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INTEGRATED SUPPLY CHAIN IMPERFECT PRODUCTION INVENTORY MODEL USING GREEN PRODUCT WITH CARBON EMISSIONS

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Abstract. Carbon emissions play an important role in global warming. Currently, some countries are alert for reducing the carbon emissions. Basically. Many countries use carbon cap & tax policies to reduce emission. The acceptance of green products is on the rise in today's competitive market because of environmental concerns. Observance this issue in mind, we considered a production quantity model for a manageable carbon emissions rate. This study explores the integration of sustainable practices within supply chains, focusing on green supply chain management (GSCM), carbon emission reduction, and demand modeling dependent on the degree of environmental sustainability. The paper examines the impact of inspection and screening processes in mitigating environmental risks and improving the efficiency of green supply chains. In particular, we investigate how the carbon emission levels during the production and deterioration stages in the supply chain. Additionally, the demand for green products is modeled as a function of the "green degree" of the product, which signifies the extent of sustainability integrated into the manufacturing process. In this paper, we consider an imperfect production inventory model for decaying items with carbon emission. This study explores how to determine the green degree of finished products that were produced within imperfect production systems. To select the perfect and imperfect items have been inspected the produced items. The defective goods are sold in an only one consignment at the stop of the 100% screening process at a lower cost. The planned process is validated with the help of a numerical example. A solution process is provided to establish the most favorable ordering policy. Finally, Sensitivity analysis of different parameters has been approved.

Mathematics Subject Classification. 90B05, 90B06.

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1. Introduction

In this era of modernity, along with environmental protection, energy consumption is also one of the important challenges, ranging from the production of inventory in the production houses of companies, workers working using technology, and it has become necessary to be vigilant to keep the inventory safe. It is well known to the business that a variety of circumstances may arise during production, which also leads to the possibility of imperfect production, but how to use a product that is not completely perfect. There is a need to work on

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Keywords. Imperfect items, deterioration, shortage, carbon emission.

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this. In which it is necessary to give utmost importance to the protection of the environment. Padiyar *et al.* [19] presented a green supply chain inventory model for imperfect items under the reliability. The review paper given by Jain *et al.* [11], this review paper is based on green inventory model and reverse logistic. Zaidi *et al.* [33] proposed a two-level trade credit policy production model having greening dependent demand. As' ad *et al.* [5] presented a two echelon supply chain inventory model for vendor managed consignment policy. With the help of this model they tried to incorporate emission from production setup, ordering process and also from the holding of inventory. The imperfect items are broken down into separate components and the damaged parts are identified, and those parts are taken apart which need replacement. Then, after the process of inspection, defective parts are sorted out and the reusable parts are refurbished. After the remanufactured goods passes through all the quality checks, they are packaged and labelled, indicating their remanufactured status, any warranty or other information associated with remanufacturing, and their environmental benefits. After this, these remanufactured goods are sent back to the market for sale. Generally, these are seen as more sustainable and cost-effective alternatives to new products. From an environmental point of view, the remanufacturing process has many environmental benefits, with its help, along with reducing waste materials, it also helps in conserving resources and reducing greenhouse gases emissions. Bhunia *et al.* [8] presented an EPQ model having imperfect product and rework with advertisement and price dependent demand. Kuraie *et al.* [16] discussed an inventory model for deteriorating items with limited storage problems along with multivariable demand under the imperfect production process. Padiyar *et al.* [19] discussed a multi echelon supply chain inventory model for deteriorating items with imperfect production process under the effect of inflation.

In this paper, an attempt has been made to develop a green inventory system which will not only protect the environment but also plays a vital role in preserving the goods and services and enables it to meet the challenges of every environment. The research paper, titled "Integrated supply chain imperfect production inventory model using green product with carbon emissions", makes important contributions in several key areas. In this paper, we consider an imperfect production inventory model for decaying items with carbon emission. This study explores how to determine the green degree of finished products that were produced within imperfect production systems. To select the perfect and imperfect items have been inspected the produced items. The defective goods are sold in an only one consignment at the stop of the 100% screening process at a lower cost. This research presents a combination of elements of inventory management, financial transactions, environmental considerations, and the impact of various conditions. This approach is likely to increase the efficiency of order fulfilment, reduce the incidence of stockouts or excess inventory, and improve overall supply chain performance. Examination of environmental priorities further increases the relevance of the model by introducing sustainability considerations into the inventory management framework. The primary objective of this research is to integrate sustainable sourcing, storage practices into inventory models, contributing to environmentally. This inventory model demonstrates eco-consciousness by choosing suppliers with eco-friendly practices, optimizing transportation routes to reduce carbon footprint, and managing inventory in a manner that minimizes environmental impact. In summary, this research has the potential to make significant contributions to both the academic literature and practical applications considering inventory management as well as environmental sustainability, with the help of which a holistic approach is achieved for supply chain optimization even in adverse circumstances. The importance of this model is that it can prove to be a game-changer for environmentally conscious businesses with imperfect production.

In the following sections, we will analyse the existing literature in depth, present the research methodology, discuss the findings, and offer recommendations for implementing a green inventory system in an inflationary environment that preserves goods, services and consumption demands. Are able to complete.

1.1. Research gap

The following research gaps can be drawn based on the existing literature:

The rest part of the paper is organized as follows.

In Section 2 discusses the research gaps and novelty of this work along with a literature review of the research done so far. Section 4 presents the problem statement, notation, and assumptions used throughout the paper.

Authors	Demand rate	Carbon emission	Screening process	Deterioration	Imperfect production
Maiti *et al.* [17]	Price-dependent	No	No	No	No
Taleizadeh [29]	Constant	No	No	No	No
Prasad and Mukherjee [22]	Stock and time dependent	No	No	No	No
Zhang *et al.* [34]	Constant	No	No	No	No
Teng *et al.* [30]	Constant	No	No	No	No
Khan *et al.* [14]	Price dependent	No	No	No	No
Shaikh *et al.* (2019) [27]	Price dependent	No	No	No	No
Rahman *et al.* [23]	Interval valued	No	No	No	No
Das *et al.* [9]	Reliability of the product dependent	No	No	No	No
	Ramp type	No	No	Yes	No
Alamri *et al.* [3]	Constant	Yes	No	No	No
Jaiswal *et al.* [12]	Triangular	Yes	No	No	No
Wang *et al.* [32]	Market demand	Yes	No	Yes	No
Alsaedi *et al.* [4]	Tringular fuzzy	Yes	No	No	No
This paper	Price and green degree dependent	Yes	Yes	Yes	Yes

The mathematical formulation of the model is represented in Section 5. The solution methodology is presented in Section 6. A numerical example with sensitivity is in Sections 7 and 8. Result discussion and managerial insight are in Section 9. The conclusion and future scope of the proposed research work are in Section 10.

2. Literature review, research gap and novelty of this study

In this section, the research work done so far, its contribution, and research gaps in the literature have been discussed. Furthermore, the novel contributions of this study are outlined in subsections, which include achieving economic benefits by reducing the cost of maintaining inventory, understanding an environment-friendly approach and use of green technology which is very essential for the supply chain model.

2.1. Literature review based on imperfect production

Remanufacturing is the process of restoring used or imperfect items or services to a usable condition through the process of remanufacturing. The remanufacturing process generally depends on the type of products, their geographical location and industry. The first step in the remanufacturing process involves collecting defective or used items that have reached the end of their initial life cycle. From where they are separated based on their condition and ability to remanufacturing. Bazan *et al.* [7] proposed a two-echelon supply chain inventory model for vendor and buyer under an imperfect production process. In their model they considered that after the process of inspection, vendor make decision regarding to scrap off, salvage at a discount rate and rework. Kumar *et al.* [15] presented an inventory model and developed optimal policies for final aspects of a product or service under imperfect production process. With the help of this model they tried to developed a decision model for determining the pricing and warranty polices of a newly launched product. Shaikh *et al.* [28] presented an EPQ model having imperfect product and rework with advertisement and price dependent demand. Aazami and Saidi-Mehrabad [1] developed a production-distribution planning (PDP) model for perishable products with a fixed lifetime. Aazami *et al.* [2] proposed a a bi-objective optimization model for the integrated production-distribution planning of perishable goods. Ruidas *et al.* [25] developed an imperfect production inventory model under the various carbon emission regulatory policies where the various carbon emission parameters. Kuraie *et al.* [16] discussed an inventory model for deteriorating items with limited storage problems along with multivariable demand under the imperfect production process. Padiyar *et al.* [19] discussed a multi echelon supply chain inventory model for deteriorating items with imperfect production process under the effect of inflation. Padiyar *et al.* [21] presented a multi echelon supply chain inventory model for perishable items having exponential demand rate under imperfect production processes. Padiyar *et al.* [20] proposed a multi echelon supply chain inventory model for deteriorating items under imperfect production process having limited storage capacity.

2.2. Literature review based on green technology

Green supply chain management or sustainable supply chain management serves as a strategic approach to supply chain processes that integrate environmentally friendly practices. Its main function is to measure the environmental impact of the supply chain from sourcing raw materials, manufacturing the product and transporting the finished product to the end customer. One of the major goals of GSCM is to promote ethically and socially responsible organizations to reduce carbon footprint and conserve resources. So that, along with protecting the environment industries can also develop smoothly. As consumers and industries have turned their attention to environmental protection, GSCM has become an important aspect of modern supply chain management.

Jauhari *et al.* [13] designed a closed loop supply chain model by considering single producer, single retailer and single collector, where the demand is linearly dependent on green technology level along with carbon cap and trade policy. Ramandi and Bafruei [24] presented an integrated inventory model for supplier and retailer having stochastic demand patterns and government regulation for reducing the carbon emissions. Vandana *et al.* [31] developed an inventory model related to the impact of energy and carbon emissions, in which they also discussed trade-credit policy and inflation. Sepehri *et al.* [26], considered depletion, trade credit, pricing, and controllable carbon emissions in their inventory model. But their model did not consider rework, imperfect production, variable demand and outsourcing. Barman *et al.* [6] presented a two stage – inventory model for deteriorating items having variable production cost. They considered that in each stage of transportation and production cost carbon cost is charged. Gautam *et al.* [10] presented a green supply chain inventory model for a vendor and a buyer under an imperfect production process. With the help of this model they tried to explore the impact of carbon emissions through production, shipments, energy usage etc. Mishra *et al.* [18] examined the effect of carbon pricing and cap mechanism on a sustainable supply chain management. They focused on the products which are energy efficient in an energy supply chain management. The existing literature on green supply chains has extensively explored sustainability strategies but has often overlooked the specific impacts of imperfect production and the associated screening costs. For instance, Alsaedi *et al.* [4] examined carbon emissions in supply chains but did not account for quality assurance processes. Similarly, Gautam *et al.* [10] discussed green technology adoption but lacked a comprehensive view of its impact on supply chain dynamics. This paper addresses these gaps by introducing a novel model that integrates sustainability-driven demand rates, screening rates for quality control, and green technology development costs, thereby offering a more holistic approach to green supply chain management. Unlike prior studies, this paper introduces a comprehensive model that incorporates demand rates affected by sustainability preferences, screening rates for quality assurance, and green technology investment costs in a supply chain context.

Based on the important details mentioned above, the literature has been extensively analyzed through multidisciplinary research and found that no one has considered imperfect production and carbon emission with green product. Thus, considering this important context, this model aims to answer the questions

(1) What is the possible effect of imperfect production in the production house with remanufacturing process on supply chain model coordination, and how the deteriorating rate can change?
(2) The inventory which is imperfect after production will be reused in the production house for re-use. And in this situation environmental protection is a big challenge, so what can be done for this challenge?

3. Problem description

In this paper, considers a centralized system of a supply chain consisting of one retailer and one manufacturer dealing with a single product. We consider an imperfect production inventory model for decaying items with carbon emission. To select the perfect and imperfect items have been inspected the produced items. The defective goods are sold in an only one consignment at the stop of the 100% screening process at a lower cost. Furthermore, our analysis acknowledges that during production, the manufacturer generates some imperfect items that are re-workable. The retailer, however, only receives flawlessly finished products from the manufacturer to meet customer demand, with customer preferences influenced by the green degree of the product and the wholesale price set by the retailer.

4. Assumptions and notations

4.1. Assumptions

(1) The demand rate (D) for buyer and the production rate (P) for supplier are known and constant, with $P > D$.
(2) The screening process rate is greater than the demand rate.
(3) It is supposed that the produced items are defective from a constant rate.
(4) Nowadays, it is observed that the customers are very health conscious to consume a commodity which is less harmful. Also, it natural tendency of customers to procure the product at the rate of cheap price. So demand rate of customers should be dependent on green degree and selling price of the product. In this respect, here the demand has been considered in the following ways: $D_r = D_0 + \gamma_1 e - \gamma_2 S_r$, where D_0 is fixed demand of customers, γ_1, γ_2 are coefficient of sensitivity of the customers about green degree(e) and selling price of product respectively and both are positive constants.
(5) To improve the green degree of the produced products, manufacturer spend some development cost in each cycle. The development cost for each cycle has been considered as the following: $D_c = d_c e^k$, $k > 0$ where d_c is the development cost in each cycle for producing products whose green degree is 1.

4.2. Notations

D	Demand rate
C_S	Set up cost per cycle
C_P	Production cost per unit
h_c	The cost to a hold the inventory (\$/unit/time) in production center
h_m	The cost to a hold of perfect quality items (\$/unit/time)
h'_d	The cost to a hold of imperfect quality items (\$/unit/time)
h_r	The cost to a hold the inventory (\$/unit/time) of retailer
A_r	Ordering cost the inventory (\$/unit/time) of retailer
C_d	Deteriorating cost of the inventory (\$/unit/time) in production center
C'_d	Deteriorating cost of perfect quality product (\$/unit/time) in production center
C''_d	Deteriorating cost of imperfect quality product (\$/unit/time) of produced items
C_{Sr}	Screening cost per unit item

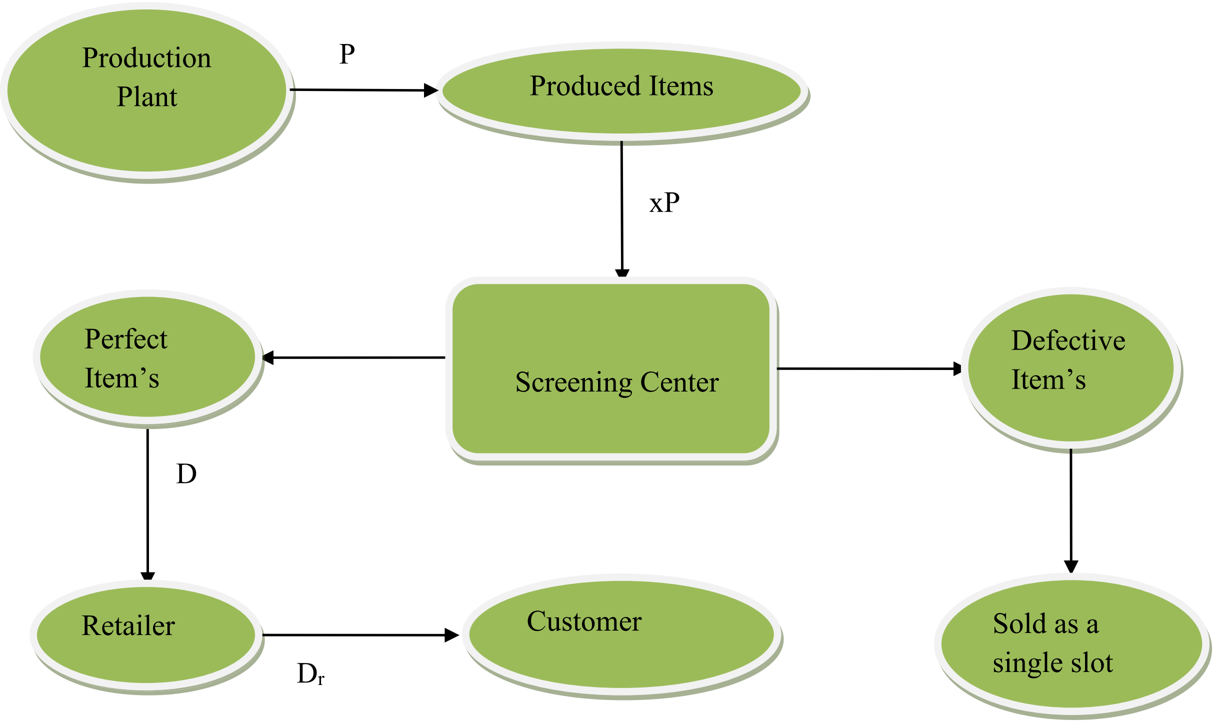


FIGURE 1. Representation of supply chain system.

D_{rc}	Deteriorating cost of items (\$/unit/time) of retailer's
S	Selling price for perfect item (\$/unit/time)
S'	Selling price for imperfect item (\$/unit/time)
u	Defective rate of produced items

Carbon emission parameter

C_{PE}	Production emission cost per unit
h_{ec}	Inventory holding emissions cost per unit
C_{de}	Disposal cost per ton of waste

Decision variable

P	Production rate
T	Cycle length of system
E	Green degree of product

5. MATHEMATICAL MODELING

We consider a production system that produces both perfect and defective items up to a time t_1. Here manufacture perform the screening process to select perfect and imperfect items up to time t_2 with a screening rate (x). Per unit time $(1-x)P$ inventory is to be gathering in screening center until a period t_2 (Fig. 1).

5.1. Mathematically representation of model in production center

At time $t = 0$, production starts and continues up to t_1. The produced goods incessantly are sending into the screening cell with the rate Px (Figs. 2 and 3).

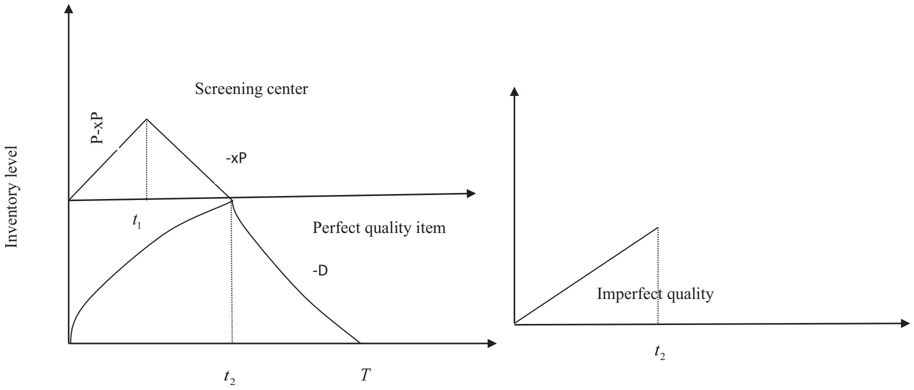


FIGURE 2. Pictorial representation of production.

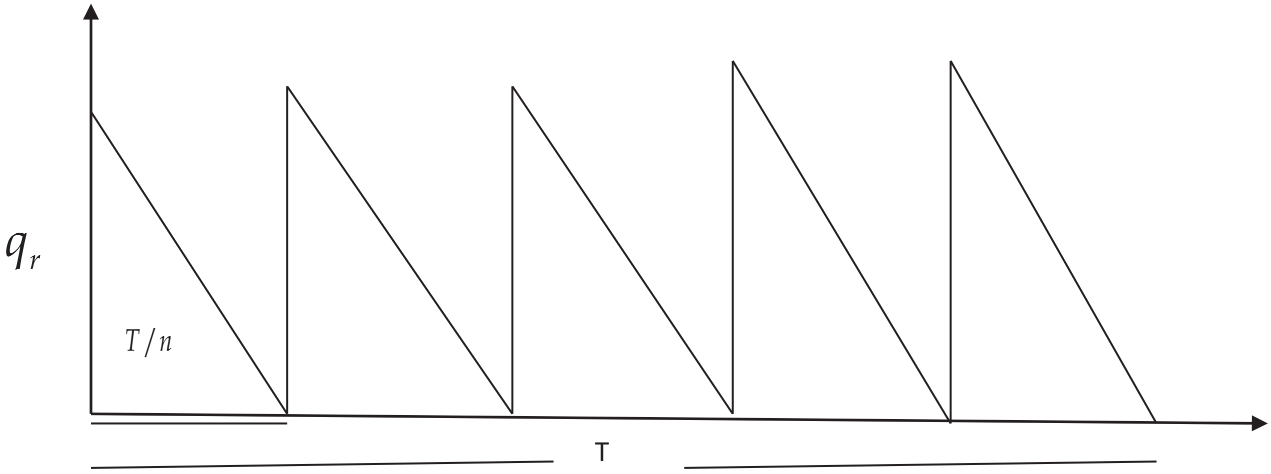


FIGURE 3. Pictorial representation of retailer's inventory model.

The behavior of inventory level w.r.t. to time can be described by the following differential equation

$$I'(t) = P - xP, \qquad 0 \le t \le t_1 \tag{1}$$

$$I'(t) = -xP, \qquad t_1 \le t \le t_2. \tag{2}$$

With the b.c. $I(0) = 0$, $I(t_1) = q$ and $I(t_2) = 0$. The solutions of the given equation are as follows

$$I(t) = (1-x)Pt, \qquad 0 \le t \le t_1 \tag{3}$$

$$I(t) = xP(t_2 - t), \qquad t_1 \le t \le t_2. \tag{4}$$

Using the condition $I(t_1) = q$, we get

$$q = (1-x)Pt_1 = xP(t_2 - t_1) \Rightarrow t_1 = xt_2. \tag{5}$$

5.2. Mathematically representation of model for perfect and imperfect quality item in a screening center

In the screening cell, at time $t = 0$ screening starts and continues up to time t_2. After the screening process, the holding rate of the faultless item is $(1 - u)Px$ which is larger than the demand of consumer. After satisfying the demand of consumer, the additional inventory $(1 - u)Px - D(t)$ per unit time is to being stock in best quality item center until period t_2. After the screening process stock becomes zero due to demand and deterioration at time T. At times t_2 imperfect items are sold in only one consignment with price S'.

So, the variation of perfect quality items $q_1(t)$ with respect to time can be described by the following differential equation:

$$\frac{\mathrm{d}I_p(t)}{\mathrm{d}t} = (1 - u)xP - \theta I_1(t) - D, \qquad 0 \le t \le t_2 \tag{6}$$

$$\frac{\mathrm{d}I_p(t)}{\mathrm{d}t} = -\theta I_2(t) - D, \qquad t_2 \le t \le T \tag{7}$$

$$\frac{\mathrm{d}I_d(t)}{\mathrm{d}t} = uxP - \theta I_d(t), \qquad 0 \le t \le t_2. \tag{8}$$

Subject to the conditions that $I_1(0) = 0$, $I_1(t_2) = I_2(0)$, $I_2(T) = 0$ s.t. $I_d(0) = 0$

$$I_p(t) = \frac{[(1 - u)xP - D]}{\theta}\left(1 - e^{-\theta t}\right) \tag{9}$$

$$I_p(t) = \frac{D}{\theta}\left(e^{\theta(T-t)} - 1\right) \tag{10}$$

$$I_d(t) = \frac{Pux}{\theta}\left(1 - e^{-\theta t}\right). \tag{11}$$

From continuity

$$\frac{[(1 - u)xP - D]}{\theta}\left(1 - e^{-\theta t_2}\right) = \frac{D}{\theta}\left(e^{\theta(T-t_2)} - 1\right) \quad \Rightarrow \quad t_2 = \frac{DT\left(1 + \frac{\theta T}{2}\right)}{[(1 - u)xP]}. \tag{12}$$

5.3. Mathematical representation of retailer's inventory model

$$\frac{\mathrm{d}I_r(t)}{\mathrm{d}t} = -\theta I_r(t) - D_r, \qquad 0 \le t \le T/n \tag{13}$$

$$I_r(t) = \frac{D_r}{\theta}\left(e^{\theta(T/n-t)} - 1\right). \tag{14}$$

At $t = 0$,

$$I_r(0) = \frac{D_r}{\theta}\left(e^{\theta(T/n)} - 1\right) = q_r.$$

Cost components of production system

$$\text{Production cost (PC)} = (C_P + C_{\mathrm{PE}})\int_0^{t_1} P\,\mathrm{d}t = (C_P + C_{\mathrm{PE}})Pt_1$$

$$\text{Screening cost (SC)} = C_{Sr}\int_0^{t_2} xP\,\mathrm{d}t = C_{Sr}Pxt_2$$

$$\text{Set up cost} = C_S.$$

Sales Revenue cost (SR): The amount of money which obtained after selling inventory is known as total revenue cost or sales revenue cost. The formula for the cost is given as follows:

$$S\int_0^T D(t)\,\mathrm{d}t + S'\int_0^{t_2} uxP\,\mathrm{d}t = SDT + S'Puxt_2.$$

Holding cost (HC): The amount of money which needs to hold the inventory is known as holding cost. In this model we considered holding cost as time dependent as the expression for finding holding cost is as follows:

$$\begin{aligned} &= (h_c + h_{ec})\int_0^{t_2} I(t)\,\mathrm{d}t + (h_m + h_{ec})\int_0^{t_2} I_1(t)\,\mathrm{d}t + (h'_m + h_{ec})\int_{t_2}^{T} I_2(t)\,\mathrm{d}t + (h_d + h_{ec})\int_0^{t_2} I_d(t)\,\mathrm{d}t \\ &= (h_c + h_{ec})\left((1-x)\frac{Pt_1^2}{2} + xP\frac{(t_2-t_1)^2}{2}\right) + (h_m + h_{ec})\frac{[(1-u)xP - D]}{\theta^2}\left[\theta t_2 + \left(e^{-\theta t_2} - 1\right)\right] \\ &\quad + (h'_m + h_{ec})\frac{D}{\theta^2}\left[\left(e^{\theta(T-t_2)} - 1\right) - \theta(T-t_2)\right] + (h_d + h_{ec})uxP\left[\frac{\theta t_2 + \left(e^{-\theta t_2} - 1\right)}{\theta^2}\right]. \end{aligned}$$

Deterioration cost (DC): The amount of money which is lost due to deterioration or damage of inventory is known as deterioration cost. The expression for deteriorating cost is given as follows:

$$\begin{aligned} &(C_d + C_{de})\int_0^{t_2} I_1(t)\,\mathrm{d}t + (C'_d + C_{de})\int_{t_2}^{T} I_2(t)\,\mathrm{d}t + (C''_d + C_{de})\int_0^{t_2} I_d(t)\,\mathrm{d}t \\ &\quad = (C_d + C_{de})\frac{[(1-u)xP - D]}{\theta^2}\left[\theta t_2 + \left(e^{-\theta t_2} - 1\right)\right] + (C'_d + C_{de})\frac{D}{\theta^2}\left[\left(e^{\theta(T-t_2)} - 1\right) - \theta(T-t_2)\right] \\ &\quad\quad + (C''_d + C_{de})uxP\left[\frac{\theta t_2 + \left(e^{-\theta t_2} - 1\right)}{\theta^2}\right]. \end{aligned}$$

Total development cost (DPM) $= D_c = d_c e^k$.
Producer average total cost (P.T.C) $= \frac{1}{T}$ (Production cost + Screening cost + Holding cost + Deterioration cost + Set up cost + Development cost − Sales Revenue cost)

$$= \frac{1}{T}\left(\begin{array}{l} (C_P + C_{\mathrm{PE}})Pt_1 + C_{Sr}Pxt_2 + C_S + (h_c + h_{ec})\left((1-x)\frac{Pt_1^2}{2} + xP\frac{(t_2-t_1)^2}{2}\right) \\ + (h_m + h_{ec} + C_d + C_{de})\frac{[(1-u)xP - D]}{\theta^2}\left[\theta t_2 + \left(e^{-\theta t_2} - 1\right)\right] \\ + (h'_m + h_{ec} + C'_d + C_{de})\frac{D}{\theta^2}\left[\left(e^{\theta(T-t_2)} - 1\right) - \theta(T-t_2)\right] \\ + (h_d + h_{ec} + C''_d + C_{de})uxP\left[\frac{\theta t_2 + \left(e^{-\theta t_2} - 1\right)}{\theta^2}\right] + d_c e^k - SDT - S'Puxt_2 \end{array}\right). \tag{15}$$

Cost components of retailer's system

Holding cost (H.C$_r$.): The amount of money which needs to hold the inventory is known as holding cost. In this model we considered holding cost as time dependent as the expression for finding holding cost is as follows:

$$= (h_r + h_{ec})\frac{n}{T}\int_0^{T/n} I_r(t)\,\mathrm{d}t = (h_r + h_{ec})\frac{n}{T}\int_0^{T/n} I_r(t)\,\mathrm{d}t = (h_r + h_{ec})\frac{nD_r}{T\theta^2}\left(e^{\theta T/n} - 1 - \frac{T\theta}{n}\right).$$

Ordering cost (O.C$_r$.): There are two components to a retailer's ordering cost. Due to placing one product A_r as is connected and the other part as A_{re} is due to carbon emissions. Lastly, the total ordering cost below the effect of the environs is

$$\mathrm{O.C}_r. = \frac{n(A_r + A_{re})}{T}.$$

Deteriorating cost (D.C$_r$.): There are two components to a retailer's deterioration cost. Due to placing one product D_r as is connected and the other part as D_{re} is due to carbon emissions. To end, the total deterioration cost below the effect of the environs is

$$\text{D.C}_r. = \frac{n}{T\theta}(D_{rc} + C_{de})D_r\left(e^{\theta T/n} - 1 - \frac{T\theta}{n}\right).$$

Sales revenue cost (S.R$_r$.): The amount of money which is obtained after selling inventory is

$$\text{S.R}_r. = \frac{n}{T}S_r\int_0^{T/n}\frac{D_r}{\theta}\left(e^{\theta(T/n)} - 1\right)\mathrm{d}t = S_r\frac{D_r}{\theta}\left(e^{\theta(T/n)} - 1\right).$$

Retailer's total cost: Ordering cost + holding cost + Deterioration cost

$$\text{R.T.C} = \frac{n}{T}\left((A_r + A_{re}) + \frac{D_r}{\theta^2}((D_{rc} + C_{de})\theta + (h_r + h_{ec}))\left(e^{\theta T/n} - 1 - \frac{T\theta}{n}\right)\right) - S_r\frac{D_r}{\theta}\left(e^{\theta(T/n)} - 1\right). \tag{16}$$

Retailer's and producers are equally responsible for determining the profit function for both demand scenarios. A centralized approach to solving this mathematical model is employed here. Let, A.T.C. be the supply chain's joint average total cost function, which is comprised of the sum of the average profit of the retailer and producer.

$$\text{A.T.C.} = \text{P.T.C.} + \text{R.T.C.}$$

6. Solution methodology

In this section we seen that average total average cost of T, e and P. The optimal values of total cost function is calculated by using following necessary and sufficient conditions:

$$\begin{aligned}\text{A.T.C.} = \frac{1}{T}&\left(\begin{array}{l}(C_P + C_{\text{PE}} + C_{Sr})Pxt_2 + C_S + (h_c + h_{ec})\left((1-x)\frac{Px^2t_2^2}{2} + xP\frac{(1-x)^2t_2^2}{2}\right)\\ + (h_m + h_{ec} + C_d + C_{de})\frac{[(1-u)xP - D]}{\theta^2}\left[\theta t_2 + \left(e^{-\theta t_2} - 1\right)\right]\\ + (h'_m + h_{ec} + C'_d + C_{de})\frac{D}{\theta^2}\left[\left(e^{\theta(T-t_2)} - 1\right) - \theta(T - t_2)\right]\\ + (h_d + h_{ec} + C''_d + C_{de})uxP\left[\frac{\theta t_2 + \left(e^{-\theta t_2} - 1\right)}{\theta^2}\right] + d_ce^k - SDT - S'Puxt_2\end{array}\right)\\ &+ \frac{n}{T}\left((A_r + A_{re}) + \frac{D_r}{\theta^2}((D_{rc} + C_{de})\theta + (h_r + h_{ec}))\left(e^{\theta T/n} - 1 - \frac{T\theta}{n}\right)\right)\\ &- S_r\frac{D_r}{\theta}\left(e^{\theta(T/n)} - 1\right).\end{aligned} \tag{17}$$

After the substituting the value of $t_1 = xt_2$.

Simplify the equation (17)

$$\text{A.T.C.} = \frac{1}{T}\left(\begin{array}{l}\Delta_1Pt_2 - SDT + \Delta_4\left(\frac{T^2}{2} - Tt_2\right) + C_S + d_ce^k\\ (\Delta_2P + \Delta_3[(1-u)xP - D] + \Delta_4D + \Delta_5P)\frac{t_2^2}{2}\end{array}\right) + \frac{T}{n}\left(\Delta_6 + \frac{D_r}{2}\Delta_7 - S_rD_r\left(1 + \frac{\theta T}{2n}\right)\right)$$

where

$$\begin{aligned}\Delta_1 &= (C_P + C_{\text{PE}} + C_{Sr} - S'u)x\\ \Delta_2 &= (h_c + h_{ec})x(1-x)\\ \Delta_3 &= (h_m + h_{ec} + C_d + C_{de})\\ \Delta_4 &= (h'_m + h_{ec} + C'_d + C_{de})\end{aligned}$$

$$
\begin{aligned}
\Delta_5 &= (h_d + h_{ec} + C''_d + C_{de})ux\\
\Delta_6 &= (A_r + A_{re})\\
\Delta_7 &= ((D_{rc} + C_{de})\theta + (h_r + h_{ec}))
\end{aligned}
$$

$$
\text{A.T.C.} = \begin{pmatrix} \Delta_1 P \tfrac{D}{((1-u)xP)}\left(1+\tfrac{\theta T}{2}\right) - SD + \Delta_4\left(\tfrac{T}{2} - \tfrac{DT}{((1-u)xP)}\left(1+\tfrac{\theta T}{2}\right)\right) + \tfrac{C_S}{T} + \tfrac{d_c e^k}{T} \\ (\Delta_2 P + \Delta_3((1-u)xP - D) + \Delta_4 D + \Delta_5 P)\tfrac{1}{2}\tfrac{D^2 T}{((1-u)xP)^2}\left(1+\tfrac{\theta T}{2}\right)^2 \end{pmatrix}
+ \frac{T}{n}\left(\Delta_6 + \frac{D_r}{2}\Delta_7 - S_r D_r\left(1+\frac{\theta T}{2n}\right)\right). \tag{18}
$$

After substituting the value of t_2.

Necessary condition: To obtain the critical points, the first-order partial derivatives concerning the decision variables are separately equal to zero *i.e.*,

$$
\frac{\partial \text{A.T.C.}}{\partial T} = \begin{pmatrix} \Delta_1 P \tfrac{D}{((1-u)xP)}\left(\tfrac{\theta}{2}\right) + \Delta_4\left(\tfrac{1}{2} - \tfrac{D}{((1-u)xP)}(1+\theta T)\right) - \tfrac{C_S}{T^2} - \tfrac{d_c e^k}{T^2} \\ (\Delta_2 P + \Delta_3((1-u)xP - D) + \Delta_4 D + \Delta_5 P)\tfrac{1}{2}\tfrac{D^2}{((1-u)xP)^2}(1+\theta T)^2 \end{pmatrix}
+ \frac{1}{n}\left(\Delta_6 + \frac{D_r}{2}\Delta_7 - S_r D_r\left(1+\frac{\theta T}{n}\right) - S_r D_r\left(\frac{\theta T}{n}\right)\right). \tag{19}
$$

To find the critical point equate the equation (19) equal to zero *i.e.*,

$$
\begin{aligned}
\frac{\partial \text{A.T.C.}}{\partial T} &= 0 \qquad (20)\\
\frac{\partial \text{A.T.C.}}{\partial e} &= \left(-\frac{k d_c e^{k-1}}{T^2}\right) + \frac{1}{n}\left(\Delta_6 + \frac{\gamma_1}{2}\Delta_7 - S_r\gamma_1\left(1+\frac{\theta T}{2n}\right) - S_r\gamma_1\left(\frac{\theta}{2n}\right)\right).
\end{aligned}
$$

To find the critical point equate the equation (20) equal to zero *i.e.*, $\frac{\partial \text{A.T.C.}}{\partial e} = 0$

$$
e = \left(-\frac{1}{k d_c}\left(T^2 \frac{1}{n}\left(1+\frac{\theta T}{n}\right)S_r\gamma_1 - \frac{\gamma_1\Delta_7}{2}\right)\right)^{1/(k-1)} \tag{21}
$$

we get

$$
\frac{\partial \text{A.T.C.}}{\partial P} = \begin{pmatrix} \Delta_1 \tfrac{D}{((1-u)xP)}\left(1+\tfrac{\theta T}{2}\right)\left(1 - \tfrac{P(1-u)x}{((1-u)xP)}\right) + \Delta_4\left(\tfrac{DT(1-u)x}{((1-u)xP)^2}\left(1+\tfrac{\theta T}{2}\right)\right) \\ (\Delta_2 + \Delta_3((1-u)x) + \Delta_5 P)\tfrac{1}{2}\tfrac{D^2 T}{((1-u)xP)^2}\left(1+\tfrac{\theta T}{2}\right)^2 \\ -(\Delta_2 P + \Delta_3((1-u)xP - D) + \Delta_4 D + \Delta_5 P)\tfrac{(1-u)xD^2 T}{((1-u)xP)^3}\left(1+\tfrac{\theta T}{2}\right)^2 \end{pmatrix}. \tag{22}
$$

To find the critical point equate the equation (22) equal to zero *i.e.*, $\frac{\partial \text{A.T.C.}}{\partial P} = 0$, we get

$$
P = -\frac{2D^2 T(2+\theta T)(\Delta_3 - \Delta_4)}{-(2+\theta T)DT\Delta_2 + 2DTx(u-1)\Delta_3 + DT^2 x\theta(u-1)\Delta_3 + 4x(1-u)\Delta_4 - (2+\theta T)DT\Delta_5}. \tag{23}
$$

Put the value of e and P from equations (21) and (23) in equation (20) and find the value of T.

Sufficient condition: The sufficient condition must be satisfied to prove the globality of the optimal solution. We obtain the convexity of the average cost function by using the Hessian matrix.

$$
\frac{\partial^2 \prod}{\partial T^2} = \frac{2C_S}{T^3} + \frac{2e^k d_c}{T^3} - \frac{2\theta D_r S_r}{n^2} + \frac{D^2 T\theta^2(P\Delta_2 + (-D + P(1-u)x)\Delta_3 + D\Delta_4 + P\Delta_5)}{4P^2(1-u)^2 x^2}
$$

Table 1. Optimal result of illustrated model.

Production run time (t_1)	Screening period (t_2)	Cycle length (T)	Production rate (P)	Green degree (e)	Average total cost (A.T.C.)
1.20688	1.5086	1.97795	16.8889	0.805327	1 581 160

$$+\left(1+\frac{T\theta}{2}\right)\frac{D^2\theta(P\Delta_2+(-D+P(1-u)x)\Delta_3+D\Delta_4+P\Delta_5)}{P^2(1-u)^2x^2}$$

$$\frac{\partial^2\prod}{\partial e^2}=\frac{e^{-2+k}(k-1)kd_c}{T}$$

$$\frac{\partial^2\prod}{\partial^2 P}=-\frac{2D\left(1+\frac{T\theta}{2}\right)\Delta_4}{P^3(1-u)x}-\frac{2D^2T\left(1+\frac{T\theta}{2}\right)^2(\Delta_2+(1-u)x\Delta_3+\Delta_5)}{P^3(1-u)^2x^2}$$
$$+\frac{3D^2T\left(1+\frac{T\theta}{2}\right)^2(P\Delta_2+(-D+P(1-u)x)\Delta_3+D\Delta_4+P\Delta_5)}{P^4(1-u)^2x^2}.$$

Similarly we find $\frac{\partial^2\prod}{\partial T\partial e}$, $\frac{\partial^2\prod}{\partial T\partial P}$, $\frac{\partial^2\prod}{\partial e\partial P}$ respectively.

The Hessian matrix (H) is calculated for obtained cost function, given as

$$H=\begin{bmatrix}\frac{\partial^2\prod}{\partial T^2} & \frac{\partial^2\prod}{\partial T\partial e} & \frac{\partial^2\prod}{\partial T\partial P}\\ \frac{\partial^2\prod}{\partial e\partial T} & \frac{\partial^2\prod}{\partial e^2} & \frac{\partial^2\prod}{\partial e\partial P}\\ \frac{\partial^2\prod}{\partial P\partial T} & \frac{\partial^2\prod}{\partial P\partial e} & \frac{\partial^2\prod}{\partial P^2}\end{bmatrix}.$$

Our main aim is to obtain the optimal cost function for the inventory model which is to be done by establishing the convexity of the considered cost function. For this, we have to prove that the above-calculated Hessian matrix is positive definite, *i.e.*, principal minors sign starting with a positive sign. Clearly, we have, $|H_1|>0, |H_2|>0$ and $|H3|>0$. This shows that obtained solution is optimal.

7. Numerical examples

In this section, a validity assessment has been performed on the proposed study in this section. There are several parameters that determine the performance of the mentioned model. The numerical values of these parameters have been taken which are purely based on the previous researches in appropriates units (Figs. 4–6).

$$C_{Sr}=50,\ C_p=50,\ C_{de}=0.05,\ D_0=30,\ \gamma_1=20,\ \gamma_2=4,\ A_r=10,\ A_{re}=2,\ \ S_r=30,\ d_c=200,\ k=3$$
$$u=0.02,\ h_r=0.5,\ C_d=0.015,\ n=5,\ h_d=0.013,\ h_c=0.25,\ h_m=10,\ D_{rc}=0.5,\ C'_d=0.25,$$
$$C''_d=0.5,\ S=75,\ S'=40,\ C_{pe}=10,\ h_{ec}=10,\ x=0.8,\ D=10,\ \theta=0.01,\ C_S=50.$$

The optimal result are given in the following Table 1.

8. Sensitivity analysis

The effect of various parameters on cost function has been observed in this part. The sensitivity analysis of these parameters has been shown in following tables.

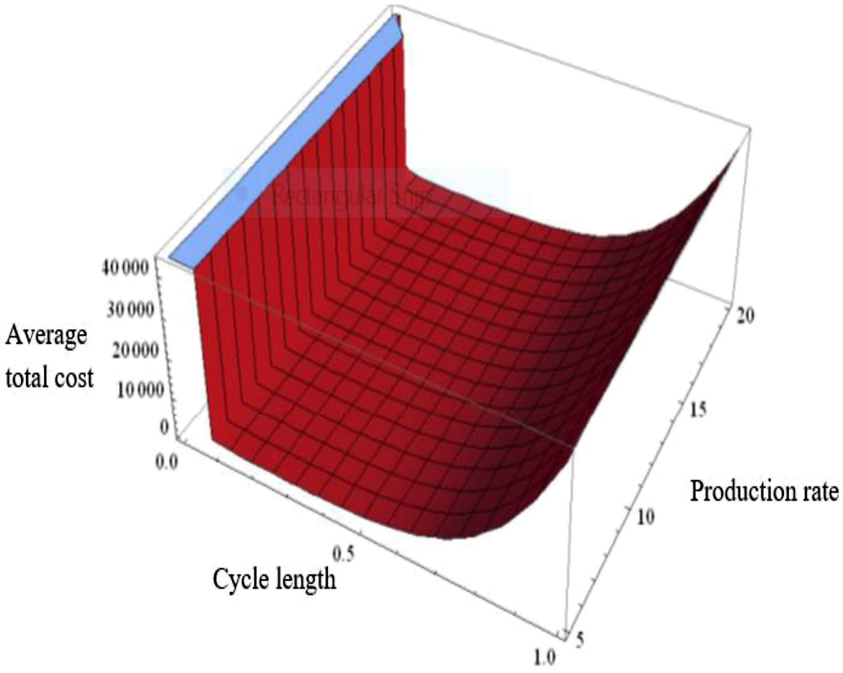


Figure 4. Convexity between green degree and Production rate.

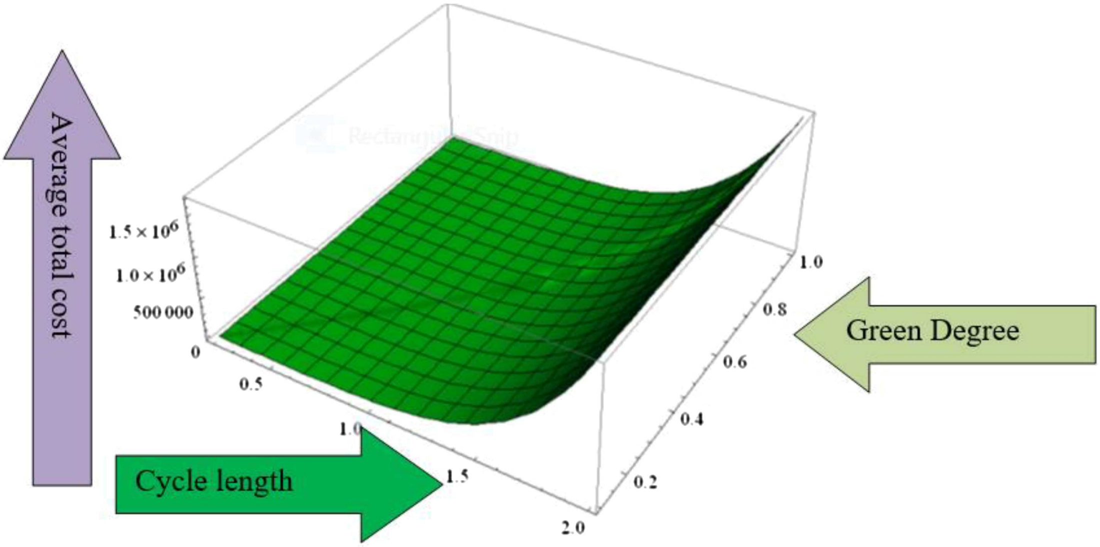


Figure 5. Convexity between green degree and cycle length.

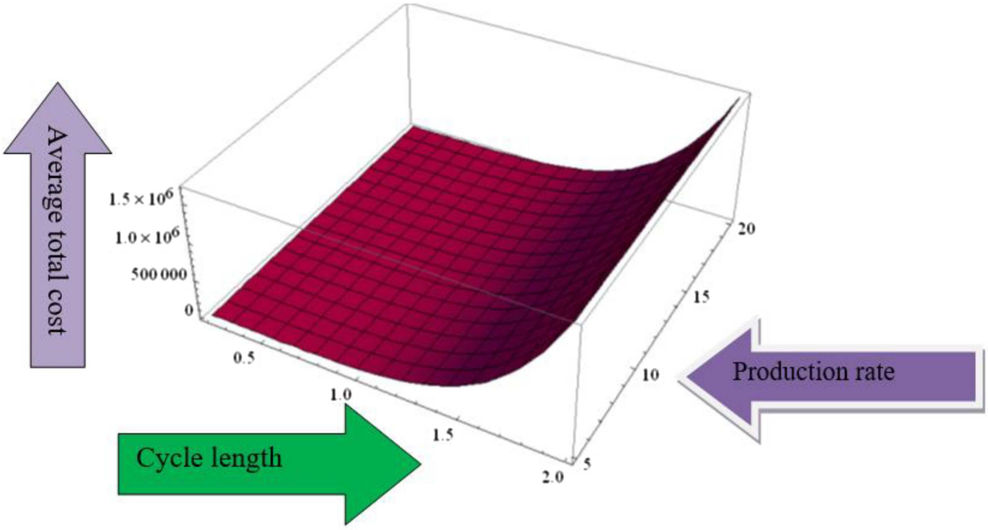


FIGURE 6. Convexity between cycle length and production rate.

TABLE 2. Sensitivity analysis w.r.t. screening rate parameter.

Variable	Value of parameter	t_1 (production run time)	t_2 (screening period)	Production Rate (P)	T (cycle length)	Green degree (e)	(Average total cost) A.T.C.
x	0.64	1.2759	1.99361	19.4922	2.40829	0.98105	4 795 900
	0.68	1.26956	1.867	18.7006	2.30022	0.936903	3 695 960
	0.72	1.256256	1.7448	18.0122	2.19309	0.893155	2 822 910
	0.76	1.2352356	1.62531	17.4121	2.08604	0.849447	2 130 170
	0.8	1.20688	1.5086	16.8889	1.97795	0.805325	1 581 140
	0.84	1.1702124	1.39311	16.4337	1.86721	0.760135	1 146 710
	0.88	1.1237248	1.27696	16.0407	1.7512	0.71279	803 217
	0.92	1.0638144	1.15632	15.7073	1.62436	0.66108	530 544
	0.96	0.9790464	1.01984	15.4403	1.47062	0.598401	307 646

9. RESULT AND DISCUSSION

(i) From Table 2, we notice that if increase the screening rate then average total cost (A.T.C.), cycle length (T), production run time (t_1), Production rate (P), green degree (e) also decrease.
(ii) From Table 3, we observe that as increase the defective rate then average total cost (A.T.C.), cycle length (T), production run time (t_1), Production rate (P), green degree (e) also increase.
(iii) From Table 4, we observe that if increase the development cost then production rate (P) slightly positive sensitive but cycle length (T), green degree (e) and total average cost (A.T.C.) slightly decrease.
(iv) From Table 5, we observe that if increase the green degree sensitive parameter then production rate (P) positive sensitive but cycle length (T), green degree (e) and total average cost (A.T.C.) decrease.

TABLE 3. Sensitivity analysis w.r.t. defective rate parameter.

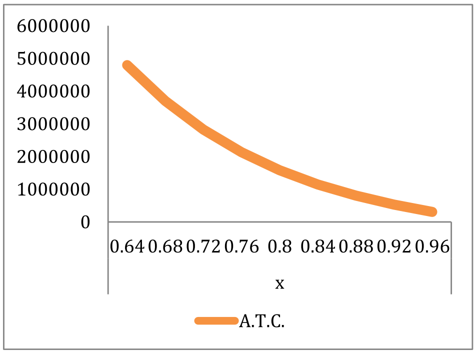
Variable	Value of parameter	t_1 (production run time)	t_2 (screening period)	Production Rate (P)	T (cycle length)	Green degree (e)	(Average total cost) A.T.C.
u	0.016	1.195872	1.49484	16.8645	1.96524	0.800139	1 525 220
	0.017	1.198632	1.49829	16.8706	1.96841	0.801433	1 539 020
	0.018	1.201376	1.50172	16.8767	1.97159	0.802729	1 552 940
	0.019	1.201376	1.50515	16.8828	1.97476	0.804027	1 566 980
	0.02	1.20688	1.5086	16.8889	1.97795	0.805325	1 581 140
	0.021	1.209632	1.51204	16.895	1.98113	0.806625	1 595 430
	0.022	1.2124	1.5155	16.9011	1.98432	0.807927	1 609 840
	0.023	1.215168	1.51896	16.9072	1.98751	0.80923	1 624 370
	0.024	1.217952	1.52244	16.9134	1.99071	0.810535	1 639 030

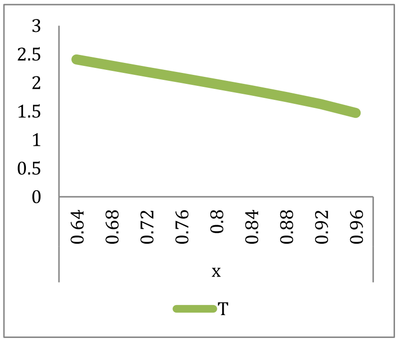
TABLE 4. Sensitivity analysis w.r.t. development cost parameter.

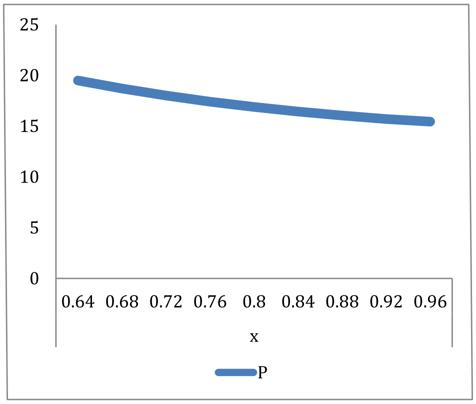
Variable	Value of parameter	t_1 (production run time)	t_2 (screening period)	Production Rate (P)	T (cycle length)	Green degree (e)	(Average total cost) A.T.C.
C_d	2.4	1.20716	1.50895	16.8859	1.97806	0.805372	1 581 660
	2.55	1.207088	1.50886	16.8866	1.97803	0.805361	1 581 530
	2.7	1.207016	1.50877	16.8874	1.978	0.805349	1 581 400
	2.85	1.206944	1.50868	16.8881	1.97798	0.805337	1 581 270
	3	1.20688	1.5086	16.8889	1.97795	0.805325	1 581 140
	3.15	1.2068	1.5085	16.8896	1.97792	0.805313	1 581 020
	3.3	1.20672	1.5084	16.8904	1.97789	0.805301	1 580 890
	3.45	1.206664	1.50833	16.8911	1.97786	0.80529	1 580 760
	3.6	1.206592	1.50824	16.8918	1.97783	0.805278	1 580 630

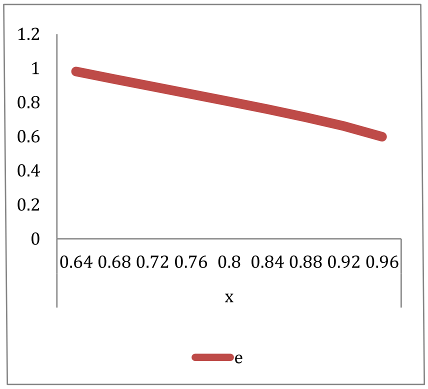
TABLE 5. Sensitivity analysis w.r.t. green degree sensitive parameter.

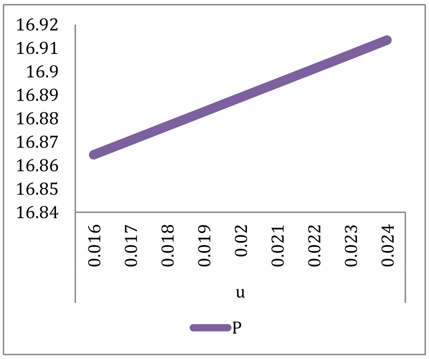
Variable	Value of parameter	t_1 (production run time)	t_2 (screening period)	Production Rate (P)	T (cycle length)	Green degree (e)	(Average total cost) A.T.C.
γ_1	16	1.851472	2.31434	16.7047	2.98637	0.98354	16 506 100
	17	1.65864	2.0733	16.7448	2.68575	0.96174	8 944 500
	18	1.490424	1.86303	16.7882	2.42277	0.936316	4 962 540
	19	1.34136	1.6767	16.8358	2.18916	0.868974	2 794 580
	20	1.20688	1.5086	16.8889	1.97795	0.805325	1 581 140
	21	1.082656	1.35332	16.9496	1.78247	0.743483	886 072
	22	0.96352	1.2044	17.0225	1.59463	0.680631	479 367
	23	0.838968	1.04871	17.1209	1.39789	0.609924	233 502
	24	0.6665584	0.833198	17.3177	1.12492	0.50121	72 609

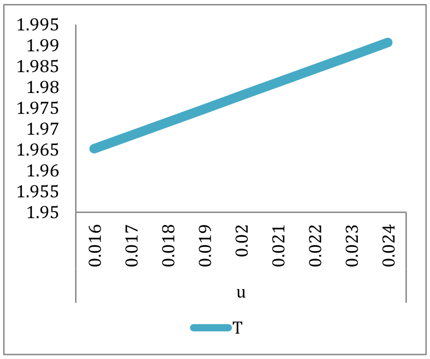








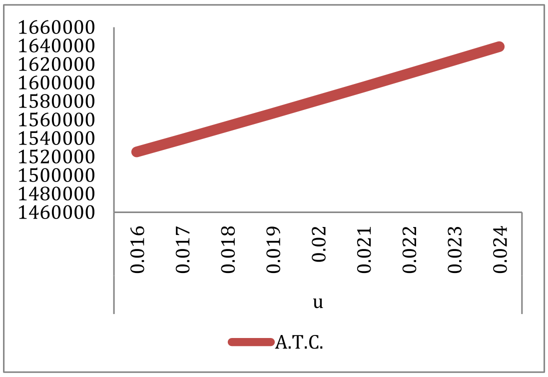


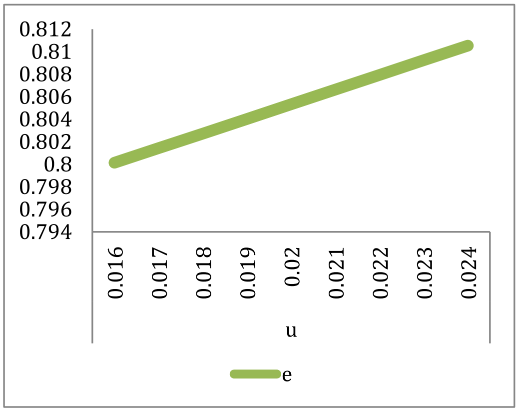


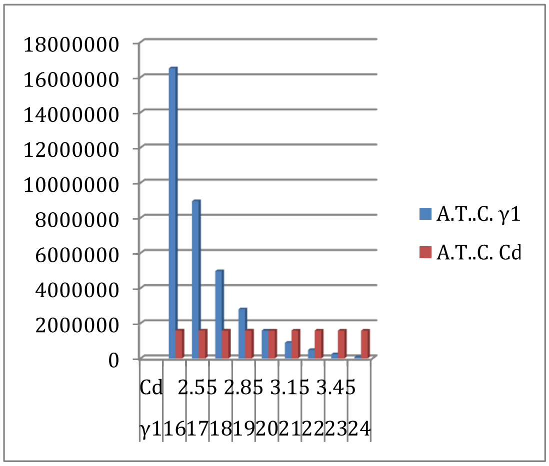
Managerial insight

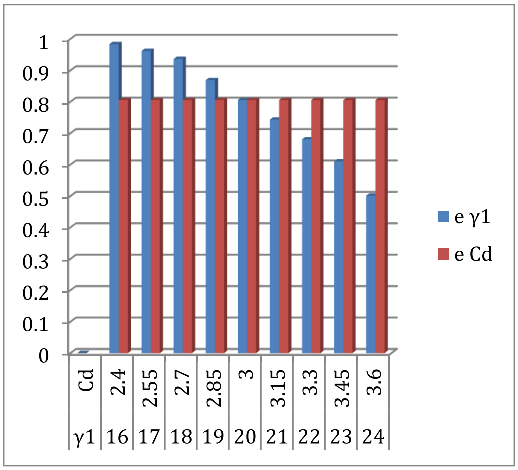
The study offers several valuable insights for supply chain managers striving to balance cost efficiency with environmental responsibility in an imperfect production environment:

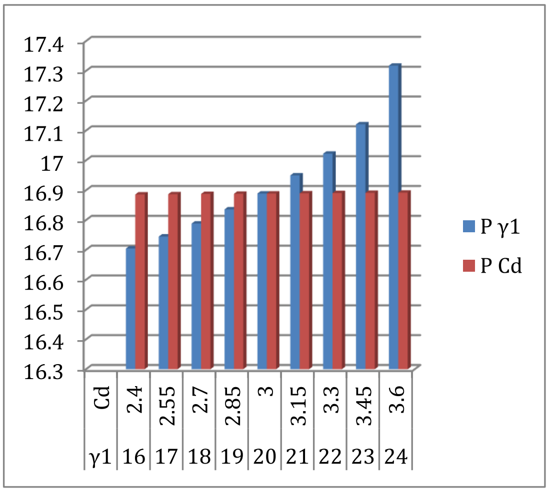
- By adopting a realistic screening approach where the screening rate is constrained between production and demand rates, managers can better handle defective products, reduce waste, and maintain product quality. This can enhance customer satisfaction while minimizing costs associated with rework or recalls.
- The inclusion of deterioration and defective product rates highlights the need for effective inventory and quality management practices. Managers can use this insight to prioritize investments in technology or processes that minimize waste and improve production quality.
- The sensitivity analyses demonstrate how the proposed model responds to variations in key parameters, such as demand, production rates, and carbon costs. This equips managers with a better understanding of the trade-offs and risks involved in adjusting these parameters.

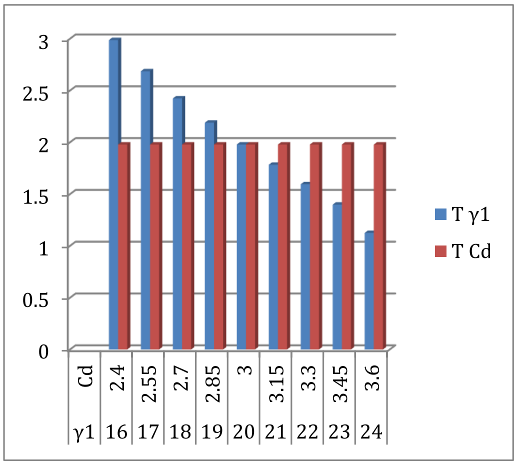












10. Conclusion

In this chapter, the screening of manufactured items is incorporated with carbon dioxide emissions in an imperfect production supply chain model. There has been an adoption of a realistic screening approach, where the screening rate is less than or equal to the production rate and greater than the demand rate. The objective is to reduce carbon emission costs and minimize the average total cost. This model includes the effect of deterioration, defective products, and carbon emissions, effect of greening of product. Lastly, numerical examples accompanied by sensitivity analyses have been provided to illustrate how the proposed model reacts to changes in the various parameters. The findings emphasize the importance of optimizing the screening rate and controlling the defective rate for more efficient and sustainable production, while balancing costs and green initiatives to achieve overall system optimization. The findings of this study have important theoretical implications for the understanding of production systems. The results align with established theories in operations management, particularly those related to production efficiency and cost optimization. For instance, the decrease in average total cost (A.T.C.) and cycle length when the screening rate increases supports the theory that improving quality control reduces waste and operational inefficiencies. Additionally, the study contributes to sustainability models in production by highlighting how an increase in the green degree can influence production rate and environmental impact. From a practical perspective, the findings provide actionable recommendations for improving real-world production processes:

Manufacturers could increase the screening rate to reduce defects, lower production costs, and enhance sustainability, directly benefiting both efficiency and environmental performance. The study also suggests that increasing the defective rate has negative consequences, emphasizing the importance of investing in quality control systems to avoid the inefficiencies associated with defects. Additionally, the slight increase in production rate with higher development costs implies that companies may need to balance investments in development with the other factors, like cycle time and cost management, to achieve optimal performance. This paper can be extended by considering technology investment, an adjustable production rate to reduce the possibility of defective and deteriorating goods. Further we can extend included trade credit policy and carbon cap policy with variable parameters. The potential extensions of the model, such as technology investments, adjustable production rates, trade credit policies, and carbon cap policies, offer avenues for strategic planning. Managers can explore these options to enhance operational flexibility, reduce environmental impact, and adapt to evolving market and regulatory conditions.

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Data availability statement

No new data/codes were created or analyzed in this study.

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