RAIRO-Oper. Res. 59 (2025) 3777–3802
https://doi.org/10.1051/ro/2025059

RAIRO Operations Research
www.rairo-ro.org

A SUPPLY CHAIN MODEL UNDER CREDIT PERIOD AND PRICE DEPENDENT DEMAND: A STACKELBERG APPROACH

Rituparna Mondal[1,2,*] and Ranjan Kumar Jana[2]

Abstract. In many industrial practices, market demand is likely to be affected by the trade credit policy. In this regard, present study aims to develop a retailer–supplier non-cooperative replenishment model with price and credit length-dependent demand and default risk, to determine the best credit period in a supplier-Stackelberg game. The perks of credit policy include attracting new buyers and avoiding long-term price rivalry. This study was conducted under three distinct decision-making scenarios. The first section demonstrates the optimal outcomes for decentralized and centralized decisions when trade credit is unavailable. Secondly, the Stackelberg game framework employs a leader-follower relationship, where the manufacturer plays the role of leader and the retailer plays the role of follower. Using a supplier Stackelberg method, the existence and uniqueness requirements for the retailer and the supplier optimal solutions have been constructed. In this supplier Stackelberg strategy, the retailer sets the best lot-size, while the supplier chooses the best credit length. A series of theorems and corollaries were designed to determine the best response under these conditions. Sensitivity analysis of few examples have been carried out to demonstrate the proposed method and the anticipated outcomes. Also, the decision variable's characteristics, obtained from the sensitivity analysis, have been figured out. According to the sensitivity analysis, the characteristic of the decision variable is provided, and the efficacy of the centralized decision-making approach is shown. Additionally, this research shows a high correlation between demand, credit length, and profit in the supply chain.

Mathematics Subject Classification. 90B05, 90B06, 35Q89 .

Received June 1, 2023. Accepted May 1, 2025.

1. Introduction

1.1. Motivation and research questions

A supply chain (SC) comprises several players who attempt to achieve economic viability by meeting consumers' requirements. Each SC participant is free to use their own strategies to meet the demand. For instance, a vendor may boost demand for the SC by investing in brand advertisements. Venders executing successful promotional campaigns stand to get a lot of attention and business from the consumers. But, in today's competitive business, traditional strategy like price discount policy, promotional campaign are becoming less successful. So, to increase the demand, SC participants adopt different trade credit policy. In a credit agreement, the buyer can

Keywords. Credit-linked demand, default risk, EOQ model, supplier-Stackelberg, trade credit.

[1] Department of Applied Science, Haldia Institute of Technology, Haldia 721657, West Bengal, India.
[2] Department of Mathematics, Sardar Vallabhbhai National Institute of Technology, Surat 395007, Gujarat, India.
*Corresponding author: parnaritumondal@gmail.com

delay the payment in return for immediate delivery of the products. No interest will be applied during this time. Consequently, a buyer might gain interest in sales revenue while the vendor loses the interest earned within this span. The vendor will charge interest on the unpaid balance if the buyer fails to pay after the expiration of its grace period. There are two ways in which the vendor benefits from the allowed delay in payment: (1) it draws new purchasers who perceive it as a form of price discount, and (2) it can be used as an option to price reduction since it doesn't incite other businesses to follow suit and drive down prices, resulting in long-term price reductions. On the other side, offering credit terms increases the vendor's default risk. In 1985, Goyal [1] first demonstrated that the allowable payment delay boosted the retailer's economic lot size and order cycle. After that, Paul and Boden [2] summarised the literature on trade credit availability and its primary justification. Currently, it is more common than short-term bank loans in almost every industrialized and developing society. Numerous empirical investigations [3] have elucidated why trade credit is more prevalent than short-term bank lending. In 2013, Zhou and Zhou [4] examined a two-echelon supply chain model using the supplier Stackelberg game, wherein a supplier distributes a product *via* a retailer with constant market demand. The supplier seeks to persuade the retailer to increase orders using trade credit to mitigate his/her inventory-related expenses. The concerns they addressed are: (1) In a stable market demand, whether the provision of trade credit may genuinely result in a reduction in the supplier's operating costs under any circumstances. (2) In what manner could the supplier formulate trade credit practices to achieve greater reductions in operating costs? Although they have considered the constant demand but in reality the price frequently determines demand, as noted by Jaggi *et al.* [5] and Barman *et al.* [6], where it is shown that the credit period positively affects market demand. Additionally, default risk is influenced by the credit period, with a longer credit duration resulting in a larger default risk. However, no one has pragmatically studied these characteristics. So, present study aims to examine the subsequent research questions:

- How does the supplier's trade credit policy affect the channel and retailer's expenses?
- How should players involved in the supply chain determine the best pricing for the product in a price-sensitive market demand?
- What impact do the parameters of the system have on the optimal profit of the supplier, retailer, and the overall supply chain under both centralized and decentralized approaches?
- How do the optimum decisions in various scenarios differ from one another?
- What decision-making approach is advantageous for supply chain participants, and in what manner may a supply chain coordination contract enhance the profitability of the supply chain system?

1.2. Literature review

Trade credit is frequently utilized as a short-term business strategy in highly competitive environments and significantly impacts corporate operations. Numerous empirical studies have evidenced the dominance of credit policy in short-term bank lending within practical contexts. According to previous research, credit opportunities influenced retailers' order strategies under constant demand. Teng *et al.* [7] generalized the fixed demand to a linearly growing demand using credit policy in which they overlooked the impact of price on demand. Again, Mahata *et al.* [8,9] formulated two-level trade credit policy with dynamic demand depends on credit period. In 2022, Choudhury and Mahata [10] introduced two level trade credit policy with price and credit sensitive customers demand in a dual channel supply chain inventory policy. Also, some research work [6, 11] assumed Price-sensitive market demand to develop the mathematical model. Shinn [12] demonstrated the buyer's order size varied depending on the credit length for price-sensitive demand. Using price-sensitivity demand and two-level credit policy, Chen and Kang [13] created integrated inventory models to find the ideal pricing and production/order strategy. In 2015, Wu and Zhao [14] offered some theoretical groundwork for an EOQ model considering inventory dependent linearly increasing time-varying demand under trade credit. Yang and Wee [15] provided a joint model with price-dependent demand considering credit policy and suggested a negotiating strategy to distribute the surplus profit between the vendor and the purchaser. Sarmah *et al.* [16] addressed the difficulty of coordinating a single supplier and several retailers at an equal cycle length using trade credit.

Table 1. Comparison of this study with other similar models.

Article	Demand pattern	Credit policy	Default risk	Nature of default risk	Model's description
[10]	Price and credit period	Yes	Yes	Exponential	Integrated
[21,22]	Price	No	No	NA	Stackelberg
[19]	Price	No	No	NA	Centralized, decentralized
[44]	Credit period	Yes	Yes	Exponential	Stackelberg
[33]	Constant	Yes	No	NA	Stackelberg
[14]	Inventory	Yes	No	NA	EOQ
[46]	Credit period	Yes	No	NA	Stackelberg
[47]	Credit period	Yes	No	NA	Newsvendor model
This Paper	Credit period, selling price	Yes	Yes	Exponential	Stackelberg

Notes. "NA" stands for Not Applicable.

Recently, Wu and Zhao [17] developed a cooperative model under trade credit for the demand that depends on inventories and changes over time throughout the limited planning horizon. In 2024, Mahato and Mahata [18] developed a model considering Stackelberg game approach with a leader-follower relationship is used where the manufacturer as a leader and the retailer as a follower. Barman *et al.* [19,20] evaluated both the centralized and decentralized marketing strategies using Stackelerg game approach. Several relevant discussion has been presented in [7,21,21–29].

However, there hasn't been a lot of focus on finding the optimal credit length from the supplier's end. Both the supplier's and the retailer's policies were established by Abad and Jaggi [30] in cooperative and non-cooperative situations. However, allowing a delay did not affect demand in their model. Jaggi *et al.* [5] prospered their model where the credit period positively affects the demand to get the optimal credit and replenishment policy of the supplier. Yet they did not provide the dealings between the supplier and the retailer; instead, they exclusively focused on the supplier. If retailers are willing to enhance their promotional activities, the supplier will extend trade credit to them to increase the size of their orders, stimulate prospective demand, and attract more people. Kim *et al.* [31] suggested a method for figuring out the best pricing for the retailer and the best credit period of the supplier in a supplier Stackelberg approach, whereas they did not focused on the credit period in its demand function. Later, Teng and Lou [32] considered a increasing demand rate of the credit length in the Stackelberg approach. Zhou and Zhou [33] determined the supplier credit period through a supplier Stackelberg game in unconditional and conditional credit opportunity. Moreover, Chern *et al.* [34] developed the supplier's ideal trade credit under a supplier-retailer Stackelberg model considering compound interest rate and lax lot-for-lot replenishment policy. Additional well-known works include those by Wang *et al.* [38], Wu *et al.* [39], Shah [35], Shah and Cardenas-Barron [36] and Su [37], among others. However, these sources do not analyze the outcomes of a supplier-Stackelberg game in the presence of credit policy, also they do not consider the effects of decentralized decision-making in the absence of credit policy (Tab. 1).

1.3. Research gap

Though credit policies are effective marketing tools for increasing product demand in the market, but credit risk always arises. Early on, Shi and Zhang [40] incorporated defaulters into the operational management decision model, where they considered fixed defaulters. But, the supplier default risk increases with the retailer's credit length. There are only a few [32,41–43] of the most renowned and up-to-date examples. Wu *et al.* [44] considered credit period demand and default risk under the Steckelberg game approach for the case where replenishment time is shorter than the credit length. Whereas they overlooked the point where the replenishment time exceeds

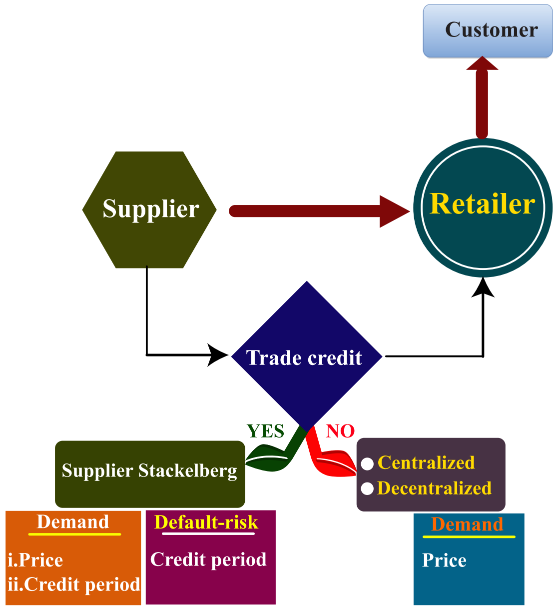

FIGURE 1. Methodological representation of identified problem statement.

the credit length. However, this phenomenon is more realistic in business. Recently, Zhang *et al.* [46] discuss the effect of trade credit period and replenishment cycle time of the retailer considering Stackelberg game approach. In 2021, Wang *et al.* [47] developed a newsvendor model under two level credit policy with credit dependent demand. But they have not considered the defaulters in their model. This study relies on existing research to conduct new, relevant studies to address the inadequacies with proper justification.

1.4. Problem statement

Modern supply chain systems involve decision-making that often requires interdependence across entities, such as manufacturers, suppliers, and retailers, aimed at reducing inefficiencies, improving coordination, and achieving greater overall profitability. This model described in Figure 1 analyzes a supplier-Stackelberg framework, including a supplier (leader) and a retailer (follower) under non cooperative replenishment dynamics, considering the interplay between demand and default risk in relation to the credit duration. Also, the retail price of the item influences market demand. In this study, three different decision-making scenarios were investigated. A demonstration of the optimal results for both decentralized and centralized decision-making in the absence of trade credit is presented in the first section. Secondly, a leader-follower relation is used in the Stackelberg game, with the retailer acting as the follower and the manufacturer as the leader. This study examines the superiority of a centralized strategy over a decentralized decision-making technique. Moreover, it can be deduced that the Supplier Stackelberg game consistently produces a greater overall advantage for the supply chain.

1.5. Contribution

This paper explores a supplier-Stackelberg model of uncooperative replenishment between retailers and suppliers when demand and default risk are coupled with credit period. Additionally, the retail price of the item has an impact on market demand. In this case, this study envisage that the supplier is more potent than the retailer. For instance, a supplier has the ability to determine its strategy and exert dominating control over the neighbouring grocery store. On the other hand, the small grocery relies only on the policies established by the dominant corporation to develop its approach. This model assess the differences between the supplier-Stackelberg model with trade credit and decentralized decision-making without trade credit. Next, analyse the two models separately to see how profit and behaviour change based on the trade credit period effect on demand and default risk. Under the non-cooperative Stackelberg equilibrium, the necessary and sufficient criteria have been developed to produce the best possible outcome for the retailer and the customer.

2. Notations and assumption

The following notations are used, and assumptions are made, throughout the work to create the suggested model, following industrial norms and our incentives to develop this model.

2.1. Notations

(i) P, production rate per year for the supplier.
(ii) $I(t)$, inventory level at any instant t.
(iii) Q_i, order quantity.
(iv) A_j, ordering cost in $ per order.
(v) h_j, inventory holding cost in $ per unit per year.
(vi) c, production cost in $.
(vii) p, wholesale price in $ per unit.
(viii) s, retail price in $ per unit.
(ix) T, replenishment time.
(x) I_e, rate of earned interest in $ per year.
(xi) I_p, rate of payable interest in $ per year.
(xii) ${Z_j}^i$, total annual profit in $.

For convenience, subscript j denotes several participants, $j = s$ denotes the supplier; $j = r$ denotes the retailer, $j = sc$ denotes the entire supply chain; $i = 0$ decentralized decision; $i = 1$ the supplier-Stackelberg game; $i = 2$ centralized decision.

2.2. Assumptions

(i) The replenishment rate is instantaneous and production rate is finite.
(ii) Time horizon is infinite.
(iii) Shortages are not permitted for increasingly fierce market competition.
(iv) To raise the order amount, the supplier gives the retailer a credit period L. The retailer can earn interest on sales revenue during the grace period at a rate of I_e and must pay interest to the bank on the outstanding balance after the grace period at a rate of I_p.
(v) Retailers likely pay their credit amount as soon as the delay period expires; otherwise, they are considered defaulters.
(vi) Default credit risk in a commercial transaction is based on the credit period. Therefore, the defaulters [44] of the supplier, $F(L), (0 \leq F(L) < 1)$ is of the form:

$$F(L) = 1 - e^{-nL}.$$

(vii) In reality, the retail price and the credit policy significantly affected market demand. Also it is known that selling price and demand are always inversely proportionate [45]. In this case, the market demand is structured so that the selling price alone influences demand without a credit policy. So, the per unit time market demand, D is in the form:

$$D = \frac{ke^{mL}}{s^{\gamma}}\cdot$$

Here, $P \geq \frac{ke^{mL}}{s^{\gamma}}$, $\quad L_{\max} = \frac{1}{m}\log(\frac{Ps^{\gamma}}{k})$ where m, n and γ are the positive real values picked by the expert for the best demand match.

3. Mathematical description of the model excluding trade credit

This section provides two inventory models without trade credit, namely, decentralized decision and centralized decision. First off, there is no coordination and no trade credit between the supplier and the retailer in a decentralized choice. Consequently, the demand rate is constant, *i.e.*, $D = \frac{k}{s^{\gamma}} = K(\text{say})$.

Under the assumption, the retailer initially orders Q units of the product at the beginning of each cycle, and inventory gradually depletes owing to market demand. Consequently, at any time, t, the inventory level, $I(t)$, can be represented mathematically as:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -D; \qquad 0 \leq t \leq T \tag{1}$$

with boundary conditions $I(0) = Q$ and $I(T) = 0$. Solving equation (1) we have

$$I(t) = D(T-t), \qquad \text{for } 0 \leq t \leq T. \tag{2}$$

$$\text{Thus, } Q = DT = KT. \tag{3}$$

The following expenses and sales are used to compute the retailer's profit:

$$\text{(i) Sales revenue, SR } = \frac{1}{T}\int_0^T sD\,\mathrm{d}t = sK, \tag{4}$$

$$\text{(ii) Ordering cost, OC} = \frac{A_r}{T}, \tag{5}$$

$$\text{(iii) Purchase cost, PC} = \frac{pQ}{T} = pK, \tag{6}$$

$$\text{(iv) Holding Cost, HC } = \frac{h_r}{T}\int_0^T I(t)\,\mathrm{d}t = \frac{h_r KT}{2}\cdot \tag{7}$$

Consequently, the retailer's annual profit is determined by

$$Z_r^0(Q) = (s-p)K - \frac{A_r K}{Q} - \frac{h_r Q}{2}\cdot \tag{8}$$

Thus, to maximize the annual profit $Z_r^0(Q)$ w.r.t. Q, the optimal order quantity is determined by

$$Q_0^* = \sqrt{\frac{2KA_r}{h_r}} \tag{9}$$

and the optimal retailer's annual profit is

$$Z_r^0 = (s-p)K - \sqrt{2KA_r h_r}. \tag{10}$$

The retailer's replenishment strategy determines the supplier's profit in a decentralized environment. The supplier's annual profit is therefore represented as

$$Z_s^0 = (p-c)K - A_s\sqrt{\frac{Kh_r}{2A_r}} - K\frac{h_s}{P}\sqrt{\frac{KA_r}{2h_r}}\cdot \tag{11}$$

Therefore, the supply chain's annual profit is given by

$$Z_{sc}^0 = (s-c)K - \sqrt{2KA_r h_r} - A_s\sqrt{\frac{Kh_r}{2A_r}} - K\frac{h_s}{P}\sqrt{\frac{KA_r}{2h_r}}\cdot \tag{12}$$

In centralized decision, the retailer and the supplier are eager to work together to create a virtual integrated firm. The replenishment strategy will be decided mutually by them. The demand rate remains constant (K) for no trade credit. Thus, the joint annual profit for the entire supply chain is

$$Z_{sc}^2(Q) = (s-c)K - (A_r + A_s)\frac{K}{Q} - Q\frac{h_r}{2} - Qh_s\frac{K}{2P}\cdot \tag{13}$$

Consequently, the supply chain's optimal joint order quantity is provided by

$$Q_2^* = \sqrt{\frac{2PK(A_r + A_s)}{Ph_r + Kh_s}}\cdot \tag{14}$$

In response to this ordering size Q_2^*, the supply chain's annual profit will be

$$Z_{sc}^2 = (s-c)K - \sqrt{\frac{2K(A_r + A_s)(Ph_r + Kh_s)}{P}}\cdot \tag{15}$$

Mathematical description of the model including trade credit in a supplier-Stackelberg game

In a supplier Stackelberg strategy, the supplier (the leader) and the retailer (the follower) do the following actions to establish the optimum credit duration and order quantity:

Step 1. The retailer seeks to maximize its profit to find its optimal Q for any given credit period L.
Step 2. To maximize its profits, the supplier imports the retailer's optimal Q into its profit function before determining its own optimal policies L. Lastly, the supplier's optimal policies L is used to determine the retailer's optimal Q. Figure 2 provides a visual depiction of the method described in the text.

Retailer's optimal decisions

The retailer's goal for every credit period L is to identify the ordered quantity Q that maximizes his/her annual profit, which is made up of the ensuing expenditures and revenues.

$$\text{(i)}\quad \text{SR} = sD \tag{16}$$

$$\text{(ii)}\quad \text{OC} = \frac{A_r}{T} \tag{17}$$

$$\text{(iii)}\quad \text{PC} = pD \tag{18}$$

$$\text{(iv)}\quad \text{HC} = \frac{h_r DT}{2}\cdot \tag{19}$$

(V) Annual interest charge (payable/earning): two possible cases could occur depending on the values of credit period (L) and replenishment time (T).

Case 1. $T < L$ and
Case 2. $T \geq L$.

For Case 1: $T < L$, the retailer obtained all of the sales revenue from the customers prior to the supplier's grace period. The retailer can thereby earn interest on the customers' payments while not having to pay any interest. The graphical depiction of this scenario is shown in Figure 3a.

Thus the interest amount paid by the retailer is expressed by

$$\text{IP} = 0 \tag{20}$$

and the retailer's earned interest is computed as follows

$$\text{IE} = \frac{1}{T}\text{IE}_1,\ \text{where} \tag{21}$$

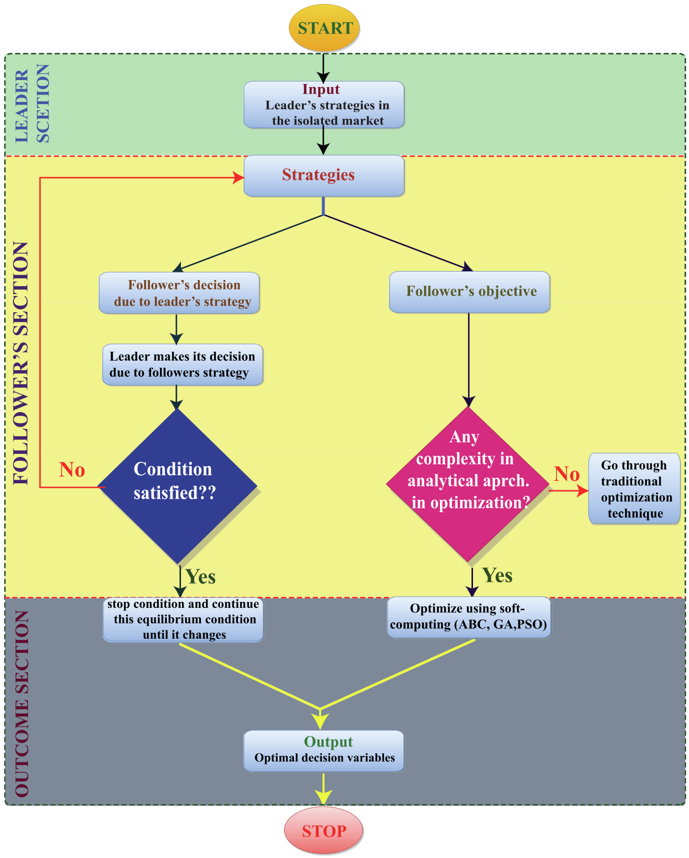

FIGURE 2. Flowchart of the Stackelberg game model.

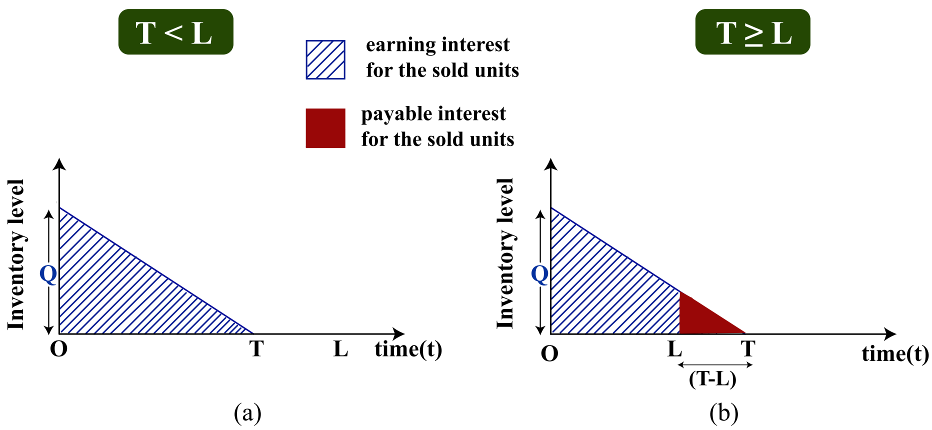

FIGURE 3. Pictorial depiction of earning and payable interest for (a) Case-1 (b) Case-2.

$$\begin{aligned}\mathrm{IE}_1 &= \text{interest earned for the sold units during } (0,T]\\ &= I_e\int_0^T sD(L-t)\,\mathrm{d}t = I_e sD\left[LT-\frac{T^2}{2}\right]\cdot\end{aligned}\tag{22}$$

For Case 2: $T \geq L$, the retailer is bound to pay interest for the products which kept as stock product. Hence, the payable interest can be expressed as

$$\mathrm{IP} = \mathrm{IP}_1/T, \text{ where}\tag{23}$$

$$\begin{aligned}\mathrm{IP}_1 &= \text{interest charged for the stock units during } (L,T]\\ &= I_c\int_L^T pD(T-t)\,\mathrm{d}t = \frac{1}{2}I_c pD(T-L)^2.\end{aligned}\tag{24}$$

Similar to case 1, the retailer is able to earn interest on the customers' collateral deposits for the units sold at $(0,L]$. The graphical depiction of this scenario is shown in Figure 3b. The yearly earned interest of the retailer becomes

$$\mathrm{IE} = \frac{1}{T}\mathrm{IE}_1, \text{ where}\tag{25}$$

$$\begin{aligned}\mathrm{IE}_1 &= \text{interest earned for the sold units during } (0,L]\\ &= I_e\int_0^L sD(L-t)\,\mathrm{d}t = I_e sD\frac{L^2}{2}\cdot\end{aligned}\tag{26}$$

Therefore, for any credit period L, the retailer's annual profit is

$$Z_r^1(T) = \begin{cases} Ds - \frac{A_r}{T} - pD - h_r D\frac{T}{2} + I_e sD\left(L-\frac{T}{2}\right), & \text{if } T<L\\ Ds - \frac{A_r}{T} - pD - h_r D\frac{T}{2} - \frac{1}{2T}I_c pD(T-L)^2 + \frac{1}{2T}I_e sDL^2, & \text{if } T\geq L.\end{cases}\tag{27}$$

To maximize the retailer's overall profit $Z_r^1(Q)$ w.r.t. Q, the criteria $\frac{\mathrm{d}Z_r^1}{\mathrm{d}Q}=0$ must be fulfilled. Hence, from the condition $\frac{\mathrm{d}Z_r^1}{\mathrm{d}Q}=0$, the optimal order size would be

$$Q_1^\star(L) = \begin{cases} Q_{r1} = \sqrt{\frac{2A_r D + (pI_c - sI_e)(DL)^2}{h_r + pI_c}}, & \text{if } 2A_r + (pI_c - sI_e)DL^2 \geq 0\\ Q_{r2} = \sqrt{\frac{2A_r D}{h_r + I_e s}}, & \text{otherwise.}\end{cases}\tag{28}$$

The retailer's optimal replenishment time under the supplier's credit period L is determined by the equation (28) and it is given as

$$T^\star = \begin{cases} \sqrt{\frac{2A_r + (pI_c - sI_e)DL^2}{D(h_r + pI_c)}}, & \text{if } 2A_r + (pI_c - sI_e)DL^2 \geq 0\\ \sqrt{\frac{2A_r}{D(h_r + I_e s)}}, & \text{otherwise.}\end{cases}\tag{29}$$

Theorem 3.1. *Let $H = H(L) = 2A_r - DL^2(h_r + I_e s)$. For any given L, we obtain:*

(i) *If $H > 0$, the optimal replenishment time $T^* > L$.*
(ii) *If $H < 0$, the optimal replenishment time $T^* < L$.*
(iii) *If $H = 0$, the optimal replenishment time $T^* = L$.*

Corollary 3.2. *If $SI_e > pI_c$, then*

(i) *Increases in A_r result in increases in H, T^* and Q^*.*
(ii) *Increases in k, m, L and h_r result in decreases in H, T^* and Q^*.*

Proof. It is clear from equation (29) and Theorem 3.1. □

3.1. The supplier's optimal decisions

As the Stackelberg leader, the supplier can monitor the retailer's optimal response for any given credit period L. After understanding the retailer's preferred response, the supplier accordingly substitutes the retailer's optimal order quantity to maximize the annual profit. The components of the supplier's profit function are:

$$\text{(i)}\quad \mathrm{SR} = (1-F(L))pD \tag{30}$$

$$\text{(ii)}\quad \mathrm{OC} = A_s\frac{D}{Q} \tag{31}$$

$$\text{(iii)}\quad \mathrm{PC} = cD \tag{32}$$

$$\text{(iv)}\quad \mathrm{HC} = \frac{h_sDQ}{2P}. \tag{33}$$

(V) Annual interest charge (payable): here, the supplier has to pay interest for the default retailer. Thus the supplier must pay interest per year as

$$\mathrm{IP} = F(L)cDI_pL. \tag{34}$$

Using equation (28), the supplier's total annual profit is represented as:

$$Z_s^1(L) = \begin{cases} (1-F(L))pD - A_s\frac{D}{Q_{r1}} - cD - \frac{h_sDQ_{r1}}{2P} - F(L)CDI_pL, & \text{if } T \geq L \\ (1-F(L))pD - A_s\frac{D}{Q_{r2}} - cD - \frac{h_sDQ_{r2}}{2P} - F(L)CDI_pL, & \text{otherwise.} \end{cases} \tag{35}$$

To maximize $Z_s^1(L)$, we obtain (if $T \leq L$)

$$\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} = \frac{(m-n)pke^{(m-n)L}}{s^\gamma} - \frac{mcke^{mL}}{s^\gamma} - \frac{mA_s}{2}\sqrt{\frac{(h_r+I_es)ke^{mL}}{2A_rs^\gamma}} - \frac{3mh_ske^{mL}}{4ps^\gamma}\sqrt{\frac{2A_rke^{mL}}{s^\gamma(h_r+I_es)}} - nck\frac{e^{(m-n)L}}{s^\gamma}I_pL - \left(1-e^{-nL}\right)\frac{cke^{mL}}{s^\gamma}I_p(mL+1). \tag{36}$$

It has only one decision variable L.

Theorem 3.3. *If $T \leq L$, the optimal credit length of the supplier is zero when (i) $n \geq m$, or (ii) $mc \geq (m-n)p$*

Proof. From equation (36), if $n \geq m$, $\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} < 0$. Hence, $L^* = 0$. Similarly, if $ac \geq (m-n)p$, we obtain the same result $\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} < 0$, and $L^* = 0$. Hence the result. □

Next, we deal with the condition $(m-n)p > mc$. According to the first derivative criteria $\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} = 0$, the optimal credit period is obtained by

$$L^{*1} = \frac{1}{\left\{m-(m-n)e^{-nL^{*1}}\right\}\frac{cke^{mL^{*1}}}{s^\gamma}I_p}\Bigg[\frac{(m-n)pke^{(m-n)L^{*1}}}{s^\gamma} - \frac{mcke^{mL^{*1}}}{s^\gamma} - \frac{mA_s}{2}\sqrt{\frac{(h_r+I_es)ke^{mL^{*1}}}{2A_rs^\gamma}} - \frac{3mh_ske^{mL^{*1}}}{4ps^\gamma}\sqrt{\frac{2A_rke^{mL^{*1}}}{s^\gamma(h_r+I_es)}} + \frac{cke^{mL^{*1}}}{s^\gamma}I_p\left\{1-e^{-nL^{*1}}\right\}\Bigg] \tag{37}$$

when

$$\frac{(m-n)pke^{(m-n)L^{*1}}}{s^\gamma} - \frac{mcke^{mL^{*1}}}{s^\gamma} - \frac{mA_s}{2}\sqrt{\frac{(h_r+I_es)ke^{mL^{*1}}}{2A_rs^\gamma}}$$

$$- \frac{3mh_s ke^{mL^{*1}}}{4ps^\gamma}\sqrt{\frac{2A_r ke^{mL^{*1}}}{s^\gamma(h_r+I_e s)}} + \frac{cke^{mL^{*1}}}{s^\gamma} I_p \left\{1 - e^{-nL^{*1}}\right\} \geq 0. \tag{38}$$

Theorem 3.4. *When* $(m-n)p - mc - \frac{mA_s}{2}\sqrt{\frac{s^\gamma(h_r+I_e s)}{2kA_r}} - \frac{3mh_s}{4p}\sqrt{\frac{2A_r k}{s^\gamma(h_r+I_e s)}} > 0$*, then*

(i) *if* $L^{*1} < L_{\max}$*, the optimal trade credit period* $L^* = L^{*1}$*;*
(ii) *if* $L^{*1} \geq L_{\max}$*, the optimal trade credit period* $L^* = L_{\max}$*.*

Proof. Here,

$$\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L}|_{L=0} > 0 \tag{39}$$

and

$$\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L}|_{L=\infty} = -\infty. \tag{40}$$

Furthermore, using the second derivative of $Z_s^1(L)$ w.r.t. L, then

$$\begin{aligned}\frac{\mathrm{d}^2 Z_s^1(L)}{\mathrm{d}L^2} &= \frac{(m-n)^2 pke^{(m-n)L}}{s^\gamma} - \frac{m^2 cke^{mL}}{s^\gamma} - \frac{m^2 A_s}{4}\sqrt{\frac{(h_r+I_e s)ke^{mL}}{2A_r s^\gamma}} \\ &\quad - \frac{9m^2 h_s ke^{mL}}{8ps^\gamma}\sqrt{\frac{2A_r ke^{mL}}{s^\gamma(h_r+I_e s)}} - \left(1-e^{-nL}\right)(2+mL)mck\frac{e^{mL}}{s^\gamma}I_p - (2+nL)nck\frac{e^{(m-n)L}}{s^\gamma}I_p \qquad (41)\\ &= \left[(m-n)^2 pe^{-nL} - m^2 c - \frac{9m^2 h_s}{8p}\sqrt{\frac{2A_r ke^{mL}}{s^\gamma(h_r+I_e s)}}\right]\frac{ke^{mL}}{s^\gamma} - \frac{m^2 A_s}{4}\sqrt{\frac{(h_r+I_e s)ke^{mL}}{2A_r s^\gamma}} \\ &\quad - \left(1-e^{-nL}\right)(2+mL)mck\frac{e^{mL}}{s^\gamma}I_p - (2+nL)nck\frac{e^{(m-n)L}}{s^\gamma}I_p. \qquad (42)\end{aligned}$$

Following that, we have two different scenarios:

(i) $(m-n)^2 p - m^2 c - \frac{9m^2 h_s}{8p}\sqrt{\frac{2A_r k}{s^\gamma(h_r+I_e s)}} - \frac{m^2 A_s}{4}\sqrt{\frac{(h_r+I_e s)s^\gamma}{2A_r k}} - 2ncI_p \leq 0$ and
(ii) $(m-n)^2 p - m^2 c - \frac{9m^2 h_s}{8p}\sqrt{\frac{2A_r k}{s^\gamma(h_r+I_e s)}} - \frac{m^2 A_s}{4}\sqrt{\frac{(h_r+I_e s)s^\gamma}{2A_r k}} - 2ncI_p > 0$.

Case 1. $(m-n)^2 p - m^2 c - \frac{9m^2 h_s}{8p}\sqrt{\frac{2A_r k}{s^\gamma(h_r+I_e s)}} - \frac{m^2 A_s}{4}\sqrt{\frac{(h_r+I_e s)s^\gamma}{2A_r k}} - 2ncI_p \leq 0$.

Clearly, $\frac{\mathrm{d}^2 Z_s^1(L)}{\mathrm{d}L^2} < 0$. Hence, $Z_s^1(L)$ is a strictly concave function in $[0,\infty)$. Therefore, from equations (39) and (40), there is a unique positive optimal solution L^{*1}(say) such that $\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} = 0$.

Case 2. $(m-n)^2 p - m^2 c - \frac{9m^2 h_s}{8p}\sqrt{\frac{2A_r k}{s^\gamma(h_r+I_e s)}} - \frac{m^2 A_s}{4}\sqrt{\frac{(h_r+I_e s)s^\gamma}{2A_r k}} - 2ncI_p > 0$.

Here, if L increases, the value of $\frac{\mathrm{d}^2 Z_s^1(L)}{\mathrm{d}L^2}$ changes from positive to negative, *i.e.*, $Z_s^1(L)$ is a convex-concave function of L. So, from equations (39) and (40), we have $Z_s^1(L)$ is a uni-modal function in $[0,\infty)$. Hence, there also have a unique positive optimal solution L^{*1} such that $\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} = 0$.

In summary, if $(m-n)p - mc - \frac{mA_s}{2}\sqrt{\frac{(h_r+I_e s)}{2A_r s^\gamma}} - \frac{3mh_s}{4ps^\gamma}\sqrt{\frac{2A_r k}{s^\gamma(h_r+I_e s)}} > 0$, L^{*1} is a unique optimal solution for $Z_s^1(L)$. Then if we look at the upper bound of L, $L_{\max}$(say). The optimal trade period is $L^* = L^{*1}$ if $L^{*1} < L_{\max}$. If $L^{*1} \geq L_{\max}$, the optimal credit period is $L^* = L_{\max}$. Hence the proof. □

Corollary 3.5. *Increases in m, p, P and decreases in n, c, A_s, h_s result in increases in* $(m-n)p - mc - \frac{mA_s}{2}\sqrt{\frac{(h_r+I_e s)}{2A_r s^{\gamma}}} - \frac{3mh_s}{4ps^{\gamma}}\sqrt{\frac{2A_r k}{s^{\gamma}(h_r+I_e s)}}$ *and* L^{*1}.

Theorem 3.6. *If $T \leq L$, the optimal credit period supplier's is zero when (i) $n \geq m$, or (ii) $mc \geq (m-n)p$, or (iii)* $(m-n)p - mc - \frac{mA_s}{2}\sqrt{\frac{(h_r+I_e s)}{2A_r s^{\gamma}}} - \frac{3mh_s}{4ps^{\gamma}}\sqrt{\frac{2A_r k}{s^{\gamma}(h_r+I_e s)}} \leq 0$.

Theorems 3.4 and 3.6, especially Theorem 3.6-(iii), shows that several factors, including the decision parameters of supplier's own operation, the default risk coefficient of retailer and demand function, affect the supplier's decision to give trade credit to the retailer. Therefore, the larger the value of the condition $(m-n)p - mc - \frac{mA_s}{2}\sqrt{\frac{(h_r+I_e s)}{2A_r s^{\gamma}}} - \frac{3mh_s}{4ps^{\gamma}}\sqrt{\frac{2A_r k}{s^{\gamma}(h_r+I_e s)}} > 0$ in Theorem 3.4, the supplier will be able to offer the more extended credit period. So, it is quite easy for the supplier to understand under which conditions they cannot supply trade credit.

Again, to maximize $Z_s^1(L)$, we get (if $T > L$)

$$\begin{aligned}
\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} &= \frac{(m-n)pke^{(m-n)L}}{s^{\gamma}} - \frac{mcke^{mL}}{s^{\gamma}} - A_s\frac{\sqrt{ke^{mL}(h_r+pI_c)}\left[mA_r s^{\gamma} - kL(pI_c - sI_e)e^{mL}\right]}{\left[2A_r s^{\gamma} + k(pI_c - sI_e)L^2 e^{mL}\right]^{\frac{3}{2}}} \\
&\quad - \frac{h_s}{2p}\left[\frac{mke^{mL}}{s^{2\gamma}}\sqrt{\frac{2A_r s^{\gamma}ke^{mL} + (pI_e - sI_e)k^2L^2e^{2mL}}{h_r+pI_c}} + \frac{k^2e^{2mL}}{s^{2\gamma}}\right. \\
&\quad \left.\times\frac{s^{\gamma}mA_r + kL(pI_e - sI_e)e^{mL}(1+mL)}{\sqrt{h_r+pI_c}\sqrt{2s^{\gamma}A_r ke^{mL} + (pI_c - sI_e)k^2L^2e^{2mL}}}\right] \\
&\quad - nck\frac{e^{(m-n)L}}{s^{\gamma}}I_pL - \left(1-e^{-nL}\right)\frac{cke^{mL}}{s^{\gamma}}I_p(mL+1) = 0.
\end{aligned} \tag{43}$$

It has only one decision variable L.

Theorem 3.7. *For $T \geq L$, the optimal credit length of the supplier is zero when (i) $n \geq m$, or (ii) $mc \geq (m-n)p$*

Proof. From equation (43), when $n \geq m$, $\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} < 0$. Hence, $L^* = 0$. Similarly, if $(m-n)p \leq mc$ or $(m-n)p \leq mc + cI_p$, we obtain the same results $\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} < 0$, and $L^* = 0$. Hence the result. □

Next, we deal another condition, $(m-n)p > mc$. According to the first derivative condition $\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} = 0$, the best credit period is determined by

$$\begin{aligned}
L^{*1} &= \frac{1}{\left[\left\{m-(m-n)e^{-nL^{*1}}\right\}\frac{cke^{aL^{*1}}I_p}{s^{\gamma}}\right]}\left[\frac{(m-n)pke^{(m-n)L^{*1}}}{s^{\gamma}} - \frac{mcke^{mL^{*1}}}{s^{\gamma}}\right. \\
&\quad - A_s\frac{\sqrt{ke^{mL}(h_r+pI_c)}\left[mA_r s^{\gamma} - kL(pI_c - sI_e)e^{kL}\right]}{\left[2A_r s^{\gamma} + k(pI_c - sI_e)L^2e^{mL}\right]^{\frac{3}{2}}} \\
&\quad - \frac{h_s}{2p}\left[\frac{mke^{mL}}{s^{2\gamma}}\sqrt{\frac{2A_r s^{\gamma}ke^{mL} + (pI_e - sI_e)k^2L^2e^{2mL}}{h_r+pI_c}} + \frac{k^2e^{2mL}}{s^{2\gamma}}\right. \\
&\quad \left.\left.\times\frac{s^{\gamma}mA_r + kL(pI_e - sI_e)e^{mL}(1+mL)}{\sqrt{h_r+pI_c}\sqrt{2s^{\gamma}A_r ke^{mL} + (pI_c - sI_e)k^2L^2e^{2mL}}}\right] + \frac{cke^{mL^{*1}}}{s^{\gamma}}I_p\left\{1-e^{-nL^{*1}}\right\}\right]
\end{aligned} \tag{44}$$

when

$$\frac{(m-n)pke^{(m-n)L^{*1}}}{s^\gamma} - \frac{mcke^{mL^{*1}}}{s^\gamma} - A_s\frac{\sqrt{ke^{mL}(h_r+pI_c)}\left[mA_rs^\gamma - kL(pI_c-sI_e)e^{kL}\right]}{\left[2A_rs^\gamma + k(pI_c-sI_e)L^2e^{mL}\right]^{\frac{3}{2}}}$$
$$-\frac{h_s}{2p}\left[\frac{mke^{mL}}{s^{2\gamma}}\sqrt{\frac{2A_rs^\gamma ke^{mL}+(pI_e-sI_e)k^2L^2e^{2mL}}{h_r+pI_c}}+\frac{k^2e^{2mL}}{s^{2\gamma}}\right.$$
$$\left.\times\frac{s^\gamma mA_r+kL(pI_e-sI_e)e^{mL}(1+mL)}{\sqrt{h_r+pI_c}\sqrt{2s^\gamma A_rke^{mL}+(pI_c-sI_e)k^2L^2e^{2mL}}}\right]+\frac{cke^{mL^{*1}}}{s^\gamma}I_p\left\{1-e^{-nL^{*1}}\right\}>0.$$

Theorem 3.8. *When*

$$(m-n)p - mc - \frac{mA_s}{2}\sqrt{\frac{s^\gamma(h_r+pI_c)}{2kA_r}} - \frac{mh_s}{2ps^\gamma\sqrt{h_r+pI_c}}\left[\sqrt{2A_rks^\gamma}+\sqrt{\frac{s^\gamma A_rk}{2}}\right]>0$$

then

(i) *if* $L^{*1} < L_{\max}$, *the optimal credit period* $L^* = L^{*1}$;
(ii) *if* $L^{*1} \geq L_{\max}$, *the optimal credit period* $L^* = L_{\max}$.

Proof. Here,

$$\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L}|_{L=0} > 0 \tag{45}$$

and

$$\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L}|_{L\to\infty} = -\infty. \tag{46}$$

Additionally, using the second derivative of $Z_s^1(L)$ w.r.t. L, we have

$$L^{*1} = \frac{1}{\left[\left\{m-(m-n)e^{-nL^{*1}}\right\}\frac{cke^{mL^{*1}}I_p}{s^\gamma}\right]}\left[\frac{(m-n)pke^{(m-n)L^{*1}}}{s^\gamma} - \frac{mcke^{mL^{*1}}}{s^\gamma}\right.$$
$$-A_s\frac{\sqrt{ke^{mL}(h_r+pI_c)}\left[mA_rs^\gamma - kL(pI_c-sI_e)e^{kL}\right]}{\left[2A_rs^\gamma + k(pI_c-sI_e)L^2e^{mL}\right]^{\frac{3}{2}}}$$
$$-\frac{h_s}{2p}\left[\frac{mke^{mL}}{s^{2\gamma}}\sqrt{\frac{2A_rs^\gamma ke^{mL}+(pI_e-sI_e)k^2L^2e^{2mL}}{h_r+pI_c}}+\frac{k^2e^{2mL}}{s^{\gamma}}\right.$$
$$\left.\left.\times\frac{s^\gamma mA_r+kL(pI_e-sI_e)e^{mL}(1+mL)}{\sqrt{h_r+pI_c}\sqrt{2s^\gamma A_rke^{mL}+(pI_c-sI_e)k^2L^2e^{2mL}}}\right]+\frac{cke^{mL^{*1}}}{s^\gamma}I_p\left\{1-e^{-nL^{*1}}\right\}\right] \tag{47}$$

$$\frac{\mathrm{d}^2Z_s^1(L)}{\mathrm{d}L^2} = \frac{(m-n)^2pke^{(m-n)L}}{s^\gamma} - \frac{m^2cke^{mL}}{s^\gamma}$$
$$-\frac{A_s}{2}\left[\frac{\sqrt{(h_r+pI_c)ke^{mL}}\left[m^2A_rs^\gamma+(3mL-2)y\right]}{x^{\frac{3}{2}}}\right.$$

$$- 3\frac{\sqrt{(h_r + pI_c)ke^{mL}}\{mA_r s^\gamma - yL\} - (2+mL)yL}{x^{\frac{5}{2}}}\Bigg]$$
$$- \frac{h_s}{2P}\left[\frac{mke^{mL}}{s^\gamma\sqrt{(h_r+pI_c)s^{2\gamma}}}\left\{\frac{3mA_r ks^\gamma + (1+2mL)kyLke^{mL}}{(xk)^{\frac{1}{2}}}\right\}\right]$$
$$+ \frac{k^2e^{2mL}}{s^\gamma\sqrt{(h_r+pI_c)s^{2\gamma}}}\Bigg[\frac{2m^2A_r s^\gamma + y[L(1+mL) + mL + (1+mL)]}{(xk)^{\frac{1}{2}}}$$
$$- \frac{[mA_r s^\gamma + (1+mL)yL][mA_r ks^\gamma + (1+mL)kyL]e^{mL}}{(xk)^{\frac{3}{2}}}\Bigg]$$
$$- \left(1 - e^{-nL}\right)(2+mL)mck\frac{e^{mL}}{s^\gamma}I_p - (2+n)nck\frac{e^{(m-n)}}{s^\gamma}I_p \tag{48}$$

where

$$x = 2A_r s^\gamma + (pI_c - sI_e)kL^2e^{mL}$$
$$y = (pI_c - sI_e)ke^{mL}.$$

Next, we have two alternative cases:

(i) $(m-n)^2p - m^2c - \frac{A_s\sqrt{(h_r+pI_c)}}{2^{\frac{5}{2}}A_r^{\frac{3}{2}}(ks^\gamma)^{\frac{1}{2}}}\left[m^2A_r s^\gamma - 2k(pI_c - sI_e) - \frac{3m}{2}\right] - \frac{h_s}{2Ps^\gamma\sqrt{h_r+pI_c}}\left[3m^2\sqrt{\frac{kA_r s^\gamma}{2}} + k\left\{\frac{2m^2A_r s^\gamma + (pI_c - sI_e)k}{\sqrt{2A_r ks^\gamma}} - \frac{m^2}{2k}\sqrt{\frac{A_r ks^\gamma}{2}}\right\}\right] \leq 0$

and

(ii) $(m-n)^2p - m^2c - \frac{A_s\sqrt{(h_r+pI_c)}}{2^{\frac{5}{2}}A_r^{\frac{3}{2}}(ks^\gamma)^{\frac{1}{2}}}\left[m^2A_r s^\gamma - 2k(pI_c - sI_e) - \frac{3m}{2}\right] - \frac{h_s}{2Ps^\gamma\sqrt{h_r+pI_c}}\left[m^2\sqrt{\frac{kA_r s^\gamma}{2}} + k\left\{\frac{2m^2A_r s^\gamma + (pI_c - sI_e)k}{\sqrt{2A_r ks^\gamma}} - \frac{m^2}{2k}\sqrt{\frac{A_r ks^\gamma}{2}}\right\}\right] > 0.$

Case 1. $(m-n)^2p - m^2c - \frac{A_s\sqrt{(h_r+pI_c)}}{2^{\frac{5}{2}}A_r^{\frac{3}{2}}(ks^\gamma)^{\frac{1}{2}}}\left[m^2A_r s^\gamma - 2k(pI_c - sI_e) - \frac{3m}{2}\right] - \frac{h_s}{2Ps^\gamma\sqrt{h_r+pI_c}}\left[m^2\sqrt{\frac{kA_r s^\gamma}{2}} + k\left\{\frac{2m^2A_r s^\gamma + (pI_c - sI_e)k}{\sqrt{2A_r ks^\gamma}} - \frac{m^2}{2k}\sqrt{\frac{A_r ks^\gamma}{2}}\right\}\right] \leq 0$, clearly, $\frac{\mathrm{d}^2Z_s^1(L)}{\mathrm{d}L^2} < 0$. So, $Z_s^1(L)$ is a strictly concave function in $[0,\infty)$. Hence, bringing equations (45) and (46) together, we know that an unique positive optimal solution L^*(say) exists such that $\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} = 0$.

Case 2. $(m-n)^2p - m^2c - \frac{A_s\sqrt{(h_r+pI_c)}}{2^{\frac{5}{2}}A_r^{\frac{3}{2}}(ks^\gamma)^{\frac{1}{2}}}\left[m^2A_r s^\gamma - 2k(pI_c - sI_e) - \frac{3m}{2}\right] - \frac{h_s}{2Ps^\gamma\sqrt{h_r+pI_c}}\left[m^2\sqrt{\frac{kA_r s^\gamma}{2}} + k\left\{\frac{2m^2A_r s^\gamma + (pI_c - sI_e)k}{\sqrt{2A_r ks^\gamma}} - \frac{m^2}{2k}\sqrt{\frac{A_r ks^\gamma}{2}}\right\}\right] > 0.$

Now, if L increases, the value of $\frac{\mathrm{d}^2Z_s^1(L)}{\mathrm{d}L^2}$ changes from positive to negative, *i.e.*, $Z_s^1(L)$ is a convex-concave function of L. So, bringing equations (45) and (46) together, we have $Z_s^1(L)$ is a uni-modal function in $[0,\infty)$. Hence a unique positive optimal solution L^*(say) in such a way that $\frac{\mathrm{d}Z_s^1(L)}{\mathrm{d}L} = 0$.

In conclusion, if $(m-n)p - mc - \frac{mA_s}{2}\sqrt{\frac{s^\gamma(h_r+I_cp)}{2kA_r}} - \frac{ah_s}{2ps^\gamma\sqrt{h_r+pI_c}}\left[\sqrt{2A_rk} + \sqrt{\frac{s^\gamma A_rk}{2}}\right] > 0$, the solution L^{*1} of equation (47) is a unique optimal solution for $Z_s^1(L)$. Then if we look at the upper bound of L, $L_{\max}$(say). The optimal credit period $L^* = L_{\max}$ when $L^{*1} < L_{\max}$. When $L^{*1} \geq L_{\max}$, the optimal credit period is $L^* = L^{*1}$. Hence the result. □

Corollary 3.9. *Increases in* n, c, A_s, h_s *result in decreases in* $(m-n)p - mc - \frac{mA_s}{2}\sqrt{\frac{s^\gamma(h_r+I_cp)}{2kA_r}} - \frac{mh_s}{2ps^\gamma\sqrt{h_r+pI_c}}\left[\sqrt{2A_rk} + \sqrt{\frac{s^\gamma A_rk}{2}}\right]$ *and* L^{*1}.

Theorem 3.7 can be amended to Theorem 3.10 according to Theorem 3.8.

Theorem 3.10. *For $T \geq L$, the supplier's optimal credit length is zero when (i) $n \geq m$, or (ii) $mc \geq (m-n)p$, or (iii)* $(m-n)p - mc - \frac{mA_s}{2}\sqrt{\frac{s^\gamma(h_r+I_cp)}{2kA_r}} - \frac{mh_s}{2ps^\gamma\sqrt{h_r+pI_c}}\left[\sqrt{2A_rk} + \sqrt{\frac{s^\gamma A_rk}{2}}\right] \leq 0.$

Note that if $Z_r^0 \geq 0$, $Z_r^1(L^*)$ increasing with L^*. Thus, $Z_r^1(L^*) \geq Z_r^0$ as long as the supplier offers the credit period. Furthermore, it is clear that $z_s^1(L^*) \geq Z_s^0$.

4. Real-life implication: case study

The current study employs optimization techniques to enhance supply chain operations, specifically in situations where a hierarchical decision-making process is followed by the leader (typically the supplier or manufacturer) and the follower (usually the retailer). The Stackelberg game model in supply chain management provides a strategic framework for analyzing and optimizing the relationships between suppliers (leaders) and retailers (followers). This approach has been successfully utilized in numerous real-world contexts to improve coordination and profitability in supply chains. The following are comprehensive case studies and data points that demonstrate these collaborations:

Partnerships between hotels and third-party booking sites

Collaboration with hotels and third-party booking platforms is necessary for the hospitality industry to maximize revenue and occupancy. The hotel owner becomes the leader who determines the room price and overbooking levels, while the booking sites play the role of followers, who adjust their marketing efforts to the leader's decisions. In addition to providing insights into optimal pricing and overbooking strategies, the model highlighted the significant impact on arrival rates and the marketing campaign's impact on costs. The room rate and the overbooking quantity are the decision factors, and the goal is to maximize the total profit of the hotel by selecting the optimal combination of these two variables. When it comes to the optimal room price, the number of guests arriving has a significant impact on it. Due to the increased number of guests arriving, the hotel owner can increase the charges accordingly. The Stackelberg game approach provides a methodical framework to enhance the profitability and operational efficiency of the hotel as well as affordable service to the guests. At the same time, the third party also benefited.

Manufacturing vendor-managed inventory (VMI) systems

Efficiently overseeing supply chain interactions between manufacturers and retailers is crucial due to pricing fluctuations and demand inconsistencies. The manufacturer typically assumes a supervisory role in the VMI system by monitoring inventory levels at the retailer's site. To analyze the interactions between producers and retailers, they utilized game theory, notably the Stackelberg and Nash equilibrium models. The manufacturer, as the leader, sets wholesale prices, while retailers, as followers, determine their order quantities. This study highlighted that the manufacturer's wholesale pricing significantly impacted the retailers' strategies and overall supply chain effectiveness. In order to maximize revenue and maintain supply chain stability, companies must consider market conditions while fixing the price.

4.1. Data points

Impact of credit periods on cash flow and sales

Previous studies indicated that retailers can order more products and boost sales by improving their cash flow by extending credit periods. Zhong and Zhou [4] demonstrate that the trade credit policy has the potential to enhance the profitability of each member as well as the overall profitability of the whole channel.

TABLE 2. Results of the decentralized and centralized models.

Example	Decentralized						Centralized				Profit due to
	D	T	Q_0^*	Z_r^0 (in \$)	Z_s^0 (in \$)	Z_{sc}^0 (in \$)	D	T	Q_2^*	Z_{sc}^2 (in \$)	collaboration (in \$)
1	2918.72	0.7121	2050.11	3515.31	2668.42	6183.73	2918.72	0.9760	2848.68	6226.25	42.52

TABLE 3. Results of the supplier Stackelberg game model.

Example	$T \leq L$					Example	$T \geq L$				
	T	L^*	Q_1^*	Z_r^1 (in \$)	Z_s^1 (in \$)		T	L^*	Q_1^*	Z_r^1(in \$)	Z_s^1(in \$)
2	0.32075	0.75895	2494.13	7709.11	3003.25	1	0.456869	0.40993	1670.71	4603.79	2682.88

Analysis of price-dependent demand

According to the literature, changing prices in response to variations in demand can have a big effect on sales volume. The efficiency of dynamic pricing techniques [48] highlights the significance of agile pricing approaches in dynamic market contexts in promptly adapting to swings in demand and optimizing profit margins.

Stackelberg Leadership

Empirical validation of the Stackelberg model has been demonstrated in multiple sectors. Assarzadegan *et al.* [49] devised a Stackelberg differential game theoretic method to examine coordinating strategies in a supply chain.

5. Numerical results

As per the derived analysis and relevant arguments of Section 4, the annual total profit of the retailer, as well as supplier won't be enhanced, if the optimal credit period is zero. Two numerical examples are taken as follows to achieve benefits from the proposed model's present policy and verify its real-life implications.

Example 1. $P = 25000$, $k = 3000$, $m = 0.55$, $n = 0.25$, $s = 3.0$, $p = 1.62$, $c = 0.6$, $A_r = 180$, $h_r = 0.25$, $I_e = 0.08$, $A_s = 200$, $h_s = 0.2$, $I_c = 0.09$, $\gamma = 0.025$.

Example 2. $P = 30000$, $k = 4000$, $m = 0.9$, $n = 0.3$, $s = 2.5$, $p = 1.5$, $c = 0.7$, $A_r = 200$, $h_r = 0.3$, $I_p = 0.09$, $A_s = 220$, $h_s = 0.25$, $I_e = 0.08$, $\gamma = 0.02$.

For Example 1, the results of Table 2 represents the optimal business solution under decentralized and centralized decision. Also, for Examples 1 and 2, the results of Table 3 represent the optimal business solution under the supplier Stackleberg game of the case 2 and case 1 respectively. These findings show that the proposed model is applicable to every retail shop, and that the decision maker may be able to determine the optimal business plan in any given circumstance. Also, from Tables 2 and 3, a Decision maker can realize the actual benefits of the supplier Stackelbeg game.

6. Sensitivity analysis

Here, the sensitivity analysis has been conducted to learn how the variations in its parameter values affect the desired values to achieve managerial insights. A unique fundamental parameter settings are used in Examples 1 and 2 for the respective centralised and decentralised decision-making and supplier stackelberg game decision-making. The obtained results are prepared in Table 4 for decentralized and centralized decision and Table 5 for supplier stackelberg game decision.

TABLE 4. Sensitivity analysis on parameters for decentarlized and centralized models.

Parameters	Values	Decentralized						Centralized			
		D	Q_0^*	T	Z_r^0(in \$)	Z_s^0(in \$)	Z_{sc}^0(in \$)	D	Q_2^*	T	Z_{sc}^2(in \$)
P	15 000	2918.72	2050.11	0.7121	3865.56	2302.22	6167.78	2918.72	2770.88	0.9493	6204.39
	20 000	2918.72	2050.11	0.7121	3865.56	2312.20	6177.76	2918.72	2818.74	0.9657	6217.98
	25 000	2918.72	2050.11	0.7121	3865.56	2318.17	6183.73	2918.72	2848.68	0.9760	6226.25
	30 000	2918.72	2050.11	0.7121	3865.56	2322.17	6187.73	2918.72	2869.18	0.9830	6231.82
	35 000	2918.72	2050.11	0.7121	3865.56	2325.02	6190.58	2918.72	2884.09	0.9881	6235.81
c	0.56	2918.72	2050.11	0.7121	3865.56	2434.93	6300.49	2918.72	2848.68	0.9760	6343.00
	0.58	2918.72	2050.11	0.7121	3865.56	2376.55	6242.11	2918.72	2848.68	0.9760	6284.63
	0.60	2918.72	2050.11	0.7121	3865.56	2318.17	6183.73	2918.72	2848.68	0.9760	6226.25
	0.62	2918.72	2050.11	0.7121	3865.56	2259.80	6125.36	2918.72	2848.68	0.9760	6167.88
	0.64	2918.72	2050.11	0.7121	3865.56	2201.43	6066.99	2918.72	2848.68	0.9760	6109.50
p	1.40	2918.72	2050.11	0.7121	4157.43	2026.31	6183.74	2918.72	2848.68	0.9760	6226.25
	1.50	2918.72	2050.11	0.7121	3865.56	2318.18	6183.74	2918.72	2848.68	0.9760	6226.25
	1.60	2918.72	2050.11	0.7121	3573.69	2610.05	6183.74	2918.72	2848.68	0.9760	6226.25
	1.70	2918.72	2050.11	0.7121	3281.81	2901.92	6183.74	2918.72	2848.68	0.9760	6226.25
	1.80	2918.72	2050.11	0.7121	2989.94	3193.80	6183.74	2918.72	2848.68	0.9760	6226.25
s	3.50	2907.50	2046.17	0.7149	5303.45	2308.76	7612.22	2907.50	2843.66	0.9780	7654.69
	3.30	2911.78	2047.67	0.71381	4729.28	2312.35	7041.64	2911.78	2845.58	0.9772	7084.12
	3.10	2916.33	2049.27	0.7127	4153.81	2316.17	6469.99	2916.33	2847.61	0.9764	6512.49
	2.90	2921.20	2050.98	0.7115	3576.93	2320.25	5897.19	2921.20	2849.78	0.9755	5939.72
	2.70	2926.42	2052.81	0.7102	2998.50	2324.64	5323.14	2926.42	2852.11	0.9746	5365.69
A_r	200	2918.72	2161.01	0.7506	3837.83	2331.50	6169.33	2918.72	2922.69	1.0013	6206.02
	190	2918.72	2106.29	0.7316	3851.51	2325.12	6176.63	2918.72	2885.92	0.9888	6216.07
	180	2918.72	2050.11	0.7121	3865.56	2318.18	6183.74	2918.72	2848.68	0.9760	6226.25
	170	2918.72	1992.35	0.6920	3880.00	2310.60	6190.60	2918.72	2810.94	0.9631	6236.57
	160	2918.72	1932.86	0.6714	3894.87	2302.28	6197.15	2918.72	2772.70	0.9500	6247.02
A_s	180	2918.72	2050.11	0.7121	3865.56	2346.65	6212.21	2918.72	2772.70	0.9500	6247.02
	190	2918.72	2050.11	0.7121	3865.56	2332.56	6197.98	2918.72	2810.95	0.9631	6236.56
	200	2918.72	2050.11	0.7121	3865.56	2318.18	6183.74	2918.72	2848.68	0.9760	6226.25
	210	2918.72	2050.11	0.7121	3865.56	2303.94	6169.50	2918.72	2885.92	0.9888	6216.07
	220	2918.72	2050.11	0.7121	3865.56	2289.70	6155.27	2918.72	2922.68	1.0013	6206.02
h_r	0.20	2918.72	2292.10	0.7961	3919.67	2345.41	6265.08	2918.72	3151.45	1.0797	6301.06
	0.23	2918.72	2137.39	0.7424	3886.49	2328.79	6215.28	2918.72	2958.99	1.0138	6255.28
	0.26	2918.72	2010.30	0.6983	3855.41	2313.01	6168.42	2918.72	2797.96	0.9586	6212.14
	0.29	2918.72	1903.48	0.6612	3826.08	2297.96	6124.04	2918.72	2660.66	0.9116	6171.22
	0.32	2918.72	1812.06	0.6294	3798.23	2283.55	6081.78	2918.72	2541.76	0.8708	6132.23
h_s	0.25	2918.72	2050.11	0.7121	3865.56	2312.19	6177.75	2918.72	2818.74	0.9657	6217.98
	0.22	2918.72	2050.11	0.7121	3865.56	2315.79	6181.35	2918.72	2836.59	0.9719	6222.93
	0.19	2918.72	2050.11	0.7121	3865.56	2319.38	6184.94	2918.72	2854.78	0.9781	6227.92
	0.16	2918.72	2050.11	0.7121	3865.56	2322.97	6188.53	2918.72	2873.33	0.9844	6232.93
	0.13	2918.72	2050.11	0.7121	3865.56	2326.56	6192.12	2918.72	2892.24	0.9909	6237.98
γ	0.22	2928.36	2053.49	0.7098	3879.17	2326.26	6205.43	2928.36	2852.98	0.9743	6247.98
	0.25	2918.72	2050.11	0.7121	3865.56	2318.17	6183.73	2918.72	2848.68	0.9760	6226.25
	0.28	2909.12	2046.73	0.7145	3852.00	2310.12	6162.12	2909.12	2844.39	0.9777	6204.60
	0.31	2899.55	2043.36	0.7168	3838.48	2302.09	6140.58	2899.55	2840.11	0.9795	6183.01
	0.34	2890.01	2040.00	0.7192	3825.01	2294.09	6119.10	2890.01	2835.82	0.9812	6161.50

Table 4 revealed some intriguing real-world occurrences for both centralized and decentralized decision-making, which are described below:

(a) If the production rate (P) increases, the order quantity (Q_0) and replenishment time (T) remains constant where as the supply chain profit (Z_{sc}^0) increase in decentralized decision but, in centralized decision, the order quantity (Q_2), replenishment time (T) and supply chain profit (Z_{sc}^2) increases. Figure 4b depicts this parameter graphically.

(b) If the production cost (c) increases then the retailer's profit (Z_r^0) remains same while the supplier profit (Z_s^0) and supply chain profit (Z_{sc}^0) decreases in both decentralized and centralized decision.

TABLE 5. Sensitivity analysis on parameters for supplier Stackelberg models.

Parameters	Values	$T \leq L$							
		D	Q_1^*	T	L^*	$F(L^*)$	Z_r^1 (in \$)	Z_s^1 (in \$)	Z_{sc}^1 (in \$)
P	35 000	7831.98	2503.11	0.3196	0.76694	0.2055	7781.75	3014.85	10 796.61
	32 000	7800.31	2498.05	0.3202	0.76244	0.2045	7740.74	3008.31	10 749.05
	29 000	7762.46	2491.98	0.3210	0.75703	0.2032	7691.76	3000.46	10 692.23
	26 000	7716.43	2484.58	0.3220	0.75042	0.2016	7632.26	2990.88	10 623.15
	23 000	7659.23	2475.35	0.3232	0.74215	0.1996	7558.42	2978.93	10 537.35
c	0.65	9257.26	2721.36	0.2940	0.95271	0.2486	9660.48	3435.08	13 095.56
	0.68	8326.97	2581.00	0.3100	0.83503	0.2216	8427.13	3166.65	11 593.78
	0.70	7775.87	2494.13	0.3207	0.75896	0.2036	7709.11	3003.25	10 712.36
	0.72	7272.84	2412.11	0.3317	0.68464	0.1857	7062.65	2850.93	9913.58
	0.74	6812.63	2334.54	0.3427	0.61201	0.1677	6479.23	2708.69	9187.92
p	1.35	5916.40	2175.57	0.3677	0.45529	0.1277	6254.80	2152.37	8407.17
	1.40	6507.24	2281.62	0.3506	0.56105	0.1549	6747.33	2418.85	9166.18
	1.45	7126.98	2387.80	0.3350	0.66213	0.1801	7233.23	2702.38	9935.62
	1.50	7775.87	2494.13	0.3207	0.75895	0.2036	7709.11	3003.25	10 712.36
	1.55	8454.16	2600.64	0.3076	0.85188	0.2255	8171.52	3321.73	11 493.24
s	2.00	7840.61	2611.11	0.3330	0.76320	0.2046	3676.63	3042.65	6719.29
	2.20	7813.75	2562.46	0.3279	0.76151	0.2042	5297.14	3026.38	8323.52
	2.40	7788.21	2516.32	0.3231	0.75980	0.2038	6907.53	3010.80	9918.34
	2.60	7763.79	2472.49	0.3185	0.75809	0.2034	8508.37	2995.83	11 504.20
	2.80	7740.32	2430.77	0.3140	0.75638	0.2030	10100.13	2981.39	13 081.52
A_r	210	7792.80	2558.50	0.3283	0.76137	0.2042	7700.19	3017.79	10 717.98
	200	7775.87	2494.13	0.3207	0.75895	0.2036	7709.11	3003.25	10 712.36
	190	7757.08	2428.04	0.3130	0.75626	0.2030	7716.34	2987.48	10 703.82
	180	7736.15	2360.09	0.3052	0.75326	0.2023	7721.58	2970.32	10 691.90
	170	7712.76	2290.12	0.2969	0.74989	0.2015	7724.45	2951.57	10 676.03
A_s	200	7876.22	2510.17	0.3187	0.77319	0.2070	7839.10	3065.80	10 904.91
	210	7826.02	2502.16	0.3197	0.76609	0.2053	7774.03	3034.47	10 808.50
	220	7775.87	2494.13	0.3207	0.75895	0.2036	7709.11	3003.25	10 712.36
	230	7725.79	2486.08	0.3218	0.75177	0.2019	7644.36	2972.12	10 616.48
	240	7675.77	2478.03	0.3228	0.74456	0.2001	7579.77	2941.09	10 520.86
h_r	0.25	7811.29	2635.03	0.3373	0.76400	0.2048	7819.09	3034.10	10 853.19
	0.30	7775.87	2494.13	0.3207	0.75895	0.2036	7709.11	3003.25	10 712.36
	0.35	7740.13	2372.59	0.3065	0.75383	0.2024	7602.16	2973.56	10 575.72
	0.40	7704.35	2266.32	0.2942	0.74868	0.2012	7498.18	2944.92	10 443.11
	0.45	7668.72	2172.37	0.2833	0.74353	0.1999	7397.07	2917.27	10 314.34
h_s	0.30	7698.79	2481.74	0.3223	0.74788	0.2010	7609.48	2987.20	10 596.68
	0.28	7729.42	2486.67	0.3217	0.75229	0.2020	7649.05	2993.59	10 642.64
	0.25	7775.87	2494.13	0.3207	0.75895	0.2036	7709.11	3003.25	10 712.36
	0.22	7822.94	2501.67	0.3198	0.76566	0.2052	7770.05	3012.99	10 783.03
	0.20	7854.67	2506.74	0.3191	0.77015	0.2063	7811.16	3019.53	10 830.69
γ	0.015	7813.22	2500.11	0.3200	0.75918	0.2037	7749.50	3018.43	10 767.92
	0.020	7775.87	2494.13	0.3207	0.75895	0.2036	7709.11	3003.25	10 712.36
	0.250	7738.70	2488.16	0.3215	0.75872	0.2036	7668.92	2988.14	10 657.06
	0.030	7701.70	2482.20	0.3223	0.75848	0.2035	7628.92	2973.10	10 602.02
	0.035	7664.86	2476.27	0.3231	0.75824	0.2035	7589.10	2958.13	10 547.23
k	2000	3714.86	1723.92	0.4641	0.70835	0.1915	3379.19	1372.60	4751.79
	2500	4727.34	1944.70	0.4113	0.72822	0.1963	4443.50	1774.61	6218.10
	3000	5742.68	2143.39	0.3732	0.74182	0.1995	5522.99	2181.19	7704.17
	3500	6759.19	2325.37	0.3440	0.75163	0.2018	6612.59	2591.02	9203.60
	4000	7775.87	2494.13	0.3207	0.75895	0.2036	7709.11	3003.25	10 712.36

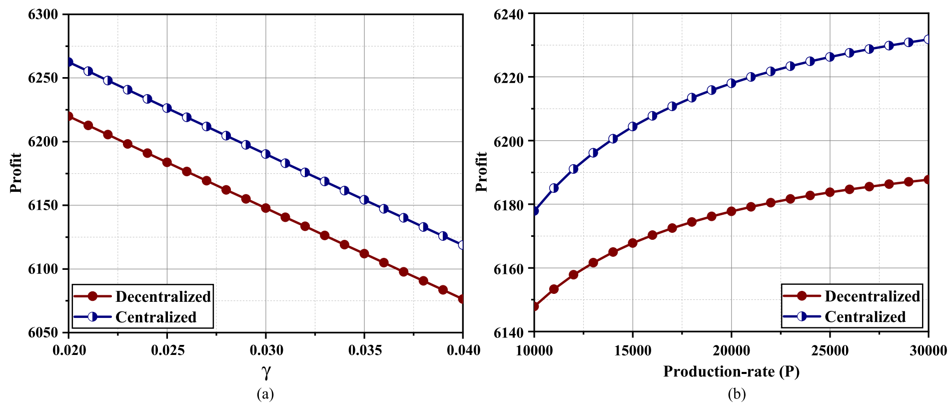

FIGURE 4. The supply chain's profit for the influence of parameters (a) γ and (b) P.

(c) In a decentralised choice, if the wholesale price (p) rises, the retailer's profit (Z_r^0) falls and the supplier's profit (Z_s^0) rises, however the supply chain profit (Z_{sc}^0) stays the same in both decisions.
(d) If the retail price (s) decreases, the order quantity (Q_0 and Q_2) increases, whereas the supply chain profit (Z_{sc}^0 and Z_{sc}^2) decreases in both the decision.
(e) If the retail's ordering cost (A_r) decreases then the order quantity (Q_0 and Q_2) decrease where as the supply chain profit (Z_{sc}^0 and Z_{sc}^2) increases in both the decision.
(f) Order quantity ($Q2$) and replenishment time (T) increase in centralised decisions when the supplier's ordering cost (As) rises, while the supply chain profit (Z_{sc}^0 and Z_{sc}^2) declines in both decisions.
(g) If the retailer's holding cost (h_r) increases then the order quantity (Q_2) and the supply chain profit (Z_{sc}^0 and Z_{sc}^2) decrease in both the decision.
(h) If the supplier's holding cost (h_s) decreases, then the order quantity (Q_2) and replenishment time (T) remain the same in the decentralized decision, whereas those increase in the centralized decision. However, the supply chain profit (Z_{sc}^0 and Z_{sc}^2) increases in both decisions.
(i) If γ increases, then the order quantity (Q_2) and the supply chain profit (Z_{sc}^0 and Z_{sc}^2) decrease in both decisions. The graphical depiction of this parameter is shown in Figure 4a.

Table 5 revealed some intriguing real-world occurrences for supplier Stackelberg decision-making, which are described below:

(a) When production rate (P) decreases, then the order quantity (Q_1), retailer's credit period (L) and the supply chain profit (Z_{sc}^1) decrease. The graphical depiction of this parameter is shown in Figure 5a. Additionally, a rise in k (the demand coefficient) leads to higher supply chain profits, as seen in Figure 5b.
(b) If the production cost (c) increases then the order quantity (Q_1), retailer's credit period (L) and supply chain profit (Z_{sc}^1) decreases.
(c) The order quantity (Q_1), retailer's credit term (L), and supply chain profit (Z_{sc}^1) all rise in response to an increase in wholesale pricing (p).
(d) If the retail price (s) rises, the profits of the retailer (Z_r^1) and the supply chain (Z_{sc}^1) rise while the profits of the suppliers (Z_s^1) fall.
(e) If the retail ordering cost (A_r) drops, the retailer's order quantity (Q_1) and credit period (L) reduce while its profit (Z_r^1) increases and the supplier's profit (Z_s^1) declines. The graphical depiction of this scenario is shown in Figure 5f. Also, Figure 6b shows that the default risk increases for the increases of A_r.
(f) If the supplier's ordering cost (A_s) goes up then the order quantity (Q_1) and supplier's credit period (L) and the supply chain profit (Z_{sc}^1) all go down.The graphical depiction of this scenario is shown in Figure 5e. Also, from Figure 6a, it is clearly visible that for the increases of A_s, the default risk deceases.

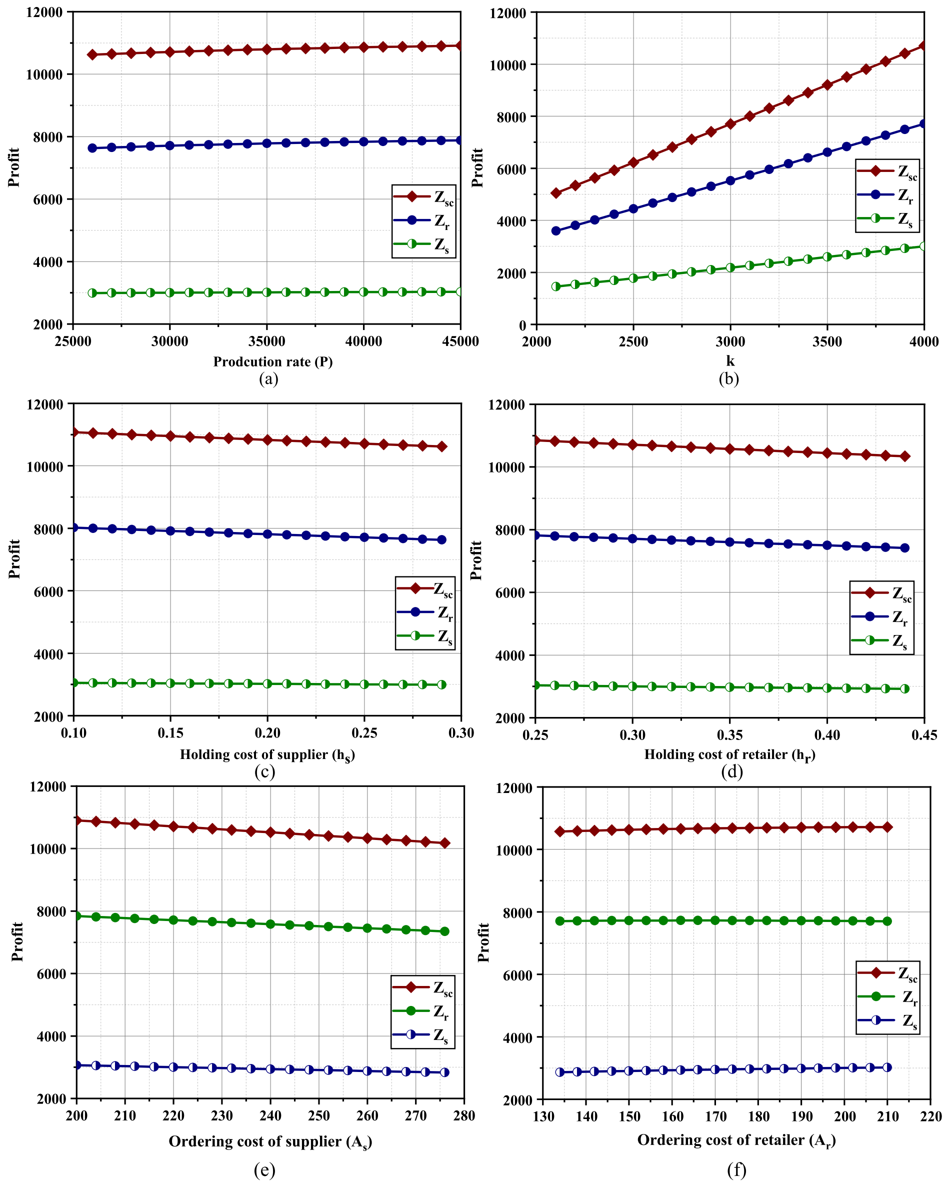

FIGURE 5. The supply chain's profit under Supplier- Stackelberg game for the influence of parameters: (a) P. (b) k. (c) h_s. (d) h_r. (e) A_s and (f) A_r.

(g) If the retailer's holding cost (h_r) increases then the order quantity (Q_1), replenishment time (T) and the supply chain profit (Z^1_{sc}) decreases. The graphical depiction of this scenario is shown in Figure 5d. Also, from Figure 7b, it is clearly visible that for the increases of h_r, the default risk deceases.

(h) If the supplier's holding cost (h_s) decreases then the order quantity (Q_1) and the supply chain profit (Z^1_{sc}) increases. The graphical depiction of this scenario is shown in Figure 5c. Also, from Figure 7a, it is clearly visible that for the increases of h_s, the default risk deceases.

(i) If γ increases then the order quantity (Q_1) and the supply chain profit (Z^1_{sc}) decreases in both the decision.

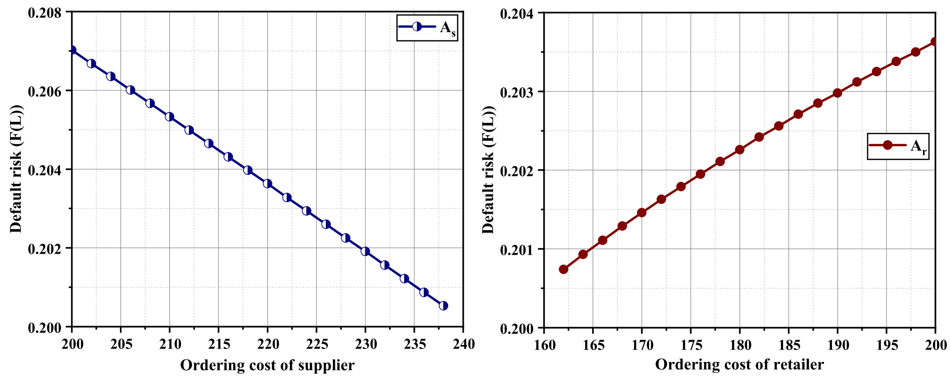

FIGURE 6. Default risk for the influence of parameters (a) A_s and (b) A_r.

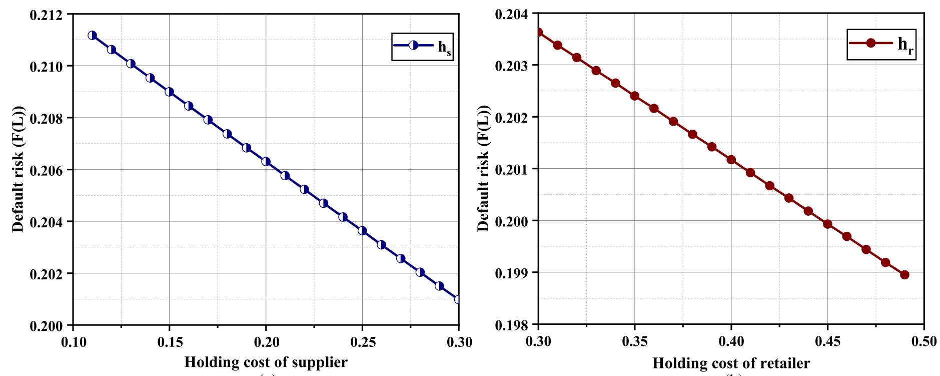

FIGURE 7. Default risk for the influence of parameters (a) h_s and (b) h_r.

The sensitivity analysis indicates that a centralized strategy is effective, as it prioritizes the global optimum through the integration of decision-making, demand data, and cost factors across the whole supply chain system. In contrast, a decentralized decision-making strategy sometimes emphasizes local optimization (*e.g.*, cost reduction at particular phases), leading to suboptimal overall performance. Also, based on the data shown in Tables 3 and 4, it can be observed that the total profit of the supply chain is consistently higher under the Supplier Stackelberg game compared to the Centralized model. Specifically, the Supplier Stackelberg game results in a higher total profit for the supply chain when a longer credit duration is offered, compared to the profit obtained from a centralized decision. This suggests that trade credit can serve as a coordinating factor.

7. Managerial significance

An integrated supply chain model facilitates the coordination of activities among many stakeholders, including suppliers, manufacturers, and retailers, to enhance operational efficiency and minimize expenses. In a Stackelberg model, a single player, known as the leader, makes the decision first, while the remaining players, referred to as followers, subsequently make their decisions, considering the leader's choice. This methodology facilitates comprehension of hierarchical decision-making within the supply chain.

- Managers can determine the most advantageous pricing and credit policies by assuming the role of leaders in the Stackelberg game. They can deliberately affect downstream participants' ordering and stocking decisions by establishing prices and credit terms. Extending the duration of credit durations or making pricing adjustments can effectively boost demand. Managers have the capacity to carefully manage these parameters to optimize both overall profitability and market share.
- Suppliers might utilize credit periods as a bargaining strategy to obtain more favorable conditions or increased orders from retailers. Improved coordination between suppliers and retailers results in more accurate demand forecasts, hence minimizing inefficiencies and costs.
- Strategic utilization of credit durations and pricing can offer a competitive advantage by enticing a larger customer base and ensuring loyalty. Integrated models enhance the overall competitiveness of the supply chain by making it more flexible and responsive to market changes.
- Gaining a comprehensive understanding of the consequences of credit terms is beneficial for effectively managing the risk of failure to pay and enhancing the management of available funds. Price-dependent demand models lessen uncertainty by enabling managers to anticipate and plan for changes in demand more accurately.

8. Conclusions

This research looked at a supplier uncooperative replenishment model with default risk and demand connected with the trade credit length in order to determine the optimal credit length in a supplier Stackelberg game. The proposed model comprises some advantageous features, as mentioned below:

- Market demand influences the selling price and the credit period. Additionally, default risk is an exponential function of the delay period.
- The credit period influences the decision.
- During absence of the trade credit, the optimal outcomes of decentralized and centralized models are considered for comparative benchmarks.
- It narrates the retailer's benefit, but it is a burden for the supplier, who has to pay the interest for given trade credit.

This article presents an EOQ model that takes into account the following realities on both the supplier and retailer sides of the supply chain: (1) the supplier will normally allow the retailer a reasonable payment delay, (2) the allowable delay of a supplier has a favorable impact on market demand, and (3) the default retailer increases with the length of the permitted delay. Further, in case of non-cooperative Stackelberg equilibrium, the necessary and sufficient criteria have been deduced to produce the best outcome for both the supplier as well as the retailer. A thorough analysis has been given for the outcomes of the supplier-Stackelberg model with trade credit to those of the decentralized and centralized models in the absence of trade credit to comprehend how a trade credit duration affects demand and default risk. Finally, a fruitful comparative performance between the Stackelberg and the Decentralized solutions has been carried out by providing few numerical attributes. From the sensitivity analysis, it can be seen that centralized strategy is effective as it emphasizes the global optimum by unifying decision-making, demand data, and cost parameters of the entire supply chain system. Conversely, a decentralized decision-making approach frequently prioritizes local optimization (*e.g.*, cost minimization at individual stages), resulting in inferior overall performance. Furthermore, it can be inferred that in all situations, the Supplier Stackelberg game yields a greater overall benefit for the supply chain when a lengthier credit period is provided, in contrast to the profit derived from a centralized decision. This implies that trade credit can function as a valuable factor for increased profitability.

The model may be expanded to incorporate additional demand functions, like quadratic trade credit period demand to conduct a future study. Additionally, one can consider deteriorating commodities, shortages, inflation, and non-coordination between a single supplier and several retailers, among other factors. As a result, future studies might consider the consequences of these new possibilities.

Data availability statement

No new data/codes were created or analyzed in this study.

References

[1] S.K. Goyal, Economic order quantity under conditions of permissible delay in payments. *J. Oper. Res. Soc.* **36** (1985) 335–338.

[2] S. Paul and R. Boden, The secret life of UK trade credit supply: setting a new research agenda. *Br. Accounting Rev.* **40** (2008) 272–281.

[3] Y.F. Huang, Optimal retailer replenishment decisions in the EPQ model for deteriorating items with two level of trade credit financing. *Eur. J. Oper. Res.* **176** (2008) 1577–1591.

[4] Y.G. Zhong and Y.W. Zhou, Improving the supply chain's performance through trade credit under inventory-dependent demand and limited storage capacity. *Int. J. Prod. Econ.* **143** (2013) 364–370.

[5] C.K. Jaggi, S.K. Goyal and S.K. Goel, Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments. *Eur. J. Oper. Res.* **190** (2008) 130–135.

[6] A. Barman, A.K. Chakraborty, A. Goswami, P. Banerjee and P.K. De, Pricing and inventory decision in a two-layer supply chain under the Weibull distribution product deterioration: an application of NSGA-II. *RAIRO Oper. Res.* **57** (2023) 2279–2300.

[7] J.T. Teng, C.T. Chang and M.S. Chern, Vendor–buyer inventory models with trade credit financing under both non-cooperative and integrated environments. *Int. J. Syst. Sci.* **43** (2012) 2050–2061.

[8] P. Mahata, G.C. Mahata and S.K. De, Optimal replenishment and credit policy in supply chain inventory model under two levels of trade credit with time- and credit-sensitive demand involving default risk. *J. Ind. Eng. Int.* **14** (2018) 31–42.

[9] A. Mukherjee and G.C. Mahata, Optimal replenishment and credit policy in an inventory model for deteriorating items under two-levels of trade credit policy when demand depends on both time and credit period involving default risk. *RAIRO Oper. Res.* **4** (2024) 1175–1200.

[10] M. Choudhury and G.C. Mahata, Dual channel supply chain inventory policies for controllable deteriorating items having dynamic demand under trade credit policy with default risk. *RAIRO Oper. Res.* **56** (2022) 2443–2473.

[11] A. Barman, R. Das and P.K. De, An analysis of optimal pricing strategy and inventory scheduling policy for a non-instantaneous deteriorating item in a two-layer supply chain. *Appl. Intell.* **52** (2022) 4626–4650.

[12] S.W. Shinn, Determining optimal retail price and lot size under day-terms supplier credit. *Comput. Ind. Eng.* **33** (1997) 717–720.

[13] L.H. Chen and F.S. Kang, Integrated inventory models considering the two-level trade credit policy and a price negotiation scheme. *Eur. J. Oper. Res.* **205** (2010) 47–58.

[14] C.F. Wu and Q.H. Zhao, An inventory model for deteriorating items with inventory-dependent and linear trend demand under trade credit. *Sci. Iran. Trans. E: Ind. Eng.* **22** (2015) 2558–2570.

[15] P.C. Yang and H.M. Wee, A collaborative inventory system with permissible delay in payment for deteriorating items. *Math. Comput. Model.* **43** (2006) 209–221.

[16] S.P. Sarmah, D. Acharya and S.K. Goyal, Coordination of a single manufacturer/multi-buyer supply chain with credit option. *Int. J. Prod. Econ.* **111** (2008) 676–685.

[17] C.F. Wu and Q.H. Zhao, Supplier–retailer inventory coordination with credit term for inventory-dependent and linear-trend demand. *Int. Trans. Oper. Res.* **21** (2014) 797–818.

[18] F. Mahato and G.C. Mahata, Two-echelon supply chain with production disruption and controllable deterioration considering carbon emission under Stackelberg game approach. *RAIRO Oper. Res.* **58** (2024) 2339–2365.

[19] A. Barman, P.K. De, A.K. Chakraborty, C.P. Lim and R. Das, Optimal pricing policy in a three-layer dual-channel supply chain under government subsidy in green manufacturing. *Math. Comput. Simul.* **204** (2023) 401–429.

[20] A. Barman, R. Das, P.K. De and S.S. Sana, Pricing policies for a competitive dual-channel supply chain with green investment and sales effort under revenue-sharing contract. *RAIRO Oper. Res.* **58** (2024) 3985–4012.

[21] F. Mahato, M. Choudhury and G.C. Mahata, Multistage supply chain inventory model for controllable deterioration and imperfect production with carbon emissions regulations under Stackelberg game approach. *Environ. Dev. Sustainability* (2024). DOI: 10.1007/s10668-024-05175-3.

[22] M. Choudhury, S.K. De and G.C. Mahata, A pollution-sensitive multistage production-inventory model for deteriorating items considering expiration date under Stackelberg game approach. *Environ. Dev. Sustainability* **25** (2023) 11847–11884.

[23] S.P. Sarmah, D. Acharya and S.K. Goyal, Coordination and profit sharing between a manufacturer and a buyer with target profit under credit option. *Eur. J. Oper. Res.* **182** (2007) 1469–1478.

[24] C.K. Huang, D.M. Tsai, J.C. Wu and K.J. Chung, An integrated vendor–buyer inventory model with order-processing cost reduction and permissible delay in payments. *Eur. J. Oper. Res.* **202** (2010) 473–478.

[25] S. Krichen, A. Laabidi and F.B. Abdelaziz, Single supplier multiple cooperative retailers inventory model with quantity discount and permissible delay in payments. *Comput. Ind. Eng.* **60** (2011) 164–172.

[26] B.C. Giri and T. Maiti, Trade credit competition between two retailers in a supply chain under credit-linked retail price and market demand. *Optim. Lett.* **11** (2013) 1–21.

[27] C.F. Wu and Q.H. Zhao, Supplier–buyer deterministic inventory coordination with trade credit and shelf-life constraint. *Int. J. Syst. Sci.: Oper. Logistics* **1** (2014) 36–46.

[28] D.A. Phan, T.L.H. Vo and A.N. Lai, Supply chain coordination under trade credit and retailer effort. *Int. J. Prod. Res.* **57** (2019) 2642–2655.

[29] C.H. Lee and B.D. Rhee, Trade credit for supply chain coordination. *Eur. J. Oper. Res.* **214** (2011) 136–146.

[30] P.I. Abad and C.K. Jaggi, A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. *Int. J. Prod. Econ.* **83** (2003) 115–122.

[31] J. Kim, H. Hwang and S. Shinn, An optimal credit policy to increase supplier's profits with price-dependent demand functions. *Prod. Planning Control* **6** (1995) 45–50.

[32] J.T. Teng and K.R. Lou, Seller's optimal credit period and replenishment time in a supply chain with up-stream and down-stream trade credits. *J. Global Optim.* **53** (2012) 417–430.

[33] Y.W. Zhou and D. Zhou, Determination of the optimal trade credit policy: a supplier-Stackelberg model. *J. Oper. Res. Soc.* **64** (2013) 1030–1048.

[34] M.S. Chern, Q.H. Pan, J.T. Teng, Y.L. Chan and S.C. Chen, Stackelberg solution in a vendor–buyer supply chain model with permissible delay in payments. *Int. J. Prod. Econ.* **144** (2013) 397–404.

[35] N.H. Shah, Retailer's replenishment and credit policies for deteriorating inventory under credit period-dependent demand and bad-debt loss. *Top* **23** (2014) 298–312.

[36] N.H. Shah and L.E. Cardenas-Barron, Retailer's decision for ordering and credit policies for deteriorating items when a supplier offers order-linked credit period or cash discount. *Appl. Math. Comput.* **259** (2015) 569–578.

[37] C.H. Su, Optimal replenishment policy for an integrated inventory system with defective items and allowable shortage under trade credit. *Int. J. Prod. Econ.* **139** (2012) 247–256.

[38] W.C. Wang, J.T. Teng and K.R. Lou, Seller's optimal credit period and cycle time in a supply chain for deteriorating items with maximum lifetime. *Eur. J. Oper. Res.* **232** (2014) 315–321.

[39] J. Wu, L.Y. Ouyang, L.E. Cardenas-Barron and S.K. Goyal, Optimal credit period and lot size for deteriorating items with expiration dates under two level trade credit financing. *Eur. J. Oper. Res.* **237** (2014) 898–908.

[40] X. Shi and S. Zhang, An incentive-compatible solution for trade credit term incorporating default risk. *Eur. J. Oper. Res.* **206** (2010) 178–196.

[41] C.Y. Dye and C.T. Yang, Sustainable trade credit and replenishment decisions with credit-linked demand under carbon emission constraints. *Eur. J. Oper. Res.* **244** (2015) 187–200.

[42] K.R. Lou and W.C. Wang, Optimal trade credit and order quantity when trade credit impacts on both demand rate and default risk. *J. Oper. Res. Soc.* **64** (2013) 1551–1556.

[43] J.T. Teng, K.R. Lou and L. Wang, Optimal trade credit and lot size policies in economic production quantity models with learning curve production costs. *Int. J. Prod. Econ.* **155** (2014) 318–323.

[44] C. Wu, Q. Zhao and M. Xi, A retailer–supplier supply chain model with trade credit default risk in a supplier-Stackelberg game. *Comput. Ind. Eng.* **112** (2017) 568–575.

[45] R.M. Mondal, R.K. Jana, P. Pramanik and M.K. Maiti, A Fuzzy EOQ model for deteriorating items under trade credit policy with unfaithfulness nature of customers. *Real Life App. Multiple Criteria Decis. Making Tech. Fuzzy Domain* **420** (2022) 453–482.

[46] C. Zhang, M. You and G. Han, Integrated supply chain decisions with credit-linked demand: a Stackelberg approach. *Sci. Iran. E* **28** (2021) 927–949.

[47] P. Wang, G. Bi and X. Yan, Optimal two-level trade credit with credit dependent demand in a newsvendor model. *Int. Trans. Oper. Res.* **29** (2022) 1915–1942.

[48] L. Kumar, K. Sharma and U.K. Khedlekar, Dynamic pricing strategies for efficient inventory management with auto-correlative stochastic demand forecasting using exponential smoothing method. *Results Control Optim.* **15** (2024) 100432.

[49] P. Assarzadegan, S.R. Hejazi and M. Rasti-Barzoki, A stackelberg differential game theoretic approach for analyzing coordination strategies in a supply chain with retailer's premium store brand. *Ann. Oper. Res.* **336** (2024) 1519–1549.

Appendix A.

Proof of Theorem 3.1. The retailer's total profit is provide by

$$Z_r^1(T) = \begin{cases} D(s-p) - \frac{A_r}{T} - (h_r + I_e s)D\frac{T}{2} + I_e sDL, & \text{if } T < L \\ D(s-p) - \frac{2A_r - (sI_e - pI_c)DL^2}{2T} - (h_r + pI_c)\frac{DT}{2} + pI_c DL, & \text{if } T \geq L. \end{cases} \tag{A.1}$$

First and second order derivative of $Z_r^1(T)$ w.r.t. T are

$$\frac{\mathrm{d}Z_r^1(T)}{\mathrm{d}T} = \begin{cases} \frac{A_r}{T^2} - (h_r + I_e s)\frac{D}{2}, & \text{if } T < L \\ \frac{2A_r - (sI_e - pI_c)DL^2}{2T^2} - \frac{D(h_r + pI_c)}{2}, & \text{if } T \geq L \end{cases} \tag{A.2}$$

and

$$\frac{\mathrm{d}^2 Z_r^1(T)}{\mathrm{d}T^2} = \begin{cases} -\frac{2A_r}{T^3}, & \text{if } T < L \\ -\frac{2A_r - (sI_e - pI_c)DL^2}{T^3}, & \text{if } T \geq L \end{cases} \tag{A.3}$$

respectively.

If $2A_r - DL^2(h_r + sI_e) > 0$, from equation (A.3), $\frac{\mathrm{d}^2 Z_r^1}{\mathrm{d}T^2} < 0$ for $T \geq L$, which imply that $\frac{\mathrm{d}Z_r^1}{\mathrm{d}T}$ is strictly decreasing in T. Again, for $T \geq L$, $\frac{\mathrm{d}Z_r^1(T)}{\mathrm{d}T}|_{T=L} = \frac{2A_r - L^2(h_r + sI_e)D}{2L^2} > 0$ and $\frac{\mathrm{d}Z_r^1(T)}{\mathrm{d}T}|_{T=\infty} = -\frac{D(h_r + pI_c)}{2} < 0$. Hence, an unique $T^* > L$ exits in such a way that

$$Z_r^1(T^*) \geq Z_r^1(T) \qquad \text{for all} \quad T \geq L. \tag{A.4}$$

Similarly, for $T \leq L$, from equations (A.2) and (A.3) we get $\frac{\mathrm{d}^2 Z_r^1(T)}{\mathrm{d}T^2} < 0$, $\frac{\mathrm{d}Z_r^1(T)}{\mathrm{d}T}|_{T=0} = \infty$ and $\frac{\mathrm{d}Z_r^1(T)}{\mathrm{d}T}|_{T=L} = \frac{2A_r - (h_r + I_e s)DL^2}{2L^2} > 0$. Hence for all $T \leq L$, $Z_r^1(T)$ is an increasing function. So, we have

$$Z_r^1(T)|_{T=L} \geq Z_r^1(T) \qquad \text{for all} \quad T \leq L. \tag{A.5}$$

By using equations (A.1), (A.4) and (A.5), we have

$$Z_r^1(T^*) \geq Z_r^1(T)|_{T=L} \geq Z_r^1(T) \qquad \text{for all} \quad T \leq L. \tag{A.6}$$

This concludes the proof of part (i) of Theorem 3.1.

Furthermore, if $2A_r - DL^2(h_r + sI_e) < 0$, then we know from equation (A.2) that $\frac{\mathrm{d}Z_r^1}{\mathrm{d}T}(T) < 0$, for all $T \leq L$. Then, we obtain

$$Z_r^1(T)|_{T=L} \geq Z_r^1(T) \qquad \text{for all} \quad T \geq L. \tag{A.7}$$

Similarly, for $T \leq L$, if $2A_r - DL^2(h_r + sI_e) < 0$, then we have from equation (A.2).

$$\frac{\mathrm{d}Z_r^1(T)}{\mathrm{d}T}|_{T=0} = \infty, \quad \text{and} \quad \frac{\mathrm{d}Z_r^1(T)}{\mathrm{d}T}|_{T=L} = \frac{2A_r - L^2(h_r + sI_e)D}{2L^2} < 0. \tag{A.8}$$

By using equations (A.3) and (A.8), we have, an unique $T^* < L$ in such a way that

$$Z_r^1(T^*) \geq Z_r^1(T) \qquad \text{for all} \quad T \leq L. \tag{A.9}$$

Again, combining equations (A.1), (A.7) and (A.9), we get

$$Z_r^1(T^*) \geq Z_r^1(T)|_{T=L} \geq Z_r^1(T) \qquad \text{for all} \quad T \geq L. \tag{A.10}$$

This concludes the proof of part (ii) of Theorem 3.1.

Part (iii) of Theorem 3.1 can be easily produced from part (i) and (ii). □