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MULTI-OBJECTIVE OPTIMIZATION BASED DECISION-MAKING PROCESS AND ITS APPLICATION TO OPTIMALLY SELECT SUITABLE GREENHOUSE SITE FOR TOMATO CROPS

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Abstract. In the modern agricultural system, the necessity for greenhouses is increasingly demanding due to unfavorable climatic conditions that hugely impact crop yields. The production of essential but very much weather-sensitive crops like tomatoes, beans, etc, can be improved by considering a greenhouse environment by regulating temperature, humidity, etc. In West Bengal's lush plains tomato production has not been able to keep up with the growing demand, and prices for tomatoes in the state's major cities have increased significantly in the recent past. However, designing the ideal greenhouse entails some ambiguity and complexity, given the variability in the climatic patterns and crop requirements. This uncertainty can be efficiently examined utilizing cylindrical neutrosophic set (CNS), which helps to manage the ambiguous and contradictory information inherent in decision-making processes, resulting in more exact and reliable greenhouse planning. Furthermore, the Dombi logarithmic law produces a very strong and consistent output result with a slight variation in operating parameters. In this research article, we have applied our proposed decision-making process to determine the best greenhouse site for cultivating tomato crops. For this purpose, we have defined Dombi logarithmic aggregation operational laws in the framework of cylindrical neutrosophic numbers (CNN) and utilized these laws to establish a new aggregation operator namely cylindrical neutrosophic Dombi weighted logarithmic aggregation operator ($\mathscr{CNDWLA}$). The said aggregation operational laws & aggregation operator have been applied to present a new and novel decision-making process where full consistency method (FUCOM) and multi-objective optimization (MOO) have been integrated and embedded fruitfully. Here, the objective functions were formulated using the concept of a single-layer neural network and then MOO and FUCOM methods are implemented to assess criterion weights. We have resolved the most favorable pareto optimal solution derived from MOO by employing simulation and the method for order of preference by similarity to ideal solution (TOPSIS) approach. We also discovered that measurement alternatives and ranking according to compromise solution (MARCOS) and the multi-objective optimization on the basis of ratio analysis (MOORA) methods have not been utilized in the CN environment. Therefore, we have applied our proposed decision-making method with MARCOS and MOORA techniques to determine the optimal greenhouse site for tomato production in West Bengal. An exhaustive sensitivity and comparison analysis have been conducted to assess the stability and robustness of our multi-criteria group decision-making (MCGDM) method. The analysis of our study points out that South Bengal is the most appropriate greenhouse place for cultivating tomatoes in West Bengal.

Keywords. Cylindrical neutrosophic Dombi weighted logarithmic aggregation operator ($\mathscr{CNDWLA}$), MCGDM, greenhouse site selection, multi-objective optimization (MOO), full consistency method (FUCOM).

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1. Introduction

In recent years, mathematical modeling in the field of uncertainty has experienced a paradigm shift due to the emergence of various mathematical frameworks that intended to manage imprecise and hesitant data. Fuzzy set theory, first presented by Zadeh [65] is one of these; it pioneered the representation of uncertainty through degrees of membership, and it provides an effective tool for making decisions in unpredictable situations. Following that, intuitionistic fuzzy set theory given by Atanassov and Stoeva [5] had improved and expanded the possibilities for uncertainty modeling and decision-making processes. Moreover, neutrosophic set (NS) theory developed by Smarandache [61] is a mathematical framework that extends the traditional concepts of fuzzy set theory, and intuitionistic fuzzy set theory, which accommodates the function of truth-membership ($\mathscr{G}_{\widehat{\Lambda}}(w)$), the function of indeterminacy-membership ($\mathscr{L}_{\widehat{\Lambda}}(w)$) and the function of falsity-membership ($\mathscr{M}_{\widehat{\Lambda}}(w)$) to capture the uncertainty through an interactive and multi-dimensional approach. In recent days, researchers have been trying their best to explore the theoretical underpinnings of NS theory to tackle intricate real-world issues where fuzzy and conventional set theory seems to be inadequate. A nonlinear programming model has been developed by Seikh and Dutta [53] to solve matrix games including single-valued neutrosophic number payoffs. Also, Seikh and Dutta [54] provided the solution of matrix games including single-valued trapezoidal neutrosophic number payoffs. Furthermore, researchers have embedded various forms of the NS that includes triangular neutrosophic sets [8], interval NS [20], trapezoidal neutrosophic number [7], pentagonal neutrosophic number [11], probability multiple valued NS [41] etc. In this context, cylindrical neutrosophic set (CNS) theory [12] has become a successful outcome, which gives a robust mechanism for dealing with uncertainties incorporating directional features by permitting the representation of uncertainty through membership degrees in a cylindrical manner.

In the intricate world of today, there is a growing demand for sophisticated decision-making systems to tackle ambiguous data. Multi-criteria group decision-making (MCGDM) has grown much with its ingenious significance in the decision science domain as it can provide a systematic approach to assess and prioritize actions based on diverse and often conflicting criteria, addressing the rising complexity of contemporary decision settings. Few MCGDM techniques in neutrosophic and cylindrical neutrosophic environments have been developed and explored. Peng *et al.* [40] implemented simplified NS applications in MCGDM. Fu *et al.* [19] described a group decision-making approach centered on group satisfaction. Durmić [15] utilized full consistency method (FUCOM) to assess the criteria for sustainable supplier selection. Fazlollahtabar *et al.* [18] implemented the FUCOM approach in group decision-making when choosing forklifts for a warehouse. Saha *et al.* [49] employed FUCOM and "measurement alternatives and ranking according to compromise solution (MARCOS)" methods for group decision-making utilizing Dombi power aggregation of dual probabilistic linguistic data. For the evaluation of pharmaceutical enterprises in a single-valued neutrosophic context, Rong *et al.* [47] suggested a hybrid group decision procedure incorporating MARCOS and regret theory. Peng *et al.* [42] formed a new MCGDM technique based on MARCOS and provided a real application through CPP selection. Amini [1] provided a fuzzy technique for order of preference by similarity to ideal solution (TOPSIS) based MCGDM methodology to sort out rural industrial sites. Hamdani and Wardoyo [24] employed the TOPSIS algorithm for complexity calculation for group decision-making. Brauers *et al.* [10] demonstrated the utilization of MOORA in location theory and MCDM. Pérez-Domínguez *et al.* [44] applied MOORA method in pythagorean fuzzy environment for MCDM. In decision-making procedures, aggregation operators play a very important and essential role, particularly when MCGDM is involved, to aggregate data from several criteria or features to give a final assessment or rating of options using these aggregation operators. Garg [21] proposed new logarithmic operational rules for pythagorean fuzzy sets and their utilizations, along with their aggregation operators. Rahman [45] developed a few novel logarithmic aggregation operators and their use in group decision-making problems based on t-norm and t-conorm. Haque *et al.* [25] provided logarithmic operational rules and aggregation operators for trapezoidal neutrosophic numbers. Ashraf *et al.* [4] developed spherical fuzzy Dombi aggregation operators and used them to group decision-making problems. Khan *et al.* [26] employed improved spherical fuzzy Dombi aggregation operators: uses in systems for decisions. Seikh and Mandal [55] provided intuitionistic fuzzy Dombi aggregation operators with its application to multiple attribute decision-making (MADM). Mandal and Seikh [32] developed an “interval-valued spherical fuzzy multi-attribute border approximation area comparison (MABAC)” method based on Dombi aggregation operators considering unknown attribute weights for selecting the best “plastic waste management process”. Seikh and Mandal [58] had manifested an interval-valued Fermatean fuzzy Dombi aggregation operator and showed its application in biomedical waste management using the “stepwise weight assessment ratio analysis (SWARA)” based “preference ranking organization method for enrichment evaluation (PROMETHEE II)” method.

However, to enhance the precision and effectiveness of decision-making processes, recent works on MCGDM have shown a noticeable trend towards the usage of “machine learning”, sophisticated optimization algorithms, and data-driven approaches etc. Multi-objective optimization (MOO) combined with data-driven techniques emphasizes complex decision-making problems where conflicting objectives need to be optimized simultaneously; it optimally gives a group of solutions known as the pareto front by identifying trade-offs between conflicting criteria. Ozcan-Deniz and Zhu [36] has employed MOO for greenhouse gas emissions from construction activities on roads. MOO methods and their application in energy-saving were reviewed by Cui *et al.* [14]. Blank and Deb [9] developed a MOO framework in Python: Pymooo. Tian *et al.* [62] surveyed the evolutionary large-scale MOO. Pereira *et al.* [43] provided a review of methods and algorithms for MOO in mechanical engineering problems. Liu *et al.* [30] has utilized “Guassian mixture model” and an “NSGA-II algorithm” in MOO for greenhouse light environments. Determining the weights of attributes in MCDM problems is a significant practical task. A popular and straightforward strategy for resolving MOO issues is the weighted sum method (WSM). Saravanan *et al.* [51] implemented WSM to select photovoltaic devices. Vujji and Dahiya [64] utilized WSM technique to predictably control the torque of the PMSM drive: real-time implementation to improve the weighting coefficient selection.

In recent years, in order to meet the needs of contemporary agriculture, greenhouses offer year-round crop growing, effective resource management, and farming, which is climate resilient. In light of shifting climates and population expansion, it provides sustainable & controlled growing conditions, guarantees food security, and lessens their negative effects on the environment. The impact of greenhouse technological developments on agriculture is highlighted by many research studies. Li *et al.* [28] have introduced intelligent sensors for automatic control and real-time monitoring to improve resource efficiency in a greenhouse. In greenhouse system Escamilla-García *et al.* [17] have introduced smart agriculture by use of artificial neural network technique. Minimized Smart Agriculture System was designed and implemented by Rubanga *et al.* [48] for small-scale greenhouse production. Ariani *et al.* [2] looks into climate-smart agriculture to boost the agriculture output and lessen greenhouse negative environmental effects. Tomato is one of the major and well-liked crops cultivated worldwide. It is produced in India on an area of 7.89 lakh hectares, yielding 197.59 lakh tonnes. With a 0.57 lakh hectare area and a productivity of 22.01 t/ha, West Bengal produces 12.65 lakh tonnes of tomatoes annually. Despite the reasonably good production figure, there has been a notable increase in tomato prices in certain areas, notably Kolkata, the capital of West Bengal, have reached Rs. 200 per kilogram. Customers have been directly impacted by the spike in tomato costs, especially those with lower incomes. Tomato growers have reported a number of concerning problems over the course of growing their produce. Reduced yields and monetary losses resulted from the plants’ problems with curled leaves, stunted growth, and cracked fruits. This study concentrates on enhancing crop-specific environmental conditions in order to raise quality and production. Cultivating tomato crops in greenhouses in West Bengal provides numerous benefits, enhancing productivity, improving quality, and boosting overall profitability for farmers. Greenhouse farming extends the period for cultivating tomatoes, providing farmers with the opportunity to grow them consistently throughout the year which helps a steady tomato supply in the market. In this study, we proposed a robust MCGDM technique to determine the ideal greenhouse site for tomato crops in West Bengal. The motivation and novelties of our study are as follows.

1.1. Motivation

From the literature available, it is apparent that the Dombi T-norm and T-conorm operations, as introduced by Dombi, exhibit significant flexibility relating to the operator's underlying parameters. The detailed advantages of this operator are as follows:

- It has the ability to modify power parameter that enables decision makers to customize the operations of T-norm and T-conorm according to their inclinations and the significance of particular criteria. For instance, the power parameter can be adjusted to capture strong or weak correlations, enabling a nuanced depiction of the relationship.
- The flexibility of Dombi T-norm and T-conorm operations guarantees that the decision model is sensitive and responsive to the intricate relationships among criteria in a MCDM scenario in the real world, where criteria may differ greatly in significance.
- Standard decision-making processes have often demonstrated a lack of flexibility and fail to deal with uncertainty in its inherent form, which might sometimes lead to suboptimal and inflexible conclusions. Dombi T-norm and T-conorm operations are highly useful in overcoming these drawbacks of standard decision-making techniques.

On the other hand, logarithmic operators can overcome the drawbacks of algebraic operations, making them a significant operational rule in the aggregate domain. The benefits of logarithmic aggregation operator can be observed in the following way:

- Algebraic operations to aggregate the criteria assume linear relationships among the underlying data, which might not be the case in real-world decision-making scenarios. For instance, population growth, economic supply and demand, climate change system, changes in stock price in the stock market etc., are all non-linear in nature.
- It is observed that logarithmic operators can compress the scale when underlying data varies significantly, which facilitates analysis [25]. This approach has special advantages in finance and various scientific applications.
- The multiplicative character of some criteria is not naturally handled by algebraic operations, but logarithmic operators convert multiplication operations into addition operations, which simplifies computations and facilitates the solution of the underlying problems. Also, logarithmic operators are fruitfully used in growth rate analysis.

Chen and Ye [13] manifested a "Dombi weighted aggregation operator" for the decision-making issues in a single-valued neutrosophic (SVN) arena. Pamucar *et al.* [38] and Mandal and Seikh [32] employed Dombi T-norm and T-conorm operations to deliver a more feasible and robust solution in their research under fuzzy and interval-valued spherical fuzzy environment respectively. Recently, Seikh and Chatterjee [52] implemented Dombi aggregation operator for the purpose of "E-learning websites selection" in intuitionistic fuzzy environment. Many researchers have made use of logarithmic operators to develop advanced and sophisticated decision-making tools. For example Li and Wei [27] developed logarithmic operational rules for IFN and developed its corresponding aggregation operators; Garg [21] made an extensive advancement in logarithmic operational rules for SVNS and used it in a MADM method; Haque *et al.* [25] introduced a logarithmic operational rule and aggregation operators for trapezoidal neutrosophic numbers and used it to propose a new MCGDM technique. However, as of now, we have not noticed any Dombi logarithmic aggregation operators in the CN environment although it is observed that cylindrical neutrosophic number (CNN) has the capability to capture the ambiguity and uncertainty of numerous real-life issues. Therefore, in our article, an attempt is made to establish Dombi logarithmic aggregation laws and operators in CN environment. Moreover, recent works on MCGDM have demonstrated a discernible trend toward the use of sophisticated computational tools like machine learning, optimization algorithms and data-driven approaches to improve the accuracy as well as efficiency of decision-making techniques. MOO emphasizes complex decision-making problems where conflicting objectives need to be optimized simultaneously. On the other hand, we noticed that neither the MARCOS nor the MOORA method has been utilized in the CN environment. Martin *et al.* [33] proposed a neutrosophic-based MARCOS method MCDM technique with single-valued triangular neutrosophic numbers to address the software selection issue and Peng and Xu [39] developed a MULTI-MOORA method for MCDM in an interval neutrosophic environment, whereas both MARCOS method and MOORA method were unexplored in the CN arena. It is to be noted that the MARCOS method has the advantage of capturing the relative performance of alternatives by taking into account both the best and worst possible solutions. This allows for a comprehensive evaluation by measuring how closely each alternative approximates the ideal and worst-case circumstances. In contrast, the MOORA method streamlines the decision-making method by normalizing and aggregating the criteria, making it simple to interpret the ranking results. All these facts encourage us to build up an MCGDM process in CN arena based on MARCOS and MOORA methods where MOO has been utilized as a conjunction. Finally, the proposed technique has been fruitfully utilized to determine the best greenhouse location for tomato cultivation in West Bengal.

1.2. Novelties

In this research article, we have presented several novel ideas and certain systematic enhancements which are enumerated below:

(i) Constituted a novel cylindrical neutrosophic Dombi weighted logarithmic aggregation ($\mathscr{CNDWLA}$) operator in CN environment.
(ii) Implemented the integrated MOO and FUCOM methods to determine attribute weights in the MCGDM model.
(iii) MARCOS and MOORA methods are employed in CN arena to rank the alternatives.
(iv) Rigorous sensitivity and comparative analysis have been carried out through an extensive numerical simulation to evaluate the validity, reliability, and utility of our proposed MCGDM model.

1.3. Arrangement of the article

The subsequent sections of our paper are arranged in the following way:

Mathematical preliminaries of CNS have been deliberated in Section 2. Dombi logarithmic aggregation operator with some properties is elaborated in Section 3. Section 4 represents the methodology of our proposed MCGDM problem in selecting the optimal greenhouse site for tomato crops. In Section 5, a numerical illustrative example is described, demonstrating the utilization of MOO, FUCOM, and decision-making techniques, specifically MARCOS and MOORA, within the MCGDM model system. Section 5 also includes the presentation of sensitivity analysis, comparative analysis, managerial implications, and limitations of our study. The conclusion of the article includes future research prospects discussed in Section 6.

2. Mathematical preliminaries

In this part, we have listed some fundamental definitions of NS and CNS for the purpose of expanding this study. Additionally, the definitions for score and accuracy functions and available Dombi operations in CNNs are also discussed.

Definition 2.1. Assume that $\widetilde{\mathscr{B}}$ presents the universal set. A Set $\widehat{\Gamma}$ in $\widetilde{\mathscr{B}}$ is specified by its membership functions representing truth, indeterminacy, and falsehood $\mathscr{G}(b)$, $\mathscr{L}(b)$, $\mathscr{M}(b)$ respectively where $\mathscr{G}(b)$, $\mathscr{L}(b)$, $\mathscr{M}(b) \in [0,1]$ satisfy the inequality $0 \leq \mathscr{G}(b)+\mathscr{L}(b)+\mathscr{M}(b) \leq 3$ for $b \in \widetilde{\mathscr{B}}$. Therefore the NS $\widehat{\Gamma}$ can be presented as $\widehat{\Gamma} = \{b; [\mathscr{G}(b), \mathscr{L}(b), \mathscr{M}(b)] : g \in \widetilde{\mathscr{B}}\}$.

Definition 2.2. Assume that $\widetilde{\mathscr{B}}$ represents the universal set of discourse. A set $\widehat{\Psi} = \{b; [\mathscr{G}(b), \mathscr{L}(b), \mathscr{M}(b)] : b \in \widetilde{\mathscr{B}}\}$ is called CNS on $\widetilde{\mathscr{B}}$, where $\mathscr{G}(b) : \widetilde{\mathscr{B}} \to [0,1]$ is expressed by the degree to which it exhibits truth membership, $\mathscr{L}(b) : \widetilde{\mathscr{B}} \to [0,1]$ is recognized by its membership value representing indeterminacy and $\mathscr{M}(b) : \widetilde{\mathscr{B}} \to [0,1]$ is referred to as the membership function for falseness and they fulfill the requirement, $(\mathscr{G}(b))^2+(\mathscr{L}(b))^2 \leq 1^2$ and $\mathscr{M}(b) \leq 1$. For ease and in most cases, we denote a single-valued number in the context of CN representation $\widehat{cnn}$ as $\widehat{cnn} = (\mathscr{G}_{\widehat{cnn}}, \mathscr{L}_{\widehat{cnn}}, \mathscr{M}_{\widehat{cnn}})$.

Definition 2.3. For any CNN $\widehat{\mathscr{J}} = \langle G, L, M\rangle$, it's score and accuracy functions [12] are defined as follows:

The score function as $\widehat{\mathscr{S}_c} = \frac{2*G^2-L^2-M^2}{2}$, where $\widehat{\mathscr{S}_c} \in [-1,1]$.
The accuracy function as $\widehat{\mathscr{A}_c} = \frac{2*G^2+L^2+M^2}{2}$, where $\widehat{\mathscr{A}_c} \in [0,2)$.

Mention that the score and accuracy function are effective tools for ordering different fuzzy numbers. Now using $\widehat{\mathscr{S}_c}$ and $\widehat{\mathscr{A}_c}$, to display the differences between two CNNs is defined in the following way:

Definition 2.4. Assume that $\widehat{\mathscr{J}_1}$ and $\widehat{\mathscr{J}_2}$ are two CNNs. Then the way of ranking is described in the following way,

(i) If $\widehat{\mathscr{S}_c}(\widehat{\mathscr{J}_1}) > \widehat{\mathscr{S}_c}(\widehat{\mathscr{J}_2})$, then $\widehat{\mathscr{J}_1} > \widehat{\mathscr{J}_2}$;
(ii) If $\widehat{\mathscr{S}_c}(\widehat{\mathscr{J}_1}) = \widehat{\mathscr{S}_c}(\widehat{\mathscr{J}_2})$ and $\widehat{\mathscr{A}_c}(\widehat{\mathscr{J}_2}) > \widehat{\mathscr{A}_c}(\widehat{\mathscr{J}_1})$, then $\widehat{\mathscr{J}_2} > \widehat{\mathscr{J}_1}$;
(iii) If $\widehat{\mathscr{S}_c}(\widehat{\mathscr{J}_1}) = \widehat{\mathscr{S}_c}(\widehat{\mathscr{J}_2})$ and $\widehat{\mathscr{A}_c}(\widehat{\mathscr{J}_2}) = \widehat{\mathscr{A}_c}(\widehat{\mathscr{J}_1})$, then $\widehat{\mathscr{J}_2} = \widehat{\mathscr{J}_1}$.

Example. Let $\widehat{\mathscr{J}_1}$ and $\widehat{\mathscr{J}_2}$ be two CNNs. Let $\widehat{\mathscr{J}_1} = \langle 0.6, 0.1, 0.2\rangle$ and $\widehat{\mathscr{J}_2} = \langle 0.8, 0.4, 0.2\rangle$, then $\widehat{\mathscr{S}_c}(\widehat{\mathscr{J}_1}) = 0.335$, $\widehat{\mathscr{S}_c}(\widehat{\mathscr{J}_2}) = 0.54$, $\widehat{\mathscr{A}_c}(\widehat{\mathscr{J}_1}) = 0.385$ and $\widehat{\mathscr{A}_c}(\widehat{\mathscr{J}_2}) = 0.74$.

Definition 2.5. Let $\widehat{\mathscr{J}_\mu} = \langle G_\mu, L_\mu, M_\mu\rangle (\mu = 1,2)$ be two CNNs and scalars $d, \alpha > 0$. Then the Dombi summation, Dombi product, Dombi scalar multiplication, and the Dombi nth power of $\widehat{\mathscr{J}_1}$ as

$$\text{(i)}\quad \widehat{\mathscr{J}_1} + \widehat{\mathscr{J}_2} = \left\langle 1 - \frac{1}{1+\left(\left(\frac{G_1}{1-G_1}\right)^\alpha + \left(\frac{G_2}{1-G_2}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\left(\frac{1-L_1}{L_1}\right)^\alpha + \left(\frac{1-L_2}{L_2}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\left(\frac{1-M_1}{M_1}\right)^\alpha + \left(\frac{1-M_2}{M_2}\right)^\alpha\right)^{\frac{1}{\alpha}}} \right\rangle$$

$$\text{(ii)}\quad \widehat{\mathscr{J}_1} \times \widehat{\mathscr{J}_2} = \left\langle \frac{1}{1+\left(\left(\frac{1-G_1}{G_1}\right)^\alpha + \left(\frac{1-G_2}{G_2}\right)^\alpha\right)^{\frac{1}{\alpha}}}, 1 - \frac{1}{1+\left(\left(\frac{L_1}{1-L_1}\right)^\alpha + \left(\frac{L_2}{1-L_2}\right)^\alpha\right)^{\frac{1}{\alpha}}}, 1 - \frac{1}{1+\left(\left(\frac{M_1}{1-M_1}\right)^\alpha + \left(\frac{M_2}{1-M_2}\right)^\alpha\right)^{\frac{1}{\alpha}}} \right\rangle \quad \text{(I)}$$

$$\text{(iii)}\quad d\widehat{\mathscr{J}_1} = \left\langle 1 - \frac{1}{1+\left(d\left(\frac{G_1}{1-G_1}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(d\left(\frac{1-L_1}{L_1}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(d\left(\frac{1-M_1}{M_1}\right)^\alpha\right)^{\frac{1}{\alpha}}} \right\rangle$$

$$\text{(iv)}\quad \widehat{\mathscr{J}_1}^d = \left\langle \frac{1}{1+\left(d\left(\frac{1-G_1}{G_1}\right)^\alpha\right)^{\frac{1}{\alpha}}}, 1 - \frac{1}{1+\left(d\left(\frac{L_1}{1-L_1}\right)^\alpha\right)^{\frac{1}{\alpha}}}, 1 - \frac{1}{1+\left(d\left(\frac{M_1}{1-M_1}\right)^\alpha\right)^{\frac{1}{\alpha}}} \right\rangle.$$

Example. Let $\widehat{\mathscr{J}_1}$ and $\widehat{\mathscr{J}_2}$ be two CNNs. Let $\widehat{\mathscr{J}_1} = \langle 0.4, 0.7, 0.8\rangle$ and $\widehat{\mathscr{J}_2} = \langle 0.6, 0.4, 0.2\rangle$, then for $d = 3$ and $\alpha = 2$,

$\widehat{\mathscr{J}_1} + \widehat{\mathscr{J}_2} = \langle 0.621, 0.391, 0.200\rangle$;
$\widehat{\mathscr{J}_1} * \widehat{\mathscr{J}_2} = \langle 0.379, 0.708, 0.800\rangle$;
$3\widehat{\mathscr{J}_1} = \langle 0.536, 0.574, 0.698\rangle$;
$\widehat{\mathscr{J}_1}^3 = \langle 0.278, 0.802, 0.874\rangle$.

3. Dombi logarithmic aggregation operator

Here, we have proposed Dombi logarithmic laws for CNN and the proposed operating laws are subsequently employed to create cylindrical neutrosophic Dombi weighted logarithmic aggregation operator. Also, the properties of this operator are studied and discussed.

3.1. Dombi logarithmic laws of cylindrical neutrosophic sets and cylindrical neutrosophic numbers

In this sub-section, Dombi logarithmic operational laws in CNS and CNN environment are discussed.

Definition 3.1. Assume that $\widehat{\mathscr{W}}$ denotes a universal set and $\widehat{\Lambda} = \{w; [\mathscr{G}_{\widehat{\Lambda}}(w), \mathscr{L}_{\widehat{\Lambda}}(w), \mathscr{M}_{\widehat{\Lambda}}(w)] : w \in \widehat{\mathscr{W}}\}$ be a CNS, then we introduce a Dombi logarithmic operational laws of CNS $\widehat{\Lambda}$ as follows:

$$\log_b \widehat{\Lambda} = \left\{ \left\langle w, 1 - \frac{1}{1 + \left(\left(\log_b \frac{\mathscr{G}_{\widehat{\Lambda}}(w)}{1-\mathscr{G}_{\widehat{\Lambda}}(w)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1 + \left(\left(\log_b \frac{1-\mathscr{L}_{\widehat{\Lambda}}(w)}{\mathscr{L}_{\widehat{\Lambda}}(w)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1 + \left(\left(\log_b \frac{1-\mathscr{M}_{\widehat{\Lambda}}(w)}{\mathscr{M}_{\widehat{\Lambda}}(w)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \right\rangle : w \in \widehat{\mathscr{W}} \right\}$$

where, $0 < b \leq \min(\mathscr{G}_{\widehat{\Lambda}}, 1 - \mathscr{L}_{\widehat{\Lambda}}, 1 - \mathscr{M}_{\widehat{\Lambda}}) < 1, b \neq 1$.

It can be easily proved that $log_b\widehat{\Lambda}$ is also CNS. Using the definition of CNS, for all $w \in \widehat{\mathscr{W}}$, the functions $\mathscr{G}_{\widehat{\Lambda}}$, $\mathscr{L}_{\widehat{\Lambda}}$, $\mathscr{M}_{\widehat{\Lambda}}$ satisfies the relations $\mathscr{G}_{\widehat{\Lambda}} : \widehat{\mathscr{W}} \to [0,1]$, $\mathscr{L}_{\widehat{\Lambda}} : \widehat{\mathscr{W}} \to [0,1]$, $\mathscr{M}_{\widehat{\Lambda}} : \widehat{\mathscr{W}} \to [0,1]$ and $(\mathscr{G}_{\widehat{\Lambda}}(w))^2 + (\mathscr{L}_{\widehat{\Lambda}}(w))^2 \leq 1^2$ and $\mathscr{M}_{\widehat{\Lambda}}(w) \leq 1$.

If $0 < b \leq \min(\mathscr{G}_{\widehat{\Lambda}}, 1-\mathscr{L}_{\widehat{\Lambda}}, 1-\mathscr{M}_{\widehat{\Lambda}}) < 1, b \neq 1$, then the function of truth membership is $1 - \frac{1}{1+((\log_b \frac{\mathscr{G}_{\widehat{\Lambda}}}{1-\mathscr{G}_{\widehat{\Lambda}}})^{\alpha})^{\frac{1}{\alpha}}} : \widehat{\mathscr{W}} \to [0,1]$, $\forall w \in \widehat{\mathscr{W}} \to 1 - \frac{1}{1+((\log_b \frac{\mathscr{G}_{\widehat{\Lambda}}(w)}{1-\mathscr{G}_{\widehat{\Lambda}}(w)})^{\alpha})^{\frac{1}{\alpha}}} \in [0,1]$; the function of indeterminacy is $\frac{1}{1+((\log_b \frac{1-\mathscr{L}_{\widehat{\Lambda}}}{\mathscr{L}_{\widehat{\Lambda}}})^{\alpha})^{\frac{1}{\alpha}}} : \widehat{\mathscr{W}} \to [0,1]$, $\forall w \in \widehat{\mathscr{W}} \to \frac{1}{1+((\log_b \frac{1-\mathscr{M}_{\widehat{\Lambda}}(w)}{\mathscr{M}_{\widehat{\Lambda}}(w)})^{\alpha})^{\frac{1}{\alpha}}} \in [0,1]$.

Therefore, $\log_b \widehat{\Lambda} = \{\langle w, 1 - \frac{1}{1+((\log_b \frac{\mathscr{G}_{\widehat{\Lambda}}(w)}{1-\mathscr{G}_{\widehat{\Lambda}}(w)})^{\alpha})^{\frac{1}{\alpha}}}, \frac{1}{1+((\log_b \frac{1-\mathscr{L}_{\widehat{\Lambda}}(w)}{\mathscr{L}_{\widehat{\Lambda}}(w)})^{\alpha})^{\frac{1}{\alpha}}}, \frac{1}{1+((\log_b \frac{1-\mathscr{M}_{\widehat{\Lambda}}(w)}{\mathscr{M}_{\widehat{\Lambda}}(w)})^{\alpha})^{\frac{1}{\alpha}}}\rangle : w \in \widehat{\mathscr{W}}\}$, where, $0 < b \leq \min(\mathscr{G}_{\widehat{\Lambda}}, 1 - \mathscr{L}_{\widehat{\Lambda}}, 1 - \mathscr{M}_{\widehat{\Lambda}}) < 1, b \neq 1$ is also CNS.

Definition 3.2. For any CNN $\widehat{\mathscr{J}} = \langle G, L, M\rangle$, $\alpha > 0$, we define,

$$\log_b \widehat{\mathscr{J}} \tag{1}$$
$$= \begin{cases} \left\langle 1 - \frac{1}{1+\left((\log_b \frac{G}{1-G})^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left((\log_b \frac{1-L}{L})^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left((\log_b \frac{1-M}{M})^{\alpha}\right)^{\frac{1}{\alpha}}} \right\rangle; & 0 < b \leq \min(G, 1-L, 1-M) < 1, b \neq 1 \\ \left\langle 1 - \frac{1}{1+\left(\left(\log_{\frac{1}{b}} \frac{G}{1-G}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\left(\log_{\frac{1}{b}} \frac{1-L}{L}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\left(\log_{\frac{1}{b}} \frac{1-M}{M}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \right\rangle; & 0 < \frac{1}{b} \leq \min(G, 1-L, 1-M) < 1, b \neq 1 \end{cases}$$

then the Dombi logarithmic operator is defined as the function $\log_b \widehat{\Lambda}$, and the value of $\log_b \widehat{\Lambda}$ is known as Dombi logarithmic CNN.

Theorem 3.3. *For CNN $\widehat{\mathscr{J}} = \langle G, L, M\rangle$, the value of the operator $\log_b \widehat{\mathscr{J}}$ is CNN.*

Proof. Assume that CNN $\widehat{\mathscr{J}} = \langle G, L, M\rangle$ satisfies $G \in [0,1]$, $L \in [0,1]$, $M \in [0,1]$ and $G^2 + L^2 \leq 1^2$ and $M \leq 1$. The two cases appear below:

(i) When $0 < b \leq \min(G, 1-L, 1-M) < 1, b \neq 1$. Therefore, $0 \leq (\log_b \frac{G}{1-G}), (\log_b \frac{1-L}{L}), (\log_b \frac{1-M}{M}) \leq 1$ and hence, $0 \leq 1 - \frac{1}{1+((\log_b \frac{G}{1-G})^{\alpha})^{\frac{1}{\alpha}}} \leq 1$, $0 \leq \frac{1}{1+((\log_b \frac{1-L}{L})^{\alpha})^{\frac{1}{\alpha}}} \leq 1$ and $0 \leq \frac{1}{1+((\log_b \frac{1-M}{M})^{\alpha})^{\frac{1}{\alpha}}} \leq 1$. Moreover, $(1 - \frac{1}{1+((\log_b \frac{G}{1-G})^{\alpha})^{\frac{1}{\alpha}}})^2 + (\frac{1}{1+((\log_b \frac{1-L}{L})^{\alpha})^{\frac{1}{\alpha}}})^2 \leq 1 - 2*1 + 1 + 1^2 = 1^2$. Therefore, $\log_b \widehat{\mathscr{J}}$ is CNN.

(ii) When $b > 1$ and $0 < \frac{1}{b} < 1$ and $\frac{1}{b} \leq \min(G, 1-L, 1-M) < 1$. Now, it can be easily proved using a similar manner as (i) that $1 - \frac{1}{1+((\log_{\frac{1}{b}} \frac{G}{1-G})^{\alpha})^{\frac{1}{\alpha}}}, \frac{1}{1+((\log_{\frac{1}{b}} \frac{1-L}{L})^{\alpha})^{\frac{1}{\alpha}}}, \frac{1}{1+((\log_{\frac{1}{b}} \frac{1-M}{M})^{\alpha})^{\frac{1}{\alpha}}}$ is also CNN.

□

3.2. Cylindrical neutrosophic Dombi weighted logarithmic aggregation ($\mathscr{CNDWLA}$) operator

Definition 3.4. Let $\widehat{\mathscr{J}_\mu} = \langle G_\mu, L_\mu, M_\mu \rangle (\mu = 1, 2, \cdots, r)$ be the assembly of CNNs, $0 < b_\mu \leq \min(G_\mu, 1 - L_\mu, 1 - M_\mu) < 1, b_\mu \neq 1$ and let $\mathscr{CNDWLA} : (\Omega)^r \to \Omega$ then $\mathscr{CNDWLA}$ operator is defined as

$$\mathscr{CNDWLA}\left(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}\right) = p_1 \log_{b1} \widehat{\mathscr{J}_1} + p_2 \log_{b2} \widehat{\mathscr{J}_2} + \cdots + p_r \log_{b_r} \widehat{\mathscr{J}_r}. \tag{2}$$

Then $\mathscr{CNDWLA}$ function is called $\mathscr{CNDWLA}$ operator where $p = (p_1, p_2, \cdots, p_r)^T$ denotes the corresponding weight vector of $\log_{b^r} \widehat{\mathscr{J}_r}$ with $p_r > 0$ and $\sum_{\mu=1}^{r} p_\mu = 1$.

Theorem 3.5. *Let $\widehat{\mathscr{J}_\mu} = \langle G_\mu, L_\mu, M_\mu \rangle (\mu = 1, 2, \cdots, r)$ be the collection of CNNs. Then the value of aggregation by applying the $\mathscr{CNDWLA}$ operator is also CNN and is given by*

$$\mathscr{CNDWLA}\left(\widehat{\mathscr{J}_1}, \cdots, \widehat{\mathscr{J}_r}\right)$$
$$= \begin{cases} \left\langle 1 - \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu \left(\log_{b_\mu} \frac{G_\mu}{1-G_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu \left(\log_{b_\mu} \frac{1-L_\mu}{L_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu \left(\log_{b_\mu} \frac{1-M_\mu}{M_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \right\rangle; \\ 0 < b_\mu \leq \min(G_\mu, 1 - L_\mu, 1 - M_\mu) < 1, b_\mu \neq 1 \\ \left\langle 1 - \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu \left(\log_{\frac{1}{b_\mu}} \frac{G_\mu}{1-G_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu \left(\log_{\frac{1}{b_\mu}} \frac{1-L_\mu}{L_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu \left(\log_{\frac{1}{b_\mu}} \frac{1-M_\mu}{M_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \right\rangle; \\ 0 < \frac{1}{b_\mu} \leq \min(G_\mu, 1 - L_\mu, 1 - M_\mu) < 1, b_\mu \neq 1. \end{cases} \tag{3}$$

Proof. We apply mathematical induction on r for $0 < b_\mu \leq \min(G_\mu, 1 - L_\mu, 1 - M_\mu) < 1, b_\mu \neq 1$ to prove the result in the above given equation in Theorem 3.5. Since for each μ, $\widehat{\mathscr{J}_\mu} = \langle G_\mu, L_\mu, M_\mu \rangle (\mu = 1, 2, \cdots, r)$ is CNN which satisfies $G_\mu, L_\mu, M_\mu \in [0, 1]$ and $(G_\mu)^2 + (L_\mu)^2 \leq 1^2$ and $(M_\mu) \leq 1$. Now we are executing mathematical induction as follows:

For $\mu = 2$, we get $\mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}) = p_1 \log_{b1} \widehat{\mathscr{J}_1} + p_2 \log_{b2} \widehat{\mathscr{J}_2}$. Since by the Definition 3.2, we can state that $\log_{b_1}$ and $\log_{b_2}$ are CNNs and hence, $p_1 \log_{b1} \widehat{\mathscr{J}_1} + p_2 \log_{b2} \widehat{\mathscr{J}_2}$ is also CNN. Now, for $\widehat{\mathscr{J}_1}$ and $\widehat{\mathscr{J}_2}$, we have,

$$\mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2})$$
$$= p_1 \log_{b1} \widehat{\mathscr{J}_1} + p_2 \log_{b2} \widehat{\mathscr{J}_2}$$
$$= \left\langle 1 - \frac{1}{1 + \left(\sum_{\mu=1}^{2} p_\mu \left(\log_{b_\mu} \frac{G_\mu}{1-G_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1 + \left(\sum_{\mu=1}^{2} p_\mu \left(\log_{b_\mu} \frac{1-L_\mu}{L_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1 + \left(\sum_{\mu=1}^{2} p_\mu \left(\log_{b_\mu} \frac{1-M_\mu}{M_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \right\rangle.$$

Thus, the outcome is satisfied for $\mu = 2$. Now assume that equation (3) is satisfied for $\mu = t$. Then,

$$\mathscr{CNDWLA}\left(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_t}\right)$$
$$= \left\langle 1 - \frac{1}{1 + \left(\sum_{\mu=1}^{t} p_\mu \left(\log_{b_\mu} \frac{G_\mu}{1-G_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1 + \left(\sum_{\mu=1}^{t} p_\mu \left(\log_{b_\mu} \frac{1-L_\mu}{L_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}, \frac{1}{1 + \left(\sum_{\mu=1}^{t} p_\mu \left(\log_{b_\mu} \frac{1-M_\mu}{M_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \right\rangle.$$

Now, we prove it for $\mu = t+1$ as follows:

$$\mathscr{CNDWLA}\left(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_{t+1}}\right)$$

$$= \bigoplus_{r=1}^{t} \log_{b_r} \widehat{\mathscr{J}_r} + \log_{b_{t+1}} \widehat{\mathscr{J}_{t+1}}$$

$$= \left\langle 1 - \frac{1}{1+\left(\sum_{\mu=1}^{t} p_\mu \left(\log_{b_\mu} \frac{G_\mu}{1-G_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\sum_{\mu=1}^{t} p_\mu \left(\log_{b_\mu} \frac{1-L_\mu}{L_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\sum_{\mu=1}^{t} p_\mu \left(\log_{b_\mu} \frac{1-M_\mu}{M_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}} \right\rangle$$

$$+ \left\langle 1 - \frac{1}{1+\left(\left(\log_{b_{t+1}} \frac{G_{t+1}}{1-G_{t+1}}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\left(\log_{b_{t+1}} \frac{1-L_{t+1}}{L_{t+1}}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\left(\log_{b_{t+1}} \frac{1-M_{t+1}}{M_{t+1}}\right)^\alpha\right)^{\frac{1}{\alpha}}} \right\rangle$$

$$= \left\langle 1 - \frac{1}{1+\left(\sum_{\mu=1}^{t+1} p_\mu \left(\log_{b_\mu} \frac{G_\mu}{1-G_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\sum_{\mu=1}^{t+1} p_\mu \left(\log_{b_\mu} \frac{1-L_\mu}{L_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\sum_{\mu=1}^{t+1} p_\mu \left(\log_{b_\mu} \frac{1-M_\mu}{M_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}} \right\rangle$$

and furthermore, the obtained value of aggregation value is CNN. Therefore, the equation (3) satisfies for $\mu = t+1$. Thus, the outcome is valid for all positive integers μ.

On another hand, if $b_\mu > 1$ and $0 < 1/b_\mu \leq \min(G_\mu, 1-L_\mu, 1-M_\mu) < 1, b_\mu \neq 1$, we can get,

$$\mathscr{CNDWLA}\left(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_t}\right)$$

$$= \left\langle 1 - \frac{1}{1+\left(\sum_{\mu=1}^{2} p_\mu \left(\log_{b_\mu} \frac{G_\mu}{1-G_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\sum_{\mu=1}^{2} p_\mu \left(\log_{b_\mu} \frac{1-L_\mu}{L_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}, \frac{1}{1+\left(\sum_{\mu=1}^{2} p_\mu \left(\log_{b_\mu} \frac{1-M_\mu}{M_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}} \right\rangle.$$

Hence, the theorem is proved. □

3.3. Properties of the $\mathscr{CNDWLA}$ operator

Let $\widehat{\mathscr{J}_\mu} = \langle G_\mu, L_\mu, M_\mu \rangle (\mu = 1, 2, \cdots, r)$ be any collection of CNNs, $0 < b_\mu \leq \min(G_\mu, 1-L_\mu, 1-M_\mu) < 1, b_\mu \neq 1$ and $p = (p_1, p_2, \cdots, p_r)^T$ indicates the weight vector of $\log_{br} \widehat{\mathscr{J}_r}$ with $p_r > 0$ and $\sum_{\mu=1}^{r} p_\mu = 1$. Then the properties of $\mathscr{CNDWLA}$ operator has been discussed below:

(i) (Commutative Law) Let, $\mathscr{CNDWLA}(\widehat{\mathscr{J}_1'}, \widehat{\mathscr{J}_2'}, \cdots, \widehat{\mathscr{J}_r'})$ be any reordering of $\mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r})$. Then,

$$\mathscr{CNDWLA}\left(\widehat{\mathscr{J}_1'}, \widehat{\mathscr{J}_2'}, \cdots, \widehat{\mathscr{J}_r'}\right) = \mathscr{CNDWLA}\left(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}\right).$$

Proof. It is apparent by the above given equation in Theorem 3.5. □

(ii) (Bounded Law) Let, $\widehat{\mathscr{J}_\mu} = \langle G_\mu, L_\mu, M_\mu \rangle (\mu = 1, 2, \cdots, r)$ represents the set of CNNs.
Let $\widehat{\mathscr{J}_{\min}} = \langle \min_\mu(G_\mu), \max_\mu(L_\mu), \max_\mu(M_\mu) \rangle$, $\widehat{\mathscr{J}_{\max}} = \langle \max_\mu(G_\mu), \min_\mu(L_\mu), \min_\mu(M_\mu) \rangle$, for $\mu = 1, 2, \cdots, r$; $\widehat{\mathscr{J}^+} = \mathscr{CNDWLA}(\widehat{\mathscr{J}_{\max}}, \widehat{\mathscr{J}_{\max}}, \cdots, \widehat{\mathscr{J}_{\max}})$

$$= \left\langle 1 - \frac{1}{1+(\sum_{\mu=1}^{r} p_\mu (\log_{b_\mu} \frac{\max_\mu(G_\mu)}{1-\max_\mu(G_\mu)})^\alpha)^{\frac{1}{\alpha}}}, \frac{1}{1+(\sum_{\mu=1}^{r} p_\mu (\log_{b_\mu} \frac{1-\min_\mu(L_\mu)}{\min_\mu(L_\mu)})^\alpha)^{\frac{1}{\alpha}}}, \frac{1}{1+(\sum_{\mu=1}^{r} p_\mu (\log_{b_\mu} \frac{1-\min_\mu(M_\mu)}{\min_\mu(M_\mu)})^\alpha)^{\frac{1}{\alpha}}} \right\rangle$$

and $\widehat{\mathscr{J}^-} = \mathscr{CNDWLA}(\widehat{\mathscr{J}_{\min}}, \widehat{\mathscr{J}_{\min}}, \cdots, \widehat{\mathscr{J}_{\min}})$

$$= \left\langle 1 - \frac{1}{1+(\sum_{\mu=1}^{r} p_\mu (\log_{b_\mu} \frac{\min_\mu(G_\mu)}{1-\min_\mu(G_\mu)})^\alpha)^{\frac{1}{\alpha}}}, \frac{1}{1+(\sum_{\mu=1}^{r} p_\mu (\log_{b_\mu} \frac{1-\max_\mu(L_\mu)}{\max_\mu(L_\mu)})^\alpha)^{\frac{1}{\alpha}}}, \frac{1}{1+(\sum_{\mu=1}^{r} p_\mu (\log_{b_\mu} \frac{1-\max_\mu(M_\mu)}{\max_\mu(M_\mu)})^\alpha)^{\frac{1}{\alpha}}} \right\rangle.$$

Then, $\widehat{\mathscr{J}^-} \leq \mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}) \leq \widehat{\mathscr{J}^+}$.

Proof. Since, $\min_\mu(G_\mu) \leq G_\mu \leq \max_\mu(G_\mu)$

$$\implies 1-\min_\mu(G_\mu) \geq 1-G_\mu \geq 1-\max_\mu(G_\mu)$$
$$\implies \frac{\min_\mu(G_\mu)}{1-\min_\mu(G_\mu)} \leq \frac{G_\mu}{1-G_\mu} \leq \frac{\max_\mu(G_\mu)}{1-\max_\mu(G_\mu)}$$
$$\implies 1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{\min_\mu(G_\mu)}{1-\min_\mu(G_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}} \leq 1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{G_\mu}{1-(G_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}} \leq 1 + \left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{\max_\mu(G_\mu)}{1-\max_\mu(G_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}$$
$$\implies \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{\min_\mu(G_\mu)}{1-\min_\mu(G_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \geq \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{G_\mu}{1-(G_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \geq \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{\max_\mu(G_\mu)}{1-\max_\mu(G_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}$$
$$\implies 1-\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{\min_\mu(G_\mu)}{1-\min_\mu(G_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \leq 1-\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{G_\mu}{1-(G_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \leq 1-\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{\max_\mu(G_\mu)}{1-\max_\mu(G_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}.$$

Since, $\max_\mu(L_\mu) \geq L_\mu \geq \min_\mu(L_\mu)$

$$\implies \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-\max_\mu(L_\mu)}{\max_\mu(L_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \leq \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-L_\mu}{L_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \leq \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-\min_\mu(L_\mu)}{\min_\mu(L_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}.$$

Since, $\max_\mu(M_\mu) \geq M_\mu \geq \min_\mu(M_\mu)$

$$\implies \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-\max_\mu(M_\mu)}{\max_\mu(M_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \leq \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-M_\mu}{M_\mu}\right)^{\alpha}\right)^{\frac{1}{\alpha}}} \leq \frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-\min_\mu(M_\mu)}{\min_\mu(M_\mu)}\right)^{\alpha}\right)^{\frac{1}{\alpha}}}.$$

Let, $\mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}) = \widehat{J} = \langle G, L, M\rangle$, $\widehat{J}^- = \langle G^-, L^-, M^-\rangle$ and $\widehat{J}^+ = \langle G^+, L^+, M^+\rangle$. Then using their score values, we get,

$$\widehat{\mathscr{S}_c}\left(\widehat{J}\right) = \frac{2 * G^2 - L^2 - M^2}{2}$$

$$= \frac{2*\left(1-\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{G_\mu}{1-(G_\mu)}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-L_\mu}{L_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-M_\mu}{M_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2}{2}$$

$$\geq \widehat{\mathscr{S}_c}\left(\widehat{J}^-\right)$$

$$= \frac{2*(G^-)^2-(L^-)^2-(M^-)^2}{2}$$

$$= \frac{2*\left(1-\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{\min_\mu(G_\mu)}{1-\min_\mu(G_\mu)}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-\max_\mu(L_\mu)}{\max_\mu(L_\mu)}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-\max_\mu(M_\mu)}{\max_\mu(M_\mu)}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2}{2}$$

$$\widehat{\mathscr{S}_c}\left(\widehat{J}\right) = \frac{2*G^2-L^2-M^2}{2}$$

$$= \frac{2*\left(1-\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{G_\mu}{1-(G_\mu)}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-L_\mu}{L_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-M_\mu}{M_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2}{2}$$

$$\leq \widehat{\mathscr{S}_c}\left(\widehat{J}^+\right)$$

$$= \frac{2*(G^+)^2-(L^+)^2-(M^+)^2}{2}$$

$$= \frac{2*\left(1-\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{\max_\mu(G_\mu)}{1-\max_\mu(G_\mu)}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-\min_\mu(L_\mu)}{\min_\mu(L_\mu)}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^{r} p_\mu\left(\log_{b_\mu}\frac{1-\min_\mu(M_\mu)}{\min_\mu(M_\mu)}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2}{2}.$$

The subsequent three cases come up,

(a) If $\widehat{\mathscr{S}_c}(\widehat{J}^-) < \widehat{\mathscr{S}_c}(\widehat{J}) < \widehat{\mathscr{S}_c}(\widehat{J}^+)$, then $\widehat{J}^- < \mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}) < \widehat{J}^+$ satisfies.

(b) If $\widehat{\mathscr{S}_c}(\widehat{J}) = \widehat{\mathscr{S}_c}(\widehat{J}^-)$, then $2*G^2-L^2-M^2 = 2*(G^-)^2-(L^-)^2-(M^-)^2 \implies G = G^-, L = L^-, M = M^-$.
Now, $\widehat{\mathscr{A}_c}(\widehat{J}) = \frac{2*G^2+L^2+M^2}{2}$ and $\widehat{\mathscr{A}_c}(\widehat{J}^-) = \frac{2*(G^-)^2+(L^-)^2+(M^-)^2}{2}$.
Therefore, $\widehat{\mathscr{A}_c}(\widehat{J}) = \widehat{\mathscr{A}_c}(\widehat{J}^-)$.
So, finally, we have, $\widehat{J} = \mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}) = \widehat{J}^-$.

(c) If $\widehat{\mathscr{S}_c}(\widehat{J}) = \widehat{\mathscr{S}_c}(\widehat{J}^+)$ then Likewise, employing the score and accuracy functions, we get $\widehat{J} = \mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}) = \widehat{J}^+$.
Based on the three cases mentioned above, it can be inferred that $\widehat{J}^- \leq \mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}) \leq \widehat{J}^+$.

□

(iii) (Monotonic Law) For considering two sets of CNNs $\widehat{\mathscr{J}_\mu} = \langle G_\mu, L_\mu, M_\mu \rangle$ and $\widehat{\mathscr{J}'_\mu} = \langle G'_\mu, L'_\mu, M'_\mu \rangle$ where $\mu = 1, 2, \cdots, r$. Also, if $G'_\mu \geq G_\mu, L'_\mu \leq L_\mu and M'_\mu \leq M_\mu$, then

$$\mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}) \leq \mathscr{CNDWLA}\left(\widehat{\mathscr{J}'_1}, \widehat{\mathscr{J}'_2}, \cdots, \widehat{\mathscr{J}'_r}\right).$$

Proof. Since, $G'_\mu \geq G_\mu$

$$\implies 1 - \frac{1}{1 + \left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{G'_\mu}{1-(G'_\mu)}\right)^\alpha\right)^{\frac{1}{\alpha}}} \geq 1 - \frac{1}{1 + \left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{G_\mu}{1-(G_\mu)}\right)^\alpha\right)^{\frac{1}{\alpha}}}.$$

Now, $L'_\mu \leq L_\mu$

$$\implies \frac{1}{1 + \left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{1-L'_\mu}{L'_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}} \leq \frac{1}{1 + \left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{1-L_\mu}{L_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}.$$

$M'_\mu \leq M_\mu$

$$\implies \frac{1}{1 + \left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{1-M'_\mu}{M'_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}} \leq \frac{1}{1 + \left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{1-M_\mu}{M_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}.$$

Let, $\widehat{\mathscr{J}_\mu} \leq \widehat{\mathscr{J}'_\mu}$, then $\mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}) = \widehat{J} = \langle G, L, M \rangle, \widehat{\mathscr{J}'} = \langle G', L', M' \rangle$. Then, using their scores values, we get

$$\begin{aligned}
\widehat{\mathscr{S}_c}\left(\widehat{J}\right) &= \frac{2 * G^2 - L^2 - M^2}{2} \\
&= \frac{2 * \left(1 - \frac{1}{1+\left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{G_\mu}{1-G_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{1-L_\mu}{L_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{1-M_\mu}{M_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2}{2} \\
&\leq \widehat{\mathscr{S}_c}\left(\widehat{\mathscr{J}'}\right) \\
&= \frac{2 * (G')^2 - (L')^2 - (M')^2}{2} \\
&= \frac{2 * \left(1 - \frac{1}{1+\left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{G'_\mu}{1-G'_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{1-L'_\mu}{L'_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2 - \left(\frac{1}{1+\left(\sum_{\mu=1}^r p_\mu \left(\log_{b_\mu} \frac{1-M'_\mu}{M'_\mu}\right)^\alpha\right)^{\frac{1}{\alpha}}}\right)^2}{2}.
\end{aligned}$$

Thus following two cases arise:

(a) If $\mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}) < \mathscr{CNDWLA}(\widehat{\mathscr{J}'_1}, \widehat{\mathscr{J}'_2}, \cdots, \widehat{\mathscr{J}'_r})$.

(b) If $\widehat{\mathscr{S}_c}(\widehat{J}) = \widehat{\mathscr{S}_c}(\widehat{\mathscr{J}'})$, then $2 * G^2 - L^2 - M^2 = 2 * (G')^2 - (L')^2 - (M')^2 \implies G = G', L = L', M = M'$.
Now, $\widehat{\mathscr{A}_c}(\widehat{J}) = \frac{2*G^2+L^2+M^2}{2} = \widehat{\mathscr{A}_c}(\widehat{\mathscr{J}'})$.
So, we have, $\mathscr{CNDWLA}(\widehat{\mathscr{J}_1}, \widehat{\mathscr{J}_2}, \cdots, \widehat{\mathscr{J}_r}) \leq \mathscr{CNDWLA}(\widehat{\mathscr{J}'_1}, \widehat{\mathscr{J}'_2}, \cdots, \widehat{\mathscr{J}'_r})$.
Hence proved.

□

4. Methodology

In a decision-making method, the collection of data performs a significant role in order to measure and assess the results. The principal issue that this study attempts to solve is the selection of an ideal site for the greenhouse establishment for tomato crops in West Bengal, taking into account the current tomato production scenarios, which include a drop in plant growth and a lower harvest. Here, we have collected the data through the direct survey method. In our model, we have proposed three types of decision-makers, namely business representatives (DM_1), agricultural experts (DM_2) and representatives from government agencies (DM_3). For the survey purpose, we have prepared a user-friendly quizzes-questionnaire where the obstacles, situations, and available managerial options for choosing greenhouse locations for tomato production in West Bengal are informed in detail. We have sent the questionnaire to a large number of decision makers as categorized above. We have received 90 valid responses from business representatives (DM_1), 120 valid responses from agricultural experts (DM_2), and 80 valid responses from representatives of government agencies (DM_3). The data collected by the survey were analyzed and average-out along with the round-off to the closest single decimal number to construct three input decision matrices namely DM_1, DM_2, and DM_3 for three types of decision makers as mentioned above. We have assigned the weights 0.32, 0.35, and 0.33 for the decision experts DM_1, DM_2 and DM_3 respectively. Next, to provide the ideal growing environment for the tomato crop or to determine the ideal weights for each attribute, we have formed a MOO problem that strikes a balance between various criteria and addressed it using the WSM approach and further, the weight upgradation is carried out using the FUCOM method. Additionally, we have implemented MARCOS and MOORA to identify the optimal alternative ranking. A numerical simulation through MATLAB has been performed employing MARCOS and MOORA procedures for recognizing variations in the ordering of the options. Figure 1 provides a pictorial representation of different steps involved in our proposed MCGDM model. A numerical illustration to identify the best greenhouse location for cultivating tomatoes in West Bengal is presented in the next section.

5. Numerical illustration

MCGDM model on CNN arena for greenhouse site selection for tomato crops in West Bengal

The production of tomato crops in West Bengal's rich plains has not kept up with the increasing demand, temperature fluctuations, and extreme weather conditions such as severe rainfall, storms, etc., especially during the off-season. Therefore, while growing their crops, tomato farmers encountered a number of issues, like decreased harvests and monetary losses as a result of a gradual decline in plant growth, causing a sharp increase in tomato prices for consumers. According to a survey conducted by the Ministry of Consumer Affairs, during last year tomatoes were offered at significantly higher prices in major cities across West Bengal. West Bengal comes under India's leading tomato-producing states; its share in the overall tomato production in 2022–2023 is approximately 6.10% [46]. Also, as per a report by the "National Bank for Agriculture and Rural Development (NABARD)" [46], tomato inflation reached 64.46% in June 2023 (month on month). Since tomatoes are one of the most commonly utilized vegetables in the world and are often regarded as a "protective food" and a rich source of nutrients. In response to this, stakeholders, comprising representatives from businesses, government agencies, and agriculture experts are engaged in the installation of greenhouses for abundant production of tomato. Regarding the greenhouse setup, site selection and positioning are critical for providing optimal

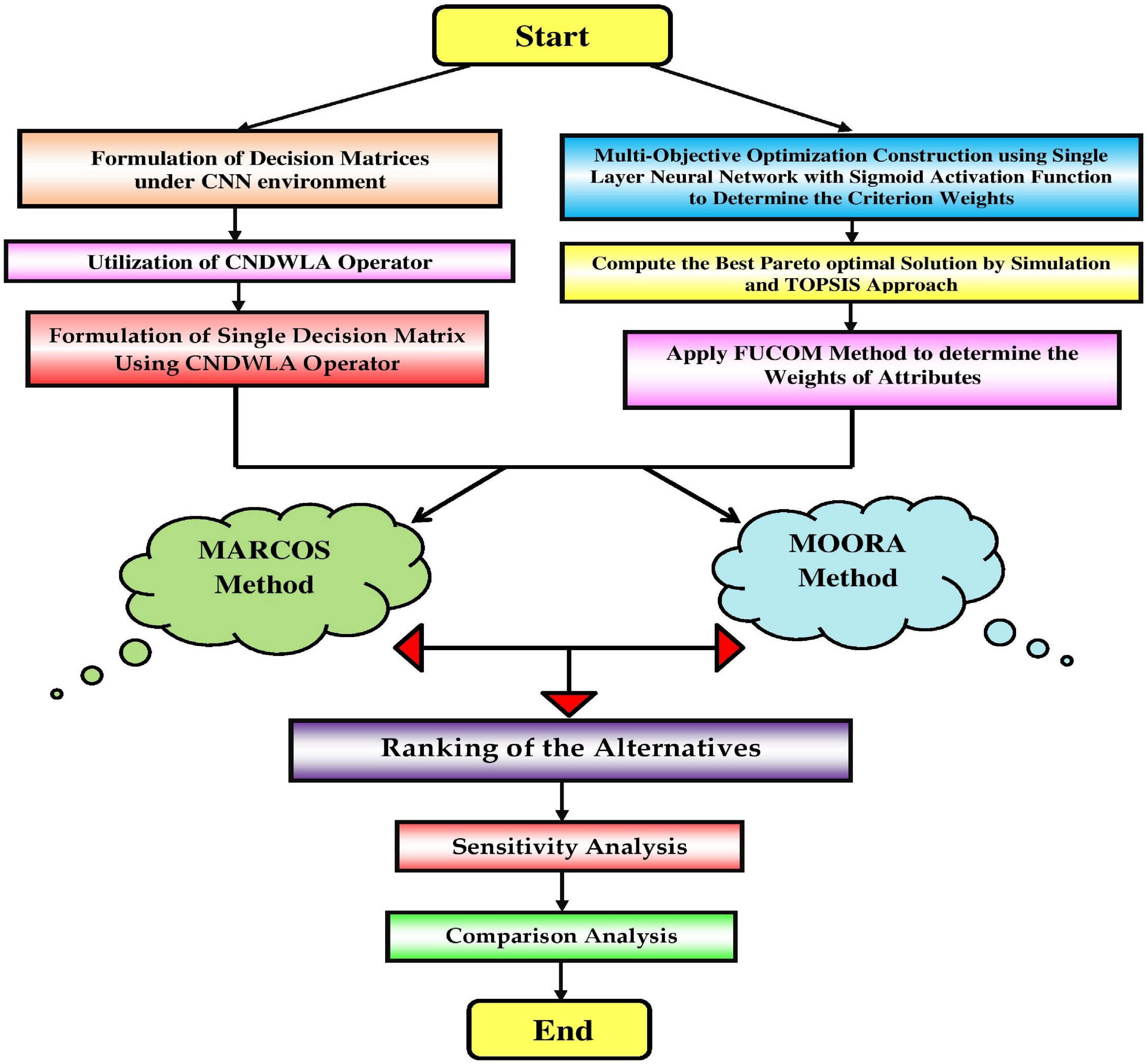

FIGURE 1. Illustration of the comprehensive methodology of our suggested MCGDM problem in selecting the optimal greenhouse site for tomato crop.

environmental conditions. According to Goldammer [22] the creation of a productive, sustainable greenhouse production system begins with effective planning before construction. Furthermore, Goldammer [22] recommended a number of crucial elements that need to be looked into before opting for greenhouse setup. Thus, there is a need for a rigorous site selection procedure that takes into account a variety of critical variables for optimal results. The decision-makers face a difficult task in assessing many criteria for site selection, they must consider temperature, humidity, VPD, slope, altitude, pH of water and soil, soluble salt content, government policies, labor access, and costing indices such as land, labor, transportation, and raw material costs.

FIGURE 2. Pictorial representation of tomato crop cultivation in the greenhouse.

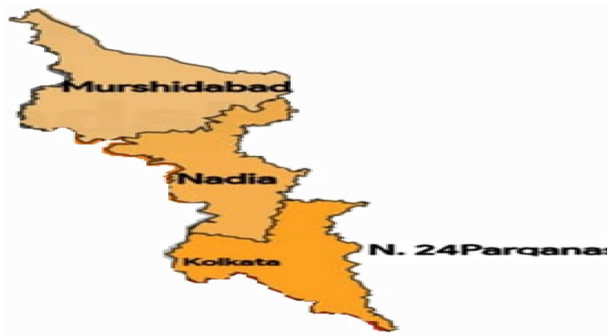

FIGURE 3. Depicts Western region of West Bengal.

FIGURE 4. Depicts Northern region of West Bengal.

Furthermore, decision-makers must consider climate and environmental factors to create optimum greenhouse structures. We have taken into consideration a MCGDM problem in a CNN context in order to overcome the greenhouse site selection problem (Fig. 2).

Here, we frame the issue in the following manner:

Western region of West Bengal ($\mathscr{A}_1$) (Fig. 3), Northern region of West Bengal ($\mathscr{A}_2$) (Fig. 4), Southern region of West Bengal ($\mathscr{A}_3$) (Fig. 5) and Eastern region of West Bengal ($\mathscr{A}_4$) (Fig. 6) are taken as the respective alternatives. Also, the considered criteria are vapor pressure density (VPD) (kPa), temperature (°C), humidity

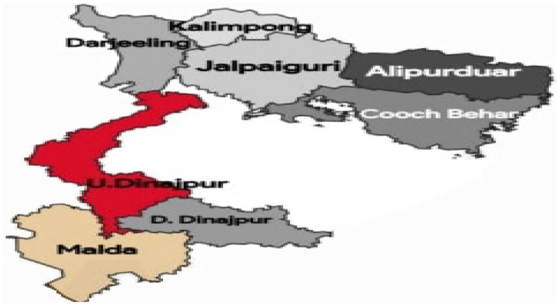

FIGURE 5. Depicts Southern region of West Bengal.

FIGURE 6. Depicts Eastern region of West Bengal.

(%RH), slope (°), altitude (km), distance from the water resource (km), pH of water, pH of soil, soluble salt content of soil (ds/m), government policies and costing index. Also, we consider three different types of decision-makers comprising business representatives, agricultural experts and representatives from government agencies (see Tab. 1 and Fig. 7).

5.1. Multi-objective optimization model

MOO considers several, often conflicting objectives, producing more realistic and balanced solutions that are better suited for dealing with real-world problems, whereas single-objective approaches simplify complex situations by focusing solely on a single criterion and frequently overlooking other important elements. The MOO technique can more effectively accommodate the diverse interests of stakeholders in complex decision-making scenarios by enabling the stakeholders to assign varying priorities to different objectives, resulting in a solution that is more palatable to a wider audience.

The principal issue that this study attempts to solve is the selection of an ideal site for the greenhouse establishment for tomato crops in West Bengal. It is a crucial decision as it impacts the overall productivity and sustainability of the crops in an agricultural operation. In this context, the primary objectives are maximizing the photosynthetic efficiency and optimizing off-season output to boost market supply. In order to produce the best growing environment for the tomato crops in West Bengal, the model balances climate, topography, water, and soil. It also takes into account the cost index, government regulations to maximize off-season production and also the market supply. VPD (d), temperature (h), humidity (w), slope (p), altitude (q), distance from the water resource (r), pH of water (s), soil pH (t), soil soluble salt content (u), government regulations (v), and costing index (z) are some of the attributes. The objective is to establish a profitable and sustainable agricultural

TABLE 1. Table of criterias.

Major factors	Criteria	Optimum range	Types of criterias
Climate	VPD	(0.5–1.2) kPa [60]	Non-Beneficial
	Temperature	(18–27) °C [50]	Beneficial
	Humidity	(60–85) %RH [60]	Beneficial
Topography	Slope	(0–2)° [34]	Beneficial
	Altitude	(0–0.3) km [50]	Beneficial
Water	Distance from resource	(0–5) km [50]	Beneficial
	pH	(5.6–6.5) [35]	Beneficial
Soil	Soluble salt	(1.5–3) ds/m [23]	Beneficial
	pH of soil	(5.8–6.8) [23]	Beneficial
Government policies	Infrastructure and labour	(0–1)	Beneficial
Costing index	Development cost	(0–1)	Non-Beneficial

system that serves both immediate market demands and long-term environmental concerns. In this case, the optimization functions are created using the single-layer neural network approach (a single-layer neural network is closely linked to the optimization function and is made up of input and output layers that are coupled to form one layer. In order to generate an output, it applies the activation function after computing the weighted sum of its inputs. In this case, the activation function, sigmoid, determines the output by using a threshold) and the MOO problem can be solved by applying the WSM to get the ideal weights for each criteria. Here $a'_1, a'_2, a'_3, a'_4, a'_5, a'_6, a'_7, a'_8, a'_9, a'_{10}, a'_{11}$ are the randomly assigned weights. The MOO model can be written as,

$$\text{Maximize: } f_1 = 1/\left(1 + e^{-(a'_1 d + a'_2 h + a'_3 w + a'_4 p + a'_5 q + a'_6 r + a'_7 s + a'_8 t + a'_9 u)}\right)$$
$$\text{Maximize: } f_2 = 1/\left(1 + e^{-(a'_1 d + a'_2 h + a'_3 w + a'_{10} v + a'_{11} z)}\right)$$

subject to the constraints,
$0.5 \le d \le 1.2, 18 \le h \le 27, 60 \le w \le 85, 0 \le p \le 2, 0 \le q \le 0.3, 0 \le r \le 5, 5.6 \le s \le 6.5, 5.8 \le t \le 6.8, 1.5 \le u \le 3, 0 \le v \le 1, 0 \le z \le 1$.

5.2. Numerical example

Step 1. At first, the numerical computation is carried out utilizing the decision matrices that the decision experts have provided. The following DM_1, DM_2 and DM_3 matrices convey the opinion of the decision-

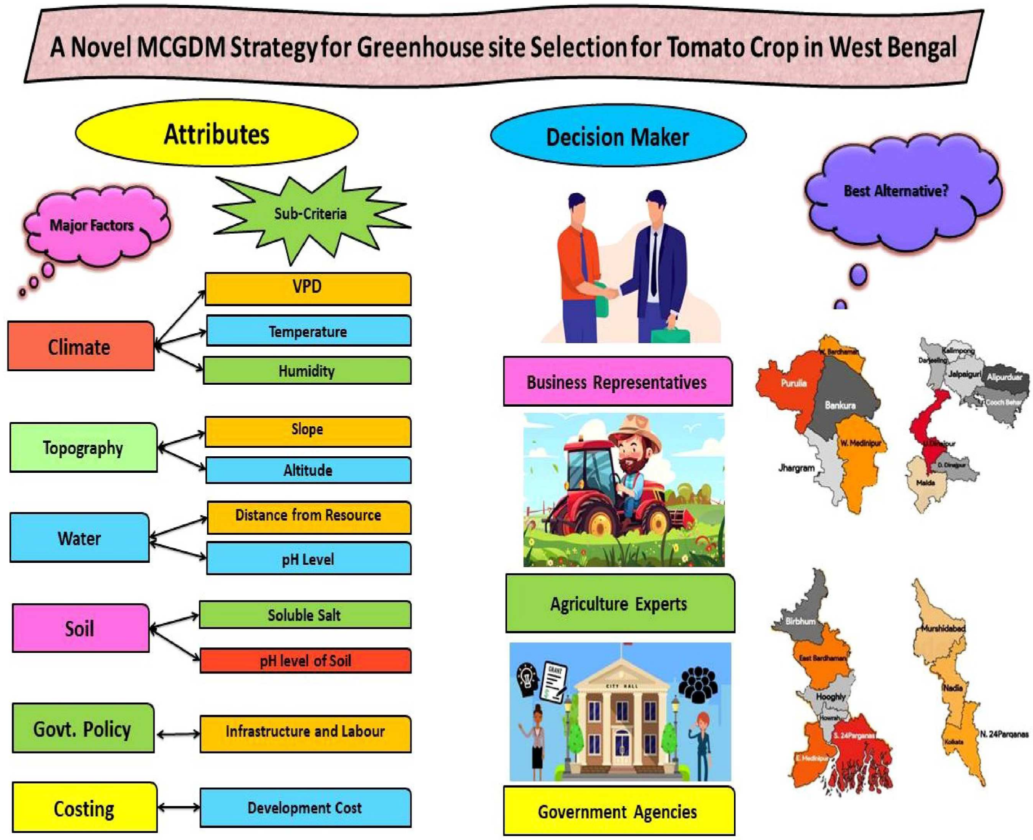

FIGURE 7. Visual representation of the attributes, decision-makers and alternatives of our proposed MCGDM model.

experts in terms of attribute *vs.* alternative mode.

$$\mathrm{DM}_1 = \begin{array}{c} \\ N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \\ N_9 \\ N_{10} \\ N_{11} \end{array} \begin{array}{c} \begin{array}{cccc} \mathscr{A}_1 & \mathscr{A}_2 & \mathscr{A}_3 & \mathscr{A}_4 \end{array} \\ \begin{pmatrix} \langle 0.3, 0.5, 0.7\rangle & \langle 0.5, 0.7, 0.8\rangle & \langle 0.7, 0.5, 0.6\rangle & \langle 0.8, 0.4, 0.7\rangle \\ \langle 0.7, 0.8, 0.7\rangle & \langle 0.5, 0.7, 0.4\rangle & \langle 0.8, 0.6, 0.7\rangle & \langle 0.4, 0.7, 0.2\rangle \\ \langle 0.7, 0.5, 0.7\rangle & \langle 0.5, 0.7, 0.6\rangle & \langle 0.6, 0.3, 0.7\rangle & \langle 0.3, 0.2, 0.8\rangle \\ \langle 0.8, 0.7, 0.3\rangle & \langle 0.5, 0.2, 0.8\rangle & \langle 0.4, 0.7, 0.7\rangle & \langle 0.8, 0.2, 0.5\rangle \\ \langle 0.3, 0.8, 0.7\rangle & \langle 0.6, 0.2, 0.4\rangle & \langle 0.5, 0.8, 0.2\rangle & \langle 0.8, 0.4, 0.7\rangle \\ \langle 0.4, 0.7, 0.7\rangle & \langle 0.7, 0.5, 0.7\rangle & \langle 0.5, 0.7, 0.4\rangle & \langle 0.8, 0.2, 0.5\rangle \\ \langle 0.7, 0.2, 0.4\rangle & \langle 0.8, 0.2, 0.4\rangle & \langle 0.2, 0.1, 0.8\rangle & \langle 0.5, 0.2, 0.8\rangle \\ \langle 0.8, 0.7, 0.3\rangle & \langle 0.3, 0.5, 0.7\rangle & \langle 0.7, 0.2, 0.3\rangle & \langle 0.5, 0.7, 0.4\rangle \\ \langle 0.7, 0.5, 0.6\rangle & \langle 0.6, 0.8, 0.5\rangle & \langle 0.8, 0.7, 0.6\rangle & \langle 0.6, 0.2, 0.4\rangle \\ \langle 0.5, 0.7, 0.8\rangle & \langle 0.7, 0.8, 0.7\rangle & \langle 0.8, 0.2, 0.5\rangle & \langle 0.3, 0.8, 0.7\rangle \\ \langle 0.6, 0.3, 0.7\rangle & \langle 0.3, 0.2, 0.8\rangle & \langle 0.5, 0.8, 0.2\rangle & \langle 0.8, 0.2, 0.5\rangle \end{pmatrix} \end{array} \tag{4}$$

$$
\mathrm{DM}_2 = \begin{array}{c} \\ N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \\ N_9 \\ N_{10} \\ N_{11} \end{array}
\begin{array}{c}
\begin{array}{cccc} \mathscr{A}_1 & \mathscr{A}_2 & \mathscr{A}_3 & \mathscr{A}_4 \end{array} \\
\begin{pmatrix}
\langle 0.8, 0.4, 0.7\rangle & \langle 0.6, 0.6, 0.7\rangle & \langle 0.4, 0.7, 0.2\rangle & \langle 0.6, 0.8, 0.5\rangle \\
\langle 0.5, 0.2, 0.8\rangle & \langle 0.3, 0.8, 0.7\rangle & \langle 0.4, 0.7, 0.7\rangle & \langle 0.6, 0.2, 0.4\rangle \\
\langle 0.5, 0.7, 0.6\rangle & \langle 0.7, 0.2, 0.3\rangle & \langle 0.3, 0.8, 0.7\rangle & \langle 0.5, 0.7, 0.8\rangle \\
\langle 0.7, 0.8, 0.7\rangle & \langle 0.5, 0.8, 0.2\rangle & \langle 0.8, 0.2, 0.4\rangle & \langle 0.5, 0.2, 0.2\rangle \\
\langle 0.7, 0.5, 0.6\rangle & \langle 0.8, 0.2, 0.5\rangle & \langle 0.2, 0.5, 0.4\rangle & \langle 0.8, 0.1, 0.7\rangle \\
\langle 0.5, 0.7, 0.4\rangle & \langle 0.8, 0.6, 0.7\rangle & \langle 0.7, 0.2, 0.4\rangle & \langle 0.2, 0.1, 0.8\rangle \\
\langle 0.3, 0.5, 0.7\rangle & \langle 0.8, 0.2, 0.4\rangle & \langle 0.6, 0.2, 0.7\rangle & \langle 0.3, 0.5, 0.7\rangle \\
\langle 0.8, 0.4, 0.7\rangle & \langle 0.4, 0.7, 0.2\rangle & \langle 0.3, 0.2, 0.8\rangle & \langle 0.7, 0.2, 0.3\rangle \\
\langle 0.6, 0.2, 0.4\rangle & \langle 0.8, 0.2, 0.5\rangle & \langle 0.4, 0.2, 0.7\rangle & \langle 0.7, 0.6, 0.7\rangle \\
\langle 0.6, 0.3, 0.7\rangle & \langle 0.5, 0.2, 0.8\rangle & \langle 0.3, 0.8, 0.7\rangle & \langle 0.5, 0.8, 0.2\rangle \\
\langle 0.7, 0.5, 0.7\rangle & \langle 0.3, 0.8, 0.7\rangle & \langle 0.4, 0.7, 0.7\rangle & \langle 0.3, 0.5, 0.7\rangle
\end{pmatrix}
\end{array} \tag{5}
$$

$$
\mathrm{DM}_3 = \begin{array}{c} \\ N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \\ N_9 \\ N_{10} \\ N_{11} \end{array}
\begin{array}{c}
\begin{array}{cccc} \mathscr{A}_1 & \mathscr{A}_2 & \mathscr{A}_3 & \mathscr{A}_4 \end{array} \\
\begin{pmatrix}
\langle 0.5, 0.7, 0.4\rangle & \langle 0.8, 0.2, 0.5\rangle & \langle 0.7, 0.2, 0.3\rangle & \langle 0.8, 0.1, 0.7\rangle \\
\langle 0.5, 0.8, 0.2\rangle & \langle 0.2, 0.5, 0.4\rangle & \langle 0.5, 0.2, 0.2\rangle & \langle 0.4, 0.7, 0.2\rangle \\
\langle 0.7, 0.5, 0.6\rangle & \langle 0.8, 0.2, 0.4\rangle & \langle 0.8, 0.7, 0.3\rangle & \langle 0.2, 0.1, 0.8\rangle \\
\langle 0.7, 0.8, 0.7\rangle & \langle 0.3, 0.8, 0.7\rangle & \langle 0.8, 0.6, 0.7\rangle & \langle 0.3, 0.2, 0.8\rangle \\
\langle 0.6, 0.8, 0.5\rangle & \langle 0.5, 0.7, 0.4\rangle & \langle 0.8, 0.1, 0.7\rangle & \langle 0.3, 0.5, 0.7\rangle \\
\langle 0.8, 0.4, 0.7\rangle & \langle 0.7, 0.5, 0.7\rangle & \langle 0.6, 0.2, 0.4\rangle & \langle 0.5, 0.7, 0.8\rangle \\
\langle 0.5, 0.7, 0.4\rangle & \langle 0.7, 0.2, 0.4\rangle & \langle 0.8, 0.2, 0.8\rangle & \langle 0.2, 0.5, 0.4\rangle \\
\langle 0.5, 0.7, 0.6\rangle & \langle 0.3, 0.8, 0.7\rangle & \langle 0.8, 0.6, 0.8\rangle & \langle 0.7, 0.8, 0.7\rangle \\
\langle 0.6, 0.3, 0.7\rangle & \langle 0.8, 0.2, 0.4\rangle & \langle 0.5, 0.2, 0.2\rangle & \langle 0.3, 0.5, 0.7\rangle \\
\langle 0.8, 0.2, 0.5\rangle & \langle 0.3, 0.8, 0.7\rangle & \langle 0.5, 0.2, 0.8\rangle & \langle 0.6, 0.8, 0.5\rangle \\
\langle 0.8, 0.6, 0.7\rangle & \langle 0.4, 0.7, 0.2\rangle & \langle 0.5, 0.7, 0.4\rangle & \langle 0.7, 0.5, 0.6\rangle
\end{pmatrix}
\end{array}. \tag{6}
$$

Step 2. Here, the aggregation of the decision matrices is computed using the $\mathscr{CNDWLA}$ operator formula as given in Theorem 3.5, with varying decision makers weight based on the relative importance given to each of them, business representatives (0.32), agricultural experts (0.35) and representatives from government agencies (0.33). The following DM is the aggregated decision matrix.

$$
\mathrm{DM} = \begin{array}{c} \\ N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \\ N_9 \\ N_{10} \\ N_{11} \end{array}
\begin{array}{c}
\begin{array}{cccc} \mathscr{A}_1 & \mathscr{A}_2 & \mathscr{A}_3 & \mathscr{A}_4 \end{array} \\
\begin{pmatrix}
\langle 0.578, 0.561, 0.487\rangle & \langle 0.545, 0.419, 0.427\rangle & \langle 0.511, 0.424, 0.414\rangle & \langle 0.623, 0.313, 0.504\rangle \\
\langle 0.409, 0.333, 0.359\rangle & \langle 0.576, 0.422, 0.537\rangle & \langle 0.542, 0.417, 0.396\rangle & \langle 0.369, 0.394, 0.378\rangle \\
\langle 0.496, 0.580, 0.543\rangle & \langle 0.576, 0.359, 0.537\rangle & \langle 0.583, 0.394, 0.450\rangle & \langle 0.573, 0.307, 0.333\rangle \\
\langle 0.602, 0.359, 0.450\rangle & \langle 0.413, 0.333, 0.360\rangle & \langle 0.627, 0.415, 0.489\rangle & \langle 0.571, 0.333, 0.378\rangle \\
\langle 0.514, 0.383, 0.564\rangle & \langle 0.551, 0.360, 0.680\rangle & \langle 0.623, 0.318, 0.421\rangle & \langle 0.641, 0.344, 0.450\rangle \\
\langle 0.545, 0.487, 0.489\rangle & \langle 0.606, 0.743, 0.450\rangle & \langle 0.444, 0.359, 0.631\rangle & \langle 0.621, 0.303, 0.378\rangle \\
\langle 0.500, 0.429, 0.537\rangle & \langle 0.641, 0.333, 0.631\rangle & \langle 0.623, 0.291, 0.361\rangle & \langle 0.576, 0.469, 0.419\rangle \\
\langle 0.621, 0.489, 0.487\rangle & \langle 0.511, 0.424, 0.394\rangle & \langle 0.604, 0.374, 0.359\rangle & \langle 0.502, 0.359, 0.485\rangle \\
\langle 0.457, 0.421, 0.541\rangle & \langle 0.627, 0.333, 0.749\rangle & \langle 0.542, 0.359, 0.417\rangle & \langle 0.515, 0.458, 0.485\rangle \\
\langle 0.545, 0.396, 0.427\rangle & \langle 0.496, 0.333, 0.394\rangle & \langle 0.573, 0.333, 0.424\rangle & \langle 0.435, 0.333, 0.422\rangle \\
\langle 0.583, 0.565, 0.450\rangle & \langle 0.514, 0.360, 0.361\rangle & \langle 0.257, 0.398, 0.419\rangle & \langle 0.602, 0.469, 0.556\rangle
\end{pmatrix}
\end{array}. \tag{7}
$$

Step 3. The formation and resolution of the MOO model occur as outlined below:

TABLE 2. Ranking values using MOORA and MARCOS.

Alternatives	MOORA		MARCOS	
	Ranking value	Ranking	Ranking value	Ranking
$\mathscr{A}_1$	-0.0278		0.292	
$\mathscr{A}_2$	-0.0057	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$	0.316	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$
$\mathscr{A}_3$	0.3773		0.868	
$\mathscr{A}_4$	0.0206		0.401	

(i) To construct the optimization functions, a single-layer neural network approach with a sigmoid activation function is employed.

$$\text{Maximize: } f_1 = 1/\left(1 + e^{-(a_1 d + a_2 h + a_3 w + a_4 p + a_5 q + a_6 r + a_7 s + a_8 t + a_9 u)}\right)$$
$$\text{Maximize: } f_2 = 1/\left(1 + e^{-(a_1 d + a_2 h + a_3 w + a_{10} v + a_{11} z)}\right)$$

subject to the constraints,
$0.5 \leq d \leq 1.2, 18 \leq h \leq 27, 60 \leq w \leq 85, 0 \leq p \leq 2, 0 \leq q \leq 0.3, 0 \leq r \leq 5, 5.6 \leq s \leq 6.5, 5.8 \leq t \leq 6.8, 1.5 \leq u \leq 3, 0 \leq v \leq 1, 0 \leq z \leq 1$.

(ii) To determine the optimum weights for each attribute, the MOO problem can be handled by utilizing the WSM and pareto optimal solution is reached by simulating the WSM a hundred times.

(iii) The TOPSIS method [3] is utilized to get the optimal outcome from pareto optimal solution. The obtained optimum weights of the attributes:
$d = 0.500, h = 20.171, w = 78.693, p = 1.175, q = 0.152, r = 3.443, s = 6.030, t = 6.303, u = 2.159, v = 0.563$,
$z = 0.525$.

Step 4. After normalization of weights [63] obtained from MOO using:

$$r'_{ij} = \begin{cases} \dfrac{r_{ij}}{\sqrt{\Sigma r_{ij}{}^2}} & r_{ij} = \text{Beneficial} \\ 1 - \dfrac{r_{ij}}{\sqrt{\Sigma r_{ij}{}^2}} & r_{ij} = \text{Non-Beneficial} \end{cases} \tag{8}$$

we get, $d' = 0.9939, h' = 0.2465, w' = 0.9618, p' = 0.0144, q' = 0.0019, r' = 0.0421, s' = 0.0737, t' = 0.0770, u' = 0.0264, v' = 0.0069, z' = 0.9936$.

Step 5. Here, the FUCOM approach is used to upgrade the normalized weights that were received from the MOO, in order to get the best possible weights for each criterion. FUCOM minimizes pairwise comparison inconsistency, which guarantees high accuracy when establishing the weights of criteria, this leads to more exact and trustworthy outcomes in decision-making [37]. The achieved upgraded weights of the attributes are
$d'' = 0.292, h'' = 0.048, w'' = 0.094, p'' = 0.049, q'' = 0.146, r'' = 0.017, s'' = 0.101, t'' = 0.079, u'' = 0.088, v'' = 0.002, z'' = 0.082$.

Step 6. Using MOORA approach [44], we obtain the alternatives ranking as $\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$, making $\mathscr{A}_3$ as the best option (see Tab. 2 and Fig. 8).

Step 7. The order of preference of the alternatives acquired utilizing MARCOS method [16] is, $\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$ (see Tab. 2 and Fig. 8) which clearly ensures that $\mathscr{A}_3$ is the optimal option.

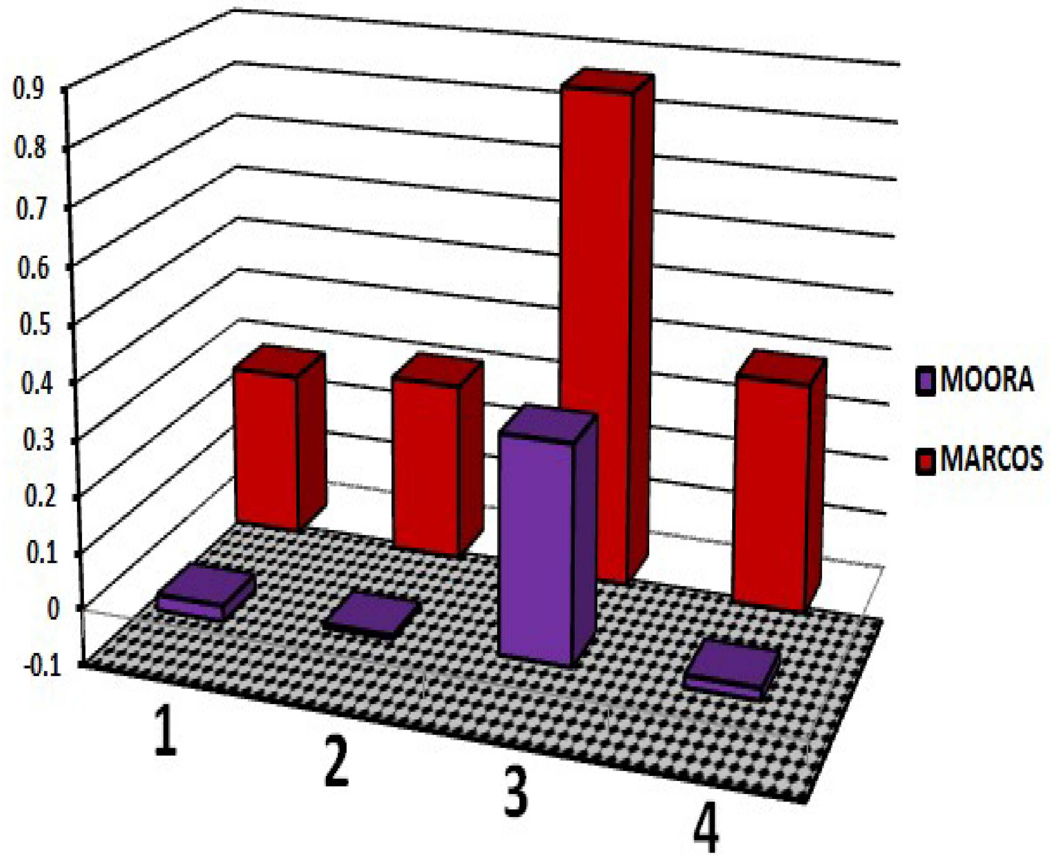

FIGURE 8. Depicts ranking of different alternatives using the decision-making processes MARCOS and MOORA.

5.3. Sensitivity analysis

Here, we evaluate a decision-making model's consistency and dependability by looking at how adjustments to input parameters or weights impact the result or alternative's ranking. We have carried out a sensitivity analysis in the context of ranking alternatives based on weight fluctuation of criteria or factors within a specific range of -5%, 5% as shown in Table 3 (see Figs. 9 and 10) and -10%, 10% as indicated in Table 4 (see Figs. 11 and 12) and observed the related changes in the order of alternatives.

The sensitivity analysis reflects a number of significant findings, from which we can infer some fundamental conclusions about this research project. Based on both tables (Tabs. 3 and 4), we can conclude that the degree of fluctuation of the weights at the range $-5\%, 5\%, -10\%, 10\%$ change has no effect on the preference of the alternatives using both MOORA and MARCOS method. Thus, the ideal site for greenhouse tomato cultivation still remains in West Bengal's Southern region.

5.4. Comparative analysis

To assess the practicality and rationale of our suggested work, we have compared it with several established techniques, as indicated in Table 5. Table 5 indicates that the approaches performed by Mercan and Sezgin [34] and Liu *et al.* [29] are unable to produce the order of the alternatives since the nature of the problem is MCDM but our achieved methodology constitutes the problem, which is MCGDM in nature. It is to be

Table 3. Senstivity analysis using MOORA and MARCOS method with −5% and 5% weight fluctuation.

Method	Alternatives	−5%	Ranking	5%	Ranking
MOORA	$\mathscr{A}_1$	−0.0259	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$	−0.0292	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$
	$\mathscr{A}_2$	−0.0049		−0.0060	
	$\mathscr{A}_3$	0.3597		0.3956	
	$\mathscr{A}_4$	0.0200		0.0213	
MARCOS	$\mathscr{A}_1$	0.2770	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$	0.2820	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$
	$\mathscr{A}_2$	0.3000		0.3080	
	$\mathscr{A}_3$	0.8240		0.8390	
	$\mathscr{A}_4$	0.3810		0.3890	

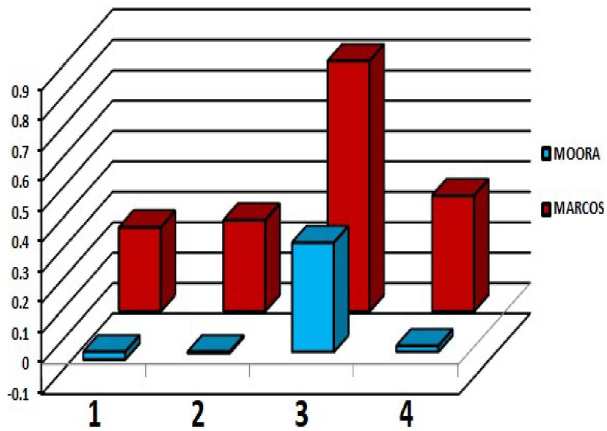

Figure 9. Ranking of alternatives at a fluctuated level of −5% using the decision-making techniques MARCOS and MOORA.

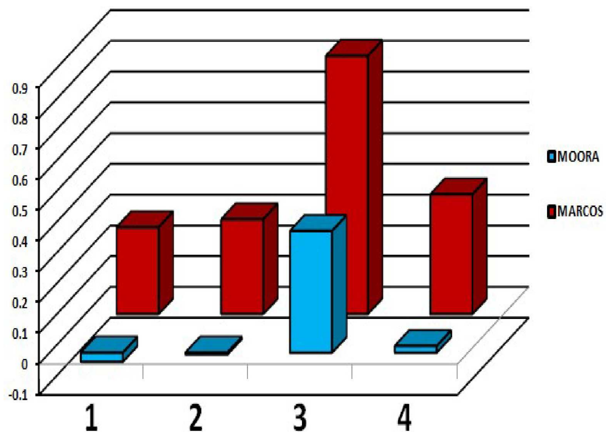

Figure 10. Ranking of alternatives at a fluctuated level of +5% using the decision-making techniques MARCOS and MOORA.

TABLE 4. Senstivity analysis using MOORA and MARCOS method with −10% and 10% weight fluctuation.

Method	Alternatives	−10%	Ranking	10%	Ranking
MOORA	$\mathscr{A}_1$	−0.0254	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$	−0.0302	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$
	$\mathscr{A}_2$	−0.0053		−0.0061	
	$\mathscr{A}_3$	0.3394		0.4152	
	$\mathscr{A}_4$	0.0179		0.0232	
MARCOS	$\mathscr{A}_1$	0.2950	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$	0.2890	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$
	$\mathscr{A}_2$	0.3210		0.3130	
	$\mathscr{A}_3$	0.8810		0.8540	
	$\mathscr{A}_4$	0.4100		0.3970	

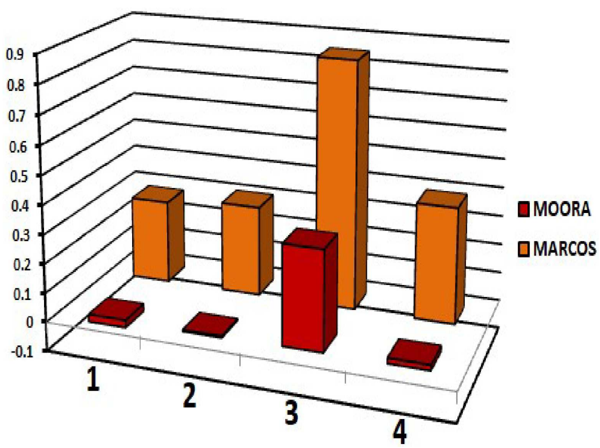

FIGURE 11. Ranking of alternatives at a fluctuated level of −10% using the decision-making techniques MARCOS and MOORA.

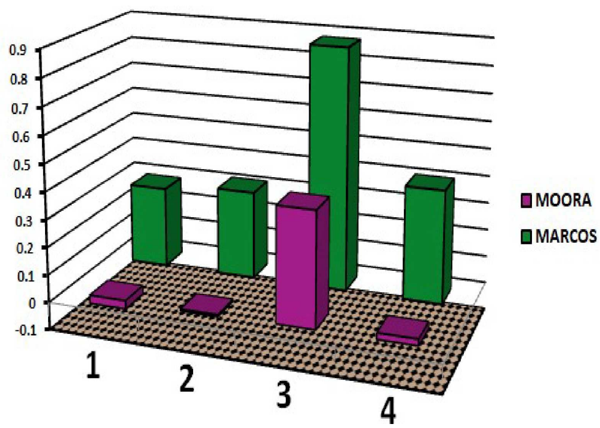

FIGURE 12. Ranking of alternatives at a fluctuated level of +10% using the decision-making techniques MARCOS and MOORA.

TABLE 5. Comparative analysis table of our proposed work and other pre-existing works.

Author	Environment & methods	Operator used	MCGDM/ MCDM	Multi objective optimization	Ranking
Mercan and Sezgin [34]	Crisp (AHP,GIS)	$\times$	MCDM	$\times$	Not Applicable
Banik *et al.* [6]	CNN	CNPAA	MCGDM	$\times$	Applicable
Liu *et al.* [29]	Crisp (DCRP)	$\times$	MCDM	✓	Not Applicable
Chakraborty *et al.* [12]	CNN	CNWA	MCGDM	$\times$	Applicable
Our proposed	CNN	CNDWLA	MCGDM	✓	Applicable

TABLE 6. Ranking of alternatives based on several methods.

Author	Method	Ranking value	Ranking of the alternatives
Banik *et al.* [6]	TOPSIS	$\mathscr{A}_1 = 0.4785$, $\mathscr{A}_2 = 0.4460$ $\mathscr{A}_3 = 0.8566$, $\mathscr{A}_4 = 0.4143$	$\mathscr{A}_3 > \mathscr{A}_1 > \mathscr{A}_2 > \mathscr{A}_4$
Banik *et al.* [6]	VIKOR	$\mathscr{A}_1 = 0.6172$, $\mathscr{A}_2 = 0.6959$ $\mathscr{A}_3 = 0.0000$, $\mathscr{A}_4 = 0.9906$	$\mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1 > \mathscr{A}_3$
Banik *et al.* [6]	EDAS	$\mathscr{A}_1 = 0.0490$, $\mathscr{A}_2 = 0.1236$ $\mathscr{A}_3 = 0.8170$, $\mathscr{A}_4 = 0.4101$	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$
Chakraborty *et al.* [12]	Score values	$\mathscr{A}_1 = 0.0470$, $\mathscr{A}_2 = 0.1009$ $\mathscr{A}_3 = 0.1338$, $\mathscr{A}_4 = 0.1638$	$\mathscr{A}_4 > \mathscr{A}_3 > \mathscr{A}_2 > \mathscr{A}_1$
Our proposed	MARCOS	$\mathscr{A}_1 = 0.2920$, $\mathscr{A}_2 = 0.3160$ $\mathscr{A}_3 = 0.8680$, $\mathscr{A}_4 = 0.4010$	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$
Our proposed	MOORA	$\mathscr{A}_1 = -0.0278$, $\mathscr{A}_2 = -0.0057$ $\mathscr{A}_3 = 0.3773$, $\mathscr{A}_4 = 0.0206$	$\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$

noted that the nature of the problems proposed by Chakraborty *et al.* [12] and Banik *et al.* [6] are MCGDM. Also, the operators used by Chakraborty *et al.* [12] and Banik *et al.* [6] are "cylindrical neutrosophic weighted aggregation (CNWA)" and "cylindrical neutrosophic power averaging aggregation (CNPAA)" respectively, in the CNN environment. Thus, it is feasible to directly compare these methods with our proposed methodology. In Table 6 we have presented a detailed comparison of these methods. Here, the results obtained by our suggested method using MARCOS and MOORA techniques have been compared with the results of Banik *et al.* [6] using TOPSIS, VIKOR, and EDAS techniques. A comparison analysis has also been performed with the results of Chakraborty *et al.* [12].

5.4.1. Comparison to the TOPSIS approach [6]

Banik *et al.* [6] implemented the TOPSIS approach in the context of CNN in the existing literature. Here, we have incorporated the remaining stages of the TOPSIS method into the weighted aggregated decision matrix (DM), given in Section 5.2. The values of alternatives are provided here as: $\mathscr{A}_1 = 0.4785$, $\mathscr{A}_2 = 0.4460$, $\mathscr{A}_3 = 0.8566$, $\mathscr{A}_4 = 0.4143$ (see Tab. 6). So, we obtain the alternatives ranking as $\mathscr{A}_3 > \mathscr{A}_1 > \mathscr{A}_2 > \mathscr{A}_4$, making $\mathscr{A}_3$ as the best option.

5.4.2. Comparison to the EDAS approach [6]

Here, we have incorporated the remaining stages of the EDAS method into the weighted aggregated decision matrix (DM), represented in Section 5.2. The values of alternatives are $\mathscr{A}_1 = 0.0490$, $\mathscr{A}_2 = 0.1236$, $\mathscr{A}_3 = 0.8170$, $\mathscr{A}_4 = 0.4101$ respectively (see Tab. 6). So, we obtain the alternatives ranking as $\mathscr{A}_3 > \mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1$, considering $\mathscr{A}_3$ as the best option.

5.4.3. Comparison to the VIKOR approach [6]

Here, we have incorporated the remaining stages of the VIKOR method into the weighted aggregated decision matrix (DM), presented in Section 5.2. The values of alternatives are presented by $\mathscr{A}_1 = 0.6172$, $\mathscr{A}_2 = 0.6959$, $\mathscr{A}_3 = 0$, $\mathscr{A}_4 = 0.9906$ (see Tab. 6). So, we obtain the alternatives ranking as $\mathscr{A}_4 > \mathscr{A}_2 > \mathscr{A}_1 > \mathscr{A}_3$, making $\mathscr{A}_4$ as the best option.

5.4.4. Comparison to the Chakraborty et al. [12] approach

In this part, we have incorporated the remaining stages of the Chakraborty *et al.* [12] method into the weighted aggregated decision matrix (DM), illustrated in Section 5.2. Here, the ranking values are computed using the score values in the CN environment. The values of alternatives are presented by $\mathscr{A}_1 = 0.0470$, $\mathscr{A}_2 = 0.1009$, $\mathscr{A}_3 = 0.1338$, $\mathscr{A}_4 = 0.1638$ (see Tab. 6). So, we get the alternatives ranking as $\mathscr{A}_4 > \mathscr{A}_3 > \mathscr{A}_2 > \mathscr{A}_1$, making $\mathscr{A}_4$ as the best option.

The graphical illustration of Table 6 is presented in Figure 13. From Table 6 and Figure 13, we have thoroughly analyzed the ranking outcomes. Here, it has been noticed that our suggested approach is equivalent to other existing methodologies employed to make the best selection. The outputs of our suggested methodology vary only for the Chakraborty *et al.* [12] method and VIKOR method [6]. Note that, Banik *et al.* [6] also highlighted multiple causes for the lower trustworthiness of the VIKOR strategy outputs.

At this point, our proposed model offers the following important advantages over the existing methodologies:

- In the TOPSIS, EDAS, VIKOR techniques [6], authors used the CNPAA operator to combine the data from various decision makers, but our proposed approach employs the $\mathscr{CNDWLA}$ operator to combine data from several decision makers. The versatility and efficacy of the proposed approach are enhanced by including of an extra parameter in the $\mathscr{CNDWLA}$ operator.
- In the TOPSIS, EDAS, VIKOR techniques [6], authors implemented the weights of attributes obtained from the survey only, the authors did not evaluate attribute weights separately. However, in our proposed approach, we have implemented the integrated MOO and FUCOM methods to determine the attributes weights. Consequently, our proposed approach is more rational, versatile and efficient.
- The authors of the Chakraborty *et al.* [12] utilized the CNWA operator to combine data collected from multiple experts and used the score function to find the best option, while our proposed approach uses the $\mathscr{CNDWLA}$ operator to combine information from numerous experts and applied decision-making techniques MARCOS and MOORA. It is pointed out that decision-making techniques are more preferred to determine the ranking of the result outcomes than using the score function for MCGDM problem. Furthermore, the versatility and effectiveness of our proposed technique are enhanced by the inclusion of an extra parameter in the $\mathscr{CNDWLA}$ operator.

5.5. Managerial implications

Some managerial implications of our proposed approach are demonstrated as follows:

(i) In this article, we have proposed a MOO based decision-making technique employing the $\mathscr{CNDWLA}$ operator in the CN environment. Here, MOO based FUCOM approach has been incorporated to ascertain the attribute weights in a dynamic way. Section 5.2 illustrates the corresponding results of our MCGDM problem which demonstrate the practicality of our proposed framework. As a consequence, our proposed approach can be utilized for sorting other real-world MCGDM scenarios with ambiguity and entirely unknown attribute weight details.

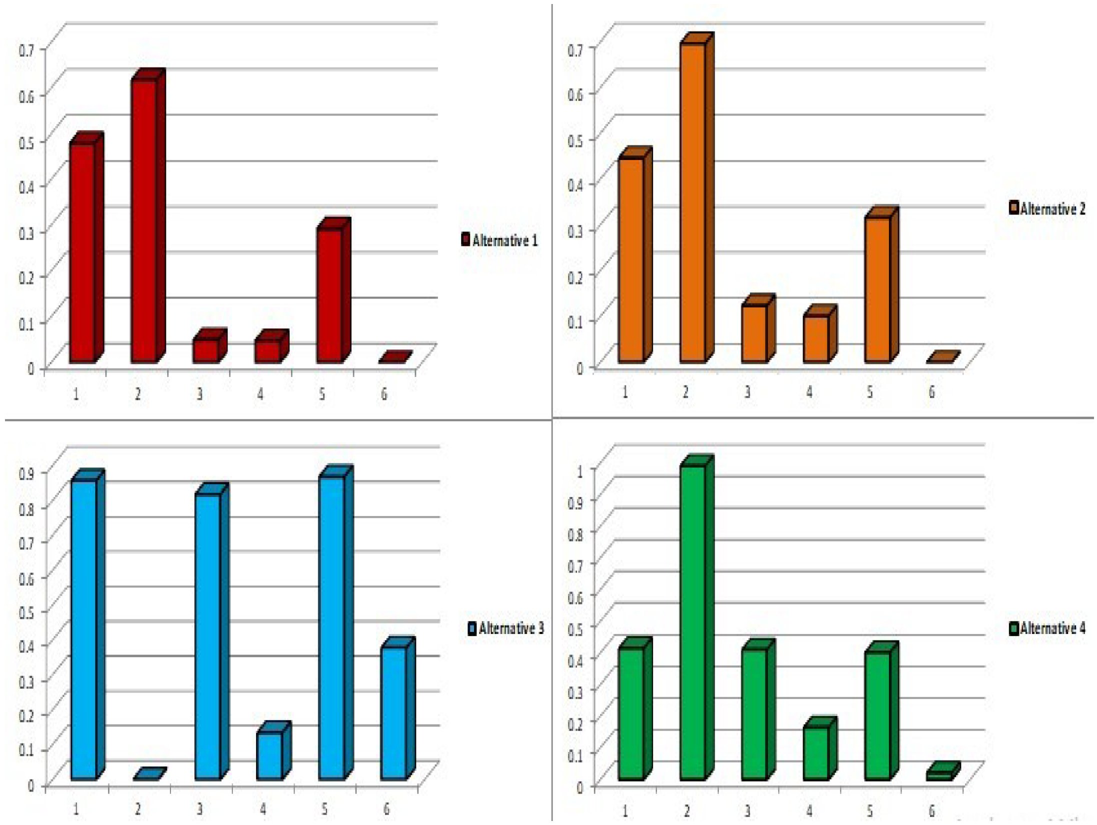

FIGURE 13. Each figure represents the ranking of different alternatives based on the outcomes of comparison Table 6.

(ii) Our proposed approach, when combined with advanced software can save laborious data collection as well as time to process for complicated decision-making situations.

5.6. Limitations

Our proposed MCGDM model has a few limitations that are discussed here:

(i) All the criterias are regarded as independent in our proposed MCGDM model. However, sometimes there are interconnections among the criteria in real-world scenarios.
(ii) Three decision makers' data about four distinct greenhouse locations for tomato crops have been employed in our proposed model. In the future, it will be able to validate the outcomes of our study by gathering information from several greenhouse locations.
(iii) A practical issue is that, to obtain full benefit from the adaptability and effectiveness of CNSs, decision-makers are required to be trained in the preference style.
(iv) Additional climate and environmental factors can be taken into consideration in creating the best greenhouse structure as environmental issues are getting more serious.

6. Conclusion and future scope

In this research study, we have established Dombi weighted logarithmic aggregation operator and explored its characteristics within the realm of CN contexts. We have studied the algebraic properties of the said operator and noticed that the said Dombi logarithmic law delivers a very robust and stable outcome with a moderate variation of operational parameters. Therefore, choosing Dombi logarithmic operations is considered more advantageous compared to other conventional logarithmic laws. Furthermore, we have implemented MOO, employing sigmoid functions as the objective functions inspired by the concept of a single-layer neural network, for the purpose of evaluating attribute weights. Ultimately, these weights are upgraded by the FUCOM method. Thus, our approach not only streamlines the attribute weight evaluation but also incorporates the FUCOM method for their continual enhancement. To address the requirements of current agricultural practices, greenhouses provide uninterrupted cultivation throughout the year, efficient resource utilization, and resilient farming practices in response to varying climatic conditions. The cultivation of tomato crops in the fertile plains of West Bengal has failed to match the growing demand, particularly during periods of the year when production is typically lower. In addition to addressing the challenges involved in growing tomato crops in West Bengal, this framework also aligns with the requirements of modern agricultural practices and establishes a sustainable agriculture model that is adaptable to a versatile range of climates. In this context, we illustrate the utilization of the $\mathscr{CNDWLA}$ operator within the CNN environment to identify the most suitable site for a greenhouse dedicated to tomato cultivation in West Bengal. The selection process of the most suitable greenhouse site in West Bengal is evaluated by using decision making techniques MARCOS and MOORA. Using both MARCOS and MOORA, we make sure that the rankings are cross-validated using two different methodologies. Thus, the final ranking of alternatives has more confidence because of this dual-method approach, which enables us to compare and confirm the consistency of the results. Additionally, to assess the stability and efficacy of our model, we have conducted a sensitivity analysis by varying attribute values across various scenarios. Following the analysis, it becomes apparent that South Bengal is the most suitable greenhouse site for tomato crops in West Bengal. Furthermore, to verify the effectiveness of our suggested methodology, we have executed a comparative analysis of our suggested research along with previous works that reflect the effectiveness and utility of our proposed methodology.

In future studies, the $\mathscr{CNDWLA}$ operator can be utilized embedding various optimization techniques and decision-making scenarios and examining its effectiveness in conjunction with alternative optimization methodologies, broadening its impact to domains beyond its present applications. Furthermore, there is potential for the creation of new advantageous operators in the realm of CNN that have yet to be established. This line of research not only advances the theoretical foundations of CNN but also creates opportunities for tackling challenging real-world issues in domains like decision analysis, uncertainty modeling and artificial intelligence. This future research trajectory offers exciting opportunities for innovation and advancement and has the potential to significantly impact the field of computational mathematics. Moreover, our suggested strategy can be used to implement a sustainable development program [31], site evaluation of software operating units [59], various fuzzy set extensions [54, 56, 57].

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Conflicts of interest

The authors declare that they have no conflict of interest regarding this research article.

Data availability statement

Data used in this work are available by request to authors.

Author contribution statement

All the authors equally contributed in this research article.

Ethical approval

None of the authors conducted any human or animal experiments for this article.

Informed consent

Every research participant was given the opportunity to give their informed consent.

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