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A REVENUE MANAGEMENT MODEL FOR AIRLINE AND RAILWAY TRANSPORTATION CONSIDERING COOPERATIVE AND COMPETITIVE GAMES

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Abstract. Simultaneous optimization of pricing and seat-allocation brings about revenue management improvement. Thus, recently, some researchers have accentuated the joint optimization of pricing and quantity decisions. Investigating customer choice behavior leads to a more precise grasp of this issues. This paper presents a game-theoretic framework, in which a rail transport company and an airline company are participants of the game. This paper's primary incentive is to study two heterogeneous participants' behavior under competition and cooperation, considering the joint optimization of pricing and quantity decisions. To that aim, firstly, two separate mixed-integer non-linear programming models are developed. The models are linked through their demand functions, and each company's demand acts based on its price and quality, as well as the competitor's price and quality. The objective functions of both models aim at maximizing the total revenue. Companies' performance is studied under both non-cooperative and cooperative games. The equal profit method and Shapley value are employed to build a coalition between participants. Ultimately, the analyses' findings indicate that both companies yield more revenue as they form a coalition rather than competing with each other. Moreover, results show that passengers may find longer train journeys inconvenient or exhausting compared to faster alternatives like flights; hence, the airline company gains more revenue in large-size instances. Furthermore, revenue increases for both airline and rail transport companies with rising demand. Besides, it turned out that price elasticity is inversely related to revenue. On the other hand, quality elasticity has a direct relation with revenue.

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1. Introduction

The term "revenue management" (RM) was initially used in Rothstein's study on airline overbooking in the early 1970s [1]. RM attempts to handle different customer classes with uncertain demand using different tools such as pricing, discount, quality, and availability to achieve the maximum expected revenue. RM has applied to a wide range of areas, such as airline, ferry, retail, car rental, theatre, and hotel management [2]. Based on several estimations, applying RM practices leads to an increase in revenue of about 4 to 5%, almost equal to many of airline companies' revenues [3]. Two main categories of RM decisions are price and quantity

Keywords. Revenue management, airline, rail transport, competition, cooperation, game theory.

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decisions [4]. Price decisions are referred to as setting the prices over time and for different products. Generally, revenue management tries to enhance profit by laying on services/products to the right customer at the right time, considering the limited capacity of services/products [5]. In contrast, quantity decisions focused more on rejection or acceptance of offers, setting the capacity to different classes of customers, and determining the booking limits. Recently, Kocabıyıkoğlu *et al.* [6] considered joint pricing and capacity allocation. They developed two separate models which are different in pricing and coordination strategy. In general, in the RM field, competition is either on the price level or the quantity level. Each of the pricing and quantity decisions can independently yield financial gains. However, simultaneous consideration of price and quantity levels in RM problems has been treated with a lack of regard. Hence, this paper tries to reduce the gap by studying the joint pricing/allocation problem of airline and rail transport companies. So, the companies compete for passengers within a revenue management setting by offering various price and quantity options. Game theory tries to investigate the acting of rational players under cooperation or competition using mathematical modeling [7]. Nevertheless, the cooperative game has not been pointed out adequately [8]. Besides, we have to bear in mind that players of a specific game are not homogeneous on all occasions. Therefore, this study tries to consider pricing and quantity decisions of two heterogeneous participants: airline and rail transport companies, under competition and cooperation, simultaneously. Under the competitive game, each participant, tries to increase its payoff by joint optimizing of pricing and seat allocation levels. On the other hand, under cooperative situations, participants play in a non-zero-sum game setting to enhance their aggregate payoff. In such a game, participants tend to dispense the revenue as equally as possible and the utility usually is considered transferable. In recent years, researchers in the RM field have given more regard to customer choice behavior. A new border of RM links with customer satisfaction and customer-centric marketing. Hence, firms must know that every passenger has their tastes, needs, and choices [9]. Nowadays, Firms devise some strategies to convince customers to purchase their products and services. In today's competitive world, companies use different levers to ensure that their efforts will pay off. Quality improvement can be a useful tool for influencing customer behavior. With this discussion, a further initiative of this paper is the consideration of customer choice behavior based on diverse factors such as price, quality, and travel time. Quality is referred to as a set of properties that customers seek such as travel-time reliability [10], implementing standard requirements, convenience of seats within classes, and service levels. Since quality-seeking customers care more about quality than price, quality improvement has always been an influential factor in demand. Thus, companies can increase customers' willingness to pay by improving quality. Another critical element that affects traveler's behavior is travel time. The travel time over each origin-destination (O-D) pair on a railway is dependent on train stop planning. By the train stop planning problem, we mean that the stopping stations for each train on the railway lines are determined to make a balance between revenue obtained from covering the demand and travel time of O-Ds. Thus, for those paths that are included in both train and airplane schedules, passengers choose between train and airplane based on their circumstances and preferences. The demand is assumed to be a function of price, quality, travel time, and the competitor's behavior. To the best of our knowledge, no paper has addressed such a problem. To sum up, the primary contributions of this study are as follows:

- Mathematical modeling of two separate systems for airplane and train;
- Simultaneous incorporation of pricing and allocation decisions into the models;
- Considering customer choice behavior based on price, quality, and travel time;
- Using the Nash equilibrium to characterize the game under competition;
- Using both equal profit method and Shapley value to characterize the game under cooperation;
- Comparing the impacts of cooperation and competition on the players' decisions.

The structure of the current paper is as follows: Section 2 reviews related publications, while Section 3 focuses on the problem statement and mathematical modeling. In Section 4, both competitive and cooperative games are provided. Detailed numerical results are discussed in Section 5, followed by practical analyses in Section 6. Lastly, Section 7 provides conclusions and several directions for future research.

2. Literature review

This section discusses the literature on both airline and railway revenue management.

2.1. Airline revenue management

Investigating the related literature, it can be witnessed that more studies have been conducted about airline RM than railway RM. For instance, Vardi *et al.* [11] developed a bi-objective model to optimize the airline RM system. They took cancelation into account, so they utilized the overbooking strategy to decrease unused seats. Finally, they used a combined approach based on the Taguchi design, which is a statistical method used to optimize processes by systematically changing factors to identify the most influential ones, and the COPRAS, which is a multi-criteria decision-making (MCDM) method used to evaluate and rank alternatives by considering multiple conflicting criteria, to optimize their model. To jointly optimize seat allocation and pricing levels, Yoon *et al.* [12] proposed a mixed-integer linear programming model. The objective function of their model was targeted at maximizing the total revenue of an airline company. Similarly, Yazdi *et al.* [13] investigated airline RM using mathematical modeling. They embedded some features – booking, overloading, and cancelation – into their model to approach the real-world problem. To deal with the randomness of data, they employed a binary differential evolution algorithm to solve their model.

Some studies have investigated airline revenue management under competition. Li and Peng [14] were among the first scholars to study this aspect. They studied the effects of dynamic pricing on airline companies. They set up a competitive game between airline companies in a way that both flights move along the same path. To complete the research path, Raza and Akgunduz [15] considered a single flight leg for two airline companies under competition such that both companies tended to optimize their quantity and pricing decisions simultaneously. Since their problem was a competitive game, they used the Nash equilibrium to solve it. They found out that market conditions can affect payoffs, quantity, and pricing decisions. Since analyzing customer choice is crucial, Lin and Sibdari [16] utilized multinomial logit modeling to define the passengers' choice behavior. They formulated a model for airline companies under competition. Finally, they applied the Nash equilibrium to their developed model to characterize the non-zero-sum game of their problem. Grauberger and Kimms [17] conducted a study to find out whether making an effort to consider competition in allocation problems is beneficial or not. They used an approximate Nash equilibrium in order to describe the competitive game. Eventually, they understood that considering the competition in allocation problems always brings about more earnings than non-competitive circumstances. Grauberger and Kimms [18] studied airline RM under competition utilizing game theory. To that aim, they developed a non-linear model with the objective of revenue maximization. They figured out that higher revenue will be gained through the price increase within high demand compared with monopolistic prices.

From reviewing the literature, it can be seen that, on the whole, researchers have not addressed the cooperative game to a satisfactory extent [8]. Because of that, few papers have studied the revenue management of transportation companies in a cooperative setting. Kimms and Çetiner [19] studied the quantity decisions of airline companies. They considered a cooperative game among airline companies. To equitably distribute the revenue between the players, they used the nucleolus-based allocation method. Clempner [20] made a significant contribution to airline RM studies by considering both pricing and allocation decisions in a cooperative setting. He investigated the efficiency of the transfer pricing mechanism for a cooperative game between airline companies. The transfer pricing mechanism was used to penalize the arrival time of passengers to the final destination. The results confirmed that in case of penalization of transfer time, the allied companies act effectively.

2.2. Railway revenue management

Railway transportation has been one of the primary modes of passenger transportation for over two centuries [21]; notwithstanding, researchers have not adequately paid attention to railway RM. From the few studies that have been conducted regarding railway RM, the below papers can be mentioned. Terabe and Ongprasert [22] studied the seat inventory control strategy for a rail transport company. The objective functions of their model

Table 1. Reviewed papers.

Authors	Transportation service		Decisions		Game		Customer choice modeling
	Airplane	Train	Pricing	Allocation	Cooperation	Competition	
Terabe and Ongprasert [22]		✓		✓			–
Li and Peng [14]	✓		✓			✓	–
Raza and Akgunduz [15]	✓		✓	✓		✓	Linear
Lin and Sibdari [16]	✓		✓			✓	Logit
Kimms and Çetiner [19]	✓			✓	✓		–
Hetrakul and Cirillo [23]		✓	✓	✓			Logit
Grauberger and Kimms [17]	✓			✓		✓	–
Hetrakul and Cirillo [24]		✓	✓	✓			Logit
Wang *et al.* [25]		✓		✓			Logit
Vardi *et al.* [11]	✓			✓			–
Grauberger and Kimms [18]	✓		✓	✓		✓	Linear
Yoon *et al.* [12]	✓		✓	✓			–
Zhang *et al.* [26]		✓	✓				Logit
Clempner [20]	✓		✓	✓	✓		–
Yazdi *et al.* [13]	✓		✓	✓			–
Current study	✓	✓	✓	✓	✓	✓	Linear

were to maximize total revenue and the average passenger load factor and minimize the number of rejections. The outcomes of their study indicated that seat allocation could lead to an increase in revenue and the average passenger load factor. Some researchers have focused on passenger choice behavior in railway RM. Hetrakul and Cirillo [23] studied a joint pricing and allocation problem for a railway company. They applied a multinomial logit approach in order to formulate customer choice behavior. They found out that a rail transport company can reach more revenue by seeking short-haul tours than long-haul tours. To cope with passenger heterogeneity, Hetrakul and Cirillo [24] formulated a new model for railway RM. They concluded that considering passenger choice in calculations results in about a 20% increase in gained profits. Wang *et al.* [25] incorporated the seat allocation problem into railway RM. Due to the uncertain nature of the problem, they stochastically formulated their model. They considered customer choice behavior in their model. Finally, to reduce the computational time, they employed the Monte Carlo approach to approximate the expected revenue using the sample average approximation algorithm. Zhang *et al.* [26] proposed a mathematical model for a rail transport company. Their model aims at jointly optimizing pricing/dispatching decisions to enhance the total revenue. They took customer choice behavior into account so that the demand function of their problem was sensitive to price shifts. They applied the railway of Guangzhou–Shenzhen as the case study to their paper. In the end, the outcomes showed the efficiency of their developed model.

2.3. Research gap

All the reviewed literature has been summarized in Table 1. According to Table 1, it can be seen that this paper is the first to jointly optimize pricing and quantity decisions for airline and rail transport companies, under competition and cooperation game. This study's primary goal is to consider the simultaneous pricing and allocation decisions of two heterogeneous participants under competitive and cooperative games. Moreover, passenger choice behavior is embedded in mathematical modeling to approach real-world circumstances.

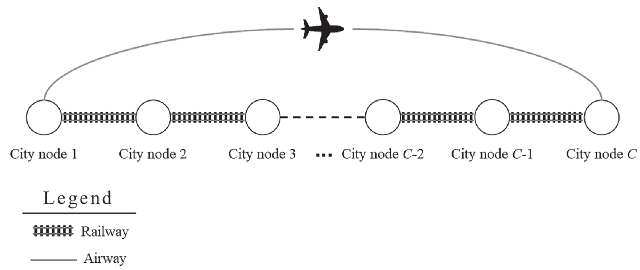

FIGURE 1. An example of city nodes and their links through railway and airway.

3. Problem definition

A multi-leg path with intermediate stations (city nodes) is considered in a way that i denotes each city node, and $i = \{1, 2, \ldots, C\}$. In the proposed model, a single-leg flight is considered for an airplane with origin city "1" and destination node C. On the other hand, the train will pass along every city node. Therefore, the airline and railway compete on origin-destination demand $(1, C)$. Concerning other O-Ds, passengers can be transported only by train. There exist multiple fare classes for each origin and destination in each transportation system. The capacity of each leg represents the total number of seats available in that leg, which is considered fixed. As we can see from Figure 1, the rail journeys consist of multiple legs, and an origin-destination itinerary utilizes the seats of multiple legs so that it is necessary to control capacities/prices on an O-D-based level. To this purpose, the station locations, booking limits, and prices are determined at the beginning of the horizon and customer classes for each O-D itinerary in each transportation system arrive with stochastic demand. The tickets are sold according to booking limits to maximize the total revenue. As mentioned before, the demand for a flight or train is affected by the price and quality of its rivals and the price and quality of its own. Due to the demand uncertainty and nature of the decision-making process in the model, the two-stage stochastic programming formulation is used. To avoid the difficulty of continuous distributions, a scenario-based approach is developed for the pricing and quality assignment in a competitive environment. Furthermore, to approach real-life problems, cancellations and customer choice behavior are embedded in the models. Ignoring each of these features can result in adverse effects; in particular, Sierag *et al.* [27] called into question those studies that have not considered cancellation. According to Sierag *et al.* [27], overbooking cancellations bring about revenue loss.

In the following, all the parameters and variables of both models are reported (Tab. 2).

3.1. Demand functions

Considering passenger preferences is one of the best ways to inform decision-makers about making a plan to attract passengers in real-world circumstances. Customer behavior theory tries to figure out passengers' choices and decisions and why they prefer one product to other products [28]. When passengers have to choose between an airplane and a train, there are plenty of factors that can affect their decisions. Services' quality, ticket price, and traveling time are examples of these factors. Taking customer choice behavior into account, the logit modeling and linear modeling approaches are two widely used approaches in the literature for modeling demand sensitivity to various factors [15, 29]. The linear modeling approach is employed in this paper to model the demand constraints. In this study, the passenger's demand is dependent on services' quality, ticket price, and traveling time. Equations (1) and (2) respectively indicate the demands of rail and air travel for O-D pair $(1, C)$. Thus, for the leg $(i = 1)$ to $(j = |\mathcal{C}|)$, passengers can choose between traveling either by train or airplane based on ticket price, quality, and traveling time. Equations (3) and (4) indicate the demands of the train for

Table 2. Notation table.

Notation	Description
Sets	
$\mathcal{C}$	Set of city node indexed by i, j, k
$\mathcal{M}$	Set of airplane class indexed by m, m'
$\mathcal{N}$	Set of train class indexed by n, n'
$\mathcal{S}$	Set of scenario indexed by s
Parameters	
ρ_s	Likelihood of sth scenario occurrence and $\sum_{s\in\mathcal{S}} \rho_s = 1$
$\alpha^{\mathbf{A}}_{ms}$	Flight cancellation rate of class m under scenario s
$cap^{\mathbf{A}}$	Capacity of airplane
$r^{\mathbf{A}}_{m}$	Flight Refund rate of class m of airplane
$b^{\mathbf{A}}_{m}$	Price elasticity of flight demand of class m
$\gamma^{\mathbf{A}}_{m}$	Quality elasticity of demand for class m of airplane
$e^{\mathbf{A}}_{m}$	Quality cost of class m in airplane
$ub^{\mathbf{A}}_{m}$	Upper bound of quality for class m in airplane
$\theta^{\mathbf{A}}_{m}$	Demand coefficient of class m in airplane and $\sum_{m\in\mathcal{M}} \theta^{\mathbf{A}}_{m} = 1$
$\psi^{\mathbf{A}}$	Cross-price elasticity of flight demand
$\alpha^{\mathbf{T}}_{ijns}$	Train ticket cancellation rate of class n of O-D (i, j) under scenario s
$cap^{\mathbf{T}}$	Capacity of train
$r^{\mathbf{T}}_{ijn}$	Train Refund rate of class n of O-D (i, j)
$b^{\mathbf{T}}_{n}$	Price elasticity of train demand of class n
$\gamma^{\mathbf{T}}_{n}$	Quality elasticity of demand for class n of train
ξ	Travel time elasticity of train demand
$e^{\mathbf{T}}_{n}$	Quality cost of class n in train
g	A big number
st	Average time of train stoppage in each station
t_j	Travel time to city node j from node $j-1$
$ub^{\mathbf{T}}_{n}$	Upper bound of quality for class n in train
$\theta^{\mathbf{T}}_{n}$	Demand coefficient of class n in train and $\sum_{n\in\mathcal{N}} \theta^{\mathbf{T}}_{n} = 1$
$\psi^{\mathbf{T}}$	Cross-price elasticity of train demand
π	Degree of customer loyalty to the airline transportation
T_s	Potential demand of O-D $(1, C)$ (the itinerary from the first city node to the last city node) under scenario s
τ_{ijs}	Potential demand of O-D (i, j) by train under scenario s $(j > i)$
Decision variables	
χ_i	1 If the train stops at station i, 0 otherwise
$D^{\mathbf{A}}_{ms}$	Flight demand of class m under scenario s
$Sl^{\mathbf{A}}_{ms}$	Flight sold ticket of class m under scenario s
$X^{\mathbf{A}}_{m}$	Flight booking limit of class m
$P^{\mathbf{A}}_{m}$	Flight ticket price of class m
$Q^{\mathbf{A}}_{m}$	Flight quality of class m
$D^{\mathbf{T}}_{ijns}$	Train demand of class n for O-D pair (i, j) under scenario s where $(j > i)$
$Sl^{\mathbf{T}}_{ijns}$	Train Sold ticket of class n for O-D pair (i, j) under scenario s where $(j > i)$
$X^{\mathbf{T}}_{ijn}$	Train booking limit of class n for O-D pair (i, j)
$P^{\mathbf{T}}_{ijn}$	Train ticket price of class n for O-D pair (i, j)
H_j	Arrival time to destination j
$Q^{\mathbf{T}}_{n}$	Quality of class n in train

the other O-Ds pairs.

$$D^{\mathbf{T}}_{ijns} = \theta^{\mathbf{T}}_n T_s \pi - b^{\mathbf{T}}_n P^{\mathbf{T}}_{ijn} + \psi^{\mathbf{T}} \sum_{n \neq n'} P^{\mathbf{T}}_{ijn'} + \sum_{m \in \mathcal{M}} b^{\mathbf{A}}_m P^{\mathbf{A}}_m + \gamma^{\mathbf{T}}_n Q^{\mathbf{T}}_n - \sum_{m \in \mathcal{M}} \gamma^{\mathbf{A}}_m Q^{\mathbf{A}}_m - \xi H_j, \qquad \forall i = 1, j = |\mathcal{C}|, n \in \mathcal{N}, s \in \mathcal{S}, \tag{1}$$

$$D^{\mathbf{A}}_{ms} = \theta^{\mathbf{A}}_m T_s (1-\pi) - b^{\mathbf{A}}_m P^{\mathbf{A}}_m + \psi^{\mathbf{A}} \sum_{m \neq m'} P^{\mathbf{A}}_m + \sum_{n \in \mathcal{N}} b^{\mathbf{T}}_n P^{\mathbf{T}}_{ijn} + \gamma^{\mathbf{A}}_m Q^{\mathbf{A}}_m - \sum_{n \in \mathcal{N}} \gamma^{\mathbf{T}}_n Q^{\mathbf{T}}_n + \xi H_j, \qquad \forall i = 1, j = |\mathcal{C}|, m \in \mathcal{M}, s \in \mathcal{S}, \tag{2}$$

$$D^{\mathbf{T}}_{ijns} = \theta^{\mathbf{T}}_n \tau_{ijs} + \psi^{\mathbf{T}} \sum_{n \neq n'} P^{\mathbf{T}}_{ijn'} - b^{\mathbf{T}}_n P^{\mathbf{T}}_{ijn} + \gamma^{\mathbf{T}}_n Q^{\mathbf{T}}_n - \xi H_j, \qquad \forall i > 1, j | j > i, m \in \mathcal{M}, n \in \mathcal{N}, s \in \mathcal{S}, \tag{3}$$

$$D^{\mathbf{T}}_{ijns} = \theta^{\mathbf{T}}_n \tau_{ijs} + \psi^{\mathbf{T}} \sum_{n \neq n'} P^{\mathbf{T}}_{ijn'} - b^{\mathbf{T}}_n P^{\mathbf{T}}_{ijn} + \gamma^{\mathbf{T}}_n Q^{\mathbf{T}}_n - \xi H_j, \qquad \forall i \in \mathcal{C}, j < \mathcal{C} | j > i, m \in \mathcal{M}, n \in \mathcal{N}, s \in \mathcal{S}. \tag{4}$$

3.2. Airline RM model

As discussed before, a one-leg flight from the first city node to the last city node is considered for the airplane. The traveling time is considered fixed under each scenario. The objective function and equations of the airplane model are as follows:

$$\max \ R^{\mathbf{A}} = \sum_{s \in \mathcal{S}} \rho_s \times \left(\sum_{m \in \mathcal{M}} Sl^{\mathbf{A}}_{ms} P^{\mathbf{A}}_m - \sum_{m \in \mathcal{M}} Sl^{\mathbf{A}}_{ms} \alpha^{\mathbf{A}}_{ms} r^{\mathbf{A}}_m \right) - \sum_{m \in \mathcal{M}} Q^{\mathbf{A}}_m e^{\mathbf{A}}_m. \tag{5}$$

Expression (5) aims at maximizing the total revenue of the airline. In the above equation, quality and refund expenses are subtracted from the profit part

$$Sl^{\mathbf{A}}_{ms}(1 - \alpha^{\mathbf{A}}_{ms}) \leqslant X^{\mathbf{A}}_m, \qquad \forall m \in \mathcal{M}, s \in \mathcal{S}, \tag{6}$$

$$Sl^{\mathbf{A}}_{ms} \leqslant D^{\mathbf{A}}_{ms}, \qquad \forall m \in \mathcal{M}, s \in \mathcal{S}, \tag{7}$$

$$\sum_{m \in \mathcal{M}} X^{\mathbf{A}}_m \leqslant cap^{\mathbf{A}}, \tag{8}$$

$$Q^{\mathbf{A}}_m \leqslant ub^{\mathbf{A}}_m, \qquad \forall m \in \mathcal{M}, \tag{9}$$

$$D^{\mathbf{A}}_{ms} \leqslant \theta^{\mathbf{A}}_m T_s, \qquad \forall m \in \mathcal{M}, s \in \mathcal{S}, \tag{10}$$

$$Q^{\mathbf{A}}_m \geqslant 0, \qquad \forall m \in \mathcal{M}, \tag{11}$$

$$P^{\mathbf{A}}_m \geqslant 0, \qquad \forall m \in \mathcal{M}, \tag{12}$$

$$X^{\mathbf{A}}_m \geqslant 0, \qquad \forall m \in \mathcal{M}, \tag{13}$$

$$D^{\mathbf{A}}_{ms} \geqslant 0, \qquad \forall m \in \mathcal{M}, s \in \mathcal{S}, \tag{14}$$

$$Sl^{\mathbf{A}}_{ms} \geqslant 0, \qquad \forall m \in \mathcal{M}, s \in \mathcal{S}. \tag{15}$$

Because of cancellation, airlines use overbooking to minimize the empty seat. Equation (6) incorporates the overbooking into the model. Moreover, this constraint imposes the flight cancellation rate, which is a key factor influencing transportation operations. Besides, equation (6) ensures that the number of sold tickets does not exceed the booking limits. Equation (7) enforces the condition that the sold tickets must not exceed the demand. The capacity is divided and assigned to each class by the partitioned booking limit. Equation (8) shows that the sum of the booking limits for all the classes must be equal to the capacity. Equation (9) demonstrates the upper bound of the service quality of the airplane. Equation (10) prevents flight demand from exceeding the demand base level. Equations (11)–(15) determine the domain of continuous variables.

3.3. Railway RM model

In contrast to the airline model presented in the last section, the train can stop at intermediate stations. However, stopping stations influence time travel, and it consequently changes the demand. Therefore, in this model, in addition to the prices and booking limits, the stopping stations are also determined. The railway RM model is as follows:

$$\max\ R^{\mathbf{T}} = \sum_{s\in\mathcal{S}} \rho_s \times \left(\sum_{i\in\mathcal{C}} \sum_{\substack{j\in\mathcal{C}\\ j>1}} \sum_{n\in\mathcal{N}} Sl^{\mathbf{T}}_{ijns} P^{\mathbf{T}}_{ijn} - \sum_{i\in\mathcal{C}} \sum_{\substack{j\in\mathcal{C}\\ j>1}} \sum_{n\in\mathcal{N}} Sl^{\mathbf{T}}_{ijns} \alpha^{\mathbf{T}}_{ijns} r^{\mathbf{T}}_{ijn} \right) - \sum_{n\in\mathcal{N}} Q^{\mathbf{T}}_n e^{\mathbf{T}}_n. \tag{16}$$

The expected revenue of railway transportation, as indicated in equation (16), is the total revenue subtracted from the total refunds of the canceled tickets and quality costs.

$$Sl^{\mathbf{T}}_{ijns}\left(1-\alpha^{\mathbf{T}}_{ijns}\right) \leqslant X^{\mathbf{T}}_{ijn}, \qquad \forall i,j\in\mathcal{C}, n\in\mathcal{N}, s\in\mathcal{S}, \tag{17}$$

$$Sl^{\mathbf{T}}_{ijns} \leqslant D^{\mathbf{T}}_{ijns}, \qquad \forall i,j\in\mathcal{C}, n\in\mathcal{N}, s\in\mathcal{S}, \tag{18}$$

$$\sum_{i\in\mathcal{C}} \sum_{n\in\mathcal{N}} Sl^{\mathbf{T}}_{ijns} \leqslant g \times \chi_j, \qquad \forall j\in\mathcal{C}, s\in\mathcal{S}, \tag{19}$$

$$\sum_{n\in\mathcal{N}} \sum_{i=1}^{k-1} \sum_{j=k+1}^{\mathcal{C}} X^{\mathbf{T}}_{ijn} \leqslant cap^{\mathbf{T}}, \qquad \forall k>1, \tag{20}$$

$$H_j = st \times \sum_{\substack{i\\ i<j}}^{\mathcal{C}} \chi_i + t_j, \qquad \forall j>1, \tag{21}$$

$$Q^{\mathbf{T}}_n \leqslant ub^{\mathbf{T}}_n, \qquad \forall n\in\mathcal{N}, \tag{22}$$

$$D^{\mathbf{T}}_{ijns} \leqslant \theta^{\mathbf{T}}_n T_s, \qquad \forall i=1, j=|\mathcal{C}|, n\in\mathcal{N}, s\in\mathcal{S}, \tag{23}$$

$$D^{\mathbf{T}}_{ijns} \leqslant \theta^{\mathbf{T}}_n \tau_{ijs}, \qquad \forall j>i, (i,j)\neq(1,|\mathcal{C}|), n\in\mathcal{N}, s\in\mathcal{S}, \tag{24}$$

$$Q^{\mathbf{T}}_n \geqslant 0, \qquad \forall n\in\mathcal{N}, \tag{25}$$

$$H_j \geqslant 0, \qquad \forall j\in\mathcal{C}, \tag{26}$$

$$P^{\mathbf{T}}_{ijn} \geqslant 0, \qquad \forall i,j\in\mathcal{C}, n\in\mathcal{N}, \tag{27}$$

$$X^{\mathbf{T}}_{ijn} \geqslant 0, \qquad \forall i,j\in\mathcal{C}, n\in\mathcal{N}, \tag{28}$$

$$Sl^{\mathbf{T}}_{ijns} \geqslant 0, \qquad \forall i,j\in\mathcal{C}, n\in\mathcal{N}, s\in\mathcal{S}, \tag{29}$$

$$D^{\mathbf{T}}_{ijns} \geqslant 0, \qquad \forall i,j\in\mathcal{C}, n\in\mathcal{N}, s\in\mathcal{S}, \tag{30}$$

$$\chi_i \in \{0,1\}, \qquad \forall i\in\mathcal{C}. \tag{31}$$

Equation (17) shows the relationship between sold tickets and booking limits under overbooking. In other words, this constraint guarantees that the number of sold tickets does not exceed the booking limits. Equation (18) imposes that sold tickets are always less than or equal to the demand. Equation (19) states the tickets can be sold for all itineraries with destination j if and only if the stop station j is opened. Equation (20) is the capacity constraint on a leg showing that the sum of the booking limits for all O-D pairs using a joint leg cannot exceed the airplane's capacity on that leg. Equation (21) indicates the arrival time to station j. In this constraint, the arrival time is equal to the sum of all stop times in stopping stations before station j and the travel time to station j without any stopping (t_j). Equation (22) demonstrates the upper bound of service quality of the train. Equations (23) and (24) demonstrate the upper bound of train demand for the different itineraries. Equations (25)–(30) determine the domain of continuous variables. Lastly, equation (31) determines the domain of discrete variables.

4. Game theoretical models

This section first presents a competitive game under the Nash game. Then, in the second part, a cooperative game is developed using two different methods, *i.e.*, equal profit method and Shapley value.

4.1. The competitive game

In the discussed competitive transportation system, each system $v \in \{A(\text{airline}), T(\text{railway})\}$, independently, tries to maximize its profit or payoff function, which leads to a non-cooperative transportation RM game. We assume competition between both rivals is under complete information. The non-cooperative game is defined by three components:

The players, who are the transportation systems in the set $\{A, T\}$;

The strategy of each system v, which corresponds to the prices and booking limits and quality vectors and stopping stations, S_v for that $S_{v=\mathbf{A}} = [\mathbf{P}^{\mathbf{A}}, \mathbf{Q}^{\mathbf{A}}, \mathbf{X}^{\mathbf{A}}]$ and $S_{v=\mathbf{T}} = [\mathbf{P}^{\mathbf{T}}, \mathbf{Q}^{\mathbf{T}}, \mathbf{X}^{\mathbf{T}}, \chi^{\mathbf{T}}]$;

The payoff function Π_v of each system v as in equation (5) or (17).

Based on the non-cooperative game, each system tries to select its best response strategy trying to maximize its payoff function, while assuming that the other player's strategies are fixed. For each player v, the best response strategy S_v^* is at least as good as every other strategy in a feasible region when the strategies of the other player, S_{-v} are fixed. For the defined game, the Nash equilibrium is then defined as follows:

Definition 1. Consider the non-cooperative transportation RM game, with payoff function Π_v given by equation (5) or (17). A vector of strategies $\mathcal{S}^*$ is a Nash equilibrium of the above-defined game, if and only if it satisfies the following conditions:

$$\Pi_v\left(S_v^*, S_{-v}^*\right) \geqslant \Pi_v\left(S_v, S_{-v}^*\right), \qquad \forall S_v, v. \tag{32}$$

A Nash equilibrium of the presented game is a position in which no player can increase his/her payoff by changing the prices and qualities while the strategies of the other player are fixed. Based on this definition, Algorithm 1 is proposed to define the Nash equilibrium for the game. It should be mentioned that this Algorithm simultaneously runs both airplane and train models under the Nash equilibrium approach. In the first step, some variables for one of the players are initialized. These variables are announced to the other player. On the other hand, these variables are fixed in the rival's model. The rival's model is solved considering these fixed strategies, and the best strategies of this player are announced to the first player. The first player optimizes his/her payoff with these announced rival's strategies. This procedure is repeated until the last iteration has occurred or the convergence condition is fulfilled. The convergence condition is shown in equation (33).

$$\left|\Pi_{\mathbf{A}}^{\text{iter}} - \Pi_{\mathbf{A}}^{\text{iter}-1}\right| + \left|\Pi_{\mathbf{T}}^{\text{iter}} - \Pi_{\mathbf{T}}^{\text{iter}-1}\right| < \varepsilon. \tag{33}$$

Algorithm 1: Nash equilibrium.

input: The airplane and train models;
begin
initialize $Q_n^{\mathbf{T}}$, H_j, and $P_{ijn}^{\mathbf{T}}$;
while *not converged* **do**
    Solve the airplane model of equations (2), and (5)–(15) using BARON solver
    Obtain the $P_m^{\mathbf{A}}$ as well as $Q_m^{\mathbf{A}}$ and put them as inputs in the train model
    Solve the train model of equations (1), (3), (4), and (16)–(31) using BARON solver
    Obtain new values of $Q_n^{\mathbf{T}}$, H_j, as well as $P_{ijn}^{\mathbf{T}}$ and put them as inputs in the airplane model
end
output: $R^{\mathbf{A}}$, $Sl_{ms}^{\mathbf{A}}$, $D_{ms}^{\mathbf{A}}$, $X_m^{\mathbf{A}}$, $P_m^{\mathbf{A}}$, $Q_m^{\mathbf{A}}$, $R^{\mathbf{T}}$, χ_i, $D_{ijns}^{\mathbf{T}}$, $Sl_{ijns}^{\mathbf{T}}$, $X_{ijn}^{\mathbf{T}}$, $P_{ijn}^{\mathbf{T}}$, H_j, $Q_n^{\mathbf{T}}$

4.2. The cooperative game

In a cooperative situation, players try to form a coalition based on mutual interests instead of competing. In such a case, participants play rationally; in other words, there is a non-zero-sum game between the participants so that each participant makes an effort to optimize its payoff. In a cooperative game, the players' profits are assumed transferable to simplify the assessment of the coalitional payoffs among the players. The fair sharing of the profits between the players can play a vital role in the long-run consistency of the coalition. From the cooperative game theory approaches, Shapley value, τ-value, core center, least core, nucleolus solution, and equal profit method (EPM) can be mentioned [30, 31]. This paper uses both EPM and Shapley value to fulfill the assumption of transferable profit.

4.2.1. Equal profit method

The EPM method consists of some advantages: first of all, it can be developed employing linear programming; second of all, this method tries to dedicate profit among the players as evenly as possible. As shown in equations (34) and (35), both airline and rail transport companies try to maximize their joint revenues. In equation (34), the objective function maximizes the sum of revenues for allied companies.

$$\max\ \mathrm{CR} \tag{34}$$

s.t.

$$\mathrm{CR} = R^{\mathbf{A}} + R^{\mathbf{T}}, \tag{35}$$

$$(1)-(4),\ (6)-(15),\ (17)-(31).$$

The pair (N, c) describes a cooperative game between players, where N denotes the grand coalition (in our case it means the set of n players) and c is the characteristic function of the coalition S. Subsequently, $c(S)$ denotes the corresponding payoff of coalition S, and $c(S)$. The EPM formulations are as follows:

$$\min\ d \tag{36}$$

s.t.

$$d \geqslant y_i/c(\{i\}) - y_j/c(\{j\}), \qquad \forall (i,j) \in N, \tag{37}$$

$$\sum_{j\in N} y_j = c(N), \tag{38}$$

$$\sum_{j\in S} y_j \geqslant c(S), \qquad \forall S \subseteq N. \tag{39}$$

In equation (36), the objective function tries to minimize the difference of the maximum discrepancy between players' relative costs (d), which is calculated by equation (37). In equation (37), y_i denotes the allocated pay off to ith player. Thus, $y_i/c(\{i\})$ denotes the relative pay-off of each player, and $y_i/c(\{i\}) - y_j/c(\{j\})$ indicates the discrepancy in relative pay-offs between players i and j. Equations (38) and (39) determine the stability of the allocations. Equation (38) is indicative of the efficiency by ensuring that the total payoff is dispensed between the participants. Moreover, equation (39) shows the rationality of the jth player. To put it in other words, the players play a rational game, since they do not pay less than their payoff when there is no coalition among them. Thus, according to Frisk *et al.* [32] and Dahlberg *et al.* [33], the solution is in the core using equations (38) and (39).

4.2.2. Shapley value

Shapley value was introduced by Shapley [34] in 1953, which is one of the most widely used profit allocation mechanisms in cooperative game theory [35]. According to the Shapley value, the amount allocated to each player in a coalition is $\phi(V) = \phi_1(V), \phi_2(V), \ldots, \phi_N(V)$; hence, $\phi_i(V)$ is the amount allocated to ith player, which is calculated using equation (40).

$$\phi_i(V) = \sum_{\substack{S\subset N \\ i\in S}} \frac{(|S|-1)!(N-|S|)!}{N!}[V(S) - V(S-\{i\})], \tag{40}$$

where $|S|$ is the cardinality of the coalition S. Moreover, N denotes the set of all players and $V(S)$ is the value of coalition S. Concerning that there are two players, *i.e.*, rail transport company and airline company, in our case, the amount of $\phi_i(V)$ for each player is calculated as shown in equations (41) and (42).

$$\phi_{\mathbf{A}}(V) = \frac{1}{2}[V(\{\mathbf{A},\mathbf{T}\}) - V(\{\mathbf{T}\})] + \frac{1}{2}[V(\{\mathbf{A}\}) - V(\emptyset)], \tag{41}$$

$$\phi_{\mathbf{T}}(V) = \frac{1}{2}[V(\{\mathbf{A},\mathbf{T}\}) - V(\{\mathbf{A}\})] + \frac{1}{2}[V(\{\mathbf{T}\}) - V(\emptyset)], \tag{42}$$

where $\phi_{\mathbf{A}}(V)$ and $\phi_{\mathbf{T}}(V)$ show the amount of allocated share to the airline company and the rail transport company, respectively. Furthermore, $V(\{\mathbf{A},\mathbf{T}\})$ denotes the aggregated benefits obtained when the airline company and the rail transport company collaborate by forming a coalition. In equations (41) and (42), $V(\{\mathbf{A}\})$ and $V(\{\mathbf{T}\})$ are the values generated by the airline company and the rail transport company operating independently, respectively. It should be mentioned that $V(\emptyset) = 0$.

5. Numerical results

In this section, we first present in Section 5.1, a realistic case study designed to validate the modeling and solution approaches. Next, several numerical experiments are generated in Section 5.2 to compare the results. It is worth noting that the developed models are coded and implemented using the GAMS software (version 24.1.2). Given the non-linear nature of the proposed models, BARON solver embedded within the GAMS software is employed to solve them.

5.1. Case study

In this section, we conduct a real-life case study based on the European railway and use real historical data for travel time (t_j) to city node j from node $j-1$. Figure 2 illustrates the locations of train stations within the European railway network. As can be seen, the travel times between two subsequent train stations are apparent in hours. It should be mentioned that the railway track from Bodø, Norway, to Bordeaux, France, is considered the baseline case, serving as the origin and destination, respectively. More specifically, this O-D pair consists of 9 nodes, including Bodø, Trondheim, Oslo, Copenhagen, Hamburg, Amsterdam, Brussels, Paris, and Bordeaux. Hence, in the baseline case, the airplane travels directly from Bodø to Bordeaux, while the train may stop at intermediate stations along the route before reaching Bordeaux. Besides, the parameter values in the numerical experiments are $\gamma_m^{\mathbf{A}} = 0.5$, $\gamma_n^{\mathbf{T}} = 0.5$, $\tau_{ijs} = \text{uniformint}(5, 20)$, $T_s = \text{uniformint}(300, 400)$, $cap^{\mathbf{A}} = 289$, and $cap^{\mathbf{T}} = 392$.

5.2. Comparative analysis and model validation

This section presents multiple numerical experiments designed to compare the performance of companies in competitive and cooperative games on various network scales, that is, small, medium, and large instances.

As can be seen, Table 3 provides information on companies' revenue under cooperation and competition. The most significant insight from the numerical experiments is that companies generate higher revenue when they form a coalition. In fact, findings demonstrate that companies earn higher revenue through collaboration, whether applying the equal profit mechanism or the Shapley value mechanism. Additionally, it was observed that the rail transport company earns more revenue than the airline company in small- and medium-sized instances. However, in large-sized instances, the airline company's revenue is significantly higher than that of the rail transport company. A possible explanation for this result might be the fact that most passengers are

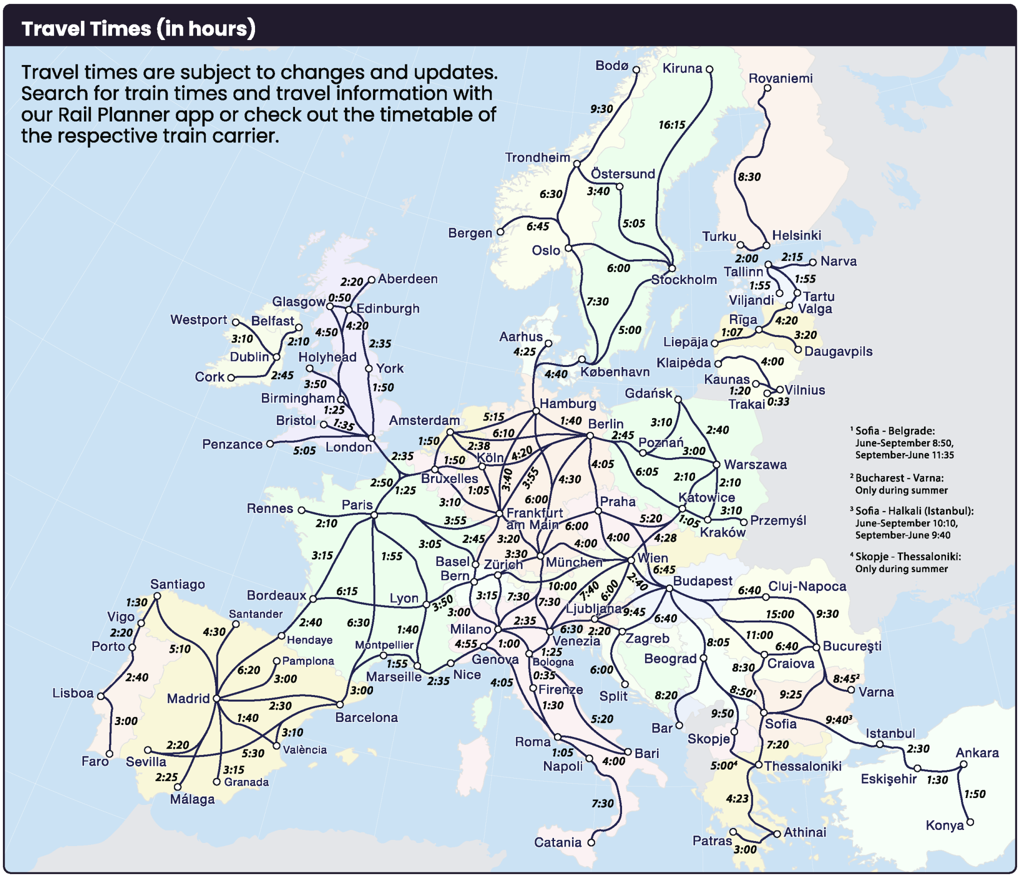

FIGURE 2. European railway map.

unwilling to endure long train journeys. As a consequence, most of them prefer air travel when the distance between origin and destination is considerably long. To summarize, this observation highlights a key difference in passenger preferences and transport efficiency across various travel distances. For small- and medium-sized instances, rail transport's revenue can be attributed to factors such as cost-effectiveness, convenience for short distances, and the availability of frequent services. However, as journey distances increase, the efficiency, travel time, and convenience of flights become more attractive, leading to a significant increase in airline revenue for large-sized instances.

6. Sensitivity analysis

Sensitivity analysis is a way to analyze the system's behavior under various conditions. In this section, several numerical experiments are generated to compare players' behavior. It should be noted that from this point forward, only the equal profit mechanism is considered for the cooperative game.

6.1. Sensitivity analysis on demand parameters of τ_{ijs} and T_s

In this section, additional experiments are conducted to examine how demand parameters affect rail transport and airline companies. Table 4 provides the solutions considering different demand cases. It should be noted that Δ measures the relative difference of revenue, either $R^{\mathbf{A}}$ or $R^{\mathbf{T}}$, with the baseline as a reference. Once

TABLE 3. Results of numerical experiments.

Instances	Size	Sets $\vert\mathcal{C}\vert$	Sets $\vert\mathcal{M}\vert$	Sets $\vert\mathcal{N}\vert$	Game	Method	Revenue: Rail transport company	Revenue: Airline company
No. 1	Small	2	3	2	Competition	Nash equilibrium	991 941.8	447 369.1
					Cooperation	Shapley value	992 882.0	448 309.3
					Cooperation	Equal profit method	993 237.7	447 953.5
No. 2	Small	4	3	2	Competition	Nash equilibrium	919 644.1	435 830.5
					Cooperation	Shapley value	973 004.1	489 190.5
					Cooperation	Equal profit method	992 049.9	470 144.5
No. 3	Medium	9	3	2	Competition	Nash equilibrium	1 503 655.3	452 116.8
					Cooperation	Shapley value	1 507 193.9	455 655.4
					Cooperation	Equal profit method	1 509 096.4	453 752.8
No. 4	Medium	11	3	2	Competition	Nash equilibrium	1 532 770.5	443 093.8
					Cooperation	Shapley value	1 537 910.0	448 233.3
					Cooperation	Equal profit method	1 540 744.3	445 398.9
No. 5	Large	17	3	2	Competition	Nash equilibrium	64 966.9	407 543.9
					Cooperation	Shapley value	1 208 121.8	1 550 698.8
					Cooperation	Equal profit method	379 318.6	2 379 502.1
No. 6	Large	21	3	2	Competition	Nash equilibrium	42 958.2	383 955.9
					Cooperation	Shapley value	1 570 212.0	1 911 209.7
					Cooperation	Equal profit method	350 317.7	3 131 103.8

TABLE 4. Amounts of revenue for rail transport and airline companies under various demand cases.

Case	T_s	τ_{ijs}	Game	Rail transport company $R^{\mathbf{T}}$	Rail transport company Δ	Airline company $R^{\mathbf{A}}$	Airline company Δ
Case expansion 1	$(250, 320)$	$(3, 8)$	Competitive	932 009.9	−38.01%	358 551.8	−20.6%
			Cooperative	932 679.5	−38.1%	358 809.4	−20.9%
Baseline	$(320, 450)$	$(10, 30)$	Competitive	1 503 655.3	–	452 116.8	–
			Cooperative	1 509 096.4	–	453 752.8	–
Case expansion 2	$(450, 550)$	$(40, 70)$	Competitive	2 164 422.3	43.9%	519 913.6	14.9%
			Cooperative	2 200 009.7	45.7%	528 462.1	16.4%

again, our findings strongly confirm that both airline and rail transport companies obtain more revenue as the demand increases. In particular, the rail transport company experiences a steeper revenue increase with increasing demand, whereas the airline company revenue grows at a more gradual rate. The initial cause of this trend is reflected in Tables 5 and 6. It is worth noting that in Table 5, the variable values of $P_{ijn}^{\mathbf{T}}$ and $X_{ijn}^{\mathbf{T}}$ have been reported exclusively for journeys that travel directly from the origin to the destination. As shown, airline and rail transport companies agree on a more expensive ticket price by making a coalition. Also, in most of the cases, an increase in demand causes a reduction in ticket prices. In addition, quality seems to be unaffected by variations in demand. Last but not least, the total number of allocated seats in both the train and the airplane proved to be unrelated to demand behavior.

TABLE 5. Amounts of $Q_n^{\mathbf{T}}$, $P_{ijn}^{\mathbf{T}}$, and $X_{ijn}^{\mathbf{T}}$ for the rail transport company under various demand cases.

Case	T_s	τ_{ijs}	Class	Competition			Cooperation		
				$Q_n^{\mathbf{T}}$	$P_{ijn}^{\mathbf{T}}$	$X_{ijn}^{\mathbf{T}}$	$Q_n^{\mathbf{T}}$	$P_{ijn}^{\mathbf{T}}$	$X_{ijn}^{\mathbf{T}}$
Case expansion 1	$(250, 320)$	$(3, 8)$	1st	841.7	2811.4	120	841.7	2813.4	120
			2nd	953.4	3049.6	43	953.4	3051.7	43
Baseline	$(320, 450)$	$(10, 30)$	1st	841.7	2687.7	150	841.7	2689.6	157
			2nd	953.4	2962.9	39	953.4	2964.8	39
Case expansion 2	$(450, 550)$	$(40, 70)$	1st	841.7	2466.8	151	841.7	2636.0	151
			2nd	953.4	2805.8	42	953.4	2975.0	42

TABLE 6. Amounts of $Q_m^{\mathbf{A}}$, $P_m^{\mathbf{A}}$, and $X_m^{\mathbf{A}}$ for the airline company under various demand cases.

Case	T_s	τ_{ijs}	Class	Competition			Cooperation		
				$Q_m^{\mathbf{A}}$	$P_m^{\mathbf{A}}$	$X_m^{\mathbf{A}}$	$Q_m^{\mathbf{A}}$	$P_m^{\mathbf{A}}$	$X_m^{\mathbf{A}}$
Case expansion 1	$(250, 320)$	$(3, 8)$	1st	936.9	1399.1	127	936.9	1400.5	158
			2nd	883.4	1396.4	81	883.4	1397.8	81
			3rd	979.8	1518.0	79	979.8	1519.3	49
Baseline	$(320, 450)$	$(10, 30)$	1st	936.9	1341.5	155	936.9	1343.7	155
			2nd	883.4	1353.6	82	883.4	1354.3	83
			3rd	979.8	1482.5	50	979.8	1483.3	50
Case expansion 2	$(450, 550)$	$(40, 70)$	1st	936.9	1237.1	126	936.9	1444.9	132
			2nd	883.4	1274.7	96	883.4	1367.7	88
			3rd	979.8	1416.4	65	979.8	1453.6	68

6.2. Sensitivity analysis on the price elasticity of demand

In this section, additional experiments are conducted to examine the effects of price elasticity on the revenue of transport and airline companies. Hence, three values of 2, 1, and 0.5 are considered for the ratio of $b_m^{\mathbf{A}}$ to $b_n^{\mathbf{T}}$. Analyzing the price elasticity of the demand shows that making a coalition is also a more beneficial approach for companies compared to a competitive setting. Table 7 shows companies' revenue under different cases. As can be seen, the airline company's revenue increases as the ratio of $\frac{b_m^{\mathbf{A}}}{b_n^{\mathbf{T}}}$ decreases. In other words, the airline company will gain a higher profit if the price elasticity of train demand is greater than that of the flight demand ($b_n^{\mathbf{T}} > b_m^{\mathbf{A}}$). On the contrary, whenever the flight demand is more sensitive to price changes compared to the train demand, the rail transportation company can achieve higher revenue. As reported, considering the second case, where the ratio of $\frac{b_m^{\mathbf{A}}}{b_n^{\mathbf{T}}}$ is equal to 1, the revenue of rail transportation company reaches its apex as both companies build a coalition. In general, current results show that the price elasticity of demand has a remarkable impact on the revenue of companies.

Tables 8 and 9 indicate the amounts of booking limit, ticket price, and quality for rail transport and airline companies, respectively. As reported in Table 8, the rail transport company offers higher-quality services by forming a coalition with the airline company. Consequently, train seats are sold at higher prices when the two companies collaborate. Furthermore, findings suggest that the rail transport company increases the ticket price to the maximum value as $\frac{b_m^{\mathbf{A}}}{b_n^{\mathbf{T}}}$ is equal to 2. As can be seen in Table 9, the flight tickets are sold at higher prices when $b_m^{\mathbf{A}}$ is equal to 0.4, *i.e.*, the second and the third cases. Moreover, findings reveal that the flight ticket

TABLE 7. Companies' revenue under different price elasticity cases.

Case	$b_m^{\mathbf{A}}$	$b_n^{\mathbf{T}}$	Ratio of $\frac{b_m^{\mathbf{A}}}{b_n^{\mathbf{T}}}$	Game	Rail transport company		Airline company	
					$R^{\mathbf{T}}$	Δ	$R^{\mathbf{A}}$	Δ
Case expansion 1	0.8	0.4	2	Competitive	1 873 382.3	23.9%	270 991.2	−20.2%
				Cooperative	1 898 169.1	−5.3%	274 576.7	−40.6%
Baseline	0.4	0.4	1	Competitive	1 472 228.9	–	339 684.3	–
				Cooperative	2 005 134.9	–	462 640.6	–
Case expansion 2	0.4	0.8	0.5	Competitive	907 216.3	−38.3%	572 396.7	68.5%
				Cooperative	926 151.5	−53.8%	584 343.6	26.3%

TABLE 8. Amounts of $Q_n^{\mathbf{T}}$, $P_{ijn}^{\mathbf{T}}$, and $X_{ijn}^{\mathbf{T}}$ under different price elasticity cases.

Case	$b_m^{\mathbf{A}}$	$b_n^{\mathbf{T}}$	Ratio of $\frac{b_m^{\mathbf{A}}}{b_n^{\mathbf{T}}}$	Class	Competition			Cooperation		
					$Q_n^{\mathbf{T}}$	$P_{ijn}^{\mathbf{T}}$	$X_{ijn}^{\mathbf{T}}$	$Q_n^{\mathbf{T}}$	$P_{ijn}^{\mathbf{T}}$	$X_{ijn}^{\mathbf{T}}$
Case expansion 1	0.8	0.4	2	1st	841.7	3347.0	157	841.7	3460.0	157
				2nd	907.8	3633.8	39	953.4	3803.8	39
Baseline	0.4	0.4	1	1st	841.7	2360.4	157	841.7	3368.4	157
				2nd	737.4	2434.3	39	953.4	3712.2	39
Case expansion 2	0.4	0.8	0.5	1st	784.2	1646.9	155	841.7	1680.2	157
				2nd	953.4	1851.9	39	953.4	1852.3	39

TABLE 9. Amounts of $Q_m^{\mathbf{A}}$, $P_m^{\mathbf{A}}$, and $X_m^{\mathbf{A}}$ under different price elasticity cases.

Case	$b_m^{\mathbf{A}}$	$b_n^{\mathbf{T}}$	Ratio of $\frac{b_m^{\mathbf{A}}}{b_n^{\mathbf{T}}}$	Class	Competition			Cooperation		
					$Q_m^{\mathbf{A}}$	$P_m^{\mathbf{A}}$	$X_m^{\mathbf{A}}$	$Q_m^{\mathbf{A}}$	$P_m^{\mathbf{A}}$	$X_m^{\mathbf{A}}$
Case expansion 1	0.8	0.4	2	1st	936.9	833.6	155	936.9	906.1	125
				2nd	883.4	841.2	82	883.4	867.4	97
				3rd	979.8	921.8	50	979.8	935.7	66
Baseline	0.4	0.4	1	1st	936.9	1011.2	155	936.9	1682.7	155
				2nd	883.4	1026.3	83	883.4	1696.0	83
				3rd	979.8	1187.4	50	979.8	1857.1	50
Case expansion 2	0.4	0.8	0.5	1st	936.9	1678.8	155	936.9	1679.4	155
				2nd	883.4	1693.8	82	883.4	1694.4	82
				3rd	979.8	1854.9	50	979.8	1855.5	50

price reaches its maximum value when $b_m^{\mathbf{A}}$ is equal to $b_n^{\mathbf{T}}$. That is why the airline company gain more revenue when the ratio of $\frac{b_m^{\mathbf{A}}}{b_n^{\mathbf{T}}}$ is less than 2. To put it another way, the airline company sets its prices at the lowest levels when the price elasticity of flight demand exceeds that of train demand.

6.3. Sensitivity analysis on quality elasticity

In this section, additional experiments are carried out to examine the effects of quality elasticity on the revenue of transport and airline companies. We used three cases to discover the role of quality elasticity. From

TABLE 10. Companies' revenue under different quality elasticity cases.

Case	$\gamma_m^{\mathbf{A}}$	$\gamma_n^{\mathbf{T}}$	Ratio of $\frac{\gamma_m^{\mathbf{A}}}{\gamma_n^{\mathbf{T}}}$	Game	Rail transport company		Airline company	
					$R^{\mathbf{T}}$	Δ	$R^{\mathbf{A}}$	Δ
Case expansion 1	0.8	0.4	2	Competitive	2 163 683.5	49.7%	888 427.5	111.5%
				Cooperative	2 186 303.5	48.0%	897 715.5	109.2%
Baseline	0.4	0.4	1	Competitive	1 444 977.1	–	419 988.4	–
				Cooperative	1 476 259.1	–	429 080.7	–
Case expansion 2	0.4	0.8	0.5	Competitive	2 215 756.8	53.3%	511 908.1	21.8%
				Cooperative	2 425 751.3	64.3%	560 423.2	30.6%

TABLE 11. Amounts of $Q_n^{\mathbf{T}}$, $P_{ijn}^{\mathbf{T}}$, and $X_{ijn}^{\mathbf{T}}$ under different quality elasticity cases.

Case	$\gamma_m^{\mathbf{A}}$	$\gamma_n^{\mathbf{T}}$	Ratio of $\frac{\gamma_m^{\mathbf{A}}}{\gamma_n^{\mathbf{T}}}$	Class	Competition			Cooperation		
					$Q_n^{\mathbf{T}}$	$P_{ijn}^{\mathbf{T}}$	$X_{ijn}^{\mathbf{T}}$	$Q_n^{\mathbf{T}}$	$P_{ijn}^{\mathbf{T}}$	$X_{ijn}^{\mathbf{T}}$
Case expansion 1	0.8	0.4	2	1st	841.7	4518.8	157	841.7	4573.0	157
				2nd	953.4	4834.7	54	953.4	4888.9	54
Baseline	0.4	0.4	1	1st	841.7	2495.0	157	841.7	2615.6	157
				2nd	953.4	2810.9	39	953.4	2931.6	39
Case expansion 2	0.4	0.8	0.5	1st	841.7	3734.7	108	841.7	3799.0	153
				2nd	767.3	3790.8	39	953.4	4226.3	39

the first case to the third case, the considered ratios of $\frac{\gamma_m^{\mathbf{A}}}{\gamma_n^{\mathbf{T}}}$ are 2, 1, and 0.5. Once again, Table 10 supports our previous result that both companies gain more revenue whenever they play in a cooperative setting, rather than a competitive one. Besides, findings show that the rail transport company gains more revenue as $\gamma_n^{\mathbf{T}}$ is higher than $\gamma_m^{\mathbf{A}}$. On the other hand, the revenue of the airline company reaches the maximum value when $\gamma_m^{\mathbf{A}}$ is higher than $\gamma_n^{\mathbf{T}}$. Interestingly, when the sensitivity of passengers' decisions to quality changes in airplane and train classes is at the same level, that is, $\frac{\gamma_m^{\mathbf{A}}}{\gamma_n^{\mathbf{T}}} = 1$, the revenue of both companies decreases.

Tables 11 and 12 provide information on the amounts of booking limit, ticket price, and quality for rail transport and airline companies, respectively. As reported in Table 11, the rail transport company allocates more seats when $\frac{\gamma_m^{\mathbf{A}}}{\gamma_n^{\mathbf{T}}}$ is equal to 2. Moreover, as mentioned earlier, the revenue of the airline company reaches the maximum level when $\gamma_m^{\mathbf{A}}$ is higher than $\gamma_n^{\mathbf{T}}$. In fact, one reason why the revenue of the airline company reaches the maximum level when $\frac{\gamma_m^{\mathbf{A}}}{\gamma_n^{\mathbf{T}}} = 2$ is that the airline company increases ticket prices. Consequently, the rail transport company sells tickets at higher prices. However, when the quality elasticity of demand for both the airplane and the train is at the same level, that is, $\frac{\gamma_m^{\mathbf{A}}}{\gamma_n^{\mathbf{T}}} = 1$, both companies sell tickets at lower prices regardless of the game type; therefore, their revenues reach the minimum level compared to other cases.

7. Conclusions and outlooks

In terms of novel contribution, this paper is the first effort to study the influences of competitive and cooperative games on airline and rail transport companies' performances under passengers' choice behavior. To that end, two mixed-integer nonlinear programming models were developed in a manner that the models were connected *via* their demand constraints. Thus, passengers choose whether to board an airplane or a train to travel from the first city node to the last one, which is the most critical O-D pair. This behavior is incorporated into the

TABLE 12. Amounts of $Q_m^{\mathbf{A}}$, $P_m^{\mathbf{A}}$, and $X_m^{\mathbf{A}}$ under different quality elasticity cases.

Case	$\gamma_m^{\mathbf{A}}$	$\gamma_n^{\mathbf{T}}$	Ratio of $\frac{\gamma_m^{\mathbf{A}}}{\gamma_n^{\mathbf{T}}}$	Class	Competition			Cooperation		
					$Q_m^{\mathbf{A}}$	$P_m^{\mathbf{A}}$	$X_m^{\mathbf{A}}$	$Q_m^{\mathbf{A}}$	$P_m^{\mathbf{A}}$	$X_m^{\mathbf{A}}$
Case expansion 1	0.8	0.4	2	1st	936.9	2590.9	155	936.9	2626.9	155
				2nd	883.4	2566.0	82	883.4	2602.0	82
				3rd	979.8	2799.1	50	979.8	2835.2	50
Baseline	0.4	0.4	1	1st	936.9	1239.7	155	936.9	1375.7	130
				2nd	883.4	1268.1	83	883.4	1320.5	94
				3rd	979.8	1405.2	50	979.8	1457.5	64
Case expansion 2	0.4	0.8	0.5	1st	936.9	1503.4	155	936.9	1558.2	149
				2nd	883.4	1531.8	83	883.4	1567.6	83
				3rd	979.8	1668.8	50	979.8	1704.6	56

proposed models by the linear modeling approach. In our study, passengers decide to travel either by train or airplane based on various factors, such as ticket price, arrival time, and service quality. Hence, both airline and rail transport companies try to attract passengers to increase their revenue. It should be noted that the train is the only choice for other O-D pairs. A Nash equilibrium was utilized to define a competitive game among airline and rail transport companies. On the other hand, to fairly divide revenues among the companies, the EPM and the Shapley value were used. We performed several cases to show the high applicability of our developed models in real-world problems. We also performed some analyses to study the companies' behavior under different conditions. We found out that revenue for both airline and rail transport companies grows as demand increases, regardless of the game type. Furthermore, our findings strongly confirm that cooperation has an advantage over competition. In other words, it turned out that both companies can make more revenue by building a coalition, rather than playing in a competitive setting. As a matter of fact, both companies can increase their revenue by establishing a profit-sharing agreement. To be more specific, such an agreement ensures that each participating company involved has a stake in the business's success. Real-world coalitions often require companies to agree on pricing strategies, such as how to set ticket prices or share revenue from jointly offered services. These negotiations can be challenging due to differing goals or market positioning. Our observations show that airline and rail transport companies tend to agree on higher prices when forming a coalition. This alignment of interests to raise ticket prices explains the resulting boost in their revenues. Also, results indicate that passengers may find long train journeys less convenient than faster options like flights, leading to higher airline revenue in large-size instances. It was shown that price elasticity is inversely related to revenue; in contrast, quality elasticity has a direct relation with revenue.

Nevertheless, this study has some limitations. These limitations can be the key component in future attempts to investigate new directions of RM. From reviewing the literature on passengers' choice behavior, it is found that most studies have neglected temporal influences in individual decision-making [36]; hence, dynamic choice modeling, dynamic service quality, and dynamic pricing can be used in future studies. Moreover, future research can consider some new attributes, such as giving a discount to loyal customers, which would be appealing and crucial in the RM field. Plus, examining common disruptive events like delays and external shocks could be an interesting avenue for future research. Additionally, sustainability concerns and multimodal transportation integration could serve as an appealing direction for future studies.

DATA AVAILABILITY STATEMENT

No new data/codes were created or analyzed in this study.

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