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THE RETAILER'S INFORMATION SHARING DECISIONS WITH MANUFACTURER'S ENCROACHMENT AND PRODUCTION LEARNING EFFECTS

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Abstract. If the manufacturer can efficiently reduce costs and boost overall profits throughout the supply chain, he can send some incremental surplus to the retailer to motivate him to share his precise demand information. We study a two-period dual-channel supply chain where a manufacturer sells through a retailer with an advantage in demand information. The manufacturer can encroach and use production learning effects to reduce costs. Based on six scenarios under the "encroachment-dominated and production-learning effect-dominated and dual-factor" approaches, we investigated how the manufacturer's encroachment and the production-learning effect affected the retailer's decisions to share information. Some interesting results are obtained. First, a single-win, win–win, or lose–lose situation is generated for the supply chain participants by the dynamic competition mechanism under the combined effects of encroachment and the production learning effect. When more ambiguous demand information is cooperatively provided, both the retailer and manufacturer will benefit from information sharing. The win–win area increases with higher direct selling costs, whereas the win–win area decreases with increased production learning effects. Second, the retailer's information sharing benefits more from a higher level of the production learning effect when demand information is unclear, and information sharing benefits more from a lower level of the production learning effect when demand information is more accurate. In addition, in terms of profitability following information sharing, we ultimately find that implementing both cost cutting and channel expansion based on unilateral payment contracts is the best incentive strategy to encourage retailers to share information. Finally, we obtain some management insights that support supply chain parties in making decisions regarding the resolution of channel conflicts and the realization of information cooperation.

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1. Introduction

The growth of e-commerce has made it easier for manufacturers to open up their direct sales channels and compete with conventional retailers. This has given them more flexibility and a competitive edge in a dual-channel strategy. It is common to refer to this competition from manufacturers expanding their direct sales

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channels as "manufacturer encroachment". However, manufacturers cannot always increase profits by expanding their markets since there is not enough information about downstream sales. Consequently, manufacturers have started aggressively collaborating with retailers on information sharing, using the retailers' informational edge to forecast market demand information and improve the efficiency of their direct sales channels. Retailers can typically get more accurate first-hand data faster during sales because they are closer to the consumer market and downstream in the supply chain. As e-commerce grows, many conventional retailers are expanding their online businesses and utilizing big data analytics and online platforms to obtain inexpensive access to various market dynamics data to forecast target market demand more precisely [1].

However, there are barriers to manufacturers and retailers sharing information. The profitability of retailers is directly threatened by manufacturers encroaching on resale channels through the opening of direct sales channels. Manufacturers with linear production costs may generate more significant incremental surpluses due to information sharing, but they cannot compensate for retailer losses [2]. Consequently, retailers will not provide manufacturers with access to their demand information. However, when a manufacturer can reduce costs, accounting for the manufacturer's self-interested cost-cutting efforts will boost retailer profits through spillover effects [3]. If the manufacturer can successfully reduce costs and increase incremental profits across the supply chain, they might use the incremental surplus to persuade retailers to provide demand information. If retailers receive a substantial profit subsidy from the manufacturer, they share demand information, resulting in a win–win situation [4].

The "production learning effect" phenomenon enables firms to attain higher profit margins and lower unit production costs by accumulating experience *via* larger outputs throughout recurrent manufacturing and production procedures gradually. Utilizing this production learning effect, in reality, has allowed manufacturers in a variety of sectors, including clothes, electronics, and automobiles, to reduce costs and optimize their production operation plans in real-time. However, most of the literature currently in publication on cost reduction [5–8] concentrates on cost learning outcomes based on a single period, ignoring the reality that the actual cost reduction process frequently involves multiple production periods. Additionally, the impact of the dynamic process of realizing reductions and cost learning on the equilibrium decision and equilibrium profit in the pre-and post-sales periods is not considered. Most research on cost reduction also tends to concentrate on the manufacturer, with few viewpoints examining the creation of channels for information-sharing and cooperation with retailers.

While much research has been done on the relationship between cost reduction and manufacturer encroachment, as well as the effects of encroachment and information sharing on supply chain performance, little is known about how the dynamic process of manufacturer encroachment and two-period production cost reduction influences retailer decision-making when it comes to information sharing and incentive schemes. In the actual world, resolving this issue can help coordinate supply chain cooperation and enhance supply chain performance since it is conducive to reducing channel disputes and attaining information cooperation. We specifically seek to respond to the following research inquiries.

(1) What are the retailer's optimal decisions regarding information sharing in the circumstances of encroachment dominating, production-learning effect dominating, and two factors of encroachment and production-learning effect, respectively?
(2) How do the supply chain parties' equilibrium decisions, equilibrium profits, and information-sharing values alter during the two periods? Is there a dynamic influencing mechanism, and if so, what is it?
(3) What is the impact of production learning effects, demand information accuracy, and direct marketing costs on the value of information sharing and the retailer's decision to share?
(4) How can incentives be chosen and implemented for a manufacturer who may encroach and implement production learning under small-bach or mass-production business strategies to encourage the retailer to share information to maximize the profitability of all supply chain members?

Based on the dimensions of manufacturer encroachment and production learning effect, this paper constructs a two-period dual-channel supply chain model with both cost uncertainty and demand uncertainty to answer the above questions. The manufacturer sells *via* a retailer who benefits from demand information advantages and can both encroach and use production learning effects to reduce costs. We use a stochastic variable to capture uncertain demand information through Bayesian updating, and the manufacturer's production costs in the second period can drop linearly with the product output in the first period at a stochastic learning rate. The manufacturer uses a dynamic pricing strategy in both periods, and the retailer chooses whether or not to share the demand information. Drawing on the perspectives of "encroachment dominance, production learning effects dominance, and two-factor", we go on to examine the equilibrium and optimal strategy for information sharing in six scenarios. Some interesting results are obtained. First, the retailer in encroachment-led and production-learning effect-led situations will not exchange demand information. Concerning the learning and realization periods of production learning, the dynamic competing mechanism under the two-factor encroachment and production learning effect causes the retailer and manufacturer to always be impacted by the inverse of encroachment in both periods, resulting in a win–win, single-win, or lose–lose state following information sharing. When more vague demand information is cooperatively provided, both the manufacturer and the retailer will benefit. The win–win area increases with higher direct selling costs, whereas the win–win area decreases with increased production learning effects. Second, the retailer's information sharing benefits more from a higher level of the production learning effect when demand information is unclear, and information sharing benefits more from a lower level of the production learning effect when demand information is more accurate. In terms of profitability following information sharing, we ultimately discover that a unilateral payment contract renders the single cost reduction strategy superior to the single channel expansion strategy. The best incentive strategy to encourage retailer information sharing is to simultaneously implement the dual strategy of cost reduction and channel expansion, which supports supply chain parties in making decisions regarding the resolution of channel conflicts and the realization of information cooperation. These findings provide further theoretical and practical guidance for information sharing decisions and incentives.

The following is the structure of the remaining portion of the paper: Section 2 reviews pertinent literature in the field of study; Section 3 presents the primary model and basic assumptions; Section 4 derives equilibrium decisions and optimal profits in the encroachment-dominated, production-learning-dominated, and dual-factor scenarios; Section 5 examines the production learning effect and economies of scale under the dual-factor scenario; In Section 6, the optimal retailer's decisions regarding information sharing are examined; Section 7 discusses payment contracts and strategic choices that encourage information sharing around the profitability of the supply chain members following information sharing; Section 8 offers specific and practical managerial insights to supply chain members; Section 9 provides an overview of the main findings and discusses the limitations and further research.

2. Literature review

The extant literature has also thoroughly examined the effects of manufacturer encroachment. The conventional wisdom holds that manufacturer encroachment can reduce retailers' earnings by cannibalizing the market and intensifying channel competition and conflict [9,10]. From the standpoints of endogenous quality [11], direct sales efficiency [12], retailer-dominated supply chains [13], product durability [14], or alternative green products [15], some academics have also confirmed that retailers may suffer from manufacturer encroachment. Nonetheless, some research indicates that contrary to the conventional wisdom stated above, manufacturer encroachment may not always be harmful to retailers if it can result in Pareto improvements for the supply chain. For instance, when examining the sales efficiency of retail operations, Arya *et al.* [16] discovered that supplier encroachment results in a drop in wholesale prices, which lessens the double marginal effect and boosts retailer profitability. Zhang *et al.* [10] examine the retailer shop branding introduction strategy, and their research indicates that the use of retailer shop branding can serve as a helpful countermeasure against annihilation. Zhang *et al.* [13] conclude that retail service investments may benefit retailer-dominated supply chains Pareto-wise if the direct sales costs are confidential information to suppliers. Regarding investment and spillovers, According to Yoon [3], the retailer's earnings rise due to the manufacturer's self-serving cost-cutting initiatives since the manufacturer can reduce costs. The majority of the literature currently available in this field applies a game modeling approach, which can be valuable in offering fresh viewpoints on solutions and research. In recent research, Goli *et al.* are enhancing supply chain management through sustainability, efficiency, and technological integration. Tirkolaee *et al.* [17] have improved closed-loop networks with algorithms like MOGWO and NSGA-II, reducing supply chain management costs and risks. In the healthcare sector, new models are addressing uncertainties in organ transplants and balancing patient satisfaction with cost considerations [18]. Energy efficiency in production is also progressing, with optimizers like MOALO offering improved solutions for scheduling and lot-sizing [19]. The dairy industry is embracing closed-loop systems for optimizing financial and physical flows, with the ABD algorithm and methods such as WSM and FMOP at the forefront [20]. Blockchain technology is increasingly important for secure and transparent transactions in international trade and supply chain management [21]. Software-defined networking (SDN) is also being explored for network management, with AI being used to tackle challenges like load imbalance [22]. The Internet of Things (IoT), combined with cloud and fog computing, is transforming data processing in supply chain management, with algorithms like HHO outperforming traditional methods [23]. These studies provide a more profound understanding of supply chain management, highlighting the role of technology in enhancing sustainability and efficiency and paving the way for future innovations with AI and blockchain.

Most of the literature concentrates on the manufacturer's encroachment in information-symmetric structures. However, in practice, the retailer possesses demand information, and the downstream manufacturer frequently encounters asymmetric market demand conditions. The research theme of information sharing under the encroachment scenario has resulted from this. A growing number of academics have been looking at the demand information sharing issue that retailers face from the standpoint of online platforms in recent years. According to research by Ha *et al.* [24], information-sharing and encroachment decisions are complementary in analyzing the strategic interactions between manufacturers and online platforms. Zheng *et al.* [25] investigate how suppliers' decisions about channel establishment interact with online retailers' decisions about information sharing in the framework of the platform economy. In uncertain demand, Tsunoda and Zennyo [26] investigate how platform information-sharing policies impact third-party suppliers' online and offline multichannel management. Lin *et al.* [27] investigate the interplay between the intermediary's information-sharing decisions and the manufacturer's preferred selling format (RS or AS) within an e-commerce supply chain consisting of an online intermediary, a manufacturer, and an online reseller. Most researchers discovered that the retailer's information-sharing could benefit retailers and manufacturers in an encroachment scenario in risk aversion [28], product quality [29], and direct selling costs [30]. A few academics also investigate the rewards associated with sharing information. Chen *et al.* [31] analyze agency contracts and resale incentives for platform information sharing. Li and Zhang [32] suggest implementing a wholesale price cap mechanism to encourage information sharing between comparatively weak retailers and strong manufacturers. Chen *et al.* [29] provide a unilateral payment incentive method to examine the reasons behind vertical information sharing. Ha *et al.* [33] studied the incentive mechanism for exchanging vertical information in competitive supply chains with manufacturing technology scale diseconomies.

This paper closely relates to one perspective on how costs influence the retailer's information-sharing decisions. A fixed entry cost may encourage retailers to give suppliers access to confidential demand information, according to research by Huang *et al.* [34]. Research indicates that the prevalence of information sharing increases with high entry costs and high channel substitution rates. According to Ha *et al.* [4], the manufacturer might provide a portion of the excess to encourage the retailer to share, creating a win–win situation if he can successfully cut costs and increase incremental profits in the supply chain.

The aforementioned research on retailers' demand for information sharing also advances the study of volume-related cost reductions. The production learning effect, a popular method of volume-based cost reduction that has also been referred to as a learning curve, experience curve, or learning-by-doing, is closely related to our research. The economic phenomenon of achieving cost reductions as production accumulates has been found in numerous studies to have significant implications for performance growth and production operation optimization in areas like financial investment, competitive interactions, market structure and trade policy, and production and inventory optimization [5]. According to the model's dimensions, learning curves are further divided into four categories: stochastic, hyperbolic, exponential, and linear [35]. The impact of cost reduction on price strategy and supply chain performance [8], operational efficiency [36], production and sourcing optimization [37], competitive interactions [38], and channel decisions [5] have all been studied extensively in recent years using multi-period linear learning curves.

The work reviewed above primarily addresses manufacturer encroachment, retailers' information sharing, and production learning effects. There is a dearth of literature that examines the dynamic influence of manufacturers' encroachment and production learning effects on retailers' demand for information sharing. Sun *et al.* [5, 6] research on the signaling game problem with a two-point distribution of market sizes under an asymmetric information structure and their discussion of the relationship between cost reduction and manufacturer encroachment is closely related to this paper. Retailers can only profit when direct selling costs are high enough, they conclude, whereas suppliers may benefit from supply chain invasion when cost reduction efficiency or direct selling costs are low. They do not consider the possibility that retailers and manufacturers may have had information-sharing mechanisms at the start, simply the scenario in which information is conveyed through signaling. While this paper's quantity-based cost reduction model is similar to that investigated by Sun *et al.* [5], their cost reduction only operates on a single period and concentrates solely on the impact of the cost reduction outcome, ignoring the reality that the cost reduction process frequently involves multiple production periods. Thus, investigating the process of dynamic cost reduction and devising a mechanism for information sharing holds significant importance. While looking at two periods of cost learning, some of the papers are less concerned with information sharing and encroachment. Shum *et al.* [8] mainly focused on pricing strategies under the production learning effect. While Gupta *et al.* [39] explored the interplay between manufacturing costs, cost learning, and encroachment, they did not discuss the role of information sharing in supply chain management. In the discussion of unilateral information sharing, Guan *et al.* [30] focused on cost information sharing by manufacturers, while Lin *et al.* [27] primarily considered demand information sharing by online intermediaries. In contrast, we emphasize the more universally relevant decision-making on information sharing by traditional retailers. It is still unclear how that mitigates channel conflict and achieves information cooperation by combining manufacturer encroachment, production learning effects, and retailer information sharing into the same framework. In three cost scenarios – no production costs, production diseconomy, or production economy – Zhao *et al.* [7] examined the effects of manufacturer encroachment and nonlinear production costs on retailers' information-sharing decisions. They discovered that sharing information benefits the manufacturer or supply chain when the production economy is relatively small. Information-sharing incentives are not discussed much, despite their research on information-sharing decision-making.

The aforementioned articles are not like our paper in a few ways. To highlight the significance of this paper, we compared the papers that are closely linked to it and summarized the similarities and differences between them, as indicated in Table 1. Specifically, we model the two-period production learning effect and examine its dynamic impact with manufacturer encroachment on the retailer's information-sharing decision. First, we consider a crucial factor the aforementioned articles have ignored: the multi-period dynamic cost reduction process. Based on the dynamic competing mechanisms of encroachment and production learning effects, it is interesting to note that, in the case of ambiguous demand information, the positive effect of the production learning period reverses the negative effect of intrusion on retailers. As a result, information sharing benefits both the manufacturer and the retailer. The win–win area increases with higher direct selling costs, whereas the win–win area decreases with increased production learning effects. Second, we employ random variables to capture uncertain demand information through Bayesian updating in this paper, in contrast to the two-point distributed signaling game approach used by Sun *et al.* [5] and Zhang and Zhang [35]. As a result, we can better evaluate the influence of information accuracy on the retailer's decisions to share information. Third, in the case of a production economy, we primarily consider the impact of linear cost reductions of two-period dynamics, not only for information-sharing decisions but also for information-sharing incentives. Zhao *et al.* [7] compared information-sharing decisions under three different economic scenarios based on nonlinear production costs. On the other hand, our results take into account how the interaction of production learning effects, demand information accuracy, and direct marketing costs impact parties involved in an information-sharing supply

TABLE 1. Similarities and differences of the most relevant studies ($\surd$: covered).

Study	Cost reduction			Encroachment	Information structure			Focus issue
	Format	Function	Period		Model	Subject	Content	
Sun *et al.* [5]	Quantity-based cost decline	Linear	One-period	$\surd$	Signal Gaming	N.A.	N.A.	Encroachment and cost decline
Sun *et al.* [6]	Production investments	Nonlinear	N.A.	$\surd$	Signal Gaming	N.A.	N.A.	Cost reduction strategy
Zhao *et al.* [7]	Production economy	Nonlinear	N.A.	$\surd$	Random variable	Retailer	Demand information sharing decision	Production diseconomy/economy
Shum *et al.* [8]	Production learning effect	Linear	Two-period	N.A.	N.A.	N.A.	N.A.	Pricing strategy
Zhang and Zhang [35]	Stochastic cost learning	Linear	Two-period	N.A.	Signal Gaming	N.A.	N.A.	Contract design
Guan *et al.* [30]	N.A.	N.A.	N.A.	$\surd$	Signal Gaming	Manufacturer	Cost information sharing decision	Production diseconomy/economy
Lin *et al.* [27]	N.A.	N.A.	N.A.	N.A.	Random variable	Online intermediary	Demand information sharing	Production diseconomy/economy
Gupta *et al.* [39]	Linear batch cost learning	Linear	Two-period	$\surd$	N.A	N.A	N.A	Supplier encroachment and cost learning
This Paper	Production learning effect	Linear	Two-period	$\surd$	Random variable	Retailer	Demand information sharing decision and incentives	Information sharing strategy

chain. Our primary findings, in contrast to theirs, indicate that a single-win, win–win, or lose–lose situation is generated for the supply chain participants by the dynamic competition mechanism under the combined effects of encroachment and the production learning effect. When more ambiguous demand information is cooperatively provided, both the retailer and manufacturer will benefit. The win–win area increases with higher direct selling costs, whereas the win–win area decreases with increased production learning effects. Furthermore, the retailer's information sharing benefits more from a higher level of the production learning effect when demand information is unclear, and information sharing benefits more from a lower level of the production learning effect when demand information is more accurate. Following information sharing, some significant management insights are obtained on selecting and implementing optimal information-sharing incentive strategies based on profitability.

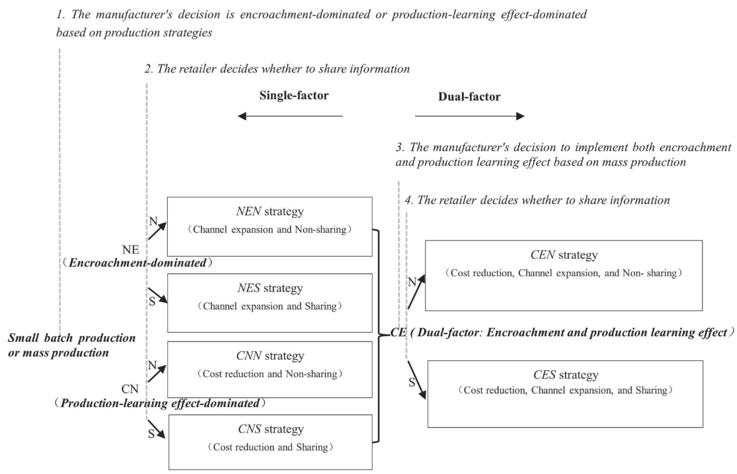

FIGURE 1. Six dynamic game models.

3. MODEL DESCRIPTION

In a supply chain when consumers, one traditional retailer, and one manufacturer are all involved. Without implementing an encroachment strategy, the manufacturer may wholesale its goods to the retailer, who then resells them to customers (denoted by the superscript N). Superscript E indicates that in addition to adopting the resale channel, the manufacturer may also be able to sell its homogeneous products directly to customers through an online channel.

The formation and realization of production learning effects may be impacted by the different production strategies used by manufacturers. It is challenging to create a production learning impact if the manufacturer intends to produce in small batches at the start of the period because the production scale is too small, and the production learning experience gained thus far is insufficient to cut costs. As a result, the manufacturer is limited to choosing the encroachment-led scenario to increase the avenue of profit realization. Large-scale production by the manufacturer will generate the production learning effect, which the manufacturer can then employ to reduce costs (shown by superscript C). At the same time, the manufacturer can decide whether or not to encroach based on this information, which applies to both the two-factor scenario of encroachment and the production learning effect and learning-led scenario. The supply chain is currently two-period ($H = 1, 2$), with the first period being the production learning phase and a production learning effect in the manufacturing process, which can be considered the industry's development period. The second period, which can be considered the industry's maturity stage, is the realization phase of production learning when cost reductions have been accomplished and the learning impact has vanished.

In both period segments, the manufacturer employs dynamic pricing strategies. The retailer can also watch for a private incomplete demand signal Y before the sales season and determine whether to share demand information with the manufacturer $S(N)$ based on encroachment and production learning details. Based on the subsequent research, it is possible to form the six dynamic game models (Fig. 1). Table 2 displays the fundamental notation and model parameters.

(1) *Notation and Hypotheses*

The model hypotheses are below, and Table 2 displays the basic notation and parameters.

Hypothesis 3.1. Market price and demand. It is assumed that the demand function is linearly inverse to represent market demand $P = a + \theta - Q$ [7], where the potential market demand consists of a deterministic component a ($a > 0$) and an uncertain random variable $\theta \sim N(0, \sigma_\theta{}^2)$ and ($\sigma_\theta{}^2 > 0$). P indicates the market clearing price, and Q denotes the total product quantity, respectively. In particular, it is assumed that $\sigma_\theta{}^2$ is sufficiently small so that the fluctuations in the uncertainty component of demand are sufficiently small to ensure that the retailer takes only one approach to demand forecasting to reduce demand volatility.

To ensure that demand is positive in each period, it is further assumed that $a > c$.

Hypothesis 3.2. Unit production costs. The manufacturer produces to order. The unit cost of production in the first period is $c_1 = c$ ($c > 0$), which is also the unit production cost in the single-period encroachment-led scenario where no production learning effect occurs. The unit cost of production in the second period is c_2. The production cost is achieved at the start of period two as a decreasing function of the output of the previous period, *i.e.*, $c_2 = c - \beta q_1$. This is a general assumption for the production learning effect [8,35], where β denotes the learning rate at which the manufacturer produces a product unit. In addition, to ensure that an optimal solution exists, this paper assumes that the production learning effect $\beta \in (0, 4/5)$ [40].

Hypothesis 3.3. The unit cost of sales. The unit selling cost of the manufacturer is assumed to be m, and $m \in (0, 3(a - c)/5)$, (Arya *et al.* [16] discovered that the manufacturer encroaches when the direct unit sales cost is adequately small). Also, the retailer's unit sales cost is assumed to be zero due to the selling advantage of the retailer's proximity to the market [7].

Hypothesis 3.4. Demand signal. Prior to the sales season, the retailer can obtain a demand signal Y, which is private and incomplete through market research or other forecasting methods. Assuming that Y is an unbiased estimate of θ, *i.e.*, $E[Y|\theta] = \theta$, the conditional expectation Y_i of θ is linear in Y, which can be represented as $E[\theta|Y] = t\sigma^2(1 + t\sigma^2)^{-1}Y$. The precision of the demand signal is $t = 1/E[Var[Y|\theta]]$. In addition, this paper defines $\tilde{y} = E[\theta|Y] = t\sigma^2(1 + t\sigma^2)^{-1}Y$, $\delta = E[E[\theta|Y]^2] = t\sigma^4(1 + t\sigma^2)^{-1}$ [7,32], so that the larger $\tilde{y}$ or δ is, the more accurate and stable the forecasted demand information is.

In addition, to guarantee optimal solutions, let $0 < \tilde{y} < 1$.

(2) *The sequence of events and decisions*

The sequence of events and decisions is as follows (M for the manufacturer and R for the retailer):

(a) In the run-up to the sales season, M considers shifting some of its incremental profits T^{kj} to encourage R to share demand signals Y and R decides whether to establish information-sharing mechanisms.
(b) After R observes its private incomplete demand signal Y, R shares Y with M if she agrees to establish an information-sharing mechanism in advance.
(c) At each period, M determines w_H^{kji} and then R determines q_{RH}^{kji}. If M adopts an encroachment strategy, the manufacturer determines q_{MH}^{kEi} according to q_{RH}^{kEi}.

Suppose there is a production learning effect at the start of the second period. In that case, production costs depend on the production learning effect to be realized $c_2 = c - \beta q_1$, where the realization depends on the production quantity in the first period.

4. Equilibrium results under single-factor and dual-factor scenarios

This section considers retailers not sharing demand information and retailers sharing demand information in encroachment-dominated, production-learning-dominated, and dual-factor considering encroachment and production learning scenarios, based on small-batch or mass-production strategies. The following six scenarios: "NEN and NES", "CNN and CNS", and "CEN and CES" – are taken into consideration to determine the equilibrium decision and expected profit of each supply chain member through the inverse solution.

TABLE 2. Summary of notations.

Notation	Description
P	Market clearing price
a	The deterministic part of market demand, where $0 < a < \widehat{a_0}$ $(\widehat{a_0} = c + m - \theta)$
θ	The variable part of market demand, where $E[\theta] = 0$
β	The rate of learning for manufacturers per unit of product produced, where $\beta \in \left(0, \frac{4}{5}\right)$
q_R	The sales quantity of the retailer
q_M	The direct sales quantity of the manufacturer if she encroaches
q_1	The sales quantity at the first period
q_2	The sales quantity in the second period
Q	The total sales quantity of the product, where $Q = q_R + q_M$
β	The production learning effect generated by the manufacturer during the first period of manufacturing, where $0 < \beta < \frac{4}{5}$
m	The manufacturer's unit sales costs in the direct channel, where $0 < m < m_0\left(m_0 = \frac{3(a-c)}{5}\right)$
σ^2	The variance of θ, $\sigma^2 > 0$
Y	Demand signal, where $E[Y\|\theta] = \theta$, $E[\theta\|Y] = \frac{t\sigma^2}{1+t\sigma^2}Y = \tilde{y}$
t	The accuracy of the demand signal, where $t = \frac{1}{E[Var[Y\|\theta]]}$
H	The notation of the sales periods, where $H = 1, 2$
J	The notation of supply chain members, the retailer and the manufacturer are denoted by R and M respectively
k	The notation of cost reduction or not, if $k = C$ indicates that the production learning effect occurs, if $k = N$ indicates no production learning effect
j	The notation of encroachment or not, if $j = E$ indicates encroachment and if $j = N$ indicates no encroachment
i	The notation of sharing or not, if $i = S$ indicates the information is shared and if $i = N$ indicates the information is not shared
q_{JH}^{kji}	The optimal sales quantity of firm J at different sales periods in the (k, j, i) setting
w_H^{kji}	The optimal wholesale price of the manufacturer at different sales periods in the (k, j, i) setting
c_H	The manufacturer's unit production costs at different sales periods, where $c_1 = c$ and $c_2 = c - \beta q_1$, $c_H > 0$
Π_{JH}^{kji}	The profit of the firm J at different sales periods in the (k, j, i) setting
Π_{SH}^{kji}	The total profit of the supply chain at different sales periods in the (k, j, i) setting
T^{kj}	The profit increment transferred by the manufacturer to facilitate the establishment of information sharing mechanisms with the retailer in the (k, j) setting

4.1. Encroachment-dominated

This section will examine the supply chain that is dominated by encroachment. At this point, it is assumed that based on a small batch production strategy, the manufacturer will only expand its channel through encroachment. This scenario is a single-period two-channel supply chain because there will be no production-learning effect. It indicates that there exist avenues in the supply chain for direct sales and resale and that there will

TABLE 3. Equilibrium decisions and optimal profits under encroachment dominance.

Cases	q_R	q_M	w	Π_R	Π_M	Π_S
NEN	$\frac{2m}{3}+\tilde{y}$	$\frac{1}{6}(3a-3c-5m)$	$\frac{1}{6}(3a+3c-m)$	$\frac{2m^2}{9}$	$\frac{1}{12}\left(A+7m^2\right)$	$\frac{1}{36}\left(3A+29m^2\right)$
NES	$\frac{2m}{3}$	$\frac{1}{6}(3a-3c-5m+3\tilde{y})$	$\frac{1}{6}(3(a+c)-m+3\tilde{y})$	$\frac{2m^2}{9}$	$\frac{1}{12}\left(A+7m^2+3\delta\right)$	$\frac{1}{36}\left(3A+29m^2+9\delta\right)$

be volume competition between them. The following sections will independently examine the two scenarios – sharing no information and sharing information.

4.1.1. NEN strategy: Non-sharing

According to the NEN strategy, the manufacturer will not be able to cut costs based on production learning effects when using a small batch production strategy. The manufacturer will therefore only encroach through direct sales channels, and the retailer won't share demand information.

At this point, $\tilde{y}=E[\theta|Y]=t\sigma^2\left(1+t\sigma^2\right)^{-1}Y$ and $E[\theta]=0$, the profit function for each member is:

$$\operatorname{Max}\Pi_M^{NEN}(E[q_M]|w,E[q_R])=(a+E[\theta]-E[q_R]-E[q_M]-c-m)E[q_M]+(w-c)E[q_R] \tag{1}$$

$$\operatorname{Max}\Pi_R^{\mathrm{NEN}}(q_R|w)=(a+E[\theta|Y]-q_R-q_M-w)q_R. \tag{2}$$

Theorem 4.1. *Under the NEN strategy, Table 3 gives the equilibrium decisions* (q_R^{NEN}, q_M^{NEN}, w^{NEN}) *and optimal profits* (Π_R^{NEN}, Π_M^{NEN}, Π_S^{NEN}).

4.1.2. NES strategy: Sharing

According to the NES strategy, the manufacturer will not be able to cut costs based on production learning effects when using a small batch production strategy. The manufacturer will therefore only encroach through direct sales channels, and the retailer will share demand information. At this point, $\tilde{y}=E[\theta|Y]=t\sigma^2\left(1+t\sigma^2\right)^{-1}Y$, the profit function for each member is:

$$\begin{aligned}\operatorname{Max}\Pi_M^{\mathrm{NES}}(E[q_M|Y])=&(a+E[\theta|Y]-E[q_R|Y]-E[q_M|Y])E[q_M|Y]\\&-(c+m)E[q_M|Y]+(w-c)E[q_R|Y]\end{aligned} \tag{3}$$

$$\operatorname{Max}\Pi_R^{\mathrm{NES}}(q_R|w)=(a+E[\theta|Y]-q_R-q_M-w)q_R. \tag{4}$$

Theorem 4.2. *Under the NES strategy, Table 3 gives the equilibrium decisions* (q_R^{NES}, q_M^{NES}, w^{NES}) *and optimal profits* (Π_R^{NES}, Π_M^{NES}, Π_S^{NES}).

4.2. Production-learning effect-dominated

4.2.1. CNN strategy: Non-sharing

According to the CNN strategy, the manufacturer will be able to cut costs based on production learning effects when using a mass-production business strategy. However, the manufacturer will not encroach, possibly due to factors such as funding and market competition. It is assumed that the retailer will not share demand information. At this point, cost reduction through the production learning effect results in a two-period game model. where the profit function for each member in the second period is:

$$\operatorname{Max}\Pi_{R2}^{CNN}(q_{R2}\mid w_2)=(a+E[\theta\mid Y]-q_{R2}-w_2)\,q_{R2} \tag{5}$$

$$\operatorname{Max}\Pi_{M2}^{\mathrm{CNN}}(w_2)=w_2E[q_{R2}]-(c-\beta q_{R1})E[q_{R2}]. \tag{6}$$

Table 4. Equilibrium decisions and optimal profits under Production-learning effect dominance.

Cases	q_{R1}	w_1	Π_{R1}	Π_{M1}
CNN	$-\frac{16\tilde{y}+(\beta+8)(a-c)}{2(\beta^2-16)}$	$\frac{1}{16}(a(\beta+8)-(\beta-8)c)$	$\frac{B_0-256(\beta^2-8)\delta}{32(\beta^2-16)^2}$	$-\frac{(a-c)^2(8+\beta)^2}{32(\beta^2-16)}$
CNS	$-\frac{8\tilde{y}+(\beta+8)(a-c)}{2(\beta^2-16)}$	$\frac{1}{16}(8\tilde{y}+a(\beta+8)+(\beta-8)(-c))$	$\frac{B_0-64(\beta^2-8)\delta}{32(\beta^2-16)^2}$	$-\frac{(\beta+8)^2(a-c)^2+64\delta}{32(\beta^2-16)}$
Cases	q_{R2}	w_2	Π_{R2}	Π_{M2}
CNN	$\frac{4((\beta-4)\beta-16)\tilde{y}+B_1(a-c)}{8(\beta^2-16)}$	$\frac{B_1c+a(-32+\beta(8+3\beta))}{4(\beta^2-16)}$	$\frac{\delta}{4}+\frac{B_1^2(a-c)^2+256\beta^2\delta}{64(\beta^2-16)^2}$	$\frac{B_1^2(a-c)^2+256\beta^2\delta}{32(\beta^2-16)^2}$
CNS	$\frac{1}{4}(-C+\tilde{y}+a-c)$	$\frac{1}{2}(C+\tilde{y}+a+c)$	$\frac{B_1^2(a-c)^2+4(\beta^4-16\beta^2+256)\delta}{64(\beta^2-16)^2}$	$\frac{B_1^2(a-c)^2+4(\beta^4-16\beta^2+256)\delta}{32(\beta^2-16)^2}$
Cases	Π_R	Π_M		Π_s
CNN	$\frac{(\beta(3\beta-16)-128)(a-c)^2+16(\beta^2-32)\delta}{64(\beta^2-16)}$	$\frac{16\beta^2\delta-(\beta(\beta(2\beta+3)-48)-128)(a-c)^2}{2(\beta^2-16)^2}$		$\frac{B_2+16(\beta^4-16\beta^2+512)\delta}{64(\beta^2-16)^2}$
CNS	$\frac{(\beta(3\beta-16)-128)(a-c)^2+4(\beta^2-32)\delta}{64(\beta^2-16)}$	$\frac{(\beta^4-32\beta^2+512)\delta-4(\beta(\beta(2\beta+3)-48)-128)(a-c)^2}{8(\beta^2-16)^2}$		$\frac{B_2+4(3\beta^4-112\beta^2+1536)\delta}{64(\beta^2-16)^2}$

The total profit function for each member for the two periods is:

$$\text{Max}\,\Pi_R^{\text{CNN}}(q_{R1}\mid w_1)=(a+E[\theta\mid Y]-q_{R1}-w_1)q_{R1}+E\left[\Pi_{R2}^{\text{CNN}}\right] \tag{7}$$

$$\text{Max}\,\Pi_M^{\text{CNN}}(w_1)=(w_1-c)E[q_{R1}]+E\left[\Pi_{M2}^{\text{CNN}}\right]. \tag{8}$$

The equilibrium solution and the expected profit under the CNN strategy are obtained using the inverse induction method, as shown in Theorem 4.3.

Theorem 4.3. *Under the CNN strategy, Table 4 gives the equilibrium decisions* (q_{RH}^{CNN}, w_H^{CNN}) *and optimal profits* (Π_{RH}^{CNN}, Π_{MH}^{CNN}, Π_R^{CNN}, Π_M^{CNN}, Π_S^{CNN}).

4.2.2. CNS strategy: Sharing

According to the CNS strategy, the manufacturer will be able to cut costs based on production learning effects when using a mass-production business strategy. However, the manufacturer will not encroach, and the retailer will share demand information. At this point $\tilde{y}=E[\theta|Y]=t\sigma^2\left(1+t\sigma^2\right)^{-1}Y$, the profit function for each member in the second period is:

$$\text{Max}\,\Pi_{R2}^{\text{CNS}}(q_{R2}|w_2)=(a+E[\theta|Y]-q_{R2}-w_2)q_{R2} \tag{9}$$

$$\text{Max}\,\Pi_{M2}^{\text{CNS}}(w_2)=w_2E[q_{R2}|Y]-(c-\beta q_{R1})E[q_{R2}|Y]. \tag{10}$$

The total profit function for each member for the two periods is:

$$\text{Max}\,\Pi_R^{\text{CNS}}(q_{R1}|w_1)=(a+E[\theta|Y]-q_{R1}-w_1)q_{R1}+E\left[\Pi_{R2}^{\text{CNS}}\right] \tag{11}$$

$$\text{Max}\,\Pi_M^{\text{CNS}}(w_1)=(w_1-c)E[q_{R1}|Y]+E\left[\Pi_{M2}^{\text{CNS}}\right]. \tag{12}$$

The equilibrium solution and the expected profit under the CNS strategy are obtained using the inverse induction method, as shown in Theorem 4.4.

Theorem 4.4. *Under the CNS strategy, Table 4 gives the equilibrium decisions* (q_{RH}^{CNS}, w_H^{CNS}) *and optimal profits* (Π_{RH}^{CNS}, Π_{MH}^{CNS}, Π_R^{CNS}, Π_M^{CNS}, Π_S^{CNS}).

4.3. Dual-factor: Encroachment and production-learning effect

This section examines the supply chain wherein the production learning effect and encroachment occur concurrently. Under the mass-production business strategy, this is a case of a two-period dual-channel supply chain since the manufacturer is expanding the channel based on the encroachment and reducing costs based on the two-period production learning effect. The two situations of sharing and not sharing information are discussed in the following section.

4.3.1. CEN strategy: Non-sharing

According to the CEN strategy, due to the mass production strategy, the manufacturer will simultaneously expand the channel based on the encroachment and reduce costs based on the two-period production learning effect. The retailer will not share demand information. At this point $\tilde{y} = E[\theta|Y] = t\sigma^2(1+t\sigma^2)^{-1}Y$ and $E[\theta] = 0$, the profit function for each member in the second period is:

$$\begin{aligned} \operatorname{Max} \Pi_{M2}^{CEN}(E[q_{M2}] \mid w_2, E[q_{R2}]) = (a + E[\theta] - E[q_{R2}] - E[q_{M2}])E[q_{M2}] \\ - (c - \beta(q_{R1} + q_{M1}) + m)E[q_{M2}] \\ + (w_2 - (c - \beta(q_{R1} + q_{M1})))E[q_{R2}] \end{aligned} \tag{13}$$

$$\operatorname{Max} \Pi_{R2}^{\text{CEN}}(q_{R2}|w_2) = (a + E[\theta|Y] - q_{R2} - q_{M2} - w_2)q_{R2}. \tag{14}$$

Given w_2 and q_{R2}, the manufacturer decides q_{M2}. It is easy to know that $\Pi_{M2}^{\text{CEN}}(q_{M2}, q_{R2})$ is a concave function of q_{M2}. The first-order optimality criteria give the manufacturer's equilibrium sales quantity as follows:

$$q_{M2}^{\text{CEN}}(q_{R2}) = \frac{1}{2}(a - c - m + \beta(q_{M1} + q_{R1}) - q_{R2}). \tag{15}$$

Substituting q_{M2} into equation (14), the retailer decides on the number of orders q_{R2}, and it is clear that $\Pi_{R2}^{\text{CEN}}(q_{R2}, w_2)$ is a concave function of q_{R2}. The optimality criteria give the retailer's equilibrium sales as follows:

$$q_{R2}^{\text{CEN}}(w_2) = \frac{1}{2}(a + c + m - \beta(q_{M1} + q_{R1}) - 2w) + \tilde{y}. \tag{16}$$

Substitute $q_{R2}^{\text{CEN}}(w_2)$ into equation (15) to obtain the manufacturer's equilibrium sales as:

$$q_{M2}^{\text{CEN}}(w_2) = \frac{1}{4}(a - 3(c + m) + 2w + 3\beta(q_{M1} + q_{R1}) + 2\tilde{y}). \tag{17}$$

Substituting $E[q_{R2}] = E[q_{R2}^{\text{CEN}}(w_2)] = \frac{1}{2}(a + c + m - \beta(q_{M1} + q_{R1}) - 2w)$, $E[q_{M2}] = E[q_{M2}^{\text{CEN}}(w_2)] = \frac{1}{4}(a - 3(c + m) + 2w + 3\beta(q_{M1} + q_{R1}))$ into equation (13), the manufacturer decides the wholesale price w_2, which is easily known $\Pi_{M2}^{\text{CEN}}(w_2)$ to be a concave function of w_2, and the equilibrium wholesale price can be obtained as:

$$w_2^{\text{CEN}} = \frac{1}{6}(3(a + c) - m - 3\beta(q_{M1} + q_{R1})). \tag{18}$$

Similar to the decision analysis process for the second period, the first-period decision is again analyzed using the reverse induction method. Where the total profit function for each member for two periods is:

$$\begin{aligned} \operatorname{Max} \Pi_{M}^{CEN}(E[q_{M1}]) = (a + E[\theta] - E[q_{R1}] - E[q_{M1}] - c - m)E[q_{M1}] \\ + (w_1 - c)E[q_{R1}] + E[\mathit{\Pi}_{M2}^{CEN}] \end{aligned} \tag{19}$$

$$\operatorname{Max} \Pi_{R}^{\text{CEN}}(q_{R1}|w_1) = (a + E[\theta|Y] - q_{R1} - q_{M1} - w_1)q_{R1} + E\left[\Pi_{R2}^{\text{CEN}}\right]. \tag{20}$$

Substituting the equilibrium solutions w_H^{CEN}, $q_{RH}^{\text{CEN}}(w_H)$, $q_{MH}^{\text{CEN}}(q_{RH})$ into equations (13)–(20), respectively, the equilibrium solutions and expected profits for each stage under the CEN strategy can be obtained as shown in Theorem 4.5.

Theorem 4.5. *Table 5 gives the equilibrium decisions* (q_{RH}^{CEN}, q_{MH}^{CEN}, w_H^{CEN}) *and optimal profits* (Π_{RH}^{CEN}, Π_{MH}^{CEN}, Π_{R}^{CEN}, Π_{M}^{CEN}, Π_{S}^{CEN}) *under the CEN strategy.*

TABLE 5. Equilibrium decisions and optimal profits under Dual-factor.

Cases	q_{R1}		q_{M1}	w_1
CEN	$-\frac{1}{12}(\beta^2-4)(3\tilde{y}+2m)$		$\frac{-a+c+m}{\beta-2}+\frac{\beta^2 m}{6}-\frac{m}{3}$	$\frac{3a(\beta-1)-3c-2\beta m+m}{3(\beta-2)}$
CES	$-\frac{2(\beta+2)m}{\beta-6}$		$\frac{(\beta+2)(-a(\beta-6)+(\beta-6)c+(\beta+2)(2\beta-5)m)-2(\beta-6)\tilde{y}}{(\beta-6)(\beta^2-4)}$	$\frac{1}{\beta-2}\left(\frac{(\beta^2-2)\tilde{y}}{\beta+2}+a(\beta-1)-c+\frac{16m}{\beta-6}+3m\right)$
Cases	q_{R2}		q_{M2}	w_2
CEN	$\frac{2m}{3}+\tilde{y}$		$-\frac{m}{3}+\frac{-a+c+m}{-2+\beta}+\frac{m\beta}{6}$	$\frac{-6c+2m+6a(-1+\beta)-m\beta(2+\beta)}{6(-2+\beta)}$
CES	$\frac{2}{3}m$		$\frac{K_0}{6(\beta-6)(\beta^2-4)}$	$\frac{1}{6}\left(2K_1+\frac{3(-4+\beta(2+\beta))\tilde{y}}{\beta^2-4}\right)$
Cases	Π_{R1}	Π_{R2}	Π_{M1}	Π_{M2}
CEN	$-\frac{1}{144}(\beta^2-4)(9\beta^2\delta+8m^2)$	$\frac{2m^2}{9}$	$-\frac{D_0}{9(\beta-2)^2}$	$D_1+\frac{m^2\beta^2}{36}+\frac{1}{64}\beta^2(-4+\beta^2)^2\delta$
CES	$\frac{4(\beta+2)m^2}{(\beta-6)^2}$	$\frac{2m^2}{9}$	$-\frac{K_2}{(\beta-2)^2}$	$\frac{K_3}{12(\beta-6)^2(\beta^2-4)^2}$
Cases	Π_R		Π_M	Π_S
CEN	$\frac{1}{144}\left(-8m^2(-8+\beta^2)-9\beta^2(-4+\beta^2)\delta\right)$		$\frac{2m^2}{3}-\frac{(-a+c+m)^2}{-2+\beta}-\frac{m^2\beta^2}{12}+\frac{1}{64}\beta^2(-4+\beta^2)^2\delta$	$\frac{D_2}{-2+\beta}$
CES	$\frac{2(\beta(\beta+6)+72)m^2}{9(\beta-6)^2}$		$-\frac{K_4}{12(2-\beta)}$	$\frac{K_5}{-2+\beta}$

4.3.2. CES strategy: Sharing

According to the CES strategy, due to the mass production strategy, the manufacturer will simultaneously expand the channel based on the encroachment and reduce costs based on the two-period production learning effect. The retailer will share demand information. At this point $E[\theta|Y]=t\sigma^2\left(1+t\sigma^2\right)^{-1}Y$, the profit function for each member in the second period is:

$$\begin{aligned}\operatorname{Max}\Pi_{M2}^{\text{CES}}(E[q_{M2}|Y]) &= (a+E[\theta|Y]-E[q_{R2}|Y]-E[q_{M2}|Y])E[q_{M2}|Y]\\ &\quad-((c-\beta(q_{R1}+q_{M1}))+m)E[q_{M2}|Y]\\ &\quad+(w_2-(c-\beta(q_{R1}+q_{M1})))E[q_{R2}|Y] \qquad (21)\\ \operatorname{Max}\Pi_{R2}^{\text{CES}}(q_{R2}|w_2) &= (a+E[\theta|Y]-q_{R2}-q_{M2}-w_2)q_{R2}. \qquad (22)\end{aligned}$$

The total profit function for each member for two periods is:

$$\begin{aligned}\operatorname{Max}\Pi_{M}^{\text{CES}}(E[q_{M1}|Y]) &= (a+E[\theta|Y]-E[q_{R1}|Y]-E[q_{M1}|Y])E[q_{M1}|Y]\\ &\quad-cE[q_{M1}|Y]-mE[q_{M1}|Y]+(w_1-c)E[q_{R1}|Y]+E\left[\Pi_{M2}^{\text{CES}}\right] \qquad (23)\\ \operatorname{Max}\Pi_{R}^{CES}(q_{R1}|w_1) &= (a+E[\theta|Y]-q_{R1}-q_{M1}-w_2)q_{R1}+E\left[\Pi_{R2}^{\text{CES}}\right]. \qquad (24)\end{aligned}$$

The equilibrium solutions and the expected profits under the CES strategy are obtained by applying the inverse induction method, as shown in Theorem 4.6.

Theorem 4.6. *When* $0<\beta<\frac{4}{5}$: $\begin{cases}0<m<\min\{m_0,\tilde{y}m_1\}\\ \tilde{y}m_1<m<(a-c)m_2+\tilde{y}m_3\end{cases}$, *Table 5 gives the equilibrium decisions* (q_{RH}^{CES}, q_{MH}^{CES}, w_H^{CES})*, and optimal profits* (Π_{RH}^{CES}, Π_{MH}^{CES}, Π_R^{CES}, Π_M^{CES}, Π_S^{CES}) *under the CES strategy.*

5. EQUILIBRIUM ANALYSIS UNDER DUAL-FACTOR

The focus of this paper is to explore the dynamic impact of the manufacturer encroachment and the production learning effect on the information sharing of retailers' demand from the dual perspective of both. Therefore, this subsection further analyses, in conjunction with Theorems 4.5 and 4.6, the impact of the production learning effect on the equilibrium decisions and optimal profits of the members of a supply chain with information sharing in the case of intrusion.

5.1. Production learning effects

Corollary 5.1. *Production learning affects supply chain members' optimal profits and equilibrium decisions under the CES strategy:*

(1) $\frac{\partial w_H^{\mathrm{CES}}}{\partial \beta} < 0$, $\frac{\partial Q_H^{\mathrm{CES}}}{\partial \beta} > 0$, $\frac{\partial P_H^{\mathrm{CES}}}{\partial \beta} < 0$, $\frac{\partial q_{M2}^{\mathrm{CES}}}{\partial \beta} > 0$, $\frac{\partial q_{R1}^{\mathrm{CES}}}{\partial \beta} > 0$, $\frac{\partial q_{R2}^{\mathrm{CES}}}{\partial \beta} = 0$.
(2) $\frac{\partial \Pi_{R1}^{\mathrm{CES}}}{\partial \beta} > 0$, $\frac{\partial \Pi_{R2}^{\mathrm{CES}}}{\partial \beta} = 0$, $\frac{\partial \Pi_{R}^{\mathrm{CES}}}{\partial \beta} > 0$, $\frac{\partial \Pi_{M2}^{\mathrm{CES}}}{\partial \beta} > 0$, $\frac{\partial \Pi_{M}^{\mathrm{CES}}}{\partial \beta} > 0$, $\frac{\partial \Pi_{S}^{\mathrm{CES}}}{\partial \beta} > 0$.
(3) $\frac{\partial^2 \Pi_{M}^{\mathrm{CES}}}{\partial \beta \partial m} > 0$, $\frac{\partial^2 \Pi_{R}^{\mathrm{CES}}}{\partial \beta \partial m} > 0$, $\frac{\partial^2 \Pi_{S}^{\mathrm{CES}}}{\partial \beta \partial m} > 0$.

Corollary 5.1 presents a counterintuitive conclusion: Despite losing the advantage, the retailer can reap more benefits from the increased learning effect following information sharing. This is because considering both encroachment and production learning can make the retailer better off. The retailer always gains from the enhanced value added of information sharing due to the improved production learning effect, especially when demand information is ambiguous.

This counterintuitive phenomenon reflects how the encroachment and production learning effect offers a dynamic competitive mechanism for the supplier chain that shares information. In particular, the manufacturer and the retailer may collaborate effectively on information thanks to the retailer's information sharing, which improves manufacturing's first-period production planning, reduces cannibalization of the resale channel, and increases the retailer's first-period sales and profits with more significant learning effects. The manufacturer lowers wholesale prices in the first period to increase sales. As the production learning effect operates in the first period, the manufacturer's profits could decline due to the negative impact of reduced wholesale pricing. In the subsequent period, the manufacturer experiences an increase in yield due to cost reductions, which also heightens competition between the two channels. The production learning effect does not influence the retailer's profitability, and cost-cutting benefits offset the drawbacks of channel competition. Instead, the two periods' earnings trends show how this competitive process may raise profit margins.

Furthermore, the greater the manufacturer's direct selling expenditures are, the more the production learning effect varies the profitability of the supplier chain members. Production learning may be carried out more successfully by manufacturing companies with significant direct selling expenses. This leads to the corresponding managerial understanding that manufacturing enterprises with high direct selling costs may carry out production learning more successfully. This kind of business is more likely to concentrate on making large profit margins by selling high-priced goods; if they can improve their production learning capabilities, they may grow their profit margins even more.

5.2. Economies of scale

Corollary 5.2. *A comparison of the CES and NES strategies leads to the conclusion that economies of scale have an impact on production learning. Production learning, in turn, realizes economies of scale, which enables the supply chain to grow profitably from economies of scale.*

(1) $q_R^{\mathrm{CES}} > q_R^{\mathrm{NES}}$, $q_M^{\mathrm{CES}} > q_M^{\mathrm{NES}}$, $Q^{\mathrm{CES}} > Q^{\mathrm{NES}}$.
(2) $\Pi_R^{\mathrm{CES}} > \Pi_R^{\mathrm{NES}}$, $\Pi_M^{\mathrm{CES}} > \Pi_M^{\mathrm{NES}}$, $\Pi_S^{\mathrm{CES}} > \Pi_S^{\mathrm{NES}}$.
(3) $\frac{\partial w^{\mathrm{CES}}}{\partial M} < 0$, $\frac{\partial \Pi_M^{\mathrm{CES}}}{\partial M} > 0$, $\frac{\partial \Pi_S^{\mathrm{CES}}}{\partial M} > 0$, $\frac{\partial \Pi_R^{\mathrm{CES}}}{\partial M} = 0$, $\frac{\partial \Pi_R^{\mathrm{CES}}}{\partial m} > 0$ *(where $M = a - c$).*

According to Corollary 5.2, such economies of scale can be understood as the discrepancy between unit production costs and market size in this study. When the market is more extensive and unit production costs are lower, the economic benefit of lower unit costs owing to large-scale realization is more prominent. On the one hand, we discover that economies of scale impact the production learning effect. It is clear from Corollary 5.2 that as scale effects grow, so will the manufacturer's and the supply chain's profitability. However, when scale effects increase, wholesale costs will fall. To meet increased demand, the manufacturer may offer reduced wholesale pricing to the retailer to encourage them to place larger orders. Remarkably, the size effect does not directly impact the shift in the retailer's profits. The cost of direct marketing has a direct impact on the retailer's profits, and scale effects also have an indirect influence by driving down wholesale pricing. In particular, the retailer makes more money when the manufacturer's direct selling expenses are higher. The retail volume is rising, giving the retailer a significant competitive edge. In contrast, economies of scale can reduce wholesale costs, lowering the retailer's cost per unit. By combining the two components, retail can increase profits when the learning effect is present.

However, in contrast to a scenario with no production-learning effect, production learning generates economies of scale from which the supply chain can realize profit growth. The production-learning manufacturer typically prioritizes maximizing sales in both channels during the first period, aiming to achieve more significant cost savings during the second period, allowing them to offer more substantial items at a reduced price. Direct and retail sales both rise as a result of this, on the one hand. On the other hand, the benefit of scale and the cost savings it offers will result in significant profits for the manufacturer and the retailer.

6. Information-sharing decisions

This paragraph describes the retailer's optimal information-sharing decision in encroachment-dominated, production-learning-dominated, and two-factor situations, considering Theorems 4.1–4.6.

Proposition 6.1. *The retailer in the production-learning effect-dominated scenario never exchanges demand information. When the encroachment predominates, the retailer losses following information sharing are offset by an additional surplus, and the manufacturer has the ability and motivation to encourage retailer information sharing.*

(1) $V_R^{\mathrm{NE}} = 0\,(\Pi_R^{\mathrm{NES}} = \Pi_R^{\mathrm{NEN}})$, $V_M^{\mathrm{NE}} > 0\,(\Pi_M^{\mathrm{NES}} > \Pi_M^{\mathrm{NEN}})$, $V_S^{\mathrm{NE}} > 0\,(\Pi_S^{\mathrm{NES}} > \Pi_S^{\mathrm{NEN}})$.
(2) $V_R^{\mathrm{CN}} < 0\,(\Pi_R^{\mathrm{CNS}} < \Pi_R^{\mathrm{CNN}})$, $V_M^{\mathrm{CN}} > 0\,(\Pi_M^{\mathrm{CNS}} > \Pi_M^{\mathrm{CNN}})$, $V_S^{\mathrm{CN}} < 0\,(\Pi_S^{\mathrm{CNS}} < \Pi_S^{\mathrm{CNN}})$.
(3) $V_S^{\mathrm{NE}} > V_S^{\mathrm{CN}}(\Pi_S^{\mathrm{NES}} - \Pi_S^{\mathrm{NEN}} > \Pi_S^{\mathrm{CNS}} - \Pi_S^{\mathrm{CNN}})$.

Proposition 6.1 states that the retailer is more motivated to share and that the encroaching dominance enhances the value of information communication. At this point, when the learning effect is dominant, information interchange is always of negative importance, and the manufacturer's income cannot compensate for the retailer's loss. Anyhow, the retailer will maintain the privacy of demand information. Following sharing, the value of information exchange in the encroachment-led scenario is non-negative, and the manufacturer can profit more from sharing without undermining the retailer's earnings. To create a win–win scenario, the manufacturer can transmit a portion of the incremental profit $T^{\mathrm{NE}}(0 < T^{\mathrm{NE}} < \delta/4)$ to the retailer to subsidize them and give them incentives to share information.

Proposition 6.2. *The following findings are derived by comparing the equilibrium decisions and equilibrium profits of the manufacturer and the retailer in every period under information sharing and non-sharing in the two-factor scenario of production learning and encroachment:*

(1) *When demand information is ambiguous* $(0 < \delta < \delta_1)$, $V_{R1}^{\mathrm{CE}} > 0\,(\Pi_{R1}^{\mathrm{CES}} > \Pi_{R1}^{\mathrm{CEN}})$ *and* $\frac{V_{R1}^{\mathrm{CE}}}{m} > 0$.
(2) *The supply chain members can achieve a win–win in the dynamic competition mechanism when the twin elements of encroachment and production learning cause the equilibrium state of the two periods after information sharing to shift oppositely.*
  (a) *When direct sales cost is high and demand information is ambiguous* $(a > c + ma_1, m > \tilde{y}m_4$ *and* $0 < \delta < \min\{\delta_1, \delta_2\})$, $w_H^{\mathrm{CES}} < w_H^{\mathrm{CEN}}$, $q_{R1}^{\mathrm{CES}} > q_{R1}^{\mathrm{CEN}}$, $q_{M1}^{\mathrm{CES}} < q_{M1}^{\mathrm{CEN}}$, $q_R^{\mathrm{CES}} > q_R^{\mathrm{CEN}}$, $q_M^{\mathrm{CES}} < q_M^{\mathrm{CEN}}$, $V_{R1}^{\mathrm{CE}} > 0\,(\Pi_{R1}^{\mathrm{CES}} > \Pi_{R1}^{\mathrm{CEN}})$, $V_{M1}^{\mathrm{CE}} < 0\,(\Pi_{M1}^{\mathrm{CES}} < \Pi_{M1}^{\mathrm{CEN}})$.
  (b) $q_{R2}^{\mathrm{CES}} < q_{R2}^{\mathrm{CEN}}$, $q_{M2}^{\mathrm{CES}} > q_{M2}^{\mathrm{CEN}}$, $V_{R2}^{\mathrm{CE}} = 0\,(\Pi_{R2}^{\mathrm{CES}} = \Pi_{R2}^{\mathrm{CEN}})$, $V_{M2}^{\mathrm{CE}} > 0\,(\Pi_{M2}^{\mathrm{CES}} > \Pi_{M2}^{\mathrm{CEN}})$.

Where

$$\begin{cases} a_1 = (24+(-12+\beta)\beta)(-6+(-2+\beta)\beta)\dfrac{1}{3}(-6+\beta)\beta, \\ m_4 = -3(-6+\beta)(-4+\beta(2+\beta))\dfrac{1}{\beta^2}\left(-4+\beta^2\right), \\ \delta_2 = m(-2+\beta)\beta(2+\beta)^2\big(-3(a-c)(-6+\beta)\beta \\ \qquad + m(24+(-12+\beta)\beta)(-6+(-2+\beta)\beta)\big)\dfrac{1}{18}(-6+\beta)^2\left(-2+\beta^2\right). \end{cases}$$

When $a > c + ma_1$ *and* $m > \tilde{y}m_4$*, then* $0 < \delta < \delta_2$*, proving that* $f_{\Pi_{M1}}(\delta) = \Pi_{M1}^{\text{CES}} - \Pi_{M1}^{\text{CEN}} < 0$. $\delta_1 = -8(60+(-14+\beta)\beta)\frac{m^2}{9}(-6+\beta)^2(-2+\beta)\beta$*, and when* $0 < \delta < \delta_1$*, proving that* $f_{\Pi_{R1}}(\delta) = \Pi_{R1}^{\text{CES}} - \Pi_{R1}^{\text{CEN}} > 0$.

In contrast to the encroachment-dominated case (in which sharing is always unprofitable), Proposition 6.2 presents an intriguing conclusion that combined with Proposition 6.1, the dual strategy of encroachment and production learning makes the retailer profitable in ambiguous information. The retailer can gain information from communication when demand information is ambiguous, and the first period of production learning, in particular, is essential in determining the retailer's willingness to share proactively. In contrast, the manufacturer always profits from the second period of cost reductions while the retailer does not.

Regarding how the dual effects of production learning and encroachment affect the equilibrium state of information sharing, they have the potential to cause a reversal in the direction of the equilibrium state of the two periods that follow information sharing, allowing supply chain members to emerge with a win in the competitive mechanism. Along with the proposal, information sharing invariably results in a decline in retail sales and an increase in direct sales during the second period, when cost reductions are realized. The manufacturer is making a profit while the retailer is losing money. In addition, information communication reduces wholesale prices, boosts retail sales, and decreases direct sales in the first period of production learning, when direct selling is more expensive, and demand information is less clear. This counteracts the double marginal effect. Information exchange might be advantageous for the retailer while incurring expenses for the manufacturer. This reversal of events allows supply chain participants to achieve a mutually beneficial outcome in exchanging information.

This paper plots the competing partitions of the information-communication value of each member in the first period of the CE scenario, as shown in Figure 2, based on the conclusions of the propositions, to more intuitively reflect the competing mechanisms brought about by the dual factors of encroachment and production learning effect for sharing. Consider the parameter $a = 500$, $c = 30$. Select any value from the exogenous parameter direct sales cost m that falls inside its valid range m. The competing regions are discovered to have altered, meaning that the co-loss region gradually shrinks and the win–win region gradually expands as the direct marketing cost m progressively grows. This impacts the range of options available to the retailer when it comes to choosing their information-sharing tactics. In order to depict the effect of changes in direct marketing expenses on competing regions, we use four values, $m = 5$, $m = 15$, $m = 40$, and $m = 100$, which are typical of stage changes.

Four zones may be distinguished from the competing states of the members in the first period in the CE scenario based on the importance of information sharing, as seen in Figure 2. The single-win region, represented by region II with better information accuracy and part I with lower information accuracy, is indicated by the red portion. Due to their differing information preferences, the retailer and the manufacturer will have a single-win rivalry in the lower and higher information accuracy scenarios. Specifically, Region I allows the retailer to profit and the manufacturer to lose profits due to sharing more ambiguous information. As a result of production learning, the manufacturer maximizes total profits at the expense of first-period benefits, while the retailer profits from the positive effects of less channel competition and information collaboration in the first period. The learning period of production learning reverses the negative effect of the encroachment on the retailer in the case of ambiguous information when the positive effect of production learning dominates. Conversely, in

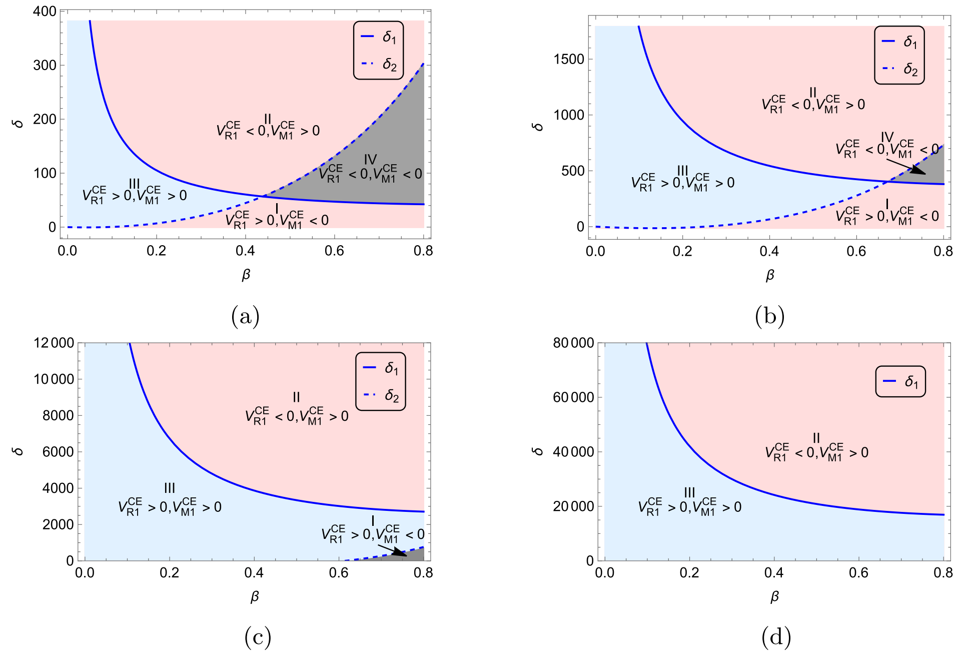

FIGURE 2. Information sharing value partitions under dual-factor. (a) $m = 5$. (b) $m = 15$. (c) $m = 40$. (d) $m = 100$.

Region II, sharing more accurate information allows the manufacturer to profit while the retailer's profits suffer. The region III, or the win–win zone, is indicated by the blue portion. In this region, information sharing can maximize benefits for the manufacturer and the retailer, provided that the learning effect is less significant when sharing more ambiguous information. The gray area denotes the loss zone (also known as region IV), wherein, without a substantial bias in the accuracy of demand information and a sizable production learning effect, both the members will experience a profit loss from sharing. The retailer shares information during the second period of cost realization reduction, intensifying channel competition following cost reduction. When the adverse effects of encroachment predominate, the production-learning realization period makes them worse for the retailer. Information sharing has no value for the retailer that loses its information advantage. In contrast, the manufacturer can consistently profit more from information sharing due to the combined effects of encroachment and production learning, with the manufacturer's profit in the second period consistently exceeding the loss in the first. As a result, in the second period, the manufacturer is always in a single-win situation.

By comparing Figures 2a–2d, which show the stage change of direct marketing cost from $m = 5$, $m = 15$, $m = 40$, to $m = 100$, as well as the proposition's conclusion, it can be observed that as m's direct marketing cost increases, the area of win–win expands (*i.e.*, the area of region III in the figure is gradually expanding) and the area of loss–loss shrinks (*i.e.*, the area of region IV in the figure shrinks until it disappears). Figure 2a ($m = 5$) illustrates how the win–win area is the smallest and the loss–loss area is the largest when direct marketing costs are small. This situation is not favorable for achieving information cooperation. On the other hand, Figure 2d ($m = 100$) illustrates how the cost of direct marketing is large, the win–win area is the largest, and the loss–loss area is the smallest, improving the situation. In the meantime, it is easy to see that the win–win zone progressively gets smaller as the production learning effect increases based on the latitude of change of the horizontal coordinates of the four graphs concerning the production learning effect. This indicates that the higher cost of direct marketing is more favorable for supply chain parties to achieve win–win through the establishment of mechanisms for information sharing, provided that the conditions of manufacturer encroachment are met. In contrast, a higher level of production learning effect is not favorable for parties to achieve a win–win through information sharing.

Proposition 6.3. *The following outcomes can be obtained by comparing the ideal total profit for the manufacturer, the retailer, and the entire supply chain with and without information sharing in the scenario of dual factors of encroachment and production learning:*

(1) *Information sharing benefits the manufacturer with encroachment, production learning capacities, and the supply chain. In this instance, the two factors maximize the supply chain's added value from information sharing.* $V_M^{\mathrm{CE}} > 0\,(\Pi_M^{\mathrm{CES}} > \Pi_M^{\mathrm{CEN}})$, $V_S^{\mathrm{CE}} > 0\,(\Pi_S^{\mathrm{CES}} > \Pi_S^{\mathrm{CEN}})$ *and* $V_S^{\mathrm{CE}} > V_S^{\mathrm{NE}} > V_S^{\mathrm{CN}}$.

(2) *When the direct sales cost is not too high* ($0 < m \leq \min\{m_0, \tilde{y}m_1\}$ *or* $\tilde{y}m_1 < m < (a-c)m_2 + \tilde{y}m_3$), *the retailer is motivated to share demand information.*

  (a) *When demand information is ambiguous* ($0 < \delta < \delta_1$), $V_R^{\mathrm{CE}} > 0\,(\Pi_R^{\mathrm{CES}} > \Pi_R^{\mathrm{CEN}})$ *and* $\frac{\partial \Pi_R^{\mathrm{CES}}}{\partial \delta} < 0$. *The retailer will proactively share demand forecasting information.*

  (b) *When demand information is more accurate* ($\delta > \delta_1$), $V_R^{\mathrm{CE}} < 0\,(\Pi_R^{\mathrm{CES}} < \Pi_R^{\mathrm{CEN}})$ *and* $\frac{\partial \Pi_M^{\mathrm{CES}}}{\partial \delta} > 0$, $\frac{\partial \Pi_S^{\mathrm{CES}}}{\partial \delta} > 0$. *The retailer can then passively share demand forecast information because the manufacturer has the ability and incentive to give up a piece of their profit increment to incentivize them to offer more precise demand information.*

Regarding information sharing, Proposition 6.3 contends that the supply chain benefits more from the dual strategy of encroachment and production learning than from the scenario in which the encroachment and the production learning impact take precedence. Information sharing with encroachment and production learning capabilities will benefit the manufacturer and the supply chain, which will gain more from more precise demand information. The retailer has additional incentives to communicate demand information thanks to the dynamic competitive cooperation mechanism based on encroachment and production learning effect. The retailer is eager to provide demand information when demand signals are unclear willingly, and their profitability increases with the ambiguity of the demand information. In particular, the direct sales channel is less effective because of the comparatively high cost of direct sales for the manufacturer. The manufacturer will lower their wholesale prices and concentrate more on resale. The retailer actively shares more ambiguous demand information, highlighting the scale advantage based on the production learning effect. Hence the increase in retail sales. At the same time, the retailer will realize lower unit costs from lower wholesale prices, which will increase profit. Competition between the two channels is fierce if the manufacturer's direct sales costs are low. The manufacturer will benefit financially, whereas the retailer loses money if they actively give the manufacturer more precise demand information without compensation. Information sharing can result in a win–win scenario for the supply chain if the manufacturer transfers some of its incremental excess to encourage the retailer to share information.

To more graphically portray the conditions for the retailer to share demand information, Figure 3 simulates the impact of the interaction between information accuracy and production learning effect on the information sharing value of each member of the supply chain under the two-factor. We choose the parameter $m = 5$ in the numerical simulation program, picking any value within the acceptable range of the exogenous parameter information accuracy δ. It is observed that there is a shift in the direction of the value of information sharing across the supply chain participants. The information-sharing value of all partners in the supply chain shifts from a rising to a diminishing trend as demand information accuracy gradually improves. The larger the size of the information sharing value of all parties increases or declines with the growth in the production learning impact. The retailer's decision to provide information is impacted by the aforementioned developments. Therefore, in order to illustrate the effect of changes in the accuracy of demand information on the information-sharing value of supply chain participants, we have chosen four values, *i.e.*, $\delta = 15$, $\delta = 45$, $\delta = 60$ and $\delta = 200$, these are typical of stage changes.

By comparing Figures 3a–3d, which show the stage change of the accuracy of demand information from $\delta = 15$, $\delta = 45$, $\delta = 60$ to $\delta = 200$ as well as the proposition's conclusion, it can be observed that as can be shown in Figure 3a ($\delta = 15$), supply chain partners' information sharing value rises progressively with the degree

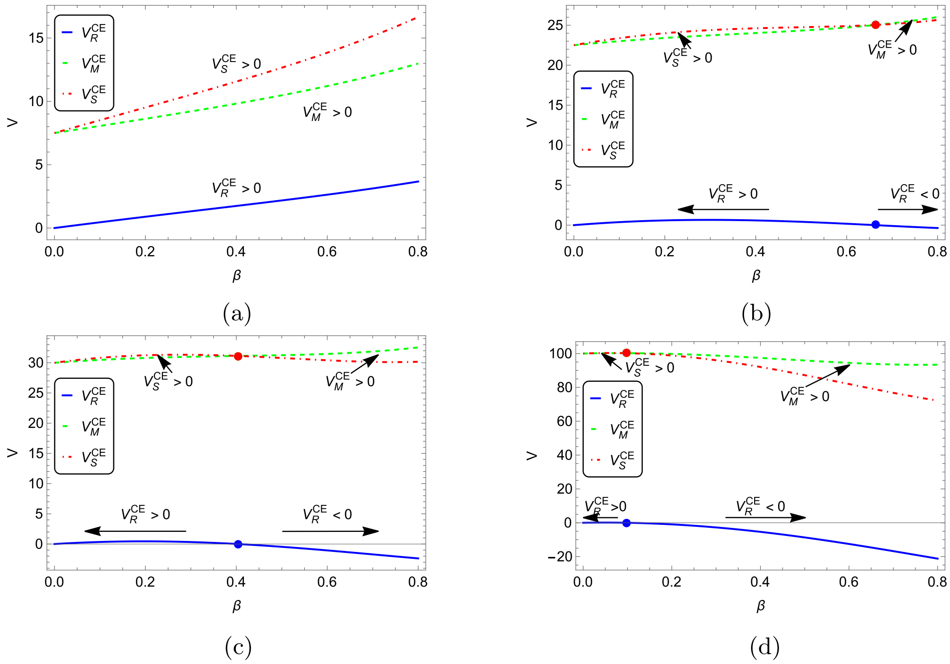

Figure 3. Information accuracy's impact on sharing value under dual-factor. (a) $\delta = 15$. (b) $\delta = 45$. (c) $\delta = 60$. (d) $\delta = 200$.

of production learning effect when demand information accuracy is poor. The trend of information sharing the value of supply chain participants changes gradually from increasing to decreasing, *i.e.*, it gradually decreases with the improvement of the production learning effect level, as demonstrated in Figures 3a–3d ($\delta = 45$, $\delta = 60$ to $\delta = 200$). When demand information is more accurate, the value of information sharing among all parties increases as the production learning effect level decreases. This means that the retailer's information sharing benefits more from a higher level of the production learning effect when demand information is unclear, and information sharing benefits more from a lower level of the production learning effect when demand information is more accurate. Specifically, More ambiguous information motivates the retailer to share proactively, even as sharing more precise demand information improves the manufacturer's capacity to shift incremental surpluses. These results can benefit all members. The retailer's informational advantage is more noticeable, resulting in a non-negative profit if the information is ambiguous. The retailer will proactively share. The shared value in the supply chain is non-negative when the demand signal is more accurate. To incentivize the retailer to be passive in information sharing, the manufacturer may transfer a portion of its incremental profit to offset the retailer's loss. Also from the latitude of change in the horizontal coordinates of the four graphs regarding the production learning effect, we discover that when the production learning effect improves, the supply chain member's information-sharing value increases due to sharing more ambiguous demand information. If more precise demand information is disclosed, the value of information sharing might only be maximized when the production learning effect is low.

7. Selection and implementation of incentive strategies: Profitability after sharing

By the non-negative value of supply chain information sharing, the retailer may share by combining Propositions 6.1 and 6.3. As per Proposition 6.1, encroachment dominance improves the manufacturer's capacity to transfer profit increments, encouraging the retailer to disseminate information passively. Furthermore, as per Proposition 6.3, production learning might increase profit increments for all supply chain members in the encroachment scenario, augmenting the retailer's incentives for sharing. Thus, this paper suggests three information-sharing incentive strategies based on the capabilities owned by the manufacturer under small-batch or mass-production business strategies, including encroachment capabilities and production learning capabilities. These strategies include a single-channel expansion strategy driven by encroachment, a single strategy of cost reduction caused by the production learning effect, and a dual strategy of channel expansion and cost reduction executed concurrently. To determine the best way to share, we cross-sectionally assess the incentive effects of three strategies on the retailer's sharing decisions around the profitability following information sharing. A unilateral payment contract implementing an optimal incentive strategy and corresponding managerial insights are presented on this basis.

Proposition 7.1. *Given a unilateral payment expense T^{CN}, if $T_0 < T^{\mathrm{CN}} < \min\{T_1, T_2\}$, the cost reduction strategy (CNS) is the optimal single strategy for sharing, than the channel expansion strategy (NES). Then $\hat{V}_R^{\text{CNS-NES}} > 0\,(\Pi_R^{\mathrm{CNS}} > \Pi_R^{\mathrm{NES}})$, $\hat{V}_M^{\text{CNS-NES}} > 0\,(\Pi_M^{\mathrm{CNS}} > \Pi_M^{\mathrm{NES}})$, $\hat{V}_S^{\text{CNS-NES}} > 0\,(\Pi_S^{\mathrm{CNS}} > \Pi_S^{\mathrm{NES}})$.*

According to Proposition 7.1, when combined with Proposition 6.1, we find that supply chain participants who share information benefit more from cost reduction strategies (CNS) than from channel expansion strategies (NES). It means that manufacturers can make supply chain parties more profitable in information sharing by implementing cost reduction strategies. However, it is important to note that since the value of supply chain information is always negative under the CNS strategy, then information cooperation is not necessarily in the best interest of all parties. However, if the manufacturer can fully compensate the retailer for the loss, then the supply chain parties can still benefit more from information sharing than by implementing a channel growth strategy.

By combining the requirements of Proposition 6.1, we create a unilateral payment contract that maximizes supply chain members' profitability while implementing the cost reduction strategy (CNS). We discover that incremental earnings T fulfill three conditions. First, the difference between the manufacturer's revenue and the transferred incremental profitability is more significant under the CNS strategy than the NES strategy. Second, the supplier chain's overall profit. The entire profit derived from the CNS strategy plus the additional profit transferred exceeds the profit from the NES strategy. Thirdly, the retailer's earnings. All the cumulative payments that are transferred and shared exceed the real gains that are not shared ($T_0 < T^{\mathrm{CN}} < \min\{T_1, T_2\}$). The best single strategy for sharing is to apply the cost reduction strategy (CNS), provided that the manufacturer and the retailer agree on payment within this incremental profit transfer.

Figure 4 compares the channel expansion single strategy (NES) to the cost reduction single strategy (CNS) and simulates the production learning effect concerning profitability following information communication. Taking parameter $a = 100$, $c = 20$, $m = 5$ and $\delta = 35$. According to the proposal, the cost-cutting single strategy (CNS) improves supply chain profitability for each member more than using the channel expansion single strategy (NES) alone under unilateral payments. Furthermore, this profitability rises as the production learning effect is enhanced. This illustrates the need for manufacturers to create mass-production strategies in an information-sharing supply chain to boost the profitability of each member of the chain.

Proposition 7.2. *When both channel expansion and cost reduction (CES) are implemented concurrently, as opposed to when only channel expansion (NES) is implemented, the profitability of all supply chain members increases, where $\hat{V}_R^{\text{CES-NES}} > 0\,(\Pi_R^{\mathrm{CES}} > \Pi_R^{\mathrm{NES}})$, $\hat{V}_M^{\text{CES-NES}} > 0\,(\Pi_M^{\mathrm{CES}} > \Pi_M^{\mathrm{NES}})$, and $\hat{V}_S^{\text{CES-NES}} > 0\,(\Pi_S^{\mathrm{CES}} > \Pi_S^{\mathrm{NES}})$. The dual strategy is the best for sharing in the event of an encroachment.*

Figure 5 compares the channel expansion single strategy (NES) to the dual strategy (CES) and simulates the production learning effect concerning profitability following sharing, taking parameters $a = 20$, $c = 10$, $m = 4$ and $\delta = 10$.

According to the proposal, the optimal strategy in an encroachment scenario is to pursue the dual strategies of channel expansion and cost reduction (CES). When a manufacturer uses an encroachment approach to grow

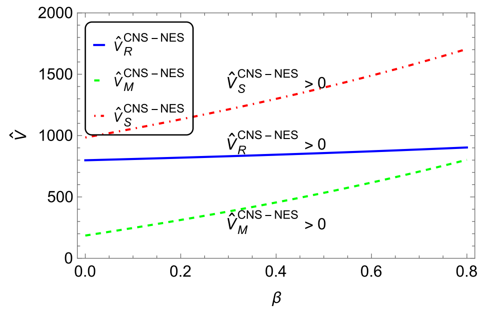

FIGURE 4. Profitability comparison: NES *vs.* CNS Strategies.

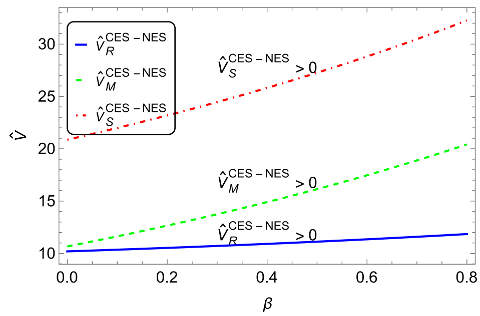

FIGURE 5. Profitability comparison: NES *vs.* CES Strategies.

their business, simultaneous production learning boosts the profitability of all supply chain members relative to the channel expansion strategy solely (NES). As the production learning effect rises, so does this ability. This implies that if the manufacturer intends to establish a direct sales channel, it should select mass production based on the production learning effect to maximize profits from cost savings in an information-sharing supply chain when presented with a strategic choice between high-selling price lean production and low-cost mass production.

Proposition 7.3. *When the cost reduction strategy and the channel expansion strategy (CES) are implemented concurrently, as opposed to a cost reduction single strategy (CNS), the manufacturer benefits more from information sharing* $\hat{V}_M^{\text{CES-CNS}} > 0\,(\Pi_M^{\text{CES}} > \Pi_M^{\text{CNS}})$*. The supplier chain and retailer may not always gain as much, however. Under certain unilateral payment circumstances, the profitability of information sharing may be optimized for all participants. The best way to motivate the retailer to contribute information is via a dual-factor (CES) strategy.*

(1) *When demand information is more accurate* $(\delta > \delta_1)$, $V_R < 0\,(\Pi_R^{\text{CES}} < \Pi_R^{\text{CEN}})$, $V_M > 0\,(\Pi_M^{\text{CES}} > \Pi_M^{\text{CEN}})$, $V_S > 0\,(\Pi_S^{\text{CES}} > \Pi_S^{\text{CEN}})$*, given a unilateral payment expense* T^{CE} $(T_3 < T^{\text{CE}} < T_4)$*. Using a dual strategy – expanding the channel while cutting costs – is the optimal strategy to get the retailer to share information. Then* $\hat{V}_R^{\text{CES-CNS}} > 0\,(\Pi_R^{\text{CES}} > \Pi_R^{\text{CNS}})$, $\hat{V}_M^{\text{CES-CNS}} > 0\,(\Pi_M^{\text{CES}} > \Pi_M^{\text{CNS}})$, $\hat{V}_S^{\text{CES-CNS}} > 0\,(\Pi_S^{\text{CES}} > \Pi_S^{\text{CNS}})$.

Proposition 7.3 focuses on the strategic choices that optimize the profitability of each supply chain member once information sharing is completed by comparing the profits of each member horizontally under the CES and CNS strategies. The proposal shows that concurrently implementing a dual strategy of channel expansion and cost reduction (CES) is always more profitable for the manufacturer than a single cost reduction strategy (CNS). Still, it is not necessarily better for the retailer and the supply chain.

Combining this with Proposition 6.3, we discover that when a manufacturer employs both a cost reduction and an encroachment strategy and when demand information is more precise, the retailer can lose money from the information-sharing mechanism. The retailer is reluctant to contribute even if the manufacturer and the supply chain stand to gain more. The benefit of sharing throughout the supply chain is now non-negative, meaning that the manufacturer makes more money than the retailer. The manufacturer is now able and willing to transmit a share of the additional profit to the retailer to encourage the retailer to take part in the information-sharing mechanism that maximizes revenues for both the retailer and the manufacturer.

The conditions of Proposition 6.3 may be combined to construct a unilateral payment contract that, by putting the best incentive strategy into place, maximizes the profitability of all supply-chain participants. We discover incremental gains that concurrently satisfy two requirements. Two requirements must be met: First, the manufacturer's profit: the gap between the profit and the incremental revenue moved under the CES strategy is more significant than the CEN strategy. Second, the retailer's profit: the sum of profit and the incremental revenue under the CES strategy and the incremental revenue is more than the CEN strategy ($T_3 < T^{\mathrm{CE}} < T_4$).

At this stage, the retailer will decide to disclose more precise information if the manufacturer and the retailer arrange payment within the parameters of the incremental profit transfer. The information-sharing mechanism will directly benefit the manufacturer by increasing incremental profits. Meanwhile, the retailer can grow their profits by taking advantage of the incremental surplus in the supply chain. Ultimately, by implementing the dual strategies of cost reduction and channel expansion, every link in the supply chain will work to increase its earnings. Accordingly, the manufacturer doing production learning may also think about pursuing an encroachment strategy, in which he may employ unilateral payment contracts to encourage the retailer to provide information and realize maximum profitability.

8. Summary of managerial implications

To a certain extent, our study resolves the issues of channel conflict and information cooperation in the supply chain by offering decision support on how the retailer with information advantages can share information in the face of potential encroachment and cost-reducing decisions by the manufacturer, as well as how the manufacturer with encroachment and production learning effects can select and implement incentives to encourage the retailer to share information to maximize the profitability of all parties in the supply chain. This research suggests the following management insights.

Firstly, our research demonstrates that the retailer's information-sharing value is always less than 0 in the one-factor scenario dominated by the production learning effect; second, the retailer's information-sharing value is always equal to 0 in the one-factor scenario dominated by the encroachment; and third, if the demand information is more ambiguous in the two-factor scenario where both the encroachment and the production learning effect coexist, the retailer's information sharing value is always greater than 0. The implication is that, without considering unilateral payment contracts, the retailer should refrain from information-sharing cooperation with the manufacturer that achieves cost reductions solely through production learning effects. Conversely, the retailer should share more ambiguous demand information if the manufacturer opens both direct sales channels and has production learning effects.

Secondly, our findings show that a single-win, win–win, or lose–lose situation is generated for the supply chain participants by the dynamic competition mechanism under the combined effects of encroachment and the production learning effect. The win–win area increases with higher direct selling costs, whereas the win–win area decreases with increased production learning effects. In other words, if a manufacturer opens up direct sales channels and experiences production learning effects at the same time, the retailer can think about setting up an information cooperation mechanism with the manufacturer that has lower production learning effects and higher direct sales costs, where there is a greater chance of a win–win outcome.

Thirdly, we also discover that when the production learning effect improves, the supply chain member's information-sharing value increases due to sharing more ambiguous demand information. If more precise demand information is disclosed, the value of information sharing might only be maximized when the production learning effect is low. This means that retailers should work with manufacturers at higher production learning effect levels when demand information is ambiguous, and conversely, when demand information is more precise, retailers should set up information-sharing mechanisms with manufacturers at lower production learning effect levels.

Finally, we show that putting into practice a dual strategy is always better than a channel expansion strategy but not always better than a cost reduction strategy. A unilateral payment contract renders the cost reduction strategy superior to the channel expansion strategy. The best incentive strategy to encourage retailer information sharing is to simultaneously implement the dual strategies of cost reduction and channel expansion. This suggests that manufacturers in industries where production learning is practiced should first consider creating a production strategy for mass production, cutting costs through production learning, and actively establishing channel channels rather than small-batch manufacturing. Manufacturers can optimize profitability in an information-sharing supply chain by using unilateral payment contracts or even sacrificing part of their profits in advance to encourage retailers to provide more precise demand information.

However, the management insights in this paper on information sharing decisions are somewhat limited. Future research could consider expanding in the following areas to improve the general applicability of management insights in this domain. In reality only the manufacturer has access to the level of the production learning effect. The bilateral asymmetry between learning speed and market size in the game could be further investigated in the future. Second, we do not take ex-post scenarios into account. Retailers may manipulate demand data to get cheaper wholesale pricing; as a result, suitable safeguards are required to guarantee the accuracy of the supplied information. Finally this work solely examines incentive strategies related to unilateral payment contracts. To improve the incentive mechanism, incentive strategies for multilateral contracts containing sharing ratios, such as two-part tariff contracts, may be further explored.

9. Conclusion

This work develops a two-period dual-channel supply chain model from the perspective of production learning effects and manufacturer encroachment from two dimensions. The manufacturer sells *via* a retailer who benefits from demand information advantages and who can both encroach and use production learning effects to achieve cost reductions. We use a stochastic variable to capture uncertain demand information through Bayesian updating, and the manufacturer's production costs in the second period can drop linearly with the product output in the first period at a stochastic learning rate. The manufacturer uses a dynamic pricing strategy in both periods, and the retailer chooses whether or not to share the demand information. Drawing on the perspectives of "encroachment dominance, production learning effects dominance, and two-factor", we go on to examine the equilibrium and optimal strategy for information sharing in six scenarios.

A substantial body of research indicates that manufacturer encroachment and the sharing of private demand information by retailers benefit manufacturers but not retailers. However, our study suggests that under the dual factors of encroachment and production learning effects, a dynamic competitive mechanism can lead to a win–win situation for both retailers and manufacturers from the cooperation of more ambiguous demand information. When direct selling costs are high, the win–win area is larger, and the greater the production learning effect, the more it benefits retailer information sharing. Interestingly, when implementing dual incentive strategies for cost reduction and channel expansion, we find that by establishing unilateral payment contracts, supply chain members can maximize their profitability through more accurate information sharing.

Specifically, The retailer in the production-learning effect-dominated scenario never exchanges demand information. When the encroachment predominates, the retailer losses following information sharing are offset by an additional surplus, and the manufacturer has the ability and motivation to encourage retailer information sharing. In contrast to the encroachment-led and production-learning effect-led scenarios, the dynamic competition mechanism based on encroachment and the production-learning effect generates higher incremental profits for all supply chain members than encroachment-dominated and production-learning-dominated scenarios. It increases incentives for the retailer to share demand information, which increases with greater economies of scale. First, encroachment and production learning make it beneficial for the retailer to provide ambiguous information proactively. Remarkably, the retailer's choice to share information proactively is determined by the first period of production learning, and the retailer may benefit from information sharing while demand information is vague. However, the retailer does not gain from the considerable cost reduction generated in the second period. Second, the production learning effect boosts the manufacturer's encroachment and raises incremental revenues in the supply chain. The manufacturer has an extraordinary ability and motive to give up a portion of the profit increase to motivate the retailer to provide precise demand information. Information-sharing in the second period is usually advantageous to the manufacturer, mainly when cost reduction is achieved. Third, from the perspective of the relationship between the retailer and the manufacturer and information sharing, encroachment and the two-period production learning effect provide a dynamic competitive mechanism for the information-sharing supply chain. This mechanism increases supply chain members' incremental profits to facilitate retailer information sharing. Specifically, depending on the learning and realization periods of production learning, the retailer and the manufacturer are always impacted by the encroachment in the opposite direction in both periods following information sharing, with a single win, win–win, or loss–loss condition. The information-sharing between the manufacturer and the retailer may result in a win–win scenario when the retailer's competitive position is in the win–win and single-win zones where demand information is ambiguous. Throughout the learning period, the sound effects of production learning offset the adverse impact of encroachment on the retailer. In addition, a more significant direct sales cost makes it easier for supply chain participants to establish a win–win situation; a more substantial production learning impact lessens the likelihood of this happening. When the production learning effect improves, the supply chain member's information-sharing value increases due to sharing more ambiguous demand information. If more precise demand information is disclosed, the value of information sharing might only be maximized when the production learning effect is low.

Depending on small-batch or mass production, manufacturers may employ either encroachment or production learning to increase profits. This paper proposes three information-sharing incentive strategies based on the manufacturer's encroachment and production learning capabilities. These strategies include an encroachment-dominated single-strategy for channel expansion, a production-learning-dominated single-strategy for cost reduction, and a dual-strategy for simultaneous channel expansion and cost reduction implementation. By comparing the profitability after information sharing, we discover that, in contrast to the channel expansion strategy, all supply chain members can gain more from implementing the cost reduction strategy if the manufacturer sacrifices a suitable profit in advance to facilitate retailer information sharing. A unilateral payment contract exists in this scenario, making the cost reduction strategy the best incentive single strategy. Interestingly, putting into practice a dual strategy is always better than a channel expansion strategy but not always better than a cost reduction strategy. Furthermore, a unilateral payment contract enables supply chain members to maximize profitability through more precise information sharing. This situation suggests that the optimal incentive strategy simultaneously implements cost reduction and channel expansion.

Even with the paper's critical conclusions, certain limitations still need more investigation. First, we assumed a linear demand function. However, the demand function may be unknown or take various forms, and suppliers and retailers may attempt to understand actual demand to maximize profits. Secondly, we assume that retailers are aware of the manufacturers' production learning levels. However, in reality, the level of production learning is mostly private information of the manufacturers. In this case, the bilateral asymmetry between the pace of learning and market size can be further examined. Third, because e-commerce platforms are so popular, they can also offer demand forecasting services. This means that e-commerce platforms can influence offline traditional retailers' decisions to share demand information, and future research can examine how e-commerce platforms affect the equilibrium results between manufacturers and traditional retailers. Fourth, we do not take ex-post scenarios into account. Retailers may manipulate demand data to get cheaper wholesale pricing; as a result, suitable safeguards are required to guarantee the accuracy of the supplied information. Finally, this work solely examines incentive strategies related to unilateral payment contracts. To improve the incentive mechanism, incentive strategies for multilateral contracts containing sharing ratios, such as two-part tariff contracts, may be further explored.

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Conflicts of interest

The authors declared no potential conflicts of interest with respect to the research, author- ship, and/or publication of this article.

Data availability statement

The data used to support the findings of this study are available from the corresponding author upon request.

References

[1] W. Shang, A.Y. Ha and S. Tong, Information sharing in a supply chain with a common retailer. *Manage. Sci.* **62** (2016) 245–263.

[2] H. Zhang, Vertical information exchange in a supply chain with duopoly retailers. *Prod. Oper. Manage.* **11** (2002) 531–546.

[3] D.-H. Yoon, Supplier encroachment and investment spillovers. *Prod. Oper. Manage.* **25** (2016) 1839–1854.

[4] A.Y. Ha, S. Tong and H. Zhang, Sharing demand information in competing supply chains with production diseconomies. *Manage. Sci.* **57** (2011) 566–581.

[5] X. Sun, W. Tang, J. Zhang and J. Chen, The impact of quantity-based cost decline on supplier encroachment. *Transp. Res. Part E: Logistics Transp. Rev.* **147** (2021) 102245.

[6] X. Sun, W. Tang, J. Chen, S. Li and J. Zhang, Manufacturer encroachment with production cost reduction under asymmetric information. *Transp. Res. Part E: Logistics Transp. Rev.* **128** (2019) 191–211.

[7] D. Zhao and Z. Li, The impact of manufacturer's encroachment and nonlinear production cost on retailer's information sharing decisions. *Ann. Oper. Res.* **264** (2018) 499–539.

[8] S. Shum, S. Tong and T. Xiao, On the impact of uncertain cost reduction when selling to strategic customers. *Manage. Sci.* **63** (2017) 843–860.

[9] Z. Li, S.M. Gilbert and G. Lai, Supplier encroachment under asymmetric information. *Manage. Sci.* **60** (2014) 449–462.

[10] Z. Zhang, H. Song, X. Gu, V. Shi and J. Zhu, How to compete with a supply chain partner: retailer's store brand vs. manufacturer's encroachment. *Omega* **103** (2021) 102412.

[11] A. Ha, X. Long and J. Nasiry, Quality in supply chain encroachment. *Manuf. Serv. Oper. Manage.* **18** (2016) 280–298.

[12] H. Yang, J. Luo and Q. Zhang, Supplier encroachment under nonlinear pricing with imperfect substitutes: bargaining power versus revenue-sharing. *Eur. J. Oper. Res.* **267** (2018) 1089–1101.

[13] S. Zhang, J. Zhang and G. Zhu, Retail service investing: an anti-encroachment strategy in a retailer-led supply chain. *Omega* **84** (2019) 212–231.

[14] W. Yan, Y. Xiong, J. Chu, G. Li and Z. Xiong, Clicks versus bricks: the role of durability in marketing channel strategy of durable goods manufacturers. *Eur. J. Oper. Res.* **265** (2018) 909–918.

[15] J. Li, Z. Hu, V. Shi and Q. Wang, Manufacturer's encroachment strategy with substitutable green products. *Int. J. Prod. Econ.* **235** (2021) 108102.

[16] A. Arya, B. Mittendorf and D.E. Sappington, The bright side of supplier encroachment. *Marketing Sci.* **26** (2007) 651–659.

[17] E.B. Tirkolaee, A. Goli, P. Ghasemi and F. Goodarzian, Designing a sustainable closed-loop supply chain network of face masks during the covid-19 pandemic: pareto-based algorithms. *J. Clean. Prod.* **333** (2022) 130056.

[18] A. Goli, A. Ala and S. Mirjalili, A robust possibilistic programming framework for designing an organ transplant supply chain under uncertainty. *Ann. Oper. Res.* **328** (2023) 493–530.

[19] A. Goli, A. Ala and M. Hajiaghaei-Keshteli, Efficient multi-objective meta-heuristic algorithms for energy-aware non-permutation flow-shop scheduling problem. *Expert Syst. App.* **213** (2023) 119077.

[20] A. Goli and E.B. Tirkolaee, Designing a portfolio-based closed-loop supply chain network for dairy products with a financial approach: accelerated benders decomposition algorithm. *Comput. Oper. Res.* **155** (2023) 106244.
[21] A. Goli, Integration of blockchain-enabled closed-loop supply chain and robust product portfolio design. *Comput. Ind. Eng.* **179** (2023) 109211.
[22] A.H. Alhilali and A. Montazerolghaem, Artificial intelligence based load balancing in SDN: a comprehensive survey. *Int. Things* **22** (2023) 100814.
[23] A. Montazerolghaem, M. Khosravi, F. Rezaee and M.R. Khayyambashi, An optimal workflow scheduling method in cloud-fog computing using three-objective harris-hawks algorithm, in 2022 12th International Conference on Computer and Knowledge Engineering (ICCKE). IEEE (2022) 300–306.
[24] A.Y. Ha, H. Luo and W. Shang, Supplier encroachment, information sharing, and channel structure in online retail platforms. *Prod. Oper. Manage.* **31** (2022) 1235–1251.
[25] H. Zheng, G. Li, X. Guan, S. Sethi and Y. Li, Downstream information sharing and sales channel selection in a platform economy. *Transp. Res. Part E: Logistics Transp. Rev.* **156** (2021) 102512.
[26] Y. Tsunoda and Y. Zennyo, Platform information transparency and effects on third-party suppliers and offline retailers. *Prod. Oper. Manage.* **30** (2021) 4219–4235.
[27] X. Lin, Q. Lin and Y.-J. Chen, Reselling/agency selling and online intermediaries' information sharing with manufacturers and resellers. *Prod. Oper. Manage.* **33** (2024) 264–281.
[28] M. Liu, E. Cao and C.K. Salifou, Pricing strategies of a dual-channel supply chain with risk aversion. *Transp. Res. Part E: Logistics Transp. Rev.* **90** (2016) 108–120.
[29] J. Zhang, S. Li, S. Zhang and R. Dai, Manufacturer encroachment with quality decision under asymmetric demand information. *Eur. J. Oper. Res.* **273** (2019) 217–236.
[30] H. Guan, Z. Yang and H. Gurnani, Value of information with manufacturer encroachment. *Prod. Oper. Manage.* **32** (2023) 780–793.
[31] X. Chen, B. Li, W. Chen and S. Wu, Influences of information sharing and online recommendations in a supply chain: reselling versus agency selling. *Ann. Oper. Res.* **329** (2023) 717–756.
[32] T. Li and H. Zhang, Gaining by ceding-bounded wholesale pricing for information sharing in a supply chain. *Prod. Oper. Manage.* **32** (2023) 829–843.
[33] A.Y. Ha, Q. Tian and S. Tong, Information sharing in competing supply chains with production cost reduction. *Manuf. Serv. Oper. Manage.* **19** (2017) 246–262.
[34] S. Huang, X. Guan and Y.-J. Chen, Retailer information sharing with supplier encroachment. *Prod. Oper. Manage.* **27** (2018) 1133–1147.
[35] S. Zhang and J. Zhang, Contract preference with stochastic cost learning in a two-period supply chain under asymmetric information. *Int. J. Prod. Econ.* **196** (2018) 226–247.
[36] Q. Zhang, W. Tang and J. Zhang, Green supply chain performance with cost learning and operational inefficiency effects. *J. Clean. Prod.* **112** (2016) 3267–3284.
[37] T. Li, S.P. Sethi and X. He, Dynamic pricing, production, and channel coordination with stochastic learning, *Prod. Oper. Manage.* **24** (2015) 857–882.
[38] S. Deng, X. Guan and J. Xu, The coopetition effect of learning-by-doing in outsourcing. *Int. J. Prod. Res.* **59** (2021) 516–541.
[39] A. Gupta, S. Jayaswal and B. Mantin, Who benefits from supplier encroachment in the presence of manufacturing cost learning? *Prod. Oper. Manage.* **33** (2024). DOI: 10.1177/10591478241253552.
[40] S. Yi and F. Guo, Fundament of Industrial Engineering (2007) 131–134.

Appendix A.

Proof of Theorem 4.1. Optimizing equation (1) gives the direct sales quantity for a given $(w,\, q_R)$ as:

$$q_M^{\mathrm{NEN}}(q_R) = \frac{a - c - m - q_R}{2}.$$

Substituting $q_M^{\mathrm{NEN}}(q_R)$ into equation (2) and optimizing it, we obtain the retail quantity:

$$q_R^{\mathrm{NEN}}(w) = \frac{a + c + m - 2w + 2\tilde{y}}{2}.$$

Substitute $q_R^{\mathrm{NEN}}(w)$ into $q_M^{\mathrm{NEN}}(q_R)$ to obtain the manufacturer's equilibrium sales:

$$q_M^{\mathrm{NEN}}(q_R) = \frac{a - 3(c + m) + 2w - 2\tilde{y}}{4}.$$

Also by substituting $E[q_M] = E[q_M^{\mathrm{NEN}}(w)] = \frac{a-3(c+m)+2w}{4}$, $E[q_R] = E[q_R^{\mathrm{NEN}}(w)] = \frac{a+c+m-2w}{2}$ into equation (1) and optimizing, we obtain the manufacturer's profit function on:

$$\mathrm{Max}\, \Pi_M^{\mathrm{NEN}}(w) = \frac{a^2 - 14ac + c^2 - 6am + 10cm + 9m^2 + 12(a + c)w - 4mw - 12w^2}{16}.$$

Optimizing this equation, equilibrium wholesale price can be obtained as:

$$w^{\mathrm{NEN}} = \frac{3a + 3c - m}{6}.$$

Substituting the above equation into $q_M^{\mathrm{NEN}}(w)$ and $q_R^{\mathrm{NEN}}(w)$, and finding the expectation, we can obtain the equilibrium sales:

$$q_M^{\mathrm{NEN}}(w) = \frac{3a - 3c - 5m}{6},$$
$$q_R^{\mathrm{NEN}}(w) = \frac{2m + 3\tilde{y}}{3}.$$

Substituting the equilibrium solutions w^{NEN}, $q_R^{\mathrm{NEN}}(w)$, $q_M^{\mathrm{NEN}}(w)$ into equations (1) and (2) respectively, the expected profits under the NEN strategy are obtained as shown in Theorem 4.1. Where $A = 3(a - c)^2 + 6(-a + c)m$. □

Proof of Theorem 4.2. Paralleled to that of Theorem 4.1, which is omitted. □

Proof of Theorem 4.3. First, given the wholesale price w_2, the retailer decides q_{R2}, which is easily known $\Pi_{R2}^{\mathrm{CNN}}(q_{R2}, w_2)$ to be a concave function of q_{R2}. The first-order optimality criteria that yield the equilibrium sales quantity of the retailer as:

$$q_{R2}^{\mathrm{CNN}}(w_2|q_{R1}) = \frac{a - w_2 + \tilde{y}}{2}.$$

Since the demand signal is not shared at this point, we get:

$$E[q_{R2}] = E\left[q_{R2}^{\mathrm{CNN}}\right] = \frac{a - w_2}{2},$$

which is substituted into equation (6), the manufacturer decides on the wholesale price w_2, which is easily known $\Pi_{M2}^{\mathrm{CNN}}(w_2)$ to be a concave function of w_2, and the equilibrium wholesale price can be obtained:

$$w_2^{\mathrm{CNN}}(q_{R1}) = \frac{a + c - \beta q_{R1}}{2}.$$

Substituting the equilibrium solution w_H^{CNN}, $q_{RH}^{\mathrm{CNN}}(w_H)$ into equations (5)–(8), respectively, the equilibrium solution and the expected profit for each stage under the CNN strategy can be obtained, as shown in Theorem 4.3.

Where $B_0 = (\beta(\beta^3 - 88\beta - 128) + 512)(a - c)^2$, $B_1 = (\beta - 8)\beta - 32$, $B_2 = (\beta(\beta(\beta(3\beta - 80) - 272) + 1792) + 6144)(a - c)^2$. □

Proof of Theorem 4.4. Paralleled to that of Theorem 4.3, which is omitted. Where $C = \beta(8\tilde{y} + (\beta + 8)(a - c))/2\left(\beta^2 - 16\right)$. □

Proof of Theorem 4.5. Where

$$\begin{aligned}
D_0 &= 9a^2(-1+\beta) + 9c^2(-1+\beta) + 3cm(-6+\beta(4+\beta)) \\
&\quad + m^2(-21+\beta(15+(-2+\beta)^2\beta)) \\
&\quad - 3a(6c(-1+\beta) + m(-6+\beta(4+\beta))), \\
D_1 &= 2m^2(-a+c+2m)(-2+\beta)(-a+c+m)^3/9, \\
D_2 &= -\left(36a^2 + 36c^2 + 72cm - 72a(c+m) + m^2(116+5(-4+\beta)\beta(2+\beta))\right)/36 \\
&\quad + (-2+\beta)^2\beta^2(2+\beta)(-8+\beta^2)\delta/64.
\end{aligned}$$

□

Proof of Theorem 4.6. Paralleled to that of Theorem 4.3, which is omitted.
Where

$$\begin{aligned}
m_1 &= \frac{3(-6+\beta)}{\beta(-8+2\beta+3\beta^2)}, \\
m_2 &= \frac{(-6+\beta)}{(2+\beta)(-5+2\beta)}, \\
m_3 &= \frac{2(-6+\beta)}{(2+\beta)^2(-5+2\beta)}, \\
K_0 &= -2(2+\beta)(3a(-6+\beta) - 3c(-6+\beta) + m(30+\beta(-17+4\beta))) + 3(-6+\beta)(-4+(-2+\beta)\beta)\tilde{y}, \\
K_1 &= 3a + m + \frac{18m}{(-6+\beta)} - \frac{3(-a+c+m)}{(-2+\beta)}, \\
K_2 &= \frac{2cm(6-5\beta)}{(-6+\beta)} + a^2(-1+\beta) + c^2(-1+\beta) + 2a\left(c + m\left(5 + \frac{24}{(-6+\beta)}\right) - c\beta\right) \\
&\quad + \frac{m^2(2+\beta)(-3+2\beta)(14+(-7+\beta)\beta)}{(-6+\beta)^2} + \frac{2\left(-2+\beta^2\right)\delta}{(2+\beta)^2}. \\
K_3 &= (2+\beta)^2\Bigg[4\Bigg(3a^2(-6+\beta)^2 + 3c^2(-6+\beta)^2 - 6cm(-6+\beta)(6+(-3+\beta)\beta) \\
&\quad + 6a(-6+\beta)(-c(-6+\beta) + m(6+(-3+\beta)\beta))\Bigg) \\
&\quad + m^2(252+\beta(-300+\beta(151-34\beta+4\beta^2)))\Bigg] \\
&\quad + 3(-6+\beta)^2(-2+\beta)^2\delta + 12(-6+\beta)^2\beta^2\delta, \\
K_4 &= 12a^2 + 12c^2 + 24cm - 24a(c+m) + \frac{4m^2(-42+\beta(11+2\beta))}{(-6+\beta)} - \frac{3(-8+\beta^2)\delta}{(2+\beta)}, \\
K5 &= 2(a-c)m + \frac{m^2(-1044-(-12+\beta)\beta(37+4\beta))}{9(-6+\beta)^2} \\
&\quad + \frac{-4(a-c)^2(2+\beta) + \left(-8+\beta^2\right)\delta}{4(2+\beta)}.
\end{aligned}$$

□

Proof of Corollary 5.1. (1)–(3) Concerning Theorem 4.6, Corollary 5.1 can be proved, which is omitted. □

Proof of Corollary 5.2. Regarding Theorems 4.2 and 4.6, Corollary 5.2 can be proved, which is omitted. □

Proof of Proposition 6.1. Concerning Theorems 4.2–4.4, Proposition 6.1 can be proved, which is omitted. □

Proof of Proposition 6.2. From Theorems 4.5 and 4.6, the following comparative results are as below:

(1) Let

$$f_{\Pi_{R1}}(\delta) = \Pi_{R1}^{\text{CES}} - \Pi_{R1}^{\text{CEN}} = \frac{4m^2(2+\beta)}{(-6+\beta)^2} + \frac{(-4+\beta^2)\left(8m^2+9\beta^2\delta\right)}{144}.$$

We get

$$\frac{\partial f_{\Pi_{R1}}(\delta)}{\partial \delta} = \frac{\beta^2(-4+\beta^2)}{16} < 0,$$
$$f_{\Pi_{R1}}(\delta_1) = \Pi_{R1}^{\text{CES}} - \Pi_{R1}^{\text{CEN}} = 0.$$

If $0 < \delta < \delta_1$, it is obvious that

$$f_{\Pi_{R1}}(\delta) = \Pi_{R1}^{\text{CES}} - \Pi_{R1}^{\text{CEN}} > 0.$$

(2) (a) Let

$$f_{w_1}(m) = w_1^{\text{CES}} - w_1^{\text{CEN}} = \frac{2m\beta}{3(-6+\beta)} + \frac{\left(-2+\beta^2\right)\tilde{y}}{(-4+\beta^2)}.$$

We get

$$\frac{\partial f_{w_1}(m)}{\partial m} = \frac{2\beta}{3(-6+\beta)} < 0,$$
$$f_{w_1}(\tilde{y}m_4) = w_1^{\text{CES}} - w_1^{\text{CEN}} = \frac{\tilde{y}(8+\beta(-6+(-2+\beta)\beta))}{\beta(-4+\beta^2)} < 0.$$

If $m > \tilde{y}m_4$, it is obvious that

$$f_{w_1}(m) = w_1^{\text{CES}} - w_1^{\text{CEN}} < 0.$$

The proof of $w_2^{\text{CES}} < w_2^{\text{CEN}}$, $q_{R1}^{\text{CES}} > q_{R1}^{\text{CEN}}$ and $q_R^{\text{CES}} > q_R^{\text{CEN}}$ are paralleled, which we omit them there.

Let

$$f_{q_{M1}}(m) = q_{M1}^{\text{CES}} - q_{M1}^{\text{CEN}} = m\left(\frac{14}{6}(-6+\beta) - \frac{\beta}{6}\right)\beta - \frac{2\tilde{y}}{(-4+\beta^2)}.$$

If $a > c + ma_1$ and $m > \tilde{y}m_4$, we get

$$\frac{\partial f_{q_{M1}}(m)}{\partial m} = -\frac{14m}{(-6+\beta)^2} - \frac{m\beta}{3} + \frac{4\beta\tilde{y}}{(-4+\beta^2)^2} < 0,$$
$$f_{q_{M1}}(\tilde{y}m_4) = q_{M1}^{\text{CES}} - q_{M1}^{\text{CEN}} = \frac{(28+\beta(-18+(-6+\beta)\beta))\tilde{y}}{2(-2+\beta)\beta} < 0.$$

If $m > \tilde{y}m_4$, it is obvious that

$$f_{q_{M1}}(m) = q_{M1}^{\text{CES}} - q_{M1}^{\text{CEN}} < 0.$$

The proof of $q_M^{\text{CES}} < q_M^{\text{CEN}}$ are paralleled, which we omit it there.

Let

$$\begin{aligned} f_{\Pi_{M1}}(\delta) &= \Pi_{M1}^{\text{CES}} - \Pi_{M1}^{\text{CEN}} \\ &= \frac{m(-2+\beta)\beta(-3(a-c)(-6+\beta)\beta}{9(-2+\beta)^2(-6+\beta)^2} \\ &\quad + \frac{m(24+(-12+\beta)\beta)(-6+(-2+\beta)\beta))}{9(-2+\beta)^2(-6+\beta)^2} \\ &\quad - \frac{18(-2+\beta^2)\delta}{9(-2+\beta)^2(2+\beta)^2}. \end{aligned}$$

We get

$$\frac{\partial f_{\Pi_{M1}}(\delta)}{\partial \delta} = -\frac{2(-2+\beta^2)}{(-4+\beta^2)^2} < 0,$$

and if $a > c + ma_1$ and $m > \tilde{y}m_4$,

$$f_{\Pi_{M1}}(\delta_2) = \Pi_{M1}^{\text{CES}} - \Pi_{M1}^{\text{CEN}} = 0.$$

So if $0 < \delta < \delta_2$, it is obvious that

$$f_{\Pi_{M1}}(\delta) = \Pi_{M1}^{\text{CES}} - \Pi_{M1}^{\text{CEN}} < 0.$$

(b) Paralleled to that of (a), and the proof of (b) is omitted. This proves Proposition 6.2. Where

$$\begin{aligned} a_1 &= \frac{(24 + (-12 + \beta)\beta)(-6 + (-2 + \beta)\beta)}{3(-6 + \beta)\beta}, \\ m_4 &= \frac{-3(-6 + \beta)(-4 + \beta(2 + \beta))}{\beta^2(-4 + \beta^2)}, \\ \delta_1 &= \frac{-8(60 + (-14 + \beta)\beta)m^2}{9(-6 + \beta)^2(-2 + \beta)\beta}, \\ \delta_2 &= \frac{m(-2 + \beta)\beta(2 + \beta)^2(-3(a - c)(-6 + \beta)\beta}{18(-6 + \beta)^2(-2 + \beta^2)} \\ &\quad + \frac{m(24 + (-12 + \beta)\beta)(-6 + (-2 + \beta)\beta))}{18(-6 + \beta)^2(-2 + \beta^2)}. \end{aligned}$$

□

Proof of Proposition 6.3. (1) Concerning Theorems 4.5 and 4.6, (1) can be proved, which is omitted.
(2) Let

$$\begin{aligned} f_R(\delta) &= V_R^{\text{CE}} = \Pi_R^{\text{CES}} - \Pi_R^{\text{CEN}} \\ &= \frac{\beta(2 + \beta)\big(8m^2(60 + (-14 + \beta)\beta) + 9(-6 + \beta)^2(-2 + \beta)\beta\delta\big)}{144(-6 + \beta)^2}. \end{aligned}$$

We get

$$\frac{\partial f_R(\delta)}{\partial \delta} = \frac{\beta^2(-4 + \beta^2)}{16} < 0,$$
$$f_R(\delta_1) = 0.$$

If $0 < \delta < \delta_1$,

$$f_R(\delta) > 0,$$

and if $\delta > \delta_1$,

$$f_R(\delta) < 0.$$

This proves Proposition 6.3.

□

Proof of Proposition 7.1. From Theorems 4.5 and 4.6, the following comparative results are as below:

$$\begin{aligned} \hat{V}_R^{\text{CNS-NES}} &= \Pi_R^{\text{CNS}} - \Pi_R^{\text{NES}} = -\frac{(-4 + \beta^2)(8m^2 + 9\beta^2\delta)}{144} > 0, \\ \hat{V}_M^{\text{CNS-NES}} &= \Pi_M^{\text{CNS}} - \Pi_M^{\text{NES}} = -\frac{a^2 - 2ac + c^2 - 2am + 2cm + 5m^2 + \delta}{4} \\ &\quad - \frac{a^2 - 2ac + c^2 + 2am - 2cm + m^2}{-2 + \beta} \\ &\quad - \frac{m^2\beta^2}{12} + \frac{\beta^2(16 - 8\beta^2 + \beta^4)\delta}{64} > 0, \\ \hat{V}_S^{\text{CNS-NES}} &= \Pi_S^{\text{CNS}} - \Pi_S^{\text{NES}} \\ &= \frac{-16(2 + \beta)(9a^2 + 9c^2 + 18cm - 18a(c + m)}{576(-2 + \beta)} \end{aligned}$$

$$+\frac{m^2(29+5(-4+\beta)\beta))+288\delta}{576(-2+\beta)}$$
$$+\frac{\beta\delta(-16+(-2+\beta)^2\beta(2+\beta)(-8+\beta^2))}{64(-2+\beta)}>0.$$

Since $\Pi_R^{\mathrm{CNS}}+T^{\mathrm{CN}}>\Pi_R^{\mathrm{CNN}}$, $\Pi_M^{\mathrm{CNS}}-T^{\mathrm{CN}}>\Pi_M^{\mathrm{NES}}$, $\Pi_S^{\mathrm{CNS}}-T^{\mathrm{CN}}>\Pi_S^{\mathrm{NES}}$, we can obtain:

$$\begin{aligned}
T^{\mathrm{CN}}>\Pi_R^{\mathrm{CNS}}-\Pi_R^{\mathrm{CNN}}&=-3(-32+\beta^2)\delta/16(-16+\beta^2)=T_0,\\
T^{\mathrm{CN}}<\Pi_M^{\mathrm{CNS}}-\Pi_M^{\mathrm{NES}}&=\big(-3(a-c)^2+6(a-c)m-7m^2-3\delta\big)/12\\
&\quad-4(a-c)^2(-128+\beta(-48+\beta(3+2\beta)))\\
&\quad+\frac{(512-32\beta^2+\beta^4)\delta}{8(-16+\beta^2)^2}\\
&=T_1,\\
T^{\mathrm{CN}}<\Pi_S^{\mathrm{CNS}}-\Pi_S^{\mathrm{NES}}&=\frac{(a-c)^2(6144+\beta(1792+\beta(-272+\beta(-80+3\beta))))}{64(-16+\beta^2)^2}\\
&\quad+\frac{4(1536-112\beta^2+3\beta^4)\delta}{64(-16+\beta^2)^2}\\
&\quad+\frac{-9(a-c)^2+18(a-c)m-29m^2-9\delta}{36}\\
&=T_2.
\end{aligned}$$

i.e., $T_0<T<\min\{T_1,T_2\}$. □

Proof of Proposition 7.2. From Theorems 4.2 and 4.6, the following comparative results are as below:

$$\begin{aligned}
\hat{V}_R^{\text{CES-NES}}&=\Pi_R^{\mathrm{CES}}-\Pi_R^{\mathrm{NES}}=\frac{4m^2(2+\beta)}{(-6+\beta)^2}>0,\\
\hat{V}_M^{\text{CES-NES}}&=\Pi_M^{\mathrm{CES}}-\Pi_M^{\mathrm{NES}}\\
&=-\frac{(2+\beta)^2}{4(-4+\beta^2)}\left(\frac{a^2+c^2+2cm-2a(c+m)}{-6+\beta}\right)\\
&\quad-\frac{(2+\beta)^2}{4(-4+\beta^2)}\left(\frac{m^2\left(-\frac{14}{-6+\beta}+\frac{5\beta}{-6+\beta}\right)}{-6+\beta}\right)\\
&\quad-\frac{(2+\beta)^2}{4(-4+\beta^2)}\cdot 4\delta>0.\\
\hat{V}_S^{CES-NES}&=\Pi_S^{CES}-\mathit{\Pi}_S^{NES}\\
&=-9(2+\beta)^2c^2(-6+\beta)^2-18(2+\beta)^2cm(-6+\beta)^2\\
&\quad-9(2+\beta)^2m^2(116+5(-12+\beta)\beta)\\
&\quad-36(-6+\beta)^2\delta/36(-6+\beta)^2(-4+\beta^2)\\
&\quad+\frac{18a(c+m)(2+\beta)-9a^2(2+\beta)}{36(\beta-2)}>0.
\end{aligned}$$

□

Proof of Proposition 7.3. From Theorems 4.4 and 4.6, the following comparative results are as follows:

$$\begin{aligned}
\Pi_M^{\mathrm{CES}}-\Pi_M^{\mathrm{CNS}}=\frac{1}{12(-2+\beta)}\big(&12a^2+12c^2+24cm-24a(c+m)\\
&+\frac{4m^2(-42+\beta(11+2\beta))}{-6+\beta}-\frac{3(-8+\beta^2)\delta}{2+\beta}\bigg)
\end{aligned}$$

$$
-\frac{1}{8(-16+\beta^2)^2}\big(-4(a-c)^2(-128+\beta(-48+\beta(3+2\beta)))
+(512-32\beta^2+\beta^4)\delta\big)>0.
$$

Since $\Pi_R^{\text{CES}}+T^{\text{CE}}>\Pi_R^{\text{CEN}}$, $\Pi_M^{\text{CES}}-T^{\text{CE}}>\Pi_M^{\text{CEN}}$, we can obtain:

$$
\begin{aligned}
T^{\text{CE}} &> \Pi_R^{\text{CEN}}-\Pi_R^{\text{CES}} \\
&= \frac{-\beta(2+\beta)\big(8m^2(60+(-14+\beta)\beta)+9(-6+\beta)^2(-2+\beta)\beta\delta\big)}{144(-6+\beta)^2} \\
&= T_3, \\
T^{\text{CE}} &< \Pi_M^{\text{CES}}-\Pi_M^{\text{CEN}} \\
&= \frac{m^2(-8+\beta)(2+\beta)}{12(-6+\beta)}-\frac{(128-80\beta^2+48\beta^4-12\beta^6+\beta^8)\delta}{64(-4+\beta^2)} \\
&= T_4,
\end{aligned}
$$

i.e., $T_3<T<T_4$. □