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POSITIVE/NEGATIVE EFFECTS OF UNDESIRABLE OUTPUT ON TECHNOLOGY GAP RATIO UNDER META-FRONTIER FRAMEWORK

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Abstract. In nonradial data envelopment analysis (DEA), the efficiency of meta-frontier may exceed that of group frontier, leading to technology gap ratio (TGR) greater than 1. Although existing methods have attempted to resolve this problem, they have severely limitations: (1) The influence of weak disposability within undesirable outputs on TGR remains unclear, particularly in distinguishing its effects on meta-frontier and group frontier; (2) Current methods fail to systematically account for undesirable outputs in projection processes, potentially distorting efficiency evaluation. According to these limitations, graphical method is used to analyze the influence of undesirable output on TGR. Then, in order not to interfere with undesirable output in projection process, new models are proposed based on the closest targets, which can control the efficiency of meta-frontier is not greater than that of group frontier. Subsequently, undesirable output possible sets are defined for different models, and thus developed to determine its minimum/maximum variation ranges under different frontiers. By determining variation range of undesirable output under variable returns to scale (VRS), the positive/negative effects on TGR are obtained. The advantages of our methods over existing methods are as follows: (1) Our methods ensure theoretical consistency in efficiency evaluation; (2) The potential impact of undesirable output on TGR is first studied; (3) Our methods offer actionable insights for efficiency improvement through undesirable output ranges. Finally, two examples are provided to illustrate the validity and effectiveness of our methods.

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1. Introduction

Data Envelopment Analysis (DEA) has been widely recognized in the literature, because it can comprehensively evaluate performance based on a limited number of assumptions and combining multiple input and output dimensions [31]. In traditional DEA method, all DMUs are in a unified technical environment for efficiency evaluation. However, there may be many factor differences in actual production activities, which make the technology sets of different decision-making units (DMUs) different. In the framework of efficiency measurement, the existence of technical heterogeneity leads to the bias of efficiency measurement. To relax the constraints of technological heterogeneity, O'Donnell *et al.* [33] proposed meta-frontier DEA analysis framework for the first time, which brought technological heterogeneity into the efficiency evaluation process. Subsequently,

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the meta-frontier framework has been widely used in efficiency evaluation in many fields, such as technological innovation [29], Supplier service [1], transportation industry [5, 43], educational development [17], customs system [2], agriculture [34].

Direction distance function (DDF), which can provide a diversified improvement direction choice for DMUs. Due to this advantage, it is often introduced into meta-frontier framework to analyze the efficiency of DMU with technical heterogeneity [8]. In addition, radial DDF model is the most common in meta-frontier framework, but in recent years, nonradial DDF model has gradually attracted the attention of many scholars. However, in nonradial DDF model, the efficiency of meta-frontier may be greater than that of group frontier [44]. To solve this problem, Wang *et al.* [37] proposed an alternative three-stage method to measure the technology gap. In the second stage, the projection point of DMU on group frontier in the first stage is used to improve, so that the improved optimization point is located on meta-frontier. Then, Wang *et al.* [38] expanded the above work, and proposed a new choice of projection direction. Wang *et al.* [37, 38] methods made the projection distance from DMU to meta-frontier always larger than that of group frontier, thus ensuring the rationality of TGR. In addition, under nonradial slacks-based measure (SBM) model, Chen *et al.* [10] proposed an alternative meta-frontier framework by introducing slack variables, which also effectively solved the problem that TGR is greater than 1. However, the above studies only ensured the rationality of TGR, ignoring the factors that lead to TGR greater than 1.

In recent years, under meta-frontier DDF, although some achievements have been made in solving the problem that TGR is greater than 1, they still have the following defects: First, at present, the inconsistency of directional vectors between meta-frontier and group frontier has been considered as the main reason for TGR being greater than 1. However, it has not been confirmed whether this is the only reason for TGR greater than 1, and whether it is affected by other factors, which have not been proposed and properly solved. Second, we find that the models proposed by Wang *et al.* [37] and Yu *et al.* [44] all contain undesirable output. Then, in the nonradial DEA, under meta-frontier framework, does the undesirable output have an effect on TGR? And, is this effect positive or negative? These questions have not been raised and properly solved in the current research. Third, at present, in the proposed method to solve the problem that TGR is greater than 1, it interferes with the optimal increase and decrease ratio of each element in the projection process, which is not conducive to measuring the affected level of TGR.

In addition, at present, most traditional DEA methods pursue the maximization of projection distance, which can find more inefficient DMUs and improve potential [35]. In recent years, closest targets have been widely used, which have provided the most relevant solution for inefficient DMU [24]. To provide the closest targets by using mathematical programming, a complementary set of nonconvex polyhedron sets must be firstly established [4]. Aparicio *et al.* [4] was the first to develop mixed integer linear program (MILP) method, and the closest distance combination was found out through different closest targets measurement methods, which was superior to the evaluated DMU. After that, many studies have been extended on this basis. For example, Wu *et al.* [40] extended it to VRS hypothesis for research. Li *et al.* [24] introduced undesirable output, established the closest targets method with undesirable output, and verified that it satisfies strong monotonicity. Li *et al.* [25] extended the closest targets to two-stage DEA. However, as far as we know, the closest targets have not been introduced into meta-frontier framework, which deserves further attention.

Therefore, this paper mainly solves the above problems from the following aspects:

- **Reveal the influence mechanism of undesirable output on TGR:** By graphic method, the influence of weak disposability of undesirable output on TGR unreasonable (TGR > 1) is systematically analyzed for the first time, and its theoretical root is clarified.
- **New models are proposed to control TGR in a reasonable range:** To interfere with the undesirable output in projection process, based on closest targets methods, NC-DDF and NR-DDF are proposed, which can effectively control TGR not to be greater than 1.
- **New methods are developed to determine minimum/maximum variation ranges:** In case of reasonable TGR value, the undesirable output possible sets are defined for different models, and some

methods are thus developed to determine its minimum/maximum variation ranges under meta-frontier and group frontier, respectively.
– **Positive/negative effects, interval value and variation range on TGR are obtained:** Under the influence of undesirable output, positive/negative effects on TGR are obtained, and the interval value and variation range of TGR are measured.

The rest of the paper is organized as follows. Section 2 introduces the basic knowledge. Some DDF models are constructed in Section 3. Section 4 develops the minimum/maximum methods of undesirable output under VRS. Section 5 makes some discussions for these methods. Two examples are provided in Section 6, while the conclusions are given in Section 7.

2. Literature review

2.1. DDF in DEA

Traditional DEA model, such as Charnes–Cooper–Rhodes (CCR) and Banker–Charnes–Cooper (BCC), can only measure the proportional improvement of input or output, but in reality, it often needs to deal with undesirable output and asymmetric adjustment. As an extension of traditional DEA model, DDF method has broken the limitations of traditional DEA, and gradually become a nonparametric research method widely used in economics and management [27]. Subsequently, Chung and Färe [12], Chambers *et al.* [7] considered the situation including bad output, which further promoted the theoretical development of DDF method. Compared with traditional DEA, the main advantages of DDF are as follows: (1) DDF can deal with the problems of excess input and insufficient output at the same time; (2) Different preferences are reflected by setting the direction vector; (3) DDF is more suitable for environmental efficiency evaluation including undesirable output.

At present, DDF has been widely used in empirical research. However, the expansion of the existing DDF method is not comprehensive, and under the meta-frontier framework, there are three critical limitations:

(1) Nonradial bias: Nonradial DDF models (*e.g.*, SBM-DDF) allow independent adjustment of variables but may violate the meta-frontier envelopment condition, causing TGR > 1.
(2) Directional arbitrariness: *Ad-hoc* direction vectors (*e.g.*, observed values or unit vectors) distort cross-frontier comparability, which may lead to the unreasonable TGR value under the framework of meta-frontier, which is not conducive to the comparison of technology gap and the decision of resource allocation.
(3) Undesirable output oversight: Most studies assume strong disposability for undesirable outputs, ignoring weak disposability's impact on frontier shapes. This directly impedes TGR validity but remains unaddressed in meta-DDF frameworks.

2.2. Meta-frontier

In traditional data envelopment analysis (DEA) method, all DMUs are evaluated in a uniform technical environment. However, because DMU often operates in different industries, regions and environments, there is a gap in its production technology. Obviously, under the assumption of unified technical environment, traditional DEA may lead to inaccurate efficiency evaluation results. Therefore, O'Donnell *et al.* [33] proposed meta-frontier framework, which can be divided into group production technology and meta-frontier production technology according to different industries or regions, and so on. This framework allows different technologies to be compared, which also becomes the advantage of meta-frontier. Among them, meta-frontier production technology represents the best production technology that DMU can adopt. In recent years, meta-frontier framework has been widely used [17].

Ensuring that the efficiency of meta-frontier is always not greater than group frontier is the main difficulty of meta-frontier framework [33], and it is often ignored by the existing methods. While Wang *et al.* [37] proposed a new method to constrain TGR within $[0, 1]$, their approach did not examine the role of undesirable outputs in distorting TGR. Furthermore, Guo *et al.* [21] revealed that weak disposability assumptions for undesirable outputs can significantly affect the efficiency evaluation results, yet no study has systematically linked this to TGR anomalies. This omission highlights the novelty of our focus on undesirable outputs' dual effects (positive/negative) on TGR.

In addition, meta-frontier has become one of the main tools for environmental efficiency assessment. However, as a "frequent visitor" to environmental efficiency assessment, undesirable output and its disposal hypothesis have not attracted enough attention. Despite progress, two fundamental gaps undermine meta-frontier reliability:

(1) Defect in efficiency evaluation: Due to the conflict in the selection of reference points for different frontiers, the efficiency of group frontier may be less than that of meta-frontier in non-radial model.
(2) Neglect of application rationality: At present, the meta-frontier framework has been widely used in environmental efficiency assessment, energy allocation optimization and other fields, and most of the proposed models contain bad outputs. However, in the non-radial DEA under the framework of meta-frontier, the influence of unexpected output on TGR has not been further explored. This leads to controversy about the efficiency evaluation of meta-frontier including unexpected output.

2.3. TGR

TGR reflects the technical efficiency gap between meta-frontier and group frontier. Since group frontiers are included in meta-frontier, in the radial DDF, the technical efficiency measured under meta-frontier must be less than that of group frontier, that is, the technology gap ratio (TGR) value in the radial DEA model is between 0 and 1 [33]. However, in the nonradial DDF model, the efficiency of meta-frontier may be greater than that of group frontier, which results in TGR is greater than 1 [44]. This contradicts that the technical efficiency under meta-frontier is less than that under group frontier. Some scholars have proposed an alternative meta-frontier framework to solve the problem of unreasonable TGR:

(1) Projection-based methods: Wang *et al.* [37, 38] proposed some new projection direction selections, which improved the projection direction from group frontier to meta-frontier, and thus ensured the rationality of TGR. On this basis, Guo and Chen [20] further ensured the rationality of TGR in three-level meta-frontier framework by improving projection direction selection.
(2) Slack-based adjustments: Chen *et al.* [11] introduced slack variables, and added constraints to cap meta-efficiency at group-efficiency levels.

Existing solutions, such as slack-based adjustments (Chen *et al.*, [11]) or directional vector improvement [38], focus on model improvement without addressing root causes. For example, Chen *et al.* [11] introduced slack variables to cap TGR at 1 but ignored how undesirable outputs' weak disposability might exacerbate TGR distortions. There are three unresolved gaps:

(1) The lack of empirical validation for directional vector inconsistency as the sole cause of TGR > 1.
(2) The unexamined role of undesirable outputs in TGR volatility, particularly in distinguishing its effects on meta-frontier and group frontier.
(3) The absence of methods to quantify undesirable outputs' variation ranges under different frontiers.

In summary of literature review, we summarize the improvements and limitations of previous research, as shown in Table 1.

3. Preliminaries

3.1. CCR model

CCR model is the basis of DEA model, and it is also the most classic DEA model. Supposed that there are n DMUs to be evaluated, and for each DMU$_d$, $x_d = (x_{1d}, x_{2d}, \ldots, x_{md}) \in R_+^m$ is the input vector, $y_d = (y_{1d}, y_{2d}, \ldots, y_{sd}) \in R_+^s$ is the desirable output vector. Then, for any DMU$_d$, CCR model can be constructed as follows.

TABLE 1. Innovations and limitations of previous research.

Methods	Advantage	Limitations	Improvement
Directional Distance Function (DDF)	Enabled asymmetric adjustments of inputs/outputs; Incorporated preferences *via* direction vectors; Improved environmental efficiency evaluation with undesirable outputs.	Direction vector selection remains subjective; May produce unreasonable TGR values under meta-frontier; Weak disposability of undesirable outputs often overlooked.	Introduced chance constrained [27]; Constructed parallel entropy dynamic DDF [42]; Combined super-efficiency model [45]; Considered null-jointness assumptions [26]; Handle nonpositive data [23].
Meta-Frontier Framework	Enabled cross-group technology comparisons; Distinguished group and meta-frontier efficiency; Determine the gap between DMU and its best technology.	Needs to guarantee meta-frontier efficiency is less than group frontier.	Introduced nash bargaining [43]; Considered benefit-of-the-doubt technique [17]; Considered non-convex [32]; Constructed meta-frontier slacks-based measure dynamics network DEA [22]; Combined hybrid super-efficiency [16].
Technology Gap Ratio (TGR)	Quantified technology gaps between frontiers; Provided benchmarks for policy targeting.	Nonradial models may violate TGR less than 1; No clear explanation for TGR greater than 1 cause; Undesirable outputs' impact on TGR remains unexplored.	Under two-level meta-frontier [37, 38]; (Chen *et al.*, [11]) and the three-level meta-frontier [20], new projection direction were proposed to solve the problem that TGR is greater than 1; Made double shared ratio selection [18].

$$
\begin{aligned}
&\text{Max } \theta_d \\
&\text{s.t. } \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_d x_{id}, \qquad i = 1, \ldots, m; \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rd}, \qquad r = 1, \ldots, s; \\
&\qquad \forall \lambda_j \ge 0
\end{aligned} \tag{1}
$$

where λ_j and θ_d are all decision variables. λ_j gives each DMU different weights. θ_d is the efficiency of DMU, and when the input-oriented CCR model is selected, its value is between 0 and 1.

3.2. Meta-frontier

In meta-frontier framework, DMUs and their possible linear combinations form a convex set. Under meta-frontier, the meta-frontier efficiency of DMU is marked as θ_d^{ME}, and the efficiency obtained by taking the meta-frontier as the evaluation benchmark. Then, meta-frontier model considering is shown as follows.

$$
\begin{aligned}
&\text{Min } \theta^{\text{ME}} \\
&\text{s.t. } \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta^{\text{ME}} x_{id}, \qquad i = 1, \ldots, m; \\
&\quad\ \ \sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{rd}, \qquad r = 1, \ldots, s; \\
&\quad\ \ \sum_{j=1}^{n} \lambda_j z_{fj} = z_{fd}, \qquad f = 1, \ldots, h; \\
&\quad\ \ \forall \lambda_j \geq 0, \qquad j = 1, 2, \ldots, n.
\end{aligned} \tag{2}
$$

Among them, the optimal solution $\theta_d^{\text{ME}^*}$ of meta-frontier can be obtained. λ_j is intensity variable, m and s represent the number of input categories and output categories, respectively.

In addition, the group efficiency set of DMU is θ^{GE}, and the efficiency obtained by taking the group frontiers as the evaluation benchmark. Supposed that there are n DMUs to be evaluated, then, these DMUs are divided into k groups ($g = 1, 2, \ldots, k$), and the number of DMUs in each group g is n^g. Thus, θ^{GE} can be obtained by the following formula.

$$
\begin{aligned}
&\text{Min } \theta^{\text{GE}} \\
&\text{s.t. } \sum_{j=1}^{n^g} \lambda_j x_{ij} \leq \theta^{\text{GE}} x_{id}, \qquad i = 1, \ldots, m; \\
&\quad\ \ \sum_{j=1}^{n^g} \lambda_j y_{rj} \leq y_{rd}, \qquad r = 1, \ldots, s; \\
&\quad\ \ \sum_{j=1}^{n^g} \lambda_j z_{fj} = z_{fd}, \qquad f = 1, \ldots, h; \\
&\quad\ \ \forall \lambda_j \geq 0, \qquad j = 1, 2, \ldots, n^g.
\end{aligned} \tag{3}
$$

Among them, the optimal solution $\theta_d^{\text{GE}^*}$ of group g can be obtained. n^g is the number of DMUs of group g, λ_j is intensity variable, k is the number of groups.

Furthermore, under the assumption of convexity, the production possible sets of meta-frontier and group frontier are as follows.

$$
\begin{aligned}
T^M &= \{(x, y) | x \text{ can produce } y;\ x \geq 0; y \geq 0\} \\
T^G &= \{(x, y) | x \text{ can produce } y \text{ with group technology } g; x \geq 0; y \geq 0\}.
\end{aligned} \tag{4}
$$

3.3. TGR

Through the calculation of model (2) and model (3), the TGR_d of DMU_d with respect to the meta-frontier and the group frontier, is calculated as follows.

$$
\text{TGR}_d = \frac{\theta_d^{\text{ME}^*}}{\theta_d^{\text{GE}^*}}. \tag{5}
$$

In equation (5), $\theta_k^{\text{ME}^*}$ and $\theta_k^{\text{GE}^*}$ are the efficiency scores of meta-frontier and group frontier, respectively. In input-oriented DEA model, the closer TGR is to 1, the smaller the technical gap between meta-frontier and group frontier, and the lower the heterogeneity of the corresponding frontier.

3.4. DDF model

DDF extends traditional DEA model, which allows the input and output to be adjusted simultaneously in the specified direction. $\overrightarrow{D_k} = (x, y; -g_x, g_y)$ can be solved by linear programming method, and the direction distance function is abbreviated as β_d. Then, for any DMU$_d$, traditional DDF model can be constructed as follows.

$$\begin{aligned}
&\text{Max } \beta_d \\
&\text{s.t. } \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{id} - \beta_d g_x, \qquad i = 1, \ldots, m; \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rd} + \beta_d g_y, \qquad r = 1, \ldots, s; \\
&\qquad \forall \lambda_j \ge 0
\end{aligned} \tag{6}$$

where λ_j gives each DMU different weights. At present, the related research mainly defines the direction vector $g = (-g_x, -g_y)$ as three different directions: (x, y), $(x, 0)$ and $(0, y)$. Different from CCR model, DDF measures the efficiency by judging the distance between DMU and its frontier. According to model (6), the efficiency value of DMU is $\frac{1-\beta_d}{1+\beta_d}$.

(1) Farthest distance projection (F-DDF)

Under traditional DDF framework, many studies have introduced undesirable output z_d, in addition, $z_d = (z_{1d}, z_{2d}, \ldots, z_{hd}) \in R_+^h$ represents undesirable output vector. In this paper, we assume that undesirable outputs z_d is weak disposability, that is, decision makers cannot reduce it without affecting inputs and desirable outputs. Chambers *et al.* [7] introduced undesirable outputs into the DDF.

$$\begin{aligned}
&\text{Max } \beta_x^m + \beta_y^m \\
&\text{s.t. } \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{id}, \qquad i = 1, \ldots, m; \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rd} + \beta_y^m g_y, \qquad r = 1, \ldots, s; \\
&\qquad \sum_{j=1}^{n} \lambda_j z_{fj} = z_{fd} - \beta_z^m g_z, \qquad f = 1, \ldots, h; \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1; \\
&\qquad \forall \lambda_j \ge 0;
\end{aligned}
\quad \text{and} \quad
\begin{aligned}
&\text{Max } \beta_x^g + \beta_y^g \\
&\text{s.t. } \sum_{j=1}^{n^g} \lambda_j x_{ij} \le x_{id}, \qquad i = 1, \ldots, m; \\
&\qquad \sum_{j=1}^{n^g} \lambda_j y_{rj} \ge y_{rd} + \beta_y^g g_y, \qquad r = 1, \ldots, s; \\
&\qquad \sum_{j=1}^{n^g} \lambda_j z_{fj} = z_{fd} - \beta_z^g g_z, \qquad f = 1, \ldots, h; \\
&\qquad \sum_{j=1}^{n^g} \lambda_j = 1; \\
&\qquad \forall \lambda_j \ge 0;
\end{aligned} \tag{7a}$$

where β_y^m and β_z^m are the technological inefficiency scores of y and z of DMU$_d$ under meta-frontier. β_y^g and β_z^g are the technological inefficiency scores of y and z of DMU$_d$ under group frontier. This paper assumes that undesirable output is weak disposability. The direction vector in meta-frontier is defined as $g^m = (y^m, -z^m)^T$, and the direction vector in group frontier is defined as $g = (y, -z)^T$. Based on it, TGR is calculated as follows.

$$\theta^{m^*} = \frac{1 - \beta_z^{m^*}}{1 + \beta_y^{m^*}} \quad \text{and} \quad \theta^{g^*} = \frac{1 - \beta_z^{g^*}}{1 + \beta_y^{g^*}} \tag{7b}$$

$$\text{TGR} = \theta^{m^*} / \theta^{g^*}. \tag{7c}$$

Among them, θ^{m^*} and θ^{g^*} are the efficiency scores of meta-frontier and group frontier, respectively. The closer TGR is to 1, the smaller the technical gap between meta-frontier and group frontier, and the lower the heterogeneity of the corresponding frontier.

(2) Closest projection distance model (N-DDF)

The closest targets, which is the minimum effort to improve the invalid DMU and achieve effectiveness. Aparicio and Pastor [3] have extended it to VRS.

$$
\begin{aligned}
&\text{Min } \left(\sum_{i=1}^{m}\beta_z+\sum_{r=1}^{s}\beta_y\right) \\
&\text{s.t. } \sum_{j=1}^{n}\lambda_j x_{ij}\le x_{id}, && i=1,\dots,m; \\
&\sum_{j=1}^{n}\lambda_j y_{rj}\ge y_{rd}+\beta_y g_r^y, && r=1,\dots,s; \\
&\sum_{j=1}^{n}\lambda_j z_{fj}\le z_{fd}-\beta_z g_f^z, && f=1,\dots,h; \\
&\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}+d_0+d_j=0 \\
&\lambda_j\le M\cdot(1-b_j) \\
&d_j\le M\cdot b_j \\
&\sum_{j=1}^{n}\lambda_j=1; \\
&\lambda_j,d_j\ge 0,b_j\in\{0,1\}, && \forall j \\
&u_r,v_i\ge\delta.
\end{aligned}
\tag{7d}
$$

Here, M is a sufficiently large positive number. d_0 is free variables. Constraints (1) and (2) are the form of DEA envelopment model. Constraint (3) constructs a strong efficient frontier, and it is the supporting facet of DDF multiplier model. When $d_j=0$, DMU_j is located on the strong efficient frontier $\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}+d_0=0$. Constraints (4) and (5) are to integrate multiplier constraint and envelope constraint. Based on it, efficiency value is calculated as follows.

$$
\theta^N=1-\left(\sum_{r=1}^{s}\beta_y/y_{rd}+\sum_{f=1}^{h}\beta_z/z_{fd}\right)/(s+h). \tag{7e}
$$

3.5. Disposability of undesirable output

Strong disposability and weak disposability of undesirable output were proposed by Färe *et al.* [15]. On this basis, Chen *et al.* [9] further proposed semi-disposability hypothesis of undesirable output. These hypotheses have been widely concerned by scholars. Based on this, this paper systematically sorts out and compares several theoretical hypotheses of undesirable output in order to provide theoretical reference and method selection basis for related research.

(1) Strong disposability assumes that decision makers are free to increase or decrease undesirable output according to demand, and strong hypothesis is defined as follows:

$$
P^S(X)=\left\{(Y,Z)\middle|\sum_{j=1}^{n}\lambda_j Y_j\ge Y,\sum_{j=1}^{n}\lambda_j Z_j=Z,\sum_{j=1}^{n}\lambda_j X_j\le X,\lambda_j\ge 0\ (j=1,2,\cdots,n)\right\}. \tag{8a}
$$

(2) Weak disposability assumes that decision makers cannot reduce undesirable outputs without affecting inputs and outputs, and weak hypothesis is defined as follows:

$$P^{w}(X)=\left\{(Y,Z)\left|\sum_{j=1}^{n}\lambda_j Y_j\geq Y,\sum_{j=1}^{n}\lambda_j Z_j\geq Z,\sum_{j=1}^{n}\lambda_j X_j\leq X,\lambda_j\geq 0\ (j=1,2,\cdots,n)\right.\right\}. \tag{8b}$$

(3) Semi-disposability assumes that decision makers can only reduce undesirable output relatively freely at the current technical level without affecting input and output, and it cannot be achieved beyond the technical level. Semi-disposability introduces the degree of non-disposability γ, which is between 0 and 1 and is obtained by Delphi method, expert meeting method, and so on. Semi-hypothesis is defined as follows:

$$P^{\mathrm{SI}}(X)=\left\{(Y,Z)\left|\sum_{j=1}^{n}\lambda_j Y_j\geq Y,\sum_{j=1}^{n}\lambda_j Z_j\leq Z,\sum_{j=1}^{n}\lambda_j Z_j\geq \gamma Z,\sum_{j=1}^{n}\lambda_j X_j\leq X,\lambda_j\geq 0\right.\right\}. \tag{8c}$$

Compared with different disposability of undesirable output, weak disposability of undesirable outputs is more popular in empirical research. When undesirable output is set to strong disposability, it is more conducive to frontier analysis, because it does not change the shape of frontier, while other disposability changes the original shape of frontier. In addition, compared with strong and weak disposability, semi-disposability needs to be considered in more complicated situations.

4. Causes of TGR greater than 1 under the influence of undesirable output

Mergoni *et al.* [30] indicated that only by effectively utilizing existing resources can environmental risks be reduced, and undesirable output is considered in DEA model, which provides a theoretical basis for sustainable development. According to the statistics of Emrouznejad *et al.* [13], a total of 311 papers from 1989 to 2022 focused on undesirable output in the measurement of ecological efficiency. Large-scale research fully confirmed the widespread concern and important position of undesirable output in the field of DEA research. Therefore, it is necessary to analyze the influence of undesirable output. By using the numerical example of Yu *et al.* [44], the influence of undesirable output on TGR greater than 1 is analyzed. Thirty DMUs with one input, one output and one undesirable output are included in this numerical example, which can be divided into three groups based on technical heterogeneity.

As shown in Figure 1, in radial DEA, the projection direction of meta-frontier is consistent with group frontier. B8B6A1A6 shows meta-frontier, A8A2A1 shows group frontier of group A (*i.e.*, A1, A2 and A8 are efficient DMUs), B6B5B3 shows group frontier of group B (*i.e.*, B3, B5, B6 and B8 are efficient DMUs), and C4C7C1 shows group frontier of group C (*i.e.*, C1, C4, C7 are efficient DMUs). A3 is an invalid DMU, and it is used as an example to explain the projection direction. In radial DEA, the projection points are all at $\mathrm{A}^{(1)}$. Obviously, in radial DDF, since group frontier is included in meta-frontier, the efficiency of meta-frontier is less than that of group frontier, and then TGR is not greater than 1.

As shown in Figure 2, the projection direction of meta-frontier is inconsistent with group frontier. $\mathrm{A}^{(1)}$ and $\mathrm{A}^{(2)}$ are the target DMUs of A3, $\overrightarrow{\mathrm{A3A}^{(1)}}$ and $\overrightarrow{\mathrm{A3A}^{(2)}}$ are the corresponding projection directions of A3. By comparing Figure 1, under meta-frontier, it can be found that DMU can obtain two different projection directions in nonradial E-DEA model, which is widely considered as the main reason for the unreasonable TGR value at present.

Then, the influence of undesirable output on the unreasonable value of TGR is analyzed. In this paper, we set the undesirable output as weak disposability. Guo *et al.* [21] think that weak disposability will overestimate the efficiency of DMU. In Figure 2, the downward trend of A1–A2, B6–B8, A1–B6 and C1–C7 reflects the weak disposability of undesirable output. Meta-frontier reference point of A3 is located in B6–B8, and its group frontier reference point is located in A1 and A2. Therefore, the weak disposability of undesirable output has

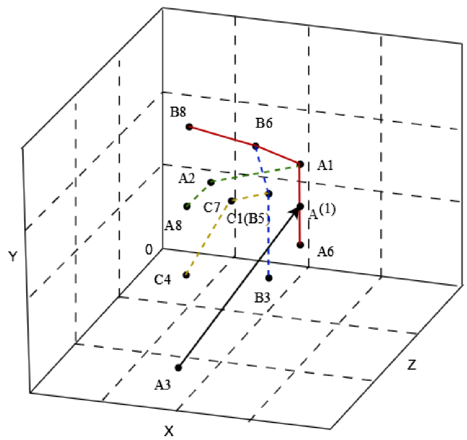


FIGURE 1. The projection direction is consistent with undesirable output.

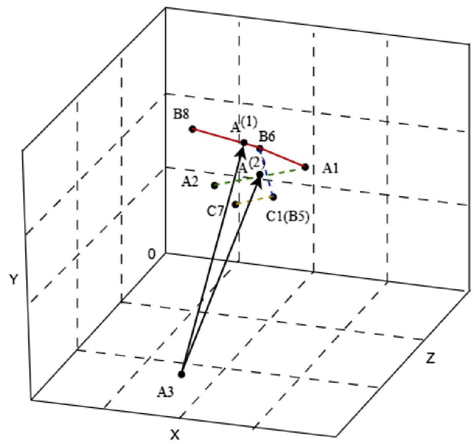


FIGURE 2. The projection direction is inconsistent with undesirable output.

an impact on A3, which makes its meta-frontier technical efficiency overestimated, and its overestimation level exceeds the efficiency of group frontier, resulting in the TGR of A3 being greater than 1. By comparing Figure 1, the reference points of A3 are all located on A1–A6, which is not affected by the weak disposability of undesirable output.

Definition 1. On the premise that the projection directions of meta-frontier and group frontier are inconsistent, the weak disposability characteristics of undesirable output will interfere with TGR, which is the direct reason that the TGR value is greater than 1.

The inconsistency of projection directions between group frontier and meta-frontier is considered as the main reason of TGR greater than 1. Therefore, at present, most methods to solve TGR greater than 1 are to unify the projection direction. However, although unifying the projection direction avoids the problem of unreasonable TGR value, it ignores the influence of undesirable output on TGR, and also neglects the choice of the optimal increase and decrease ratio of undesirable and desirable output. Therefore, on the basis of not interfering with the optimal increase and decrease ratio of undesirable output and desirable output, how to control the TGR value within a reasonable range is key to measure the influence of undesirable output on TGR.

5. Relieve of TGR greater than 1 under the influence of undesirable output

The farthest distance projection is used for both meta-frontier and group frontier. To relieve that TGR is greater than 1, this paper uses the closest targets for the group frontier. First, this method can control TGR within a reasonable range, because the nearest distance projection method minimizes the slack variable and distance to effective frontier, thus reducing the difference of DMU projection between meta-frontier and group frontier, so as to better distinguish the influence level of undesirable output on TGR.

5.1. Conservative DDF model based on closest targets (NC-DDF)

Cooper *et al.* [6] propose a range adjustment model based on relaxation variables, which can get strong efficient projection. NC-DDF model first ensures that α_y^g and α_z^m reach the minimum, and then minimizes the values of S_i^-, S_r^+ and S_f^-, which is a conservative efficiency measurement method.

$$
\begin{aligned}
&\text{Min}\ \left[\left(\alpha_y^g+\alpha_z^g\right)+\varepsilon\left(\sum_{i=1}^{m}\frac{S_i^-}{R_i}+\sum_{r=1}^{s}\frac{S_r^+}{R_r}+\sum_{f=1}^{h}\frac{S_f^-}{R_f}\right)/(m+s+h)\right]\\
&\text{s.t.}\ \sum_{j=1}^{n}\lambda_j^g x_{ij}=x_{id}-S_i^-, && i=1,\dots,m;\\
&\qquad \sum_{j=1}^{n}\lambda_j^g y_{rj}=y_{rd}+\alpha_y^g g_r^y+S_r^+, && r=1,\dots,s;\\
&\qquad \sum_{j=1}^{n}\lambda_j^g z_{fj}=z_{fd}-\alpha_z^m g_f^z-S_f^-, && f=1,\dots,h;\\
&\qquad \sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}-\sum_{f=1}^{h}w_f z_{fj}+d_0+d_j=0\\
&\qquad \lambda_j^g\le M\cdot(1-b_j)\\
&\qquad d_j\le M\cdot b_j\\
&\qquad \sum_{j=1}^{n}\lambda_j^g=1;\\
&\qquad \lambda_j,d_j\ge 0,b_j\in\{0,1\}, && \forall j\\
&\qquad S_i^-,S_r^+,S_f^-\ge 0, && \forall i,r,f\\
&\qquad u_r,v_i,w_f\ge\delta.
\end{aligned}
\tag{9}
$$

Here, λ represents intensity variables. We set the directional distance vector equal to ($g_r^y = y$, $g_r^z = z$). ε is a non-Archimedean infinitesimal. When the projection directions are inconsistent, α_y^g and α_z^g represent technological inefficiency scores of desirable output and undesirable output respectively. s_r^+ and s_r^+ are slack variables of desirable output and undesirable output, respectively. $R_i = \max_j(x_{ij})-\min_j(x_{ij})$, $R_r = \max_j(y_{rj})-\min_j(y_{rj})$, $R_f = \max_j(z_{fj}) - \min_j(y_{fj})$ are the allowable adjustment range of input, desirable output and undesirable output. δ is a parameter between 0 and 1, which is different from model (7a), and it is because the proportional change of relative weight will not break the constraint.

Assume that $(\alpha_y^{g^*}, \alpha_y^{g^*}, S_i^-, S_r^+, S_f^-)$ are the optimal solution of model (9), then the efficiency of DMU_d based on NC-DDF can be set as follows.

$$
\theta_g^{\text{NC}} = 1-\left(\sum_{i=1}^{m}S_i^-/R_i+\sum_{r=1}^{s}S_r^+/R_r+\sum_{f=1}^{h}S_f^-/R_f\right)/(m+s+h),\quad g=1,\dots,k. \tag{10}
$$

Then, the $\mathrm{TGR}^{\mathrm{NC}}$ of DMU_d based on NC-DDF can be set as follows.

$$\mathrm{TGR}^{\mathrm{NC}} = \theta^{m^*}/\theta^{\mathrm{NC}^*}. \tag{11}$$

Theorem 1. *NC-DDF can relieve the situation that TGR is greater than 1.*

Proof. See Appendix A. $\square$

5.2. Radical DDF model based on closest targets (NR-DDF)

Compared with NC-DDF, NR-DDF directly minimizes the values of S_i^-, S_r^+ and S_f^-, and this model only pursues the smallest improvement path, which is a radical efficiency evaluation method.

$$\begin{aligned}
&\mathrm{Min}\ \left(\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ + \sum_{f=1}^{h} S_f^-\right) && \\
&\text{s.t.}\ \sum_{j=1}^{n} \lambda_j^g x_{ij} = x_{id} - S_i^-, && i = 1,\ldots,m;\\
&\sum_{j=1}^{n} \lambda_j^g y_{rj} = y_{rd} + \alpha_y^g g_r^y + S_r^+, && r = 1,\ldots,s;\\
&\sum_{j=1}^{n} \lambda_j^g z_{fj} = z_{fd} - \alpha_z^g g_f^z - S_f^-, && f = 1,\ldots,h;\\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \sum_{f=1}^{h} w_f z_{fj} + d_0 + d_j = 0 && \\
&\lambda_j^g \le M \cdot (1 - b_j) && \\
&d_j \le M \cdot b_j && \\
&\sum_{j=1}^{n} \lambda_j^g = 1; && \\
&\lambda_j, d_j \ge 0, b_j \in \{0,1\}, && \forall j\\
&S_i^-, S_r^+, S_f^- \ge 0, && \forall i, r, f\\
&u_r, v_i, w_f \ge \delta. &&
\end{aligned} \tag{12}$$

Assume that $(\alpha_y^{g^*}, \alpha_y^{g^*}, S_i^-, S_r^+, S_f^-)$ are the optimal solution of model (12), then the efficiency of DMU_d based on NR-DDF can be set as follows.

$$\theta^{\mathrm{NR}} = 1 - \left(\sum_{r=1}^{s} \alpha_y^g / y_{rd} + \sum_{f=1}^{h} \alpha_z^g / z_{fd}\right) / (s+h), \qquad g = 1,\ldots,k. \tag{13}$$

Then, the $\mathrm{TGR}^{\mathrm{NR}}$ of DMU_d based on NC-DDF can be obtained by model (11).

Theorem 2. *NR-DDF can relieve the situation that TGR is greater than 1.*

Proof. See Appendix B. $\square$

6. Variation range of undesirable output based on closest targets

6.1. Variation range of undesirable output under group frontier

Section 6.1 is to keep meta-frontier as the farthest distance projection (F-DDF) unchanged, and group frontier based on the closest targets (N-DDF), so as to control TGR not to be greater than 1. Then, by minimizing/maximizing the range of undesirable output, the influence of undesirable output on the efficiency of group frontier and TGR is measured.

(1) Minimize/ maximize the variation range of undesirable output based on NC-DDF (NC-CI and NC-CA)

Based on Chen *et al.* [11], the key to determining the variation range of undesirable output under VRS is to define the undesirable output possible set (UPS) of all DMUs. According to the definition of PPS^{V}, the PPS^{NC} and UPS^{NC} under VRS can be defined as follows.

$$\text{PPS}^N = \left\{ X, Y, Z \;\middle|\; \sum_{j=1}^{n} \lambda_j x_{ij} = x_{id} - S_i^-; \sum_{j=1}^{n} \lambda_j y_{rj} = y_{rd} + \beta_y g_y + S_r^+; \right.$$
$$\left. \sum_{j=1}^{n} \lambda_j Z_j = Z - \beta_Z - S_f^-; \sum_{j=1}^{n} \lambda_j = 1; \lambda_j \geq 0 \right\}$$
$$\text{UPS}^N = \left\{ Z \;\middle|\; \sum_{j=1}^{n} \lambda_j Z_j = Z - \alpha_z g_f^z - S_f^-; \sum_{j=1}^{n} \lambda_j = 1; \lambda_j \geq 0; j = 1, \ldots, n \right\}. \tag{14}$$

According to the improved path $K_d^{\text{NC}} = \beta_z g_z + S_f^-$ of undesirable output, then a new UPS^{NC} can be obtained.

$$\text{UPS}^{\text{NC-CI}} = \text{UPS}^{\text{NC}} \cap \left\{ Z \mid Z \geq Z - K_d^{\text{NC}} \right\}. \tag{15}$$

Theorem 3. *For the change range of undesirable output K_d^{NC}, when it is in UPS^{NC}, it has a feasible solution, that is, the corresponding efficiency values of each factor can be obtained.*

Proof. See Appendix C. □

Then, under group frontier, by minimizing/maximizing K_d^{NC}, we can find the range of minimizing/maximizing the undesirable output.

First, minimize the variation range of undesirable output based on NC-CI.

$$K_d^{\text{NC-CI}^*} = \left(K_{1d}^{\text{NC-CI}^*}, \ldots, K_{fd}^{\text{NC-CI}^*} \right) = \min \left\{ \left(K_{1d}^{\text{NC-CI}}, \ldots, K_{fd}^{\text{NC-CI}} \right) : Z - K_d^{\text{NC-CI}} \in \text{UPS}^{\text{NC}} \right\}. \tag{16}$$

The model (16) can be transformed into a single objective linear programming, that is, the model (16) is transformed into a model (17).

$$K_d^{\text{NC-CI}^*} = \min \left\{ \left(\sum_{r=1}^{s} \alpha_y^g + \sum_{f=1}^{h} \alpha_z^g \right) + \varepsilon \left(\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ + \sum_{f=1}^{h} S_f^- \right) \right\}. \tag{17}$$

According to model (17), the optimal improved path of undesirable output is $\bar{Z}_d^* = Z_d - (\beta^* g_z + S_f^{-*}) = Z_d - K_d^{\text{NC-CI}^*}$; Let $\delta_d = \beta_y g_y + S_r^+$ then the new PPS^{NC} can be obtained.

$$\text{PPS}^{\text{NC-CI}} = \text{PPS}^{\text{NC}} \cap \left\{ (X, Y, Z) \;\middle|\; X \geq X - S_i^-; Y \leq Y + \delta_d; Z \geq Z - K_d^{\text{NC}} \geq Z - K_d^{\text{NC-CI}^*} \right\}. \tag{18}$$

Therefore, the optimal solution $(S_i^{-*}, \delta_d^*, K_d^{\mathrm{NC}^*})$ can be obtained as follows.

$$\begin{aligned}\left(S_i^{-*}, \delta_d^*, K_d^{\text{NC-CI}^*}\right) = \min\Big\{&(S_{1d},\ldots,S_{id});(\delta_{1d},\ldots,\delta_{rd});\left(K_{1d}^{\mathrm{NC}},\ldots,K_{fd}^{\mathrm{NC}}\right): \\ &\left(X - S_i^-, Y+\delta_d, Z - K_d^{\mathrm{NC}^*}\right)\in \mathrm{PPS}^{\text{NC-CI}}\Big\}.\end{aligned} \tag{19}$$

The model (19) is a multi-objective linear programming (MOLP) model, which refers to simultaneously optimizing multiple conflicting objective functions, and it can be transformed into a single objective linear programming, that is, the model (19) is transformed into a model (20).

$$\left(S_i^{-*}, \delta_d^*, K_d^{\text{NC-CI}^*}\right) = \min\left\{\left(\alpha_y^g + \alpha_z^g\right) + \varepsilon\left(\sum_{i=1}^m \frac{S_i^-}{R_i} + \sum_{r=1}^s \frac{S_r^+}{R_r} + \sum_{f=1}^h \frac{S_f^-}{R_f}\right)/(m+s+h)\right\}. \tag{20}$$

The optimal solution $(S_i^{-*}, \delta_d^*, K_d^{\text{NC-CI}^*})$ can be obtained by model (20), and thus the efficiency value $\theta^{\text{NC-CI}^*}$ and $\mathrm{TGR}^{\text{NC-CI}^*}$ can be calculated by models (10) and (11).

Similarly, maximize the variation range of undesirable output based on NC-CA.

$$K_d^{\text{NC-CA}^*} = \left(K_{1d}^{\text{NC-CA}^*},\ldots,K_{fd}^{\text{NC-CA}^*}\right) = \max\left\{\left(K_{1d}^{\text{NC-CA}},\ldots,K_{fd}^{\text{NC-CA}}\right): Z - K_d^{\text{NC-CA}} \in \mathrm{UPS}^{\mathrm{NC}}\right\}. \tag{21}$$

The model (21) is transformed into a model (23).

$$K_d^{\text{NC-CA}^*} = \max\left\{\left(\sum_{r=1}^s \alpha_y^g + \sum_{f=1}^h \alpha_z^g\right) + \varepsilon\left(\sum_{i=1}^m S_i^- + \sum_{r=1}^s S_r^+ + \sum_{f=1}^h S_f^-\right)\right\}. \tag{22}$$

The new $\mathrm{PPS}^{\text{NC-CA}}$ is similar to formula (18), and the optimal solution $(S_i^{-*}, \delta_d^*, K_d^{\text{NC-CA}^*})$ can be obtained by model (22), thus, the efficiency value $\theta^{\text{NC-CA}^*}$ and $\mathrm{TGR}^{\text{NC-CA}^*}$ can be calculated by model (10) and model (11).

(2) Minimize/maximize the variation range of undesirable output based on NR-DDF (NR-RI and NR-RA)

According to model (14), based on the radical efficiency evaluation method, the variation range $K_d^{\mathrm{NR}} = \beta_z g_z + S_f^-$ of undesirable output, then a new $\mathrm{UPS}^{\mathrm{NR}}$ can be obtained.

$$\mathrm{UPS}^{\mathrm{NR}} = \mathrm{UPS}^{N} \cap \left\{Z \mid Z \geq Z - K_d^{\mathrm{NR}}\right\}. \tag{23}$$

Theorem 4. *For the variation range of undesirable output K_d^{NR}, when it is in $\mathrm{UPS}^{\mathrm{NR}}$, it has a feasible solution, that is, the corresponding efficiency values of each factor can be obtained.*

Proof. See Appendix D. □

Then, under group frontier, by minimizing/maximizing K_d^{NC}, we can find the range of minimizing/maximizing the undesirable output.

First, minimize the variation range of undesirable output based on NC-DDF (NC-CI).

$$K_d^{\text{NR-RI}^*} = \min\left\{\sum_{i=1}^m S_i^- + \sum_{r=1}^s S_r^+ + \sum_{f=1}^h S_f^-\right\}. \tag{24}$$

According to model (24), the optimal improved path of undesirable output is $\bar{Z}_d^* = Z_d - (\beta^* g_z + S_f^{-*}) = Z_d - K_d^{\text{NR-RI}^*}$; Let $\delta_d = \beta_y g_y + S_r^+$ then the new $\text{PPS}^{\text{NR-RI}}$ can be obtained.

$$\text{PPS}^{\text{NR-RI}} = \text{PPS}^{\text{V}} \cap \left\{ (X,Y,Z) \;\middle|\; X \geq X - S_i^-; Y \leq Y + \delta_d; Z \geq Z - K_d^{\text{NR}} \geq Z - K_d^{\text{NR-RI}^*} \right\}. \tag{25}$$

Therefore, the optimal solution $(S_i^{-*}, \delta_d^*, K_d^{\text{NR-RI}^*})$ can be obtained.

$$\left(S_i^{-*}, \delta_d^*, K_d^{\text{NR-RI}^*}\right) = \min \Big\{ (S_{1d}, \ldots, S_{id}); (\delta_{1d}, \ldots, \delta_{rd}); \left(K_{1d}^{\text{NR-RI}}, \ldots, K_{fd}^{\text{NR-RI}}\right) : \left(X - S_i^-, Y + \delta_d, Z - K_d^{\text{NR-RI}^*}\right) \in \text{PPS}^{\text{NR-RI}} \Big\}. \tag{26}$$

The model (26) is a MOLP model, and it can be transformed into a single objective linear programming, that is, the model (26) is transformed into a model (27).

$$\left(S_i^{-*}, \delta_d^*, K_d^{\text{NR-RI}^*}\right) = \min \left\{ \sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ + \sum_{f=1}^{h} S_f^- \right\}. \tag{27}$$

The optimal solution $(S_i^{-*}, \delta_d^*, K_d^{\text{NR-RI}^*})$ can be obtained by model (27), thus, $\theta^{\text{NR-RI}^*}$ and $\text{TGR}^{\text{NR-RI}^*}$ can be calculated by model (13) and model (11).

Similarly, maximize the variation range of undesirable output based on NR-RA.

$$K_d^{\text{NR-RA}^*} = \max \left\{ \sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ + \sum_{f=1}^{h} S_f^- \right\}. \tag{28}$$

The new $\text{PPS}^{\text{NR-RA}}$ is similar to formula (25), and the optimal solution $(S_i^{-*}, \delta_d^*, K_d^{\text{NR-RA}^*})$ can be obtained by model (28), thus, $\theta^{\text{NR-RA}^*}$ and $\text{TGR}^{\text{NR-RA}*}$ can be calculated by model (13) and model (11).

6.2. Variation range of undesirable output under meta-frontier

Section 6.2 is to keep group frontier as the closest targets (NC-DDF) unchanged, and meta-frontier based on the farthest distance projection (F-DDF), so as to control TGR not to be greater than 1. Then, by minimizing/maximizing the range of undesirable output, the influence of undesirable output on the efficiency of meta- frontier and TGR is measured. By maximizing/minimizing the range, the potential impact of undesirable output on TGR can be clarified, which can help decision makers to better understand the potential impact of different undesirable outputs on current technological progress, so as to make more informed decisions, that is, to give priority to reducing which undesirable output through technological optimization can be more conducive to shortening the gap with the best technology.

(1) Conservative minimize/maximize the variation range of undesirable output based on F-DDF (F-CI and F-CA)

Based on the farthest distance projection, F-CI is the conservative minimum variation range of undesirable output, which is obtained by conservative objective function of meta-frontier, and it is in case of the projection of group frontier unchanged, while F-CA is the conservative maximum variation range of undesirable output obtained in this case. Under F-DDF, the PPS^{F} and UPS^{F} under VRS can be defined as follows.

$$\text{PPS}^{\text{F}} = \left\{ X, Y, Z \;\middle|\; \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{id}; \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rd} + \beta_y g_y; \sum_{j=1}^{n} \lambda_j Z_j = Z - \beta_Z; \sum_{j=1}^{n} \lambda_j = 1; \lambda_j \geq 0 \right\}$$

$$\mathrm{UPS}^{\mathrm{F}} = \left\{ Z \;\middle|\; \sum_{j=1}^{n} \lambda_j Z_j = Z - \beta_Z - S_f^-; \sum_{j=1}^{n} \lambda_j = 1; \lambda_j \geq 0; j = 1, \ldots, n \right\}. \tag{29}$$

Then a new $\mathrm{UPS}^{\mathrm{F}}$ can be obtained, as shown below.

$$\mathrm{UPS}^{\text{F-CI}} = \mathrm{UPS}^{\mathrm{V}} \cap \left\{ Z \mid Z \geq Z - K_d^{\mathrm{F}} \right\}. \tag{30}$$

Under meta-frontier, by minimizing/maximizing K_d^{F}, we can find the range of minimizing/maximizing the undesirable output.

First, minimize the variation range of undesirable output based on F-CI.

$$K_d^{\text{F-CI}^*} = \left(K_{1d}^{\text{F-CI}^*}, \ldots, K_{fd}^{\text{F-CI}^*} \right) = \min\left\{ \left(K_{1d}^{\text{F-CI}}, \ldots, K_{fd}^{\text{NC-CI}} \right) : Z - K_d^{\text{F-CI}} \in \mathrm{UPS}^{\mathrm{F}} \right\}. \tag{31}$$

The model (31) can be transformed into model (32).

$$K_d^{\text{F-CI}^*} = \min\left\{ \left(\sum_{r=1}^{s} \alpha_y^g + \sum_{f=1}^{h} \alpha_z^g \right) + \varepsilon \left(\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ + \sum_{f=1}^{h} S_f^- \right) \right\}. \tag{32}$$

According to model (32), the optimal variation range of undesirable output is $\bar{Z}_d^* = Z_d - (\beta^* g_z + S_f^{-*}) = Z_d - K_d^{\text{F-CI}^*}$; Let $\delta_d = \beta_y g_y$ then the new $\mathrm{PPS}^{\text{F-CI}}$ can be obtained.

$$\mathrm{PPS}^{\text{F-CI}} = \mathrm{PPS}^{\mathrm{V}} \cap \left\{ (Y, Z) \;\middle|\; Y \leq Y + \delta_d; Z \geq Z - K_d^{\mathrm{F}} \geq Z - K_d^{\text{F-CI}^*} \right\}. \tag{33}$$

Therefore, the optimal solution $(\delta_d^*, K_d^{\text{F-CI}^*})$ can be obtained.

$$\left(\delta_d^*, K_d^{\text{NC-CI}^*} \right) = \max\left\{ (\delta_{1d}, \ldots, \delta_{rd}); \left(K_{1d}^{\text{F-CI}}, \ldots, K_{fd}^{\text{F-CI}} \right) : \left(Y + \delta_d, Z - K_d^{\text{F-CI}^*} \right) \in \mathrm{PPS}^{\text{F-CI}} \right\}. \tag{34}$$

The model (34) can be transformed into model (35).

$$\left(\delta_d^*, K_d^{\text{F-CI}^*} \right) = \max\left\{ \beta_y^m + \beta_z^m \right\}. \tag{35}$$

The optimal solution $(\delta_d^*, K_d^{\text{F-CI}^*})$ can be obtained by model (35), thus, $\theta^{\text{F-CI}^*}$ and $\mathrm{TGR}^{\text{F-CI}^*}$ can be calculated by calculated by equation (5) and (6).

Similarly, maximize the variation range of undesirable output based on F-DDF (F-CA).

$$K_d^{\text{F-CA}^*} = \left(K_{1d}^{\text{F-CA}^*}, \ldots, K_{fd}^{\text{F-CA}^*} \right) = \max\left\{ \left(K_{1d}^{\text{F-CA}}, \ldots, K_{fd}^{\text{F-CA}} \right) : Z - K_d^{\text{F-CA}} \in \mathrm{UPS}^{\mathrm{F}} \right\}. \tag{36}$$

The model (36) is transformed into a model (37).

$$K_d^{\text{NC-CA}^*} = \max\left\{ \left(\sum_{r=1}^{s} \alpha_y^g + \sum_{f=1}^{h} \alpha_z^g \right) + \varepsilon \left(\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ + \sum_{f=1}^{h} S_f^- \right) \right\}. \tag{37}$$

The new $\mathrm{PPS}^{\text{F-CA}}$ is similar to formula (34), and the optimal solution $(\delta_d^*, K_d^{\text{F-CA}^*})$ can be obtained by model (36), thus, $\theta^{\text{F-CA}^*}$ and $\mathrm{TGR}^{\text{F-CA}^*}$ can be calculated by equations (5) and (6).

(2) Radical minimize/ maximize the variation range of undesirable output based on F-DDF (F-RI and F-RA)

According to equations (29) and (30), under meta-frontier, by radical minimizing/maximizing K_d^{F}, we can find the range of radical minimizing/maximizing the undesirable output.

First, radical minimize the variation range of undesirable output based on F-RI.

$$K_d^{\text{F-RI}^*} = \left(K_{1d}^{\text{F-RI}^*}, \ldots, K_{fd}^{\text{F-RI}^*}\right) = \min\left\{\left(K_{1d}^{\text{F-RI}}, \ldots, K_{fd}^{NC-RI}\right) : Z - K_d^{\text{F-RI}} \in \mathrm{UPS}^{\mathrm{F}}\right\}. \tag{38}$$

The model (38) can be transformed into model (39).

$$K_d^{\text{F-RI}^*} = \min\left\{\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ + \sum_{f=1}^{h} S_f^-\right\}. \tag{39}$$

According to model (39), the optimal variation range of undesirable output is $\bar{Z}_d^* = Z_d - (\beta^* g_z + S_f^{-*}) = Z_d - K_d^{\text{F-RI}^*}$; Let $\delta_d = \beta_y g_y$, then the new $\mathrm{PPS}^{\text{F-RI}}$ can be obtained as follows.

$$\mathrm{PPS}^{\text{F-RI}} = \mathrm{PPS}^{\mathrm{F}} \cap \left\{(Y, Z) \,\middle|\, Y \le Y + \delta_d; Z \ge Z - K_d^{\mathrm{F}} \ge Z - K_d^{\text{F-RI}^*}\right\}. \tag{40}$$

Therefore, the optimal solution $(\delta_d^*, K_d^{\text{F-RI}^*})$ can be obtained.

$$\left(\delta_d^*, K_d^{\text{NC-RI}^*}\right) = \max\left\{(\delta_{1d}, \ldots, \delta_{rd}); \left(K_{1d}^{\text{F-RI}}, \ldots, K_{fd}^{\text{F-RI}}\right) : \left(Y + \delta_d, Z - K_d^{\text{F-RI}^*}\right) \in \mathrm{PPS}^{\text{F-RI}}\right\}. \tag{41}$$

The model (41) can be transformed into model (42).

$$\left(\delta_d^*, K_d^{\text{F-RI}^*}\right) = \max\left\{\beta_y^m + \beta_z^m\right\}. \tag{42}$$

The optimal solution $(\delta_d^*, K_d^{\text{F-RI}^*})$ can be obtained by model (42), thus, $\theta^{\text{F-RI}^*}$ and $\mathrm{TGR}^{\text{F-RI}^*}$ can be calculated by equations (5) and (6).

Similarly, radical maximize the variation range of undesirable output based on F-RA.

$$K_d^{\text{F-RA}^*} = \left(K_{1d}^{\text{F-RA}^*}, \ldots, K_{fd}^{\text{F-RA}^*}\right) = \max\left\{\left(K_{1d}^{\text{F-RA}}, \ldots, K_{fd}^{\text{F-RA}}\right) : Z - K_d^{\text{F-RA}} \in \mathrm{UPS}^{\mathrm{F}}\right\}. \tag{43}$$

The model (43) is transformed into a model (44).

$$K_d^{\text{F-RA}^*} = \max\left\{\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ + \sum_{f=1}^{h} S_f^-\right\}. \tag{44}$$

The new $\mathrm{PPS}^{\text{F-RA}}$ is similar to formula (41), $(\delta_d^*, K_d^{\text{F-RA}^*})$ can be obtained by model (43), thus, $\theta^{\text{F-CA}^*}$ and $\mathrm{TGR}^{\text{F-CA}^*}$can be calculated by equations (5) and (6).

6.3. Expansion: TGR impact analysis based on dynamic slacks-based DEA

Tone and Tsutsui [36] first combined slack variables with dynamic DEA, and constructed dynamic slacks-based DEA model. Then, dynamic slacks-based measure (SBM) model has been wildly used in environmental efficiency analyses [28]. In addition to its advantages in dynamic analysis, Emrouznejad *et al.* [14] pointed out that SBM is also very mature in dealing with undesirable output. At present, there are two main SBM methods to solve undesirable output: (1) SBM model for handling desirable output and undesirable output is distinguished; (2) SBM model of jointly dealing with desirable output and undesirable output. Obviously, SBM model can deal with undesirable output flexibly and efficiently. Therefore, we further expand this research method of this paper, and combine dynamic SBM with NC-DDF and NR-DDF respectively, so that we can deeply analyze the dynamic changes of TGR caused by undesirable output.

First, we combine dynamic SBM with NC-DDF, and the new model SNC-DDF is constructed as follows:

$$
\begin{aligned}
&\text{Min } \left[\left(\alpha_y^g + \alpha_z^g\right) + \varepsilon \left(\sum_{i=1}^{m} \frac{S_{it}^-}{R_{it}} + \sum_{r=1}^{s} \frac{S_{rt}^+}{R_{rt}} + \sum_{f=1}^{h} \frac{S_{ft}^-}{R_{ft}} + \sum_{i=1}^{\text{ninput}} \frac{S_{it}^{\text{ninput}}}{R_{it}} \right) / (m + s + h + \text{ninput}) \right] \\
&\text{s.t. } \sum_{j=1}^{n} \lambda_{jt}^g \pi_{ijt}^{\partial} = \sum_{j=1}^{n} \lambda_{j(t+1)}^g \pi_{ijt}^{\partial}, && t = 1, \dots, T; \\
&\sum_{j=1}^{n} \lambda_{jt}^g x_{ijt} = x_{idt} - S_{it}^-, && i = 1, \dots, m; \\
&\sum_{j=1}^{n} \lambda_{jt}^g x_{ijt}^{\text{input}} = x_{idt}^{\text{input}} - S_{it}^{\text{input}}, i = 1, \dots, \text{ninput}; \\
&\sum_{j=1}^{n} \lambda_{jt}^g y_{rjt} = y_{rdt} + \alpha_{yt}^g g_{rt}^y + S_{rt}^+, && r = 1, \dots, s; \\
&\sum_{j=1}^{n} \lambda_{jt}^g z_{ftj} = z_{fdt} - \alpha_{zt}^m g_{ft}^z - S_{ft}^-, && f = 1, \dots, h; \\
&\sum_{r=1}^{s} u_{rt} y_{rjt} - \sum_{i=1}^{m} v_{it} x_{ijt} - \sum_{f=1}^{h} w_{ft} z_{ftj} + d_{0t} + d_{jt} = 0 \\
&\sum_{j=1}^{n} \lambda_{jt}^g z_{ftj} \le C_t, && f = 1, \dots, h; \\
&\lambda_{jt}^g \le M \cdot (1 - b_{jt}) \\
&d_{jt} \le M \cdot b_{jt} \\
&\sum_{j=1}^{n} \lambda_{jt}^g = 1; \\
&\lambda_{jt}, d_{jt} \ge 0, b_{jt} \in \{0, 1\}, && \forall j \\
&S_{it}^-, S_{rt}^+, S_{ft}^- \ge 0, u_{rt}, v_{it}, w_{ft} \ge \delta_t. && (45)
\end{aligned}
$$

Among them, constraint (1) is the continuity of link flows (carry-overs) between terms t and $t+1$, and x_{ijt}^{input} is carried over from period t to period $t+1$. ε is a non-Archimedean infinitesimal. When the projection directions are inconsistent, α_{yt}^g and α_{zt}^g represent technological inefficiency scores of desirable output and undesirable output in times t, respectively. s_{rt}^+ and s_{rt}^+ are slack variables of desirable output and undesirable output in times t, respectively.

Second, we combine dynamic SBM with NR-DDF, and the new model SNR-DDF is constructed as follows:

$$
\begin{aligned}
&\text{Min } \left(\sum_{i=1}^{m} S_{it}^- + \sum_{r=1}^{s} S_{rt}^+ + \sum_{f=1}^{h} S_{ft}^- + \sum_{i=1}^{\text{ninput}} S_{it}^{\text{input}} \right) \\
&\text{s.t. } \sum_{j=1}^{n} \lambda_{jt}^g \pi_{ijt}^{\partial} = \sum_{j=1}^{n} \lambda_{j(t+1)}^g \pi_{ijt}^{\partial}, && t = 1, \dots, T;
\end{aligned}
$$

$$\begin{aligned}
&\sum_{j=1}^{n} \lambda_{jt}^{g} x_{ijt} = x_{idt} - S_{it}^{-}, && i = 1, \ldots, m;\\
&\sum_{j=1}^{n} \lambda_{jt}^{g} x_{ijt}^{\text{input}} = x_{idt}^{\text{input}} - S_{it}^{\text{input}}, && i = 1, \ldots, \text{ninput};\\
&\sum_{j=1}^{n} \lambda_{jt}^{g} y_{rjt} = y_{rdt} + \alpha_{yt}^{g} g_{rt}^{y} + S_{rt}^{+}, && r = 1, \ldots, s;\\
&\sum_{j=1}^{n} \lambda_{jt}^{g} z_{ftj} = z_{fdt} - \alpha_{zt}^{m} g_{ft}^{z} - S_{ft}^{-}, && f = 1, \ldots, h;\\
&\sum_{r=1}^{s} u_{rt} y_{rjt} - \sum_{i=1}^{m} v_{it} x_{ijt} - \sum_{f=1}^{h} w_{ft} z_{ftj} + d_{0t} + d_{jt} = 0\\
&\sum_{j=1}^{n} \lambda_{jt}^{g} z_{ftj} \leq C_t, && f = 1, \ldots, h;\\
&\lambda_{jt}^{g} \leq M \cdot (1 - b_{jt})\\
&d_{jt} \leq M \cdot b_{jt}\\
&\sum_{j=1}^{n} \lambda_{jt}^{g} = 1;\\
&\lambda_{jt}, d_{jt} \geq 0, b_{jt} \in \{0, 1\}, && \forall j\\
&S_{it}^{-}, S_{rt}^{+}, S_{ft}^{-} \geq 0, u_{rt}, v_{it}, w_{ft} \geq \delta_t.
\end{aligned} \tag{46}$$

SNC-DDF and SNR-DDF replace NC-DDF and NR-DDF, respectively. On this basis, the range of undesirable output is measured according to Section 6. Then, combined with Section 7, the range of TGR can be calculated. According to the above process, the method proposed in this paper can be further extended to dynamic DEA for analysis.

7. Discussion on NC-DDF and NR-DDF

The influence of undesirable output is defined as follows:

Definition 2. Under the undesirable output influence, the value interval of new TGR is as follows.

$$\left[\min\left(\text{TGR}^{\text{lower}}, \text{TGR}^{\text{original}}\right), \max\left(\text{TGR}^{\text{upper}}, \text{TGR}^{\text{original}}\right)\right]. \tag{47}$$

Definition 3. When meta-frontier keeps using F-DDF unchanged, on the basis of NC-DDF and NR-DDF, conservative minimization/maximization and radical minimization/maximization are proposed, and they are used to measure the variation range of undesirable output in group frontier. Therefore, the variation range of the newly obtained TGR is as follows.

$$\begin{aligned}
&\left[0, \text{TGR}^{\text{new}} - \text{TGR}^{\text{original}}\right],\ \text{TGR}^{\text{new}} - \text{TGR}^{\text{original}} \geq 0\\
&\left[\text{TGR}^{\text{new}} - \text{TGR}^{\text{original}}, 0\right],\ \text{TGR}^{\text{new}} - \text{TGR}^{\text{original}} \leq 0.
\end{aligned} \tag{48}$$

In the above case, when TGR^{new} is obtained by NC-CI or NC-CA, $\text{TGR}^{\text{original}}$ stands for $\text{TGR}^{\text{NC-DDF}}$; When TGR^{new} is obtained by NR-RI or NR-RA, $\text{TGR}^{\text{original}}$ stands for $\text{TGR}^{\text{NR-DDF}}$.

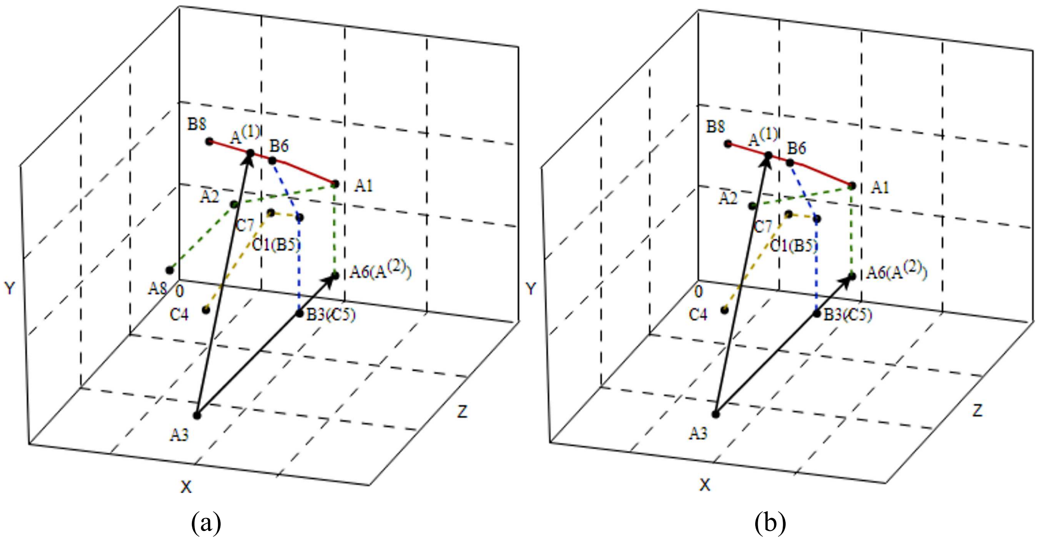


FIGURE 3. Projections of NC-DDF and NR-DDF.

On the basis of F-DDF, conservative minimization/maximization and radical minimization/maximization are proposed, and they are used to measure the variation range of undesirable output in meta-frontier. Therefore, the variation range of the newly obtained TGR is the same as Formula (48). However, $\mathrm{TGR}^{\text{original}}$ stands for $\mathrm{TGR}^{\text{NC-DDF}}$ at this time.

Definition 4. Positive effects, that is, when $\mathrm{TGR}^{\text{new}} - \mathrm{TGR}^{\text{original}} \geq 0$, the variation range of TGR is positive. In this case, maintaining or appropriately reducing undesirable output is beneficial to reducing technological heterogeneity.

Definition 5. Negative effects, that is, when $\mathrm{TGR}^{\text{new}} - \mathrm{TGR}^{\text{original}} \leq 0$, the variation range of TGR is negative. In this case, it is necessary to strictly control the undesirable output, otherwise, it will lead to an increase in technological heterogeneity.

7.1. Advantages of NC-DDF and NR-DDF in controlling TGR value reasonably

The key to measure the influence of undesirable output on TGR is to find out its variable range. Before that, we must ensure that the TGR value is reasonable, while both NC-DDF and NR-DDF can effectively relieve the unreasonable TGR value. Based on the closest targets, NC-DDF and NR-DDF are in the case of not interfering with the optimal increase or decrease ratio of each factor in the projection process, which is beneficial to measure the influence of undesirable output on TGR.

According to the numerical example of Yu *et al.* [44], the TGR of A3 is greater than 1. Thus, taking A3 as an example. In Figure 3a, the farthest distance projection of meta-frontier is kept unchanged, $\mathrm{A}^{(1)}$ is the projection point of A3 on meta-frontier. $\overrightarrow{\mathrm{A3A}^{(2)}}$ is the projection direction of A3, which is the closest targets of group frontier. In Figure 3b, $\overrightarrow{\mathrm{A3A}^{(2)}}$ is the projection direction of A3 on group frontier in NR-DDF, and $\mathrm{A}^{(2)}$ is the projection point of A3 on group frontier. Obviously, in NC-DDF and NR-DDF, the closest targets point of A3 is A6 ($\mathrm{A}^{(2)}$). In the case of keeping the farthest distance projection of meta-frontier unchanged, both NC-DDF and NR-DDF project at the closest targets, which will not interfere with the optimal increase or decrease ratio of undesirable output and expected output in projection process.

In Table 2, both TGR^{NC} and TGR^{NR} are not greater than 1, and it is controlled within a reasonable range.

Table 2. TGR of NC-DDF and NR-DDF.

DMU	TGR^{NC}	TGR^{NR}	DMU	TGR^{NC}	TGR^{NR}	DMU	TGR^{NC}	TGR^{NR}
A1	1.0000	1.0000	B1	0.7941	0.3871	C1	1.0000	0.5000
A2	0.6000	0.6000	B2	1.0000	0.5625	C2	0.1667	0.1626
A3	0.1250	0.1851	B3	0.5000	0.3333	C3	1.0244	0.5143
A4	0.5714	0.3236	B4	0.6750	0.3125	C4	0.5455	0.2246
A5	0.6000	0.3169	B5	1.0000	0.5000	C5	0.5000	0.3333
A6	1.0000	0.5000	B6	1.0000	1.0000	C6	0.2763	0.2000
A7	0.2500	0.2069	B7	0.4286	0.3817	C7	1.0000	0.6667
A8	0.3333	0.3333	B8	1.0000	1.0000	C8	0.2059	0.2229
A9	0.1667	0.2262	B9	1.0000	0.5000	C9	0.5000	0.3333
A10	0.6000	0.3169	B10	0.2903	0.4000	C10	0.5455	0.2246

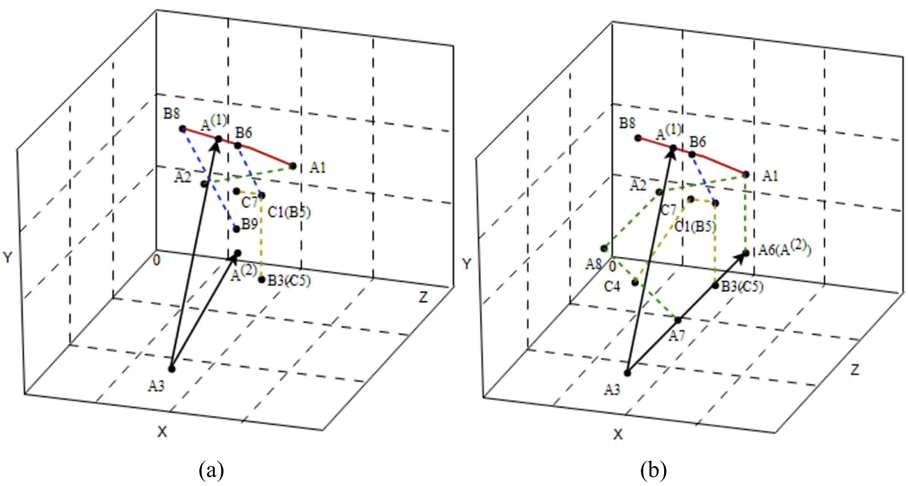


Figure 4. Minimum change of undesirable output of NC-CI and NR-RI.

7.2. Advantage of the variation range of undesirable output to measure TGR

(1) NC-CI and NR-RI of the minimum change of undesirable output

The projection path of A3 is shown in Figure 4. Comparing Figures 4a and 3a, obviously, the projection path of A3 has changed under the minimum change of undesirable output. After minimizing the undesirable output, NC-DDF mainly makes group frontier extend vertically by relaxing variables, and then obtains the projection reference point of A3.

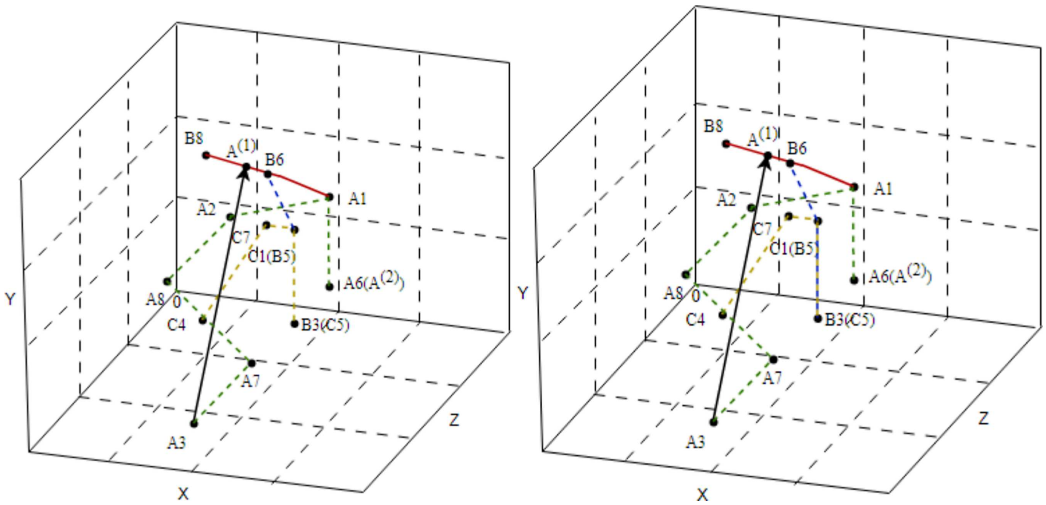
In Table 3, although the variation range of undesirable output is minimized, its TGR values are all within a reasonable range. Meanwhile, after minimizing the range of undesirable output, the TGR value fluctuates to some extent, so it can be preliminarily judged that undesirable output has an impact on the TGR value.

(2) NC-CA and NR-RA of the maximum change of undesirable output

Based on the closest targets, the variation range of undesirable output is maximized, and the projection path of A3 is shown in Figure 5. Obviously, when the range of undesirable output reaches the maximum, A3 is located on frontier of group A in NC-CA and NR-RA, that is, A3 is an effective DMU. Meanwhile, we can also find that after the range of undesirable output is maximized, the efficiency of group frontier decreases to some

TABLE 3. Upper limit of TGR for NC-CI and NR-RI.

DMU	$TGR^{NC\text{-}CI}$	$TGR^{NR\text{-}RI}$	DMU	$TGR^{NC\text{-}CI}$	$TGR^{NR\text{-}RI}$	DMU	$TGR^{NC\text{-}CI}$	$TGR^{NR\text{-}RI}$
A1	1.0000	1.0000	B1	0.8438	0.7500	C1	1.0000	1.0000
A2	0.6000	0.6000	B2	1.0286	1.0000	C2	0.1667	0.1161
A3	0.1250	0.1322	B3	0.5455	1.0000	C3	1.0244	1.0000
A4	0.5714	0.5744	B4	0.7200	0.6383	C4	0.5455	0.4468
A5	0.6000	0.6000	B5	1.0000	1.0000	C5	0.5000	0.5000
A6	1.0000	1.0000	B6	1.0000	1.0000	C6	0.2763	0.2667
A7	0.2500	0.2500	B7	0.3913	0.3827	C7	1.0000	1.0000
A8	0.3333	0.3333	B8	1.0000	1.0000	C8	0.2059	0.1765
A9	0.1667	0.1791	B9	1.0000	1.0000	C9	0.5000	0.5000
A10	0.6000	0.6000	B10	0.3103	0.5000	C10	0.5455	0.4468



(a) (b)

FIGURE 5. Maximum change of undesirable output of NC-CA and NR-RA.

extent, so some TGR are improved. In case of keeping TGR not greater than 1, $TGR^{NC\text{-}CA}$ and $TGR^{NR\text{-}RA}$ in Table 4 are the lower limit of TGR change caused by undesirable output.

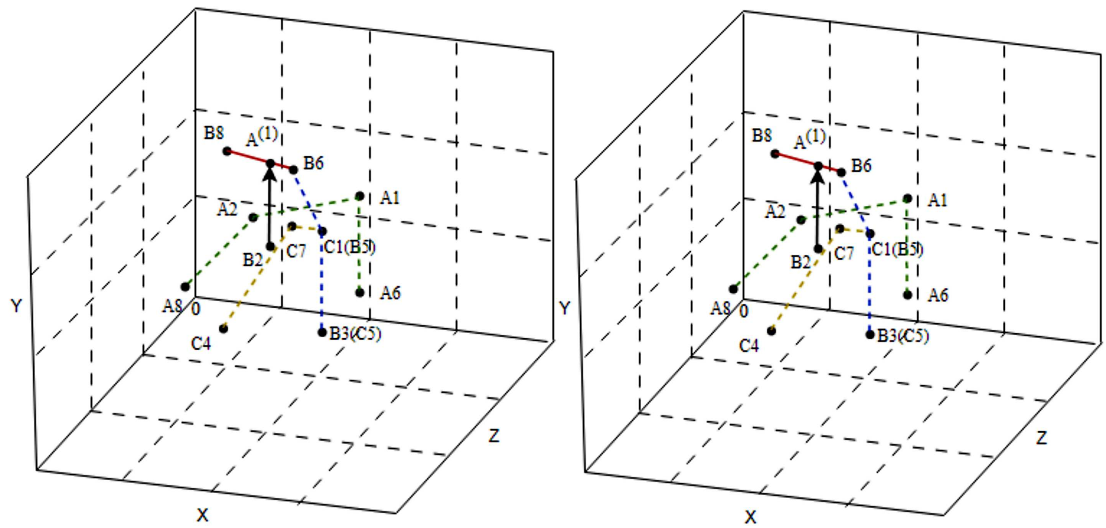
Taking A3 as an example, in NR-RA/RI, that is $[0.1250, 0.1851]$; and A9 is $[0.1667, 0.2262]$ in NR-RA/RI, however, in NC-CA/CI, the $TGR^{NC\text{-}CA}$ of A9 remains unchanged; C2 is $[0.1111, 0.1626]$ and C8 is $[0.1667, 0.2229]$ in NR-RA/RI, and they are $[0.1111, 0.1667]$ and $[0.5000, 0.5000]$ in NC-CA/CI, respectively.

(3) F-CI and F-RI of the minimum change of undesirable output

A3 is insensitive to the variation range of undesirable output under meta-frontier, we take B2 as an example. In Figure 6, under F-CI and F-RI, meta-frontier is composed of B6 and B8, and B2 is projected in the same way in F-CI and F-RI. In addition, we further compare $TGR^{F\text{-}CI}$ and $TGR^{F\text{-}RI}$, and find that they are completely the same, which shows that the F-DDF model is insensitive to the conservative and radical minimum variation range of undesirable output under meta-frontier. Moreover, both $TGR^{F\text{-}CI}$ and $TGR^{F\text{-}RI}$ are not greater than 1. As shown in Table 5.

TABLE 4. Lower limit of TGR for NC-CA and NR-RA.

DMU	$TGR^{NC\text{-}CA}$	$TGR^{NR\text{-}RA}$	DMU	$TGR^{NC\text{-}CA}$	$TGR^{NR\text{-}RA}$	DMU	$TGR^{NC\text{-}CA}$	$TGR^{NR\text{-}RA}$
A1	1.0000	1.0000	B1	0.7500	0.7500	C1	1.0000	1.0000
A2	0.6000	0.6000	B2	1.0000	1.0000	C2	0.1111	0.1111
A3	0.1250	0.1250	B3	0.5455	0.5000	C3	1.0000	1.0000
A4	0.5714	0.5714	B4	0.6000	0.6000	C4	0.4286	0.4286
A5	0.6000	0.6000	B5	1.0000	1.0000	C5	0.5000	0.5000
A6	1.0000	1.0000	B6	1.0000	1.0000	C6	0.2500	0.2500
A7	0.2500	0.2500	B7	0.3750	0.3750	C7	1.0000	1.0000
A8	0.3333	0.3333	B8	1.0000	1.0000	C8	0.1667	0.1667
A9	0.1667	0.1667	B9	1.0000	1.0000	C9	0.5000	0.5000
A10	0.6000	0.6000	B10	0.2727	0.2500	C10	0.4286	0.4286

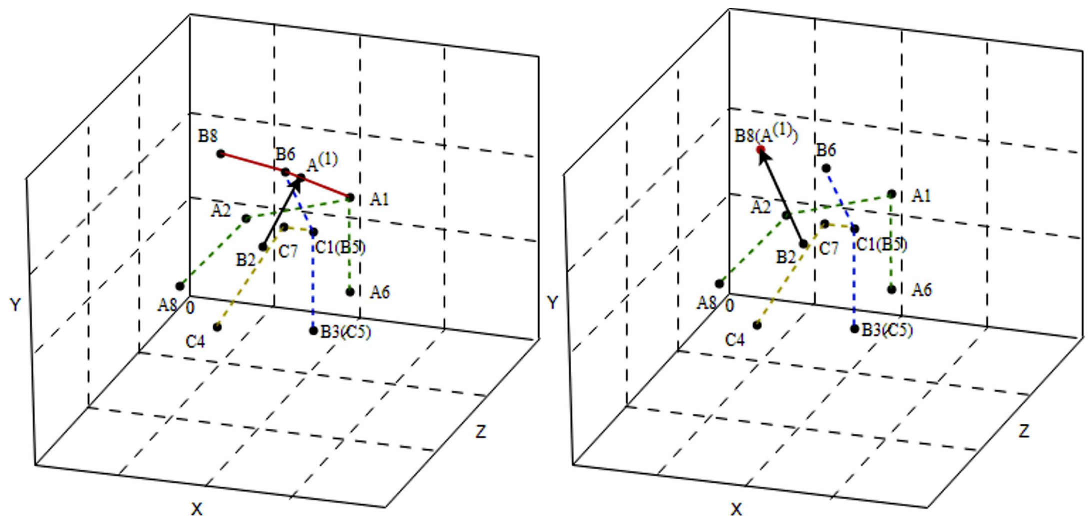


(a) (b)

FIGURE 6. Minimum change of undesirable output of F-CI and F-RI.

TABLE 5. Upper limit of TGR for F-CI and F-RI.

DMU	$TGR^{F\text{-}CI}$	$TGR^{F\text{-}RI}$	DMU	$TGR^{F\text{-}CI}$	$TGR^{F\text{-}RI}$	DMU	$TGR^{F\text{-}CI}$	$TGR^{F\text{-}RI}$
A1	0.5000	0.5000	B1	0.5294	0.5294	C1	0.5000	0.5000
A2	0.2000	0.2000	B2	0.7500	0.7500	C2	0.3000	0.3000
A3	0.2000	0.2000	B3	0.2500	0.2500	C3	0.5122	0.5122
A4	0.1500	0.1500	B4	0.1125	0.1125	C4	0.1273	0.1273
A5	0.1000	0.1000	B5	0.5000	0.5000	C5	0.2500	0.2500
A6	0.2500	0.2500	B6	1.0000	1.0000	C6	0.2763	0.2763
A7	0.2500	0.2500	B7	0.2286	0.2286	C7	0.7500	0.7500
A8	0.2000	0.2000	B8	1.0000	1.0000	C8	0.2471	0.2471
A9	0.2000	0.2000	B9	0.5000	0.5000	C9	0.2500	0.2500
A10	0.1000	0.1000	B10	0.2903	0.2903	C10	0.1273	0.1273



(a) (b)

FIGURE 7. Maximum change of undesirable output of F-CA and F-RA.

TABLE 6. Lower limit of TGR for F-CA and F-RA.

DMU	$TGR^{F\text{-}CA}$	$TGR^{F\text{-}RA}$	DMU	$TGR^{F\text{-}CA}$	$TGR^{F\text{-}RA}$	DMU	$TGR^{F\text{-}CA}$	$TGR^{F\text{-}RA}$
A1	1.0000	0.4000	B1	0.3971	0.4235	C1	0.5000	0.4000
A2	0.6000	0.8000	B2	0.5625	0.6000	C2	0.3000	0.3000
A3	0.2000	0.2000	B3	0.3333	0.2000	C3	0.5122	0.4098
A4	0.4500	0.6000	B4	0.4500	0.4500	C4	0.5091	0.5091
A5	0.4000	0.4000	B5	0.5000	0.4000	C5	0.3333	0.2000
A6	0.5000	0.2000	B6	1.0000	0.8000	C6	0.2456	0.2211
A7	0.2222	0.2000	B7	0.6857	0.9143	C7	0.6667	0.6000
A8	0.6000	0.8000	B8	1.0000	1.0000	C8	0.2471	0.2471
A9	0.2000	0.2000	B9	0.5000	0.4000	C9	0.3333	0.2000
A10	0.4000	0.4000	B10	0.2581	0.2323	C10	0.5091	0.5091

(4) F-CA and F-RA of the maximum change of undesirable output

In Figure 7, the frontier facet of F-CA is composed of A1, B6 and B8. Due to the influence of the range of undesirable output, the projection path of B2 in F-CA and F-CI has changed. In addition, $TGR^{F\text{-}CI}$ is 0.7500 and $TGR^{F\text{-}CA}$ is 0.5625. Obviously, undesirable output has a significant impact on TGR, and when the range of undesirable output is maximized by conservative method, the efficiency of group frontier increases, and then TGR decreases. That is, in case of TGR is not greater than 1, $TGR^{F\text{-}CA}$ and $TGR^{F\text{-}RA}$ in Table 6 are the lower limits of TGR change caused by undesirable output.

Taking B2 as an example, due to the influence of undesirable output, in F-DDF, that is $[0.5625, 1.0000]$ in F-CI/CA; and A9 is $[0.1667, 0.2262]$ in F-CI/CA, however, in F-RI/RA, the range of TGR of A9 is $[0.2000, 0.2262]$; A3 is $[0.1250, 0.2000]$ and C8 is $[0.2059, 0.2471]$ in F-CI/CA, and they are $[0.1250, 0.2000]$ and $[0.2229, 0.2471]$ in F-RI/RA, respectively.

Table 7. The variation range of TGR in different methods.

	$\text{TGR}^{\text{NC-CI}}$	$\text{TGR}^{\text{NR-RI}}$	$\text{TGR}^{\text{NC-CA}}$	$\text{TGR}^{\text{NR-RA}}$
Group A	0	$[0, 0.1260]$	$[-0.1240, 0]$	$[0, 0.1237]$
Group B	$[0, 0.0151]$	$[0, 0.2894]$	$[-0.1900, 0]$	$[0, 0.2098]$
Group C	0	$[0, 0.2071]$	$[-0.2382, 0]$	$[0, 0.2003]$
	$\text{TGR}^{\text{F-CI}}$	$\text{TGR}^{\text{F-RI}}$	$\text{TGR}^{\text{F-CA}}$	$\text{TGR}^{\text{F-RA}}$
Group A	$[-0.3096, 0]$	$[-0.3096, 0]$	$[-0.0674, 0]$	$[-0.1046, 0]$
Group B	$[-0.2527, 0]$	$[-0.2527, 0]$	$[-0.2001, 0]$	$[-0.2268, 0]$
Group C	$[-0.2424, 0]$	$[-0.2424, 0]$	$[-0.1608, 0]$	$[-0.2168, 0]$

7.3. Advantages comparison of different methods

Under the influence of undesirable output, the variation range of TGR caused by it is further calculated, and the results are shown in Table 7.

In Table 7, when the closest targets of meta-frontier unchanged, based on F-DDF, $\text{TGR}^{\text{F-CI}}$, $\text{TGR}^{\text{F-RI}}$, $\text{TGR}^{\text{F-CA}}$ and $\text{TGR}^{\text{F-RA}}$ will be reduced to some extent by adjusting undesirable output, and it is due to the weak disposability may overestimate the efficiency value on meta-frontier.

Similarly, when the projection of the farthest distance of meta-frontier unchanged, due to the weak disposability of the undesirable output, the efficiency value may be overestimated. By minimizing the undesirable output, the efficiency value of group frontier may be reduced, so that the range of $\text{TGR}^{\text{NC-CI}}$ and $\text{TGR}^{\text{NR-RI}}$ increase to some degrees. In addition, the variation range of $\text{TGR}^{\text{NC-CA}}$ has decreased, which is because after conservatively maximizing the variation range of undesirable output, its undesirable output has decreased, and then group frontier efficiency has increased, thus the TGR has decreased. This also shows that the NC-CA method can measure the lower limit of TGR under NC-DDF. Moreover, compared with group frontier, the variation change of undesirable output has a greater impact on TGR of meta-frontier.

7.4. Monte Carlo simulations

In this section, we use Monte Carlo simulation method to verify the robustness of new method in solving unreasonable value of TGR, and further analyze the influence of carbon pricing mechanism on TGR.

(1) Robustness test

We use Monte Carlo simulation method to check the robustness of the new method, and compare it with the existing methods to verify the reliability of the new method. First, under VRS, 200, 2000 and 6000 DMUs are randomly generated in the interval $[1, 10\,000]$, respectively, and then they are randomly divided into two groups, each DMU contains an input, an output and an undesirable output. On the premise of using F-DDF in meta-frontier, we test conservative and radical DDF models, respectively, and gradually increased the number of DMUs to check whether TGR is less than 1, and further verified the traditional DDF model by the same steps.

In Figure 8, after Monte Carlo simulation, traditional DDF has many unreasonable TGR values. This is due to traditional DDF does not constrain DMU projection path, which leads to the situation that the efficiency of meta-frontier is greater than that of group frontier. With increase of DMU simulation number, the number of unreasonable TGR in traditional DDF also increases, which shows that traditional DDF cannot maintain the rationality of TGR. In addition, with the increase of DMU simulation number, the unreasonable value of TGR becomes larger and larger. When the number of DMUs in the group is 3000, the maximum value of TGR is greater than 200, far exceeding TGR reasonable range $[0, 1]$, which further proves that the traditional DDF cannot guarantee the theoretical consistency of TGR.

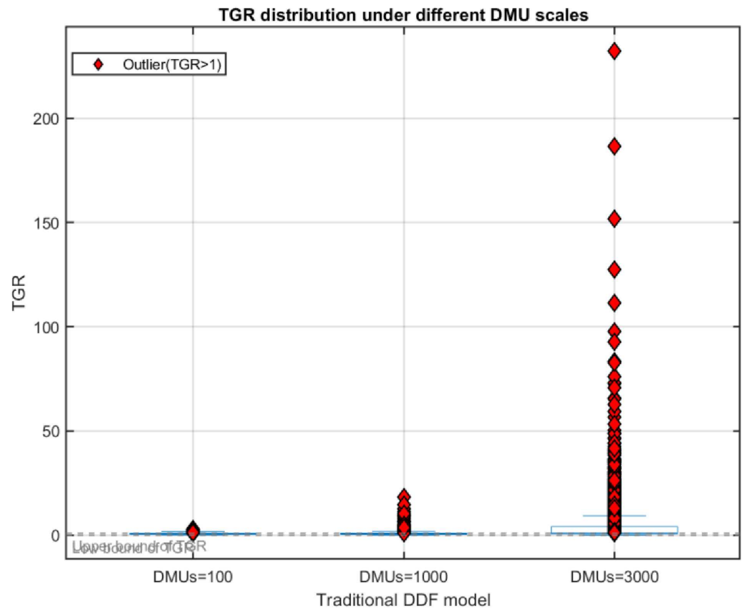


FIGURE 8. Test results of traditional DDF models.

In Figure 9, after Monte Carlo simulation, with the increase of DMU simulation number, the TGR of conservative DDF is strictly not greater than 1, which keeps the theoretical consistency well. However, when the number of DMUs in the group is 100 and 1000, the radical DDF has detected a TGR greater than 1. With the increase of DMUs to 3000, the number of unreasonable TGR becomes three, that is, the probability of reasonable TGR measured by radical DDF is 99%, which may be caused by the idea that radical DDF only pursues the minimum improvement path.

By comparing Figures 8 and 9, there are a lot of unreasonable TGR phenomena in traditional DDF, and the TGR deviates greatly. In contrast, the robustness and conservative DDF proposed in this paper control the rationality of TGR above 95%, showing better robustness in rationality measuring TGR.

(2) Carbon pricing mechanism analysis

We use Monte Carlo simulation method to analyze the influence of carbon pricing mechanism on TGR. First, under VRS, 2000 DMUs are randomly generated in the interval of $[1, 10\,000]$, and then they are randomly and evenly divided into two groups, each DMU contains an input, an output and an undesirable output. We use model (49) which introduces relaxation variables to measure the efficiency of meta-frontier. Because of the introduction of carbon price, when measuring the efficiency of group frontier and meta-frontier, we change the objective function of model (7d) and model (49) to Max PS_f^-, respectively. Then, we test TGR when carbon price (P) is 100 USD/ton, 120 USD/ton, 140 USD/ton, 160 USD/ton, 180 USD/ton and 200 USD/ton respectively, and analyze the convergence of TGR.

$$
\begin{aligned}
&\text{Max } \beta_d \\
&\text{s.t. } \sum_{j=1}^{n} \lambda_j x_{ij} = x_{id} - S_i^-, && i = 1, \ldots, m; \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} = y_{rd} + \beta_d g_y + S_r^+, && r = 1, \ldots, s;
\end{aligned}
$$

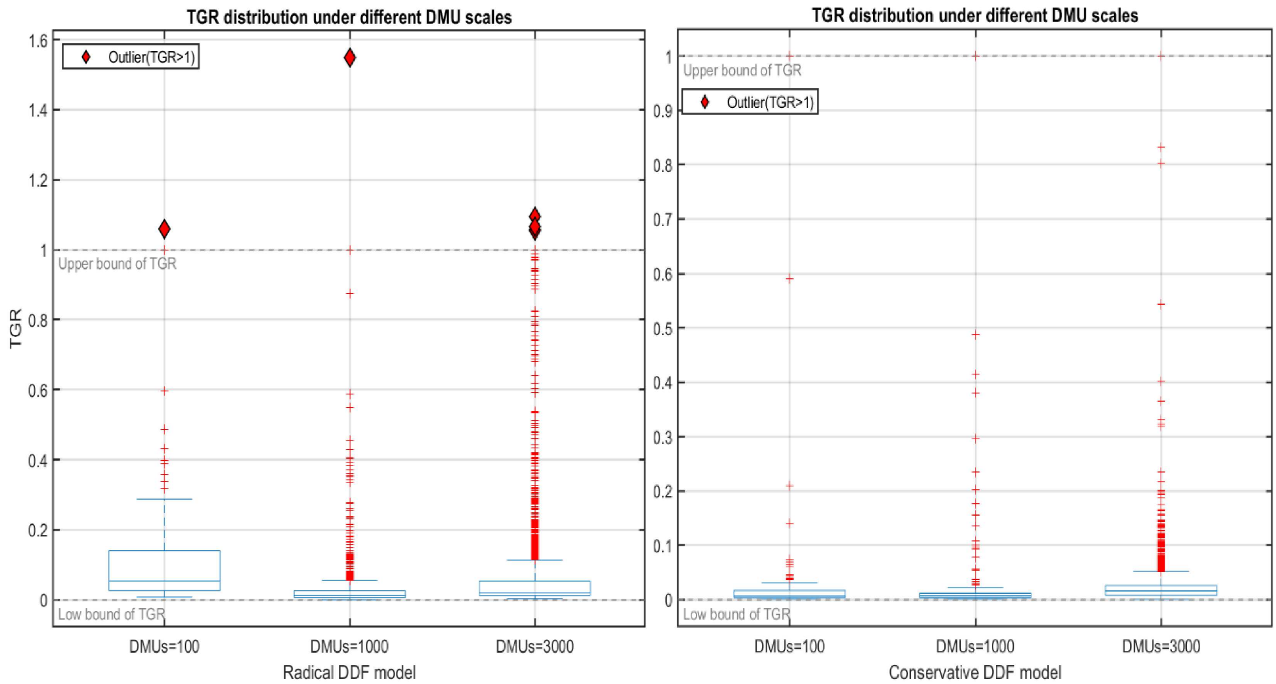


FIGURE 9. Test results of radical and conservative DDF models proposed in this paper.

$$\begin{aligned}
&\sum_{j=1}^{n} \lambda_j z_{fj} = z_{fd} - \beta_d g_z - S_f^-, \qquad f = 1, \ldots, h;\\
&\sum_{j=1}^{n} \lambda_j = 1;\\
&\forall \lambda_j, S_i^-, S_r^+, S_f^- \geq 0.
\end{aligned} \tag{49}$$

In Figure 10, with the increase of carbon price, the mean of TGR has not fluctuated obviously, which indicates that carbon price may have little influence on the absolute level of technology growth rate. In contrast, the median of TGR fluctuated greatly under different carbon prices. When $P = 200$, the difference between median and mean of TGR is the largest, indicating that there are many TGR close to 1, which indicates that the technical gap between group frontier and meta-frontier is reduced in high carbon price. In addition, the standard deviation of TGR fluctuates obviously under different carbon price, and standard deviation is the largest when $P = 200$, indicating that TGR has the largest fluctuation when $P = 200$.

We further evaluate the coefficient of variation (CV) at different carbon prices. CV quantifies the relative dispersion of TGR, with lower values indicating stronger convergence in TGR. In Figure 11, CV is the lowest at $P = 200$, which indicates that TGR is relatively stable and has the best convergence at this price. When $160 < P < 180$, CV increases the most, which may lead to polarization of technical response, that is, efficient enterprises reduce input and undesirable output through innovation, and thus maintain TGR; However, inefficient enterprises have reduced overall TGR due to insufficient investment in technology. Therefore, carbon pricing has a significant impact on the technical level, and paying close attention to its impact on TGR is important, which enables enterprises across varying capability levels to develop tailored and effective strategies.

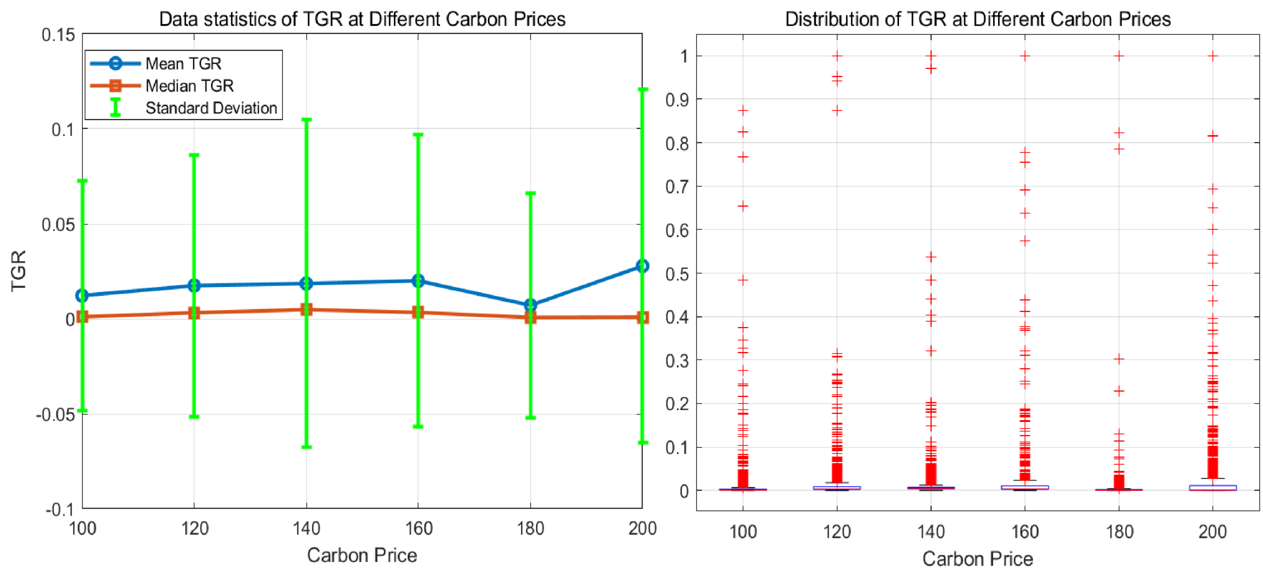


FIGURE 10. TGR evaluation under different carbon pricing.

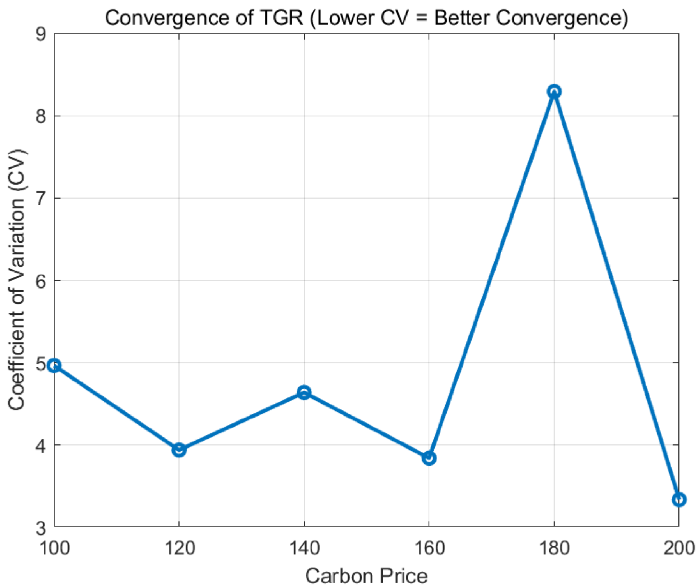


FIGURE 11. Coefficient variation of TGR.

8. Empirical application

8.1. Example description

Under meta-frontier, Wang *et al.* [38] select energy, labor and capital stock as input indicators, and they are often regarded as input in other carbon emission studies [39]. Furthermore, gross domestic product (GDP), which refers to the total market value of all final goods and services produced by a country or region in a certain

TABLE 8. Variables selection.

Variables	Variables	Unit	Data source
Primary	Net agricultural production (x^1)	Input indicator of primary industry (1 000 000 metric tons)	Nation Master
	Added value of agriculture, forestry, animal husbandry and fishery (y^1)	Desirable output indicators of primary industry (100 000 000 USD)	World Bank
	Agricultural carbon emission (z^1)	Undesirable output indicator of primary industry development (MtCO2e)	Climate Watch
Secondary	Material consumption of fossil fuels (x^2)	Input indicator of secondary industry (1 000 000 metric tons)	Nation Master
	Added value of manufacturing industry (y^2)	Desirable output indicators of secondary industry (100 000 000 USD)	World Bank
	Carbon emissions from manufacturing and construction industries (z^2)	Undesirable output indicator of secondary industry development (MtCO2e)	Climate Watch
Tertiary	Number of air passengers carried (x^3)	Input indicator of tertiary industry (1 000 000 metric tons)	Nation Master
	Added value of service industry (y^3)	Desirable output indicators of tertiary industry (100 000 000 USD)	World Bank
	Carbon emission from transportation industry (z^3)	Undesirable output indicator of tertiary industry development (MtCO2e)	Climate Watch

period of time. It is usually as a desirable output, and in the study of Wang *et al.* [38], industrial output value as desirable output factor, and CO_2 emissions as undesirable outputs [41]. According to the above analysis, the variable selection of this paper refers to the inputs/desirable outputs/undesirable outputs framework of Wang *et al.* [38], and further refines the inputs/desirable outputs/undesirable outputs indicators according to the characteristics of three industries, such as the primary industry replacing the traditional energy input indicators with net agricultural output, and replacing the traditional undesirable output indicators with agricultural carbon emissions. By measuring the inputs, desirable outputs and undesirable outputs of three industries, the influence of undesirable outputs on TGR is further measured. According to the mentioned analysis, the corresponding indicators were selected as shown in Table 8.

In this paper, we collect relevant data from 36 countries, which include 12 European countries, 12 Asian countries and 12 African countries. Summarize the above indicators in Table 9, as shown below.

8.2. Empirical analysis

(1) TGR of three industries based on NC-DDF and NR-DDF

In Table 10, values calculated by NC-DDF and NR-DDF are within a reasonable range, which means that they can well measure the technological gap of the three industries.

From the calculation results of NC-DDF model of Asia, the tertiary industry is more efficient than other industries in adopting the best available technology, followed by the secondary industry, and the overall utilization efficiency of the primary industry is the lowest. The overall ranking is consistent with NR-DDF; From the calculation results of NC-DDF and NR-DDF in Europe, the tertiary industry is more efficient than other industries in adopting the best available technology, followed by the secondary industry, and the overall utilization efficiency of the primary industry is the lowest; From the calculation results of NC-DDF and NR-DDF in Africa, the secondary industry is more efficient than other industries in adopting the best available technology, followed by the tertiary industry, and the overall utilization efficiency of the primary industry is the lowest.

TABLE 9. Descriptive statistics of the variables.

Variables	Sample size	Std. Dev	Min	Max	Mean
Net agricultural production (x^1)	$1*36$	847.24	0.29	6075.38	258.03
Material consumption of fossil fuels (x^2)	$1*36$	653.43	0.13	4907.08	177.23
Number of air passengers carried (x^3)	$1*36$	915.96	0.08	6596.29	386.68
Added value of agriculture, forestry, animal husbandry and fishery (y^1)	$1*36$	1428.44	1.09	10 200.00	407.69
Added value of manufacturing industry (y^2)	$1*36$	7402.33	2.48	55 100.00	2342.16
Added value of service industry (y^3)	$1*36$	11 425.67	15.51	77 500.00	4456.72
CO_2 emissions (z^1)	$1*36$	125.01	0.02	728.42	48.60
CO_2 emissions (z^2)	$1*36$	366.24	0.02	2797.19	80.70
CO_2 emissions (z^3)	$1*36$	135.60	0.20	965.51	59.14

The TGR value calculated by NR-DDF model is between 0 and 1, which can also effectively measure technical heterogeneity.

(2) TGR of three industries based on NC-CI and NR-RI

In Table 11, after minimizing the variation range of undesirable output, NC-CI and NR-RI for unreasonable TGR are still applicable. In Asia, based on NC-CI, the efficiency of adopting the best available technology is the highest in the tertiary industry. The above TGR ranking is consistent with that in NC-CA. Meanwhile, influenced by the minimization of undesirable output, based on NC-CI, the variation range of TGR of primary industry is $[0, 0.0013]$, that of secondary industry is $[0, 0.0025]$, and that of tertiary industry is $[0, 0.0066]$; Similarly, in Europe, based on NC-CI, the efficiency of adopting the best available technology is the highest in the tertiary industry. The above TGR ranking is consistent with NC-CA. Affected by the minimization of undesirable output, based on NC-CI, the variation range of TGR of the primary industry in Europe is $[0, 0.0004]$, that of the secondary industry is $[0, 0.0005]$ and that of the tertiary industry is $[0, 0.0061]$; In Africa, based on NC-CI and NR-RI, the efficiency of adopting the best available technology is the highest in the primary industry. Affected by the minimization of undesirable output, based on NC-CI, the variation range of TGR of the primary industry in Europe is $[-0.0024, 0]$, that of the secondary industry is $[-0.0054, 0]$ and that of the tertiary industry is $[-0.0017, 0]$.

(3) TGR of three industries based on F-CI and F-CA

In Table 12, the TGR values calculated by F-CI and F-CA are not greater than 1. The overall average value of TGR in F-CI and F-CA industries is different. The average TGR of the primary industry, in Asia, the secondary industry and the tertiary industry of F-CI is 0.1476, 0.3074 and 0.4099, respectively, while that of F-CA is 0.0559, 0.3335 and 0.3561. This means that the change of undesirable output has a significant impact on TGR. Meanwhile, in Asia, influenced by conservative minimize/ maximize of undesirable output, based on F-CI, the variation range of TGR_1 of primary industry is $[-0.0018, 0]$, that of TGR_2 of secondary industry is $[0, 0.0392]$, and that of TGR_3 the tertiary industry is $[0, 0.0052]$; In Europe, influenced by conservative minimize/maximize of undesirable output, based on F-CI, the variation range of TGR_1 of primary industry is $[0, 0.0004]$, that of TGR_2 of secondary industry is $[0, 0.0005]$, and that of TGR_3 the tertiary industry is $[0, 0.0063]$; In Africa, influenced by conservative minimize/maximize of undesirable output, based on F-CI, the variation range of TGR_1 of primary industry is $[-0.0022, 0]$, that of TGR_2 of secondary industry is $[-0.0054, 0]$, and that of TGR_3 the tertiary industry is $[-0.0017, 0]$.

Table 10. TGR of NC-DDF and NR-DDF.

		NC-DDF			NR-DDF		
		TGR_1	TGR_2	TGR_3	TGR_1	TGR_2	TGR_3
Asia	India	0.0574	0.0723	0.1484	0.0710	0.0831	0.1754
	Japan	0.1177	0.5483	1.0000	0.1177	0.5483	1.0000
	South Korea	0.0865	0.5102	0.4828	0.0866	0.5154	0.4939
	Saudi Arabia	0.0956	0.3068	0.2025	0.0956	0.3115	0.2120
	Singapore	1.0000	0.3174	1.0000	1.0000	0.3192	1.0000
	China	0.1695	0.2221	1.0000	0.1695	0.2221	1.0000
	Thailand	0.0421	0.2431	0.2242	0.0428	0.2447	0.2353
	Kazakhstan	0.0188	0.1252	0.3190	0.0188	0.1263	0.3197
	Malaysia	0.0975	0.2638	0.1588	0.0980	0.2643	0.1656
	Iraq	0.0151	0.4519	0.1196	0.0152	0.4541	0.1208
	Iran	0.0571	0.0740	0.0767	0.0575	0.0758	0.0802
	Viet Nam	0.0356	0.0833	0.1243	0.0365	0.0839	0.1276
Europe	Germany	0.0290	0.6151	0.8470	0.0298	0.6151	0.8470
	Greece	0.0385	0.1766	0.3686	0.0386	0.1770	0.3670
	Italy	0.0589	0.6837	1.0000	0.0593	0.6837	1.0000
	France	0.0348	0.6885	0.8442	0.0357	0.6885	0.8442
	Spain	0.0441	0.5356	0.5452	0.0448	0.5326	0.5539
	The Netherlands	0.0376	0.4421	0.9559	0.0378	0.4398	0.9559
	Russia	0.0365	0.1244	0.2064	0.0379	0.1329	0.2266
	Poland	0.0225	0.3216	0.3553	0.0228	0.3237	0.3592
	Ukraine	0.0218	0.0590	0.2001	0.0222	0.0588	0.1998
	Austria	0.0240	0.3316	0.4979	0.0241	0.3320	0.5011
	Hungary	0.0257	0.2200	0.2428	0.0257	0.2201	0.2425
	Ireland	0.0089	1.0000	0.4594	0.0088	1.0000	0.4979
Africa	South Africa	0.0156	0.1135	0.2828	0.0156	0.1155	0.2866
	Algeria	0.0734	0.2360	0.1074	0.0735	0.2333	0.1081
	Morocco	0.0427	0.1642	0.1866	0.0428	0.1607	0.1866
	Nigeria	0.0735	0.4579	0.2258	0.0735	0.4579	0.2285
	Jamaica	0.0487	0.0378	0.1078	0.0487	0.0378	0.1078
	Mauritius	0.0685	0.1838	0.2144	0.0685	0.1838	0.2144
	Paraguay	0.0077	1.0000	0.0730	0.0076	1.0000	0.0714
	Namibia	0.0050	0.2995	0.1008	0.0047	0.2995	0.1008
	Gabon	0.0437	0.3237	1.0000	0.0437	0.3237	1.0000
	Albania	0.0283	0.1605	0.0952	0.0283	0.1000	0.0952
	Suriname	0.0209	0.3859	0.0743	0.0154	0.3860	0.0706
	Belize	0.0341	1.0000	0.0941	0.0114	1.0000	0.0721

8.3. Models comparison

In Table 13, when the variation range of undesirable output is minimized, the TGR of the tertiary industry in Asia is most affected by NC-CI. Because its variation change has increased, it shows that if undesirable output of the tertiary industry in Asia decreases, the heterogeneity of the tertiary industry will decrease significantly. Under F-CI, the secondary industry in Africa has been most affected, and its TGR is decline, which means that it is necessary to strictly control the increase of undesirable output in this region, otherwise, the heterogeneity of the secondary industry will be adversely affected. Under NR-RA, the tertiary industry in Europe has been most affected. Because its variation range has decreased, it is similar to the secondary industry in Africa, and it is necessary to strictly supervise the undesirable output of the tertiary industry in Europe.

Table 11. TGR of NC-CI and NR-RI.

		NC-CI			NR-RI		
		TGR_1	TGR_2	TGR_3	TGR_1	TGR_2	TGR_3
Asia	India	0.0710	0.0831	0.1758	0.0574	0.0723	0.1485
	Japan	0.1177	0.5483	1.0000	0.1177	0.5483	1.0000
	South Korea	0.0865	0.5154	0.4989	0.0865	0.5102	0.4842
	Saudi Arabia	0.0956	0.3115	0.2120	0.0956	0.3068	0.2027
	Singapore	1.0000	0.3192	1.0000	1.0000	0.3174	1.0000
	China	0.1695	0.2221	1.0000	0.1695	0.2221	1.0000
	Thailand	0.0428	0.2447	0.2353	0.0421	0.2431	0.2250
	Kazakhstan	0.0188	0.1263	0.3190	0.0188	0.1252	0.3190
	Malaysia	0.0975	0.2643	0.1656	0.0975	0.2632	0.1592
	Iraq	0.0152	0.4541	0.1208	0.0151	0.4519	0.1209
	Iran	0.0575	0.0758	0.0805	0.0571	0.0740	0.0767
	Viet Nam	0.0365	0.0839	0.1280	0.0357	0.0833	0.1247
Europe	Germany	0.0298	0.6151	0.8523	0.0291	0.6151	0.8472
	Greece	0.0385	0.1770	0.3670	0.0385	0.1766	0.3665
	Italy	0.0593	0.6837	1.0000	0.0589	0.6837	1.0000
	France	0.0357	0.6885	0.8450	0.0348	0.6885	0.8443
	Spain	0.0448	0.5326	0.5595	0.0442	0.5356	0.5470
	The Netherlands	0.0378	0.4398	0.9559	0.0376	0.4428	0.9559
	Russia	0.0379	0.1329	0.2288	0.0366	0.1244	0.2066
	Poland	0.0228	0.3237	0.3592	0.0225	0.3216	0.3557
	Ukraine	0.0222	0.0588	0.2002	0.0218	0.0590	0.2015
	Austria	0.0241	0.3320	0.4927	0.0240	0.3316	0.4927
	Hungary	0.0257	0.2201	0.2424	0.0256	0.2200	0.2387
	Ireland	0.0088	1.0000	0.4932	0.0089	1.0000	0.4594
Africa	South Africa	0.0156	0.1155	0.2866	0.0156	0.1135	0.2836
	Algeria	0.0734	0.2333	0.1083	0.0734	0.2326	0.1077
	Morocco	0.0428	0.1607	0.1861	0.0427	0.1650	0.1858
	Nigeria	0.0735	0.4579	0.2285	0.0735	0.4579	0.2264
	Jamaica	0.0487	0.0378	0.1079	0.0487	0.0378	0.1078
	Mauritius	0.0685	0.1838	0.2145	0.0703	0.1851	0.2144
	Paraguay	0.0076	1.0000	0.0714	0.0077	1.0000	0.0743
	Namibia	0.0047	0.2995	0.1008	0.0050	0.2995	0.1008
	Gabon	0.0437	0.3237	1.0000	0.0437	0.3237	1.0000
	Albania	0.0283	0.1000	0.0952	0.0283	0.1605	0.0952
	Suriname	0.0154	0.3860	0.0706	0.0241	0.3859	0.0743
	Belize	0.0114	1.0000	0.0721	0.0424	1.0000	0.0941

9. Conclusions

In this paper, the direct reason of unreasonable TGR value is explored, which makes up the blank of exploring the influence of undesirable output on technological heterogeneity. By introducing the closest targets into meta-frontier framework, and determining the variation range of undesirable output under VRS, the positive/negative effects on TGR are obtained. Based on the calculation results, the following conclusions can be drawn.

– **Methodological advancements:** We propose NC-DDF and NR-DDF, two improved models that strictly constrain TGR not to be greater than 1, resolving a longstanding issue of inflated TGR values in traditional approaches; Eight derived methods (NC-CI, NC-CA, NR-RI, NR-RA, F-CI, F-CA, F-RI, F-RA) are validated to remain statistically and computationally robust even after minimizing/ maximizing undesirable output ranges, ensuring their applicability across diverse empirical settings.

Table 12. TGR of F-CI and F-CA.

		FCI			FCA		
		TGR_1	TGR_2	TGR_3	TGR_1	TGR_2	TGR_3
Asia	India	0.0598	0.0831	0.1641	0.1242	0.2256	0.2136
	Japan	0.1177	0.5483	1.0000	0.0648	0.8358	1.0000
	South Korea	0.0866	0.5154	0.4939	0.0521	0.3875	0.3533
	Saudi Arabia	0.0956	0.3115	0.2120	0.0812	0.3008	0.2584
	Singapore	1.0000	0.3192	1.0000	0.0282	0.2002	0.1599
	China	0.1430	0.6625	1.0000	0.1390	0.8592	1.0000
	Thailand	0.0428	0.2447	0.2353	0.0317	0.2687	0.1372
	Kazakhstan	0.0188	0.1263	0.3197	0.0179	0.0986	0.2740
	Malaysia	0.0980	0.2643	0.1656	0.0335	0.2494	0.0980
	Iraq	0.0152	0.4541	0.1208	0.0413	0.2898	0.5765
	Iran	0.0575	0.0758	0.0802	0.0297	0.0794	0.1422
	Viet Nam	0.0365	0.0839	0.1276	0.0278	0.2070	0.0598
Europe	Germany	0.0298	0.6151	0.8470	0.0212	0.6880	0.7809
	Greece	0.0386	0.1770	0.3670	0.0202	0.1101	0.1793
	Italy	0.0593	0.6837	1.0000	0.0288	0.6505	1.0000
	France	0.0357	0.6885	0.8442	0.0278	0.7702	0.8387
	Spain	0.0448	0.5326	0.5539	0.0248	0.6855	0.3650
	The Netherlands	0.0378	0.4398	0.9559	0.0211	0.4760	0.3671
	Russia	0.0379	0.1329	0.2266	0.0334	0.2528	0.3111
	Poland	0.0228	0.3237	0.3592	0.0150	0.2483	0.6539
	Ukraine	0.0222	0.0588	0.1998	0.0103	0.1083	0.2078
	Austria	0.0241	0.3320	0.5011	0.0192	0.6572	0.1643
	Hungary	0.0257	0.2201	0.2470	0.0188	0.2473	0.0591
	Ireland	0.0088	1.0000	0.4979	0.0131	1.0000	0.0642
Africa	South Africa	0.0156	0.1155	0.2866	0.0120	0.0902	0.2062
	Algeria	0.0735	0.2333	0.1081	0.0421	0.2100	0.2202
	Morocco	0.0445	0.1607	0.1866	0.0304	0.1926	0.1422
	Nigeria	0.0735	0.4579	0.2285	0.0605	0.9250	0.6339
	Jamaica	0.0487	0.0378	0.1078	0.0236	0.1409	0.5430
	Mauritius	0.0685	0.1838	0.2144	0.0183	0.1190	0.0731
	Paraguay	0.0076	1.0000	0.0714	0.0112	0.6799	0.3680
	Namibia	0.0047	0.2995	0.1008	0.0255	0.2347	0.3695
	Gabon	0.0437	0.3237	1.0000	0.0351	0.5682	1.0000
	Albania	0.0283	0.1000	0.0952	0.0244	0.2267	0.2768
	Suriname	0.0154	0.3860	0.0706	0.0233	0.1464	0.1028
	Belize	0.0114	1.0000	0.0721	0.0098	1.0000	0.0156

- **Theoretical insights:** From the degree of influence on TGR, compared with group frontier, under the influence of meta-frontier, the change of undesirable output has a greater influence on TGR, which is embodied in its wider variation range of TGR.
- **TGR bounds identification:** From the variation range of TGR, the lower limit of TGR can be obtained by maximizing the variation range of undesirable output. Similarly, the upper limit of TGR can be obtained by minimizing the range of undesirable output. These results provide policymakers with a practical tool to assess worst- and best-case efficiency scenarios.
- **Empirical insights:** From the practical application of these methods, these methods can measure the level of industrial heterogeneity of different industries in different countries, which are affected by undesirable output, and can further clarify whether it is positive or negative impact, so as to propose targeted suggestions.

Table 13. The variation range of TGR in different methods.

	NC-CI			NR-RI		
	TGR1	TGR2	TGR3	TGR1	TGR2	TGR3
Asia	$[0, 0.0013]$	$[0, 0.0025]$	$[0, 0.0066]$	$[-0.0013, 0]$	$[-0.0026, 0]$	$[-0.0058, 0]$
Europe	$[0, 0.0004]$	$[0, 0.0005]$	$[0, 0.0061]$	$[-0.0004, 0]$	$[-0.0004, 0]$	$[-0.0066, 0]$
Africa	$[-0.0024, 0]$	$[-0.0054, 0]$	$[-0.0017, 0]$	$[0, 0.0038]$	$[0, 0.0053]$	$[0, 0.0018]$
	NC-CA			NR-RA		
	TGR1	TGR2	TGR3	TGR1	TGR2	TGR3
Asia	0	$[0, 0.0016]$	$[0, 0.0039]$	$[-0.0014, 0]$	$[-0.0029, 0]$	$[-0.0064, 0]$
Europe	$[0, 0.0002]$	$[0, 0.0001]$	$[0, 0.0044]$	$[-0.0005, 0]$	$[-0.0012, 0]$	$[-0.0072, 0]$
Africa	$[-0.0024, 0]$	$[-0.0054, 0]$	$[-0.0024, 0]$	$[0, 0.0001]$	$[-0.0002, 0]$	$[0, 0.0054]$
	F-CI			F-RI		
	TGR1	TGR2	TGR3	TGR1	TGR2	TGR3
Asia	$[-0.0018, 0]$	$[0, 0.0392]$	$[0, 0.0052]$	$[-0.0018, 0]$	$[0, 0.0392]$	$[0, 0.0052]$
Europe	$[0, 0.0004]$	$[0, 0.0005]$	$[0, 0.0064]$	$[0, 0.0004]$	$[0, 0.0005]$	$[0, 0.0064]$
Africa	$[-0.0022, 0]$	$[-0.0054, 0]$	$[-0.0017, 0]$	$[-0.0022, 0]$	$[-0.0054, 0]$	$[-0.0017, 0]$
	F-CA			F-RA		
	TGR1	TGR2	TGR3	TGR1	TGR2	TGR3
Asia	$[-0.0934, 0]$	$[0, 0.0653]$	$[-0.0486, 0]$	$[-0.0934, 0]$	$[0, 0.0653]$	$[-0.0486, 0]$
Europe	$[-0.0107, 0]$	$[0, 0.0580]$	$[-0.1276, 0]$	$[-0.0107, 0]$	$[0, 0.0580]$	$[-0.1276, 0]$
Africa	$[-0.0122, 0]$	$[0, 0.0142]$	$[0, 0.1158]$	$[-0.0122, 0]$	$[0, 0.0142]$	$[0, 0.1158]$

Although this paper has contributed to meta-frontier framework, some limitations are worth noting: first, this study explores the influence of undesirable output on TGR from static level, and only considers theoretical influence, without further considering external influences, such as national policies promulgated in reality. In the future, the dynamic meta-frontier model can be further developed to track technological evolution under policy shocks, and it can also be extended to multi-objective optimization scenarios, for example, to explore how to balance the impact of economic development and undesirable output on TGR.

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Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships, and all authors have approved the manuscript for submission.

Data availability statement

The code used in this paper is available online in a Zenodo repository: https://doi.org/10.5281/zenodo.15173171: [19].

Author contribution statement

All authors contributed to the study conception and design. Conceptualization, Methodology, Software, and Writingoriginal draft were performed by *Xu Guo*, Conceptualization, Methodology, Supervision, Validation, and Writingreview & editing were performed by *Lei Chen*. All authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Ethical approval

All the studies involved in this study are in line with ethical standards.

Informed consent

The authors of this paper all volunteered to participate in the submission and publication of this paper. The authors of this paper have been informed and agreed to the submission and publication of this paper.

References

[1] B. Adenso-Diaz and S. Lozano, A metafrontier analysis approach for assessing the efficiency of freight service providers. *Int. J. Syst. Sci.-Oper. Logistics* **10** (2023) 2177896.

[2] E. Alda, V. Gimènez, I.G. Paz Castro and A.I. Zamora Torres, Modernization plans for the Mexican customs system: have they really worked? A productivity impact assessment. *Appl. Econ.* **56** (2024) 796–811.

[3] J. Aparicio and J.T. Pastor, Closest targets and strong monotonicity on the strongly efficient frontier in DEA. *Omega* **44** (2014) 51–57.

[4] J. Aparicio, J.L. Ruiz and I. Sirvent, Closest targets and minimum distance to the Pareto-efficient frontier in DEA. *J. Prod. Anal.* **28** (2007) 209–218.

[5] C.Q. Bai, Z.J. Chen and D.P. Wang, Transportation carbon emission reduction potential and mitigation strategy in China. *Sci. Total Environ.* **873** (2023) 162074.

[6] W.W. Cooper, K.S. Park and J.T. Pastor, RAM: a range adjusted measure of inefficiency for use with additive models, and relations to other models and measures in DEA. *J. Prod. Anal.* **11** (1999) 5–42.

[7] R. Chambers, Y.H. Chung and R. Färe, Profit, Directional distance functions, and Nerlovian efficiency. *J. Optim. Theory App.* **98** (1998) 351–364.

[8] L. Chen and Y.M. Wang, Game directional distance function in meta-frontier data envelopment analysis. *Omega* **121** (2023) 102935.

[9] L. Chen, Y.M. Wang and F.J. Lai, Semi-disposability of undesirable outputs in data envelopment analysis for environmental assessments. *Eur. J. Oper. Res.* **260** (2016) 655–664.

[10] Y. Chen, W. Xu, Q. Zhou and Z. Zhou, Total factor energy efficiency, carbon emission efficiency, and technology gap: evidence from sub-industries of Anhui Province in China. *Sustainability* **12** (2020) 1402.

[11] Y. Chen, M.Y. Wang, C.P. Feng, H. Zhou and K. Wang, Total factor energy efficiency in Chinese manufacturing industry under industry and regional heterogeneities. *Res. Conserv. Recycl.* **168** (2021) 105255.

[12] Y.H. Chung, R. Färe and S. Grosskopf, Productivity and undesirable outputs: a directional distance function approach. *J. Environ. Manage.* **51** (1997) 229–240.

[13] A. Emrouznejad, M. Marra, G.L. Yang and M. Michali, Eco-efficiency considering NetZero and data envelopment analysis: a critical literature review. *IMA J. Manage. Math.* **34** (2023) 599–632.

[14] A. Emrouznejad, L. Brzezicki and C.X. Lu, Development and evolution of slacks-based measure models in Data Envelopment Analysis: a comprehensive review of the literature. *J. Econ. Surv.* (2025). DOI: 10.1111/joes.12682.

[15] R. Färe, S. Grosskopf, C. Lovell and C. Pasurka, Multilateral productivity comparisons when some outputs are undesirable: a nonparametric approach. *Rev. Econ. Stat.* **71** (1989) 90–98.

[16] M.C. Gouveia and C.O. Henriques, Assessing potential reductions of agricultural GHG in countries with different land productivities: long-term integrated efficiency in DEA hybrid meta-frontier model. *Sustain. Energy Technol. Assess.* **81** (2025) 104430.

[17] R. Gulati, V. Charles and S. Kumar, School education development index: a meta-frontier range directional measure benefit-of-the-doubt model. *Soc.-Econ. Planning Sci.* **92** (2024) 101823.

[18] X. Guo and L. Chen, Double shared ratio selection of two-stage meta-frontier DEA methods based on shared input. *Comput. Appl. Math.* **44** (2025) 319.

[19] X. Guo and L. Chen, Positive/negative effects of undesirable output on technology gap ratio under meta-frontier framework (2025). Code in a Zenodo repository: https://doi.org/10.5281/zenodo.15173171.

[20] X. Guo and L. Chen, Extension and improvement of three-level meta-frontier framework based on multiple combined projection methods. *Comput. Appl. Math.* **43** (2024) 314.

[21] X. Guo, L. Chen, J.C. Wang and L. Liao, The impact of disposability characteristics on carbon efficiency from a potential emissions reduction perspective. *J. Clean. Prod.* **408** (2023) 137180.

[22] K. He and L. Jie, Evaluation of the eco-efficiency of the industrial system in China considering regional heterogeneity and dynamics. *Oper. Res.* **25** (2025) 2.

[23] H.S. Lee, Variable range measure: a new range measure for super-efficiency model based on DDF in presence of nonpositive data. *Omega* **134** (2025) 103295.

[24] X.C. Li, F. Li, N.G. Zhao and Q. Zhu, Measuring environmental sustainability performance of freight transportation seaports in China: a data envelopment analysis approach based on the closest targets. *Expert Syst.* **37** (2020) e12334.

[25] F. Li, P. Zhu, L. Liang and G. Kou, A two-stage DEA efficiency evaluation approach based on closest targets. *Chin. Manage. Sci.* **30** (2022) 198–209.

[26] R.Y. Lin and Z.X. Li, Infeasibility conditions and resolution strategies for super-efficiency models under weak disposability and null-jointness: a directional distance function approach with endogenous directions. *Eur. J. Oper. Res.* **326** (2025) 585–596.

[27] S. Lozano, E. Gutiérrez and E. Aguilera, Environmental efficiency of rainfed and irrigated wheat crops in Spain. A stochastic DEA metafrontier approach. *OR Spect.* (2024). DOI: 10.1007/s00291-024-00791-x.

[28] L.C. Lu, S.Y. Chiu, Y.H. Chiu, T.H. Chang and W. Tang, Energy performance of European countries by considering the role of forest. *Environ. Sci. Pollut. Res.* **29** (2022) 44162–44174.

[29] Y.S. Luo, L. Xu and C. Wu, Internet development and regional innovation efficiency: the moderating effect of intellectual property protection. *Technol. Anal. Strat. Manage.* **36** (2024) 3147–3165.

[30] A. Mergoni, A.R. Dipierro and C. Colamartino, European agricultural sector: the tortuous path across efficiency, sustainability and environmental risk. *Soc.-Econ. Planning Sci.* **92** (2024) 101848.

[31] A. Mergoni, A. Emrouznejad and K. De Witte, Fifty years of Data Envelopment Analysis. *Eur. J. Oper. Res.* **326** (2025) 389–412.

[32] M.A.T. Nguyen and M.M. Yu, Disentangling technology gap using a non-convex three-hierarchy meta-frontier framework: insights into the contemporary cruise segment. *Appl. Econ.* (2025) 1–17. DOI: 10.1080/00036846.2025.2449617.

[33] C. O'Donnell, D. Rao and G. Battese, Metafrontier frameworks for the study of firm-level efficiencies and technology ratios. *Empirical Econ.* **34** (2008) 231–255.

[34] T.S. Olasehinde, F.B. Qiao and S.P. Mao, Impact of improved maize varieties on production efficiency in Nigeria: separating technology from managerial gaps. *Agric.-Basel* **13** (2023) 611.

[35] M.C.A.S. Portela, P.C. Borges and E. Thanassoulis, Finding closest targets in non-oriented DEA models: the case of convex and non-convex technologies. *J. Prod. Anal.* **19** (2003) 251–269.

[36] K. Tone and M. Tsutsui, Dynamic DEA: a slacks-based measure approach. *Omega* **38** (2010) 145–156.

[37] Q.W. Wang, B. Su, P. Zhou and C.R. Chiu, Measuring total-factor CO_2 emission performance and technology gaps using a non-radial directional distance function: a modified approach. *Energy Econ.* **56** (2016) 475–482.

[38] Q.W. Wang, Y. Hang, J.L. Hu and C.R. Chiu, An alternative metafrontier framework for measuring the heterogeneity of technology. *Nav. Res. Logistics* **65** (2018) 427–445.

[39] F.Q. Wei, X.Q. Zhang, J.F. Chu, F. Yang and Z. Yuan, Energy and environmental efficiency of China's transportation sectors considering CO_2 emission uncertainty. *Transp. Res. Part D-Transp. Environ.* **97** (2021) 102955.

[40] J. Wu, Y.F. Yu, Q.Y. Zhu, Q. An and L. Liang, Closest target for the orientation-free context-dependent DEA under variable returns to scale. *J. Oper. Res. Soc.* **69** (2018) 1819–1833.

[41] H.J. Xiao, Y. Zhou, N. Zhang, D. Wang, Y. Shan and J. Ren, CO_2 emission reduction potential in China from combined effects of structural adjustment of economy and efficiency improvement. *Res. Conserv. Recycl.* **174** (2021) 105760.

[42] L. Yang, S.Y. Chen, Y.H. Chiu, T.H. Chang and Y. Wang, Reassessment of industrial eco-efficiency in China under the sustainable development goals: a meta two-stage parallel entropy dynamic DDF-DEA model. *J. Clean. Prod.* **447** (2024) 141275.

[43] M.M. Yu and I. Rakshit, How to establish input and output targets for airlines with heterogeneous production technologies: a nash bargaining DEA approach within the meta-frontier framework. *J. Air Transp. Manage.* **116** (2024) 102576.

[44] M.M. Yu, K.F. See and B. Hsiao, Integrating group frontier and metafrontier directional distance functions to evaluate the efficiency of production units. *Eur. J. Oper. Res.* **301** (2022) 254–276.

[45] W.W. Zhu, Z.W. Bai and Y. Yu, A super-efficiency model based on the directional distance function under a new direction. *Oper. Res.* **25** (2025) 20.

Appendix A.

Theorem A.1. *NC-DDF can relieve the situation that TGR is greater than 1.*

Proof. From Section 3.1, it can be seen that undesirable output can affect TGR, and overestimate the efficiency of DMU. When its overestimation level of meta- frontier efficiency exceeds that of group frontier, TGR will be greater than 1. Therefore, NC-DDF is based on the closest targets, the efficiency of group frontier is obtained on the strong efficient frontier, that is, in $\mathrm{TGR}^{\mathrm{NC}} = \theta^{m^*}/\theta^{\mathrm{NC}^*}$, θ^{NC^*} will increase and θ^{m^*} remain unchanged, so as to relieve TGR, and it also can maintain the optimal increase and decrease ratio of each factor in the projection process. □

Appendix B.

Theorem B.1. *NR-DDF can relieve the situation that TGR is greater than 1.*

Proof. Based on the closest targets, NR-DDF minimizes the improved path of each element in group frontier, and on the effective frontier, the target DMU of the evaluated DMU is the production combination closest to its input/output level. That is, in $\mathrm{TGR}^{\mathrm{NC}} = \theta^{m^*}/\theta^{\mathrm{NC}^*}$, by minimizing S_i^-, S_r^+ and S_f^-, θ^{NC^*} is increased and θ^{m^*} remains unchanged, so that the optimal increase and decrease ratio of each element in the projection process is not disturbed, and the TGR is relieved to be greater than 1. □

Appendix C.

Theorem C.1. *For the change range of undesirable output K_d^{NC}, when it is in UPS$^{\mathrm{NC}}$, it has a feasible solution, that is, the corresponding efficiency values of each factor can be obtained.*

Proof. According to model (9), $K_d = \beta_z g_z + S_f^-$. if K_d is in UPS$^{\mathrm{V}}$, then $\sum_{j=1}^n \lambda_j^g z_{fj} \leq z_{fd} - K_d$ in the case of $\sum_{j=1}^n \lambda_j^g = 1$ $(\lambda_j^g \geq 0)$. Meanwhile, $K_d \geq 0$, and K_d is also in PPS$^{\mathrm{V}}$. Then, there must be a corresponding K_d^* to satisfy $\sum_{j=1}^n \lambda_j^g z_{fj} \leq z_{fd} - K_d$, that is, NC-DDF has the optimal feasible solution under VRS. Similarly, if K_d^{NC} is in UPS$^{\mathrm{NC}}$, then $\sum_{j=1}^n \lambda_j^g z_{fj} \leq z_{fd} - K_d^{\mathrm{NC}}$ in the case of $\sum_{j=1}^n \lambda_j^g = 1$ $(\lambda_j^g \geq 0)$. Meanwhile, $K_d^{\mathrm{NC}} \geq 0$, and K_d^{NC} is also in PPS$^{\mathrm{NC}}$. Then, there must be a corresponding $K_d^{\mathrm{NC}^*}$ to satisfy $\sum_{j=1}^n \lambda_j^g z_{fj} \leq z_{fd} - K_d^{\mathrm{NC}}$. Therefore, NC-DDF can get the optimal solution with UPS$^{\mathrm{NC}}$, and with K_d^{NC}, the change of undesirable output $\bar{Z}_d^{\mathrm{NC}}$ can affect the efficiency of group frontier, and then its influence on TGR is measured. □

Appendix D.

Theorem D.1. *For the variation range of undesirable output K_d^{NR}, when it is in UPS$^{\mathrm{NR}}$, it has a feasible solution, that is, the corresponding efficiency values of each factor can be obtained.*

Proof. From the proof process of Theorem C.1, obviously, if K_d^{NR} is in $\mathrm{UPS}^{\mathrm{NR}}$, then $\sum_{j=1}^n \lambda_j^g z_{fj} \leq z_{fd} - K_d^{\mathrm{NR}}$ in the case of $\sum_{j=1}^n \lambda_j^g = 1$ ($\lambda_j^g \geq 0$). Meanwhile, $K_d^{\mathrm{NR}} \geq 0$, and K_d^{NR} is also in $\mathrm{PPS}^{\mathrm{NR}}$. Then, there must be a corresponding K_d^{NR*} to satisfy $\sum_{j=1}^n \lambda_j^g z_{fj} \leq z_{fd} - K_d^{\mathrm{NR}}$. Therefore, NC-DDF can get the optimal solution with $\mathrm{UPS}^{\mathrm{NR}}$, and with K_d^{NR}, the change of undesirable output $\bar{Z}_d^{\mathrm{NR}}$ can affect the efficiency of group frontier, and then its influence on TGR is measured. $\square$