ON BUILDING REMANUFACTURING OUTSOURCING STRATEGIES UNDER RECYCLING COMPETITION AND GOVERNMENT SUBSIDIES IN CLOSED-LOOP SUPPLY CHAINS

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Abstract. Based on the competition in the recycling market, this paper investigates the technology licensing outsourcing strategy and the remanufacturing strategy of a closed-loop supply chain, and analyses the effect of the implementation of government subsidy policies. This paper considers a closed-loop supply chain consisting of a manufacturer, a retailer and two competing recyclers (one specializing in sorted recycling and the other in traditional recycling). Using the Stackelberg game model, eight decision-making models under different outsourcing modes, remanufacturing modes and government subsidies are constructed. Through the comparative analysis of the equilibrium results, the optimal outsourcing authorization and remanufacturing strategies for the closed-loop supply chain are proposed, and the facilitating effect of the government's double subsidy policy is verified. Our findings show that manufacturers tend to choose the unit-pay outsourcing model in outsourcing and remanufacturing decisions, although this may lead to profit inequality among other members. Meanwhile, increasing government subsidies for sorted recycling and remanufacturing favors the profitability of sorted recyclers, however, it is of concern that this is not conducive to increasing the profitability of traditional recyclers. Furthermore, at low recycling prices, an increase in dual subsidies is beneficial in boosting market demand and profits for all members, however, at high recycling prices, the equilibrium outcomes and member profits change in the opposite direction as under the fixed-pay outsourcing model. We also find that government inputs of double subsidies can increase member profits, promote recycling and remanufacturing, and maintain market stability, especially the fixed-pay outsourcing *versus* full remanufacturing model is more conducive to achieving these objectives. The research in this paper contributes to the construction and development of the remanufacturing cycle system in the market.

1. Introduction

With the rapid advancement of industrialization and the swift turnover of products, the quantity of waste and discarded items is increasing rapidly, posing numerous negative impacts on both humans and the environment [21]. The global community is actively seeking efficient methods to facilitate the recycling of waste products,

aiming to foster sustainable development in both economic and environmental dimensions [26]. Furthermore, remanufacturing, as a form of circular and sustainable production method, not only reduces production costs and resource utilization but also alleviates the harmful environmental impact of electronic waste [36]. The United States has enacted the "Federal Vehicle Repair Cost Savings Act of 2015", which encourages the use of remanufactured components. Japan has formulated and implemented the "Promotion of Recycling of Small Waste Electrical and Electronic Equipment", which regulates manufacturers' involvement in the recycling of electronic products [2].

With the continuous development of technology, recycling methods have gradually evolved to include new approaches such as classified recycling and "Internet Plus" recycling. The entry of numerous emerging recycling methods can lead to intense competition within the recycling market [23]. Part of the recycling industry, due to factors such as larger scale, more abundant funds, and advanced equipment, actively adapts to the demand for classified recycling. Moreover, classified recycling, as a prelude to resource recycling and recycling-based manufacturing, is a crucial step influencing the efficiency of waste management [13]. The re-manufacturing process is widely regarded as the best method to put an end to pollution caused by discarded products. Despite the fact that re-manufactured goods are typically priced 30–40% lower than new products, the production costs for re-manufactured items are 40–65% lower than those of new products [8]. For example, Xerox's Green World Alliance program has saved the company 40–65% in manufacturing costs through the reuse of parts and materials [28].

In practice, while manufacturers have technological and brand advantages in remanufacturing activities, when manufacturers are located far from the product sales points, it inevitably leads to higher transportation and inventory costs [16]. Therefore, the manufacturer must incur outsourcing expenses to subcontract remanufacturing to third parties. Due to the lack of remanufacturing technology and startup funds, businesses may show a lack of enthusiasm for the recycling and remanufacturing of discarded products. In this situation, the government should play a regulatory and guiding role to promote the healthy development of recycling and remanufacturing industries for discarded products worldwide [25].

This study attempts to address the following questions through the examination of different outsourcing remanufacturing models in a closed-loop supply chain. What impact do different outsourcing remanufacturing models have on the level of recycling competition, the revenue coefficient of classified recycling, and the price coefficient of recycling? How do different outsourcing and remanufacturing models influence the choice of a closed-loop supply chain model? Additionally, how does the economic benefit change before and after the dual subsidies from the government are introduced?

The remaining part of the article is as follows: Section 1 outlines the background to the research. Section 2 summarises the relevant literature. Section 3 describes the problem and the model assumptions. Section 4 describes the model construction and provides the equilibrium solution. As the model develops, Section 5 analyses the supply chain structure, while Section 6 investigates the model construction with the introduction of government subsidy inputs. Section 7 validates the conclusions using numerical simulations. The conclusion of the paper is reached in the last section.

2. Literature review

The effects of competition in sales in supply chains [3, 5] and the impact of government policies on supply chain decisions have been the subject of considerable research. Some scholars have focused on the impact of government subsidy policies on green supply chain operations [6], agricultural supply chain operations [10], and blockchain technology adoption strategies [12]. Others have concentrated on the different impacts of subsidy and tax policies on supply chain greening transformation and carbon emission reduction effects [4, 11]. However, in light of the growing salience of resource and environmental concerns, this paper turns its attention to the competition between recycling channels in closed-loop supply chains, encompassing recycling and remanufacturing. Additionally, it examines the influence of government subsidy policies on these dynamics. Therefore, this paper combines the social reality and future development needs, combines the recycling competition, sorting recycling,

and outsourcing remanufacturing and other related literature, tries to solve the above research problems, and provides theoretical and practical basis for supply chain outsourcing and remanufacturing decision-making.

2.1. Research status of competition in closed-loop supply chain

Competition in closed-loop supply chains can be broadly categorized into three types: competition in production and manufacturing, competition in sales, and competition in recycling. In the examination of competition in the sales link, Savaskan et al. [27] analyzed the interaction between manufacturers' pricing decisions on recycling waste products in the forward and reverse channels when retailers compete with each other in the closed-loop supply chain. In the competition within manufacturing links, Liu et al. [24] studied the influence of competition behavior among original manufacturers and product goodwill on the decision-making of closed-loop supply chain members. In the study of recycling competition, Toyasaki et al. [29] analyzed the impact of competition between two manufacturers and two recyclers on the competitive scheme of WEEE recycling. Liu et al. [24] developed a quality-based price-competitive recycling model based on the WEEE recycling market and investigated the influence of the equilibrium purchase price and government subsidies of the two recycling channels. Xing et al. [34], considering the characteristics of risk avoidance in a closed-loop supply chain composed of two competing third parties, examined changes in expected utility brought about by cooperative emission reduction decisions through a Stackelberg game study. Zhou et al. [41] established a two-phase model in which the sales cycle is managed by the manufacturer and the recovery cycle by two recyclers with different levels of competitiveness. The study found that legislation improved environmental performance but reduced the total profit of the manufacturer and recycler.

2.2. Remanufacturing mode of the closed-loop supply chain

In reality, the implementation of waste recycling and remanufacturing involves original equipment manufacturers, third-party remanufacturers, and product distributors, namely retailers. The following three models are summarized. In the case of retailers as the subject of remanufacturing, Wang et al. [31] took Game Stop as an example to compare and analyze the impact of retailers' remanufacturing and outsourced remanufacturing on revenue and the environment. Based on the remanufacturing model of authorized retailers, Zhang et al. [38] discussed the boundary conditions of manufacturers' remanufacturing model selection and the range of government subsidy parameters. In the case of the remanufacturer as the subject of remanufacturing, Yan et al. [35] constructed two manufacturing models involving manufacturers outsourcing remanufacturing to remanufacturers and suppliers. They explored the impact of different outsourcing object selections on green operation issues related to the economy, environment, and social demand. Huang et al. [18] analyzed the impact of consumer preferences and third-party remanufacturing costs on the three remanufacturing scenarios of no remanufacturing, partial remanufacturing, and complete remanufacturing based on the manufacturer's authorization of a third party for remanufacturing. Based on a competitive closed-loop supply chain consisting of two OEMs and two third-party remanufacturers, Zhang et al. [37] considered two third-party remanufacturing modes of outsourcing and authorization, finding that enterprises are more inclined to adopt outsourcing strategies. When the subject of remanufacturing is the manufacturer, Zhao et al. [39] established a closed-loop supply chain decision-making model in which the manufacturer remanufactures and commissions the retailer to remanufacture under different technology licenses. They found that the remanufacturing model, after the retailer pays a fixed technology license fee, achieves profit maximization. Feng et al. [7] constructed a closed-loop supply chain model for manufacturers and third-party remanufacturers and studied the impact of green consumption and environmental responsibility behaviors on the two remanufacturing strategies of manufacturers. Zhou et al. [41] studied three remanufacturing strategies of contract manufacturers according to different waste quality conditions based on the cooperation between original equipment manufacturers and contract manufacturers for new product production and remanufacturing through outsourcing or licensing. These strategies are no manufacturing, partial remanufacturing, and complete remanufacturing.

TABLE 1. A comparison between this article and related research is presented below.

Research work	Remanufacturing	Recycling competition	Separate recycling	Technology license	Government subsidies
Feng *et al.* [7]	✓			✓	
Han *et al.* [13]	✓	✓	✓		
Huang *et al.* [18]	✓			✓	
Liu *et al.* [22]	✓	✓			✓
Liu *et al.* [23]	✓	✓		✓	
Mitra *et al.* [25]	✓	✓			✓
Zhang *et al.* [38]	✓				✓
Zhang *et al.* [36]	✓	✓			
Zhao *et al.* [39]	✓			✓	
This work	✓	✓	✓	✓	✓

2.3. Outsourcing mode of the closed-loop supply chain

The term "outsourcing" was first proposed in 1982, specifically in the context of manufacturing outsourcing (1999), and gained widespread usage in the mid-1990s (2007), gradually maturing over time. Heywood [15] defined manufacturing outsourcing as the transfer of one or several non-core businesses by an enterprise to a third party, who provides services within a certain period of time at a fixed but limited price. Antras *et al.* [1] introduced the global situation of business outsourcing and conducted some preliminary theoretical analysis. In recent years, foreign scholars have delved into more in-depth and realistic research on outsourcing. Guo *et al.* [9] discussed the impact of different outsourcing modes on the supply chain based on information distortion. Wang *et al.* [30] respectively constructed three different non-cooperative pricing strategies and discussed each member's preference for the outsourcing mode under the two outsourcing modes. Kayis *et al.* [20], in the field of manufacturing outsourcing, studied the influence of manufacturers' choice of different outsourcing methods under the condition of asymmetric production costs. Huang *et al.* [18] studied the benefits of information sharing among members in the closed-loop supply chain based on the remanufacturing situation of manufacturers, distributors, and third parties under the technology outsourcing license. Based on asymmetric advertising costs, Xie *et al.* [33] studied the advertising outsourcing plan of manufacturers considering demand uncertainty.

2.4. Comparison between this article and related research

The following is a comparison of the literature most relevant to our paper. Han *et al.* [13] and Xing *et al.* [34] studied the impact of recycling competition on recycling programmes, member co-operation and prices, but did not yet focus on the asymmetric situation of recycling competition; Zhou *et al.* [40] and Zhao *et al.* [39] focused on the different licensing targets of technology outsourcing and the selection of different remanufacturing strategies, but did not consider the situation where technology outsourcing licensing targets are recyclers, and rarely considered the combination of remanufacturing models and technology outsourcing; Zhang *et al.* [37] and Zhang *et al.* [38] explored the impact of different government subsidy policies on closed-loop supply chains, and rarely considered the combination of subsidies for both recycling and remanufacturing. Literature related to the above-mentioned topics is compared in Table 1.

Overall, our work extends existing research on supply chain management to include more recyclers and a wider range of recycling methods, taking into account the different ways in which manufacturers authorize and empower sorted recyclers to recover, and the mechanisms through which government subsidies influence the construction of supply chain recycling systems, can provide meaningful decision guidance for managers.

3. Symbol description and model assumptions

The closed-loop supply chain consists of a manufacturer, a retailer, two competing classified recyclers, and traditional recyclers. The manufacturer is the channel leader, and all four of them play the Stackelberg game under complete information. The decision models for unit-paid outsourcing part remanufacturing (model), fixed-paid outsourcing part remanufacturing (model), unit-paid outsourcing all remanufacturing (model), and fixed-paid outsourcing all remanufacturing (model) are constructed, respectively.

Under the partial remanufacturing model, the manufacturer is responsible for the production of new products and the partial remanufacturing of reprocessed items in the forward supply chain, and the retailer is responsible for the sales of new products. In the reverse supply chain, classified recyclers are responsible for recycling and partially remanufacturing waste products, while traditional recyclers are responsible for recycling waste products, as shown in Figure 1a. Under the total remanufacturing model, the manufacturer is responsible for the production of new products, and the retailer is responsible for the sales of new products. In the reverse supply chain, classified recyclers are responsible for recycling and fully remanufacturing waste products, while traditional recyclers are responsible for recycling waste products, as shown in Figure 1b.

3.1. Model symbols and descriptions

The model symbols and its descriptions in this paper are shown in Table 2.

3.2. Fundamental assumption

Assumption 1. *Recycling and remanufacturing can restore waste products to a new state, providing the same guarantee as new products when sold. This allows new products to be marketed without any discernible difference. In other words, this paper assumes that there is no distinction between recycled or remanufactured products and new products in terms of appearance, function, performance, and other aspects, and that consumer acceptance of them remains consistent [25].*

Assumption 2. *The demand function of the closed-loop supply chain is assumed to be for $q = a - \beta p$: where a represents the market capacity, and $a > 0$; β represents the consumer to the new product price sensitivity coefficient, and $\beta > 0$; At the same time, in order to ensure that the members are profitable and feasible, the conditions should be met $a > \beta$.*

Assumption 3. *Suppose that the recovery rate when the recycler recovers WEEE is $\tau_i = \sqrt{I_i/k}$, where $i \in \{1, 2\}$, $k > 0$ denotes the scale parameter, representing the difficulty of recovering WEEE, and its recovery cost is $I = k\tau_i^2$; The recovery input cost function of two competitive recyclers is $I_i = I_i^* + \alpha I_j$, $i, j \in \{1, 2\}$ and $i \neq j$, I_i^* represents an effective input from one of the third parties, I_i represents the total input cost of one of the third parties; $\alpha_i (0 \leq \alpha_i \leq 1)$ represents the recovery competition coefficient between two recyclers. To simplify the operation, this paper assumes that the cross-influence coefficient of recovery cost between two recyclers is consistent, i.e. $\alpha_1 = \alpha_2 = \alpha$. Thus, the recovery effort cost function of the two competing recoveries is, respectively $I_1 = \frac{k(\tau_1^2 + \alpha\tau_2^2)}{1-\alpha^2}$, $I_2 = \frac{k(\tau_2^2 + \alpha\tau_1^2)}{1-\alpha^2}$, [19, 27].*

Assumption 4. *Classification recovery cost is a quadratic function of the effort of classification recovery, that is, the recovery cost borne by the classification recycler is $g\theta^2/2$. Assuming that the benefit of the effort per unit is proportional to the effort of the manufacturer, that is, the benefit of the effort per unit to the manufacturer is $\delta\theta$ [14].*

4. Closed-loop supply chain decision model of different outsourcing remanufacturing models under recycling competition

In this paper, four Stackelberg game decision models are constructed, in which the manufacturer outsources part of the remanufacturing business to the sorted recycler and pays the franchise outsourcing fee and an
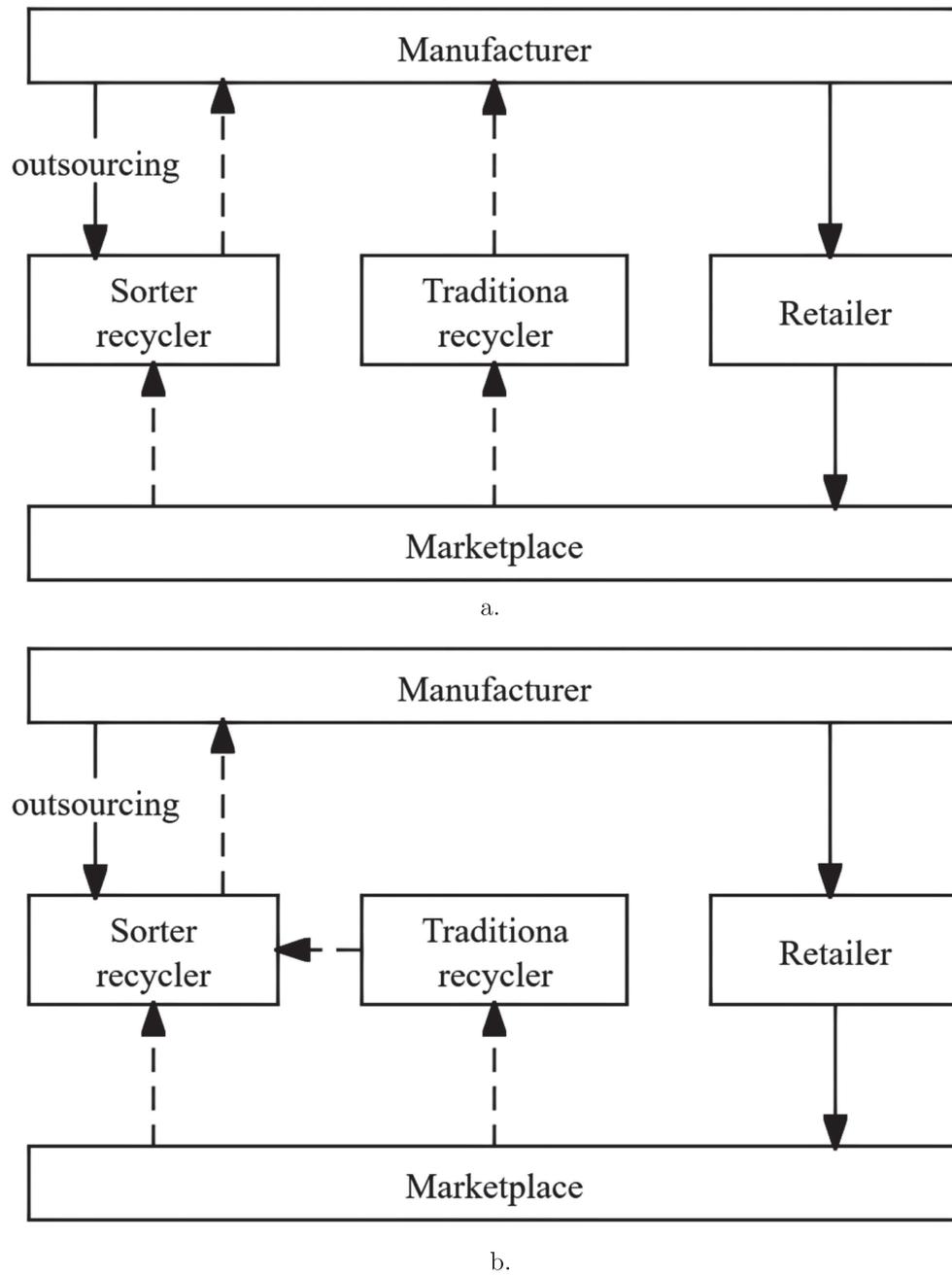
FIGURE 1. Structure diagram of the partial / total remanufacturing model of the closed-loop supply chain. (a) Partial remanufacturing under government subsidies. (b) Partial remanufacturing under government subsidies.

TABLE 2. List of notations.

Symbol	Definition and assumption
c_m	Unit production cost of the new products
c_r	Unit production cost of re-products
b	The transfer price that the manufacturer needs to pay to the recycler per unit of waste products
A	What recyclers have to pay for their waste products from consumers
w	Unit wholesale price of the products, $w > c_m$
p	Unit sales price of the product
τ_i	Recycling rate of waste products, $0 \leq \tau \leq 1$
α	Competition coefficient between the two third parties
θ	Recycler unit recovery classification effort, $\theta > 0$
g	Cost coefficient of unit classification effort, $g > 0$
δ	When the recycler classifies and recovers, the income coefficient of unit classification recovers, $\delta > 0$
H	Manufacturers are required to pay a one-time fixed outsourcing fee
h	The manufacturer needs to pay a certain unit outsourcing fee
μ	Manufacturers need to provide additional recovery price coefficient for each unit of classified waste products

outsourcing fee, and the manufacturer outsources all the remanufacturing business to the sorted recycler and pays the franchise outsourcing fee and an outsourcing fee, so as to obtain the optimal strategy and the maximum decision of the closed-loop supply chain.

4.1. Unit paid otsourcing partial remanufacturing decision model

In the model Sh, the manufacturer outsources part of the remanufacturing business to the recycler and pays the franchise outsourcing fee h. The recycler is responsible for the recycling and remanufacturing of its own waste products, while the manufacturer and retailer remain responsible for the sales of the products. All members of the closed-loop supply chain use profit maximization as the decision-making criterion. The profits of manufacturers, retailers, classified recyclers, and traditional recyclers are respectively:

$$\Pi_m^{Sh}(w) = (w - c_m)q + (c_m - c_r - b)\tau_2 q + (c_m - \mu b - h)\tau_1 q \tag{1}$$

$$\Pi_r^{Sh}(p) = (p - w)q \tag{2}$$

$$\Pi_{T1}^{Sh}(\tau_1) = (\mu b - c_r - A + \delta\theta + h)\tau_1 q - \frac{g\theta^2}{2} - \frac{k(\tau_1^2 + \alpha\tau_2^2)}{1 - \alpha^2} \tag{3}$$

$$\Pi_{T2}^{Sh}(\tau_2) = (b - A)\tau_2 q - \frac{k(\tau_2^2 + \alpha\tau_1^2)}{1 - \alpha^2}. \tag{4}$$

Theorem 1. *Under the model sh, the optimal strategy and the maximum profit are respectively*

$$w^{Sh*} = \frac{(\alpha^2 - 1)a\beta\Delta_1 - 2k(\beta c_m + a)}{(\alpha^2 - 1)\beta^2\Delta_1 - 4\beta k}, \quad p^{Sh*} = \frac{(\alpha^2 - 1)a\beta\Delta_1 - 2k(\beta c_m + 3a)}{(\alpha^2 - 1)\beta^2\Delta_1 - 4\beta k},$$

$$\tau_1^{Sh*} = \frac{(1 - \alpha^2)\Delta_2(c_r - \mu b - \delta\theta - h)}{2(\alpha^2 - 1)\beta\Delta_1 - 8k}, \quad \tau_2^{Sh*} = \frac{(\alpha^2 - 1)b\Delta_2}{2(\alpha^2 - 1)\beta\Delta_1 - 8k}, \quad q^{Sh*} = -\frac{k\Delta_2}{(\alpha^2 - 1)\beta\Delta_1 - 4k},$$

$$\Pi_m^{Sh*} = -\frac{k\Delta_2}{2(\alpha^2 - 1)\beta^2\Delta_1 - 8\beta k}, \quad \Pi_r^{Sh*} = \frac{k^2\Delta_2}{((\alpha^2 - 1)\beta\Delta_1 - 4k)^2\beta},$$

$$\Pi_{T2}^{Sh^*} = \frac{k\Delta_2{}^2(\alpha^2 - 1)\left((-\mu b - \delta\theta - h + c_r)^2\alpha - b^2\right)}{4((\alpha^2 - 1)\beta\Delta_1 - 4k)^2}, \quad \Pi_{T1}^{Sh^*} = \frac{2(1 - \alpha^2)\Delta_6 + 64(1 - \alpha)g\theta^2 k^2}{4(4k - (\alpha^2 - 1)\beta\Delta_1)^2}.$$

Among them

$$
\begin{aligned}
\Delta_1 &= (\mu^2 + 1)b^2 + ((\delta\theta + 2h - c_m - c_r)\mu - c_m + c_r)b + (-\delta\theta - h + c_r)(c_m - h), \\
\Delta_2 &= (\mu^2 + 1)b^2 + ((-\mu - 1)c_m + (-\mu + 1)c_r + 2\mu h)b + (h - c_r)(h - c_m), \\
\Delta_3 &= (\mu^2 - \alpha)b^2 + 2\mu(\delta\theta + h - c_r)b + (\delta\theta + h - c_r)^2, \quad \Delta_4 = \delta(-b\mu - h + c_r)\theta - \Delta_5, \\
\Delta_5 &= (-b\mu - \delta\theta - h + c_r)^2 - \alpha b^2, \\
\Delta_6 &= \left(\frac{\Delta_6 c_m{}^2 k}{b} + g(\alpha^2 - 1)(\theta(\Delta_7))^4\right)\beta^2 - k^2(-8g\theta^2\Delta_7 + a\Delta_8 c_m)\beta + ka^2\Delta_8(\alpha - 1).
\end{aligned}
$$

Proof. According to the game relationship of each member, the decision order is as follows: the manufacturer first decides the wholesale price of new products w and the recycling price of waste products b, and then the retailer decides the retail price p, classifying the recyclers and traditional recyclers τ_1, τ_2. It was solved by backward recur method. First of (2)–(4), easy to get $\frac{\partial^2\Pi_r^{Sh}(p)}{\partial p^2} = -2\beta < 0$, $\frac{\partial^2\Pi_{T1}^{Sh}(\tau_1)}{\partial\tau_1^2} = \frac{\partial^2\Pi_{T2}^{Sh}(\tau_2)}{\partial\tau_2^2} = \frac{2k}{\alpha^2 - 1} < 0$. The known $\Pi_r^{Sh}(p)$, $\Pi_{T1}^{Sh}(\tau_1)$, $\Pi_{T1}^{Sh}(\tau_1)$, is a strictly concave function about p, τ_1 and τ_2. Further, add τ_1^{Sh}, τ_2^{Sh}, p^{Sh}, separately into equation (1), easy evidence $\Pi_m^R(w)$, is a strict concave function. According to the first-order condition, $\frac{\partial\Pi_m^{Sh}(w)}{\partial w} = 0$. Can find the manufacturer's optimal decision w^{Sh^*}, and then put w^{Sh^*} into the expression of τ_1^{Sh}, τ_2^{Sh}, p^{Sh}, can obtain $\tau_1^{Sh^*}$, $\tau_2^{Sh^*}$, p^{Sh^*}. Then you pass $q = a - \beta p$, you can get q^{Sh^*}. Finally, the above equilibrium variables are substituted into the profit function of each member enterprise, that is, to obtain the maximum profit of the two members, and the certificate. □

4.2. Fixed paid outsourcing partial remanufacturing decision model

In the model SH, the manufacturer outsources part of the remanufacturing business to the classified recycler and pays an outsourcing fee H. The classified recycler is responsible for the recycling and remanufacturing of its own waste, while the manufacturer and retailer remain responsible for the sales of the products. All members of the closed-loop supply chain take profit maximization as the decision-making criterion, and the profits of manufacturers, retailers, classified recyclers, and traditional recyclers are respectively:

$$\Pi_m^{SH}(w) = (w - c_m)q + (c_m - c_r - b)\tau_2 q + (c_m - \mu b)\tau_1 q - H \tag{5}$$

$$\Pi_r^{SH}(p) = (p - w)q \tag{6}$$

$$\Pi_{T1}^{SH}(\tau_1) = (\mu b - c_r - A + \delta\theta)\tau_1 q - \frac{g\theta^2}{2} - \frac{k(\tau_1^2 + \alpha\tau_2^2)}{1 - \alpha^2} + H \tag{7}$$

$$\Pi_{T2}^{SH}(\tau_2) = (b - A)\tau_2 q - \frac{k(\tau_2^2 + \alpha\tau_1^2)}{1 - \alpha^2}. \tag{8}$$

Theorem 2. *Under the model SH, the optimal strategy and the maximum profit are respectively*

$$
\begin{aligned}
w^{SH^*} &= \frac{(\alpha^2 - 1)a\beta\Delta_7 + 2k(\beta c_m + a)}{(\alpha^2 - 1)\beta^2\Delta_7 + 4\beta k}, \quad p^{SH^*} = \frac{(\alpha^2 - 1)a\beta\Delta_7 + k(\beta c_m + 3a)}{(\alpha^2 - 1)\beta^2\Delta_7 + 4\beta k}, \\
\tau_1^{SH^*} &= \frac{(1 - \alpha^2)\Delta_2(c_r - \mu b - \delta\theta)}{2(\alpha^2 - 1)\beta\Delta_7 + 8k}, \quad \tau_2^{SH^*} = \frac{(\alpha^2 - 1)b\Delta_2}{2(\alpha^2 - 1)\beta\Delta_7 + 8k}, \quad q^{SH^*} = \frac{k\Delta_2}{(\alpha^2 - 1)\beta\Delta_7 + 4k}, \\
\Pi_m^{SH^*} &= \frac{(c_m{}^2\beta^2 - 8(1/4ac_m + H)\beta + a^2)k + 2(\alpha^2 - 1)\Delta_7 H\beta^2}{2(\alpha^2 - 1)\beta^2\Delta_7 + 8\beta k}, \quad \Pi_r^{SH^*} = \frac{k^2\Delta_2}{((\alpha^2 - 1)\beta\Delta_7 + 4k)^2\beta},
\end{aligned}
$$

$$\Pi_{T2}^{SH^*} = \frac{k\Delta_2{}^2(\alpha^2-1)\left((-\mu b-\delta\theta+c_r)^2\alpha-b^2\right)}{4\left((\alpha^2-1)\beta\Delta_7+4k\right)^2},$$

$$\Pi_{T1}^{SH^*} = \frac{4\left(\left(\alpha\left(b^3\Delta_8+\left(4\alpha^2\Delta_{10}+\Delta_9-4\Delta_{10}\right)b^2+(\delta\theta-c_r)\Delta_{11}b-(\delta\theta-c_r)\Delta_{12}c_m{}^2\right)\right)^2\right)^2}{4\left((\alpha^2-1)\beta\Delta_7+4k\right)^2}.$$

Since the decision order and solving method of member enterprises in the closed-loop supply chain are similar to the proof process in Section 4.1, the detailed solving process is omitted here.

$$\Delta_7 = (-\mu^2-1)b^2+((-\delta\theta+c_m+c_r)\mu+c_m-c_r)b-c_m(-\delta\theta+c_r),$$

$$\Delta_8 = b\mu^2+(2\delta\theta-2c_m-2c_r)\mu+b-2c_m+2c_r,$$

$$\Delta_9 = -4\theta\delta\left(-\frac{1}{2}g\theta^2+H\right)\left(\left(\frac{c_m}{2}-\frac{c_r}{2}\right)\mu+\frac{c_m}{2}\right)+\left(\left(\frac{(-c_m{}^2-4c_mc_r-c_r{}^2)g}{2}+H\delta^2\right)\mu^2\right.$$
$$\left.+(-c_m{}^2g+c_r{}^2g)\mu-\frac{g(c_m{}^2+c_r{}^2)}{2}\right)\theta^2-\frac{g\mu^2\delta^2\theta^4}{2},$$

$$\Delta_{10} = \left((c_m{}^2+4c_rc_m+c_r{}^2)\mu^2\right)H\alpha^2+\frac{c_m{}^2k\alpha}{4}+\left(\left(-H-\frac{k}{4}\right)c_m{}^2-4Hc_mc_r\right)\mu^2+(-c_m{}^2+2c_r{}^2)2H\mu,$$

$$\Delta_{11} = \left(-g(\mu+1)\theta^2+2H(\mu+1)\alpha^2+2(-\mu-1)H-\frac{k\mu}{2}\right)c_m+(-\theta\mu\delta+(\mu-1)c_r),$$

$$\Delta_{12} = (-g\alpha^2+g)\theta^2+2H\alpha^2-2H-\frac{k}{2},$$

$$\Delta_{13} = \left(-4g(\mu+1)\theta^2+a\mu\delta\theta+(-ac_r+8H)\mu+8H\right)4c_m-\theta\mu\delta32\left(\frac{-g\theta^2}{2}+H\right),$$

$$\Delta_{14} = \left(2a(\mu^2-\alpha)c_m-32\left(\frac{-g\theta^2}{2}+H\right)(\mu^2+1)\right)b^2-32\left(\frac{-g\theta^2}{2}+\frac{\theta a\delta-ac_r}{16}+H\right),$$

$$\Delta_{15} = -(\alpha^2-1)\left(-\alpha b^2+(-b\mu-\delta\theta+c_r)^2\right)a^2+64\left(-1/2g\theta^2+H\right)k.$$

4.3. Unit paid outsourcing all remanufacturing decision model

In the model Ah, the manufacturer is solely accountable for new product production, outsourcing all remanufacturing operations to a classified recycler and paying a franchise outsourcing fee h. The classified recycler is responsible for recycling its own waste products and remanufacturing all waste products. Classified recyclers handle the recycling and remanufacturing of their own waste, while manufacturers and retailers remain responsible for the sales of recycled products. All members of the closed-loop supply chain prioritize profit maximization as the decision-making criterion, with profits distributed among manufacturers, retailers, classified recyclers, and traditional recyclers, respectively:

$$\Pi_m^{Ah}(w) = (w-c_m)q+(c_m-h-\mu b)(\tau_1+\tau_2)q \tag{9}$$

$$\Pi_r^{Ah}(p) = (p-w)q \tag{10}$$

$$\Pi_{T1}^{Ah}(\tau_1) = (\mu b-c_r+\delta\theta+h)(\tau_1+\tau_2)q-A\tau_1q-b\tau_2q-\frac{g\theta^2}{2}-\frac{k(\tau_1^2+\alpha\tau_2^2)}{1-\alpha^2} \tag{11}$$

$$\Pi_{T2}^{Ah}(\tau_2) = (b-A)\tau_2q-\frac{k(\tau_2^2+\alpha\tau_1^2)}{1-\alpha^2}. \tag{12}$$

Theorem 3. *Under the model Ah, the optimal strategy and the maximum profit are respectively*

$$w^{Ah^*} = \frac{a\beta\Omega_1 + 2k(\beta c_m + a)}{\beta^2\Omega_1 - 4\beta k}, \quad p^{Ah^*} = \frac{a\beta\Omega_1 + k(\beta c_m + 3a)}{\beta^2\Omega_1 - 4\beta k}, \quad \tau_1^{Sh^*} = \frac{\Delta_2\left(1 - \alpha^2\right)(c_r - \mu b - \delta\theta - h)}{2\beta\Omega_1 - 8k},$$

$$\tau_2^{Ah^*} = \frac{\left(\alpha^2 - 1\right)b\Delta_2}{2\beta\Omega_1 - 8k}, \quad q^{Ah^*} = \frac{k\Delta_2}{4k - \beta\Omega_1}, \quad \Pi_m^{Ah^*} = \frac{k\Delta_2}{8\beta k - 2\beta^2\Omega_1},$$

$$\Pi_r^{Ah^*} = \frac{k^2\Delta_2}{(\beta\Omega_1 - 4k)^2\beta}, \quad \Pi_{T2}^{Ah^*} = \frac{k\Delta_2{}^2(\alpha^2 - 1)\left((-\mu b - \delta\theta - h + c_r)^2\alpha - b^2\right)}{4(\beta\Omega_1 - 4k)^2},$$

$$\Pi_{T1}^{Ah^*} = \frac{\left(2\Omega_5\beta\left(\alpha^2 - 1\right) - \Omega_6\right)k + \left(1 - \alpha^2\right)\Omega_4\beta^2}{4(4k - \beta\Omega_1)^2}.$$

Since the decision order and solving method of member enterprises in the closed-loop supply chain are similar to the proof process in Section 5.1, the detailed solving process is omitted here.

$$\Delta_{16} = (-b\mu - h + c_m)((-\mu - 1)b - \delta\theta + c_r - h)(\alpha^2 - 1),$$

$$\Delta_{17} = \left(\mu^2 - \alpha + 2\mu - 2\right)b^2 + 2(\mu + 1)(-\delta\theta - h + c_r)b + (-\delta\theta - h + c_r)^2,$$

$$\Delta_{18} = 2\left(\alpha^2 - 1\right)((\mu + 1)b + \delta\theta - c_r + h)^2(b\mu + h - c_m)^2 g + c_m{}^2 k\delta^2,$$

$$\Delta_{19} = \theta^2\Delta_{18} + (\Delta_{17} + (h - c_r)(\delta\theta + h - c_r))kc_m{}^2,$$

$$\Delta_{20} = ac_m\left(\Delta_{17} + (\delta\theta + h - c_r)^2\right) + 8\theta^2 g(b\mu + \delta\theta + b + h - c_r)(b\mu + h - c_m),$$

$$\Delta_{21} = \left(\Delta_{17} + (\delta\theta + h - c_r)^2\right)(\alpha^2 - 1)a^2 + 32gk\theta^2.$$

4.4. Fixed-paid outsourcing all-remanufacturing decision model

In the model AH, the manufacturer is solely responsible for the production of new products, outsourcing all remanufacturing operations to a classified recycler and paying an outsourcing fee H. The classified recycler is responsible for recycling its own waste products and remanufacturing all waste products. Classified recyclers handle the recycling and remanufacturing of their own waste, while manufacturers and retailers remain responsible for the sales of recycled products. All members of the closed-loop supply chain prioritize profit maximization as the decision-making criterion, with profits distributed among manufacturers, retailers, classified recyclers, and traditional recyclers, respectively:

$$\Pi_m^{AH}(w) = (w - c_m)q + (c_m - \mu b)(\tau_1 + \tau_2)q - H \tag{13}$$

$$\Pi_r^{AH}(p) = (p - w)q \tag{14}$$

$$\Pi_{T1}^{AH}(\tau_1) = (\mu b - c_r + \delta\theta)(\tau_1 + \tau_2)q - A\tau_1 q - b\tau_2 q - \frac{g\theta^2}{2} - \frac{k\left(\tau_1^2 + \alpha\tau_2^2\right)}{1 - \alpha^2} + H \tag{15}$$

$$\Pi_{T2}^{AH}(\tau_2) = (b - A)\tau_2 q - \frac{k\left(\tau_2^2 + \alpha\tau_1^2\right)}{1 - \alpha^2}. \tag{16}$$

Theorem 4. *Under the model AH, the optimal strategy and the maximum profit are respectively*

$$w^{AH^*} = \frac{a\beta\Delta_{22} + 2k(\beta c_m + a)}{\beta^2\Delta_{22} + 4\beta k},$$

$$p^{AH^*} = \frac{a\beta\Delta_{22} + k(\beta c_m + 3a)}{\beta^2\Delta_{22} + 4\beta k}, \quad \tau_1^{AH^*} = \frac{\left(1 - \alpha^2\right)\Delta_2(c_r - \mu b - \delta\theta)}{2\beta\Delta_{22} + 8k},$$

$$\tau_2^{AH^*} = \frac{\left(1 - \alpha^2\right)b\Delta_2}{2\beta\Delta_{22} + 8k}, \quad q^{AH^*} = \frac{k\Delta_2}{\beta\Delta_{22} + 4k}, \quad \Pi_m^{AH^*} = \frac{\left(-c_m{}^2\beta^2 + 8(1/4ac_m + H)\beta - a^2\right)k - 2\Delta_{22}H\beta^2}{2\beta^2\Delta_{22} - 8\beta k},$$

TABLE 3. Effects of recycling competition degree α on equilibrium solution and profit margin.

	Parameter range	w^*	p^*	τ_1^*	τ_2^*	q^*	Π_m^*	Π_r^*	Π_{T2}^*	Parameter range	Π_{T1}^*
Sh	$b^2 > \gamma_1$	↑	↑	↓	↓	↓	↓	↓	↓	$b^2 > \gamma_2$	↓
	$b^2 < \gamma_1$	↓	↓	↓	↓	↑	↑	↑	↓	$b^2 < \gamma_2$	↑
SH	$b^2 > \gamma_3$	↑	↑	↓	↓	↓	↓	↓	↓	$b^2 > \gamma_4$	↓
	$b^2 < \gamma_3$	↓	↓	↓	↓	↑	↑	↑	↓	$b^2 < \gamma_4$	↑
Ah	$b^2 > \gamma_4$	↑	↑	↓	↓	↓	↓	↓	↓	$b > \gamma_5$	↓
	$b^2 < \gamma_4$	↓	↓	↓	↓	↑	↑	↑	↓	$b < \gamma_5$	↑
AH	–	↑	↑	↓	↓	↓	↓	↓	↓	–	↓

$$\Pi_r^{AH^*} = \frac{k^2\Delta_2}{(\beta\Delta_{22} + 4k)^2\beta}, \quad \Pi_{T2}^{AH^*} = \frac{k\Delta_2{}^2(\alpha^2 - 1)\left((-\mu b - \delta\theta + c_r)^2\alpha - b^2\right)}{4(\beta\Delta_{22} + 4k)^2},$$

$$\Pi_{T1}^{AH^*} = \frac{64k^2\left(-1/2\,g\theta^2 + H\right) + (\alpha^2 - 1)\left((\Delta_{29}a^2 + \Delta_{28}\beta)k + 4\Delta_{26}\beta^2\right)}{4(\beta\Delta_{22} + 4k)^2}.$$

Since the decision order and solving method of member enterprises in the closed-loop supply chain are similar to the proof process in Section 5.1, the detailed solving process is omitted here.

$$\Delta_{22} = (b\mu - c_m)(\alpha^2 - 1)((-\mu - 1)b - \delta\theta + c_r), \quad \Delta_{23} = \left(-\frac{g\theta^2}{2} + H\right)\alpha^2 - H - \frac{k}{4} + \frac{g\theta^2}{2},$$

$$\Delta_{24} = (\alpha^2 - 1)\left(-\frac{g\theta^2}{2} + H\right)(\delta\theta - c_r),$$

$$\Delta_{25} = b^3\mu(\alpha^2 - 1)(\mu + 1)\left(\mu^2 b + (2\delta\theta + b - 2c_m - 2c_r)\mu - 2c_m\right)\left(-\frac{g\theta^2}{2} + H\right),$$

$$\Delta_{26} = ((\mu + 1)b + \delta\theta - c_r)^2\Delta_{23}c_m{}^2 - 4((\mu + 1)b + 1/2\,\delta\theta - 1/2\,c_r)\Delta_{24}b\mu c_m,$$

$$\Delta_{27} = \left(2a(\mu^2 - \alpha + 2\mu - 2)c_m - 32\mu(\mu + 1)(-1/2\,g\theta^2 + H)\right)b^2 + 32\left(\frac{\theta a\delta - 4g\theta^2}{8}\right),$$

$$\Delta_{28} = \Delta_{27}b - 32\left(-1/2\,g\theta^2 + 1/16\,\theta a\delta - 1/16\,c_r a + H\right)(-\delta\theta + c_r)c_m,$$

$$\Delta_{29} = -(\mu^2 - \alpha + 2\mu - 2)b^2 + 2(\mu + 1)(-\delta\theta + c_r)b - (-\delta\theta + c_r)^2.$$

5. BALANCED OUTCOME ANALYSIS

5.1. The impact of the recovery competition degree on the optimal strategy and the profit

Proposition 1. *The marginal analysis of each parameter and the corresponding profits of the supply chain members through the degree of recovery competition is obtained, as shown in Table 3.*

Proof. Take the partial parameters of the case AH in property 1 as an example.

$$\frac{\partial w^{AH^*}}{\partial\alpha} = \frac{4(b\mu - c_m)(-\beta c_m + a)((-\mu - 1)b - \delta\theta + c_r)\alpha k}{((b\mu - c_m)(\alpha - 1)(\alpha + 1)((-\mu - 1)b - \delta\theta + c_r)\beta + 4k)^2} > 0,$$

TABLE 4. Effects of classification return coefficient δ on equilibrium solution and profit margin.

	Parameter range	w^*	p^*	τ_1^*	τ_2^*	q^*	Π_m^*	Π_r^*	Π_{T1}^*	Π_{T2}^*
Sh/Ah	$b > \dfrac{c_m - h}{\mu}$	↑	↑	↑	↓	↓	↓	↓	↓	↓
	$0 < b < \dfrac{c_m - h}{\mu}$	↓	↓	↑	↑	↑	↑	↑	↑	↓
SH/AH	$-$	↓	↓	↑	↑	↑	↑	↑	↑	↓

$$\frac{\partial p^{AH^*}}{\partial \alpha} = \frac{2(b\mu - c_m)(-\beta c_m + a)((-\mu - 1)b - \delta\theta + c_r)\alpha k}{((b\mu - c_m)(\alpha - 1)(\alpha + 1)((-\mu - 1)b - \delta\theta + c_r)\beta + 4k)^2} > 0,$$

$$\frac{\partial \tau_1^{AH^*}}{\partial \alpha} = \frac{4(-b\mu - \delta\theta + c_r)(-\beta c_m + a)\alpha k}{((b\mu - c_m)(\alpha - 1)(\alpha + 1)((-\mu - 1)b - \delta\theta + c_r)\beta + 4k)^2} < 0,$$

$$\frac{\partial q^{AH^*}}{\partial \alpha} = \frac{-2\beta(b\mu - c_m)(-\beta c_m + a)((-\mu - 1)b - \delta\theta + c_r)\alpha k}{((b\mu - c_m)(\alpha - 1)(\alpha + 1)((-\mu - 1)b - \delta\theta + c_r)\beta + 4k)^2} < 0.$$

$\square$

Since the proof process of *Sh*, *SH*, *Ah* is similar to the current situation *AH*, the detailed proof process is omitted here.

$$\gamma_1 = \frac{[(1 + \mu)c_m + (-\delta\theta - 2h + c_r)\mu - c_r]b - (\delta\theta + h - c_r)(h - c_m)}{(-\mu^2 - 1)},$$

$$\gamma_2 = \frac{-2(b\mu + \delta\theta + h - c_r)^2\alpha - b^2}{3\alpha^2},$$

$$\gamma_3 = \frac{((\mu + 1)c_m + (-\delta\theta + c_r)\mu - c_r)b + c_m(\delta\theta - c_r)}{\mu^2 + 1}, \quad \gamma_4 = \frac{2(b\mu + \delta\theta - c_r)^2\alpha + b^2}{3\alpha^2}, \quad \gamma_5 = \frac{c_m - h}{\mu}.$$

Table 3 reveals that, under the condition of total remanufacturing-fixed payment, an escalation in recycling competition diminishes the classification and traditional recycling rates, resulting in decreased profits for manufacturers, retailers, and traditional recyclers. To safeguard their interests, manufacturers, as channel leaders, opt for a strategy of raising wholesale prices, compelling follower retailers to increase retail prices to protect their interests. This chain reaction reduces the market demand for the product. In the case of partial remanufacturing-unit pay/fixed pay and full remanufacturing-unit pay, when the recovery price, classification recovery effort, and coefficient are greater than zero, the parameters and members' profits mirror those in the case of full remanufacturing-fixed pay. However, when the recovery price, classification recovery effort, and the coefficient of the threshold interval are less than zero, the situation is entirely reversed. It can be concluded that intense recycling competition may adversely impact market development. For instance, recycling platforms such as Aihui, Changyi, and Huishou Bao have engaged in detrimental recycling competition to vie for limited market share. Some platforms hoard goods through price competition, selling them at high prices, leading to chaos in the recycling market. These behaviors affect the stable development of the overall market.

5.2. The impact of the classification return coefficient on the optimal strategy and the profit

Proposition 2. *Involves the marginal analysis of each parameter and the profits of supply chain members by classifying the revenue coefficient, resulting in the determination of the feasible interval, as shown in Table 4.*

TABLE 5. Influence of recovery price coefficient μ on equilibrium solution and profit margin.

	Parameter scope	w^*	p^*	τ_1^*	τ_2^*	q^*	Π_m^*	Π_r^*	Π_{T1}^*	Π_{T2}^*
$Sh/SH/Ah/AH$	$b > \gamma^i$	↑	↑	↑	↓	↓	↓	↓	↑	↓
	$b < \gamma^i$	↓	↓	↑	↑	↑	↑	↑	↑	↓

From Proposition 2, it can be observed that the income coefficient of classified recovery is directly proportional to the classified recovery rate and the profit of classified recyclers, while it is inversely proportional to the profit of traditional recyclers. In the outsourcing mode of unit payment, the influence of the revenue coefficient on the equilibrium result and member profit depends on the new product cost, outsourcing cost, recovery price, and coefficient. When the recovery price exceeds the threshold value, the income coefficient of classified recovery is directly proportional to the wholesale price and retail price, and inversely proportional to the traditional recovery rate, market demand, as well as the profit of manufacturers and retailers. This is because with the increase in the profit coefficient of classification and recycling, although manufacturers can obtain higher profits, the profits of manufacturers will decline due to the large unit costs generated by outsourcing, and the profits of follower retailers will also decline. When the recovery price is less than the threshold value, the reverse is true, and the recovery coefficient is inversely proportional to the wholesale price and retail price, and directly proportional to the traditional recovery rate, market demand, and the profits of manufacturers and retailers. This is because with the increase in the income coefficient of classified recycling, the manufacturer not only obtains higher income but also incurs only a small unit cost. The spillover effect leads to an increase in the profit of downstream retailers, thus promoting recycling.

Under the fixed payment outsourcing model, an increase in the income coefficient of classified recycling is conducive to the recycling of waste products. This, in turn, leads to an increase in the market share of products, a reduction in the wholesale and retail prices of products, and an increase in the profits of manufacturers and retailers as the income coefficient of classified recycling rises. It can be concluded that improving the profit coefficient of classification and recycling for manufacturers helps increase the efforts of classification and recycling, leading to a "win–win situation" of economic and environmental benefits. Manufacturers should actively engage in technology outsourcing and joint research and development with recyclers to enhance the profit coefficient of classification and recycling, ensuring the smooth operation of the recycling market.

5.3. The impact of the classification return coefficient on the optimal strategy and the profit

Proposition 3. *Marginal analysis of each parameter and the profits of supply chain members is carried out through the recovery price coefficient, and feasible intervals are obtained, as shown in Table 5.*

Since the Proposition 3 proof process is similar to the property 1 proof process, the detailed proof process is omitted here. Where $\gamma^{Sh} = -\frac{\delta\theta + 2h - c_m - c_r}{2\mu}$, $\gamma^{SH} = -\frac{\delta\theta - c_m - c_r}{2\mu}$, $\gamma^{Ah} = -\frac{\delta\theta + 2h + b - c_m - c_r}{2\mu}$, $\gamma^{AH} = -\frac{\delta\theta + b - c_m - c_r}{2\mu}$.

According to Proposition 3, the impact of the recovery price coefficient on the equilibrium result and member profit depends on classification recovery income, new product production, and remanufacturing cost. An increase in the recovery price factor will enhance the sorting recovery rate, thereby increasing the profit of the sorting recyclers. However, due to the fixed market share in the recycling market, when the recovery price is at different interval thresholds, although the increase in the recovery price coefficient will have a two-way impact on the traditional recovery rate, the increase in the recovery price coefficient and the classification recovery rate will result in a reduction in the profit of traditional recyclers. When the recovery price is greater than the threshold, as the recovery price coefficient increases, the market demand for the product will decrease, leading to a reduction in the profits of manufacturers and retailers. To protect their own interests, both choose to increase the wholesale and retail prices. However, due to the great influence of recovery price coefficient, the profit is difficult to

TABLE 6. Comparison of equilibrium results under the unit payment model.

	Parameter scope	w^*	p^*	τ_1^*	τ_2^*	q^*	Π_m^*	Parameter scope	Π_r^*	Π_{T1}^*	Π_{T2}^*
$Sh-Ah$	$b > \gamma_1^{Sh-Ah}$	$-$	$-$	$+$	$+$	$+$	$+$	$b > \gamma_i^{Sh-Ah}$	$+$	$+$	$+$
	$b < \gamma_1^{Sh-Ah}$	$+$	$+$	$-$	$-$	$-$	$-$	$b < \gamma_i^{Sh-Ah}$	$-$	$-$	$-$

maintain balance; On the contrary, when the recovery price is less than the threshold value, as the recovery price coefficient increases, the market demand for products increases, and the profits of manufacturers and retailers also increase accordingly. To further expand profits, manufacturers adopt the strategy of "small profits and quick sales", rapidly expanding the market by reducing the wholesale price. Meanwhile, retailers choose to reduce the retail price to obtain greater profits. This strategy drives market expansion under the influence of the recovery price factor.

5.4. Research on optimal remanufacturing mode selection

Result 1. The parameters of the two outsourcing remanufacturing modes and the corresponding profits of supply chain members are compared, and the feasible range is obtained, as shown in Table 6.

Proof. Take some parameters in Result 1 as an example

$$w^{Sh^*} - w^{Ah^*} = \frac{-2(-\beta c_m + a)(\alpha^2 - 1)bk((-\mu+1)b + c_r - h)}{((\alpha^2 - 1)\beta^2\Delta_1 - 4\beta k)(\beta^2\Delta_{16} - 4\beta k)},$$

$$p^{Sh^*} - p^{Ah^*} = \frac{-(-\beta c_m + a)(\alpha^2 - 1)bk((-\mu+1)b + c_r - h)}{((\alpha^2 - 1)\beta^2\Delta_1 - 4\beta k)(\beta^2\Delta_{16} - 4\beta k)},$$

$$q^{Sh^*} - q^{Ah^*} = \frac{\beta(-\beta c_m + a)k(\alpha^2 - 1)b((-\mu+1)b + c_r - h)}{(\beta\Delta_{16} - 4k)((\alpha^2 - 1)\beta\Delta_1 - 4k)}.$$

□

Since the profit proof process of other parameters and members in Nature Proposition 4 is similar to this process, the detailed proof process is omitted here. Where $i = \{r, T1, T2\}$, $\gamma_1^{Sh-Ah} = \frac{c_r - h}{(\mu-1)}$, $\gamma_r^{Sh-Ah} = -\frac{\delta\theta + h - c_m - c_r}{2\mu}$, $\gamma_{T1}^{Sh-Ah} = -\frac{\delta\theta + h - c_r}{\mu-1}$, $\gamma_{T2}^{Sh-Ah} = -\frac{2\delta\theta + h - 2c_r}{2\mu}$.

According to Result 1, the decision of the remanufacturing mode in the unit paid outsourcing mode is affected by the recovery price, remanufacturing cost, and unit outsourcing cost. However, when the unit cost is higher than the specific threshold, the profit of wholesale price, retail price, retailers, and traditional recyclers is lower in the partial remanufacturing mode, while the recovery rate of waste products, market demand, and manufacturers' profit are higher. On the contrary, when the unit outsourcing cost is less than a specific threshold, the profit of wholesale price, retail price, retailers, and traditional recyclers under full remanufacturing is lower, while the recovery rate of waste products, market demand, and manufacturers' profit are higher. For the manufacturer, if the unit outsourcing cost is too large, the manufacturer will incur a high outsourcing cost when choosing the full remanufacturing mode. Therefore, the manufacturer will opt for the partial remanufacturing mode. When the unit cost is small, the manufacturer is more willing to choose the full remanufacturing mode to carry out all new product production and product sales.

Result 2. Under the fixed payment outsourcing mode, parameters of the two outsourcing remanufacturing modes and corresponding profits of supply chain members are compared to obtain the feasible range, as shown in Table 7.

TABLE 7. Comparison of equilibrium results in fixed-payment mode.

	Parameter range	w^*	p^*	τ_1^*	τ_2^*	q^*	Π_m^*	Parameter range	Π_r^*	Π_{T1}^*	Π_{T2}^*
$SH-AH$	$b > \gamma_1^{SH-AH}$	−	−	+	+	+	+	$b > \gamma_2^{SH-AH}$	+	+	+
	$b < \gamma_1^{SH-AH}$	+	+	−	−	−	−	$b < \gamma_2^{SH-AH}$	−	−	−

TABLE 8. Comparison of equilibrium results under different remanufacturing modes.

	Parameter range	w^*	p^*	τ_1^*	τ_2^*	q^*
S/A	$b > \gamma^i$	$w^{ih^*} > w^{iH^*}$	$p^{ih^*} > p^{iH^*}$	$\tau_1^{ih^*} > \tau_1^{iH^*}$	$\tau_2^{ih^*} < \tau_2^{iH^*}$	$q^{ih^*} < q^{iH^*}$
	$b < \gamma^i$	$w^{ih^*} < w^{iH^*}$	$p^{ih^*} < p^{iH^*}$		$\tau_2^{ih^*} > \tau_2^{iH^*}$	$q^{ih^*} > q^{iH^*}$

Since the proof process of Result 2 is similar to the proof process of Result 1, the detailed proof process is omitted here. Where $\gamma_1^{SH-AH} = -\frac{\delta\theta - c_m - c_r}{2\mu + 1}$, $\gamma_2^{SH-AH} = \frac{c_r}{\mu - 1}$.

According to Result 2, the decision of remanufacturing mode under fixed payment outsourcing mode is affected by new product and remanufactured product costs, recovery price, and coefficient. When the recovery price is higher than the specific threshold, the wholesale price and retail price are at their lowest, the recovery rate and market demand are the highest, and the profits of manufacturers, retailers, and traditional recyclers are higher. When the recovery price is less than a specific threshold, the wholesale price and retail price in the manufacturer's total remanufacturing mode are the lowest, the recovery rate and market demand are the highest, and the profits of manufacturers, retailers, and traditional recyclers are also higher. When the manufacturer chooses the fixed-fee outsourcing model, compared with the unit fee outsourcing model, the manufacturer only needs to consider an appropriate fixed fee, and the outsourcing decision is more influenced by other factors. Therefore, the manufacturer will develop different outsourcing strategies according to different recycling prices.

5.5. Research on optimal outsourcing mode selection

Result 3. In the case of partial/full remanufacturing mode, parameters of the two paid outsourcing modes were compared to obtain the feasible range, as shown in Table 8.

Since the proof process of Result 3 is similar to the proof process of Result 1, the detailed proof process is omitted here. Where $i = \{S, A\}$, $\gamma^S = -\frac{\delta\theta + h - c_m - c_r}{2\mu}$, $\gamma^A = -\frac{\delta\theta + h - c_m - c_r}{2\mu + 1}$.

It can be seen from Result 3 that when the recovery price is greater than the threshold, the wholesale price, retail price, and classification recovery rate are the highest under the unit paid outsourcing model, and the traditional recovery rate and market demand are the lowest, followed by the fixed paid outsourcing model. On the contrary, when the recovery price is less than the threshold value, the wholesale price and retail price under the fixed payment outsourcing model are relatively high, while the classification, traditional recovery rate, and market demand are relatively low, with the unit payment outsourcing model coming next. Under the unit pay outsourcing model, the classification recovery rate is always higher than that of the fixed pay outsourcing model. This is because the outsourcing remanufacturing strategy adopted by the manufacturer promotes the improvement of the classification recovery rate. The manufacturer chooses outsourcing remanufacturing as the classification recycler who undertakes part or all of the remanufacturing task, thus significantly improving the classification recovery rate under this mode.

Result 4. In the partial/full remanufacturing mode, the profit of supply chain members corresponding to the three paid outsourcing modes is compared to obtain the feasible range, as shown in Table 9.

TABLE 9. Profit comparison of members under different remanufacturing modes.

	Parameter range	Π_m^*	Π_r^*	Π_{T2}^*
S/A	$b > \gamma^i$	$\Pi_m^{ih^*} > \Pi_m^{iH^*}$	$\Pi_r^{iH^*} > \Pi_r^{ih^*}$	$\Pi_{T2}^{iH^*} > \Pi_{T2}^{ih^*}$
	$b < \gamma^i$	$\Pi_m^{ih^*} > \Pi_m^{iH^*}$	$\Pi_r^{iH^*} < \Pi_r^{ih^*}$	

Since the proof process of Result 4 is similar to that of Result 1, the detailed proof process is omitted here. Where $i = \{S, A\}$, $\gamma^S = -\frac{\delta\theta + h - c_m - c_r}{2\mu}$, $\gamma^A = -\frac{\delta\theta + h - c_m - c_r}{2\mu + 1}$.

According to Result 4, the highest profit among traditional recyclers is observed in the fixed payment outsourcing mode, with the unit payment outsourcing mode securing the second-highest profit. This is attributed to the fact that, under the fixed-fee outsourcing model, traditional recyclers can more effectively estimate recycling costs, leading to higher profits. However, in the unit pay outsourcing model, costs are subject to certain fluctuations, impacting the profit levels of recyclers. When the recovery price exceeds the threshold value, manufacturers achieve the highest profit in the unit payment outsourcing mode, followed by the fixed payment outsourcing mode. The fixed payment outsourcing model yields the highest profit for retailers, with the unit payment outsourcing model following closely. When the recovery price falls below the threshold value, manufacturers attain the highest profit in the unit payment outsourcing mode and the lowest in the fixed payment outsourcing mode. The highest profit for retailers is achieved under the unit pay outsourcing model, with the fixed pay outsourcing model following closely.

The unit pay outsourcing model consistently yields the highest profit for manufacturers. When the recycling price is high, the product demand under the unit pay outsourcing model tends to be low. This prompts manufacturers to sustain their profit by increasing the wholesale price, leading retailers to subsequently raise the selling price of the product. In the scenario of a low recycling price, the reduced wholesale price associated with the unit pay outsourcing model stimulates an increase in market demand. This attracts more consumers, leading to a "rise" in manufacturer profits. Subsequently, retailers adjust by lowering the retail price to maximize their profits. However, in diverse market environments, the impact of various paid outsourcing models on participant profits can vary. This discrepancy is influenced by a combination of factors, including market demand, cost structure, and pricing strategy.

6. RESEARCH ON CLOSED-LOOP SUPPLY CHAIN DECISION-MAKING OF DIFFERENT OUTSOURCING REMANUFACTURING MODELS UNDER RECYCLING COMPETITION AND GOVERNMENT SUBSIDIES

With the emergence of innovative recycling methods like separate recycling, the economic and environmental benefits derived from efficient waste utilization have significantly increased. In this landscape, external government intervention and guidance on the classification recycling and remanufacturing process become crucial. This chapter delves into the influence of the government's dual subsidies, targeting "classified recycling + remanufacturing", on the decision-making processes within closed-loop supply chain outsourcing remanufacturing under recycling competition.

To foster increased enthusiasm for remanufacturing and sorting and recycling, the government implements subsidies for remanufacturing entities. Specifically, a unit remanufacturing subsidy v is provided at the remanufacturing stage. In the context of the manufacturer's partial remanufacturing mode, both the manufacturer and the sorting and recycling company receive the unit remanufacturing subsidy v provided by the government. This initiative aims to incentivize and support efforts towards more sustainable practices in remanufacturing and recycling within the supply chain. In the scenario where the manufacturer opts for the full remanufacturing model, the government extends the unit remanufacturing subsidy (v) exclusively to the remanufacturing under-

taker, the sorting recycler. Additionally, at the sorting and recycling stage, it is assumed that the government offers a unit sorting and recycling subsidy (s) to the sorting and recycling operators.

6.1. The decision model for partially outsourcing remanufacturing with unit payment under government subsidy

In the model $\overline{Sh}$, the sorting recycler is responsible for a portion of the remanufacturing process, while the manufacturer adopts the royalty payment model. The recovery rate is initially determined by both classified recyclers and traditional recyclers. Subsequently, retailers establish the retail price, and finally, the manufacturer determines the wholesale price of the product. Members of the closed-loop supply chain take profit maximization as the decision-making criterion, and the profits of manufacturers, retailers, classified recyclers and traditional recyclers are:

$$\Pi_m^{\overline{Sh}}(w) = (w - c_m)q + (c_m - c_r - b + v)\tau_2 q + (c_m - \mu b - h)\tau_1 q, \tag{17}$$

$$\Pi_r^{\overline{Sh}}(p) = (p - w)q, \tag{18}$$

$$\Pi_{T1}^{\overline{Sh}}(\tau_1) = (\mu b - c_r + v - A + \delta\theta + s + h)\tau_1 q - \frac{g\theta^2}{2} - \frac{k(\tau_1^2 + \alpha\tau_2^2)}{1 - \alpha^2}, \tag{19}$$

$$\Pi_{T2}^{R}(\tau_2) = (b - A)\tau_2 q - \frac{k(\tau_2^2 + \alpha\tau_1^2)}{1 - \alpha^2}. \tag{20}$$

Theorem 5. *Under the model $\overline{Sh}$, when $k > \frac{\Delta_{25}\beta(\alpha^2-1)}{4}$, $b\mu + \delta\theta + h + s + v - c_r > 0$ and $(-b\mu - \delta\theta - h + c_r - s - v)^2\alpha - b^2 < 0$, the optimal strategy and the maximum profit are respectively*

$$w^{\overline{Sh}*} = \frac{(\Delta_{30}(\alpha^2 - 1)a + 2c_m k)\beta + 2ak}{\beta(\Delta_{30}(\alpha^2 - 1)\beta + 4k)}, \quad p^{\overline{Sh}*} = \frac{(\Delta_{30}(\alpha^2 - 1)a + 2c_m k)\beta + 3ak}{\beta(\Delta_{30}(\alpha^2 - 1)\beta + 4k)},$$

$$\tau_1^{\overline{Sh}*} = \frac{(b\mu + \delta\theta + h + s + v - c_r)\Delta_2(\alpha^2 - 1)}{(-2\alpha^2 + 2)\beta\Delta_{30} + 8k}, \quad \tau_2^{\overline{Sh}*} = \frac{b\Delta_2(1 - \alpha^2)}{(-2\alpha^2 + 2)\beta\Delta_{30} + 8k}, \quad \tau_2^{\overline{Sh}*} = \frac{b\Delta_2(1 - \alpha^2)}{(-2\alpha^2 + 2)\beta\Delta_{30} + 8k},$$

$$q^{\overline{Sh}*} = \frac{k\Delta_2}{(-\alpha^2 + 1)\beta\Delta_{30} + 4k}, \quad \Pi_m^{\overline{Sh}*} = \frac{\Delta_2^2 k}{2\beta(\Delta_{30}(\alpha^2 - 1)\beta + 4k)}, \quad \Pi_r^{\overline{Sh}*} = \frac{\Delta_2^2 k^2}{((-\alpha^2 + 1)\Delta_{25}\beta + 4k)^2\beta},$$

$$\Pi_{T1}^{\overline{Sh}*} = \frac{-32gk^2\theta^2 + (\alpha^2 - 1)((\alpha b^2 - (\mu b + \theta\delta + h + s + v - c_r)^2)a^2 - 2\Delta_{37}\beta)k}{4(\Delta_{30}(\alpha^2 - 1)\beta + 4k)^2},$$

$$\Pi_{T2}^{\overline{Sh}*} = \frac{(\alpha^2 - 1)\Delta_2^2 k(\alpha(b\mu + \delta\theta + h + s + v - c_r)^2 - b^2)}{4(\Delta_{30}(\alpha^2 - 1)\beta + 4k)^2}.$$

Since the decision order and solution method of member enterprises in the closed-loop supply chain under this section are similar to the proof process in Section 4.1, the detailed solution process is omitted here (Fig. 2).

$$\Delta_{30} = ((\mu + 1)c_m + (-\delta\theta - 2h - s - v + c_r)\mu - c_r + v)b + (c_m - h)(\delta\theta + h + s + v - c_r),$$

$$\Delta_{31} = (\mu^2 + 1)b^2 + ((-c_r + 2h + s + v)\mu + c_r - v)b + (c_r - h - s - v)(c_m - h),$$

$$\Delta_{32} = (\mu^2 - \alpha)b^2 - 2\mu(c_r - h - s - v)b + (c_r - h - s - v)^2,$$

$$\Delta_{33} = -2(\alpha + 1)^3(\delta(\mu b + h)\theta + \Delta_{31})g\delta\theta^3 c_m + (\alpha^2 - 1)(\delta(\mu b + h)\theta + \Delta_{32})^2 g\theta^2,$$

$$\Delta_{34} = 1/2(2(g(\alpha^2 - 1)\theta^2 + k/2)\theta^2\delta^2 + 2k\delta(\mu b + h + s + v - c_r)\theta + \Delta_{32}k)c_m^2 - \Delta_{33},$$

$$\Delta_{35} = \delta(\mu b + h)\theta + (\mu^2 + 1)b^2 + ((-c_r + 2h + s + v)\mu + c_r - v)b - h(c_r - h - s - v),$$

$$\Delta_{36} = -\theta^2\delta^2 + 2\delta(-\mu b - h - s - v + c_r)\theta + 2\mu(c_r - h - s - v)b - (c_r - h - s - v)^2,$$

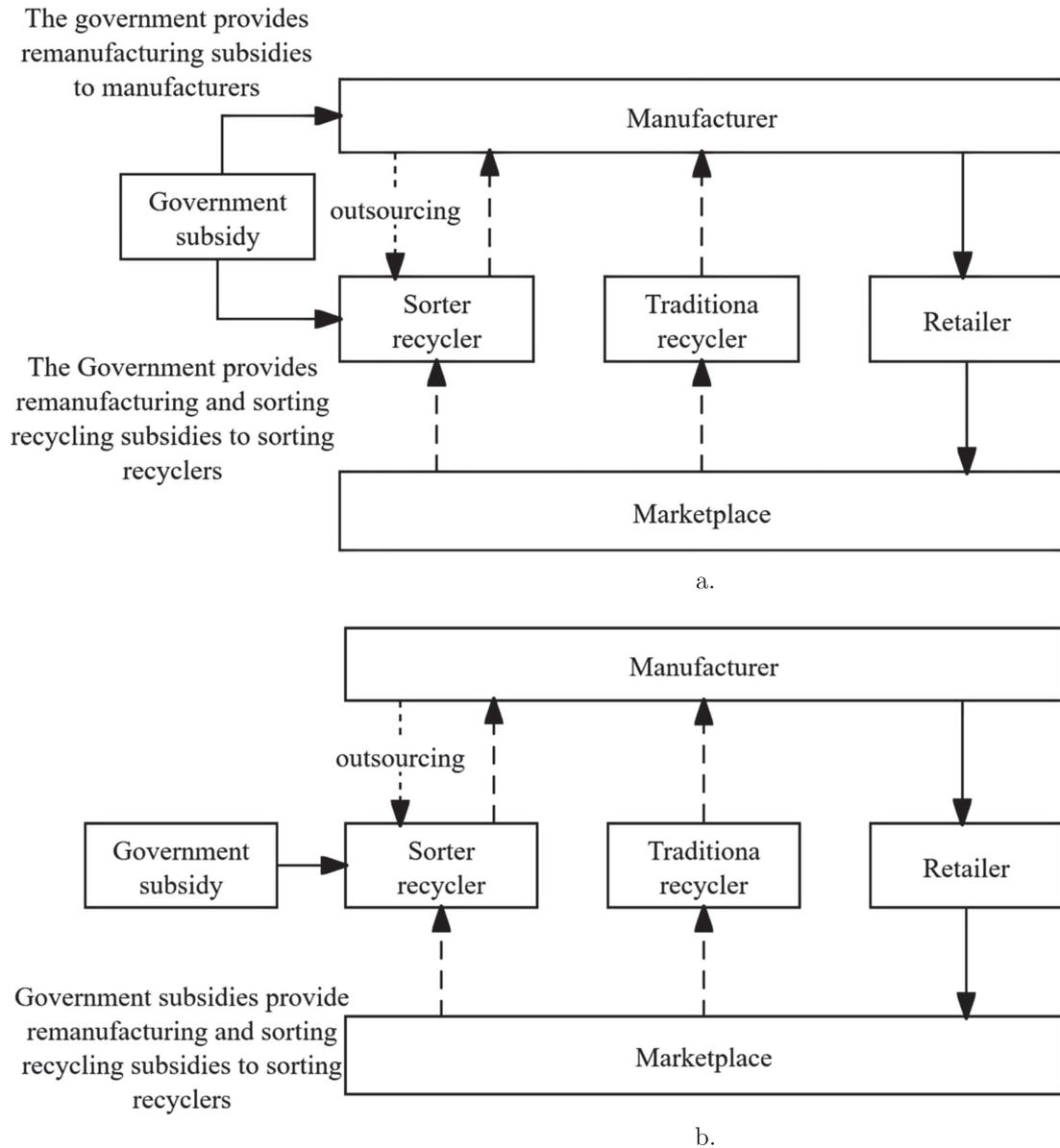$$\Delta_{37} = (\Delta_{36}a + 8g(\theta\delta + (\mu + 1)b - c_r + h + s + v)\theta^2)c_m - 8g\Delta_{35}\theta^2.$$

The government provides
remanufacturing subsidies
to manufacturers

Government
subsidy

Manufacturer

outsourcing

Sorter
recycler

Traditiona
recycler

Retailer

The Government provides
remanufacturing and sorting
recycling subsidies to sorting
recyclers

Marketplace

a.

Manufacturer

outsourcing

Government
subsidy

Sorter
recycler

Traditiona
recycler

Retailer

Government subsidies provide
remanufacturing and sorting
recycling subsidies to sorting
recyclers

Marketplace

b.

FIGURE 2. Remanufacturing processes under government subsidies. (a) Partial remanufacturing
under government subsidies. (b) Partial remanufacturing under government subsidies.

Result 5. By comparing the optimal decision-making level and profits of closed-loop supply chain members in
the payment mode of partial remanufacturing units before and after government subsidies, as shown in Table 10.

Since the proof procedure of Result 5 is similar to that of Result 1, the detailed proof procedure is omitted
here.

According to Result 5, under the partial remanufacturing unit payment model, affected by the subsidy policy,
the classification recovery rate after subsidies continues to be higher than the level without subsidies, but the
profits of traditional recyclers show a downward trend. When the subsidy ratio exceeds the specific threshold,

TABLE 10. Changes in the equilibrium results before and after government subsidies.

	Parameter range	w^*	p^*	τ_1^*	τ_2^*	q^*	Π_m^*	Π_r^*	Π_{T2}^*
$Sh - \overline{Sh}$	$\dfrac{v}{v+s} > \dfrac{\mu b + h - c_m}{b}$	+	+	−	−	−	−	−	+
	$\dfrac{v}{v+s} < \dfrac{\mu b + h - c_m}{b}$	−	−	−	+	+	+	+	+

the government subsidy policy consistently contributes to reducing the wholesale and retail prices, increasing the traditional recovery rate and market demand, and yielding higher profits for manufacturers and retailers compared to the period before the subsidy. When the subsidy ratio falls below a certain threshold, government subsidies will decrease the profits of manufacturers and retailers. To safeguard their interests, manufacturers will adopt the strategy of raising wholesale prices, while retailers will pursue increasing retail prices to maintain their profits, thereby diminishing market demand for the products.

6.2. The decision model for partially outsourcing remanufacturing with fixed payment under government subsidy

In the model $\overline{SH}$, the sorting recycler is responsible for a portion of the remanufacturing, while the manufacturer adopts the fixed-fee outsourcing model. The decision-making structure mirrors that of the model $\overline{SH}$. Members of the closed-loop supply chain take profit maximization as the decision-making criterion, and the profits of manufacturers, retailers, classified recyclers and traditional recyclers are:

$$\Pi_m^{\overline{SH}}(w) = (w - c_m)q + (c_m - c_r - b + v)\tau_2 q + (c_m - \mu b)\tau_1 q - H, \tag{21}$$

$$\Pi_r^{\overline{SH}}(p) = (p - w)q, \tag{22}$$

$$\Pi_{T1}^{\overline{SH}}(\tau_1) = (\mu b - c_r + v - A + \delta\theta + s)\tau_1 q - \frac{g\theta^2}{2} - \frac{k(\tau_1^2 + \alpha\tau_2^2)}{1 - \alpha^2} + H, \tag{23}$$

$$\Pi_{T2}^{\overline{SH}}(\tau_2) = (b - A)\tau_2 q - \frac{k(\tau_2^2 + \alpha\tau_1^2)}{1 - \alpha^2}. \tag{24}$$

Theorem 6. *Under the model* $\overline{SH}$, *when* $k > \frac{\Delta_{28}\beta}{4}$, $b\mu + \delta\theta + s + v - c_r > 0$, *and* $(-b\mu - \delta\theta + c_r - s - v)^2\alpha - b^2 < 0$, *the optimal strategy and the maximum profit are respectively*

$$w^{\overline{SH}^*} = \frac{2(\beta c_m + a)k - a\beta\Delta_{38}}{\beta^2\Delta_{38} + 4\beta k}, \quad p^{\overline{SH}^*} = \frac{(\beta c_m + 3a)k + a\beta\Delta_{38}}{\beta^2\Delta_{38} + 4\beta k},$$

$$\tau_1^{\overline{SH}^*} = \frac{(b\mu + \delta\theta + s + v - c_r)\Delta_2(1 - \alpha^2)}{2\Delta_{38}\beta + 8k}, \quad \tau_2^{\overline{SH}^*} = \frac{b\Delta_2(1 - \alpha^2)}{2\Delta_{38}\beta + 8k}, \quad q^{\overline{SH}^*} = \frac{k\Delta_2}{\Delta_{38}\beta + 4k},$$

$$\Pi_m^{\overline{SH}^*} = \frac{(c_m^2 k - 2\Delta_{38}H)\beta^2 - 8k(1/4 a c_m + H)\beta + a^2 k}{2\beta^2\Delta_{38} + 8\beta k}, \quad \Pi_r^{\overline{SH}^*} = \frac{k^2\Delta_2^2}{\beta(\Delta_{38}\beta + 4k)^2},$$

$$\Pi_{T1}^{\overline{SH}^*} = 1/4\frac{\Delta_{43} + (\alpha^2 - 1)(-\Delta_{40}c_m^2\beta^2 + \Delta_{43})k - 4(\alpha^2 - 1)\Delta_{45}\beta^2}{(\Delta_{38}\beta + 4k)^2},$$

$$\Pi_{T2}^{\overline{SH}^*} = 1/4\frac{(\alpha^2 - 1)k(\alpha(b\mu + \delta\theta + s + v - c_r)^2 - b^2)\Delta_2^2}{(\Delta_{38}\beta + 4k)^2}.$$

Since the decision order and solution method of member enterprises in the closed-loop supply chain under this section are similar to the proof process in Section 4.1, the detailed solution process is omitted here.

$$\Delta_{38} = (\mu^2 - \alpha)b^2 + 2\mu(\delta\theta + s + v - c_r)b + (\delta\theta + s + v - c_r)^2,$$

TABLE 11. Changes in the equilibrium results before and after government subsidies.

	Parameter range	w^*	p^*	τ_1^*	τ_2^*	q^*	Π_m^*	Π_r^*	Π_{T2}^*
$Sh - \overline{Sh}$	$\dfrac{v}{v+s} > \dfrac{\mu b - c_m}{b}$	+	+	−	−	−	−	−	+
	$\dfrac{v}{v+s} < \dfrac{\mu b - c_m}{b}$	−	−	−	+	+	+	+	+

$$\Delta_{39} = \delta\theta + s + v - c_r,$$
$$\Delta_{40} = \Delta_{39}\mu - v + c_r,$$
$$\Delta_{41} = \left(-32g\theta^2 + 64H\right)k^2 - 2bk\mu\Delta_{37}\Delta_{39},$$
$$\Delta_{42} = 2\big((\Delta_{38}a - 16\Delta_{37}((-\mu-1)b + \Delta_{39}))c_m - 16\Delta_{37}b\big((\mu^2+1)b + \Delta_{40}\big)\big)\beta,$$
$$\Delta_{43} = \left(-\Delta_{37}(\mu+1)^2(\alpha^2-1)c_m{}^2 + 4(\alpha^2-1)\big(\Delta_{39}\mu^2 + 1/2(\mu+1)(\delta\theta+s)\big)\Delta_{37}c_m\right)b^2,$$
$$\Delta_{44} = 2\Delta_{39}c_m\big(\big(((-\mu-1)\alpha^2+\mu)c_m + \Delta_{31}(\alpha^2-1)\big)\Delta_{37} + \Delta_{40}c_m\big)b + \Delta_{39}{}^2c_m{}^2\big(-\alpha^2+1\big)\Delta_{37},$$
$$\Delta_{45} = -\Delta_{37}b^3(\mu^2+1)(\alpha^2-1)\big((-2\mu-2)c_m + b^4(\mu^2+1) + 2\Delta_{40}\big) + \Delta_{43} + \Delta_{44},$$
$$\Delta_{46} = ((\mu+1)b + \delta\theta - c_r + h + s + v)(b\mu + h - c_m)(\alpha^2-1)\beta,$$
$$\Delta_{47} = (\alpha^2-1)\left(\big(\alpha b^2 - (\mu b + \theta\delta + h + s + v - c_r)^2\big)a^2 - 2\Delta_{37}\beta\right)k - 2\alpha^2\beta^2\Delta_{34} + 2\Delta_{34}\beta^2.$$

Result 6. The optimal decision-making level and closed-loop supply chain member profits under the fixed payment mode of partial remanufacturing before and after government subsidies were compared, as shown in Table 11.

Since the proof procedure of Result 6 is similar to that of Result 1, the detailed proof procedure is omitted here.

According to Result 6, under the fixed payment model of partial remanufacturing, the classification recovery rate after subsidies is higher than that without subsidies, and the profits of traditional recyclers show a downward trend. This is due to the fact that government subsidies effectively incentivize manufacturers and recyclers to actively engage in classified recycling and remanufacturing, while the profits of traditional recyclers have declined due to the impact of classified recyclers. When the ratio of remanufacturing subsidies to double subsidies exceeds a specific threshold, the increase in subsidies will lead to a decrease in wholesale and retail prices. Additionally, traditional recovery rates and market demand will increase, resulting in higher profits for manufacturers and retailers compared to the period before subsidies. The opposite holds true when the ratio of remanufacturing subsidies to double subsidies falls below a specific threshold. Despite the substantial benefits of generous subsidies to manufacturers, the reliance on external assistance for decision-making renders manufacturers seemingly hesitant, leading them, as industry leaders, to avoid such decisions.

6.3. The decision model for full outsourcing remanufacturing with unit payment under government subsidy

In the model $\overline{Ah}$, the sorting recycler is responsible for a portion of the remanufacturing, while the manufacturer adopts the fixed-fee outsourcing model. The decision-making structure mirrors that of the model $\overline{Sh}$. Members of the closed-loop supply chain take profit maximization as the decision-making criterion, and the profits of manufacturers, retailers, classified recyclers and traditional recyclers are:

$$\Pi_m^{\overline{Ah}}(w) = (w - c_m)q + (c_m - h - \mu b)(\tau_1 + \tau_2)q \tag{25}$$
$$\Pi_r^{\overline{Ah}}(p) = (p - w)q \tag{26}$$

TABLE 12. Changes in the equilibrium results before and after government subsidies.

	Parameter range	w^*	p^*	τ_1^*	τ_2^*	q^*	Π_m^*	Π_r^*	Π_{T2}^*
$Ah - \overline{Ah}$	$0 < h < c_m - \mu b$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$
	$h > c_m - \mu b$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$

$$\Pi_{T1}^{\overline{Ah}}(\tau_1) = (\mu b - c_r + v + \delta\theta + s + h)(\tau_1 + \tau_2)q - A\tau_1 q - b\tau_2 q - \frac{g\theta^2}{2} - \frac{k(\tau_1^2 + \alpha\tau_2^2)}{1 - \alpha^2} \tag{27}$$

$$\Pi_{T2}^{\overline{Ah}}(\tau_2) = (b - A)\tau_2 q - \frac{k(\tau_2^2 + \alpha\tau_1^2)}{1 - \alpha^2}. \tag{28}$$

Theorem 7. *Under the model $\overline{Ah}$, when $k > \frac{\Delta_{38}\beta}{4}$, $b\mu + \delta\theta + h - c_r + s + v > 0$ and $(-b\mu - \delta\theta - h + c_r - s - v)^2\alpha - b^2 < 0$, the optimal strategy and the maximum profit are respectively*

$$w^{\overline{Ah}^*} = \frac{2(\beta c_m + a)k + a\beta\Delta_{48}}{\beta(\Delta_{48}\beta + 4k)}, \quad p^{\overline{Ah}^*} = \frac{(\beta c_m + 3a)k + a\beta\Delta_{48}}{\beta(\Delta_{48}\beta + 4k)},$$

$$p^{\overline{Ah}^*} = \frac{(\beta c_m + 3a)k + a\beta\Delta_{48}}{\beta(\Delta_{48}\beta + 4k)}, \quad \tau_2^{\overline{Ah}^*} = \frac{\Delta_2 b(1 - \alpha^2)}{2\Delta_{48}\beta + 8k},$$

$$q^{\overline{Ah}^*} = \frac{k\Delta_2}{\Delta_{48}\beta + 4k}, \Pi_m^{\overline{Ah}^*} = \frac{\Delta_2{}^2 k}{2\beta(\Delta_{48}\beta + 4k)}, \quad \Pi_r^{\overline{Ah}^*} = \frac{k^2\Delta_2{}^2}{\beta(\Delta_{48}\beta + 4k)^2},$$

$$\Pi_{T1}^{\overline{Ah}^*} = \frac{(\Delta_{52} + \Delta_{51})k - 2\Delta_{49}{}^2 g\theta^2(\alpha^2 - 1)(-\delta\theta + \Delta_{50})^2}{4(\Delta_{48}\beta + 4k)^2},$$

$$\Pi_{T2}^{\overline{Ah}^*} = \frac{(\alpha^2 - 1)\Delta_2{}^2 k\left(\alpha(b\mu + \delta\theta + h + s + v - c_r)^2 - b^2\right)}{4(\Delta_{48}\beta + 4k)^2}.$$

Since the decision order and solution method of member enterprises in the closed-loop supply chain under this section are similar to the proof process in Section 4.1, the detailed solution process is omitted here.

$$\Delta_{48} = -(b\mu + h - c_m)(b\mu + \delta\theta + b + h + s + v - c_r)(\alpha^2 - 1),$$
$$\Delta_{49} = -b\mu - h + c_m,$$
$$\Delta_{50} = (-\mu - 1)b + c_r - h - s - v,$$
$$\Delta_{51} = (-c_m{}^2 + 1)\theta^2\delta^2 - 2\Delta_{50}\delta\theta + \Delta_{50}{}^2 - (\alpha + 3)b^2,$$
$$\Delta_{52} = \left(\Delta_{50}\left(b^2\beta(\alpha + 3)c_m^2 + \left((\delta\theta - \Delta_{50})^2 - (\alpha + 3)b^2\right)c_m + 26\Delta_{49}g\theta^2\right)\right)^2(\alpha^2 - 1)\beta.$$

Result 7. By comparing the optimal decision-making level and closed-loop supply chain member profits under the payment mode of all remanufacturing units before and after government subsidies, as shown in Table 12.

Since the proof procedure of Result 7 is similar to that of Result 1, the detailed proof procedure is omitted here.

As evident from Result 7, when the manufacturer outsources all remanufacturing to sorting recyclers, the government grants full subsidies to these sorting recyclers. Consequently, the sorting recovery rate experiences a substantial increase, intensifying competition among sorting recyclers. This heightened competition adversely affects the profits of traditional recyclers. In the unit outsourcing payment model, if the unit cost is lower than the difference between the new product cost and the recovery commission cost, the introduction of government subsidies can lead to a reduction in wholesale and retail prices. Additionally, it can contribute to an increase

in traditional recovery rates and market demand, ultimately enhancing the profits of both manufacturers and retailers. Indeed, when the unit cost surpasses the difference between the new product cost and the recovery commission cost, the infusion of government subsidies results in an increase in wholesale and retail prices. Moreover, this scenario leads to a decrease in traditional recovery rates, market demand, and the profits of both manufacturers and retailers. Consequently, manufacturers are inclined to adopt the unit pay outsourcing full remanufacturing model only when the unit cost remains relatively low.

6.4. The secision model for full outsourcing remanufacturing with fixed payment under government subsidy

In the model $\overline{AH}$, the sorting recycler is responsible for a portion of the remanufacturing, while the manufacturer adopts the fixed-fee outsourcing model. The decision-making structure mirrors that of the model $\overline{Sh}$. Members of the closed-loop supply chain take profit maximization as the decision-making criterion, and the profits of manufacturers, retailers, classified recyclers and traditional recyclers are:

$$\Pi_m^{\overline{AH}}(w) = (w - c_m)q + (c_m - \mu b)(\tau_1 + \tau_2)q - H \tag{29}$$

$$\Pi_r^{\overline{AH}}(p) = (p - w)q \tag{30}$$

$$\Pi_{T1}^{\overline{AH}}(\tau_1) = (\mu b - c_r + v + \delta\theta + s)(\tau_1 + \tau_2)q - A\tau_1 q - b\tau_2 q - \frac{g\theta^2}{2} - \frac{k(\tau_1^2 + \alpha\tau_2^2)}{1 - \alpha^2} + H \tag{31}$$

$$\Pi_{T2}^{\overline{AH}}(\tau_2) = (b - A)\tau_2 q - \frac{k(\tau_2^2 + \alpha\tau_1^2)}{1 - \alpha^2}. \tag{32}$$

Theorem 8. *Under the model $\overline{AH}$, when $k > \frac{\Delta_{43}\beta}{4}$, $b\mu + \delta\theta - c_r + s + v > 0$ and $(-b\mu - \delta\theta + c_r - s - v)^2\alpha - b^2 < 0$, the optimal strategy and the maximum profit are respectively*

$$w^{\overline{AH}*} = \frac{(2\beta c_m + 2a)k + a\beta\Delta_{53}}{(\beta\Delta_{53} + 4k)\beta}, \quad p^{\overline{AH}*} = \frac{(\beta c_m + 3a)k + a\beta\Delta_{53}}{(\beta\Delta_{53} + 4k)\beta}, \quad \tau_1^{\overline{AH}*} = \frac{\Delta_2(1 - \alpha^2)(b\mu + \delta\theta + s + v - c_r)}{2\beta\Delta_{53} + 8k},$$

$$\tau_2^{\overline{AH}*} = \frac{b\Delta_2(1 - \alpha^2)}{2\beta\Delta_{53} + 8k}, \quad \tau_2^{\overline{AH}*} = \frac{b\Delta_2(1 - \alpha^2)}{2\beta\Delta_{53} + 8k}, \quad \Pi_m^{\overline{AH}*} = \frac{((-\beta c_m + a)^2 - 8\beta H)k - 2H\beta^2\Delta_{53}}{2(\beta\Delta_{53} + 4k)\beta},$$

$$\Pi_r^{\overline{AH}*} = \frac{k^2\Delta_2^2}{(\beta\Delta_{53} + 4k)^2\beta}, \quad \Pi_{T1}^{\overline{AH}*} = \frac{64k^2\Delta_{49} + \Delta_{58}}{4(\beta\Delta_{53} + 4k)^2}, \quad \Pi_{T2}^{\overline{AH}*} = \frac{(\alpha^2 - 1)(\alpha(b\mu + \delta\theta + s + v - c_r)^2 - b^2)k\Delta_2^2}{4(-\beta\Delta_{53} + 4k)^2}.$$

Since the decision order and solution method of member enterprises in the closed-loop supply chain under this section are similar to the proof process in Section 4.1, the detailed solution process is omitted here.

$$\Delta_{53} = -(b\mu - c_m)(b\mu + \delta\theta + b + s + v - c_r)(\alpha^2 - 1),$$

$$\Delta_{54} = (4(\alpha^2 + 1)\Delta_{49} - k(\mu^2 - \alpha + 2\mu - 2))c_m^2 + \Delta_{51}\Delta_{49}\mu(\alpha^2 - 1)(\mu\Delta_{51} - 4\mu c_m - 4c_m),$$

$$\Delta_{55} = \Delta_{54}b^2 - c_m(((-4\alpha^2 - 4)\Delta_{49} + k)(\Delta_{49}^2 + 2b\Delta_{51}(\mu + 1))c_m + 8\Delta_{51}^2 b\Delta_{49}\mu(\alpha^2 - 1)),$$

$$\Delta_{56} = (2a(\mu^2 - \alpha + 2\mu - 2)b^2 + 4(\mu + 1)(\Delta_{51}a + 8\Delta_{49})b + 2\Delta_{51}^2 a + 32\Delta_{51}\Delta_{49})c_m,$$

$$\Delta_{57} = ((\alpha^2 - 1)(\Delta_{46} - 32b((\mu + 1)b + \Delta_{51})\Delta_{49}\mu)\beta - (\alpha^2 - 1)(\Delta_{50} + 2b\Delta_{51})a^2)k,$$

$$\Delta_{58} = \Delta_{57} + (\alpha^2 - 1)(4b^3(\alpha^2 - 1)(\mu^2 b + (b - 2\Delta_{51})\mu - 2c_m)\mu\Delta_{49}(\mu + 1) + \Delta_{55})\beta^2.$$

Result 8. By comparing the optimal decision-making level and closed-loop supply chain member profits under the fixed payment mode of total remanufacturing before and after government subsidies, as shown in Table 13.

Since the proof procedure of Result 8 is similar to that of Result 1, the detailed proof procedure is omitted here.

Result 8 suggests that in the total remanufacturing outsourcing mode, the government extends remanufacturing and sorting recycling subsidies to sorting recyclers. This subsidy structure serves to enhance the sorting

TABLE 13. Changes in the equilibrium results before and after government subsidies.

	w^*	p^*	τ_1^*	τ_2^*	q^*	Π_m^*	Π_r^*	Π_{T2}^*
$AH - \overline{AH}$	+	+	−	−	−	−	−	+

recovery rate within the system. Indeed, when manufacturers opt for a flat fee payment model, the government's sorted recycling subsidy can effectively boost recycling rates. Simultaneously, this subsidy mechanism tends to lower wholesale and retail prices, thereby contributing to increased profits for both manufacturers and retailers. The primary objective of government subsidies in this context is to incentivize classified recycling and remanufacturing practices while ensuring market stability. In comparison, the fixed-fee outsourcing full remanufacturing model, supported by government subsidies, is particularly advantageous in attaining these objectives.

7. NUMERICAL SIMULATION

This section will further elucidate the management and practical significance of the model through numerical simulation. The paper analyzes the effects of the degree of recycling competition, the profit coefficient of classified recycling, and the price coefficient of recycling on the equilibrium outcome of the partially closed-loop supply chain and the profits of its members. Parameter Settings should meet the assumptions in the theoretical model, refer to Huang *et al.* [19] and He *et al.* [14]. Assume that the cost paid by the recycler to obtain the unit of waste product from the consumer $A = 0$, and $a = 200$, $k = 600$, $c_m = 25$, $c_r = 25$, $b = 5$, $g = 10$, $\beta = 4$. When sensitivity analysis is performed on the degree of recycling competition, the order $\delta = 2.5$, $\mu = 1.2$. When the sensitivity analysis of the classification return coefficient is carried out, the order $\alpha = 0.5$, $\mu = 1.2$. When the sensitivity analysis of the recovery price coefficient is performed, the order $\alpha = 0.5$, $\delta = 2.5$.

7.1. The impact of the degree of recycling competition on partial equilibrium results

As depicted in Figure 3, in scenario Ah, the initial market demand, along with the profits of traditional recyclers and manufacturers, are higher initially but exhibit a more rapid decline compared to other scenarios. In contrast to the other scenarios, only in case SH does the market demand and manufacturer's profit show an increase with the rise in recycling competition. For case AH, it's noteworthy that the profit of classified recyclers is the highest, yet the rate of decline is also the most pronounced. In case Ah, the profit even diminishes to zero as the degree of recycling competition intensifies. This is because the traditional and classified recyclers, as the main participants in the recycling competition, will obviously be more affected. The profits of the two recyclers will decrease with the increase in recycling competition, and the reduction rates are different. It can be seen that intense recycling competition is not conducive to the performance of a closed-loop supply chain. If it is unavoidable, manufacturers can choose scenario Ah to ensure their own profits.

7.2. The impact of the classification recovery coefficient on partial equilibrium results

It can be seen from Figure 4 that under the four outsourcing remanufacturing modes, the profit coefficient of classified recycling is inversely proportional to that of traditional recyclers. This is mainly because, with the improvement of the income coefficient of classification and recycling, classification and recycling will enhance the classification recovery rate through efficient recycling, thus weakening the traditional recovery rate. The greater the benefits of classified recycling to manufacturers and classified recyclers, the smaller the benefits to traditional recyclers. In addition, in the SH and AH models, the manufacturer's profit increases with the increase of the classification recovery profit coefficient, while in other cases, it is inversely proportional because the manufacturer pays a larger recovery price. In case Sh, the manufacturer's profit is the best in the initial case, but with the increase of the return coefficient of classification and recycling, case AH has more room for an increase. This indicates that the profit coefficient of classification recovery has a significant impact on
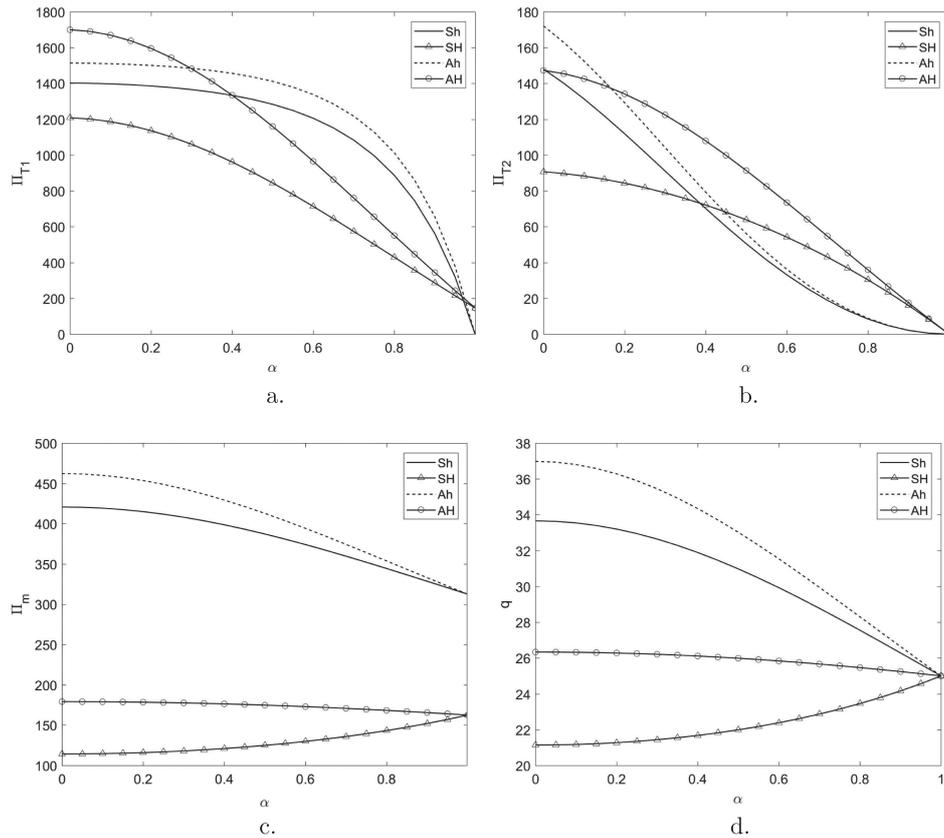
FIGURE 3. The impact of the degree of recycling competition on partial equilibrium results. (a) The profit of the classified recycler. (b) The profit of the traditional recycler. (c) The profit of the manufacturer. (d) Market demand.

manufacturers' profits and may cause different changes in the rate of increase under different circumstances. Overall, the profit coefficient of classified recycling has a more significant impact on recyclers' profits, indirectly affecting manufacturers' profits and market demand.

7.3. The impact of the recovery price coefficient on partial equilibrium result

As can be seen from Figure 5, the profit of sorting recyclers is proportional to the recycling price coefficient. In the face of an increased recovery price factor, classified recyclers are able to adjust their recycling strategies more flexibly, thereby achieving higher profits. Through comprehensive analysis, the recovery price coefficient is directly proportional to market demand and the income brought by the coefficient in SH and AH modes, and inversely proportional to the wholesale price. The main reason is that when the recovery price factor increases, manufacturers will stimulate market demand by reducing the wholesale price and increase their own profits. The above analysis can verify the relevant conclusions.

7.4. The profit changes for some members before and after government subsidies

As can be seen from Figure 6, the amount of government subsidy is always positively correlated with the profit increment of sorting recyclers. Furthermore, compared with other paid outsourcing models, the increment of sorting recyclers in the fixed-paid outsourcing model is the most significant. The subsidy policy has obviously
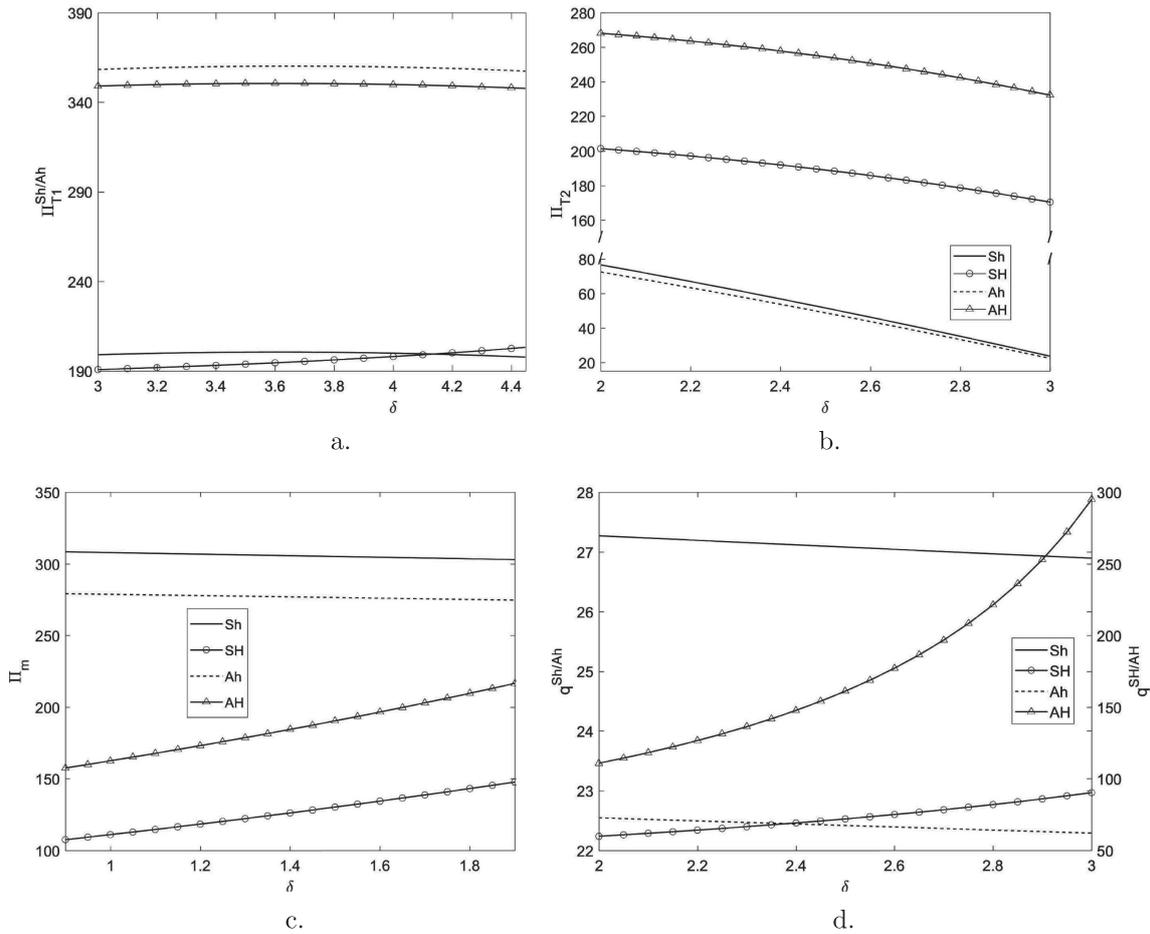
FIGURE 4. The impact of the classification recovery coefficient on partial equilibrium results. (a) The profit of the classified recycler. (b) The profit of the traditional recycler. (c) The profit of the manufacturer. (d) Market demand.

effectively promoted recycling and remanufacturing, especially among the sorting recyclers who are the main undertakers of this process. Their own profits have also increased significantly. For manufacturers, there is also a positive correlation between profits and government subsidies. Although the manufacturer may not engage in large-scale remanufacturing under the two outsourcing remanufacturing models, and may not even undertake remanufacturing production. But as the decision-makers of the closed-loop supply chain and beneficiaries of sorting, recycling, and remanufacturing, manufacturers have significantly increased their own profits by making sound outsourced remanufacturing decisions and partnering with sorting recyclers. As a result, government subsidies are profitable for both manufacturers and sorting recyclers, incentivizing them to accept government subsidies to promote more sorting, recycling, and remanufacturing activities.

8. Conclusion

Based on the recycling market competition, four closed-loop supply chain decision-making models-partial remanufacturing-unit pay, partial remanufacturing-fixed pay, full remanufacturing-unit pay, and full remanufacturing-fixed pay-are constructed, considering the technology outsourcing strategy and remanufactur-
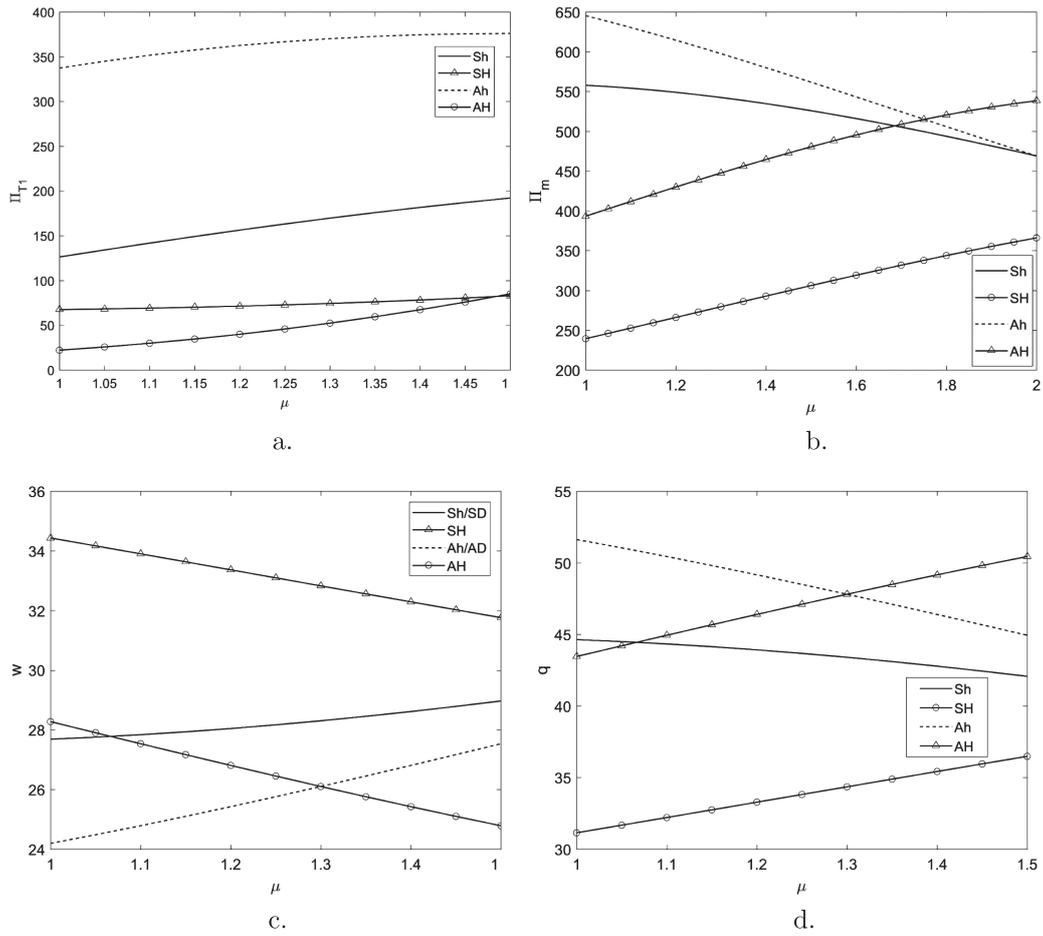
FIGURE 5. The impact of the recovery price coefficient on partial equilibrium results. (a) The profit of the classified recycler. (b) The profit of the manufacturer. (c) Wholesale price. (d) Market demand.

FIGURE 6. The impact of the recovery price coefficient on partial equilibrium results. (a) The profit of the classified recycler. (b) The profit of the manufacturer.

ing strategy of the manufacturer to the classified recyclers. The effects of competition degree of recycling, income coefficient of sorting recycling, and price coefficient of recycling on decision-making and profit of closed-loop supply chain members are analyzed. Further, under the dual subsidy strategy of "classified recycling + reman-ufacturing", four closed-loop supply chain decision-making models considering different technology outsourcing and remanufacturing strategies are constructed. This paper analyzes the effects of government subsidies on system decision-making and profit and puts forward the optimal technology authorization outsourcing strategy and remanufacturing strategy of manufacturers, respectively, to obtain the optimal decision and maximum profit of closed-loop supply chain members.

The results show that: First, the increase in the recovery price coefficient is beneficial for improving the sorting recovery rate and the profit of sorting recyclers. However, it is detrimental to traditional recyclers' profits. When the recovery price is high, an increase in the recovery price coefficient leads to higher profits for manufacturers and retailers. Conversely, when the recovery price is low, the situation is reversed. Second, regardless of the outsourcing remanufacturing mode, an increase in the return coefficient of sorting and recycling benefits the improvement of the sorting recovery rate but results in a decline in the profit of traditional recyclers. When the recovery price is low, enhancing the profit coefficient of sorting recovery is conducive to increasing the profits of manufacturers, retailers, and sorting recyclers. When the recycling price is high, most outsourcing remanufac-turing models exhibit the opposite trend. However, the fixed-fee outsourcing model behaves similarly to what occurs when recycling prices are lower. Third, when the recycling price surpasses the threshold, intense recycling competition will result in a reduction of members' profits and have a negative impact on market demand. How-ever, when the recycling price is below the threshold, the changing trend of most outsourcing remanufacturing patterns is opposite to that observed when the recycling price is above the threshold. Nevertheless, the fixed payment outsourcing total remanufacturing model remains unaffected by the fluctuation of recovery prices and continues to exhibit the same change trend as observed when it is above the threshold value. Fourth, at high recovery costs, members tend to prefer the partial remanufacturing mode, while at low recovery costs, they tend to favor the total remanufacturing mode. Regarding the decision on the paid outsourcing model, manufac-turers tend to prefer the unit payment outsourcing model, while traditional recyclers tend to lean towards the fixed payment outsourcing model. Among them, retailers choose the unit payment outsourcing model under low recovery costs and opt for the fixed payment outsourcing model under high recovery costs. It can be observed that making the best decision on the outsourcing remanufacturing mode within a closed-loop supply chain is challenging. Fifth, in any decision-making model, increasing sorting and remanufacturing subsidies is always conducive to improving sorting recovery rates and sorting recyclers' profits, but not to traditional recyclers' profits. Under the unit pay outsourcing model, when the recovery price is low, the increase in the subsidy level always enhances member profits and expands market demand. The opposite is true when recovery prices are high. Under the fixed-fee outsourcing model, the increase in double subsidies is always directly proportional to the traditional recovery rate, market demand, and the profits of manufacturers and retailers, and inversely proportional to the wholesale and retail prices. Sixth, the decision-making situation under government subsidies is similar to that under non-government subsidies but is no longer limited by the recovery price. It is mainly influenced by subsidies for remanufacturing and sorting and recycling. Whether viewed from the perspective of enhancing the profits of each member, expanding market demand, or improving the recovery rate, the fixed payment outsourcing total remanufacturing model is always conducive to achieving the aforementioned goals.

For manufacturers, remanufacturing strategies should be decided in relation to different conditions, partially outsourced remanufacturing or fully outsourced remanufacturing, as well as outsourcing licensing strategies, unit-pay or fixed-pay. For the two types of recyclers, the sorted recyclers have more flexibility in their profit composition because they undertake the remanufacturing work, and at the same time they have positive spill-over effects on the upstream companies, which often give them more advantages in the recycling competition, and the traditional recyclers should positively transform themselves into the sorted recyclers. Under certain conditions, too much competition in the recycling market is not only unfavorable to the recycling of the reverse channel, but also raises the price of the positive channel and reduces the profits of enterprises, while the government's "double subsidy" policy can effectively eliminate the negative effects of recycling competition, so the government can help

the market to establish a healthier recycling system by formulating reasonable subsidy policies. Therefore, the government can help the market to establish a healthier recycling system by formulating a reasonable subsidy policy. The research work in this paper expands the scope of research in the field of closed-loop supply chain, and provides a mathematical model and analysis of the results to provide theoretical references and decision-making support for enterprises to build their own recycling system. However, there are some possible extensions to this work. Firstly, in this paper we assume that new and remanufactured products are sold at the same price, while future research could explore scenarios where these products are sold at different prices. Secondly, this paper assumes that the manufacturer is the Stackelberg leader, and in the future different power structures in the market may be considered, such as recycler-led or retailer-led. Finally, the study in this paper assumes that information in the closed-loop supply chain system, such as manufacturing/remanufacturing costs and recycling efficiency, is symmetric for all members, the impact of information asymmetry, which was not addressed in this paper, could be explored in future studies.

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Conflicts of interest

No potential conflict of interest was reported by the authors.

Data availability statement

The research data associated with this article are included in the article.

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