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RESOURCE ALLOCATION IN PARALLEL-SERIES PRODUCTION SYSTEMS WITH SHARED INPUTS AND OUTPUTS: A DATA ENVELOPMENT ANALYSIS APPROACH

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Abstract. The problem of resource allocation and reallocation in management science and production theory has drawn much attention among researchers and decision-makers. In this contribution, we focus on this problem in production processes in which two parallel stages are serially connected to a third stage. We assume that in addition to stage-specific inputs, we have shared inputs between the two parallel stages. In this sense, a linear programming-based model is proposed to calculate the technical efficiency of the whole process along with an optimal split of shared resources. To demonstrate the real-world applicability of the proposed approach, a case study on 22 Indian insurance companies is conducted. The analysis reveals that only three companies are efficient across all scenarios. Furthermore, narrow bounds on shared resources and intermediate products lead to more accurate and reliable results compared to medium and wide bounds, highlighting the importance of precise resource allocation constraints.

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1. Introduction

Assessing financial efficiency is a crucial aspect of evaluating the performance of entities in a market economy. For insurance companies, financial efficiency and performance are shaped by several factors, including the scope of their insurance activities and the types of risks they underwrite [28]. Proper evaluation of an insurer's financial effectiveness is essential not only for understanding its market position but also for formulating strategies to enhance financial performance. These strategies can lead to improved operational plans, which, in turn, may boost sales and financial outcomes in the future.

Given this importance, there is a growing need for robust research methods to assess the financial situation of insurance companies effectively. Based on a review of the literature, these methods can be divided into two main categories.

Keywords. Data envelopment analysis, three-stage network production, shared resources, parallel system.

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The first category includes basic methods, such as simplified ratio analysis and comprehensive ratio analysis, with metrics tailored specifically to insurance operations [1]. The second category involves advanced methods, notably the use of DEA (Data Envelopment Analysis) models and Tobit models [23].

These advanced approaches provide deeper insights and allow for a more sophisticated evaluation of financial efficiency, making them valuable tools for understanding and improving the financial health of insurers.

In recent years, DEA has been applied to a variety of fields, from assessing Canadian banks [5] to improving the efficiency of insurance companies [15,19] and even predicting corporate success and failure [38]. Its versatility makes it a vital tool for solving real-world problems, particularly in complex industries like insurance. Recent studies explore various dimensions of operational efficiency and optimization across industries, utilizing innovative methodologies and contextual factors. For instance, Goyal *et al.* [20] investigated the inventory management under imperfect production and imprecise demand, highlighting the effects of learning and advertisement-driven demand adjustments on economic order quantities. Similarly, Kumar *et al.* [27] examined the influence of pricing strategies and promotional efforts on inventory policies, considering trade credit, inflation, and time-dependent holding costs. On the sustainability front, Yadav *et al.* [46] emphasized reducing waste and carbon emissions by integrating cross-price elasticity of demand and preservation technology in supply chain design. These studies collectively underscore the growing relevance of multi-dimensional approaches to resource management. By applying a DEA-based framework, your research complements these findings, offering a system-wide perspective on resource allocation in complex production networks, which bridges operational efficiency with shared-resource optimization.

For example, Tone *et al.* [42] introduced a dynamic two-stage network DEA model to analyze the overall performance of insurance companies. Building on this, Omrani *et al.* [33] developed a more advanced model that could handle negative data and undesirable outputs, applying it to evaluate 22 insurance companies. Later, Omrani *et al.* [34] proposed a mixed-integer network DEA model to analyze the insurance sector by incorporating shared inputs and undesirable outputs. Similarly, Yang [47] suggested a centralized method for allocating resources between different stages of two-stage network systems.

When it comes to managing performance, researchers have explored various approaches tailored to the insurance industry. Hatami-Marbini and Saati [21] proposed a common-weights network DEA model for evaluating non-life insurance companies, while Zhang *et al.* [49] introduced a model designed for two-stage parallel-series structures involving 24 non-life insurers.

While resource allocation in parallel-series production systems has been widely studied, many existing approaches fall short when it comes to addressing the complexities of shared inputs and outputs between interconnected processes. Most traditional DEA models focus on evaluating efficiency in isolation, ignoring the intricate dependencies that naturally arise in such systems. Moreover, these methods rarely provide a unified framework for tackling both resource allocation and performance evaluation, particularly in multi-tiered production setups. This leaves a critical gap in practical solutions for industries, including the insurance industry, aiming to optimize shared resources while maintaining overall efficiency.

To address this gap, our study presents a fresh DEA-based approach specifically designed for parallel-series production systems. Our contributions include: (1) developing a model that effectively accounts for shared inputs and outputs, reflecting the inherent interconnections within these systems; (2) proposing a balanced resource allocation method that improves operational performance across all units; and (3) extending the traditional DEA framework to provide decision-makers with a practical tool for assessing and enhancing efficiency in complex production environments. Furthermore, the proposed model has been analyzed with particular attention to its applicability within the insurance industry, showcasing its relevance in real-world contexts. By tackling these challenges, our research not only fills an important gap in the literature but also offers valuable insights for practitioners in resource-driven industries, and especially those in the insurance sector, who face similar resource allocation complexities. This kind of network system reflects the complexity of many real-world situations. By building on methods developed by Papaionnou and Podinovski [35] and Podinovski [37], our approach can be applied to similar systems, offering practical solutions for resource allocation and performance management in interconnected environments like the insurance industry.

The rest of this paper is organized as follows: Section 2 reviews the related literature. In Section 3, we state the main problem, present the deterministic version of the production possibility set, and introduce models for evaluating overall efficiency. In Section 4, we apply our proposed model to the simple numerical example. In Section 5, we apply our proposed model to the real application, *i.e.*, the Indian insurance companies. Conclusions appear in Section 6.

2. Literature review

2.1. DEA

DEA is a powerful optimization tool that helps evaluate the performance of similar organizations or systems, known as decision-making units (DMUs), by analyzing their inputs and outputs [13]. One of its key strengths is that it does not require assumptions about how inputs relate to outputs, making it a flexible and widely used approach [7, 12, 31]. Beyond simply measuring efficiency, DEA provides valuable insights that help decision-makers allocate resources more effectively.

For example, an insurance company's efficiency depends on interconnected processes such as risk assessment, claims management, and policy underwriting, which traditional models often fail to account for. DEA remains a highly versatile tool for analyzing the efficiency of insurance companies [8]. It does not rely on prior assumptions about the relationships between inputs and outputs, can evaluate multiple inputs and outputs simultaneously, and performs effectively even when key financial data, such as premium costs or claim expenses, is incomplete or unavailable. Moreover, it is flexible enough to compare insurance companies of varying sizes and scales, rank their performance, and provide actionable benchmarks for improving underperforming entities [12].

2.2. Network DEA

Traditional DEA research often overlooks the internal structures of DMUs and treats the entire DMU system as a black box. Recent focus has shifted towards evaluating the performance and efficiency of network systems. Kao and Hwang [26] categorize network DEA models into three groups. The first group acknowledges network processes but computes overall and individual efficiencies independently using conventional DEA models, with Seiford and Zhu [40] serving as a typical example. The second group considers interconnected process interactions when calculating system efficiency [29, 41]. Alves and Meza [2] provided a systematic review of slack-based measures (SBM) for network DEA models which can shed light on the wide range of applications of SBM. The third group assumes some form of mathematical relationship between the overall system efficiency and component efficiencies. For instance, Kao and Hwang [25], An *et al.* [4], Wu *et al.* [44], Sahoo *et al.* [39], Despotis *et al.* [16], and Liang *et al.* [30] use a multiplicative form, while Chen *et al.* [11] and Cook *et al.* [14] opt for a weighted average approach.

In the context of production systems with a network structure, one important area of study involves the allocation of shared resources between production stages. Amirteimoori *et al.* [3] presented additive models that gauge the performance of two-stage network DEA processes involving shared inputs. Chao *et al.* [10] introduced a model for decomposing shipping service production in a container shipping company with shared inputs. Wang *et al.* [43] delved into the examination of the monotonicity of decomposition weights in a two-stage DEA model with shared resources. Zhao *et al.* [50] explored operational costs as shared resources for assessing performance in the Chinese banking industry with a network structure.

A fundamental type of network structure is the parallel system, where a production process or DMU consists of a group of sub-units. Currently, all relevant research in this field has assumed that all parallel sub-units' function independently, using the same inputs to generate corresponding outputs. The inputs and outputs of each DMU can be derived by aggregating those of its individual sub-units [9, 24]. Du *et al.* [18] introduced a series of DEA models designed to accommodate scenarios where non-homogenous subunits operate within parallel network structures with intermediate measures or links. Imanirad *et al.* [22] suggest that inputs (or resources) can be divided and allocated across sub-units in a parallel system. However, this assumption may constrain the broader applicability of the methodology in cases where non-separable input measures are involved.

In their study, Ma and Chen [32] conceptualize an insurance company as a two-stage system composed of three sub-systems. The first stage consists of two independent sub-systems operating in parallel, and these are then connected to the second stage or the third sub-system in series. To demonstrate the efficiency formation mechanism of the parallel-series system when evaluating efficiency measures, they propose the combined application of additive and multiplicative DEA models.

Considering the allocation of shared resources in parallel network production system is an important area that merits attention. Bi *et al.* [6] and Xiong *et al.* [45] investigate the generation of shared resource allocation and target setting plans in parallel network system. Ding *et al.* [17] introduce a new model in a parallel network production system based on the multiplier- type frameworks and resource allocation variable. Yu *et al.* [48] introduce a novel approach demonstrating the appropriate allocation of fixed costs across all DMUs within a parallel network structure when considering efficiency. Phung *et al.* [36] introduced a new DEA modelling technique to address a mixed network structure, encompassing both serial and parallel processes with shared input resources.

3. Three-stage network process with shared inputs and intermediate products

3.1. Problem description

Problem Description Resource allocation in parallel-series production systems presents significant challenges, as these systems involve complex interactions between processes and the sharing of resources. A key problem is how to efficiently allocate these shared resources across interconnected processes to maximize overall system efficiency. Traditional resource allocation methods, like standard DEA models, fall short in this context. They often ignore the critical interdependencies created by shared inputs and outputs, and they do not offer a unified approach for both allocating resources and evaluating performance. Existing techniques also struggle with these challenges in multi-tiered systems. Consequently, a gap exists in providing practical optimization tools for systems characterized by shared resource needs across their interconnected processes. This paper addresses this problem with a novel DEA-based approach that explicitly considers shared inputs and outputs within parallel-series production systems. This method aims to provide managers with a way to optimize resource allocation and accurately assess performance within the complex setting of interconnected production processes. The challenge of optimal resource allocation in parallel-series production systems, characterized by the interdependence of production lines and shared inputs and outputs, is therefore the central focus of this research.

3.2. Technology set

This section is devoted to examining the distinctive production possibility set of our network process depicted in Figure 1. Our network process consists of three stages arranged in series with the first stage being a combination of two parallel stages. Specifically, stages 1 and 2 are fed by their dedicated inputs along with a shared resource (which is jointly consumed by these two parallel stages) to produce an intermediate product that is used as input to the third stage. The final outputs are produced from the third stage. Note that both parallel components (stages 1 and 2) are involved in the generation of intermediate products.

In the following, we present the appropriate production technology set to the above-mentioned production system. Consider production technology $T \subset R_+^{m+h+f+s}$ with the m-tuple initial inputs X_j, h-tuple shared intermediate products Z_j, and s-tuple final outputs Y_j. The technology set T consists of all $(X, Z, Y) \in T$ where $X \in R_+^m$, $Z \in R_+^h$, and $Y \in R_+^s$ are vectors of inputs, intermediate product, and outputs, respectively. Specifically, assume that we have n observed DMUs denoted by (X_j, Z_j, Y_j), $j \in \{1, \ldots, n\}$ with $X_j \neq 0$, $Z_j \neq 0$, and $Y_j \neq 0$ for all j. Due to the existence of shared inputs and intermediate products in parallel section, the set of input variables is partitioned as $I = I^S \cup I^{D1} \cup I^{D2}$, in which I^S , I^{D1} and I^{D2} are the index sets of

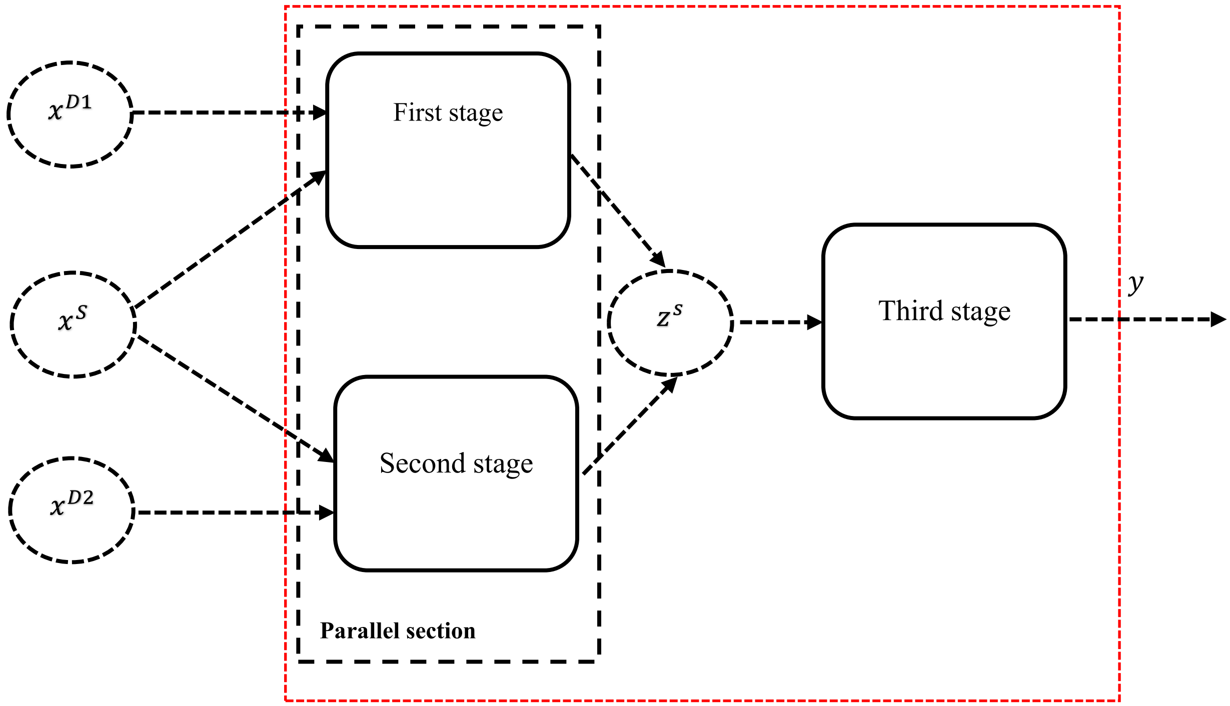

FIGURE 1. Three-stage network process with shared inputs.

shared resources and dedicated resources to the first and second stages, respectively. X^{D1}, X^{D2} and X^S are respectively the dedicated and shared input vectors in parallel section.

In performance evaluation using tools such as DEA, the selection of underlying model is important. When most of the DMUs do not perform in optimal scale, the variable returns to scale models are more appropriate to analyze the performances of the DMUs than the constant returns to scale models. In this sense, we focus our discussion on the case of variable returns to scale (VRS). The production technology set in the parallel section with dedicated and shared inputs and shared intermediate products satisfies the following axioms:

Axiom 1 (Feasibility of Observed DMUs). $(X_j, Z_j) \in T^{parallel}$, for any j.

Axiom 2 (Strong Disposability). If $(X, Z) \in T^{parallel}$, $X \leq \acute{X}$ and $\acute{Z} \leq Z$ then $\left(\acute{X}, \acute{Z}\right) \in T^{parallel}$.

Defining convex combination of stages in parallel section that share certain inputs and intermediate products in unknown proportions is clearly problematic. We used multicomponent convexity proposed by Papaioannou and Podinovski [35].

Axiom 3 (Multicomponent convexity). Consider any vectors $\lambda^k = \left(\lambda_1^k, \lambda_2^k, \ldots, \lambda_n^k\right) \in R_+^n$, $k = 1,2$; such that $\sum_{j=1}^{n} \lambda_j^k = 1$, $k = 1,2$. Then, the vector $\left(X^D, X^S, Z^S\right)$ with the following components belongs to $T^{parallel}$.

$$X^D = \sum_{j=1}^{n} \lambda_j^k X_j^{Dk}, \quad k = 1,2,$$

$$X^S = \sum_{j=1}^{n} \bar{\Lambda}_j X_j^{Sk}, \quad k = 1,2$$

$$Z^S = \sum_{j=1}^{n} \underline{\Lambda}_j Z_j^{Sk}, \quad k = 1, 2. \tag{1}$$

The $\bar{\Lambda}_j$ and $\underline{\Lambda}_j$ are the optimal objective values of the following two linear programming problems:

$$\overline{\Lambda}_j = \text{Max} \left\{ \sum_{j=1}^{n} \lambda_j^k \alpha_j^k \Big| \sum_{j=1}^{n} \alpha_j^k = 1, \underline{\alpha}_j^k \le \alpha_j^k \le \overline{\alpha}_j^k, \alpha_j^k \ge 0, \quad k = 1, 2 \right\} \tag{2}$$

$$\underline{\Lambda}_j = \text{Min} \left\{ \sum_{j=1}^{n} \lambda_j^k \beta_j^k \Big| \sum_{j=1}^{n} \beta_j^k = 1, \underline{\beta}_j^k \le \beta_j^k \le \overline{\beta}_j^k, \beta_j^k \ge 0, \quad k = 1, 2 \right\} \tag{3}$$

α_j^k and β_j^k denote the proportion of the shared inputs X_j^s and the shared intermediate products Z_j^s, where $0 \le \underline{\alpha}_j^k \le \overline{\alpha}_j^k \le 1$ and $0 \le \underline{\beta}_j^k \le \overline{\beta}_j^k \le 1$, for all j and $k = 1, 2$.

We are currently focused on determining the maximum quantities of shared inputs and minimum quantities of shared outputs achievable across all potential distributions of the shared measures among the parallel processes. These quantities remain unaffected by inputs and shared intermediary products and are consistent with the optimal values $\bar{\Lambda}_j$ and $\underline{\Lambda}_j$ in the subsequent linear programs formulated individually for each j. In these programs, the optimization is carried out concerning variables (α_j^k) and (β_j^k) (with the subscripts j and i omitted as they are unnecessary), while the components λ_j^k remain constant for all j and $(k \in \{1, 2\})$ [35].

The conditions (1) (X^S and Z^S) are not linear because at the first step the calculation of the terms $\overline{\Lambda}_j$ and $\underline{\Lambda}_j$ can be obtained by solving the dual program to programs (2) and (3). Linearized form of the $\overline{\Lambda}_j$ and $\underline{\Lambda}_j$ are (in which the terms λ_j^k are constant, for all j and $(k \in \{1, 2\})$ [35]):

$$\begin{aligned} \overline{\Lambda}_j = \text{Min}\ & \varphi_j - \sum_{k=1}^{2} \underline{\alpha}_j^k \pi_j^k + \sum_{k=1}^{2} \overline{\alpha}_j^k \rho_j^k \\ \text{s.t.}\ & \varphi_j - \pi_j^k + \rho_j^k \ge \lambda_j^k, \quad k = 1, 2, \\ & \varphi_j, \pi_j^k, \rho_j^k \ge 0, \quad k = 1, 2. \end{aligned} \tag{4}$$

$$\begin{aligned} \underline{\Lambda}_j = \text{Max}\ & \psi_j - \sum_{k=1}^{2} \underline{\beta}_j^k \sigma_j^k + \sum_{k=1}^{2} \overline{\beta}_j^k \tau_j^k \\ \text{s.t.}\ & \psi_j + \sigma_j^k - \tau_j^k \le \lambda_j^k, \quad k = 1, 2, \\ & \psi_j, \sigma_j^k, \tau_j^k \ge 0, \quad k = 1, 2. \end{aligned} \tag{5}$$

Technology $T^{parallel}$ is the set of all DMUs $(X, Z, Y) \in R_+^{m+h+f}$ for which there exist vectors $\lambda^k \in R_+^n$ $(k = 1, 2)$ and scalers $\varphi_j, \psi_j, \pi_j^k, \sigma_j^k, \rho_j^k$ and τ_j^k, $k = 1, 2$, such that

$$\begin{aligned} T^{parallel} = \Bigg\{ & \sum_{j=1}^{n} \left(\varphi_j - \sum_{k=1}^{2} \underline{\alpha}_j^k \pi_j^k + \sum_{k=1}^{2} \overline{\alpha}_j^k \rho_j^k \right) X_j^{Sk} \le X^S, \\ & \sum_{j=1}^{n} \lambda_j^k X_j^{Dk} \le X^D, \quad k = 1, 2, \\ & \sum_{j=1}^{n} \left(\psi_j - \sum_{k=1}^{2} \underline{\beta}_j^k \sigma_j^k + \sum_{k=1}^{2} \overline{\beta}_j^k \tau_j^k \right) Z_j^{Sk} \ge Z^S, \\ & \sum_{j=1}^{n} \lambda_j^k = 1, \quad k = 1, 2, \end{aligned}$$

$$\begin{aligned}
&\lambda_j^k - \varphi_j + \pi_j^k - \rho_j^k \leq 0, \quad k=1,2,\\
&-\lambda_j^k + \psi_j + \sigma_j^k - \tau_j^k \leq 0, \quad k=1,2\\
&\lambda_j^k, \varphi_j, \psi_j, \pi_j^k, \sigma_j^k, \rho_j^k, \tau_j^k \geq 0, \quad k=1,2\big\}.
\end{aligned} \tag{6}$$

The production technology in the final section with intermediate products as inputs and final outputs satisfies the following axioms:

Axiom 1* (Feasibility of Observed DMUs). $(Z_j, Y_j) \in T^{final}$, for any j.

Axiom 2* (Strong Disposability). If $(Z, Y) \in T^{final}$, $\acute{Z} \geq Z$ and $\acute{Y} \leq Y$ then $\left(\acute{Z}, \acute{Y}\right) \in T^{final}$.

Axiom 3* (Convexity). Let $\left(\tilde{Z},\tilde{Y}\right) \in T^{final}$ and $\left(\hat{Z},\hat{Y}\right) \in T^{final}$ then $\omega\left(\tilde{Z},\tilde{Y}\right) + (1-\omega)\left(\hat{Z},\hat{Y}\right) \in T^{final}$ for any $\omega \in [0,1]$.

$$T^{final} = \left\{ Z^S \geq \sum_{\text{j}} \lambda_j^3 Z_j, \quad Y \leq \sum_{\text{j}} \lambda_j^3 Y_j, \quad \sum_{\text{j}} \lambda_j^3 = 1 \right\}. \tag{7}$$

According to the $T^{parallel}$ and T^{final}, the overall production technology of the three-stage network system can be defined as the following:

$$\begin{aligned}
T^{overall} = \Bigg\{ &\sum_{j=1}^{n} \left(\varphi_j - \sum_{k=1}^{2} \underline{\alpha}_j^k \pi_j^k + \sum_{k=1}^{2} \overline{\alpha}_j^k \rho_j^k \right) X_j^{Sk} \leq X^S,\\
&\sum_{j=1}^{n} \lambda_j^k X_j^{Dk} \leq X^D,\\
&\sum_{j=1}^{n} \left(\psi_j - \sum_{k=1}^{2} \underline{\beta}_j^k \sigma_j^k + \sum_{k=1}^{2} \overline{\beta}_j^k \tau_j^k \right) Z_j^{Sk} \geq \sum_{\text{j}} \lambda_j^3 Z_j,\\
&\sum_{\text{j}} \lambda_j^3 Y_j \geq Y\\
&\sum_{j=1}^{n} \lambda_j^k = 1, \quad k=1,2,3,\\
&\lambda_j^k - \varphi_j + \pi_j^k - \rho_j^k \leq 0, \quad k=1,2\\
&-\lambda_j^k + \psi_j + \sigma_j^k - \tau_j^k \leq 0, \quad k=1,2\\
&\lambda_j^3, \lambda_j^k, \varphi_j, \psi_j, \pi_j^k, \sigma_j^k, \rho_j^k, \tau_j^k \geq 0, \quad k=1,2\big\}.
\end{aligned} \tag{8}$$

3.3. Resource allocation network model with Shared inputs and outputs

Based on the technology $T^{overall}$, we evaluate the input radial efficiency for the production system as shown in Figure 1:

$$\begin{aligned}
\theta^* = &\text{ Min } \theta\\
&\text{s.t} \sum_{j=1}^{n} \lambda_j^k X_j^k \leq \theta X_o^k, \;\; k=1,2,\\
&\sum_{j=1}^{n} (\varphi_j - \sum_{k=1}^{2} \underline{\alpha}_j^k \pi_j^k + \sum_{k=1}^{2} \overline{\alpha}_j^k \rho_j^k) X_j^s \leq \theta X_o^s,
\end{aligned}$$

Table 1. DMUs in the numerical example.

DMU	$X^{(1)}$	$X^{(2)}$	X^s	Z	Y
A	10	10	2	10	8
B	5	6	4	12	7
C	4	5	3	8	10
D	10	6	9	6	4

$$
\begin{aligned}
&-\sum_{j=1}^{n}(\psi_j - \sum_{k=1}^{2}\underline{\beta}_j^k\sigma_j^k + \sum_{k=1}^{2}\overline{\beta}_j^k\tau_j^k)Z_j + \sum_{j=1}^{n}\lambda_j^3 Z_j \leq 0\\
&\sum_{j=1}^{n}\lambda_j^3 Y_j \geq Y_o\\
&\sum_{j=1}^{n}\lambda_j^k = 1, \quad k = 1,2,3,\\
&\lambda_j^k - \varphi_j + \pi_j^k - \rho_j^k \leq 0,\ j = 1,\ldots,n;\ k = 1,2\\
&-\lambda_j^k + \psi_j + \sigma_j^k - \tau_j^k \leq 0, \quad j = 1,\ldots,n; k = 1,2\\
&\lambda_j^3, \lambda_j^k, \varphi_j, \psi_j, \pi_j^k, \sigma_j^k, \rho_j^k, \tau_j^k \geq 0, \quad j = 1,\ldots,n; k = 1,2.
\end{aligned}
\tag{9}
$$

We assume that the bounds $\underline{\alpha}_j^k$, $\overline{\alpha}_j^k$, $\underline{\beta}_j^k$, and $\overline{\beta}_j^k$ are unknown and it is necessary for these is that $\sum_{k=1}^{2}\underline{\alpha}_j^k \leq 1$, $\sum_{k=1}^{2}\overline{\alpha}_j^k \geq 1$, $\sum_{k=1}^{2}\underline{\beta}_j^k \leq 1$ and $\sum_{k=1}^{2}\overline{\beta}_j^k \geq 1$.

4. Simple numerical example

This section focuses on a simple numerical example for a three-stage network process. We use four DMUs with three inputs $(X^{(1)}, X^{(2)}, X^s)$, one intermediate outputs (Z), and one final output (Y). The data is listed in Table 1.

Dedicated input $X^{(1)}$ is specific to the first stage, dedicated input $X^{(2)}$ is specific to the second stage, and Y is the final output. Z is the intermediate variable that is produced from one stage and is used as input by the later stage. The input X^s is shared input and both stages are fed by this resource. To explore the effect of bounds on the discriminating power of the resulting model, we consider three sets of bounds. We first use wide bounds in which, for each DMU_j in the parallel stage $k = 1, 2$, we take both lower bounds $\underline{\alpha}_j^k$ and $\underline{\beta}_j^k$ equal to 0.1 and we take the upper bounds $\overline{\alpha}_j^k$ and $\overline{\beta}_j^k$ equal to 1. This means that the first and second stages use at least 10% of the shared inputs. In medium bounds, we take both lower bounds $\underline{\alpha}_j^k$ and $\underline{\beta}_j^k$ equal to 0.3 and we take the upper bounds $\overline{\alpha}_j^k$ and $\overline{\beta}_j^k$ equal to 1. This also means that the first and second stages use no less than 30% of the shared inputs. Finally, in narrow bound, we take both lower bounds $\underline{\alpha}_j^k$ and $\underline{\beta}_j^k$ equal to 0.5 and the upper bounds $\overline{\alpha}_j^k$ and $\overline{\beta}_j^k$ equal to 1. Apart from all these bounds, we can consider $\underline{\alpha}_j^k = \underline{\beta}_j^k = 0$ and $\overline{\alpha}_j^k = \overline{\beta}_j^k = 1$, for all DMUs. If assume that $\underline{\alpha}_j^k = \underline{\beta}_j^k = \overline{\alpha}_j^k = \overline{\beta}_j^k = 0$, this means the model (1) cannot allocate shared inputs and intermediate products to the stages. Four different scenarios have been considered as bounds in the envelopment model (1), the wide, medium, narrow along with zero bounds $\underline{\alpha}_j^k, \underline{\beta}_j^k, \overline{\alpha}_j^k$ and $\overline{\beta}_j^k$. The technical efficiencies obtained from these models are denoted by θ^{wide}, θ^{medium}, θ^{narrow}, and θ^{O}, respectively. The results are given in Table 2.

As the results show, out of four DMUs, only one DMU is efficient in the scenario η^{O}. DMU A, in wide, medium, and narrow bounds is efficient but when $\underline{\alpha}_j^k = \underline{\beta}_j^k = \overline{\alpha}_j^k = \overline{\beta}_j^k = 0$ it prevailed as inefficient. Consider narrow bounds, after solving the model, the first DMU use 2 units of the shared input in the first stage and 3 units of the shared input in the second stage, and the second DMU use 2 units of the shared input in the first stage and 3 units of the shared input in the second stage. The production of total amount of shared intermediate product in the first stage and second stage are 34 and 34, respectively. In this case, regarding model (1), the following customized optimization model is solved for first

TABLE 2. Output radial efficiency in the VRS models.

DMU	θ^{O}	θ^{wide}	θ^{medium}	θ^{narrow}
A	0.5	1	1	1
B	0.83	0.83	0.83	0.83
C	1	1	1	1
D	0.83	0.83	0.83	0.83

DMU with wide bounded:

$$\begin{aligned}
&\theta_A^* = \min\ \theta \\
&\text{Subject to,} \\
&\text{Life stage constraints :} \\
&\text{Input } (X^L): \\
&10\lambda_1^1 + 5\lambda_2^1 + 4\lambda_3^1 + 10\lambda_4^1 \leq 10\theta \\
&\text{Non-life stage constraints :} \\
&\text{Input } (X^{NL}): \\
&10\lambda_1^2 + 6\lambda_2^2 + 5\lambda_3^2 + 6\lambda_2^2 \leq 10\theta \\
&\text{Shared input}(X^s): \\
&2\Big(\varphi_1 - \big(0.1\pi_1^1 + 0.1\pi_1^2\big) + \big(\rho_1^1 + \rho_1^2\big)\Big) + 4\Big(\varphi_2 - \big(0.1\pi_2^1 + 0.1\pi_2^2\big) + \big(\rho_2^1 + \rho_2^2\big)\Big) + \\
&3\Big(\varphi_3 - \big(0.1\pi_3^1 + 0.1\pi_3^2\big) + \big(\rho_3^1 + \rho_3^2\big)\Big) + 9\Big(\varphi_1 - \big(0.1\pi_1^1 + 0.1\pi_1^2\big) + \big(\rho_1^1 + \rho_1^2\big)\Big) \leq 2 \\
&\text{Shared intermediate product } (Z): \\
&-10\Big(\psi_1 - \big(0.1\sigma_1^1 + 0.1\sigma_1^2\big) + \big(\tau_1^1 + \tau_1^2\big)\Big) - 12\Big(\psi_2 - \big(0.1\sigma_2^1 + 0.1\sigma_2^2\big) + \big(\tau_2^1 + \tau_2^2\big)\Big) - \\
&8\Big(\psi_3 - \big(0.1\sigma_3^1 + 0.1\sigma_3^2\big) + \big(\tau_3^1 + \tau_3^2\big)\Big) - 6\Big(\psi_4 - \big(0.1\sigma_4^1 + 0.1\sigma_4^2\big) + \big(\tau_4^1 + \tau_4^2\big)\Big) + \\
&\Big(10\lambda_1^3 + 12\lambda_2^3 + 8\lambda_3^3 + 6\lambda_4^3\Big) \leq 0 \\
&\text{Third stage output } (Y): \\
&8\lambda_1^3 + 7\lambda_2^3 + 10\lambda_3^3 + 4\lambda_4^3 \geq 8 \\
&\text{Intensity constraint :} \\
&\lambda_1^1 + \lambda_2^1 + \lambda_3^1 + \lambda_4^1 = 1, \quad \lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_4^2 = 1, \quad \lambda_1^3 + \lambda_2^3 + \lambda_3^3 + \lambda_4^3 = 1 \\
&\lambda_1^1 - \varphi_1 + \pi_1^1 - \rho_1^1 \leq 0, \quad \lambda_1^2 - \varphi_1 + \pi_1^2 - \rho_1^2 \leq 0, \quad \lambda_2^1 - \varphi_2 + \pi_2^1 - \rho_2^1 \leq 0, \\
&\lambda_2^2 - \varphi_2 + \pi_2^2 - \rho_2^2 \leq 0, \quad \lambda_3^1 - \varphi_3 + \pi_3^1 - \rho_3^1 \leq 0, \quad \lambda_3^2 - \varphi_3 + \pi_3^2 - \rho_3^2 \leq 0, \\
&\lambda_4^1 - \varphi_4 + \pi_4^1 - \rho_4^1 \leq 0, \quad \lambda_4^2 - \varphi_4 + \pi_4^2 - \rho_4^2 \leq 0.
\end{aligned}$$

5. An application in insurance sector

In this section, the real applicability of the proposed approach is demonstrated by a real case on 22 Indian insurance companies in 2020. In each insurance company, life and non-life insurance sectors do business separately and parallely to produce income and the obtained income are used to invest in the investment sector. The first stage (life insurance sector) uses operational expenses (OPEXP), claim inquired (CLMINQ), and part of staff expenses as inputs to produce total income (TOTINC) as intermediate outputs. These incomes are used to invest in the third stage (investment sector). Incomes from investment are considered as final outputs. The work process is depicted in Figure 2. Table 3 summarizes the descriptive statistics of the data.

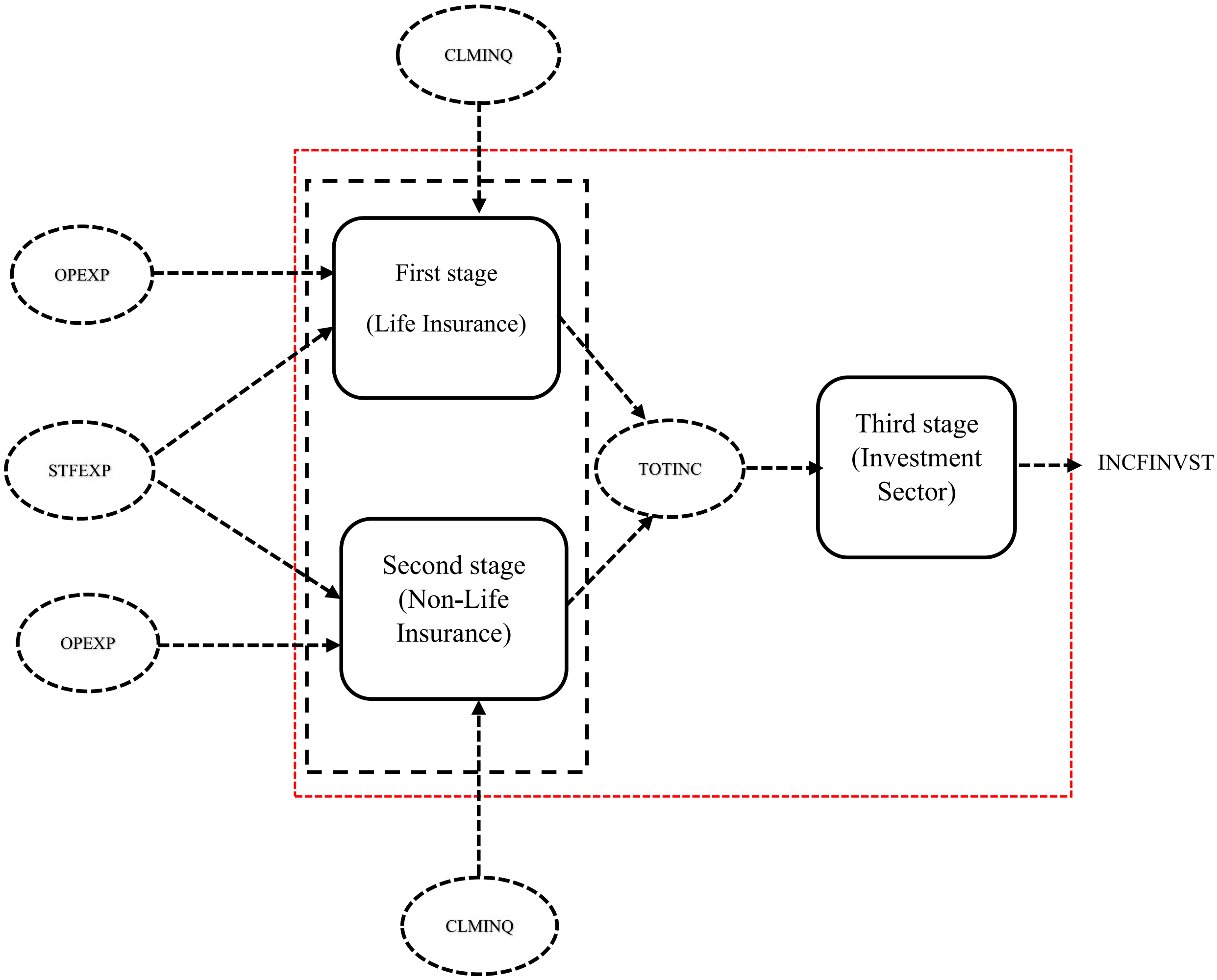

FIGURE 2. Three-stage network process with shared inputs.

We have considered three sets of bounds to explore the effect of bounds on the discriminating power of the resulting models. We first considered wide bounds: for each insurance company j and each of its parallel stages k, we take both lower bounds $\underline{\alpha}_j^k$ and $\underline{\beta}_j^k$ equal to 0.002 and we take the upper bounds $\overline{\alpha}_j^k$ and $\overline{\beta}_j^k$ equal to 1. In the second attempt, we defined medium bounds $\underline{\alpha}_j^k$ and $\underline{\beta}_j^k$ to be equal to 0.003 and we take the upper bounds $\overline{\alpha}_j^k$ and $\overline{\beta}_j^k$ equal to 0.68. Finally, we defined narrow bounds $\underline{\alpha}_j^k$ and $\underline{\beta}_j^k$ to be equal to 0.037 and we take the upper bounds $\overline{\alpha}_j^k$ and $\overline{\beta}_j^k$ as 0.54. The results are given in Table 4.

As the results show, three companies, 2, 11, and 12, are efficient in all four scenarios. However, there are companies that are efficient in some scenarios, while inefficient in others.

This observation also highlighted the intricate relationship between efficiency and resource allocation outcomes, shedding light on the trade-offs involved when making decisions under varying levels of allocation. It underscores the importance of carefully considering allocating levels in resource allocation strategies.

TABLE 3. Data for insurance companies.

	Inputs				Intermediate measure	Final output	
Company	OPEXP-LI	STFEXP	OPEXP-NLI	CLMINQ-L	CLMINQ-NL	TOT-INCOM	INCOMFINVST
1	918	2767	1244	1090	3665	13367	3894
2	245	852	421	280	374	3347	347
3	408	1333	634	1090	1921	7549	1571
4	1346	3980	1763	6714	8477	30016	5802
5	535	1653	757	394	499	5505	689
6	254	997	525	1723	2550	8485	1430
7	487	1659	809	1462	1653	8196	1608
8	219	1075	621	455	516	4217	554
9	1281	7101	4267	3173	4662	27347	9363
10	1617	5714	2847	9303	10074	39494	11533
11	183	548	245	697	349	2744	711
12	229	923	492	3094	154	6724	1586
13	553	2575	1459	3572	1854	13526	2196
14	2761	8101	3568	22549	25016	82573	237878
15	1204	4541	2344	1621	2131	15896	5467
16	542	1855	907	1335	1605	8658	1827
17	237	726	330	394	525	3144	278
18	1327	3748	1601	3084	3272	17506	3613
19	1103	4501	2413	6175	8780	30826	9410
20	187	891	509	686	987	4423	475
21	211	740	367	1035	1441	5453	809
22	435	2594	1592	2367	2690	13176	2399

5.1. Wide bounded

Figure 3 depicts a comparison of inefficiency scores in two scenarios where the bounds for shared inputs and shared intermediate products are widely considered and when no bounds are considered. Figure 3 provides a visual representation of the contrasting inefficiency measures resulting from these different approaches. Examining the disparities in inefficiency scores highlighted the importance of incorporating appropriate bounds in the analysis. Companies 2, 11, 12, 14, and 17 are efficient in wide bounded cases but unit 14 is inefficient when no bound is considered.

5.2. Medium bounded

Figure 4 depicts a comparison of inefficiency scores in two scenarios where the bounds for shared inputs and shared intermediate products are considered as medium and no bounds. Comparing the results of medium bounds with wide bounds, the efficiency scores gained from the model (1) when considering medium bounds are closer to time when no bounds are considered. In this case, companies 2, 11, 12, 14, and 17 are efficient.

5.3. Narrow bounded

The outcomes of model (1), when considering a narrow bound, closely align with those obtained when no bound is considered for the shared input and shared intermediate product (Fig. 5). Companies 2, 11, 12, and 14 are efficient in this scenario. Upon examining these three bounds within model (1), the reduction in shared resource bounds within model (1) appears to demonstrate greater efficacy in pinpointing inefficiency factors.

Tables 5–7 present the image points of shared input and shared intermediate product within the wide, medium, and narrow bounds, respectively. It is evident that as the selected bounds converge, the results appear increasingly rational. Additionally, Figure 6 provides a comparison of the total shared input and total shared intermediate product within the three distinct bounds. In the narrow-bound mode, a more suitable total amount of shared intermediate product is achieved while consuming a lesser the total amount of shared input.

TABLE 4. Output radial efficiency in the VRS models.

DMU	η^{O}	η^{wide}	η^{medium}	η^{narrow}
1	0.2653835	0.3434599	0.3429146	0.2967904
2	1	1	1	1
3	0.5028059	0.5032219	0.5032219	0.5028059
4	0.1389677	0.1794278	0.1767807	0.1724582
5	0.7106599	0.7106599	0.7106599	0.7106599
6	0.7204724	0.7227424	0.7204724	0.7204724
7	0.406931	0.4069348	0.406931	0.406931
8	0.9999331	0.9999331	0.9999331	0.9999331
9	0.1635306	0.2132333	0.2052592	0.1635348
10	0.1131725	0.1523923	0.1523923	0.1474528
11	1	1	1	1
12	1	1	1	1
13	0.3309222	0.3330756	0.3309222	0.3309222
14	0.06866592	1	1	1
15	0.183587	0.248161	0.248149	0.1983587
16	0.3870829	0.3870855	0.3870829	0.3870829
17	0.9697218	1	1	0.9705495
18	0.1605295	0.2099532	0.2052855	0.1922978
19	0.1659112	0.1905691	0.1883003	0.1826199
20	0.9918182	0.9918182	0.9918182	0.9918182
21	0.8672986	0.8676710	0.8672986	0.8672986
22	0.4206897	0.4238014	0.4206897	0.4206897

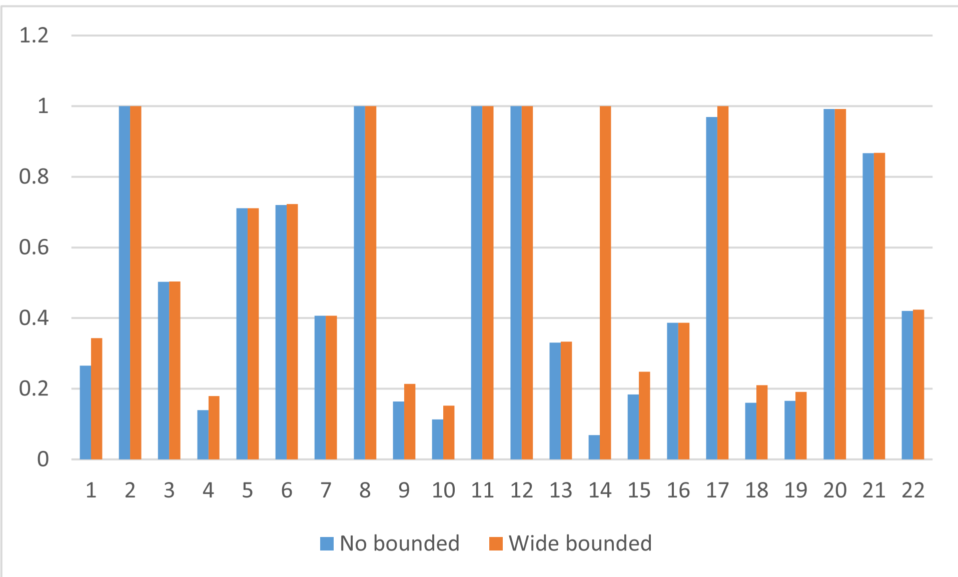

FIGURE 3. Input efficiency trends of wide and without bounds of commercial banking.

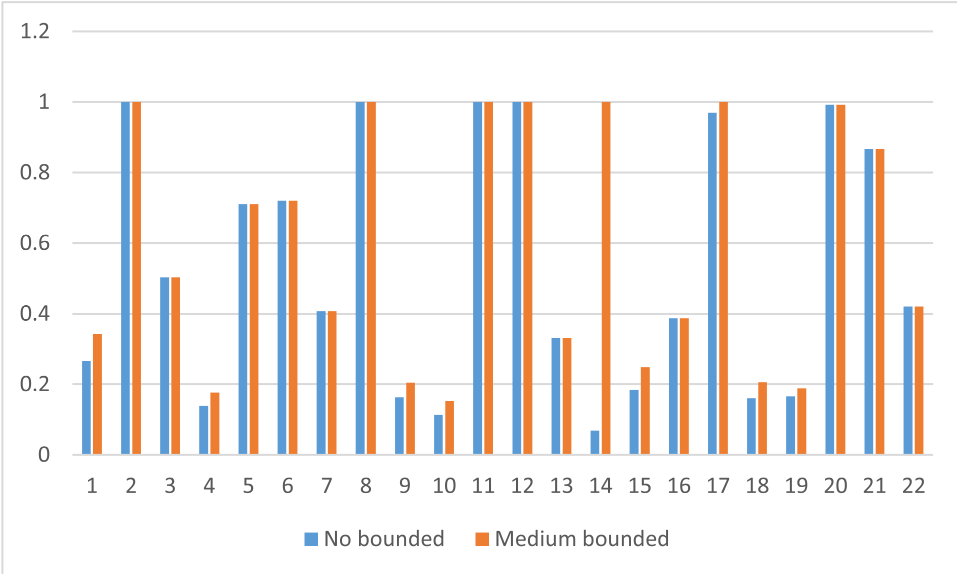

FIGURE 4. Input efficiency trends of medium and without bounds of commercial banking.

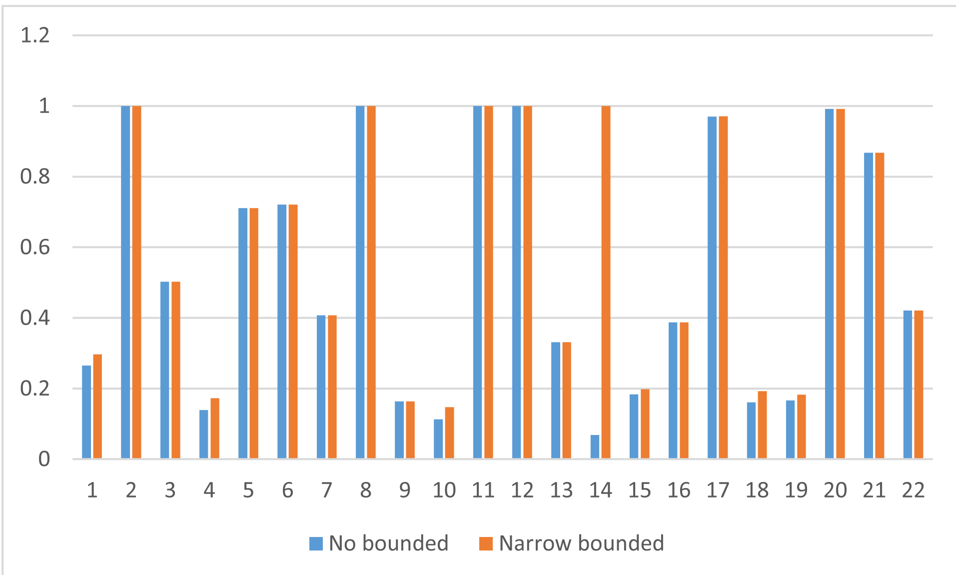

FIGURE 5. Input efficiency trends of narrow and without bounds of commercial banking.

As evident in Figure 7, the enhancement of the total shared input corresponds to an improvement in the production of the total shared intermediate product with initial values.

The results indicated that when we use the wider bounds on shared inputs and intermediate products, our model determines the portion of both stages equally. While wide bounds may be perceived as preferable to having no bounds, they might be overly cautious and could potentially be narrowed without compromising the model's realism. The medium bounds, on the other hand, offer greater stringency while still maintaining a sense of reliability. Narrow bounds may

Table 5. The optimal values of shared inputs and shared intermediate products with wide bounds.

DMU	Shared input		Shared intermediate product	
	First stage	Second stage	First stage	Second stage
1	903.7293	903.7293	3815.379	3815.379
2	852	852	6660.145	6660.145
3	670.7948	670.7948	3033.471	3033.471
4	710.1319	710.1319	4457.601	4457.601
5	852	1590.053	3347	8975.599
6	597.44	597.44	2986.011	2986.011
7	522.8484	522.8484	2466.39	2466.39
8	724.4443	724.4443	3093.987	3093.987
9	1400.183	1400.183	5656.212	5656.212
10	870.7694	870.7694	6386.621	6386.621
11	548	548	2744	2744
12	923	923	6724	6724
13	650.1119	650.1119	3243.842	3243.842
14	8101	8101	82573	82573
15	1126.526	1126.526	4344.841	4344.841
16	715.2984	715.2984	3119.639	3119.639
17	726	726	3144	3144
18	703.7586	703.7586	3720.796	3720.796
19	857.7358	857.7358	5672.032	5672.032
20	560.111	560.111	2768.023	2768.023
21	712.2828	712.2828	2776.986	2776.986
22	664.0706	664.0706	3312.171	3312.171
Total	49522.53		337720.9	

Table 6. The optimal values of shared inputs and shared intermediate products with medium bounds.

DMU	Shared input		Shared intermediate product	
	First stage	Second stage	First stage	Second stage
1	956.6609	956.6609	3958.65	3958.65
2	852	852	3347	3347
3	670.7948	670.7948	3033.471	3033.471
4	724.6767	652.9144	4982.745	4224.276
5	852	858.2547	3347	3644.494
6	548	622.4341	2744	3530.707
7	622.4084	619.6493	2891.593	2913.323
8	724.4443	724.4443	3093.987	3093.987
9	1342.404	1497.472	5436.37	6242.231
10	381.5678	381.5678	2781.734	2781.734
11	548	548	2744	2744
12	923	923	6724	6724
13	548	851.3383	2744	5837.219
14	8101	8101	82573	82573
15	1126.609	1126.609	4345.436	4345.436
16	679.4012	714.7704	3004.641	3378.465

TABLE 6. Continued.

DMU	Shared input		Shared intermediate product	
	First stage	Second stage	First stage	Second stage
17	726	726	3144	3144
18	780.5158	701.0793	4019.669	3733.499
19	769.5505	879.9982	5312.927	6480.269
20	560.111	696.0034	2768.023	4204.294
21	548	558.1454	2744	2851.229
22	548	828.8035	2744	4738.82
Total	48024.08		326008.4	

TABLE 7. The optimal values of shared inputs and shared intermediate products with narrow bounds.

DMU	Shared input		Shared intermediate product	
	First stage	Second stage	First stage	Second stage
1	803.9194	748.1977	3269.537	4865.434
2	852	852	3347	3347
3	656.5809	671.4281	2959.376	3121.644
4	724.2152	640.9213	4935.409	4055.061
5	852	858.2547	3347	3644.494
6	548	765.0995	2744	3559.092
7	622.4084	716.2395	2891.593	3684.156
8	724.4443	773.5816	3093.987	3443.057
9	812.299	1307.609	3432.275	8863.837
10	843.6398	822.8182	6627.06	6296.253
11	548	548	2744	2744
12	837.4674	923	6129.946	6724
13	548	722.0007	2744	3958.503
14	8101	8101	82573	82573
15	821.717	854.4199	3286.932	6110.837
16	679.4012	702.4444	3004.641	3256.487
17	776.1786	548	3197.887	2744
18	740.3705	645.3296	3744.708	3963.302
19	691.4798	926.7909	4515.597	7066.644
20	560.111	816.5513	2768.023	3752.266
21	548	640.5203	2744	3091.364
22	548	1202.862	2744	5202.655
Total	47626.3		332911.1	

be justifiable, but further research into the allocation of shared resources across all insurances would be necessary to validate their realism [35].

6. CONCLUDING REMARKS

One of the most widely used extensions of DEA is the performance analysis in two-stage production processes using DEA. The existence of parallel and serially connected production processes makes the process complicated and there is a need to provide models and methods to evaluate the performance of such processes. In this paper, we considered a parallel-series production processes in which two parallel processes are serially connected to the third component and in addition to component-specific resources, there is a shared resource that must be shared between the two parallel components.

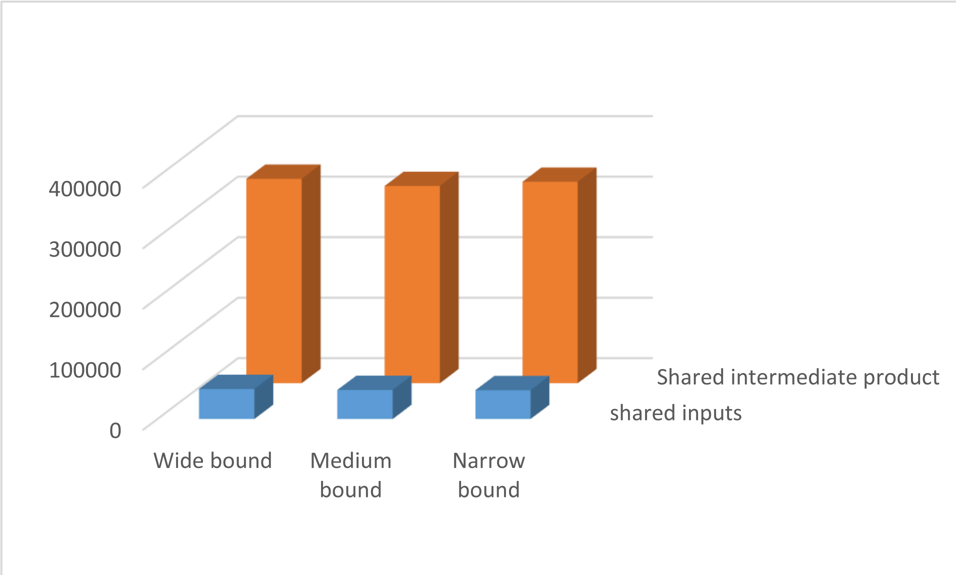

FIGURE 6. Comparing total amount of shared input and shared intermediate product in different levels.

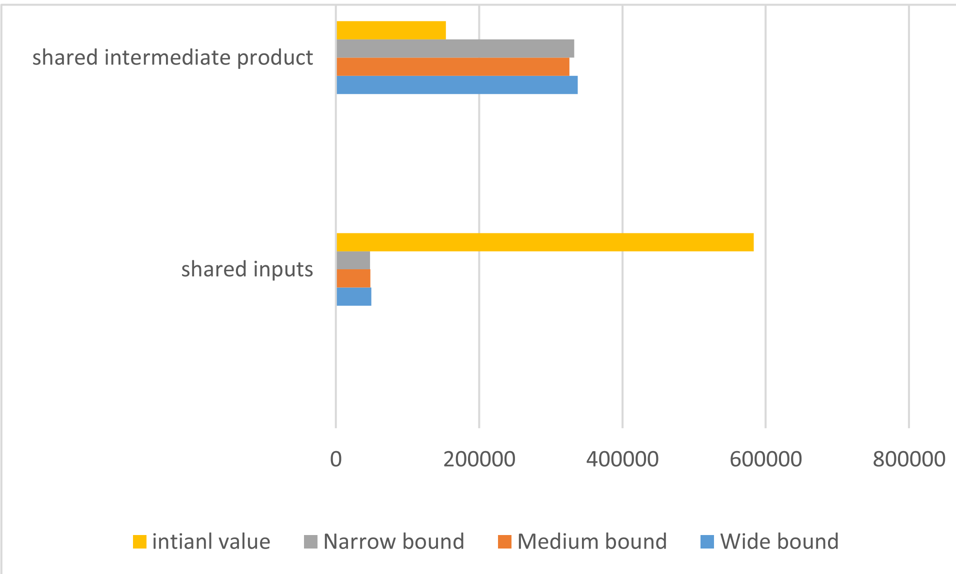

FIGURE 7. Comparing the total amount of shared input and shared intermediate product in different levels.

A DEA-based model is proposed to determine the relative efficiency and optimal allocation of shared resources. To demonstrate the real applicability of the theoretical approach, we have used data on 22 Indian insurance companies. We have used linear bounds to provide restricted allocation proportions in the Indian life and non-life insurance sector. The numerical results revealed that the narrower the bounds, the more realistic the results.

A research study could be conducted on the problem of shared resources allocation across DMUs to demonstrate that narrow bounds may be more justifiable than wide bounds and this needs to be validated.

In the approach we proposed in this paper, all inputs are reducible, and all outputs are expandable in a deterministic environment. However, in some real cases, there are undesirable outputs in the process in an uncertain environment, and they need to be minimized. As a topic for future study, we suggest considering undesirable products in the process in the presence of uncertainty in the data. This is our future research.

Data availability statement

No new data/codes were created or analyzed in this study.

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