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RESEARCH ON PROMOTIONAL PRICING DECISIONS OF RETAILERS CONSIDERING CUSTOMERS' ADD-ON ITEMS RETURN BEHAVIOR

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Abstract. This study aims to investigate a retailer's optimal decisions under "Value Increasing" promotion, where speculative consumers will deliberately purchase add-on items to qualify for discounts if the purchase amount is less than the "Value Increasing" promotional threshold and then return the add-on items after successful payment. The models without and with the "Value Increasing" promotion are established to investigate the effects of speculative consumers' add-on items refund behavior on the optimal pricing strategies and the optimal profits. The results show that participating in the "Value Increasing" promotional campaigns does not always benefit retailers. When the promotional discounts degree meets the incentive compatibility conditions, a low probability of the product being added by speculative consumers or a small proportion of speculative consumers makes retailers benefit more from participating in the "Value Increasing" promotional campaigns. However, when these conditions are not met, not participating in the "Value Increasing" promotion is better for retailers. Moreover, compared to without "Value Increasing" promotional campaigns, retailers will set a higher regular price to offset the losses associated with speculative returns under the "Value Increasing" promotional campaigns, which may result in consumers' final price after the discount not necessarily be lower than the price they would pay under non-promotional campaigns.

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1. Introduction

In recent years, the emergence of Internet information technology has greatly promoted the development of online retail. According to a report conducted by Statista, China retail e-commerce sales reached \$3.17 trillion with an increase of 10% in 2023[1]. To achieve this, "Value Increasing" promotional campaigns have been adopted as a common marketing strategy by more and more retailers to attract users' online shopping, thereby increasing revenues [1–3]. "Value Increasing" promotional campaigns refer to a strategy that platform decides the price discounts and retailers decide whether to participate. If a retailer participates, his products are sold according to the price discounts set by the platform. For example, in the cross-store "300 minus 30" activity implemented by Tmall.com, the platform decides the promotion discount of "300 minus 30", and retailers decide whether to

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[1] https://www.emarketer.com/content/china-ecommerce-forecast-2022.

participate or not. Participating retailers sell products in accordance with the rule of "300 minus 30". If the consumers' purchase amounts are greater than or equal to 300 yuan, they can enjoy 30-yuan discounts, and the discounts will be allocated to each commodity[2]. Otherwise, consumers will face two options: purchase the product at the original price directly or purchase add-on items from the retailers that participate in "Value Increasing" promotion activity to qualify for price discounts.

The implementation of the "Value Increasing" promotion will undoubtedly stimulate demand and increase product sales for retailers [4–6]. However, it also breeds the add-on items refund behavior of speculative consumers whose purchase amounts are less than the full reduction threshold. Take two e-commerce giants in China as an example, JD.com and Tmall.com officially announced that the total turnover of "Double 11" reached 769.7 billion yuan at 0:00 on November 12, 2020. However, on the first day after the "Double 11" promotion festival, "refund" airborne first place on the Weibo hot topics list[2]. In the discussion area of this topic, there was a voting activity about "What is the reason for your refund?". The result shows that the vote for a refund reason that purchases add-on items to get discounts and then returns the add-on items after successful payment ranks third, which means that consumers' add-on items refund behavior has become a common phenomenon. However, due to the surge of consumer purchasing behavior during the "Double 11" period, which involves a lot of storage costs and labor costs, retailers need to increase staff in advance to cope with the "double 11". The customer's add-on items refund behavior on the one hand will affect the retailers' calculation of inventory resources. On the other hand, the behavior also inhibits the normal online shopping of other consumers to some extent. Especially for some small and medium-sized retailers, participating in "Double 11" "Value Increasing" promotion activity to a large extent is just for advertising. However, they will encounter such "Add-on items refund", which is quite helpless.

Therefore, there are many interesting questions worth exploring (1) Why does a retailer participates in "Value Increasing" promotion activity? Will participation in "Value Increasing" promotion activity always be beneficial to retailers in the presence of speculative consumers? (2) Taking into account speculative consumers' add-on items refund behavior, how should retailers set optimal pricing decisions under different sales modes and what factors will affect them? (3) Will the retailer with "Value Increasing" promotion have a better pricing strategy than a retailer without one?

To solve the above problems, considering consumers' speculative behavior, we first examine the scenario without "Value Increasing" promotion as a benchmark. Then we build a decision-making model with "Value Increasing" promotion and analyze the impact of speculative consumers' add-on items refund behavior on the optimal price decision of retailers. We further explore the influence of the speculative customers' behavior on profits by a comparison analysis. Finally, the numerical analysis supports and complements our theoretical analysis, providing further managerial insights.

Several main results emerge from our study. First, participating in the "Value Increasing" promotion activity does not always benefit retailers. The promotional discounts degree, the probability of the product being added by speculative consumers, and the proportion of speculative consumers are key factors that determine the promotional choice of retailers. In particular, when the promotional discount degree meets the incentive compatibility conditions, a low probability of the product being added by speculative consumers or a small proportion of speculative consumers makes retailers benefit more from participating in the "Value Increasing" promotion. In contrast, when these conditions are not met, not participating in the "Value Increasing" promotion is more advantageous for retailers.

Second, under the "Value Increasing" promotional campaign, an increase in the proportion of speculative consumers prompts retailers to set higher promotional prices. However, an increase in the probability of the product being added by speculative consumers or an increase in the sensitivity coefficient of consumers to the discount amounts leads retailers to raise promotional prices only within specific parameter ranges. In contrast, under non-"Value Increasing" promotional campaign, the product price is determined solely by the characteristics of

[2] https://www.sohu.com/a/431950057_641950.

the product itself. Furthermore, the "Value Increasing" promotional campaign motivates retailers to set a higher price compared to the situation where such a promotion is not used.

This study contributes to the literature mainly in two areas: First, we provide new insights into the literature on "Value Increasing" promotion. That is, we not only consider the positive effect of "Value Increasing" promotion on retailers but also analyze the negative effect of speculative customer's add-on items refund behavior bred by "Value Increasing" promotion on retailers. Under the combined effect of the two situations, we find that participating in the "Value Increasing" promotion is not always a dominant strategy, and under certain conditions, not participating "Value Increasing" promotion will lead to the more profits. We provide theoretical basis and practical suggestions for retailers to effectively imply the "Value Increasing" promotion.

Second, we extend the research on customer behavior. We have taken into account the impact of speculative consumers' deliberate add-on items refund behavior on retailers' decisions. We reveal that speculative consumers' deliberate add-on items refund behavior plays a critical role in making pricing strategies in "Value Increasing" promotion. In the presence of consumers speculative behavior, the retailers need to consider the balance between the positive effect and the negative effect of "Value Increasing" promotion.

The rest of this paper is organized as follows: Section 2 reviews the relevant literature about promotion strategy and customer behavior. Section 3 describes and constructs the mathematical models without and with "Value Increasing" promotion. Section 4 compares the optimal profits of the retailer under two scenarios. Section 5 provides a numerical analysis. Section 6 draws the conclusions and provides suggestions for the optimal decisions of retailers.

2. Literature review

There are two streams of research related to this paper: promotion strategy and customer behavior.

2.1. Promotion strategy

Promotion has been a popular marketing strategy adopted by online retailers, and has been widely researched by scholars. Tian *et al.* [7] investigated the impact of retailers' assortment strategies on profits and consumer behavior in community group-buying channels. Huang *et al.* [8] showed that group buying could rapidly generate high demand for firms by offering substantial price discounts, which not only attract more consumers who may be unfamiliar with the products but also increase their willingness to make purchases. Aviv *et al.* [9] taking retailers selling seasonal products as the research object, analyzed how retailers should price in the promotion period when facing strategic consumers. Chen *et al.* [10] investigated the strategic usage of instant discount and gift card promotions between suppliers and retailers in different covered markets. Feng *et al.* [11] examined the promotional strategies a platform could adopt for its first-party products when third-party sellers offered identical products, and found that the promotional campaign did not always result in higher demand for the first-party product despite market expansion could increase the number of price-sensitive consumers. Zhang *et al.* [12] discussed how advance selling and customers' regret affect retailers and revealed that when action regret is ignored, retailers suffer moderate profit losses, but ignoring inaction regret leads to greater losses. Xu *et al.* [13] examined the strategy of premium advance selling in the context of customers' regret and analyzed these conditions of adopting this strategy for retailers. Jiang *et al.* [14] investigated the sellers' optimal rebate provision strategies in a supply chain setting wherein two competing sellers offer substitutable products *via* a common platform, and can choose whether to provide rebates to boost their respective sales. Li *et al.* [15] explored the action mechanism of coupon promotion on omnichannel price and optimal decisions of retailer by constructing a theoretical model. Wang *et al.* [16] investigated the influence of rebate promotion provided by the supplier on the retailer.

The existing literature primarily explores different promotional models, such as group buying, discount, coupon, advance selling, and rebates, and their impact on retailers' optimal decisions. In contrast, our study focuses on "Value-Increasing" promotional campaigns. We examine both the positive impact of this promotion on product sales and the negative effect of consumers' speculative add-on return behavior on the optimal pricing and the optimal profits of retailers.

2.2. Customer behavior

In the research on customer behavior, scholars have pointed out that customer behavior is a key factor influencing retailers' decision-making, and there are significant differences in the impact of different consumer behaviors on retailers' price strategies. Barta *et al.* [17] investigated the origins and impact of consumer regret on retailers' marketing strategies, particularly in managing negative online reviews. The results provided valuable managerial insights on how to effectively address consumer regret in order to boost sales. Duan and Feng [18] explored the optimal price decisions in a social network, where consumers undergo a network impact that relies on their peers' consumptions and a reference price that their peers receive the average price. Zaman and Zaccour [19] found that the equilibrium price will be higher when consumers act strategically than when they act myopically. Wang *et al.* [20] considered the retailer's optimal pricing decision when the strategy and myopic consumers coexist, and conclude that the quick response policy can effectively increase the retailer's profits. Pang *et al.* [21] explored how strategic consumer behavior emerging from consumer-to-consumer resale activities affects market incumbents from both economic and environmental perspectives. The study found that all market participants can achieve a win–win situation at the expense of exacerbating environmental impacts. Chaab *et al.* [22] developed a new product diffusion model based on the consideration of social influence and strategic consumer behavior, and the results of the study showed that the firm adopts a penetration pricing strategy in the presence of strategic consumers, whereas it decreases the price first and then increases it in face of myopic consumers. Moreover, some scholars have also studied consumers' opportunistic behavior. Fan *et al.* [23] designed a joint policy for retailers regarding price, refund amount and type of refund based on the purpose of dealing with opportunistic consumer returns. Khouja and Hammami [24] analyzed the optimization problem of product price, order quantity and return strategy when there is opportunistic consumer behavior in the market. Different from these studies, we study the speculative return behavior in which consumers buy the add-on product with the full intention of returning it. In other words, these consumers do not have the intention to keep the add-on product at all, they purchase it solely to meet the promotional threshold.

Considering customers' speculative return behavior, this paper constructs theoretical models for a retailer without and with the "Value Increasing" promotion and obtain the optimal pricing strategies under the two modes respectively. Through the comparative analysis under two models, this paper explores the impact of the customers' speculative return behavior on the retailer's optimal price and profit, as well as the critical condition for the retailer to choose to (not) participate in the "Value Increasing" promotion. Finally, some practical suggestions are provided for the retailer to make the optimal decision. Different from the existing research, this study not only considers the positive effect of "Value Increasing" promotion, but also analyzes the negative effect of consumers' speculative return behavior on retailers. This paper examines how retailers should set product prices under the combined influence of these two factors and identifies the key determinants that affect pricing decisions. The findings provide a strategic decision-making framework to guide retailers in the effective implementation of "Value-Increasing" promotional campaigns.

Table 1 compares this work with related previous researches. Our main contributions are threefold: First, most existing research on promotional strategy mainly focuses on discounts, coupons, advance selling, cashback, and rebates. In contrast, we analyze "Value-Increasing" promotional campaigns. In this framework, "Value-Increasing" promotion can stimulate unplanned purchases, thereby increasing sales. However, with the protection of return policies, some opportunistic consumers may exploit this strategy by purchasing multiple products to meet the promotion requirements and then returning the items that were not originally intended for purchase. This behavior can result in additional return-related losses for retailers.

Second, although some scholars, such as Song *et al.* [26], have studied "Value-Increasing" promotions, their work focuses on a retailer selling two complementary products and assumes that product 2 is an add-on item. However, in reality, each product has the potential to be purchased by consumers as either a core product or an add-on item. In this paper, we examine a more realistic scenario where the product has a certain probability of

TABLE 1. Comparison with related studies.

Articles	Decision maker and product type	Promotional type	Speculative behavior	Discount-sensitivity	Research question	Key determinants of promotional strategies
Feng *et al.* [11]	Identical product provided by a platform vendor and third-party seller	Discount	No	Yes	Platform's promotional strategy	(a) Market expansion effect, (b) proportion of price-sensitive consumers, (c) redemption cost, (d) commission rate
Nie *et al.* [25]	A retailer selling the core product and add-on product	Bundle or add-on discount	No	No	Sales pricing models based on return	(a) add-on product substandard rate, (b) core product price and degree of discount
Song *et al.* [26]	Two retailers selling complementary products	Cross-store full-reduction promotion	Yes	No	Optimal pricing strategy of retailers	(a) The proportion of speculative consumer, (b) discount amount allocated to a product, (c) commission rate
Liang *et al.* [27]	Retailers with green products	Limited-time and limited-quantity	No	No	The effect of the limited-time *vs.* limited-quantity on sharing intention	(a) Products' identity signaling attributes, (b) sharing intention
Bauner *et al.* [28]	Manufacturer's and retailer's national brands, private label	Coupon	No	Yes	Coupon strategies	(a) Quality, (b) degree of feature differentiation
Jiang *et al.* [14]	Two sellers selling substitutable products	Rebate	No	Yes	Rebate strategies	(a) Sales format, (b) commission rate differentiation, (c) the ratio of sensitive consumers
Zhang *et al.* [29]	Retailer with a brand or two retailers with substitute brands	Coupon	No	Yes	Coupon duration on brand profitability	(a) Coupon face value, (b) coupon duration, (c) redemption cost
Xu *et al.* [13]	A retailer selling a time-sensitive product	Advance selling	No	No	Premium pricing and capacity rationing	(a) The relative strength of regret, (b) the difference between premium and spot pricing
This study	A retailer selling a product that may be treated as add-on item	"Value Increasing" promotion	Yes	Yes	"Value Increasing" promotion and pricing decisions	(a) The probability of the product being treated as add-on item, (b) the proportion of speculative consumer

being classified as either an add-on item or a core product. We explore whether retailers should participate in threshold-based discount "Value-Increasing" promotions and how to determine optimal pricing strategies. This analysis aligns more closely with reality and offers significant practical applicability and general value.

Third, we provide a theoretical analysis that uncovers the mechanisms by which retailers, despite knowing the existence of speculative return behaviors among consumers, still choose to allow such behavior, expanding the current research by providing a fresh viewpoint on the issue.

3. Problem description and modeling

Consider a shopping scenario composed of a retailer and a group of consumers, where the retailer sells products at regular price p in a single sales period. During the shopping carnival, the retailer needs to decide whether to participates in the "Value Increasing" promotional activity provided by the third-party platform, which sets the promotion threshold and discounts, in order to promote its products, or not to participates in the "Value Increasing" promotional activity. Following Nie *et al.* [25], we assume $\theta (0 < \theta < 1)$ is the discount degree in promotional campaigns. The retailer incurs a total cost, denoted as C, which includes production cost, sales cost and commission fee paid to a platform etc. [30].

After observing retailer's sales price and promotional discount degree, consumers make their purchase decisions. If the product price is greater than or equal to the "Value Increasing" promotion threshold, the consumers can enjoy discounts by directly purchasing the product. Otherwise, in terms of reflecting the comprehensive nature of the shopping experience, consumers either purchase at regular price or choose to purchase add-on items in order to reach the "Value Increasing" promotion threshold and then get discounts. Due to the characteristics of product sales, there are two types of customers in the market. One is the consumers who deliberately purchase add-on items to get discounts and then return the add-on items after successful payment, which are defined as the speculative consumers. The other is the consumers who do not intentionally purchase add-on items to get discounts and then return it, which are defined as ordinary consumers. Denote λ and $1-\lambda$ represent the proportion of speculative consumers and ordinary consumers in the market, respectively. Since each product has the potential to be used by speculative consumers as an add-on product, we assume that the probability of retailer's product being used as an add-on product by speculative consumers is $\beta(0 < \beta < 1)$, which is closer to the reality, and is also one of the innovations of this paper.

If the retailer participates in the "Value Increasing" promotion activity, the retailer will entail the unit return cost of $h(0 < h < C)$ for the products returned by the speculative consumers. For simplicity, we assume that returned products are not resold, and any unsold products at the end of the sales cycle are assigned a residual value of zero.

Following Siqin *et al.* [31], He *et al.* [32], and Xu *et al.* [33], we assume that the market demands for products is a linear function of the price. Let d_n and d_t represent the retailer's demands under the two sales models of not participating and participating in the "Value Increasing" promotion, respectively; S represents the initial market demand for the product; and k is the demand price elasticities, where $S, k > 0$.

According to Liu *et al.* [34] and the law of diminishing marginal effect, under the sales mode of not participating in the "Value Increasing" promotion campaign, the demands function for the product is as follows:

$$d_n = s - kp_n. \tag{1}$$

Under the sales mode of participating in the "Value Increasing" promotion campaigns, following Liu *et al.* [34], the demand functions for the product purchased at regular price d_{to} and the product purchased at promotional price d_{ta} are, respectively:

$$d_{to} = s - kp_t \tag{2}$$

$$d_{ta} = s - k(p_t - \mu(1-\theta)p_t) \tag{3}$$

where $\mu(0 < \mu < 1)$ is the sensitivity coefficient of consumers to price discounts, which is consistent with Feng *et al.* [11], and $\theta(0 < \theta < 1)$ is the price discount degree in the "Value Increasing" promotion. Table 2 lists the symbols used in this paper.

3.1. Benchmark model

To investigate the impact of consumers' speculative behavior on retailers' decision with "Value Increasing" promotion, we need to analyze the retailer's optimal strategy without "Value Increasing" promotion as a benchmark model. In this case, both speculative consumers and ordinary consumers will purchase products at the original price. The profit function of the retailer is as follows:

$$\begin{aligned}\Pi_n &= (p_n - C)d_n \\ &= (p_n - C)(S - kp_n).\end{aligned} \tag{4}$$

Proposition 1. *When the retailer does not participate in "Value Increasing" promotion campaigns, there is the optimal original price p_n^*, which maximizes the profit of the retailer Π_n^*. Where p_n^* and Π_n^* are respectively:*

$$p_n^* = \frac{S + kC}{2k} \tag{5}$$

TABLE 2. Summary of notations and symbols.

Notations	Description
p_n	Product price without "Value Increasing" promotion
p_t	Product price with "Value Increasing" promotion
d_n	Product demands without "Value Increasing" promotion
d_{to}	Demands for product purchased at original price with "Value Increasing" promotion
d_{ta}	Demands for product purchased at promotion price with "Value Increasing" promotion
S	Initial market demands of products
k	The demands price elasticities of products
μ	The sensitivity coefficient of consumers to discount amounts
θ	Price discount degree $(0 < \theta < 1)$
λ	The proportion of speculative consumers $(0 < \lambda < 1)$
β	The probability of products added by speculative consumers $(0 \le \beta \le 1)$
C	The total cost of a product sold by retailer (including production cost, sales cost etc.)
h	The unit return cost $(h < C)$

$$\Pi_n^* = \frac{(S - kC)^2}{4k} \cdot \tag{6}$$

The proof is provided in the Appendix A.

Proposition 1 shows that when the retailer does not participate in the "Value Increasing" promotion activity, the retailer's profits are a univariate quadratic function with a unique peak value about price, that is, there is an optimal solution to maximize the retailer's profits.

3.2. The model with "Value Increasing" promotion

This subsection constructs the decision models for the retailer participating in "Value Increasing" promotion campaigns. To distinguish from the symbols in the basic decision model, the symbols used in this model are marked with the subscript "t".

When the retailer participates in "Value Increasing" promotional campaigns, on the one hand, "Value Increasing" promotion can stimulate consumers to make additional purchases and increase sales. On the other hand, the "Value Increasing" promotion also breeds the behavior of speculative customers' add-on refund, which leads to the losses of retailers. Therefore, under the combined effect of the two factors, the retailer's profit function is expressed as:

$$\Pi_t = (1-\lambda)(p_t - c)d_{to} + (1-\lambda)(\theta p_t - c)d_{ta} + \lambda(1-\beta)(\theta p_t - c)d_{ta} - \lambda\beta h d_{ta} \tag{7}$$

where the first term represents the benefits derived from regular consumers purchasing products at the original price; The second term reflects the benefits derived from regular consumers purchasing products at promotional price; The third term represents the benefits obtained by speculative consumers purchasing products at promotional prices; The final term represents the losses incurred when speculative consumers use the product as add-ons to meet the promotion conditions.

Proposition 2. *When the retailer participates in "Value Increasing" promotional campaigns, there is the optimal price p_t^*, which maximizes the profit of the retailer Π_t^*. Where p_t^* and Π_t^* are respectively:*

$$p_t^* = \frac{1}{2k} \frac{\begin{array}{l}(1-\lambda)(S+kC) + ((1-\lambda) \\ +\lambda(1-\beta))(\theta S + kC(1-\mu(1-\theta))) + \lambda\beta hk(1-\mu(1-\theta))\end{array}}{(1-\lambda) + \theta((1-\lambda)+\lambda(1-\beta))(1-\mu(1-\theta))} \tag{8}$$

$$\Pi_t^* = \frac{1}{4k}\frac{((1-\lambda)(S+kC)+((1-\lambda\beta))(\theta S+kC(1-\mu(1-\theta)))+\lambda\beta hk(1-\mu(1-\theta)))^2}{(1-\lambda)+\theta((1-\lambda)+\lambda(1-\beta))(1-\mu(1-\theta))} - ((1-\lambda)+(1-\lambda\beta))CS - \lambda\beta hS. \tag{9}$$

The proof is provided in the Appendix A.

It can be seen from Proposition 2 that the proportion of the speculative consumers, the probability of the product added by speculative consumers and the consumer's sensitivity to the discount amounts will affect the optimal price. Specifically, we obtain the Corollary 1.

Corollary 1. *Under the "Value Increasing" promotional activity:*

(1) *The price* p_t^* *increases in the proportion of speculative consumers* (λ).

(2) *When (i)* $\theta > \frac{-hk\mu+s\mu+ck(-1+2\mu)+\sqrt{c^2k^2-2k(chk-cs+2hs)\mu+(hk+s)^2\mu^2}}{2(ck+s)\mu}$ *or (ii)* $\theta < \frac{-hk\mu+s\mu+ck(-1+2\mu)+\sqrt{c^2k^2-2k(chk-cs+2hs)\mu+(hk+s)^2\mu^2}}{2(ck+s)\mu}$ *and* $\lambda > \frac{s(-1+\theta)\theta\mu+ck(-1+\theta)(1+(-1+\theta)\mu)+hk(1+(-1+\theta)\mu)(1+\theta+(-1+\theta)\theta\mu)}{k(h+c(-1+\theta))+(-1+\theta)(hk+ck(-1+\theta)+s\theta)\mu}$, *the price* p_t^* *increases in the probability of the product added by speculative consumers* (β).

(3) *When* $\theta < -\frac{ck+s-\sqrt{c^2k^2+6cks+s^2}}{2s}$ *and* $\beta > \frac{(c-h)k-(ck+s)\theta}{s\theta^2}$, *if* $\lambda > \overline{\lambda}$, *the price* p_t^* *increases in the sensitivity coefficient of consumers to the promotion amounts* (μ). *Otherwise, if* $\lambda < \overline{\lambda}$, *the price* p_t^* *decreases in the sensitivity coefficient of consumers to the promotion amounts* (μ).

The expression of $\overline{\lambda}$ and proof are provided in the Appendix A.

From Corollary 1, it can be seen that: (1) When the proportion of speculative consumers λ gradually increases, it shows that "Value Increasing" promotion has bred a large number of consumers' speculation behavior. In order to alleviate the losses caused by speculation consumers, retailers will raise the promotion price. (2) When the promotional discount rate θ is high, the retailer's optimal product price increases with the increase in the probability of the product added by speculative consumers β. And when the promotional discount rate is small θ, if the proportion of speculative consumers exceeds the threshold λ, with the increase of the probability that the product added by speculative consumers β, the retailer also increases the product price. (iii) When the promotional discount rate provided by the platform θ is smaller and the probability of the products added by speculative consumers is more than the critical value β, if the proportion of speculative consumers λ is larger, a higher sensitivity coefficient of consumers to promotion discount amounts μ will prompt the retailer to raise product price to offset the return loss and obtain a high marginal revenue. Otherwise, if the proportion of speculative consumers is small λ, the retailer will reduce the price as the sensitivity coefficient of consumers to promotion discount amounts μ increases to attract more consumers to purchase products.

Corollary 2. *The retailers' profit* Π_t^* *decreases in the proportion of speculative consumers* λ *in the market, i.e.,* $\frac{\partial \Pi_t^*}{\partial \lambda} < 0$.

The proof is provided in the Appendix A.

Corollary 2 demonstrates that a higher proportion of speculative consumers increases results in a lower profit of the retailer. The reasoning is that while the retailer's "Value Increasing" promotion can attract more consumers and increase revenue, it may be open to abuse and motivates some consumers to behave speculatively where consumers buy the product from the retailer with the full intention of returning it in order to meet the "Value Increasing" promotion threshold, thereby causing losses for the retailer. As the proportion of speculative consumers rises, the additional revenue from the promotion is no longer enough to compensate for the losses incurred from returns, ultimately causing a reduction in the retailer's profit.

4. Comparative analysis

To further investigate the impact of "Value Increasing" promotion on retailers' profit, we first compare optimal equilibrium price under non-"Value Increasing" sales model and "Value Increasing" sales model. Corollary 3 presents this result.

Corollary 3. *By comparing the optimal price under two sales models, we can get (i)* $p_t^* > p_n^*$*, and (ii) if* $\theta < \theta_o$*,* $\theta p_t^* < p_n^*$*; otherwise,* $\theta p_t^* \geq p_n^*$*.*

The proof is provided in the Appendix A.

Corollary 3 suggests that retailers often increase the regular price of products when implementing "Value Increasing" promotion. A noteworthy finding is that, under such promotions, consumers may not always enjoy a lower price compared to non-promotional periods. In other words, compared to retailers who do not offer "Value Increasing" promotions, consumers may not always enjoy a lower discount price at retailers offering such promotion. In particular, when the platform offers a higher discount degree (θ is low), consumers can obtain a lower final price at retailers with promotion (*i.e.*, $\theta p_t^* < p_n^*$). However, if the platform's discount degree is lower (θ is high), consumers may find lower prices at retailers who do not offer such promotion (*i.e.*, $\theta p_t^* > p_n^*$). The underlying reason for this phenomenon is that retailers will raise regular prices to offset the losses associated with speculative returns, which results in consumers' final price after the discount not necessarily being lower than the price they would pay during non-promotional periods.

What will happen to the profits of retailer in the presence of speculative consumers' add-on items return behavior? Is it beneficial for retailer to participate in "Value Increasing" promotion? This section will analyze these problems in detail.

By comparing the profits of retailer under the two modes of not participating in and participating in "Value Increasing" promotion, we can get the difference of profits $\prod_l$ under two modes:

$$\begin{aligned}\prod_l &= \Pi_t^* - \Pi_n^* \\ &= \frac{1}{4k}\frac{((1-\lambda)(S+kC)+(1-\lambda\beta)(\theta S+kC(1-\mu(1-\theta)))+\lambda\beta hk(1-\mu(1-\theta)))^2}{(1-\lambda)+\theta((1-\lambda)+\lambda(1-\beta))(1-\mu(1-\theta))} \\ &\quad -((1-\lambda)+(1-\lambda\beta))CS-\lambda\beta hS-\frac{(S-kC)^2}{4k}. \end{aligned} \tag{10}$$

Since the focus of this section is to explore the critical conditions for retailers to participate in and not participate in "Value Increasing" promotion, according to the difference of the situation that product is added and the proportion of speculative consumers, we can obtain Corollary 4:

Corollary 4. (i) *When* $\theta > \underline{\theta}, \beta < \underline{\beta}$*, we can get* $\Pi_t^* > \Pi_n^*$ *in the feasible range of* $0 < \lambda < 1$*.*
(ii) *When* $\theta > \underline{\theta}$*, if* $\beta > \underline{\beta}$ *and* $\lambda > \underline{\lambda}$*, we can get* $\Pi_t^* < \Pi_n^*$*; otherwise* $\Pi_t^* > \Pi_n^*$*.*

Where $\theta > \underline{\theta}$ is to satisfy the incentive compatibility condition of $\Pi_t^* > \Pi_n^*$ when $\lambda = 0$, that is, when there is no speculative consumer in the market, the retailer will participate in "Value Increasing" promotion only if he gets more profits in this case.

The expressions of $\underline{\theta}$, $\underline{\beta}$, $\underline{\lambda}$, and proof are provided in the Appendix A.

Corollary 4 (i) shows that, when the promotion discount rate set by the platform meets the incentive compatibility conditions, if the probability of the product added by speculative consumers is smaller, no matter how much proportion of the speculative consumers exist in the market, the profit of the retailer participating in "Value Increasing" promotion is always greater than that of not participating in "Value Increasing" promotion.

Corollary 4 (ii) also shows that, when the promotion discount rate set by the platform meets the incentive compatibility conditions, if the probability of the product added by speculative consumers is greater than $\underline{\beta}$ and the proportion of speculative consumers is greater than $\underline{\lambda}$, the retailer can gain more profits in the case of not participating in "Value Increasing" promotion. Otherwise, the retailer can gain more profits in the case of participating in "Value Increasing" promotion. This conclusion is consistent with the reality.

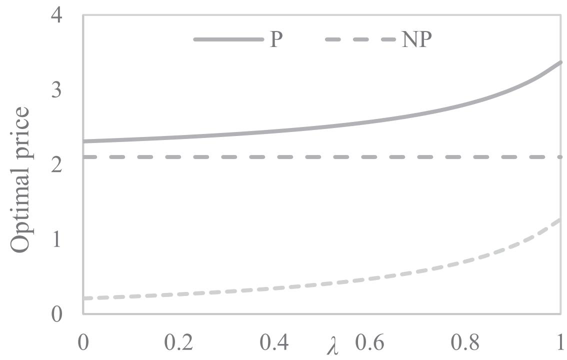

FIGURE 1. The impact of λ on optimal price (Note: P, NP represent participating and not participating in "Value Increasing" promotion, respectively).

5. Numerical analysis

To support and complement our theoretical analysis, we conduct a numerical study to explore the influence of key factors, such as the proportion of speculative consumers, the probability of product being added by speculative consumers, and consumers' sensitivity coefficient to promotion discount amounts, on the equilibrium decisions for the retailer under two sales modes. Referring to previous literature [25,34,35], together with some other news reports in the e-commerce industry[3,4], some key model parameters are set to $S = 5, k = 0.5, C = 0.2, \theta = 0.5, h = 0.5, \mu = 0.5, \lambda = \{0.5, 0.9\}, \beta = \{0.1, 0.5, 0.95\}$.

5.1. The impact of relevant parameters on retailers' optimal price

(1) The impact of the proportion of speculative customers on retailer's optimal price.

The relationship between the proportion of speculative customers and the optimal price of retailer is shown in Figure 1.

It can be seen from Figure 1 that when the retailer does not participate in "Value Increasing" promotion, the optimal price of the product is determined by the characteristics of the product itself, and is not affected by the behavior of speculative consumers. When retailer participates in "Value Increasing" promotion, the retailer's optimal product price increases with the increase of the proportion of speculative customers, and is always higher than that of the sales model without "Value Increasing" promotion. This is consistent with a phenomenon existing in the trading market, that is, when the retailers participate in "Value Increasing" promotion, they adopt the marketing strategy of first increasing price and then lowering price.

(2) The impact of the probability of product added by speculative consumers on the optimal price of retailer.

When $\theta > \frac{-hk\mu+s\mu+ck(-1+2\mu)+\sqrt{c^2k^2-2k(chk-cs+2hs)\mu+(hk+s)^2\mu^2}}{2(ck+s)\mu}$, the relationship between β and the optimal price of retailer is shown in Figure 2.

When $\theta < \frac{-hk\mu+s\mu+ck(-1+2\mu)+\sqrt{c^2k^2-2k(chk-cs+2hs)\mu+(hk+s)^2\mu^2}}{2(ck+s)\mu}$ and $\lambda > \frac{s(-1+\theta)\theta\mu+ck(-1+\theta)(1+(-1+\theta)\mu)+hk(1+(-1+\theta)\mu)(1+\theta+(-1+\theta)\theta\mu)}{k(h+c(-1+\theta))+(-1+\theta)(hk+ck(-1+\theta)+s\theta)\mu}$, the relationship between β and the optimal price of retailer is shown in Figure 3.

It can be seen from Figure 2 that when the promotion discount rate provided by the platform is smaller, the retailer's optimal price increases with the increase of the probability of product added by speculative consumers.

[3] https://www.cifnews.com/article/118032.
[4] https://news.qq.com/rain/a/20241109A0474Y00.

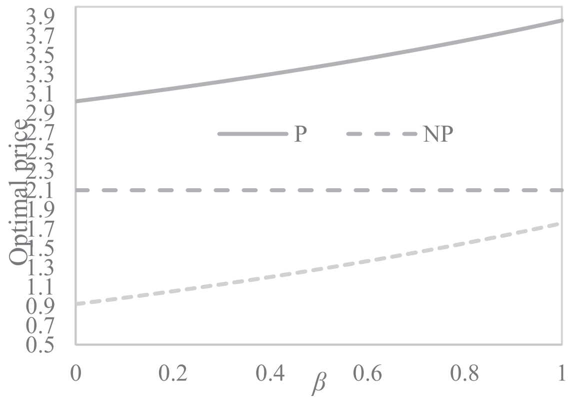

FIGURE 2. The impact of β on optimal price.

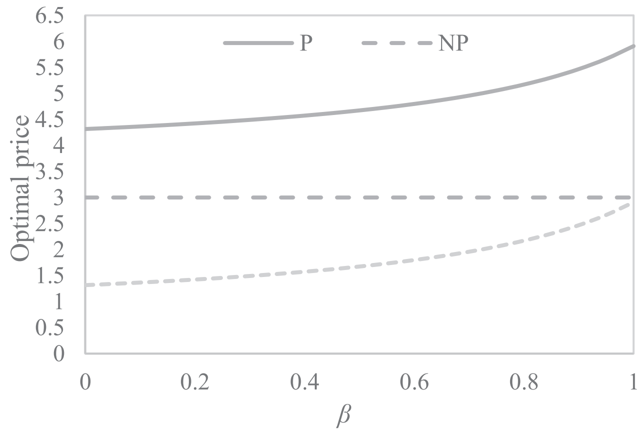

FIGURE 3. The impact of β on optimal price.

This is because the smaller the discount rate is, the greater the discount amounts are. The purpose of retailers participating in higher level sales promotion is to stimulate sales and achieve the goal of "small profits and quick turnover". However, in the case of certain proportion of speculative consumers, as the probability of products being added by speculative consumers increases, the number of deliberate add-on items refunds of consumers increase, resulting in increasing losses. Retailers setting higher product prices on the one hand can effectively restrain the consumers' add-on item refund behavior to a certain extent. On the other hand, it can make up for the loss caused by speculative consumers, so as to achieve the purpose of increasing profits. At the same time, it can be seen from Figure 3 that when the promotion discount rate provided by the platform is greater than the critical value, if the proportion of speculative customers exceeds the threshold, the retailer's optimal price increases with the increase of the probability of the product added by speculative consumers. This is due to the fact that although the amounts of promotion discount amounts are smaller, the retailer will still suffer greater losses when the quantity of products added is larger. In this case, the retailer should also raise the product price, and the internal mechanism is consistent with the above case.

(3) The impact of consumers' sensitivity coefficient to promotion discount amounts on retailer's optimal price.

When $\theta < \frac{-ck-s+\sqrt{c^2k^2+6cks+s^2}}{2s}$, $\lambda > \overline{\lambda}$ and $\beta > \frac{(c-h)k-(ck+s)\theta}{s\theta^2}$, the relationship between consumers' sensitivity coefficient to promotion discount amounts and the optimal price of retailer is shown in Figure 4.

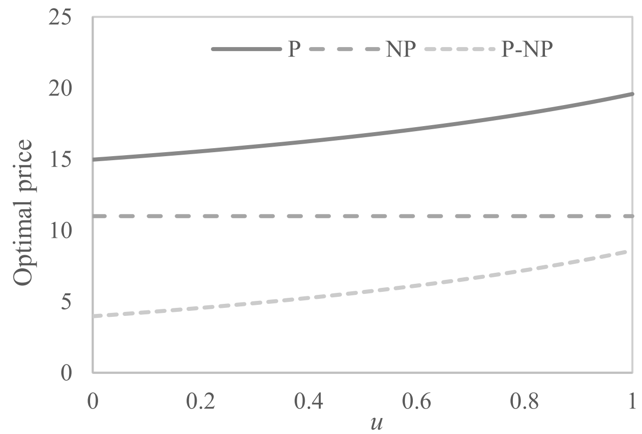

FIGURE 4. The impact of μ on optimal price.

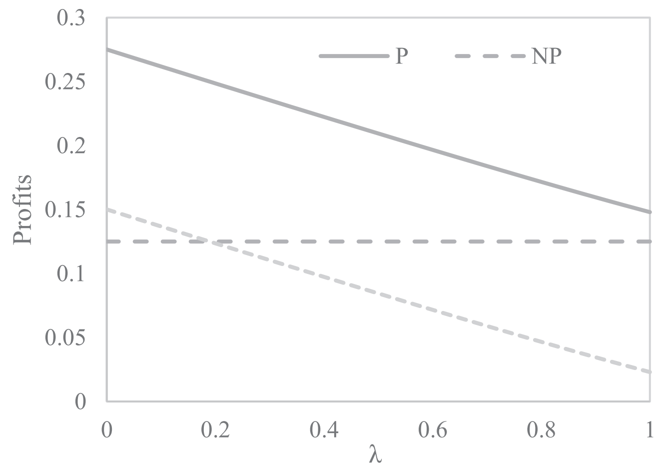

FIGURE 5. The impact of λ on profits.

It can be seen from Figure 4 that when the promotion discount rate is smaller and the proportion of speculative customers is larger in the market, if the probability that the product is added by speculative customers is greater than the critical value, the optimal price of the retailer increases with the increase of the consumers' sensitivity coefficient to promotion discount amounts. This is because the lower the promotional discount rate, the greater the discount amounts. When the sensitivity coefficient of consumers to promotion discount amounts increases, it means that promotions can attract more ordinary consumers or speculative consumers to make add-on items purchases. However, if there are a large number of speculative consumers in the market, the more probability the product is to be added and then returned. That is to say, in this case, the promotion leads to a large number of speculative customers to purchase the add-on items, which increases the loss of retailers. In order to alleviate the negative impact of a large number of speculative customers behavior, retailers will increase the price of products. Otherwise, retailers will lower the product price to achieve the effect of "small profit but quick turnover".

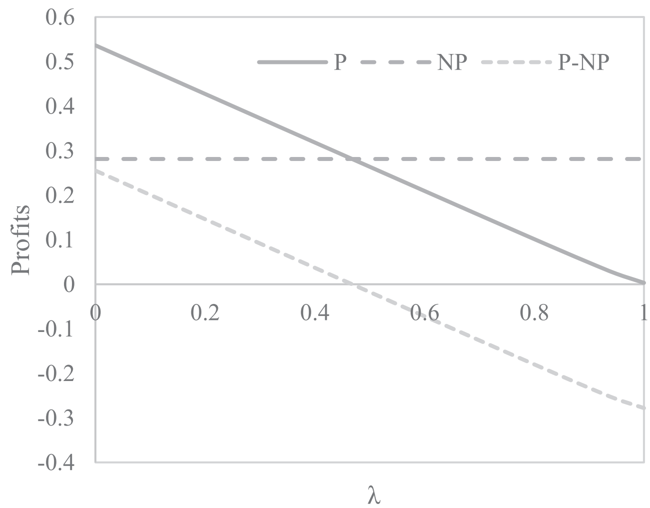

FIGURE 6. The impact of λ on profits.

5.2. The impact of relevant parameters on retailer's profit

(1) When $\theta > \underline{\theta}$ and $\beta < \underline{\beta}$, the relationship between the proportion of speculative customers and the profits of the retailer is shown in Figure 5.
(2) When $\theta > \underline{\theta}$ and $\beta > \underline{\beta}$, the relationship between the proportion of speculative customers and the profits of the retailer is shown in Figure 6.

It can be seen from Figures 5 and 6 that, under the premise that the promotion discount rate provided by the platform meets the incentive compatibility condition, if the probability of the product being added by speculative customers is less than the critical value, no matter what proportion of the speculative customers exits in the market, the profits obtained with "Value Increasing" promotion are higher than those without "Value Increasing" promotion. This is because when the promotion discount rate is larger and the probability of the product being added by speculative customers is smaller, the extra profits brought by participating in the "Value Increasing" promotion is higher than the loss caused by a small number of speculative customers' behavior. "Value Increasing" promotion plays a positive role. However, when the probability of product being added by speculative customers is greater than the critical value and the number of speculative customers exceeds a certain threshold, the profits of retailer with "Value Increasing" promotion can no longer compensate for the loss caused by a large number of speculative customers' behavior, and the negative impact of "Value Increasing" promotion is dominant. Therefore, retailers do not necessarily get higher profits with "Value Increasing" promotion, nor do they necessarily get lower profits without "Value Increasing" promotion. The retailer's optimal strategy is: when the discount rate provided by the platform meets the incentive compatibility condition, if the probability of product being added deliberately by speculative customers is larger and the number of speculative customers in the market is more, the retailer should try not to participate in "Value Increasing" promotion; otherwise, try to participate in "Value Increasing" promotion.

6. CONCLUSION

On the one hand, retailers participating in "Value Increasing" promotion can stimulate consumers' additional purchases, thereby increasing the total revenue. On the other hand, it also breeds speculative consumers' add-on item refund behavior, which causes losses. This paper makes an in-depth study by building a theoretical model. The main results are as follows: (1) By comparing the retailer's profits under the two sales modes, we find that participating in the "Value Increasing" promotion does not always benefit the retailer. A low probability of the product being added by speculative consumers, or fewer speculative consumers, is crucial for the retailer to participate in the "Value Increasing" promotion. (2) When the retailer does not participate in "Value Increasing" promotion, the optimal price of the product is solely determined by the characteristics of the product itself. However, when the retailer participates in the "Value Increasing" promotion, the optimal product price increases with a higher proportion of speculative consumers. Additionally, under certain conditions, the optimal price also rises with an increase in the probability of the product being added by speculative consumers and the sensitivity coefficient of consumers to the discount amounts. (3) The price under the "Value Increasing" promotion mode is higher than that under not "Value Increasing" promotion mode.

6.1. Managerial insights

On the basis of the research conclusions, the managerial insights for retailers are as follows:

First, although the "Value Increasing" promotion can attract consumer attention and enhance their purchasing intentions, retailers should not simply follow the trend of participating in such promotions. Instead, they must carefully weigh both the potential benefits and drawbacks of the "Value Increasing" promotion in order to make informed, strategic decisions.

Second, retailers should carefully assess how consumers' add-on item refund behavior and their sensitivity to discount amounts impact the retailer's optimal pricing and promotional strategy. Understanding these factors is crucial for optimizing both profitability and consumer engagement.

Third, when retailers participate in the "Value Increasing" promotion, they should, under certain conditions, raise prices strategically to counterbalance potential losses stemming from speculative customers' return behavior on add-on items. This strategy can help protect profitability while still benefiting from the promotion's overall appeal.

6.2. Future research

Several aspects of our work deserve further research. First, this paper only considers the influence of consumer psychology prior to purchase on the retailers' decision-making in order to explore the impact of the speculative customers' add-on item refund behavior on retailers' optimal decisions. It is interesting to consider the potential impact of post-purchase dissatisfaction and returns in future research. In addition, we study the sole retailer's optimal decisions considering speculative returns. Actually, the platform and the retailers operate within a competitive or cooperative relationship. It is valuable to explore the pricing strategy of the supply chain subject under the competition and cooperation relationship.

Data availability statement

No new data/codes were created or analyzed in this study.

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Appendix A.

Proof of the Proposition 1. Based on the principle of maximizing the profits of the retailer, calculate the first-order partial derivatives of p_n in equation (4), and then make it be equal to 0. We can obtain:

$$\frac{\partial \Pi_n}{\partial p_n} = S - 2kp_n + kC. \tag{A.1}$$

So, we get:

$$p_n^* = \frac{S + kC}{2k}. \tag{A.2}$$

Calculate the second-order partial derivatives of p_n in equation (4), the following equation can be obtained:

$$\frac{\partial^2 \Pi_n}{\partial p_n^2} = -2k < 0. \tag{A.3}$$

So, the profit of the retailer is a concave function of the price, that is, there is the optimal price to maximize the profit of retailer. □

Proof of the Proposition 2. Similarly, based on the principle of maximizing the profit of the retailer, calculate the first-order partial derivatives of p_t in equation (7) and make it be equal to 0. Then, we can obtain:

$$\begin{aligned}\frac{\partial \Pi_t}{\partial p_t} &= (1-\lambda)(S - kp_t - k(p_t - C)) + ((1-\lambda) + \lambda(1-\beta))\big(\theta(S - k(1-\mu(1-\theta))p_t) \\ &\quad - k(1-\mu(1-\theta))(\theta p_t - C)\big) + \lambda\beta hk(1-\mu(1-\theta)).\end{aligned} \tag{A.4}$$

So, we get:

$$p_t^* = \frac{1}{2k}\,\frac{(1-\lambda)(S+kC) + ((1-\lambda)+\lambda(1-\beta))(\theta S + kC(1-\mu(1-\theta))) + \lambda\beta hk(1-\mu(1-\theta))}{(1-\lambda) + \theta((1-\lambda)+\lambda(1-\beta))(1-\mu(1-\theta))}.$$

Calculate the second-order partial derivatives of p_t in equation (7), the following equation can be obtained:

$$\frac{\partial \Pi_t^2}{\partial p_t^2} = -4k(1-\lambda) - 2k\lambda(1-\beta)(1-\mu(1-\theta)) < 0. \tag{A.5}$$

So, the profit of retailer is a concave function of price. That is, there is the optimal price to maximize the profits of retailer. □

Proof of the Corollary 1. (i) Calculate the first-order partial derivatives of λ in equation p_t^*, we can obtain:

$$\frac{\partial p_t^*}{\partial \lambda}=\frac{1}{2k}\frac{(1-\beta)(\theta S\mu(1-\theta)+kC(1-\theta)(1-\mu(1-\theta)))+\beta hk(1-\mu(1-\theta))+\beta hk\theta(1-\mu(1-\theta))^2}{(1-\lambda+\theta(1-\lambda\beta)(1-\mu(1-\theta)))^2}. \tag{A.6}$$

Because of $0<\beta,\theta,\mu<1,k,C,S,h>0$, we can get $\frac{\partial p_t^*}{\partial \lambda}>0$. Therefore, p_t^* is a monotone increasing function of λ.

(ii) Calculate the first-order partial derivatives of β in equation p_t^*, we can obtain:

$$\frac{\partial p_t^*}{\partial \beta}=\frac{1}{2k}\frac{\lambda(1-\lambda)(\theta S\mu(1-\theta)+kC(1-\theta)(1-\mu(1-\theta))-hk(1-\mu(1-\theta)))-\lambda hk\theta(1-\mu(1-\theta))^2}{(1-\lambda+\theta(1-\lambda\beta)(1-\mu(1-\theta)))^2}. \tag{A.7}$$

Let $F=(1-\lambda)(\theta S\mu(1-\theta)+kC(1-\theta)(1-\mu(1-\theta))-hk(1-\mu(1-\theta)))-hk\theta(1-\mu(1-\theta))^2$.
When $\theta S\mu(1-\theta)+kC(1-\theta)(1-\mu(1-\theta))-hk(1-\mu(1-\theta))>0$, that is $\theta>\frac{-hk\mu+s\mu+ck(-1+2\mu)+\sqrt{c^2k^2-2k(chk-cs+2hs)\mu+(hk+s)^2\mu^2}}{2(ck+s)\mu}$. It can be concluded that $F>0$ in the range of $0<\lambda<1$. When $\theta S\mu(1-\theta)+kC(1-\theta)(1-\mu(1-\theta))-hk(1-\mu(1-\theta))<0$,that is $\theta<\frac{-hk\mu+s\mu+ck(-1+2\mu)+\sqrt{c^2k^2-2k(chk-cs+2hs)\mu+(hk+s)^2\mu^2}}{2(ck+s)\mu}$, if $\lambda>\frac{-\theta S\mu(1-\theta)-kC(1-\theta)(1-\mu(1-\theta))+hk(1-\mu(1-\theta))+hk\theta(1-\mu(1-\theta))^2}{-\theta S\mu(1-\theta)-kC(1-\theta)(1-\mu(1-\theta))+hk(1-\mu(1-\theta))}$, we can get $F>0$. The proof process of (iii) is similar to that of (i) and (ii), we omit it.

The expressions used in Corollary 1 are as follows.

$$\overline{\lambda}=\frac{B+\sqrt{B^2-4AE}}{2A} \tag{A.8}$$

$$A=\beta(\theta S+kC\theta-kC+hk)+\beta^2\theta^2S \tag{A.9}$$

$$B=(1+\beta)(\theta S+kC\theta-kC)+\beta hk+2\beta\theta^2S \tag{A.10}$$

$$E=\theta S+kC\theta-kC+\theta^2S. \tag{A.11}$$

□

Proof of the Corollary 2. We have derived the first-order partial derivatives of Π_t^* with respect to λ:

$$\begin{aligned}\frac{\partial \Pi_t^*}{\partial \lambda} = {} & ((1+\beta\theta(1+(-1+\theta)\mu))(s^2(-1+\lambda+\theta(-1+\beta\lambda))^2\\ & +h^2k^2\beta^2\lambda^2(1+(-1+\theta)\mu)^2+c^2k^2(-2+\lambda+\beta\lambda+(-1+\theta)(-1+\beta\lambda)\mu)^2\\ & +2hks\beta\lambda(-1-\theta+\lambda+\beta\theta\lambda+(-1+\theta)(1-\lambda+\theta(-1+\beta\lambda))\mu)\\ & -2ck(s(-2+\lambda+\beta\lambda)(-1+\lambda+\theta(-1+\beta\lambda))\\ & +s(-1+\theta)(-1+\beta\lambda)(1-\lambda+\theta(-3+(2+\beta)\lambda))\mu\\ & +hk\beta\lambda(1+(-1+\theta)\mu)(-2+\lambda+\beta\lambda+(-1+\theta)(-1+\beta\lambda)\mu)))\\ & -2(-1+\lambda+\theta(-1+\beta\lambda)(1+(-1+\theta)\mu))(s^2(1+\beta\theta)(-1-\theta+\lambda+\beta\theta\lambda)\\ & +h^2k^2\beta^2\lambda(1+(-1+\theta)\mu)^2\\ & +c^2k^2(1+\beta+\beta(-1+\theta)\mu)(-2+\lambda+\beta\lambda+(-1+\theta)(-1+\beta\lambda)\mu)\\ & +hks\beta(-1-\theta+2\lambda+2\beta\theta\lambda+(-1+\theta)(1-\theta-2\lambda+2\beta\theta\lambda)\mu)\\ & +ck(s(3+\beta+\theta+3\beta\theta-2\lambda-2\beta\lambda-2\beta\theta\lambda-2\beta^2\theta\lambda-(-1+\theta)(1+\beta-2\theta-4\beta\theta\\ & +2\beta(-1+(2+\beta)\theta)\lambda)\mu)\\ & -hk\beta(1+(-1+\theta)\mu)(-2+\mu-\theta\mu+2\lambda(1+\beta+\beta(-1+\theta)\mu)))))\\ & \Big/\left(4k\begin{matrix}(-1+\lambda+\theta(-1+\beta\lambda)^2\\(1+(-1+\theta)\mu))\end{matrix}\right).\end{aligned}$$

According to the premise conditions to ensure all equilibrium solutions are positive, it is easy to get that $\frac{\partial \Pi_t^*}{\partial \lambda}<0$ always holds. □

Proof of the Corollary 3. (i) Let $\nabla_1 p=p_t^*-p_n^*$. Then, we have derived the first-order partial derivatives of $\nabla_1 p$ with respect to λ:

$$\frac{\partial \nabla_1 p}{\partial \lambda}$$

$$= \frac{s(-1+\beta)(-1+\theta)\theta\mu + ck(-1+\beta)(-1+\theta)(1+(-1+\theta)\mu) + hk\beta(1+(-1+\theta)\mu)(1+\theta+(-1+\theta)\theta\mu)}{2k(-1+\lambda+\theta(-1+\beta\lambda)(1+(-1+\theta)\mu))^2}.$$

From the expression of $\frac{\partial \nabla_1 p}{\partial \lambda}$, we can get $\frac{\partial \nabla_1 p}{\partial \lambda} > 0$. ∇p increases in λ. Then, we have $\nabla_1 p(\lambda = 0) = \frac{s(1+\theta)+ck(2+(-1+\theta)\mu)}{2k(1+\theta+(-1+\theta)\theta\mu)} > 0$. Thus, we can get $\nabla_1 p > 0$ always holds for any feasible region of λ.

(ii) Denote $\nabla_2 p = \theta p_t^* - p_n^*$. We can get that $\nabla_2 p(\theta = 1) = \frac{1}{2}\left(-\frac{h\beta\lambda}{-2+\lambda+\beta\lambda}\right) > 0$, and $\nabla_2 p(\theta = 0) = -\frac{1}{2}\left(c + \frac{s}{k}\right) < 0$. Then, we can proof that $\frac{\partial \nabla_2 p}{\partial \theta} > 0$. Thus, there exists a θ_o that makes $\nabla_2 p(\theta = \theta_o) = 0$. When $\theta<\theta_o$, $\nabla_2 p = \theta p_t^* - p_n^* < 0$; and when $\theta>\theta_o$, $\nabla_2 p = \theta p_t^* - p_n^* > 0$.

Where,

$$\theta_o = \frac{1}{2(s(-1+\beta\lambda)(-1+\mu)+hk\beta\lambda\mu)}\Big(ck(-1+\lambda) + s\lambda - hk\beta\lambda - s\beta\lambda - s\mu + hk\beta\lambda\mu + s\beta\lambda\mu + \sqrt{(ck(-1+\lambda)+s(\lambda+\beta\lambda(-1+\mu)-\mu)+hk\beta\lambda(-1+\mu))^2 - 4(ck+s)(-1+\lambda)(s(-1+\beta\lambda)(-1+\mu)+hk\beta\lambda\mu)}\Big).$$

□

Proof of the Corollary 4. (i) According to the condition of incentive compatibility, when $\lambda = 0$, we can get $\Pi_t^* > \Pi_n^*$. Because as λ increases, $\prod_l$ decreases. And the denominator of $\prod_l$ is a decreasing function of λ, so we can deduce that the numerator of $\prod_l$ is also a decreasing function of λ. Therefore, as long as it is satisfied that when all consumers in the market are speculative consumers, $\prod_l$ is still greater than 0, we can get $\Pi_t^* > \Pi_n^*$ in the case of $0 < \lambda < 1$. Specifically, substituting $\lambda = 1$ into equation (10), we can get:

$$\prod_l = \frac{1}{4k}\frac{((1-\beta)(\theta S + kC(1-\mu(1-\theta))) + \beta hk(1-\mu(1-\theta)))^2}{\theta(1-\beta)(1-\mu(1-\theta))} - (1-\beta)CS - \beta hS - \frac{(S-kC)^2}{4k}. \tag{A.12}$$

Make the equations (A.12) be greater than 0, we can get $\beta < \underline{\beta}, \Pi_t^* > \Pi_n^*$. The proof of (ii) is similar to that of (i). Corollary 4 is proved.

The expressions used in Corollary 4 are as follows.

$$\underline{\theta} = \frac{-(2GJ+(\mu-1)H) + \sqrt{(2GJ+(\mu-1)H)^2 - 4(G^2-\mu H)(J^2-H)}}{2(G^2-\mu H)} \tag{A.13}$$

$$\underline{\beta} = \frac{-L+\sqrt{L^2-4MN}}{2M} \tag{A.14}$$

$$\underline{\lambda} = \frac{-(2XZ+HW-4kY(1+\theta d)) + \sqrt{(2XZ+HW-4kY(1+\theta d)))^2 - 4(X^2+4kYW)(Z^2-(1+\theta d)H)}}{2(X^2+4kYW)} \tag{A.15}$$

$$G = S + kC\mu \tag{A.16}$$

$$H = S^2 + 6kCS + k^2C^2 \tag{A.17}$$

$$J = S + 2kC - kC\mu \tag{A.18}$$

$$d = 1 - \mu(1-\theta) \tag{A.19}$$

$$L = 2((h-C)kd - \theta S)(\theta S + kCd - (4k\theta S(h-2C) - (S-kC)^2\theta)d) \tag{A.20}$$

$$M = ((h-C)kd - \theta S)^2 + 4k\theta S(h-C)d \tag{A.21}$$

$$N = (k^2C^2d - S^2\theta)(d-\theta) \tag{A.22}$$

$$W = 1 + \beta\theta d \tag{A.23}$$

$$X = \beta(hkd - kcd - \theta S) - S - kC \tag{A.24}$$

$$Y = \beta(h-C)S - CS \tag{A.25}$$

$$Z = S + \theta S + kC + kCda. \tag{A.26}$$

□