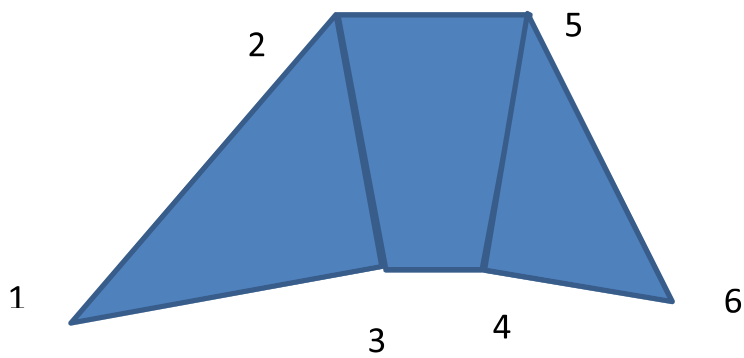

FIGURE 1. Constructing E_j.

adapted from an article by Jahanshahloo *et al.* [10]. Jahanshahloo *et al.* have presented an algorithm to finding strong defining hyperplanes of PPS. We will generalize the method presented by them and use it in the process of determining the units that can be ignored. We will briefly explain the method presented by them.

Suppose among all DMUs, L of them are extreme efficient; then all of these DMUs are on some strong supporting hyperplane. Because of strong efficiency, there exist some $(u^*, v^*) > 0$ then the extreme efficient DMU lies on $-v^{*t}x_0 + u^{*t}y_0 + u_0^* = 0$; and this hyperplane is strong, according to its definition. Using the following theorems, it can be determined which DMUs lie on the same supporting hyperplane.

Theorem 2. *Assume that $(x_p, y_p), (x_q, y_q)$ are DMUs observed on a supporting strong hyper plane then any convex combination of them is on the same hyper plane.*

Proof. Refer to Jahanshahloo *et al.* [10]. □

Theorem 3. *Assume that $(x_p, y_p), (x_q, y_q)$ are DMUs observed on different hyper planes (except for their intersection; if not empty). Then any point obtained from their convex combination will be an internal point and therefore will be radially inefficient.*

Proof. Refer to Jahanshahloo *et al.* [10]. □

Suppose we have L DMU that they are all extreme efficient. We can assume that extreme efficient DMU are $\text{DMU}_1, \ldots, \text{DMU}_l$. Suppose $E = \{1, \ldots, l\}$ is a set. We consider a distinct pair DMU_p and DMU_q that $p, q \in E$ and create a virtual DMU as follows:

$$\text{DMU}_k = \frac{1}{2}\text{DMU}_p + \frac{1}{2}\text{DMU}_q.$$

Using DEA models, it can be determined whether DMU_k is efficient or inefficient. If it is efficient, using the Theorem 2, it can be concluded that DMU_p and DMU_q are on a hyper plane.

For each member $j(j = 1, \ldots, l)$ of E a new set E_j we will construct. E_j is a subset of E that its member are coplanar. This means there exist some hyperplane contains DMU_j and some DMUs in E_j. It is obvious that $j \in E_j$.

Example 1. In Figure 1, suppose $1, 2, \ldots, 6$ are extreme efficient DMUs. Since there is a plane the passes through 1 and 2 then $1 \in E_2$ and $2 \in E_1$; and since there is a plane that passes through 2 and 5 then $2 \in E_5$ and $5 \in E_2$; and so on. Then

$$E_1 = \{1, 2, 3\} \qquad E_2 = \{1, 2, 3, 4, 5\} \qquad E_3 = \{1, 2, 3, 4, 5\}$$

$$E_4 = \{2,3,4,5,6\} \qquad E_5 = \{2,3,4,5,6\} \qquad E_6 = \{4,5,6\}.$$

Note: For DMUs with m inputs and s outputs, PPS will be a subset of R^{m+s}. Therefore, each supporting hyper plane of PPS is at least independent of $m+s$ Affine DMU.

In order to determine the set α members, we define a new set called $\bar{E}_j$ which using $\bar{E}_j = E - E_j$.

Example 2. For the above example data, $\bar{E}_j$ can be obtained as follows:

$$\begin{array}{ll} \bar{E}_1 = \{4,5,6\} & \bar{E}_2 = \{6\} \\ \bar{E}_3 = \{6\} & \bar{E}_4 = \{1\} \\ \bar{E}_5 = \{1\} & \bar{E}_6 = \{1,2,3\}. \end{array}$$

We will define the set α as follows:

$$\alpha = \left\{ j \middle| j \in E,\ \left|\bar{E}_j\right| = \text{Max.} \right\}$$

The best units to ignore are units in the set of $\bar{E}_j$, that there are more units. In other words, the cardinality of the set of $\bar{E}_j$ is the maximum. A lower value of this numerical value for each unit indicates a lower possibility to ignore. If, $\bar{E}_j = \phi$ it means that this unit cannot be ignored.

3.1.1. Algorithm for determining the set α members

Step 1. Determine extreme efficient units and assume $|E| = l$.
Step 2. For each $p, q \in E$ that $p \neq q$, evaluate $\text{DMU}_k = \frac{1}{2}\text{DMU}_p + \frac{1}{2}\text{DMU}_q$. If it is efficient, $p \in E_q, q \in E_p$.
Step 3. Calculate $\bar{E}_j = E - E_j$ for each $j (j = 1, \ldots, l)$.
Step 4. Determine $\alpha = \left\{ j \middle| j \in E, \left|\bar{E}_j\right| = \text{Max} \right\}$ (The member of set α are $j \in E$ that Cardinal numbers of $\bar{E}_j$ is max).

3.2. Forming the model

Once the process of determining α members, the manager can use his analysis and the information he already has from the system to select the unit or units that should not be used as a benchmark for other units ($\bar{\alpha}$) through which he will apply his opinion in order to make the process of determining the benchmark and target as real as possible. So $\bar{\alpha} \subseteq \alpha \subset E_{\text{VRS}}$.

Now, by determining the members of set $\bar{\alpha}$ by the system manager, model (6) can be used by substituting $\bar{\alpha}$ instead of E and adding limits (6.4) and (6.5) and considering $\alpha'_{\text{VRS}} = E_{\text{VRS}} - \bar{\alpha}_{\text{VRS}}$ to determine alternative models for the units under evaluation.

Now the following model is obtained to determine the closest alternative model by adding constraints (6.4) and (6.5) and considering $\alpha'_{\text{VRS}} = E_{\text{VRS}} - \bar{\alpha}_{\text{VRS}}$:

$$\min z = \sum_{i=1}^{m} w_i^- s_{io}^- + \sum_{r=1}^{s} w_r^+ s_{ro}^+$$

s.t.

$$\sum_{j \in \alpha'_{\text{VRS}}} \lambda_j x_{ij} = x_{io} - s_{io}^- \qquad i = 1, \ldots, m \tag{6.1}$$

$$\sum_{j \in \alpha'_{\text{VRS}}} \lambda_j y_{rj} = y_{ro} + s_{ro}^+ \qquad r = 1, \ldots, s \tag{6.2}$$

$$\sum_{j \in \alpha'_{\text{VRS}}} \lambda_j = 1 \tag{6.3}$$

$$
\begin{aligned}
& -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + u_0 + d_j \leq 0 && j \in \bar{\alpha}_{\text{VRS}} && (6.4)\\
& -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + u_0 + d_j = 0 && j \notin \bar{\alpha}_{\text{VRS}},\quad j \in E_{\text{VRS}} && (6.5)\\
& v_i \geq w_i^- && i = 1,\ldots,m && (6.6)\\
& u_r \geq w_r^+ && r = 1,\ldots,s && (6.7)\\
& s_i^- \geq 0 && i = 1,\ldots,m && (6.8)\\
& s_r^+ \geq 0 && r = 1,\ldots,s && (6.9)\\
& \lambda_j \geq 0 && j \in \alpha'_{\text{VRS}} && (6.10)\\
& d_j \leq M b_j && j \in \alpha'_{\text{VRS}} && (6.11)\\
& \lambda_j \leq 1 - b_j && j \in \alpha'_{\text{VRS}} && (6.12)\\
& d_j \geq 0 && j \in \alpha'_{\text{VRS}} && (6.13)\\
& b_j \in \{0,1\} \cdot d_j, \lambda_j \geq 0 && j \in \alpha'_{\text{VRS}} && (6.14)\\
& u_0 \ \text{ free.} && && (6.15)
\end{aligned} \tag{6}
$$

If the production possibility set of model (6) is not zero, it is a the hyperplane where all DMUs belonging to α'_{VRS}, are located. This is guaranteed by the set of constraints (6.5). The set of constraints (6.4) guaranteed that all DMUs are in $\bar{\alpha}$ are on one side of the hyperplane.

By ignoring some efficient units in the benchmarking process, the obtained efficiency frontier will be a subset of the initial frontier $\partial^{\alpha}(T) \subset \partial(T)$.

Definition 3. DMU_0 with input–output vector (x_0, y_0) is said to be $\bar{\alpha}$-efficient if, and only if, $(x_0, y_0) \in \partial^{\bar{\alpha}}(T)$.

Now the new frontier obtained $(\partial^{\bar{\alpha}}(T))$ is the frontier of the best performance taking into account the opinion of the system manager because some efficient units have been ignored in the benchmarking process. As regards $\partial^{\bar{\alpha}}(T) \subseteq \partial(T)$ therefore, some DMUs may be located on the frontier of $\partial^{\bar{\alpha}}(T)$, that is, they are efficient from the point of view of the previous frontier, but they are not efficient from the point of view of the new frontier, and they are not located on the frontier of $\partial^{\bar{\alpha}}(T)$. Of course, for such units, determining the targets and benchmarking may be accompanied by the deterioration of some inputs or outputs, in order to reach the new efficiency frontier. In other words, when manager's opinion are incorporated into the analysis, dominance does not necessarily fulfill. On the other hand, when management is aware that the evaluated inefficient unit, in practice is not possible to achieve the performance of some efficient units in the system for various reasons, so it is better for the unit to think about determining other benchmarks through the closest alternative benchmarks before doing anything and wasting its energy and resources. Attempting to achieve the performance of units that are not justified by the manager will be a futile effort, hence avoiding this even if the process of benchmarking and target setting leads to worsening some inputs and outputs will be a good thing because ignoring such units sometimes leads to select the closest model outside DMU dominant region.

Due to the non-fulfillment of dominance, the model (6) can be written as follows for benchmarking and determining alternative targets.

$$
\min\ z^B = \sum_{i=1}^{m} w_i^- \left|s_{io}^-\right| + \sum_{r=1}^{s} w_r^+ \left|s_{ro}^+\right|
$$

s.t.

$$
\sum_{j \in \alpha'_{\text{VRS}}}^{.} \lambda_j x_{ij} = x_{io} - s_{io}^- \qquad i = 1,\ldots,m \tag{7.1}
$$

$$
\begin{aligned}
&\sum_{j\in\alpha'_{\mathrm{VRS}}} \lambda_j y_{rj} = y_{ro} + s^+_{ro} && r = 1,\ldots,s && (7.2)\\
&\sum_{j\in\alpha'_{\mathrm{VRS}}} \lambda_j = 1 && && (7.3)\\
&-\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + u_0 + d_j \le 0 && j \in \bar{\alpha}_{\mathrm{VRS}} && (7.4)\\
&-\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + u_0 + d_j = 0 && j \notin \bar{\alpha}_{\mathrm{VRS}},\quad j \in E_{\mathrm{VRS}} && (7.5) \qquad (7)\\
&v_i \ge w^-_i && i = 1,\ldots,m && (7.6)\\
&u_r \ge w^+_r && r = 1,\ldots,s && (7.7)\\
&s^-_i \ge 0 && i = 1,\ldots,m && (7.8)\\
&s^+_r \ge 0 && r = 1,\ldots,s && (7.9)\\
&\lambda_j \ge 0 && j \in \alpha'_{\mathrm{VRS}} && (7.10)\\
&d_j \le M b_j && j \in \alpha'_{\mathrm{VRS}} && (7.11)\\
&\lambda_j \le 1 - b_j && j \in \alpha'_{\mathrm{VRS}} && (7.12)\\
&d_j \ge 0 && j \in \alpha'_{\mathrm{VRS}} && (7.13)\\
&b_j \in \{0,1\} \cdot d_j, \lambda_j \ge 0 && j \in \alpha'_{\mathrm{VRS}} && (7.14)\\
&u_0 \ \text{ free} && && (7.15)\\
&s^-_{io}, s^+_{ro} \ \text{free} && \forall i, r.
\end{aligned}
$$

Model (7) minimizes the $L1$-distance ($\|.\|_1$) from DMU_0 to $\partial^{\bar{\alpha}}(T)$. As said before, if there is no real benchmark in the dominant region that conforms to management, the dominant element is not required for benchmarking and target setting because slacks are not non-negative but free variables and DMUs are allowed to increase some of their inputs if necessary and some reduce their outputs to reach frontier $\partial^{\bar{\alpha}}(T)$. The DMUs α-efficient are the ones that can play a role as potential benchmarks for the rest of the units.

Due to the absolute value in model (7), this model is a nonlinear model that can be easily linearized. For this purpose, new decision-making variables $\varphi^{I+}_{io}, \varphi^{I-}_{io} \ge 0$ $i = 1,\ldots,m$, and $\varphi^{o+}_{ro}, \varphi^{o-}_{ro} \ge 0$ $r = 1,\ldots,s$: add to the set of constraints of (5) and $s^+_{ro} = \varphi^{o+}_{ro} - \varphi^{o-}_{ro}$ $r = 1,\ldots,s$.

$s^-_{io} = \varphi^{I+}_{io} - \varphi^{I-}_{io}$ $i = 1,\ldots,m$ then, minimizing the non-linear objective in (5) is equivalent to minimizing the linear objective function $\sum_{i=1}^{m}(\varphi^{I+}_{io} + \varphi^{I-}_{io}) + \sum_{r=1}^{s}(\varphi^{o+}_{ro} + \varphi^{o-}_{ro})$ subject to the resulting set of constraints.

4. Numerical example

Table 1 shows the input and output values for eleven DMUs. In this part, we implemented the proposed algorithm on the data in Table 1 and first by specifying set α (permissible efficient units to be ignored), then we select set $\bar{\alpha}$ with the manager's opinion, and then we will introduce the closest alternative model for the units under evaluation.

PPS generated from DMUs in Table 1 are shown in Figure 2.

Extreme efficient DMUs (E) are obtained from article Charnes *et al.* [5] as follows:

$$
\begin{aligned}
&\text{Min } \lambda_o\\
&\text{s.t.}\\
&\qquad \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io}, \qquad i = 1,\ldots,m
\end{aligned}
$$

Table 1. Input and output values for eleven DMU.

DMU	A	B	C	D	E	F	G	H	I	J	K
x_1	1	3	3	5	3	3	5	4	10	10	9
x_2	4	2	6	1	6	5	5	10	3	3	2
y	5	4	6	6	3	8	9	7	5	10	6

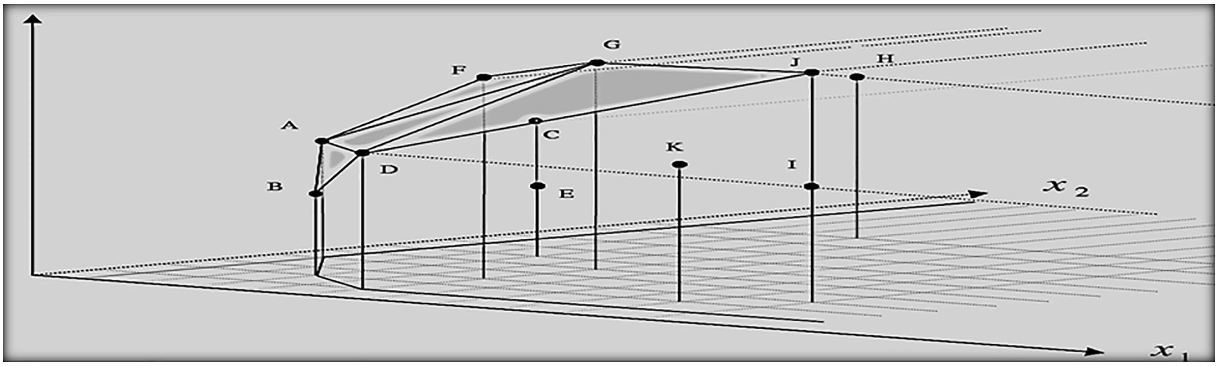

Figure 2. Position of DMUs in Table 1.

$$\begin{aligned} &\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, && r = 1, \ldots, s \\ &\sum_{j=1}^{n} \lambda_j = 1 && j = 1, \ldots, n \\ &\lambda_j \geq 0 && j = 1, \ldots, n. \end{aligned} \tag{8}$$

Assume that λ_o^* is the optimal value of the above benchmark, if $\lambda_o^* = 1$, then DMU_o is an extreme efficient unit. In the above example:

$$E = \{A, B, D, F, G, J\}.$$

Using the second and third steps of the presented algorithm, we determine the sets E_j and $\bar{E}_j$ for each member of E.

$$\begin{aligned} &E_A = \{A, B, D, F, G\} && \bar{E}_A = \{J\} \\ &E_B = \{A, B, D\} && \bar{E}_B = \{F, G, J\} \\ &E_D = \{A, B, D, G, J\} && \bar{E}_D = \{F\} \\ &E_F = \{A, F, G\} && \bar{E}_F = \{B, D, J\} \\ &E_G = \{A, D, F, G, J\} && \bar{E}_G = \{B\} \\ &E_J = \{D, G, J\} && \bar{E}_J = \{A, B, F\}. \end{aligned}$$

Then, we select subsets $m + s = 3$ as E member with the conditions mentioned on Jahanshahloo *et al.* [10], in which case the units of strong full-dimensional strong components will be as follows:

$$P1 = \{A, B, D\}, \quad P2 = \{A, F, G\}, \quad P3 = \{A, G, D\}, \quad P4 = \{D, G, J\}.$$

TABLE 2. Benchmarks and target points using model (4) (Aparicio *et al.*).

Inefficient unit	$j\|\lambda_j \geq 0$	Target point	S_1^-, S_2^-, S_1^+
$K(9,2,6)$	$D(0.5)$ $J(0.5)$	$(7.5,2,8)$	$(1.5,0,2)$
$I(10,3,5)$	$J1$	$(10,3,10)$	$(0,0,5)$
$H(4,10,7)$	$F(0.5)$ $G(0.5)$	$(4,5,8.5)$	$(0,5,1.5)$
$C(3,6,6)$	$F1$	$(3,5,8)$	$(0,1,2)$
$E(3,6,3)$	$B1$	$(3,2,4)$	$(0,4,1)$

TABLE 3. Alternative targets and benchmarks by ignoring inappropriate benchmark.

Inefficient unit	$j\|\lambda_j \geq 0$	Target point	$\varphi_1^{I+}, \varphi_1^{I-} \varphi_2^{I+} \varphi_2^{I-}, \varphi_1^{o+}, \varphi_1^{o-}$
$K(9,2,6)$	$D(0.5)$ $G(0.5)$	$(5,2,6.75)$	$\varphi_1^{I+}=4, \varphi_1^{I-}=0, \varphi_2^{I+}=0, \varphi_2^{I-}=0, \varphi_1^{o+}=0.75, \varphi_1^{o-}=0$
$I(5,3,7.5)$	$D(0.5)$ $G(0.5)$	$I(10,3,5)$	$\varphi_1^{I+}=5, \varphi_1^{I-}=0, \varphi_2^{I+}=0, \varphi_2^{I-}=0, \varphi_1^{o+}=2.5, \varphi_1^{o-}=0$
$H(4,10,7)$	$F(0.5)$ $G(0.5)$	$(4,5,8.5)$	$\varphi_1^{I+}=0, \varphi_1^{I-}=0, \varphi_2^{I+}=5, \varphi_2^{I-}=0, \varphi_1^{o+}=1.5, \varphi_1^{o-}=0$
$C(3,6,6)$	$F1$	$(3,5,8)$	$\varphi_1^{I+}=0, \varphi_1^{I-}=0, \varphi_2^{I+}=1, \varphi_2^{I-}=0, \varphi_1^{o+}=2, \varphi_1^{o-}=0$
$E(3,6,3)$	$F1$	$(3,5,8)$	$\varphi_1^{I+}=0, \varphi_1^{I-}=0, \varphi_2^{I+}=1, \varphi_2^{I-}=0, \varphi_1^{o+}=5, \varphi_1^{o-}=0$

Now according to the algorithm (units allowed to be ignored by management) α is determined as follows.

$$\alpha = \left\{ j \middle| j \in E, \left|\bar{E}_j\right| = \text{Max} \right\} = \{B, F, J\}.$$

In order to show the algorithm and model presented in this article and make a favorable comparison, we first obtain the target for inefficient units by using the model of Aparicio *et al.* then, for the same units, using the proposed algorithm, we introduce an alternative target and benchmarks in Table 3.

So, in summary, as it is clear from Tables 2 and 3 it can be said: In Aparicio *et al.*'s method, reaching the efficiency frontier for the unit under evaluation is definitely done by improving the input or output or both, but in the method presented by us, the improvement may not be done depending on the conditions, but the target of determining, it is closer to the target determined by the Aparicio method and achieves the efficiency frontier with the least changes in input and output. The reason that we may sometimes have to increase the input is that the need for production (output) may be much more evaluated than the dominant area per unit. Or, it is necessary to increase one of the inputs, for example, to increase employment, so it is necessary to increase the input in the closest targets.

The closest targets and benchmarks for inefficient units using the model provided by Aparicio *et al.* [3] with $\|0\|_1$ are listed in the Table 2.

In order to show the given approach in this article, we assume that management with its previous knowledge and information considers two efficient Units J and B ($\bar{\alpha} = \{B, J\}$) as inappropriate units for benchmarking among the units of the set α. Because model (4) of Aparicio *et al.* is not effective on determining the benchmark, hence our proposed model in this article can determine the closest alternative model by ignoring the units of the set $\bar{\alpha}$ in benchmarking process for inefficient units. The Table 3 shows benchmarking and target setting by ignoring the two units mentioned in this process that the results are as follows.

The results of Table 3 show that inefficient Unit K, by ignoring Unit J in benchmarking process, replaces it with another unit called Unit G and the data in Table 2 shows that the final target for unit K is $(7.5, 2, 8)$, but by applying the presented algorithm, the closest alternative target for this unit is $(5, 2, 6.75)$. For the ineffective unit I, the benchmark introduced by model (4) is unit J, and by removing the units of set $\bar{\alpha}$, the replacement benchmark for I is units D and G. But the benchmark of Unit C, which is the efficient Unit F, remains unchanged. This shows that ignoring Units $\bar{\alpha}$ in benchmarking process has no effect on the selected benchmark of Unit C because the benchmarks of Unit C are not inappropriate *i.e.* $\bar{\alpha}$ set so remain unchanged. But the closest benchmark for Unit E using model (4) is Unit B, which is located on the efficient side (A, B, D). Ignoring Unit B, J by management, the inefficient unit E is forced to choose other units used as benchmarks for itself. By applying the proposed model in this article, the closest alternative benchmark to the mentioned unit is Unit F, which is on the efficient side (A, F, G). This means that the proposed model introduces a new benchmark for inefficient units in the system if their former benchmark is a subset of set α. In other words, if their previous benchmark is a member of set $\bar{\alpha}$, it should be replaced in the benchmarking process.

5. Conclusion

In this article, an algorithm was presented that can determine the closest alternative benchmark for the inefficient units in the system if they have an inappropriate benchmark from the point of view of management. This algorithm had three main steps: (1) Specify, allowed benchmarks to ignore; (2) Choosing among these benchmarks by the manager based on his knowledge and expertise; (3) Determining the closest alternative benchmark for the units under evaluation. This prevents the extra effort of the unit to achieve efficiency and determines the best and fastest way to achieve efficiency from an alternative path for inefficient units. This approach can have many advantages in the real world and provide justified targets and benchmarks technically and in terms of management knowledge, experience and belief to introduce to inefficient units in the system so that targets are fully consistent with reality and prevent waste of resources. Your sentence is mostly clear, but it could benefit from a slight rephrasing for improved clarity and grammatical accuracy. For future work, it is recommended to consider selecting the closest target that is both technically and managerially justified, as well as identifying the closest benchmark units based on cost efficiency. Additionally, it is suggested to explore determining the closest target in the presence of undesirable outputs and under uncertain circumstances.

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