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FINDING ALTERNATIVE STEPWISE IMPROVEMENT PATHS WITH DIFFERENT IMPROVEMENT EFFECTS

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Abstract. Targets on the data-envelopment-analysis (DEA) efficient frontier show the direction of inefficient decision-making units' (DMUs) improvements. Some authors have correlated the improvement effects of targets with management needs. Nevertheless, targets are unattainable for DMUs with poor performance. Hence, stepwise improvement paths with different improvement effects should be provided to solve the problem of unachievable targets and flexibly meet decision-makers' (DMs) diverse management needs in performance improvement. For this purpose, we propose novel stepwise DEA benchmarking approaches and a search algorithm to find progress-improvement strategies with different improvement effects. The contributions of our approaches are as follows. (1) Several stepwise improvement paths with different improvement effects are found, which provides DMs with more abundant alternatives for performance optimization in various management situations. (2) The upper bound for efforts in all intermediate improvement steps can be effectively controlled, which overcomes the problem of unachievable targets and makes stepwise improvement paths more feasible. To illustrate our approaches, we conducted a case study on 18 ports in China. In this case study of Chinese ports, we further highlighted the advantages of our methods by comparing them with existing DEA models. In sum, our approaches are suitable for poorly performing units that aspire to align performance improvements with management needs.

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1. Introduction

In many industries, benchmarking has been used to identify best practices and use them as targets or benchmarks to assess and improve the performance of peer firms. Data envelopment analysis (DEA) has been proven to play an important role in benchmarking. This method can find the targets with best practices on the efficient frontier of a production possibility set (PPS) consisting of all observed homogeneous decision-making units (DMUs), and it can further use targets as benchmarks for evaluating and improving actual performance [1]. These targets represent the best input-output levels that inefficient DMUs aspire to attain, thus illustrating directions for improvement [2]. The efficient frontier is also known as the "best-practice frontier" [3]. In terms of performance improvement, the direction for improvement determines the improvement effects, and the target on the efficient frontier represents the direct improvement path with the specific improvement effect. Hence, target setting is one of the most important functions of DEA.

Target setting should match the actual management needs of DMUs. In other words, the direction of improvement shown by the target should be consistent with the management goals associated with the decrease or increase in certain inputs or outputs. Currently, some DEA methods can generate a series of alternative targets (or direct improvement paths) with different effects for improvement. For example, Lozano and Soltani [4] proposed a new multidirectional directional distance function (DDF) approach that uses a grid-based method to sample all possible improvement directions from a given inefficient operating DMU. In the framework of cross-benchmarking, Ramón *et al.* [5] developed a new DEA method that can set alternative targets with different improvement directions determined by a broad range of common peer groups. Ruiz and Sirvent [6] proposed bi-objective benchmarking methods that search for alternative closer targets by identifying different directions for improvement within additional efforts allowed by decision-makers (DMs). In sum, the alternative targets provided by these studies achieve different improvement effects. However, as Ruiz and Sirvent [6] stated, for DMUs with poor performance, since there exists a large gap between their actual performance and best-practice targets (even if the target is closest to the actual performance), targets with different effects remain unattainable in the short term. The same view was echoed by Ramón *et al.* [5]. In other words, reaching the target requires huge reductions in inputs and increases in output [7]. Owing to the restriction of management expertise, available resources, and other real factors, it may be practically infeasible for DMUs with poor performance to achieve the target in a single step [8,9]. In brief, the targets (or direct improvement paths) showing different improvement directions/effects may still be unachievable for many DMUs with poor performance. Hence, it is necessary to find alternative stepwise improvement paths that consider both different improvement effects and intermediate targets, to achieve performance improvement in various management needs and deal with overambitious targets. To this end, this paper presents novel stepwise benchmarking approaches and a search algorithm to search for stepwise improvement paths with different improvement effects. The contributions of the proposed approaches and algorithm are as follows. (1) Several alternative stepwise improvement paths with different improvement effects can be identified based on differences in improvement direction. This can provide DMs with all kinds of different alternatives to address performance improvement needs in different management situations. (2) The upper bound for the improvement efforts of all intermediate steps can be adjusted and controlled according to DMs' requirements, which makes the local performance improvement of each intermediate step easier. In other words, this overcomes the problem of overambitious targets and guarantees the feasibility of the step-by-step plans. In sum, our stepwise improvement plans can both solve the problem of unachievable targets proposed by Ruiz and Sirvent [6] and can further provide several gradual improvement plans with different improvement effects for DMs. Hence, our stepwise DEA is suitable for low-performing DMUs, especially when these units are subject to various practical factors (*e.g.*, management expertise and available resources) but want performance improvements to meet their actual management needs.

The rest of this paper is organized as follows. In Section 2, we first propose novel multi-objective stepwise DEA benchmarking approaches and a search algorithm for finding several stepwise improvement paths with different improvement effects. In Section 3, the proposed novel stepwise DEA approaches and search algorithm are applied to a case study consisting of 18 ports in China. The conclusion is presented in Section 4.

2. Literature review

Although DEA models are useful for benchmarking, many additive or weighted DEA models, which always maximize slacks, often provide unattainable targets that are too divergent from actual performance [10]. To solve this problem, some scholars have proposed new DEA approaches to find the closest targets [11–15]. Among them, one simpler and more efficient approach is noteworthy; that is, a new DEA model proposed by Aparicio *et al.* [16] can be employed to identify the target that is closest to the actual performance of an inefficient DMU. The closest target can be achieved with less effort than the one provided by additive or weighed DEA models. Since the closest target presents a direct path with better feasibility (compared with the farther target identified by additive or weighted DEA approaches), the DEA method of Aparicio *et al.* [16] has been expanded and applied in many ways: closest target setting with expert preferences [17], closest targets satisfying strong monotonicity [18], output-oriented closest targets [19], closest targets identified under the framework of common benchmarking [20, 21], carbon emission abatement considering closest targets [22], closest target setting on the framework of cross-benchmarking [5], finding the closest targets similar to management goals [23, 24], and peer identification that is similar to the closest target [6]. However, as many authors say [5, 6, 10, 25], frequently, the closest targets are still unachievable for some inefficient DMUs (or DMUs with poor performance), at least in the short term. This paper focuses on these DMUs with poor performance.

To solve the problem of unachievable targets (including closest targets), several studies have developed various stepwise DEA benchmarking approaches based on sequential benchmarking. These stepwise DEA methods aim to set successive intermediate targets between final targets (with best practices) and actual performance in different ways. Successive intermediate targets advance toward the final target and further form a stepwise improvement path. There are several types of DEA-based studies for stepwise paths. The first type of research is mainly based on the stratification algorithm proposed by Seiford and Zhu [26]. To be specific, this type of study first clusters all DMUs into several frontiers with different performance levels by running a stratification algorithm, and then it identifies (at least one) intermediate target on each intermediate frontier according to different criteria. For example, Seiford and Zhu [26] selected one of the real DMUs on the intermediate frontier as an intermediate target according to attractiveness and progress. Based on the study of Seiford and Zhu [26], Lim *et al.* [9] added new criteria, called infeasibility, to select one as an intermediate target from all real DMUs on a middle frontier. Similarly, Wu *et al.* [27] regarded the closeness of the target as a new criterion to find a suitable intermediate target, considering attractiveness and progress. Kwon *et al.* [28] proposed a "better-practice" benchmarking approach that can identify intermediate targets on each layer's middle frontier by introducing two functions to the neural network approach: adaptive learning and prediction capability. Ramón *et al.* [10] proposed a two-step benchmarking method, and this DEA method searches for (at least one) alternative intermediate target on the intermediate frontier by adjusting the importance coefficient related to the distances between the intermediate target and final target and between the intermediate target and actual performance. Kim *et al.* [29] first proposed a new DEA model to identify the closest final target among existing efficient DMUs and then determined the shortest stepwise paths consisting of sequential midterm targets based on hierarchical algorithms and Dijkstra's algorithm. Different from the first type of literature based on the layering algorithm, the second type can identify successive intermediate targets based on known input-output changes. Lozano and Villa [7] obtained a sequence of intermediate targets by iteratively solving a novel DEA model that considers given bounds on the rates of change in inputs and outputs that a DMU under evaluation can accomplish in each intermediate step. In the context of common benchmarking (or similar circumstances), based on the given changes in inputs and outputs, Ji *et al.* [30] developed a DEA model and iteratively solved this method to find successive medial targets and determine the stepwise path to achieving targets closest to actual management goals. Based on the work of Ji *et al.* [30], Ji *et al.* [31] first proposed a DEA model and iteratively ran it to find the stepwise path to reach best-practice targets. These targets are closest to their respective corresponding actual management goals and can ensure the improvement of the total amount of each input/output. In addition to the first two types of studies, some authors have determined the stepwise path by combining the hierarchical algorithm with the given rate of input-output change. For instance, An *et al.* [32]

first clustered all DMUs into several efficient frontiers with different performance levels by running hierarchical algorithm, and then they proposed a bound-change target-setting approach that identifies an intermediate target based on the minimum distance to the next intermediate frontier and the given bounds on the rates of change in inputs and outputs. Through the method of An *et al.* [32], three different paths are obtained: cross-level path, one-step path, and step-by-step path. Apart from the first three studies based on frontier stratification and given changes in inputs and outputs, many authors have identified stepwise benchmarking paths from various perspectives. For example, Lozano and Calzada-Infante [4] proposed a new stepwise benchmarking approach, and this DEA method can determine the successive intermediate targets based on the concept of efficiency field potential given by a continuous and differentiable function that decreases monotonously as inputs decrease and outputs increase. Furthermore, Nasrabadi *et al.* [33] proposed an algorithm that can provide a stepwise improvement path of targets for each inefficient DMU. All DMUs on this path are better than the DMU under evaluation in terms of the efficiency scores defined for interval scale data. Dehnokhalaji and Soltani [34] set a sequence of intermediate targets according to three criteria: the same returns to scale between targets and DMUs under evaluation, the minimum distance from intermediate targets to the part of the efficient frontier, and nondecreasing efficiency values. Kadziński *et al.* [35] proposed a stepwise benchmarking method based on the framework of multiple criteria sorting to find the corresponding progress paths.

Setting the stepwise improvement paths consisting of successive intermediate targets is a feasible way to address the problem of unachievable or overambitious targets. However, most of the above stepwise DEA approaches can only provide a unique stepwise path whose improvement effects may not match the actual management situations of DMs. Although a few studies, such as that of Ramón *et al.* [10], have produced multiple alternative step-by-step paths, they have also failed to meet various management needs (this is specifically analyzed in Sect. 4.3.1). Moreover, the stepwise DEA methods proposed by the existing studies do not effectively limit the intermediate improvement efforts (*i.e.*, to reduce intermediate improvement efforts as much as possible to the extent permitted by DMs), which may reduce the feasibility of the path. To fill these gaps, we aim to make two contributions through our proposed stepwise DEA methods: (1) to provide multiple progressive improvement paths with different improvement functions to adapt to different management requirements and (2) to effectively limit the upper bound of intermediate improvement efforts for each stepwise path to ensure that poor performance can be better improved.

3. Stepwise DEA benchmarking approaches

3.1. DEA efficiency frontiers with different levels of performance

In this section, we first consider DMUs, where each $\text{DMU}_j (j = 1, \ldots, n)$ can produce s desirable outputs, denoted by $Y_j = (y_{1j}, \cdots, y_{rj}, \cdots, y_{sj}) \geq 0$ using m inputs that are denoted by $X_j = (x_{1j}, \cdots, x_{ij}, \cdots, x_{mj}) \geq 0$. (X_j, Y_j) represents an actual performance of DMU_j. For the stepwise benchmarking, we construct the following PPS under various-returns-to-scale (VRS) technology [36]:

$$T = \left\{ (X, Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \right\}. \quad (1)$$

The DEA efficient frontier of PPS T is denoted by $\partial(T)$. λ_j is the intensity (or weight) of $\text{DMU}_j, j \in T$. $\sum_{j=1}^{n} \lambda_j = 1$ means that T is the PPS under VRS. Based on $\sum_{j=1}^{n} \lambda_j = 1$, $(\sum_{j=1}^{n} \lambda_j X_j, \sum_{j=1}^{n} \lambda_j Y_j)$ is a result of the linear convex combination of all $(X_j, Y_j), j \in T$ and can also be seen as a possible target for DMU_j. According to relevant studies [37, 38], the weak disposal should also be considered in (1) when undesirable outputs exist. For this reason, this paper also establishes novel stepwise benchmarking approaches considering the weak disposal of undesirable outputs (see Appendix 1).

Next, through the layering algorithm in Seiford and Zhu [26], we stratify all DMUs in T into several nested DEA efficiency frontiers with different levels of performance. The specific process is as follows:

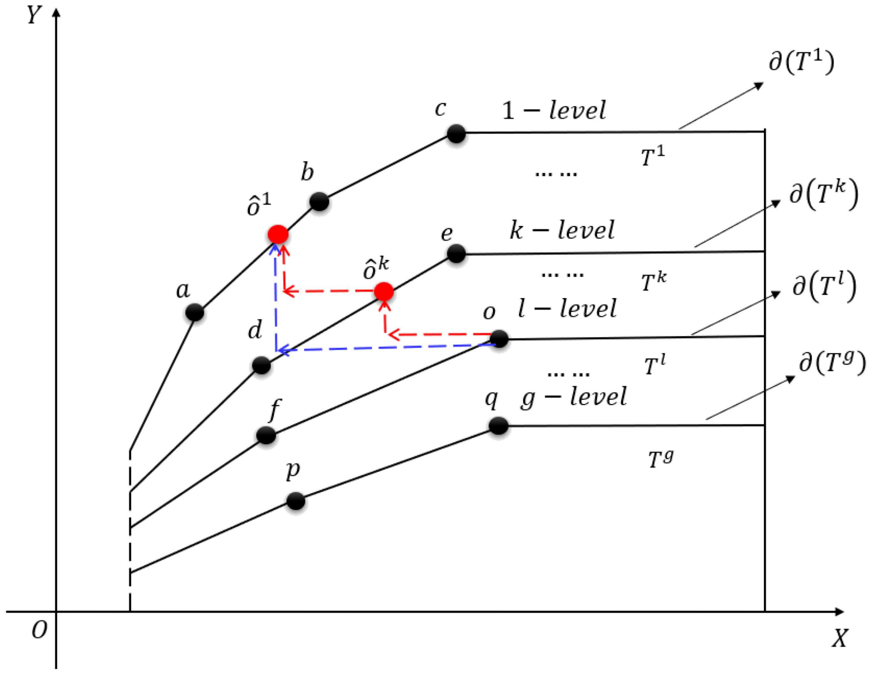

FIGURE 1. Efficient frontier with different levels of performance.

Stratification algorithm [26]

Step 1. Set $k = 1$, and $T^1 = T$.

Step 2. Calculate the efficiency values[1] for all DMUs in T^1, obtain the efficient DMUs, and set $E^1 = \{\text{All efficient DMUs in } T^1\}$ and $\partial(T^1)$ (or the first-level or best-practice frontier).

Step 3. Exclude efficient DMUs by setting $T^{k+1} = T^k - E^k$.

Step 4. Set $k = k + 1$ and go to Step 2.

Stopping standard: $T^k = \emptyset$.

Suppose we obtain g layers efficient frontiers with different levels of performance, which can be illustrated in Figure 1.

In Figure 1, a, b, and c are on the first-level efficient frontier $\partial(T^1)$ (or the efficient frontier with best practices), and $a, b, c \in E^1$. Likewise, d and e are on the k-level intermediate frontier $\partial(T^k)$, and $d, e \in E^k$. When $l > 2$, units (f and o) on $\partial(T^l)$ have poor performance and need to reach the best practice targets by setting a series of middle targets (see red dots in Fig. 1). The same principle also applies to units (p, q) on $\partial(T^g)$. For instance, to reach the best practice level, o needs to find the intermediate target $\hat{o}^k$ on $\partial(T^k)$ and efficient target $\hat{o}^1$ on $\partial(T^1)$ (see red dotted arrows in Fig. 1). The change from o and $\hat{o}^1$ will form a stepwise improvement path (see red dotted arrows in Fig. 1). It should be emphasized again that the global improvement direction of the stepwise path is determined by $\hat{o}^1$ because $\hat{o}^1$ determines the cumulative percentage improvement of inputs and outputs from o to $\hat{o}^k$ (see the blue dotted arrow in Fig. 1).

[1]The efficiency value of the DMU can be calculated using non-radial DEA models [9, 39]. When weak disposal of undesirable output is considered, (A.3) in Appendix 1 can be used as a tool to measure efficiency (although (A.3) is mainly used to find the closest target considering weak disposal in this paper).

3.2. Stepwise improvement paths

According to Figure 1, for every DMU on $\partial(T^l)(2 < l \le g)$, to attain the best performance level on $\partial(T^1)$, they need to find a sequence of intermediate targets $(\hat{X}_0^k, \hat{Y}_0^k)$ on the intermediate frontier $\partial(T^k)(2 \le k \le l-1)$, as well as the final target $(\hat{X}_0^1, \hat{Y}_0^1)$ on $\partial(T^1)$. These sequential targets form the stepwise improvement paths, and the final target determines the improvement direction (or cumulative improvement percentage of inputs and outputs). From (X_0^l, Y_0^l) to $(\hat{X}_0^1, \hat{Y}_0^1)$, to implement the stepwise improvement path with less effort, we should both minimize the gaps between $(\hat{X}_0^k, \hat{Y}_0^k)$ and $(\hat{X}_0^{k+1}, \hat{Y}_0^{k+1})(1 \le k \le l-2)$ and minimize the gap between $(\hat{X}_0^{l-1}, \hat{Y}_0^{l-1})$ and (X_0^l, Y_0^l). This can be implemented by the following multi-objective programming:

$$
\begin{aligned}
&\text{Min } \left\{e_0^1, \dots e_0^k, \dots, e_0^{l-1}\right\} \\
&\ \text{s.t.} \\
&\qquad \left(\hat{X}_0^k, \hat{Y}_0^k\right) \in \partial(T^k) && k = 1, \dots, l-1 \\
&\qquad \left(X_0^l, Y_0^l\right) \in \partial(T^l) \\
&\qquad \left(\hat{X}_0^k, \hat{Y}_0^k\right) \text{ dominates } \left(\hat{X}_0^{k+1}, \hat{Y}_0^{k+1}\right) && k = 1, \dots, l-1 \\
&\qquad \left(\hat{X}_0^{l-1}, \hat{Y}_0^{l-1}\right) \text{ dominates } \left(X_0^l, Y_0^l\right) \\
&\qquad e_0^k = \left\| \left(\hat{X}_0^{k+1}, \hat{Y}_0^{k+1}\right) - \left(\hat{X}_0^k, \hat{Y}_0^k\right) \right\|_1^\omega && k = 1, \dots, l-1 \\
&\qquad e_0^{l-1} = \left\| \left(X_0^l, Y_0^l\right) - \left(\hat{X}_0^{l-1}, \hat{Y}_0^{l-1}\right) \right\|_1^\omega \\
&\qquad e_0^k \le e_0^{\text{closest}} && k = 1, \dots, l-1
\end{aligned}
\tag{2}
$$

wherein $(\hat{X}_0^k, \hat{Y}_0^k)$ is intermediate target on $\partial(T^k)$, and (X_0^l, Y_0^l) is the actual performance of DMU_0^l. $(\hat{X}_0^k, \hat{Y}_0^k)$ dominates $(\hat{X}_0^{k+}, \hat{Y}_0^{k+1})$ and $(\hat{X}_0^{l-1}, \hat{Y}_0^{l-1})$ dominates (X_0^l, Y_0^l); these relationships indicate that $(-\hat{X}_0^k, \hat{Y}_0^k) \ge (-\hat{X}_0^{k+1}, \hat{Y}_0^{k+1})$ and $(-\hat{X}_0^{l-1}, \hat{Y}_0^{l-1}) \ge (-\hat{X}_0^l, \hat{Y}_0^l)$, respectively. In the weighted L_1-norm $\|(\hat{X}_0^{k+1}, \hat{Y}_0^{k+1}) - (\hat{X}_0^k, \hat{Y}_0^k)\|_1^\omega = \sum_{i=1}^m \frac{\hat{x}_{i0}^{k+1} - \hat{x}_{i0}^k}{x_{i0}^l} + \sum_{r=1}^s \frac{\hat{y}_{r0}^k - \hat{y}_{r0}^{k+1}}{y_{r0}^l}$, $\|(X_0^l, Y_0^l) - (\hat{X}_0^{l-1}, \hat{Y}_0^{l-1})\|_1^\omega = \sum_{i=1}^m \frac{x_{i0}^l - \hat{x}_{i0}^{l-1}}{x_{i0}^l} + \sum_{r=1}^s \frac{\hat{y}_{r0}^{l-1} - \hat{y}_{r0}^l}{y_{r0}^l}$, all the deviations are assumed to be nonnegative to define a strategy of continuous improvements. e_0^k represents the efforts to improve $(\hat{X}_0^{k+1}, \hat{Y}_0^{k+1})$ to $(\hat{X}_0^k, \hat{Y}_0^k)$, and e_0^{l-1} represents the efforts to improve (X_0^l, Y_0^l) to $(\hat{X}_0^{l-1}, \hat{Y}_0^{l-1})$. Moreover, e_0^{closest} represents the minimum efforts or gaps to directly achieve the closest target on $\partial(T^1)$, which can be calculated using the method of Aparicio *et al.* [16]. The constraint $e_0^k \le e_0^{\text{closest}}$ forces the efforts in all improvement steps to be less than e_0^{closest}; otherwise, there is no need to perform the stepwise improvement path (because an inefficient DMU may directly reach $\partial(T^1)$ through efforts less than e_0^k in a single step).

To solve (2), we transform it into the following programming problem:

$$
\begin{aligned}
&\text{Min } e_0^u - \varepsilon * \sum_{k=1}^{l-1} t_0^k \\
&\ \text{s.t.} \\
&\qquad \text{Constraints in (2)} \\
&\qquad e_0^k + t_0^k = e_0^u && 1 \le k \le l-1
\end{aligned}
\tag{3}
$$

wherein e_0^u is the upper bound for all e_0^k and represents the maximum (or most demanding) efforts in all intermediate improvement steps. t_0^k represents the slack variables between e_0^u and e_0^k. ε is an arbitrarily small positive number (10^{-3}–10^{-6}). In this paper, we can set ε to 10^{-6}. The objective function of (3) is essentially a weighted objective function in which the weight of the first objective function (Min e_0^u) is larger (equal to 1) than the one of the second objective function, so it has the highest priority to be minimized. The second objective function (Min $-\varepsilon * \sum_{k=1}^{l-1} t_0^k$) has minimal weight ($\varepsilon = 10^{-6}$), so it has the lowest priority to be minimized. Hence, according to the lexicographic optimization method[2], the objective function of (3) can also be equivalently written in a new form: Lex $\min\{e_0^u, -\sum_{k=1}^{l-1} t_0^k\}$. In sum, the process of solving (3) is as follows: Through the first objective function, we can minimize the most demanding efforts e_0^u. However, the solution generated from the first objective function is not guaranteed to be Pareto-efficient because the efforts in some intermediate steps may be further reduced. Based on this, on the premise of minimizing e_0^u, we further run the second objective function (Min $-\varepsilon * \sum_{k=1}^{l-1} t_0^k$). In the end, the effort in some (at least one) intermediate steps is equal to e_0^u and the effort in others may be strictly less than e_0^u. In sum, the objective function in (3) will generate the Pareto-efficient solution, which can be explained in the following proposition:

Proposition 1. *The vector* $(e_0^{1*}, \dots e_0^{k*}, \dots, e_0^{l-1*})$ *corresponding to the Pareto-efficient solution of* (3) *is also the nondominant vector of* (2).

Proof. Suppose that there exists an alternative vector $(\acute{e}_0^{1*}, \dots \acute{e}_0^{k*}, \dots, \acute{e}_0^{l-1*})$ generated from (2) that satisfies $(\acute{e}_0^{1*}, \dots \acute{e}_0^{k*}, \dots, \acute{e}_0^{l-1*}, \acute{e}_0^{u*}) \leq (e_0^{1*}, \dots e_0^{k*}, \dots, e_0^{l-1*}, e_0^{u*})$ with at least one strict inequality existing. Then, we can deduce $\acute{e}_0^{u*} + \varepsilon * \sum_{k=1}^{l-1} \acute{e}_0^{k*} < e_0^{u*} + \varepsilon * \sum_{k=1}^{l-1} e_0^{k*}$. This condition contradicts the initial assumption that the vector $(e_0^{1*}, \dots e_0^{k*}, \dots, e_0^{l-1*})$ corresponding to the Pareto-efficient solution of (3) is also the nondominant vector of (2). Therefore, this proposition holds. □

The idea of (3) can be described in detail in Figure 2.

In Figure 2, $\hat{o}^{\text{closest}}$ is the closest target that can, in principle, be achieved directly with minimum effort (see the black dotted arrow in Fig. 2). Meanwhile, $e_0^k = \|\hat{o}^k - \hat{o}^{k+1}\|_1^\omega$ is the intermediate effort to improve from $\hat{o}^{k+1}$ to $\hat{o}^k$($e_0^{l-1} = \|\hat{o}^{l-1} - o\|_1^\omega$ is the intermediate effort to improve from actual performance o to $\hat{o}^{l-1}$). It can be easily seen that $e_0^u = \max\{e_0^k \mid k = 1, \dots, l-1\}$ is the upper bound for all intermediate efforts. When o cannot reach $\hat{o}^{\text{closest}}$, the stepwise path ($o \to \cdots \to \hat{o}^k \to \cdots \to \hat{o}^1$) becomes necessary. Hence, e_0^u should be no more than $\hat{o}^{\text{closest}}$, which means that the stepwise path is feasible. To further enhance the feasibility, we should minimize e_0^u first and then reduce e_0^k further.

Next, based on the characterization of $\partial(T)$ in terms of a set of linear constraints in the study of Aparicio *et al.* [16], we establish the following operational formulation of (4):

$$\text{Min } e_0^u - \varepsilon * \sum_{k=1}^{l-1} t_0^k$$

s.t.

$$\sum_{j \in E^k} \lambda_j^k x_{ij}^k = x_{i0}^l - \sum_{k}^{l-1} s_{i0}^{k-} \qquad i = 1, \dots, m; \quad k = 1, \dots, l-1 \tag{4.1}$$

$$\sum_{j \in E^k} \lambda_j^k y_{rj}^k = y_{r0}^l + \sum_{k}^{l-1} s_{i0}^{k+} \qquad r = 1, \dots, s; \quad k = 1, \dots, l-1 \tag{4.2}$$

$$\sum_{j \in E^k} \lambda_j^k = 1 \qquad k = 1, \dots, l-1 \tag{4.3}$$

[2]Specifically, the objective function of the lexicographic optimization method is as follows: Lex $\min\{f_1(w), f_2(w), \dots, f_l(w)\}$, where w represents the variables. Lex $\min\{f_1(w), f_2(w), \dots, f_l(w)\}$ runs as follows: We optimize the first objective function (with the highest priority) to obtain Min $f_1(w) = f_1^*$. Then, we optimize the second objective function $f_2(w)$ by adding the new constraint $f_1(w) = f_1^*$ (to keep the optimal solution of $f_1(w)$). After obtaining Min $f_2(w) = f_2^*$, we optimize the third objective function $f_3(w)$ by adding both $f_1(w) = f_1^*$ and $f_2(w) = f_2^*$ to the constraint section (to keep the optimal solution of $f_1(w)$ and $f_2(w)$). We do this until we finish all the remaining objective functions $(f_4(w), \dots, f_l(w))$.

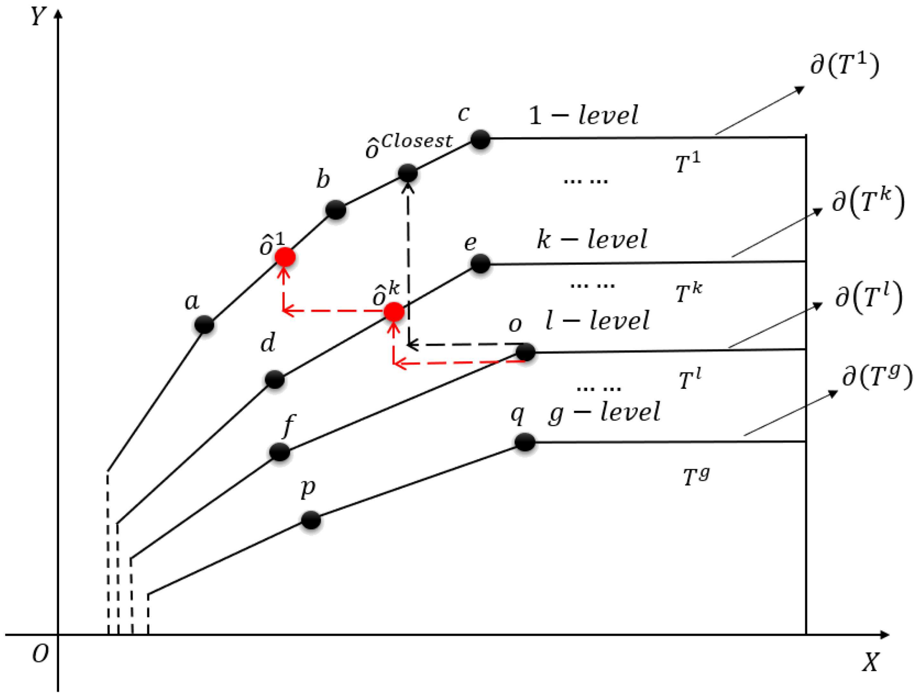

FIGURE 2. Stepwise improvement path.

$$-\sum_{i=1}^{m} v_i^k x_{ij}^k + \sum_{r=1}^{s} u_r^k y_{rj}^k + d_j^k + u_0^k = 0 \qquad j \in E^k;\quad \forall k \tag{4.4}$$

$$v_i^k x_{i0}^l \geq 1 \qquad i = 1,\dots,m;\quad k = 1,\dots,l-1 \tag{4.5}$$

$$u_r^k y_{r0}^l \geq 1 \qquad r = 1,\dots,s;\quad k = 1,\dots,l-1 \tag{4.6}$$

$$\lambda_j^k d_j^k = 0 \qquad j \in E^k;\quad \forall k \tag{4.7}$$

$$\sum_{i=1}^{m} \frac{s_{i0}^{k-}}{x_{i0}^l} + \sum_{r=1}^{s} \frac{s_{r0}^{k+}}{y_{r0}^l} - e_0^k = 0 \qquad \forall i, r, f, k \tag{4.8}$$

$$e_0^k + t_0^k = e_0^u \qquad \forall k \tag{4.9}$$

$$e_0^u \leq e_0^{\text{closest}} \qquad \forall k \tag{4.10}$$

$$s_{i0}^{k-}, s_{r0}^{k+}, \lambda_j^k, d_j^k \geq 0 \qquad j \in E^k;\quad \forall i, r, f, k$$

$$u_0^k \qquad \text{free};\quad k = 1,\dots,l-1 \tag{4.11}$$

where

(1) In constraints (4.1) and (4.2), (x_{i0}^l, y_{r0}^l) is the actual performance of DMU_0^l under analysis; (x_{ij}^k, y_{rj}^k) is the actual performance for $\text{DMU}_j^k (j \in E^k)$ on the middle frontier $\partial(T^k)$; and $(\sum_{j\in E^k} \lambda_j^k x_{ij}^k, \sum_{j\in E^k} \lambda_j^k y_{rj}^k)$ is the possible intermediate target on $\partial(T^k)$ that is a result of the combination of all $\text{DMU}_j^k, j \in E^k$. That is, the intermediate target $(\hat{x}_{i0}^k, \hat{y}_{r0}^k) = (\sum_{j\in E^k} \lambda_j^k x_{ij}^k, \sum_{j\in E^k} \lambda_j^k y_{rj}^k)$; $(\sum_k^{l-1} s_{i0}^{k-}, \sum_k^{l-1} s_{i0}^{k+})$ is the distance (slack variable) from (x_{i0}^l, y_{r0}^l) to $(\hat{x}_{i0}^k, \hat{y}_{r0}^k)$; and $(s_{i0}^{k-}, s_{i0}^{k+})$ is the gap from $(\hat{x}_{i0}^{k+1}, \hat{y}_{r0}^{k+1})$ to $(\hat{x}_{i0}^k, \hat{y}_{r0}^k)$,

$(s_{i0}^{k-}, s_{i0}^{k+}) = (\hat{x}_{i0}^{k} - \hat{x}_{i0}^{k+1}, \hat{y}_{r0}^{k+1} - \hat{y}_{r0}^{k})$. Here, it is clear that if $k=1$, $(\hat{x}_{i0}^{1}, \hat{y}_{r0}^{1}) = (\sum_{j\in E^k} \lambda_j^1 x_{ij}^1, \sum_{j\in E^k} \lambda_j^1 y_{rj}^1)$ is the final target on $\partial(T^1)$, and $(\sum_k^{l-1} s_{i0}^{k-}, \sum_k^{l-1} s_{i0}^{k+})$ is the improvement direction of the stepwise path.

(2) $-\sum_{i=1}^{m} v_i^k x_{ij}^k + \sum_{r=1}^{s} u_r^k y_{rj}^k + u_0^k = 0$ in constraint (4.4) represents a facet of $\partial(T^k)$, (v_i^k, u_r^k, u_0^k) is the nonzero coefficient related to this facet, and u_0^k is also the free variable. Constraints (4.5) and (4.6) ensure the range of the nonzero coefficient.

(3) Nonlinear constraint (4.7) $\lambda_j^k d_j^k = 0$ can be replaced by the special ordered set (SOS) type 1, which makes it possible to solve nonlinear constraints. The SOS type 1 is a set of variables in which at most one is nonzero positive; that is, λ_j^k and d_j^k cannot both be positive. Through $\lambda_j^k d_j^k = 0$ and constraints (4.1)–(4.6), we can ensure that $(\hat{X}_0^k, \hat{Y}_0^k)$ are on a facet of $\partial(T^k)$. Specifically, when $\lambda_j^k > 0$, $d_j^k = 0$, $(\hat{X}_0^k, \hat{Y}_0^k)$, and members (X_j^k, Y_j^k) in E^k are on the same hyperplane $-\sum_{i=1}^{m} v_i^k x_{ij}^k + \sum_{r=1}^{s} u_r^k y_{rj}^k + u_0^k = 0$ (see constraint (4.4)). In other words, all (X_j^k, Y_j^k) whose λ_j^k is nonzero positive are on the same facet as $(\hat{X}_0^k, \hat{Y}_0^k)$. On the contrary, if $\lambda_j^k > 0$, then $d_j^k > 0$. All (X_j^k, Y_j^k) whose λ_j^k is equal to zero are not on the same face $(-\sum_{i=1}^{m} v_i^k x_{ij}^k + \sum_{r=1}^{s} u_r^k y_{rj}^k + u_0^k = 0)$ as $(\hat{X}_0^k, \hat{Y}_0^k)$. The SOS type 1 has been widely used in numerous studies (*e.g.*, [6, 10, 32]) and also be easily solved by LINGO, CPLEX, and PYTHON.

(4) Based on the analysis of constraints (4.1) and (4.2), $e_0^k = \sum_{i=1}^{m} \frac{s_{i0}^{k-}}{x_{i0}^l} + \sum_{r=1}^{s} \frac{s_{r0}^{k+}}{y_{r0}^l}$ in constraint (4.8) represent the improvement efforts from $(\hat{x}_{i0}^{k+1}, \hat{y}_{r0}^{k+1})$ to $(\hat{x}_{i0}^{k}, \hat{y}_{r0}^{k})$. Constraints (4.9) and (4.10) can be understood through (3).

By solving (4), we can find the sequential targets $(\hat{X}_0^{k*}, \hat{Y}_0^{k*})(k = 1, \ldots, l-1)$:

$$\left(\hat{X}_0^{k*}, \hat{Y}_0^{k*}\right) = \left(\sum_{j\in E^k} \lambda_j^{k*}\, x_{ij}^k, \sum_{j\in E^k} \lambda_j^{k*}\, y_{rj}^k\right) = \left(x_{i0}^l - \sum_k^{l-1} s_{i0}^{k-*}, y_{r0}^l + \sum_k^{l-1} s_{r0}^{k+*}\right). \tag{5}$$

These sequential targets can form the stepwise improvement path. As can be seen from (5), the final target $(\hat{X}_0^{1*}, \hat{Y}_0^{1*})$ represents the global cumulative improvement (which can be denoted by $(\sum_k^{l-1} S_{i0}^{k-*}, \sum_{lk}^{l-1} S_{r0}^{k+*})$) of (X_0^l, Y_0^l), so it can show the global direction for improvement.

3.3. Finding more alternative stepwise improvement paths with different improvement effects

Next, based on the premise of obtaining a stepwise improvement path generated from (4), we intend to further search for more alternative stepwise improvement paths with different improvement effects. To this end, we propose three criteria for finding the new alternative improvement paths. First, the final target corresponding to new alternative stepwise improvement paths can show the new direction for improvement. Second, the upper bound of efforts of all intermediate improvement steps should be minimized as much as possible within a range of strictly less than the e_0^{closest}. Third, the effort in every intermediate step should be minimized as much as possible. The first criterion is similar to the ones in the study of Ruiz and Sirvent [6] that are used for identifying alternative targets with a new improvement direction. The main difference is that the improvement direction shown by the final target in this paper is a cumulative improvement of inputs and outputs from actual performance to the final target. This is illustrated in Figure 3.

In Figure 3, (X, Y) is the actual performance of o; (X^1, Y^1) is the final target corresponding to the first stepwise path $(o \to \cdots \to \hat{o}^k \to \cdots \hat{o}^1)$ (see the red dotted arrow in Fig. 3); and $(X^{1'}, Y^{1'})$ is the final target related to the second stepwise path $(o \to \cdots \to \hat{o}^{k'} \to \cdots \to \hat{o}^{1'})$ (see the blue dotted arrow in Fig. 3). According to Figure 3, $(\frac{X-X^1}{X}, \frac{Y^1-Y}{Y})$ represents the improvement direction of the stepwise path $(o \to \cdots \to \hat{o}^k \to \cdots \to \hat{o}^1)$. Likewise, $(\frac{X-X^{1'}}{X}, \frac{Y^{1'}-Y}{Y})$ is the improvement direction shown by the stepwise path $(o \to \cdots \to \hat{o}^{k'} \to \cdots \to \hat{o}^{1'})$. It can be easily seen that the difference in the improvement direction (the cumulative improvement of inputs and outputs from (X, Y) to (X^1, Y^1) and $(X^{1'}, Y^{1'})$) between two stepwise

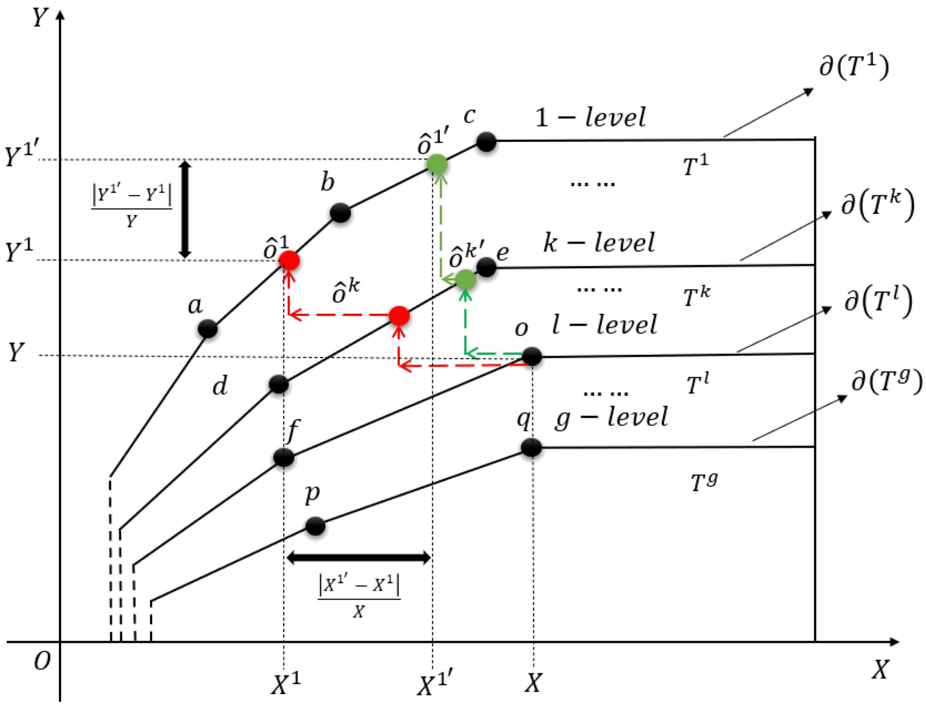

FIGURE 3. Difference in improvement direction.

paths can be denoted by $\left(\frac{|X^{1'}-X^1|}{X}+\frac{|Y^{1'}-Y^1|}{Y}\right)$ (see two black two-way arrows in Fig. 3). This also reflects the difference in the improvement of input-output indicators between the two stepwise paths.

Based on the above three criteria, we can establish the following multi-objective programming:

$$\begin{array}{lll}
\text{Max} \left\{ \delta_0^a, -e_0^{u(a)}, \sum_{k=1}^{l-1} t_0^{k(a)} \right\} & & \\
\text{s.t.} & & \\
\left(\hat{X}_0^{k(a)}, \hat{Y}_0^{k(a)}\right) \in \partial\left(T^k\right) & k=1,\ldots,l-1 & \\
\left(\hat{X}_0^{k(a)}, \hat{Y}_0^{k(a)}\right) \text{ dominates } \left(\hat{X}_0^{k+1(a)}, \hat{Y}_0^{k+1(a)}\right) & k=1,\ldots,l-2 & \\
\left(\hat{X}_0^{l-1(a)}, \hat{Y}_0^{l-1(a)}\right) \in \partial\left(T^{l-1}\right) & k=l-1 & \\
\left(\hat{X}_0^{l-1(a)}, \hat{Y}_0^{l-1(a)}\right) \text{ dominates } \left(X_0^{l-1}, Y_0^{l-1}\right) & k=l-1 & (6) \\
\delta_0^a = \left\| \left(\hat{X}_0^{1(a)}, \hat{Y}_0^{1(a)}\right) - TS_0^{a-1} \right\|_1^{\omega} & & \\
TS_0^{a-1} = \left\{ (\hat{X}_0^{1(\xi)*}, \hat{Y}_0^{1(\xi)*}) \right\} | \xi = 0,1,\ldots,a-1 \} & & \\
e_0^{k(a)} = \left\| \left(\hat{X}_0^{k+1(a)}, \hat{Y}_0^{k+1(a)}\right) - \left(\hat{X}_0^{k(a)}, \hat{Y}_0^{k(a)}\right) \right\|_1^{\omega} & k=1,\ldots,l-2 & \\
e_0^{l-1(a)} = \left\| (X_0^l, Y_0^l) - \left(\hat{X}_0^{l-1(a)}, \hat{Y}_0^{l-1(a)}\right) \right\|_1^{\omega} & k=l-1 & \\
e_0^{k(a)} + t_0^{k(a)} = e_0^{u(a)} & k=1,\ldots,l-1 &
\end{array}$$

$$e_0^{u(a)} \le e_0^{\text{closest}}$$

where

(1) $(\hat{X}_0^{k(a)}, \hat{Y}_0^{k(a)})$ is an intermediate $(k = 2, \ldots, l-1)$ on $\partial(T^k)$ and a final target $(k = 1)$ $\partial(T^1)$ corresponding to the ath stepwise improvement path. $(\hat{X}_0^{l-1(a)}, \hat{Y}_0^{l-1(a)})$ is also an intermediate target on $\partial(T^{l-1})$ related to the ath stepwise improvement path. Constraints "$(\hat{X}_0^{k(a)}, \hat{Y}_0^{k(a)})$ dominates $(\hat{X}_0^{k+1(a)}, \hat{Y}_0^{k+1(a)})$" and "$(\hat{X}_0^{l-1(a)}, \hat{Y}_0^{l-1(a)})$ dominates (X_0^{l-1}, Y_0^{l-1})" can be understood based on (2).

(2) TS_0^{a-1} is the set of final targets $(\hat{X}_0^{1(\xi)*}, \hat{Y}_0^{1(\xi)*})(\xi = 0, 1, \ldots, a-1)$ associated with all previously obtained (from 1th to ξth) stepwise improvement paths. δ_0^a is the absolute distance from $(\hat{X}_0^{1(a)}, \hat{Y}_0^{1(a)})$ to TS_0^{a-1}, and it can also be rewritten as: $|(\hat{X}_0^{1(a)}, \hat{Y}_0^{1(a)}) - TS_0^{a-1}|_1^{\omega} = \text{Min}\{|(\hat{X}_0^{1(a)}, \hat{Y}_0^{1(a)}) - (\hat{X}_0^{1(\xi)*}, \hat{Y}_0^{1(\xi)*})|_1^{\omega}. \mid \xi = 0, 1, \ldots, a-1\} = \text{Min}\{\frac{|\hat{X}_0^{1(a)} - \hat{X}_0^{1(\xi)*}|}{X_0^l} + \frac{|\hat{Y}_0^{1(a)} - \hat{Y}_0^{1(\xi)*}|}{Y_0^l} \mid \xi = 0, 1, \ldots, a-1\}$. In other words, δ_0^a represents the differences in the improvement direction between the ath target $(\hat{X}_0^{1(a)}, \hat{Y}_0^{1(a)})$ and previously existing target $(\hat{X}_0^{1(\xi)*}, \hat{Y}_0^{1(\xi)*})(\xi = 0, 1, \ldots, a-1)$. $e_0^{k(a)}$, $e_0^{l-1(a)}$, $e_0^{u(a)}$, $e_0^{k(a)}$, and $t_0^{k(a)}$ can be easily understood *via* (2) and (3).

(3) Based on δ_0^a, the first objective function in (6) aims at maximizing the improvement direction differences between $(\hat{X}_0^{1(a)}, \hat{Y}_0^{1(a)})$, and all final targets $(\hat{X}_0^{1(\xi)*}, \hat{Y}_0^{1(\xi)*})(\xi = 0, 1, \ldots, a-1)$. In other words, the first objective function is to follow the proposed first criterion. Constraint $e_0^{u(a)} \le e_0^{\text{closest}}$ means that the upper bound of efforts of all intermediate steps must be less than e_0^{closest}. As a consequence, the second objective function can implement the proposed second criterion. Based on constraint $e_0^{k(a)} + t_0^{k(a)} = e_0^{u(a)}$, the third objective function aims to follow the proposed third criteria. Moreover, $e_0^{k(a)}$, $e_0^{l-1(a)}$, and $t_0^{k(a)}$ are similar to e_0^k, e_0^{l-1}, and t_0^k in (2), respectively. Eventually, we convert (6) into the following lexicographic linear programming:

$$\begin{aligned} &\text{Lex max}\left\{\delta_0^a + \varepsilon * c, \sum_{k=1}^{l-1} t_0^{k(a)}\right\} \\ &\text{s.t.} \\ &\quad \text{Constraints in (6)} \\ &\quad e_0^{u(a)} + c = \rho \\ &\quad \rho \in \left[e_0^{u*}, e_0^{\text{closest}}\right] \end{aligned} \tag{7}$$

where ε is an arbitrarily small positive number $(10^{-3} - 10^{-6})$, and we also set ε equal to 10^{-6} here. This means that δ_0^a has priority over c. Based on this, the objective function of (3) can also be written as Lex $\max\{e_0^u, c, \sum_{k=1}^{l-1} t_0^{k(a)}\}$. ρ is the upper bound for $e_0^{u(a)}$, and c is the slack variable between ρ and $e_0^{u(a)}$. Constraint $\rho \in [e_0^{u*}, e_0^{\text{closest}}]$ means that $e_0^{u(a)}$ must be strictly less than e_0^{closest}; otherwise, the new stepwise improvement path will be redundant (because the inefficient DMU can directly reach $\partial(T^1)$ with the effort equal to $e_0^{u(a)}$ in one step). The lexicographic objective function in (7) first finds the new alternative final target that is different from previously determined final targets in the global improvement direction. Then it further minimizes the most demanding effort within a range of strictly less than the e_0^{closest}, and it finally minimizes efforts in intermediate steps. To implement the lexicographic objective function in (7), we first regard ρ as a threshold that can be adjusted and then divide the distance (which can be denoted by $(e_0^{\text{closest}} - e_0^{u*})$ from e_0^{u*} and e_0^{closest} into τ equal intervals and further obtain $(\tau + 1)$ interval points. Next, we propose a search algorithm to search for more alternative final targets and further obtain several stepwise improvement paths with different improvement effects. The proposed search algorithm is as follows:

Step 1. Solve constraint (4) and the model of Aparicio *et al.* [16], and obtain the e_0^{closest}, the initial stepwise improvement paths $(\hat{X}_0^{k*}, \hat{Y}_0^{k*})(k = 1, \ldots, l-1)$, the final targets $(\hat{X}_0^{1*}, \hat{Y}_0^{1*})$, and e_0^{u*}.

Step 2. If $e_0^{u*} = e_0^{\text{closest}}$, stop; otherwise, go to step 3.

Step 3. Divide $(e_0^{\text{closest}} - e_0^{u*})$ into τ equal intervals, and set $\rho = e_0^{u*} + \frac{(ee_0^{\text{closest}} - e_0^{u*})}{\tau} * \sigma$.

Step 4. Set $a = 1$, $\sigma = 0$.

Step 5. If $\sigma > \tau$, stop; otherwise, go to step 6.

Step 6. Solve the operative formulation of (7) (following (9)) and find $(\hat{X}_0^{1(a)*}, \hat{Y}_0^{1(a)*})$.

Step 7. If $(\hat{X}_0^{1(a)*}, \hat{Y}_0^{1(a)*})$ fails to show the new direction for improvement, set $\sigma = \sigma + 1$, and go to step 5. Otherwise, go to step 8.

Step 8. Set $TS_0^{a-1} = TS_0^{a-1} \cup (\hat{X}_0^{1(a)*}, \hat{Y}_0^{1(a)*})$, and regard $(\hat{X}_0^{k(a)*}, \hat{Y}_0^{k(a)*})(k = 1, \ldots, l-1)$ as the new alternative stepwise improvement path.

Step 9. If the search is to be continued, then set $a = a + 1$ and again run steps 5–8 until $\sigma > \tau$.

In sum, by running the above-proposed search algorithm, we can obtain several alternative stepwise improvement paths with different improvement effects. To this end, we establish the operative formulation of (7):

$$\text{Lex } \max\left\{\delta_0^a + \varepsilon * c, \sum_{k=1}^{l-1} t_0^{k(a)}\right\}$$

s.t.

$$\sum_{j\in E^k} \lambda_j^{k(a)} x_{ij}^k = x_{i0}^l - \sum_{k}^{l-1} s_{i0}^{k(a)-} \qquad i = 1,\ldots,m; \quad k = 1,\ldots,l-1 \tag{8.1}$$

$$\sum_{j\in E^k} \lambda_j^{k(a)} y_{rj}^k = y_{r0}^l + \sum_{k}^{l-1} s_{r0}^{k(a)+} \qquad r = 1,\ldots,s; \quad k = 1,\ldots,l-1 \tag{8.2}$$

$$\sum_{j\in E^k} \lambda_j^{k(a)} = 1 \qquad k = 1,\ldots,l-1 \tag{8.3}$$

$$-\sum_{i=1}^{m} v_i^{k(a)} x_{ij}^k + \sum_{r=1}^{s} u_r^{k(a)} y_{rj}^k + d_j^{k(a)} + u_0^{k(a)} = 0 \qquad j \in E^k; \quad \forall k \tag{8.4}$$

$$v_i^{k(a)} x_{i0}^l \geq 1 \tag{8.5}$$

$$u_r^{k(a)} y_{r0}^l \geq 1 \tag{8.6}$$

$$\sum_{i=1}^{m} \frac{\left|\hat{x}_{i0}^{1(a)} - \hat{x}_{i0}^{1(\xi)*}\right|}{x_{i0}^l} + \sum_{r=1}^{s} \frac{\left|\hat{y}_{r0}^{1(a)} - \hat{y}_{r0}^{1(\xi)*}\right|}{y_{r0}^l} \geq \delta_0^a \qquad \xi = 0,1,\ldots,a-1; \quad \forall k \tag{8.7}$$

$$\lambda_j^{k(a)} d_j^{k(a)} = 0 \tag{8.8}$$

$$\sum_{i=1}^{m} \frac{s_{i0}^{(a)k-}}{x_{i0}^l} + \sum_{r=1}^{s} \frac{s_{r0}^{(a)k+}}{y_{r0}^l} - e_0^{k(a)} = 0 \qquad \forall i, r, k \tag{8.9}$$

$$e_0^{u(a)} + c = \rho \tag{8.10}$$

$$e_0^{k(a)} + t_0^{k(a)} = e_0^{u(a)} \qquad \forall k \tag{8.11}$$

$$s_{i0}^{k(a)-}, s_{r0}^{k(a)+}, \lambda_j^{k(a)}, d_j^{k(a)} \geq 0 \qquad j \in E^k; \quad \forall i, r, k$$

$$u_0^{k(a)} \qquad \text{free}; \quad k = 1,\ldots,l-1$$

where the variables in constraints (8.1)–(8.6), (8.8), (8.9), and (8.11) can be understood through the explanation for constraints in (4). Constraint (8.7) is equivalent to $\delta_0^a = \text{Min}\{\frac{\hat{x}_0^{1(a)} - \hat{x}_0^{1(\xi)*}}{x_0^l} + \frac{|\hat{Y}_0^{1(a)} - \hat{Y}_0^{1(\xi)*}|}{Y_0^l} \mid \xi = 0, 1, \ldots, a -$

$1\} = \mathrm{Min}\{\sum_{i=1}^{m} \frac{|\hat{x}_{i0}^{1(a)} - \hat{x}_{i0}^{1(\xi)*}|}{x_{i0}^{i}} + \sum_{r=1}^{s} \frac{|\hat{y}_{r0}^{1(a)} - \hat{y}_{i0}^{1(\xi)*}|}{y_{r0}^{i}} | \xi = 0, 1, \ldots, a-1\}$, and the previously found final targets $(\hat{x}_{i0}^{1(\xi)*}, \hat{y}_{r0}^{1(\xi)*})(\xi = 0, 1, \ldots, a-1)$ can be written as $(x_{i0}^{l} + \sum_{k=1}^{l-1} s_{i0}^{k(\xi)-*},\ y_{r0}^{l} + \sum k = 1^{l-1}\ s_{i0}^{k(\xi)-*})$. Similarly, ath new final targets $(\hat{x}_{i0}^{1(a)*}, \hat{y}_{r0}^{1(a)*})$ can be denoted by $(x_{i0}^{l} + \sum_{k=1}^{l-1} s_{i0}^{k(a)-*}, y_{r0}^{l} + \sum_{k=1}^{l-1}\ s_{r0}^{k(a)+*})$. Constraint (8.10) has been explained in (7). Moreover, (8) can make the following propositions hold:

Proposition 2. *For any values of ρ, the vector $(-\delta_0^{a*}, e_0^{u(a)*}, e_0^{1(a)*}, \ldots, e_0^{k(a)*}, \ldots, e_0^{l-1(a)*})$ corresponding to the optimal solution of (8) is nondominant.*

Proof. Under some value of ρ, when δ_0^{a*} is obtained, suppose there is an alternative vector $(-\acute{\delta}_0^{a*}, \acute{e}_0^{u(a)*}, \acute{e}_0^{1(a)*}, \ldots, \acute{e}_0^{k(a)*}, \ldots, \acute{e}_0^{l-1(a)*})$ that satisfies the condition: $(-\acute{\delta}_0^{a*}, \acute{e}_0^{u(a)*}, \acute{e}_0^{1(a)*}, \ldots, \acute{e}_0^{k(a)*}, \ldots, \acute{e}_0^{l-1(a)*}) \leq (-\delta_0^{a*}, e_0^{u(a)*}, e_0^{1(a)*}, \ldots, e_0^{k(a)*}, \ldots, e_0^{l-1(a)*})$ with at least one strict inequality existing. Then, there will be two cases to analyze:

Case 1. When $-\acute{\delta}_0^{a*} < -\delta_0^{a*}$ or $\acute{e}_0^{u(a)*} < e_0^{u(a)*}$, then $\acute{\delta}_0^{a*} > \delta_0^{a*}$ or $\grave{c}^* > c^*$, and we can deduce $\acute{\delta}_0^{a*} + \varepsilon * \grave{c}^* > \delta_0^{a*} + \varepsilon * c^*$.

Case 2. When $-\acute{\delta}_0^{a*} = -\delta_0^{a*}, \acute{e}_0^{u(a)*} = e_0^{u(a)*}$ and $(\acute{e}_0^{1(a)*}, \ldots, \acute{e}_0^{k(a)*}, \ldots, \acute{e}_0^{l-1(a)*}) \leq (e_0^{1(a)*}, \ldots, e_0^{k(a)*}, \ldots, e_0^{l-1(a)*})$ with at least one strict inequality existing. Then, we can deduce $\sum_{k-1}^{l-1} \acute{e}_0^{1(a)*} < \sum_{k=1}^{l-1} e_0^{1(a)*}$ or $\sum_{k=1}^{l-1} \acute{t}_0^{1(a)*} > \sum_{k=1}^{l-1} t_0^{1(a)*}$.

In sum, both the two cases contradict the initial assumption that $(-\delta_0^{a*}, e_0^{u(a)*}, e_0^{1(a)*}, \ldots, e_0^{k(a)*}, \ldots, e_0^{l-1(a)*})$ is nondominant. Therefore, Proposition 2 holds. □

By Proposition 2, we know that (8) can provide the Pareto-efficient solution.

Proposition 3. *For any value of ρ in $[e_0^{u*}, e_0^{\text{closest}}]$, $\delta_0^{a*} \leq \delta_0^{(a-1)*}$.*

For some value of ρ, when $(\hat{X}_0^{1(a-1)*}, \hat{Y}_0^{1(a-1)*})$ and $(\hat{X}_0^{k(a-1)*}, \hat{Y}_0^{k(a-1)*})(k = 1, \ldots, l-1)$ are determined, $(\hat{X}_0^{1(a-1)*}, \hat{Y}_0^{1(a-1)*})$ is farthest from TS_0^{a-1} on $\partial(T^1)$. Then, as we continue to obtain $(\hat{X}_0^{1(a)*}, \hat{Y}_0^{1(a)*})$, it can be seen that $(\hat{X}_0^{1(a)*}, \hat{Y}_0^{1(a)*})$ is the farthest target point from TS_0^{a-1} after $(\hat{X}_0^{1(a-1)*}, \hat{Y}_0^{1(a-1)*})$ on $\partial(T^1)$. Therefore, this proposition holds.

According to Proposition 3, under the same value of ρ, the more alternative final targets have been found, the less chance there is to determine the next new alternative final target.

In addition to propositions related to (8), the nonlinear form in (8.7) can be handled by introducing new nonnegative variables $p_{i0}^{x(\xi)+}, n_{i0}^{x(\xi)-}, p_{r0}^{y(\xi)+}, n_{i0}^{y(\xi)-}$ that can satisfy the following new constraints: $|\hat{x}_{i0}^{1(a)} - \hat{x}_{i0}^{1(\xi)*}| = p_{i0}^{x(\xi)+} + n_{i0}^{x(\xi)-}$, $|\hat{y}_{r0}^{1(a)} - \hat{y}_{r0}^{1(\xi)*}| = p_{r0}^{y(\xi)+} + n_{r0}^{y(\xi)-}$, $p_{i0}^{x(\xi)+} + n_{i0}^{x(\xi)-} = 0$, and $p_{i0}^{y(\xi)+} + n_{i0}^{y(\xi)-} = 0$. Through these new variables, we further transform (8) into the following (9):

$$\text{Lex}\ \max\left\{\delta_0^a + \varepsilon * c, \sum_{k=1}^{l-1} t_0^{k(a)}\right\}$$

s.t.

$$\begin{aligned}
&\text{Constraints (8.1)–(8.6), (8.8)–(8.11)} && \\
&\sum_{l=1}^{m} \frac{p_{i0}^{x(\xi)+} + n_{i0}^{x(\xi)-}}{x_{i0}^{l}} + \sum_{r=1}^{s} \frac{p_{r0}^{y(\xi)+} + n_{r0}^{y(\xi)-}}{y_{r0}^{l}} \geq \delta_0^a && \forall i, r, \xi \\
&\hat{x}_{i0}^{1(a)} - \hat{x}_{i0}^{1(\xi)*} = p_{i0}^{x(\xi)+} - n_{i0}^{x(\xi)-} && i = 1, \ldots, m;\quad \xi = 0, 1, \ldots, a-1 \\
&\hat{y}_{r0}^{1(a)} - \hat{y}_{r0}^{1(\xi)*} = p_{r0}^{y(\xi)+} - n_{r0}^{y(\xi)-} && r = 1, \ldots, s;\quad \xi = 0, 1, \ldots, a-1 \\
&s_{i0}^{k(a)-}, s_{r0}^{k(a)+}, \lambda_j^{k(a)}, d_j^{k(a)} \geq 0 && j \in E^k; \forall i, r, k
\end{aligned} \tag{9}$$

TABLE 1. DMUs with different performance levels.

Classified ports	Ports with different levels of performance
Ports on $\partial(T^1)$	Zhaoshang, Shanghai, Yantian, Ningbo, Jinzhou, Xiamen, Zhuhai, and Nanjing
Ports on $\partial(T^2)$	Qingdao, Tianjin, Liaoning, Qinhuangdao, Tangshan, Beibu, Chongqing, Lianyungang
Ports on $\partial(T^3)$	Rizhao, Guangzhou

$$p_{i0}^{x(\xi)+}, n_{i0}^{x(\xi)-}, p_{r0}^{y(\xi)+}, n_{r0}^{y(\xi)-} \geq 0 \qquad \xi = 0, 1, \ldots, a-1$$
$$u_0^{k(a)} \qquad \text{free;} \quad k = 1, \ldots, l-1.$$

In (9), $p_{i0}^{x(\xi)+} n_{i0}^{x(\xi)-} = 0$ and $p_{i0}^{y(\xi)+} n_{i0}^{y(\xi)-} = 0$ can also be handled by SOS1. Eventually, we obtain the corresponding solution by solving (9).

4. CASE STUDY

In this section, we illustrate the effectiveness of the proposed stepwise DEA approaches and search algorithm through a case study consisting of 18 ports in China. The dataset for 18 Chinese ports was mainly collected from the China Port Statistical Yearbook (2020) and the Annual Report of port enterprises in 2019 (see Appendix 2). In this dataset, labor, operating costs, total assets, and CO_2 emission are inputs, and operating revenue is an output. Note that CO_2 emission here is equivalent to the environmental input costs needed to obtain operating revenue. In other words, it can be "freely disposable." A similar example can also be found in some industries. For instance, most generators have pollution-control systems that reduce SO2. However, SO2 can also be freely increased, at least to some extent, by shutting down the pollution-control system (see Liu *et al.* [40] for a detailed discussion). Moreover, free disposability can only hold up to some extent, and undesirable outputs should be weakly disposable in many cases [37, 38, 40]. Port performance has been recently investigated in many studies relating to DEA models (*e.g.*, [41–44]). However, few studies have been concerned with gradual improvement for ports showing poor performance, especially when these ports have only limited potential for improvement. Therefore, setting more alternative stepwise improvement paths with different improvement effects represents a feasible scheme to improve the poor performance of ports.

4.1. Ports with different performance levels

By using the layering algorithm, we obtained ports with different levels of performance, which are shown in Table 1.

As shown in Table 1, Rizhao and Guangzhou showed poor performance, and we took Rizhao as a representative case to explain the function of our approaches and search algorithm. Specifically, our stepwise DEA approaches provided several (at least one) alternative stepwise improvement paths for Rizhao. Specifically, the performance improvement of Rizhao port was divided into two intermediate steps: $(X_0^3, Y_0^3) \xrightarrow{\text{Step 1}} (\hat{X}_0^{2*}, \hat{Y}_0^{2*}) \xrightarrow{\text{Step 2}} (\hat{X}_0^{1*}, \hat{Y}_0^{1*})$, where $(\hat{X}_0^{2*}, \hat{Y}_0^{2*})$ and $(\hat{X}_0^{1*}, \hat{Y}_0^{1*})$ are the intermediate and final targets, respectively. Next, our stepwise DEA approaches provided several (at least one) alternative stepwise improvement paths for Rizhao.

4.2. Alternative stepwise improvement paths with different improvement effects

4.2.1. Generation of alternatives

Table 2 reports the stepwise improvement paths provided by the search algorithm based on the proposed stepwise DEA approaches. In Table 2, column 1 records the values of σ, and column 2 shows the found alternative stepwise improvement paths (denoted by $A0$, $A1$, $A2$, $A3$...). For each alternative stepwise improvement path, line 1 ("Inter") and line 2 ("Final") show the cumulative improvement percentage of inputs and outputs needed

TABLE 2. Rizhao: Alternative stepwise improvement paths.

σ	Alternatives		Labor	Operating costs	Total assets	CO_2 emission	Operating revenue	δ_0^a	$e_0^{u(a)}$	$e_0^{1(a)}$	$e_0^{2(a)}$
		Actual	5581	3934	23 248	242 830	5246				
	$A0$	Inter	0.0%	0.0%	28.5%	54.3%	0.0%	–	0.829	0.829	0.828
		Final	48.4%	0.0%	45.8%	71.4%	0.0%				
0.00		Inter	0.0%	0.0%	28.5%	54.3%	0.0%	0.005	0.829	0.829	0.828
		Final	48.6%	0.0%	45.9%	71.2%	0.0%				
1.00	$A1$	Inter	0.0%	0.0%	28.4%	54.4%	0.0%	0.431	0.912	0.912	0.828
		Final	65.0%	0.0%	28.4%	76.6%	3.9%				
1.00	$A2$	Inter	0.0%	0.0%	28.5%	54.3%	0.0%	0.406	0.912	0.912	0.828
		Final	62.8%	0.9%	54.9%	55.3%	0.0%				
2.00	$A3$	Inter	0.0%	0.0%	28.5%	54.3%	0.0%	0.320	0.912	0.912	0.828
		Final	55.6%	0.0%	35.4%	70.0%	12.9%				
3.00		Inter	0.0%	0.0%	28.5%	54.3%	0.0%	0.280	0.912	0.912	0.828
		Final	66.6%	0.0%	40.8%	66.6%	0.0%				
3.00		Inter	0.0%	0.0%	28.5%	54.3%	0.0%	0.412	0.994	0.994	0.828
		Final	62.6%	14.5%	38.6%	66.3%	0.2%				
3.00	$A4$	Inter	0.0%	0.0%	28.5%	54.3%	0.0%	0.511	1.077	1.077	0.828
		Final	68.0%	23.0%	36.3%	63.3%	0.0%				

to achieve intermediate and final performance targets, respectively (see columns 3–8). Note that in Table 2, the improvement in input refers to a decrease, and the improvement in output refers to an increase. δ_0^a in column 9 reflects the difference in improvement direction between this stepwise path and all previously identified ones. $e_0^{(a)}$ in column 10 is the upper bound for efforts of two intermediate steps ($e_0^{2(a)}$ and $e_0^{1(a)}$). $e_0^{2(a)}$ in column 12 and $e_0^{1(a)}$ in column 11 are the improvement efforts in step 1 (from (X_0^3, Y_0^3) to $(\hat{X}_0^{2*}, \hat{Y}_0^{2*})$) and step 2 (from $(\hat{X}_0^{2*}, \hat{Y}_0^{2*})$ to $(\hat{X}_0^{1*}, \hat{Y}_0^{1*})$), respectively.

According to step 1 in the proposed search algorithm, we calculated the value of e_0^{closest} (1.656) and found the first stepwise improvement path (denoted by $A0$), final target, and $e_0^{u*} = 0.829$ *via* the method of Aparicio *et al.* [16] and (4). Since $e_0^{u*}(0.829) < e_0^{\text{closest}}(1.656)$, we proceeded to step 3. In step 3, we divided $(e_0^{\text{closest}} - e_0^{u*})$ into 10 equal intervals (*i.e.*, $\tau = 10$) and obtained 11 grid points (the value of σ ranged from 0 to 10), and let $\rho = \frac{(e_0^{\text{closest}} - e_0^{u*})}{10} * \sigma$. Next, we continued with the remaining steps (steps 4–9) in the proposed search algorithm.

In step 4, set $a = 1$ and $\sigma = 0$. In step 5, $\sigma < \tau$. In steps 6 and 7, we could obtain the new solution by solving (9). However, it can be seen that the final target of the new optimal solution was similar to that of $A0$ ($\delta_0^{1*} = 0.005$, see cumulative percentage improvement for all inputs and outputs in the line "Final" under $\sigma = 0$), so no new direction was shown. According to step 7, we set $\sigma = \sigma + 1 = 0 + 1 = 1$ and returned to step 5. In step 5, σ was strictly less than τ. Then, when $a = 1$ and $\sigma = 1$, step 6 gave the new optimal solution. In step 7, we can easily see that $e_0^{u(1)*}(0.912) < e_0^{\text{closest}}(1.656)$ and that final target of the optimal solution was different from that of $A0$ in the direction of improvement ($\delta_0^{1*} = 0.431$). Based on this, we proceeded to step 8 and could obtain the second new stepwise path $A1$. Compared with $A0$, $A1$ showed big differences in improvement effects in labor and total assets and little distinctions in improving CO_2 emission and operating revenue (see cumulative percentage improvement for all inputs and outputs in the line "Final" under $\sigma = 1$). To be specific, in terms of major improvement differences, reductions in labor and total assets were 48.4% and 45.8% in $A0$ and 65% and 28.4% in $A1$, respectively. After obtaining $A0$ and $A1$, we let $a = a + 1 = 1 + 1 = 2$ in step 9 and returned to step 5 again. In step 5, it was still true that $\sigma < \tau$. On account of steps 6 and 7, when $a = 2$ and $\sigma = 1$, we could obtain the third new stepwise path $A2$ that showed the new direction for improvement ($\delta_0^{2*} = 0.406$, see cumulative percentage improvement for all inputs and outputs in the line "Final" corresponding to $A2$ under $\sigma = 1$). Moreover, $e_0^{u(2)*}(0.912) < e_0^{\text{closest}}(1.656)$. In $A2$, unlike $A0$ and $A1$, the improvement degree of total assets (54.9%) was significantly more demanding, while the reduction of CO_2 emission was relatively small (55.3%). In addition, there were some small gaps in labor and operating costs

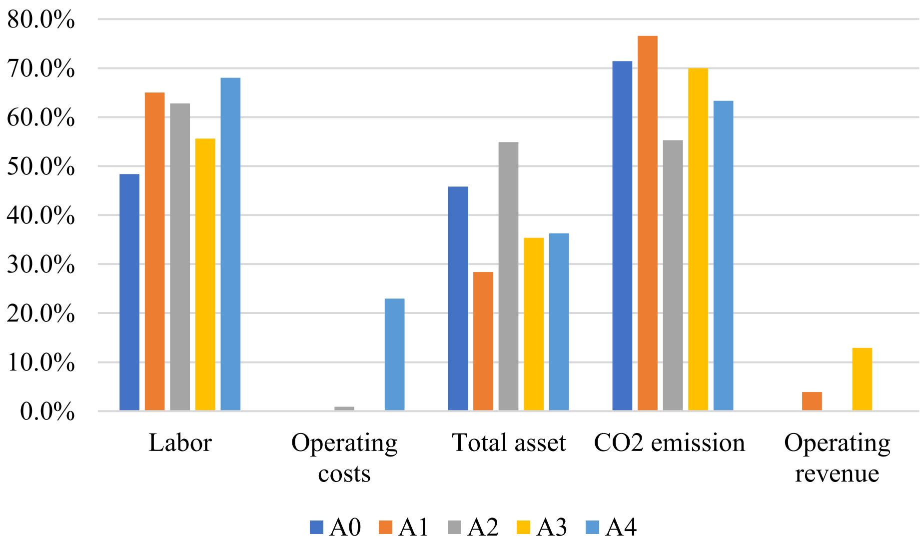

FIGURE 4. Improvement effects of the stepwise improvement paths in this paper.

between $A2$ and the previous two schemes ($A0$ and $A1$). Similarly to how we obtained $A1$ and $A2$, we could obtain $A3$ and $A4$ by repeating steps 5–8. Specifically, when $a = 3$ and $\sigma = 1$, $A3$ could be found owing to its new improvement effect ($\delta_0^{3*} = 0.320$, see cumulative percentage improvement for all inputs and outputs in the line "Final" corresponding to $A3$ under $\sigma = 2$) and $e_0^{u(3)*}(0.912) < e_0^{\text{closest}}(1.656)$. Differently from the previous step-by-step paths ($A0$, $A1$, and $A2$), there was obvious room (12.9%) for increasing operating revenue in $A3$. Meanwhile, other inputs (labor, total assets, and CO_2 emission) also had different potentials for cumulative improvement. Finally, $A4$ could be identified when $a = 4$, $\sigma = 3$, and $e_0^{u(4)*}(1.077) < e_0^{\text{closest}}(1.656)$, and a clear different improvement effect could be found in operating costs, which was more demanding than $A0$, $A1$, $A2$, and $A3$ ($\delta_0^{4*} = 0.511$; see the cumulative percentage improvement for all inputs and outputs in the line "Final" corresponding to $A4$ under $\sigma = 3$). The remaining inputs (labor, total assets, and CO_2 emission) in $A4$ also showed significantly distinctive cumulative improvement percentages.

In short, we could obtain five alternative step-by-step improvement schemes, which is sufficient for the DMs, as too many schemes could make decision-making on choosing difficult [6]. Another important reason is that as the value of σ increases, $e_0^{u(a)*}$ may be closer to $e_0^{\text{closest}}(e_0^{u*}(0.829) < e_0^{u(1)*}(0.912) < e_0^{u(2)*}(0.912) < e_0^{u(3)*}(0.912) < e_0^{u(4)*}(1.077))$, which means that the efforts (which are equal to $e_0^{u(a)*}$) in some (at least one, $e_0^{1(a)*}$ or $e_0^{2(a)*}$) intermediate steps will be more demanding. Based on these, the search could be stopped when $\sigma = 3$.

4.2.2. Difference analysis of alternatives

The improvement direction shown by the stepwise path determines its improvement effect. The differences in the improvement effects (or the cumulative improvement percentage of inputs and outputs) of all stepwise schemes ($A0$, $A1$, $A2$, $A3$, and $A4$) found in Table 2 are depicted in Figure 4.

As shown in Figure 4, differences in the improvement effects of all inputs/outputs were significant in the five step-by-step schemes ($A0$, $A1$, $A2$, $A3$, and $A4$). This means that DMs can obtain several different improvement options. For example, if Rizhao's DMs focus on decreasing operating costs while gradually improving its performance, then $A4$ will be the most suitable alternative. When Rizhao's DMs intend to expand operating revenue through progressive performance improvement, $A3$ will be a better choice. All alternatives in Figure 4 are available if DMs plan to reduce the enormous stress caused by too many employees by improving performance. Likewise, when DMs have the management goal that CO_2 emission can be reduced, all stepwise plans in Figure 4 are available. In short, DMs can choose corresponding gradual improvement plans based on their own management needs.

4.2.3. Feasibility analysis of alternatives

In addition, every incremental improvement path had an intermediate target (see the "Inter" row for $A0$, $A1$, $A2$, $A3$, and $A4$ in Tab. 2) that divided total efforts into two intermediate steps. For example, in $A0$, the improvements in total assets and CO_2 emission were divided into two steps. Concretely speaking, in step 1, to reach the intermediate target, we had to reduce the total assets and CO_2 emissions by 28.5% and 54.3%, respectively. In step 2, to further achieve the final target, we had to reduce the total assets and CO_2 emission by 17.3%(= 45.8% − 28.5%) and 17.1%(= 54.3% − 17.1%), respectively. Meanwhile, for $A0$, $e_0^{2*}(0.828) < e_0^{1*}(0.829) = e_0^{u*}(0.829) < e_0^{\text{closest}}(1.656)$. This indicates that the effort in two intermediate steps is significantly lower than e_0^{closest} because their upper bound (e_0^{u*}) is effectively limited and strictly less than e_0^{closest} (see the last three columns in Tab. 2). Likewise, according to $A1$, in step 1, to achieve the intermediate target, Rizhao first reduced total assets by 28.4% and CO_2 emissions by 54.5%. In step 2, to further achieve the intermediate target, Rizhao had to keep down labor by 65% and CO_2 emission by 22.2%(= 76.6% − 54.4%) and increase operating revenue by 3.9%. For $A1$, $e_0^{2(1)*}(0.828) < e_0^{1(1)*}(0.912) = e_0^{u(1)*}(0.912) < e_0^{\text{closest}}(1.656)$. On the basis of this, the upper bound ($e_0^{u(1)*}$) was also limited and strictly less than e_0^{closest}. Similar advantages also applied in $A2$, $A3$, and $A4$. In sum, in all stepwise schemes, the efforts in every step were significantly lower than e_0^{closest} (1.656) because $e_0^{u(a)*}$ was strictly less than $e_0^{\text{closest}}(1.656)$ (see the last three columns in Tab. 2). Although Rizhao's performance was poor, the port could more easily reach the final target with best practices through these more feasible step-by-step schemes.

4.3. Comparative analysis with the existing DEA methods

4.3.1. Comparison with the method of Ramón et al. [10]

Ramón *et al.* [10] developed a two-step benchmarking method determined by two-layer DEA efficient frontiers (*i.e.*, first-level efficient frontier with best practices $\partial(T^1)$ and second-level intermediate frontier with better practices $\partial(T^2)$). According to the theoretical framework of Ramón *et al.* [10], all DMUs in T were classified as one of three unit groups with different levels of performance: DMUs (which can be denoted by $\text{DMU}_j^1(X_j^1, Y_j^1), j \in E^1$) in $\partial(T^1)$ with the best performance levels, DMUs (which can be denoted by $\text{DMU}_j^2(X_j^2, Y_j^2), j \in E^1$) in $\partial(T^2)$ with good performance levels, and inefficient DMUs with poor performance (which can be denoted by $\text{DMU}_j^3(X_j^3, Y_j^3), j \in T^3$) in $T^3(T^3 = T - E^1 - E^2)$. Based on this, the objective function of the method of Ramón *et al.* [10] could equivalently be written as $Min\, \alpha\|(\hat{X}_0^1, \hat{Y}_0^1) - (\hat{X}_0^2, \hat{Y}_0^2)\|_1^\omega + (1-\alpha)\|(\hat{X}_0^2, \hat{Y}_0^2) - (X_0^3, Y_0^3)\|_1^\omega$, where $\alpha \in [0, 1]$ represents the coefficient for weights related to $\|(\hat{X}_0^1, \hat{Y}_0^1) - (\hat{X}_0^2, \hat{Y}_0^2)\|_1^\omega$ and $\|(\hat{X}_0^2, \hat{Y}_0^2) - (X_0^3, Y_0^3)\|_1^\omega$; $(\hat{X}_0^1, \hat{Y}_0^1)$ and $(\hat{X}_0^2, \hat{Y}_0^2)$ are the final and intermediate targets, respectively; and (X_0^3, Y_0^3) is the actual performance. This DEA method aims to find the alternative intermediate target on the intermediate frontier by adjusting the importance coefficient related to the distances between the intermediate and final targets and between the intermediate target and actual performance.

The two-step benchmarking method could be applied to the gradual performance improvement of Rizhao in Table 1. By solving this method, the stepwise improvement paths are reported in Table 3.

In Table 3, each value of α in column 1 represents an alternative step-by-step scheme. For each scheme, the contents in row 1 ("Inter") and row 2 ("Final") are the cumulative improvement percentage of inputs and outputs needed to achieve intermediate and final targets, respectively (see columns 2–7). e_0^2 in column 9 and e_0^1 in column 8 are the improvement efforts in step 1 (from (X_0^3, Y_0^3) to $(\hat{X}_0^{2*}, \hat{Y}_0^{2*})$) and step 2 (from $(\hat{X}_0^{2*}, \hat{Y}_0^{2*})$ to $(\hat{X}_0^{1*}, \hat{Y}_0^{1*})$), respectively.

According to Table 3, the effort of the second intermediate step (e_0^1) under $\alpha = 0.25, 0.00$ was more demanding ($e_0^1 = 1.396$). In contrast, since $e_0^{u(a)}$ values of all stepwise schemes in Table 2 were strictly less than 1.396 (see column 10), the efforts of all middle steps in Table 2 were also strictly lower than 1.396 (see columns 11 and 12). Compared with alternatives for $\alpha = 0.25, 0.00$, our stepwise paths were more feasible. In addition, the efforts

TABLE 3. Rizhao: Stepwise improvement paths under different values of α provided by Ramón *et al.* [10].

α		Labor	Operating costs	Total assets	CO_2 emission	Operating revenue	e_0^1	e_0^2
	Actual	5581	3934	23,248	242,830	5246		
1.00	Inter	0.0%	0.0%	28.5%	54.3%	0.0%	0.828	0.828
	Final	48.4%	0.0%	45.8%	71.4%	0.0%		
0.75	Inter	0.0%	0.0%	28.5%	54.3%	0.0%	0.828	0.828
	Final	48.4%	0.0%	45.8%	71.4%	0.0%		
0.50	Inter	6.4%	0.0%	30.1%	46.1%	0.0%	0.831	0.825
	Final	48.4%	0.0%	45.8%	71.4%	0.0%		
0.25	Inter	0.0%	0.0%	0.0%	21.6%	8.7%	1.396	0.304
	Final	50.9%	0.0%	37.1%	73.3%	8.7%		
0.00	Inter	0.0%	0.0%	0.0%	21.6%	8.7%	1.396	0.304
	Final	50.9%	0.0%	37.1%	73.3%	8.7%		

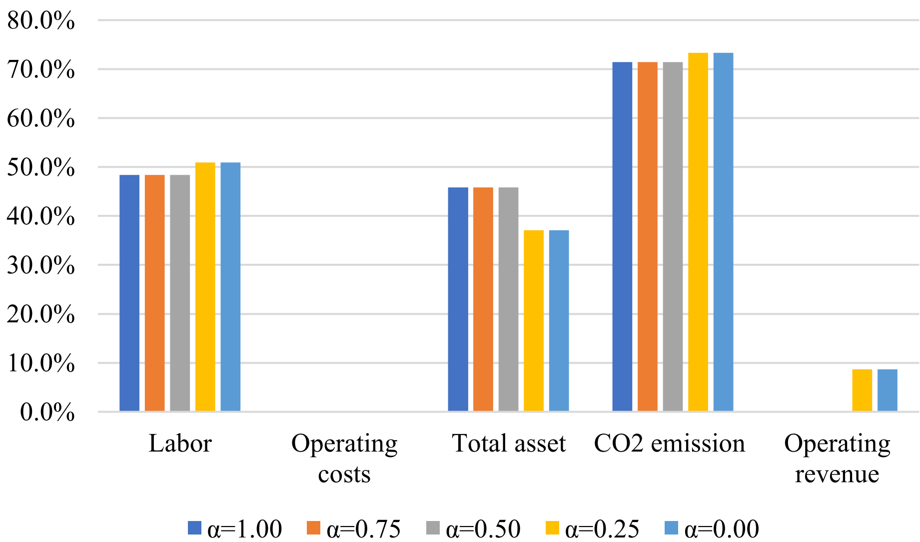

FIGURE 5. Improvement effects of the two-step improvement plans in Ramón *et al.* [10].

of intermediate steps of stepwise paths in Table 3 were less demanding under other values of α (1.00, 0.75, and 0.50). Hence, the feasibilities of the step-by-step plans under $\alpha = 1.00, 0.75, 0.50$ were better.

Comparing Figures 4 and 5, except for total assets and operating revenue, we found that all five alternatives provided by the two-step benchmarking method were generally similar in terms of improvement effects of other inputs (see Fig. 5). In contrast, the alternative stepwise improvement paths in Figure 4 showed obvious differences in the improvement effects of all inputs/outputs.

In sum, part of the gradual schemes ($\alpha = 1.00, 0.75$, and 0.50) generated by Ramón *et al.* [10] had better feasibility owing to the less demanding improvement efforts in intermediate steps. However, in terms of improvement effect, the diversities between these two-step schemes could not be guaranteed. Instead, since the upper bound of the intermediate improvement effort was effectively limited, stepwise paths provided by our stepwise methods and search algorithm were also feasible. Furthermore, our progressive paths enabled various improvement effects.

TABLE 4. Rizhao: Alternative targets with different directions for improvement provided by Ruiz and Sirvent [6].

Δe	Alternative targets	Labor	Operating costs	Total assets	CO_2 emission	Operating revenue	z_0^a	$e_0^a = e_0^{\text{closest}} + \Delta e$
	Actual	5581	3934	23 248	242 830	5246		
Closest target	AT0	48.4%	0.0%	45.8%	71.4%	0.0%	–	1.656
0%		48.6%	0.0%	45.9%	71.3%	0.0%	0.004	1.657
1%		50.4%	0.0%	47.2%	69.1%	0.0%	0.057	1.667
2%		52.3%	0.0%	48.5%	66.9%	0.0%	0.11	1.677
3%		54.1%	0.0%	49.8%	64.8%	0.0%	0.163	1.687
4%		56.0%	0.0%	51.1%	62.6%	0.0%	0.216	1.697
5%		57.8%	0.0%	52.4%	60.5%	0.0%	0.269	1.707
6%	AT1	59.7%	0.0%	53.7%	58.3%	0.0%	0.322	1.717
6%	AT2	63.0%	0.0%	32.9%	75.6%	0.2%	0.319	1.717
6%	AT3	52.6%	0.0%	35.4%	72.6%	11.1%	0.269	1.717
6%		59.6%	0.0%	41.2%	67.3%	4.7%	0.246	1.727
7%		59.7%	1.2%	41.4%	66.6%	4.9%	0.266	1.737
8%		59.6%	2.8%	41.2%	66.4%	4.7%	0.282	1.747
9%		59.4%	4.5%	41.0%	66.3%	4.5%	0.299	1.757
10%		59.6%	0.0%	41.2%	67.3%	4.7%	0.246	1.727

4.3.2. Comparison with the methods of Ruiz and Sirvent [6]

Ruiz and Sirvent [6] proposed bi-objective DEA methods that aim to find alternative targets with different improvement directions (or effects) within the range from e_0^{closest} to $e_0^{\text{closest}} + \Delta e(\Delta e \geq 0)$. These targets are on the efficient frontier with the best practice and need to be directly reached (from actual performance directly to final targets with best practices). According to Ruiz and Sirvent [6], Δe represents a small amount of additional effort and can be controlled within additional efforts allowed by DMs. We applied the method proposed by Ruiz and Sirvent [6] to the performance improvement of Rizhao in Table 1. By solving this DEA method, we could report the stepwise improvement paths (Tab. 4).

In Table 4, let Δe take values in the range from 0% to 10% (see columns 1); four alternative targets and direct improvement paths (denoted by AT0, AT1, ..., AT3; see columns 1) with different improvement directions can be obtained. For each target, the contents in columns 3–7 are the improvement percentage of inputs and outputs needed to directly achieve this target. z_0^a in column 9 reflects the difference in the improvement effect between this target and all previously identified ones. $e_0^a = e_0^{\text{closest}} + \Delta e$ in column 2 is the effort needed to directly reach this target (*i.e.*, e_0^0 is that efforts AT0, e_0^1 is that efforts AT1, ...). Obviously, $e_0^{\text{closest}} = 1.656$, which was calculated by Aparicio *et al.* [16].

Table 4 shows the alternative final targets and their corresponding direct improvement directions. These targets on $\partial(T^1)$ (or $\partial(T)$) were provided by the DEA method proposed by Ruiz and Sirvent [6], which was implemented based on the new direction for improvement and controlled additional efforts. As shown in Table 4, the five alternative targets could be found within the allowable little additional efforts ($\Delta e \leq 0.1$)[3]. These alternative targets show different directions for improvement and therefore can provide DMs with more abundant alternatives, which is clearly shown in Figure 6.

According to Figure 6, there are clear differences in the improvement effects of labor, total assets, CO_2 emission, and operating revenue and no reduction in operating costs. In terms of improvement effects, this is more similar to Figure 4 and quite different from Figure 5. However, in Table 4, it can be seen that the efforts

[3]In the research of Ruiz and Sirvent [6], the maximum value for Δe was 0.05 (5%). However, to ensure that at least three alternatives can be obtained in Table 4, we set the maximum value for Δe to 0.1 (1%).

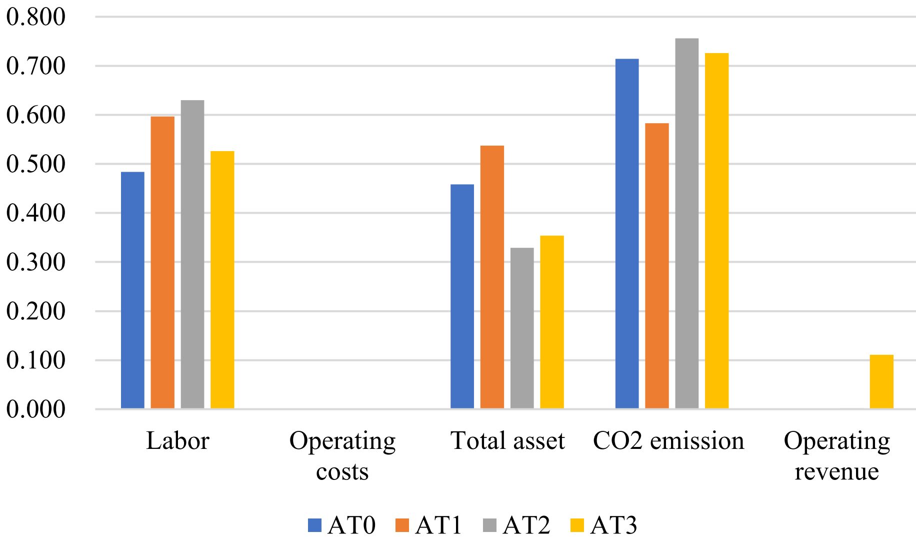

FIGURE 6. Alternative targets in the study of Ruiz and Sirvent [6].

for all alternative targets are greater than or equal to e_0^{closest}. That is, $e_0^{\text{closest}}(1.656) < e_0^0(1.656) < e_0^1(1.717) = e_0^2(1.717) = e_0^3(1.717)$ (see column 9 in Tab. 4). As Ruiz and Sirvent [6] said, although setting alternative targets with different improvement directions only needs a little additional effort ($\Delta e = 1 - 10\%$), these targets (even if the closest target was e_0^{closest}) would still be unattainable for DMUs with poor performance in many cases. By comparison, since $e_0^{1(a)}/e_0^{1(a)} \leq e_0^{u(a)} < e_0^{\text{closest}}$ in Table 2, our schemes ($A0$, $A1$, $A2$, $A3$, and $A4$ in Tab. 2) not only had different improvement effects but also solved the problem of unachievable targets. This makes it easier for inefficient DMUs to gradually improve their performance in several intermediate steps when e_0^{closest} (or $e_0^{\text{closest}} + \Delta e$) cannot be implemented in a single step.

In short, the DEA method of Ruiz and Sirvent [6] provides several best-practice targets with different improvement effects. However, these targets need to be directly reached in a single step, and this is difficult for DMUs with poor performance. On the contrary, stepwise plans generated from our stepwise approaches and search algorithm consider both various improvement effects and intermediate targets.

4.4. Stepwise improvement paths considering weak disposability

Section 4.2 explains the function of our stepwise DEA approaches, which are based on the assumption that undesirable outputs (CO_2 emission) are freely disposable. However, we also emphasized that weak disposability should apply in many situations. Since the proposed methods in Section 3 cannot deal with weak disposability, we further developed new stepwise DEA models (see (A.2) and (A.4) in Appendix 1) and analyzed their effectiveness in the following subsection.

4.4.1. Ports of different performance levels under weak disposability

By running the stratification algorithm, we categorized 18 ports into three-layer frontiers with different performance levels; these are shown in Table 5.

Differently from in Table 1, in Table 5, Zhuhai was located at $\partial(T^2)$, which indicates that the weak disposal might affect the performance level of individual ports. However, it is clear that no matter in Table 1 or Table 5, both Rizhao and Guangzhou were on $\partial(T^3)$, so they both had poor performance. For the convenience of comparison, we once again took Rizhao as an example to illustrate the effectiveness of (A.2) and (A.4) in Appendix 1. The performance improvement of Rizhao under weak disposal was also divided into two intermediate

TABLE 5. DMUs with different performance levels.

Classified ports	Ports with different levels of performance
Ports on $\partial(T^1)$	Zhaoshang, Shanghai, Yantian, Ningbo, Jinzhou, Xiamen, Nanjing
Ports on $\partial(T^2)$	Qingdao, Tianjin, Liaoning, Qinhuangdao, Tangshan, Beibu, Chongqing, Lianyungang, Zhuhai
Ports on $\partial(T^3)$	Rizhao, Guangzhou

TABLE 6. Rizhao: Alternative stepwise improvement paths.

σ	Alternatives		Labor	Operating costs	Total assets	CO_2 emission	Operating revenue	δ_0^a	$e_0^{u(a)}$	$e_0^{1(a)}$	$e_0^{2(a)}$
		Actual	5581	3934	23 248	242 830	5246				
	AW0	Inter	22.6%	0.0%	0.0%	0.0%	50.0%	–	0.726	0.726	0.726
		Final	64.2%	0.0%	0.0%	0.0%	80.9%				
0.00		Inter	25.5%	0.0%	0.2%	0.0%	46.9%	0.003	0.726	0.726	0.726
		Final	64.2%	0.0%	0.2%	0.0%	80.8%				
1.00	AW1	Inter	11.7%	0.0%	17.1%	0.0%	51.1%	0.432	0.799	0.799	0.799
		Final	63.3%	0.0%	28.9%	0.0%	67.5%				
1.00	AW2	Inter	27.1%	0.0%	5.2%	0.0%	47.6%	0.294	0.799	0.799	0.799
		Final	63.8%	0.0%	14.2%	7.8%	74.0%				
2.00	AW3	Inter	23.5%	0.0%	13.8%	0.0%	49.3%	0.442	0.871	0.871	0.865
		Final	63.8%	0.0%	13.8%	22.6%	73.5%				
3.00		Inter	23.3%	0.0%	14.3%	0.0%	49.3%	0.435	0.871	0.871	0.870
		Final	67.5%	17.6%	14.3%	1.0%	73.6%				
3.00	AW4	Inter	4.6%	0.0%	31.6%	0.0%	50.4%	0.401	0.871	0.871	0.861
		Final	62.6%	0.0%	55.7%	0.0%	55.0%				

steps: $(X_0^3, Y_0^3) \xrightarrow{\text{Step 1}} (\hat{X}_0^{2*}, \hat{Y}_0^{2*}) \xrightarrow{\text{Step 2}} (\hat{X}_0^{1*}, \hat{Y}_0^{1*})$, where $(\hat{X}_0^{2*}, \hat{Y}_0^{2*})$ and $(\hat{X}_0^{1*}, \hat{Y}_0^{1*})$ are the intermediate and final targets, respectively.

4.4.2. Stepwise improvement paths under weak disposability

The search algorithm proposed in Section 3.3 is also applicable to (A.2), (A.3), and (A.4) in Appendix 1. Based on this, we could calculate the value of e_0^{closest} (1.452) by solving (A.3) and further acquiring five alternatives through (A.2) and (A.3). All stepwise plans (*i.e.*, AW0, AW1, AW2, AW3, and AW4) under weak disposability are listed in Table 6. It is important to note that the format of Tables 2 and 6 is the same.

(1) Difference of alternatives under weak disposability.

As shown in Figure 7, on the premise of considering weak disposal of CO_2 emissions, there were obvious differences in the improvement effects of all step-by-step schemes (AW0, AW1, AW2, AW3, and AW4). By comparing Figure 7 with Figure 4, it can be seen that we could guarantee that the stepwise schemes would vary in terms of improvement effects under both free and weak disposal of CO_2. To avoid a large amount of duplication, we did not analyze the differences between the solutions in Figure 7. In summary, Rizhao had access to several different improvement options that were considered weak disposal.

(2) Feasibility of alternative under weak disposability.

As in Table 2, every incremental scheme in Table 6 had a corresponding intermediate target (see the "Inter" row for AW0, AW1, AW2, AW3, and AW4 in Tab. 6) that spread the performance improvement over two steps. In other words, the intermediate targets of alternatives in Tables 2 and 6 had the same effect on incremental performance improvement, which allowed the decrease and increase in some inputs and outputs to be implemented in two steps. Based on this, we did not perform a repeat analysis.

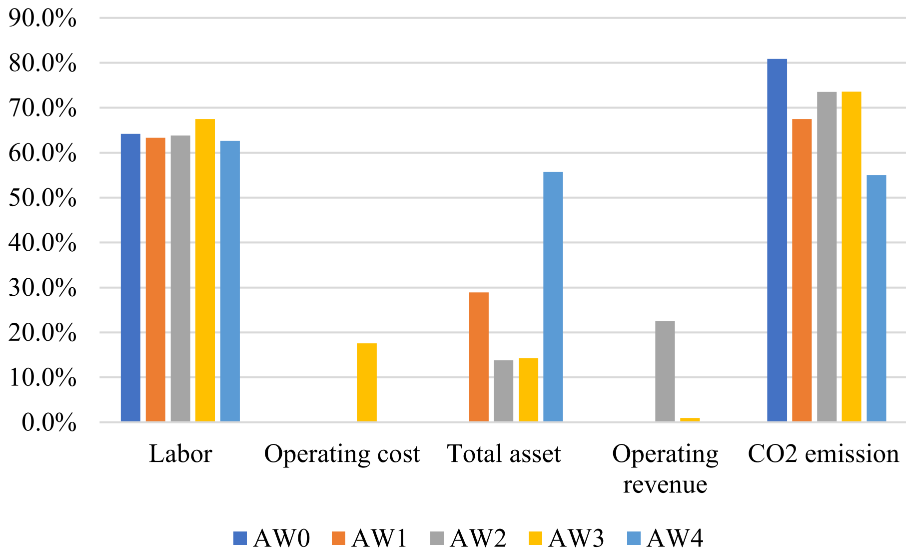

FIGURE 7. Improvement effects of the stepwise plans under weak disposal.

Furthermore, in all stepwise schemes considering weak disposal of CO_2, the upper bound $e_0^{u(a)*}$ for intermediate improvement efforts was significantly lower than e_0^{closest} (1.452) (see column 10 in Tab. 6). This indicates that the efforts of each intermediate step were strictly less than e_0^{closest} (1.452) (see columns 11 and 12 in Tab. 6). Although Rizhao's performance was also poorer while weak disposal of CO_2 was considered, the port could more easily reach the final target with best practices through these more feasible step-by-step schemes.

4.5. Managerial implications

There are always many units (or enterprises) whose actual performance is far away from best-practice performance in any industry (*e.g.*, banks, universities, coal, aviation, shipping, and ports). The poor performance means that these units have great limitations in management experience and resource utilization ability, which, in turn, restricts their ability to improve actual performance to efficient target. As an example, some ports cannot achieve their best performance target because of limited technology progress, management level, and exogenous factors. In addition, the actual management situation experienced by the units may lead them to focus more on the decrease or increase in certain inputs or outputs in performance improvement. Furthermore, taking ports as a classic case, when there are too many workers in a port enterprise, the DMs hope that the performance improvement can greatly reduce the number of employees. If a port enterprise has the management goal of increasing operating revenue in the next year's work plan, the DMs will hope that performance improvement can promote the extension of operating revenue. If the redundancies of operating costs and total assets of port enterprises are too large, the performance improvement should reduce the operating cost and total assets. In summary, ports and other industries experience many of the above situations.

Whether free or weak disposal is considered, the step-by-step paths provided in this study can address the abovementioned issues. First, the proposed stepwise alternatives can more flexibly meet the increasing and decreasing demands of various input-output indicators under different management needs of DMs. Second, the upper bound for the intermediate improvement effort can be effectively limited so that the inefficient DMUs (*e.g.*, Rizhao port) have relatively less improvement pressure in each intermediate step. The managerial implication of our gradual paths is verified in Sections 4.2.2 and 4.2.3.

5. Conclusions

Targets with best practices can show the clear improvement direction for an inefficient DMU, which determines the improvement effects of the direct path shown by the target. For this reason, some scholars have proposed novel DEA methods to find alternative targets (or direct improvement paths) with different improvement effects [4–6], which can satisfy various management needs. However, for DMUs with poor performance, it is difficult to reach the target (even if the target is closest) owing to the large gap between actual performance and the target and the limitation of practical factors. To supplement this deficiency, we established the novel stepwise DEA methods and a search algorithm to search for alternative step-by-step improvement paths with different improvement effects. These stepwise paths provided by our DEA methods can both deal with performance improvement under different management needs and solve the problem of unattainable targets. Based on this, the two main contributions of this study are as follows. (1) Multiple gradual improvement strategies with different improvement effects are found, which provide various schemes and can satisfy the management requirements of DMs. (2) By restricting the upper bound for improvement efforts of all intermediate steps, our stepwise plans make it more feasible to improve poor performance. Our study results also have practical implications. Concretely, the methods proposed in this paper can contribute to poorly performing enterprises (or units) in many industries (*e.g.*, banks, universities, coal, aviation, shipping, and ports). These units are constrained by practical factors (*e.g.*, management expertise and exogenous factors) and always want performance improvements to match actual management objectives. The practical advantages of our approach are demonstrated in the case of Rizhao Port in Section 4.2.

There are still some limitations (*e.g.*, peer identification, the given change rates in inputs and outputs, cost efficiency, and the closeness between best-practice targets and ideal points) in our stepwise DEA approaches, which provide some potential directions for future research. First, similar to what Ruiz and Sirvent [6] advocated, we should extend the proposed stepwise benchmarking approach to the new stepwise DEA methods that can consider the identification of suitable peers in intermediate steps. This allows us to not only find alternative stepwise improvement paths with different improvement effects, but also identify the sequential peers to learn. Second, our approaches should also consider the given rates of change in inputs and outputs that DMUs can implement in each improvement step, in line with the research from Lozano and Villa [7] and An *et al.* [32]. Our stepwise DEA approaches can also consider incorporating research related to cost efficiency [45, 46]. Last, as with Lozano *et al.* [47], the similarities between final targets with best practices and ideal points should also be further considered.

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Data availability statement

The research data associated with this article are included in the article.

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Appendix A Stepwise benchmarking DEA approaches including the weak disposal of undesirable outputs

We consider n DMUs, where each $\text{DMU}_j (j = 1, \ldots, n)$ can produce s desirable outputs, denoted by $Y_j = (y_{1j}, \ldots, y_{rj}, \ldots, y_{sj}) \geq 0$, and h undesirable outputs, denoted by $B_j = (b_{1j}, \ldots, b_{fj}, \ldots, b_{hj}) \geq 0$, through m inputs, denoted by $X_j = (x_{1j}, \ldots, x_{ij}, \ldots, x_{mj}) \geq 0$. (X_j, Y_j, B_j) represents the actual performance of DMU_j. On the premise of considering the weak disposal of undesirable outputs, we construct the following PPS under VRS technology:

$$T = \left\{ \begin{matrix} (X,Y,B)|X \geq \sum_{j=1}^{n}(\lambda_j + \gamma_j)X_j, Y \leq \sum_{j=1}^{n}\lambda_j Y_j, B = \sum_{j=1}^{n}\lambda_j B_j, \\ \sum_{j=1}^{n}(\lambda_j + \gamma_j) = 1, \lambda_j \geq 0, \gamma_j \geq 0, \quad \forall j \in n \end{matrix} \right\}. \tag{A.1}$$

Next, we can propose the following programming to seek the initial stepwise improvement path:

$$\text{Min } e_0^u - \varepsilon * \sum_{k=1}^{l-1} t_0^k$$

s.t.

$$\sum_{j \in E^k} \left(\lambda_j^k + \gamma_j^k\right) x_{ij}^k = x_{i0}^l - \sum_{k}^{l-1} s_{i0}^{k-} \qquad i = 1, \ldots, m; \quad k = 1, \ldots, l-1 \tag{A2.1}$$

$$\sum_{j \in E^k} \lambda_j^k \, y_{rj}^k = y_{r0}^l + \sum_{k}^{l-1} s_{r0}^{k+} \qquad r = 1, \ldots, s; \quad k = 1, \ldots, l-1 \tag{A2.2}$$

$$\sum_{j \in E^k} \lambda_j^k \, b_{fj}^k = b_{f0}^l + \sum_{k}^{l-1} c_{f0}^{k-} \qquad f = 1, \ldots, h; \quad k = 1, \ldots, l-1 \tag{A2.3}$$

$$\sum_{j \in E^k} \left(\lambda_j^k + \gamma_j^k\right) = 1 \qquad k = 1, \ldots, l-1 \tag{A2.4}$$

$$-\sum_{i=1}^{m} v_i^k \, x_{ij}^k + \sum_{r=1}^{s} u_r^k \, y_{rj}^k - \sum_{i=1}^{m} z_f^k \, b_{fj}^k + d_j^k + u_0^k = 0 \qquad j \in E^k; \quad \forall k \tag{A2.5}$$

$$x_{io}^l v_i^k \geq 1 \qquad i = 1, \ldots, m; \quad k = 1, \ldots, l-1 \tag{A2.6}$$

$$y_{r0}^l u_r^k \geq 1 \qquad r = 1, \ldots, s; \quad k = 1, \ldots, l-1 \tag{A2.7}$$

$$b_{f0}^L z_f^k \geq 1 \qquad f = 1, \ldots, h; \quad k = 1, \ldots, l-1 \tag{A2.8}$$

$$\left(\lambda_j^k + \gamma_j^k\right) d_j^k = 0 \qquad j \in E^k; \quad \forall k \tag{A2.9}$$

$$\sum_{i=1}^{m} \frac{s_{i0}^{k-}}{x_{i0}^l} + \sum_{r=1}^{s} \frac{s_{r0}^{k+}}{y_{r0}^l} + \sum_{f=1}^{h} \frac{c_{f0}^{k-}}{b_{f0}^l} - e_0^k = 0 \qquad \forall i, r, f, k \tag{A2.10}$$

$$e_0^k + t_0^k = e_0^u \qquad \forall k \tag{A2.11}$$

$$e_0^u \leq e_0^{\text{closest}} \tag{A2.12}$$

$$s_{i0}^{k-}, s_{r0}^{k+}, c_{f0}^{k-}, \lambda_j^k, d_j^k \geq 0 \qquad j \in E^k; \quad \forall i, r, f, k$$

$$u_0^k \qquad \text{free}; \quad k = 1, \ldots, g-1$$

where (A2.9) can also be handled using SOS1. In addition to the weak disposal of undesirable outputs, the core function of (A.1) is the same as that of (4), and Proposition 1 can also be applied to (A.1). By solving (A.1), we can obtain the sequential targets $(\hat{x}_{ij}^{k*}, \hat{y}_{rj}^{k*}, \hat{D}_{fj}^{k*})(k = 1, \ldots, l-1)$ that can form the stepwise improvement path. Meanwhile, e_0^{closest} can be calculated in advance using the following model (A.3):

$$\text{Min } e_0^{\text{closest}} = \sum_{i=1}^{m} \frac{s_{i0}^-}{x_{i0}} + \sum_{r=1}^{s} \frac{s_{r0}^+}{y_{r0}} + \sum_{r=1}^{s} \frac{c_{f0}^-}{b_{f0}}$$

s.t.

$$\sum_{j \in E} \left(\lambda_j + \gamma_j\right) x_{ij} = x_{i0} - s_{i0}^- \qquad i = 1, \ldots, m \tag{A3.1}$$

$$\sum_{j \in E} \lambda_j \, y_{rj} = y_{r0} + s_{i0}^+ \qquad r = 1, \ldots, s \tag{A3.2}$$

$$\sum_{j\in E}\lambda_j\, b_{fj} = b_{f0} + c_{f0}^{-} \qquad f=1,\ldots,h \qquad \text{(A3.3)}$$

$$\sum_{j\in E}\left(\lambda_j+\gamma_j\right) = 1, \qquad \text{(A3.4)}$$

$$-\sum_{i=1}^{m} v_i\, x_{ij} + \sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} z_f\, b_{fj} + d_j + u_0 = 0 \qquad j\in E^k \qquad \text{(A3.5)}$$

$$x_{i0}v_i \geq 1 \qquad i=1,\ldots,m \qquad \text{(A3.6)}$$

$$y_{r0}u_r \geq 1 \qquad r=1,\ldots,s \qquad \text{(A3.7)}$$

$$b_{f0}z_f \geq 1 \qquad f=1,\ldots,h \qquad \text{(A3.8)}$$

$$(\lambda_j+\gamma_j)d_j = 0 \qquad j\in E \qquad \text{(A3.9)}$$

$$s_{i0}^{-}, s_{r0}^{+}, c_{f0}^{-}, \lambda_j, d_j \geq 0 \qquad j\in E;\quad \forall i,r,f,k$$

$$u_0 \qquad \text{free.}$$

Similar to (A2.9), (A3.9) can also be handled by SOS1. Solving (A.3) can provide the closest targets for inefficient DMUs. Next, to find more stepwise improvement paths showing different improvement effects, we still establish the following programming (A.4):

$$\text{Lex max}\left\{\delta_0^a + \varepsilon * c, \sum_{k=1}^{l-1} t_0^{k(a)}\right\}$$

s.t.

$$\sum_{j\in E^k}\left(\lambda_j^{k(a)}+\gamma_j^{k(a)}\right)x_{ij}^k = x_{i0}^l - \sum_{k}^{l-1} s_{i0}^{k(a)-} \qquad i=1,\ldots,m;\quad k=1,\ldots,l-1 \qquad \text{(A4.1)}$$

$$\sum_{j\in E^k}\lambda_j^{k(a)} y_{rj}^k = y_{r0}^l + \sum_{k}^{l-1} s_{r0}^{k(a)+} \qquad r=1,\ldots,s;\quad k=1,\ldots,l-1 \qquad \text{(A4.2)}$$

$$\sum_{j\in E^k}\lambda_j^{k(a)}\, b_{fj}^k = b_{f0}^l - \sum_{k}^{l-1} c_{f0}^{k(a)-} \qquad f=1,\ldots,h;\quad k=1,\ldots,l-1 \qquad \text{(A4.3)}$$

$$\sum_{j\in E^k}\left(\lambda_j^{k(a)}+\gamma_j^{k(a)}\right) = 1 \qquad k=1,\ldots,l-1 \qquad \text{(A4.4)}$$

$$-\sum_{i=1}^{m} v_i^{k(a)}\, x_{ij}^k + \sum_{r=1}^{s} u_r^{k(a)}\, y_{rj}^k$$
$$-\sum_{i=1}^{m} z_f^{k(a)}\, b_{fj}^k + d_j^{k(a)} + u_0^{k(a)} = 0 \qquad j\in E^k;\quad \forall k \qquad \text{(A4.5)}$$

$$x_{i0}^l v_i^{k(a)} \geq 1 \qquad i=1,\ldots,m;\quad k=1,\ldots,l-1 \qquad \text{(A4.6)}$$

$$y_{r0}^l u_r^{k(a)} \geq 1 \qquad r=1,\ldots,s;\quad k=1,\ldots,l-1 \qquad \text{(A4.7)}$$

$$b_{f0}^l z_f^{k(a)} \geq 1 \qquad f=1,\ldots,h;\quad k=1,\ldots,l-1 \qquad \text{(A4.8)}$$

$$\sum_{i=1}^{m}\frac{\left|\hat{x}_{i,0}^{1(a)}-\hat{x}_{i,0}^{1(\xi)*}\right|}{x_{i,0}^l} + \sum_{r=1}^{s}\frac{\left|\hat{y}_{r,0}^{1(a)}-\hat{y}_{r,0}^{1(\xi)*}\right|}{y_{r,0}^l},$$
$$+\sum_{f=1}^{h}\frac{\left|\hat{b}_{i0}^{1(a)}-\hat{b}_{i0}^{1(\xi)*}\right|}{b_{f0}^l} - \delta_0^a = 0 \qquad \forall k \qquad \text{(A4.9)}$$

$$\left(\lambda_j^{k(a)}+\gamma_j^{k(a)}\right)d_j^{k(a)} = 0 \qquad j\in E^k;\quad \forall k \qquad \text{(A4.10)}$$

$$\sum_{i=1}^{m}\frac{s_{i0}^{(a)k-}}{x_{i0}^{l}}+\sum_{r=1}^{s}\frac{s_{r0}^{(a)k+}}{y_{r0}^{l}}+\sum_{f=1}^{h}\frac{c_{f0}^{(a)k-}}{b_{f0}^{l}}-e_{0}^{k(a)}=0 \quad \forall i,r,f,k \tag{A4.11}$$

$$e_0^{k(a)}+t_0^{k(a)}=e_0^{u(a)} \quad \forall k \tag{A4.12}$$

$$e_0^{u(a)}+c=\rho \tag{A4.13}$$

$$s_{i0}^{k(a)-},s_{r0}^{k(a)+},c_{f0}^{k(a)-},\lambda_j^{k(a)},\gamma_j^k,d_j^k\geq 0 \quad j\in E^k;\ \ \forall i,r,f,k$$

$$u_0^{k(a)} \quad \text{free};\ \ k=1,\ldots,g-1.$$

As the constraint (8.7) in (8), the nonlinear form in (A4.10) can also be transformed by deviation variables, and (A4.10) can be solved by SOS1. Except for the weak disposal, the objective functions in (A.4) have the same meaning as those in (8). Hence, Propositions 2 and 3 can also be applied to (A.4). By solving (A.4), we can obtain new stepwise improvement paths with improvement effects.

Finally, we apply (A.2), (A.3), and (A.4) to the proposed search algorithm, and then more stepwise improvement paths consisting of sequential targets $(\hat{X}_{ij}^{k(\xi)*},\hat{y}_{rj}^{k(\xi)*},\hat{D}_{fj}^{k(\xi)*})\xi=0,1,\ldots,\alpha$ can be found.

Appendix B Raw data and descriptive summary of input-output indicator for 18 ports in China

Ports	Labor (person)	Operating cost (106 yuan)	Total asset (106 yuan)	CO_2 emission (104 ton)	Operating revenue (106 yuan)
Zhaoshang	15 619	7649	156 697	59 130	12 124
Shanghai	14 068	25 016	142 177	375 301	36 102
Yantian	848	316	32 016	12 222	5934
Ningbo	17 790	18 582	71 779	586 479	24 322
Qingdao	8981	8179	52 785	302 306	12 164
Tianjin	7254	10 244	35 107	257 716	12 885
Liaoning	3774	4655	50 319	191 852	6646
Qinhuangdao	11 235	3844	25 480	114 564	6723
Tangshan	3853	8783	24 659	343 869	11 209
Rizhao	5581	3934	23 248	242 830	5246
Beibu	7509	2930	18 149	133 874	4792
Guangzhou	8808	8032	28800	317 385	10 420
Jinzhou	1578	6139	17 353	59 376	7033
Chongqing	2553	4365	12 353	89 677	4777
Xiamen	4907	13 274	9770	111 757	14 155
Lianyungang	3198	1044	9462	122 816	1425
Zhuhai	2883	2794	9251	72 456	3322
Nanjing	1058	383	4657	134 508	737
Maximum	17 790	25 015.77	156 696.92	586 479.12	36 101.63
Minimum	848	316	4657.07	12 222.29	736.82
Mean	6749.83	7231.27	40 225.61	196 006.54	10 000.75
St. Dev.	5118.78	6436.98	43 435.58	146 128.36	8578.72