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FLEXIBLE MANUFACTURING SYSTEM FOR HIGH-TECH PRODUCT WITH REWORKING, VARIABLE DEMAND, AND INVESTMENT IN CARBON REDUCTION

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Abstract. Due to the technological drift in the current century, environmental issues are associated with high-tech electronics products like tablets, Android phones, desktops, laptops, refrigerators, etc. Governments everywhere are taking initiatives to reduce the carbon footprint that these high-tech products leave behind during production process. Further, eco-friendly electronics items have significant impact on the demand of the customers. Current study develops a flexible manufacturing system for high-tech products, considering imperfect manufacturing processes. Reworking is considered in the model for imperfect items to reduce the waste from the system. Partial outsourcing of products is considered in the model because an imperfect manufacturing procedure hurts a manufacturer's credibility in the marketplace. In this model, it is assumed that the demand of high-tech product is affected by its selling price and the amount of money invested in green technology to make it environmentally friendly. Several activities like setup preparation, production, deterioration, transportation, reworking, and carrying the products in stock result carbon emissions. To limit the carbon footprint, government governed regulatory body imposes carbon tax policy together with investment in green technology. An algorithm is built to find the optimal values for the decisive variables. To support the established model, numerical examples and sensitive analysis are provided. Managerial perspectives are also presented in the model. Analysis demonstrates that investing in green technology increases demand, and due to this, the profit of the system rises by 75%. In addition to this, it is observed that the adaptation of green technology reduces the carbon footprint by 5.5%, and due to this, the profit of the system increases. More outsourced items result in less profit for the system. Different elastic demand factors have a favorable effect on demand and system profitability.

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1. Introduction

The manufacturing and retail sectors both experience technological improvement in the twenty-first century. Different electronic devices including TVs, adapters, tablets, electronic gamepads, refrigerators, smart watches, etc. all exhibit sharp changes in their design and functioning. The development of technology has caused several environmental problems for humanity. A carbon footprint is left behind during the manufacturing of high-tech devices[1]. It is observed that 10 tonnes of CO_2 are emitted because of the production of a tonne of laptops, and in addition to this, the production of laptops requires "rare earth" material. A collaborative document, "A New Circular Vision for Electronic Time for a Global Reboot", was released in 2019 to assess the negative effects that electronic devices have on the environment. With the intention to establish international standards for electrical and electronic devices, the IEC (International Electrotechnical Commission) was established in 1906. Electronic devices require energy to operate. Energy for electronic devices is generated from fossil fuels, wind, or solar energy. Utilization of these resources has a negative impact on the environment. All of this encourages the manufacturing company to invest in environment friendly products. Therefore, it is crucial to consider government-imposed carbon tax policy and green investment for the benefit of the environment and, by consequently, humanity.

The government is aware from the different studies that environmentally conscious customers are more inclined towards eco-friendly products. Thus, investment by the manufacturer in green technology attracts more customers. Following the pandemic, it has been noted that consumer preferences have become more ecologically conscious, and today's businesses are responding appropriately to compete in the cutthroat market[2]. In the 2019 Global Consumer Insight Survey, 35% of consumers chose environmentally sustainable products, while in June 2021, the Global Consumer Insights Pulse Survey shows that this number has increased to 50%. Additionally, Zhao *et al.* [58] assert that a product's pricing has a significant impact on consumer demand. So, while making a purchase decision, especially for high-tech items, price is crucial. Technological advancement leads to lower prices for high-tech products. For example, nowadays, in PC business, the same level of functionality can be achieved at a lower price because of technological advancement. For the manufacturer, it is essential to take demand into account as a function of green investment and pricing, especially when working with high-tech products.

Imperfect production is an inevitable problem linked to the manufacturing system because of interruptions in power, equipment faults, unskilled workforce, machine ageing, etc. Particularly when it comes to high-tech products, the manufacturer suffers significant financial losses as a result. High-tech items require "rare earth" materials and a lot of energy during the manufacturing process negatively affects the ecosystem. Therefore, for the manufacturing of high-tech products, a flexible manufacturing system, *i.e.*, a smart manufacturing system, is more viable in place of traditional manufacturing system. Because of the smart manufacturing process, different processes associated with the manufacturing system can be controlled. In addition to this, with the help of the reworking process, the negative impact of the imperfect manufacturing process of high-tech products on the environment can be reduced as much as possible. Flexible manufacturing techniques and the reworking of defective goods must be considered to create realistic inventory models.

The remainder of the proposed study is as follows: In Section 2, a literature review is offered together with the novelty of the current work, and Section 3 comprises research hypotheses and notations for the creation of the model. The mathematical model is presented in Section 4, and the method to identify the most optimal outcomes is described in Section 5. Section 6 includes a section on numerical analysis; sensitivity analysis is also the part of Section 6. In Section 7, managerial concepts are discussed. Conclusions and possible extensions are presented in Section 8.

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[1] https://www.genevaenvironmentnetwork.org/resources/updates/the-growing-environmental-risks-of-e-waste/#scroll-nav__1 visited on 19.02.2023.

[2] https://www.strategy-business.com/article/The-rise-of-the-eco-friendly-consumer visited on 19.02.2023.

2. Literature review

A systematic review is carried out in this section related to important keywords. The contribution of each paper is elaborated in the direction of important keywords.

2.1. Inventory models with carbon emissions

Despite numerous difficulties on a global scale, such the COVID-19 epidemic, the war in Ukraine, etc., customers are more concerned about the climate change[3]. However, according to the Emissions Gap Report 2022, all global efforts will not be enough to meet the Paris Agreement's target, and it is anticipated that the average global temperature could reach 2.80 °C by the end of the century. Therefore, carbon emissions are still a major worry for governments across the globe, particularly due to the manufacturing process. Researchers have recently focused especially on carbon tax policy, both experimentally and conceptually, to reduce carbon emissions from the production processes. According to Baranzini *et al.* [4], a carbon tax policy is straightforward and simple to put into practice, and the money collected from the tax can be utilized to limit the carbon emissions. It is believed that Jaber *et al.* [22] was one of the first studies to propose a carbon tax policy to lower carbon emissions in an inventory system. Toptal *et al.* [49] devised an inventory replenishment policy and investigated several emissions control techniques, such as carbon tax and carbon cap-and-trade, to lessen carbon footprints in the production system. They discovered that carbon tax initiatives performed better than the substitutes. In the pull system inventory model, Lin and Sarker [30] theoretically showed that for a powerful retailer, number of shipments, order quantity, and expected profit are affected by a progressive or flat carbon tax. Shen *et al.* [47] examined a production inventory model while taking into consideration carbon tax policy and preservation technology. They found that as carbon emissions from shipping, setup, production, and ordering rise, investments in preservation technologies and the volume of orders rise but the system's overall profit falls. The research work of Shen *et al.* [47] was further expanded upon by Lu *et al.* [31], who used the Stackelberg game strategy to arrive at the optimal solution. They discovered that reducing material deterioration contributes to a decrease in carbon emissions. According to Yadav and Khanna [54], a carbon tax policy is a useful instrument for a greener environment. Numerically, they shown that carbon levels are mitigated by 50% by considering carbon tax. A flexible manufacturing inventory model modelled by Yadav *et al.* [55] and numerically they observed that pollution tax legislation results in a 0.62% reduction in pollution. The research by Mishra *et al.* [38], Lu *et al.* [32], Kugele and Sarkar [27], Govindan *et al.* [18], and others is crucial in this area.

2.2. Inventory models with variable demand

The ClimatePartner[4] report from 2022 states that "51% of participants say carbon neutral or climate-friendly products influence their purchasing decisions". Practically speaking, companies who provide sustainable products are faced with a challenging situation: although most consumers prefer eco-friendly items, they are not yet willing to make a financial commitment. As a result, in the present context, green investment and selling price have a significant impact on consumer demand. Many researchers explore the impact of green investment on demand [2,3,7,37]. They all noted that green investment benefits both the environment and the system's profit. More scholars study the inventory issue in the literature while taking price-dependent demand into account [6,15,25,39,42,43]. They have all demonstrated a negative correlation between demand and the product's selling price. According to Khan *et al.* [24], an increase in selling price may not necessarily result in an increase in the inventory system's profit. According to Giri and Dash [16], products with a high level of greening and less expensive attracts more customers and hence raises demand for the product. As a result, the store places a larger number of orders. Jani *et al.* [23] found through numerical analysis that an upsurge in product price indicates that relatively few consumers are interested in buying the goods.

[3]https://www.unep.org/resources/emissions-gap-report-2022?gclid=Cj0KCQjwxYOiBhC9ARIsANiEIfZyysMHGbXMsXL_udEn2U8i_5m0a2n0Ge4AQBk4h529ndxMjpkKKuMaAq2EEALw_wcB dated 19.04.2023.

[4]https://www.climatepartner.com/en/resources/climate-action-awareness-report-2022 dated 19.04.2023.

2.3. Flexible smart manufacturing process

The manufacturing of high-tech items like smart watches, virtual reality goggles, 3D printers, intelligent home appliances, etc. is linked to numerous environmental problems. The demand for high-tech goods is skyrocketing now. Further, imperfect output is inevitable due to the lengthy operating times of machinery, energy fluctuations, human mistake, etc. A flexible manufacturing method is useful in this situation to manage the faulty production process, satisfy the customer's demand, and enhance the customer satisfaction. Sana *et al.* [44] created a flexible manufacturing system with a flawed manufacturing process. They noticed that a higher rate of manufacturing causes high production of defective goods, which lowers the system's overall profit. Production rates are assumed to follow the Cobb-Douglas rule by Panda *et al.* [41]. They claimed that the market's customer demand might be satisfied with the aid of a variable manufacturing rate. Flexibility aids in balancing excessive stock levels and demand. According to Sarkar and Chung [45], a regulated manufacturing rate can help to reduce the number of flawed items and holding expenses. They recommended that businesses use flexible production within certain parameters to reduce the overall cost of inventories. A production-inventory model was thoroughly reviewed by Glock and Grosse [17] considering flexible manufacturing system. They also examined the model's effects on machine breakdowns, product quality, shelf life, stockout risk, greenhouse gas emissions, and energy usage. A manufacturing inventory model that considers volume flexibility, online payment, and a pre-order discount option was presented by Singh and Singh [48]. Yadav *et al.* [57] developed an environmentally friendly smart production-inventory system including partial backlogging. They discovered that the tax on carbon had a sizable effect on the cost of inventory system. Bachar *et al.* [3] suggested that smart flexible manufacturing systems must be considered for industry 4.0. They noticed that reworking with smart production techniques and global outsourcing helped the company make more money.

2.4. Inventory models with outsourcing

The idea of outsourcing is particularly beneficial for manufacturing companies to survive in a cutthroat industry. It aids in sustaining the market reputation of manufacturing companies and, as a result, their level of customer service [5]. When a production system is inefficient, outsourcing is quite beneficial. Low transportation costs and a good level of service, according to Cheng [8], are important aspects of outsourcing transportation. Multiple carbon policies were developed into an integrated model by Li *et al.* [29]. They recommended that the decision-maker outsource additional transport services if the cost of energy keeps rising. A mathematical model was created by Chiu *et al.* [9] for manufacturers considering multi-shipment, outsourcing, and rework. Alkahtani [1] modelled an integrated inventory model and performed the screening for all outsourced and produced items. They recommended that to manage outsourcing in an integrated system, managers must investigate the important cost criteria. A green production-inventory model created by Bachar *et al.* [2] revealed that outsourcing lowers operational costs and boosts profits for manufacturing companies. In this direction, Bachar *et al.* [3] created a rework- and outsourcing-based sustainable, flexible manufacturing system. They recommended outsourcing as a highly effective way to deal with substandard output. In instances of price inflation and post-pandemic industry regression, this approach is particularly beneficial.

2.5. Research gaps and novelty of current study

From the Table 1, it is observed that numerous researchers investigated the flexible production-inventory model while individually considering diverse factors like green investment, reworking, deterioration, carbon emissions, variable demand rate, and outsourcing. These challenges are all vital to the production process and are connected to one another. As far as we are aware, no researcher has attempted to address every one of these issues at once. Thus, the following are some ways that current study advances the literature:

(1) Smart flexible manufacturing processes for high-tech products are considered with deterioration.
(2) Reworking is considered in the model as imperfect high-tech products cause environmental issues.
(3) According to the concept, demand is influenced by both product selling price and investments in green technologies.

TABLE 1. Author's contribution table.

Author(s)	Production Process	Demand	Volume flexibility	Reworking	Outsourcing	Carbon Policy
Sana *et al.* [44]	Imperfect	Constant	Yes	No	No	No
Panda and Maity [40]	Imperfect	Price dependent	Yes	No	No	No
Guchhait *et al.* [19]	Perfect	Stock and time dependent	Yes	No	No	No
Mahata [33]	Imperfect	Constant	No	Yes	No	No
Datta [11]	Imperfect	Price Dependent	Yes	Yes	No	Carbon tax
De and Mahata [12]	Imperfect	Constant	No	No	No	No
De and Mahata [13]	Perfect	Constant	No	No	No	No
Mahata and Mahata [34]	Imperfect	Constant	No	No	No	No
De and Mahata [14]	Imperfect	Constant	No	No	No	No
Khanna *et al.* [26]	Imperfect	Price dependent	No	Yes	No	Carbon tax
Heydari *et al.* [21]	Perfect	Constant	No	No	Yes	No
Mahato *et al.* [36]	Imperfect	Constant	Yes	Yes	No	Carbon cab, carbon cap and trade, carbon tax
Bachar *et al.* [2]	Perfect	Selling price dependent	No	Yes	Yes	No
Alkahtani [1]	Imperfect	Constant	No	Yes	Yes	No
Yadav *et al.* [57]	Imperfect	Constant	Yes	Yes	No	Carbon tax
Mahata *et al.* [35]	Imperfect	Constant	No	No	No	No
Vandana *et al.* [53]	Perfect	Time dependent	Yes	No	No	Carbon tax
Bachar *et al.* [3]	Imperfect	Price dependent	Yes	Yes	Yes	No
This paper	Imperfect	Price and green investment dependent	Yes	Yes	Yes	Carbon tax

(4) The production system's operational actions that result in carbon emissions are considered.
(5) Government imposed carbon tax mechanism is adopted to mitigate the carbon level in the environment.
(6) To make the production system ecologically friendly, investment in green technology is made.
(7) Partial outsourcing is considered to keep the manufacturing firm competitive.

3. Problem descriptions, research hypothesis, and notations

Along with the necessary hypothesis and notations for the mathematical formulation, a detailed description of the problem is given in this section.

3.1. Problem description

Smart manufacturing system is considered in this study to manufacture high-tech products. Although the manufacturing process is not flawless, it is assumed that the production rate may be controlled. The reworking process is considered in the model to mitigate the detrimental effects of defective high-tech products on the environment. The deteriorating process is considered in the model, and unit production cost is considered as the function of the production rate. The carbon emissions caused by various operational activities are assessed. Green investment is considered in the model with the aim to reduce the amount of carbon released from the operational activities associated with production system. In the model, high-tech product demand is thought to be influenced by green technology investment and selling price. Partial outsourcing is considered to offset the detrimental effects of imperfect manufacturing on customer service. The current study's goal is to increase the manufacturing company's profit as much as possible.

3.2. Research hypothesis

The necessary research hypothesis to develop a mathematical model for a producer are presented in this section. These are listed below:

(1) The mathematical model consists of a single manufacturer dealing in a single item. Imperfect manufacturing process is considered in the model.
(2) In-house reworking is considered, and it is assumed that a reworked item is perfect.
(3) A certain fraction of the production lot size is outsourced to prevent shortages because of imperfect manufacturing. These items are of the good quality and utilize to satisfy the customers demand during the production downtime. This helps to maintain the goodwill of manufacturer in front of customers [2,3].
(4) Carbon emissions tax is imposed by the government to mitigate the level of carbon emissions that takes place during the setup preparation for production process, manufacturing process, reworking process of imperfect items, transportation of outsourced products, deterioration process, and holding the products in stock. This help to reduce the adverse impact of greenhouse gases on the environment [11].
(5) The production system's operational activities have led to the manufacturer planning to invest in green technologies to reduce carbon emissions. Following is the proportion of carbon that has been reduced:

$$C(\varphi) = \alpha\left(1 - e^{-\beta\varphi}\right)$$

β is efficiency parameter and φ is the investment in green technology [3].
(6) Green investment and selling price can influence the demand of customers. Thus, demand is considered as follows:

$$D(\varphi, s) = D_0 + \alpha_1\varphi + \alpha_2\left(\frac{s_{\max} - s}{s - s_{\min}}\right)$$

where D_0 is base demand, $s_{\max}$ maximum selling price, $s_{\min}$ minimum selling price, α_1, α_2 are scaling parameters [16].

(7) Production rate is considered as variable which can be set within the following limit [57]:

$$P_{\min} \leq P \leq P_{\max}.$$

(8) Variable unit production costs are based on the price of raw materials, the cost of development, and the price of tools and dies. Unit production is as follows:

$$M(P) = \beta_1 + \frac{\beta_2}{P} + \beta_3 P$$

where β_1, β_2, and β_3 are the scaling parameters of raw material cost, development cost, and tool/die cost respectively [41].

(9) Inventory deteriorates because of its physical presence in the stock.

3.3. Notations

The necessary notations for developing a mathematical model for the manufacturer are contained in this section. These are listed below:

s	Selling price (\$/unit)
P	Production rate (unit/unit time)
C_T	Transportation cost because of outsourcing the products (\$/unit)
λ	Fraction of imperfect production
R	Reworking rate (unit/unit time)
S_c	Setup (\$/setup)
$M(P)$	$\left(= \beta_1 + \frac{\beta_2}{P} + \beta_3 P\right)$ Production cost (\$/unit)
$\beta_1 > 0$	Scaling parameters of raw material cost
$\beta_2 > 0$	Scaling parameters of development cost
$\beta_3 > 0$	Scaling parameters of tool/die cost
Q	Production lot size (units)
Q_1	Level of serviceable inventory on hand at the end of the production process (units)
Q_2	Level of serviceable inventory on hand at the end of the reworking process (units)
ρ	Outsourcing portion of the production lot size
$D(\varphi, s)$	$\left(= D_0 + \alpha_1\varphi + \alpha_2\left(\frac{s_{\max} - s}{s - s_{\min}}\right)\right)$ Demand rate (Unit/unit time)
D_0	Base demand
$s_{\min}$	Minimum selling price (\$/unit)
$s_{\max}$	Maximum selling price (\$/unit)
$\alpha_1, \alpha_2 > 0$	Scaling parameters
θ	Deterioration rate (unit/per unit time)
a_1	Connecting parameter between setup cost and constant outsourcing cost
a_2	Connecting parameter between production cost and variable outsourcing cost
C_R	Reworking cost (\$/unit)
φ	Investment in green technology (\$/per cycle)
C_H	Holding cost (\$/unit/per unit time)
C_D	Deterioration cost (\$/unit)
e_{ST}	Carbon emissions due to setup (Kg/setup)
e_{PP}	Carbon emissions due to production process (Kg/unit)
e_T	Carbon emissions due to transportation (Kg/unit)
e_{DP}	Carbon emissions due to deterioration (Kg/unit)
e_{RP}	Carbon emissions due to reworking (Kg/unit)
e_{CC}	Carbon emissions due to holding the inventory (Kg/unit)
σ	Carbon tax (\$/Kg)

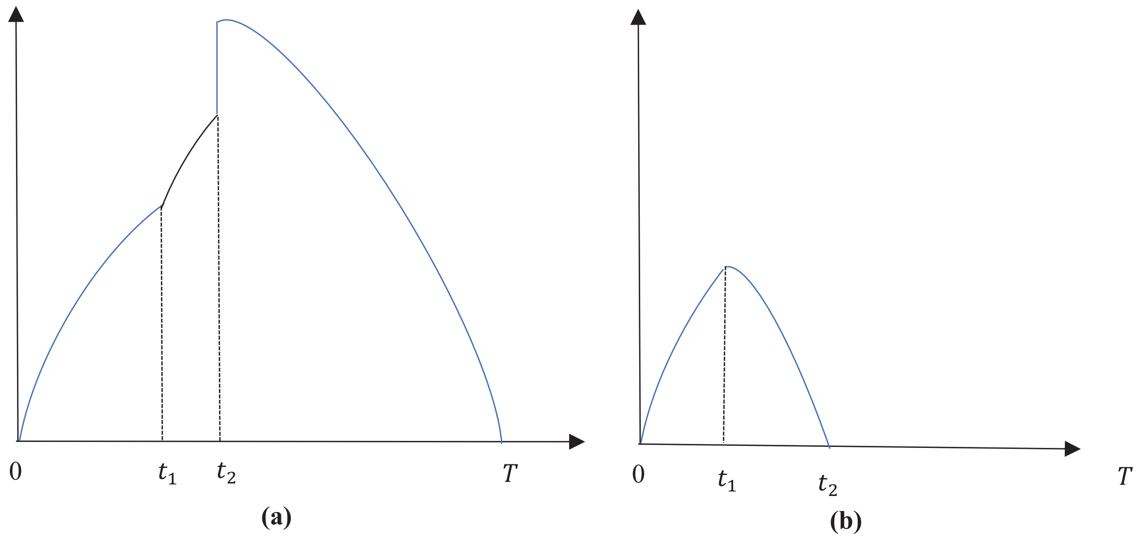

FIGURE 1. (a) Inventory level of serviceable item. (b) Inventory level of non-serviceable item.

4. Mathematical model of flexible manufacturing system (FMS)

A mathematical model for a flexible manufacturing system is developed in this section considering manufacturing process is not perfect. Manufactured products are divided into two categories: serviceable and non-serviceable. Items that are not serviceable are reworked to bring them up to par with perfect items. A certain fraction of the production lot size is outsourced to avoid shortages because of imperfect manufacturing process. Here, demand is dependent on green investment and selling prices. A deterioration process also takes place in the system because of the physical presence of inventory in stock. Different costs associated with the manufacturing system are also calculated along with the total carbon emissions from the manufacturing system. See Figures 1a and 1b for the graphical representation of inventory system.

During the period $[0, t_1]$, inventory levels increase due to production and decline due to demand and deterioration Mathematically, this can be illustrated as follows:

$$\frac{\mathrm{d}I_1(t)}{\mathrm{d}t} = (1-\lambda)P - \theta I_1(t) - D(\varphi, s), \qquad 0 \le t \le t_1, \quad I_1(0) = 0. \tag{1}$$

As the production process is not perfect, so during the period $[0, t_1]$ non-serviceable items pileup at the rate λP. At the end of t_1, production process stops and reworking process starts at the rate R which continue upto t_2. Thus, during $[0, t_2]$ inventory rises due to reworking and decline due to combined effect of demand and deterioration. Mathematically, it can be illustrated as follows:

$$\frac{\mathrm{d}I_2(t)}{\mathrm{d}t} = R - \theta I_2(t) - D(\varphi, s), \qquad t_1 \le t \le t_2, \quad I_1(t_1) = I_2(t_1). \tag{2}$$

At the end of t_2, outsourced items $(= \rho Q)$ are attached to the system. Thus, during $[t_2, T]$ inventory level drops due to demand and deterioration. Mathematically, it can be illustrated as follows:

$$\frac{\mathrm{d}I_3(t)}{\mathrm{d}t} = -\theta I_3(t) - D(\varphi, s), \qquad t_2 \le t \le T, \quad I_3(T) = 0. \tag{3}$$

Non-serviceable items stored at reworking station during $[0, t_1]$. Its inventory level can be represented mathematically as follows:

$$\frac{\mathrm{d}I_4(t)}{\mathrm{d}t} = \lambda P - \theta I_4(t), \qquad 0 \le t \le t_1, \quad I_4(0) = 0. \tag{4}$$

Reworking process takes place during $[t_1, t_2]$. Mathematically, it can be represented as follows:

$$\frac{\mathrm{d}I_5(t)}{\mathrm{d}t} = -R - \theta I_5(t), \qquad t_1 \le t \le t_2, \quad I_5(t_2) = 0. \tag{5}$$

On solving equations (1)–(5), we get

$$I_1(t) = \frac{(1-\lambda)P - D(\varphi,s)}{\theta}\left(1 - e^{-\theta t}\right), \qquad 0 \le t \le t_1 \tag{6}$$

$$I_2(t) = \frac{((1-\lambda)P - D(\varphi,s))\left(e^{-\theta(t-t_1)} - e^{-\theta t}\right)}{\theta} + \frac{(R - D(\varphi,s))\left(1 - e^{-\theta(t-t_1)}\right)}{\theta}, \qquad t_1 \le t \le t_2 \tag{7}$$

$$I_3(t) = \frac{D(\varphi,s)}{\theta}\left(e^{\theta(T-t)} - 1\right), \qquad t_2 \le t \le T \tag{8}$$

$$I_4(t) = \frac{\lambda P}{\theta}\left(1 - e^{-\theta t}\right), \qquad 0 \le t \le t_1 \tag{9}$$

$$I_5(t) = \frac{R}{\theta}\left(e^{\theta(t_2-t)} - 1\right), \qquad t_1 \le t \le t_2. \tag{10}$$

At the end of the production process, the level of serviceable inventory is

$$Q_1 = \frac{(1-\lambda)P - D(\varphi,s)}{\theta}\left(1 - e^{-\theta t_1}\right). \tag{11}$$

At the end of the reworking process, the level of serviceable items in stock is

$$Q_2 = \frac{((1-\lambda)P - D(\varphi,s))\left(e^{-\theta(t_2-t_1)} - e^{-\theta t_2}\right)}{\theta} + \frac{(R - D(\varphi,s))\left(1 - e^{-\theta(t_2-t_1)}\right)}{\theta}\cdot \tag{12}$$

On-hand inventory level of serviceable items after receiving the outsourced items is

$$Q_3 = Q_2 + \rho Q. \tag{13}$$

Production period is given by

$$t_1 = \frac{(1-\rho)Q}{P}\cdot \tag{14}$$

Reworking period is given by

$$t_2 - t_1 = \frac{I_4(t_1)}{R} = \frac{\lambda P}{R\theta}\left(1 - e^{-\theta t_1}\right) \text{ or } t_2 = t_1 + \frac{\lambda P}{R\theta}\left(1 - e^{-\theta t_1}\right). \tag{15}$$

Relation $I_3(t_2) = I_2(t_2) + \rho Q$, gives

$$T = t_2 + \log\left(1 + \frac{((1-\lambda)P - D(\varphi,s))\left(e^{-\theta(t_2-t_1)} - e^{-\theta t_2}\right)}{D(\varphi,s)} + \frac{(R - D(\varphi,s))\left(1 - e^{-\theta(t_2-t_1)}\right)}{D(\varphi,s)}\right)\cdot \tag{16}$$

4.1. Relevant production inventory cost

The goal of current research is to increase the profit connected to the production system after selling the serviceable and reworked items under environmental consideration. Therefore, first we evaluate the different costs associated with the production-inventory system.

Revenue

Production firms generate revenue after selling perfect items, reworked items, and outsourced items. Thus, revenue generated by the production firm is

$$\mathrm{RG} = sD(\varphi,s)T.$$

Production setup cost

To configure the machine for production purposes, an investment is made in the form of setup costs. Thus, the setup cost for the manufacturer is

$$\mathrm{PSC} = S_c.$$

Production cost

Here, it is assumed that the production rate affects the cost per unit of an item. Costs for raw materials, tools and dies, and development are included in production costs. Thus, the cost of production is

$$\mathrm{PC} = M(P)\int_0^{t_1} P\,\mathrm{d}t = M(P)Pt_1.$$

Outsourcing cost

A fixed component and a variable component make up the cost of outsourcing. Fixed component is some proportion of setup cost "S_c" and variable component is some proportion of "$M(P)$". Thus, outsourcing cost is

$$\mathrm{OC} = (1+a_1)S_c + (1+a_2)M(P)\rho Q.$$

Transportation cost

To transport the outsourced product, an additional cost is incurred in form of transportation cost. Thus, transportation cost is $\rho Q C_T$.

Reworking cost

Here, the production process is not perfect. So, produced items are bifurcated into serviceable items and non-serviceable items. Non-serviceable items are reworked and make good as serviceable items. Thus, reworking cost is

$$\mathrm{RC} = C_R\int_{t_1}^{t_2} R\,\mathrm{d}t = C_R R(t_2-t_1).$$

Green investment

The production firm is very concerned about the environment. In this model, green investment is considered to control the amount of carbon emitted. Thus, green investment is φ.

Holding cost

Carrying serviceable and non-serviceable objects that go through the reworking process results in holding expenses. Thus, holding cost is

$$\begin{aligned}
\mathrm{HC} &= C_H\left(\int_0^{t_1} I_1(t)\,\mathrm{d}t + \int_{t_1}^{t_2} I_2(t)\,\mathrm{d}t + \int_{t_2}^{T} I_3(t)\,\mathrm{d}t + \int_0^{t_1} I_4(t)\,\mathrm{d}t + \int_{t_1}^{t_2} I_5(t)\,\mathrm{d}t\right)\\
&= C_H\left\{\frac{(1-\lambda)P - D(\varphi,s)}{\theta}\left(t_1 + \frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right) + \frac{((1-\lambda)P - D(\varphi,s))}{\theta}\right.\\
&\quad\times\left(\frac{1}{\theta}\left(1-e^{-\theta(t_2-t_1)}\right) + \frac{1}{\theta}\left(e^{-\theta t_2}-e^{-\theta t_1}\right)\right) + \frac{(R-D(\varphi,s))}{\theta}\left(t_2 - t_1 + \frac{1}{\theta}\left(e^{-\theta(t_2-t_1)}-1\right)\right)\\
&\quad+\frac{D(\varphi,s)}{\theta}\left(\frac{1}{\theta}\left(e^{\theta(T-t_2)}-1\right) - (T-t_2)\right) + \frac{\lambda P}{\theta}\left(t_1 + \frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)\\
&\quad\left.+\frac{R}{\theta}\left(\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right) - (t_2-t_1)\right)\right\}.
\end{aligned}$$

Deterioration cost

Serviceable and non-serviceable products both deteriorate with time, which results in deterioration costs. Thus, the deterioration cost is

$$\begin{aligned}\mathrm{DC} &= C_D\left(\int_0^{t_1}\theta I_1(t)\,\mathrm{d}t+\int_{t_1}^{t_2}\theta I_2(t)\,\mathrm{d}t+\int_{t_2}^{T}\theta I_3(t)\,\mathrm{d}t+\int_0^{t_1}\theta I_4(t)\,\mathrm{d}t+\int_{t_1}^{t_2}\theta I_5(t)\,\mathrm{d}t\right)\\
&= C_D\Bigg\{((1-\lambda)P-D(\varphi,s))\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)+((1-\lambda)P-D(\varphi,s))\\
&\quad\times\left(\frac{1}{\theta}\left(1-e^{-\theta(t_2-t_1)}\right)+\frac{1}{\theta}\left(e^{-\theta t_2}-e^{-\theta t_1}\right)\right)+(R-D(\varphi,s))\left(t_2-t_1+\frac{1}{\theta}\left(e^{-\theta(t_2-t_1)}-1\right)\right)\\
&\quad+D(\varphi,s)\left(\frac{1}{\theta}\left(e^{\theta(T-t_2)}-1\right)-(T-t_2)\right)+\lambda P\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)\\
&\quad+R\left(\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)-(t_2-t_1)\right)\Bigg\}.\end{aligned}$$

4.2. Carbon emissions tax

As earlier stated, because of production activities like setup, production, reworking, carrying the stock, and deterioration, carbon emissions is unavoidable. Governments are very much concerned to control the carbon emissions. They adopted different mechanisms to mitigate the carbon emissions. Carbon tax is very common and effective mechanism in this direction. Therefore, to evaluate the carbon tax first we evaluate the unit of carbon emitted due to the above-mentioned process.

- Carbon emissions takes place to setup the machines for the production process. Thus, emitted unit of carbon because of setup of production process is e_{ST}.
- Similarly, due to production process carbon emissions also takes place. Thus, emitted unit of carbon because of production process is $e_{\mathrm{PP}}Pt_1$.
- An amount ρQ is outsourced by the manufacturer to satisfy the demand of the customers. To transport these items from the external source to the manufacturer, carbon emissions take place. Thus, emitted unit of carbon due to the transportation is $e_T\rho Q$.
- Certain proportion of produced items are defective which are reworked at the rate R. Thus, emitted unit of carbon due to the reworking process is $e_{\mathrm{RP}}R(t_2-t_1)$.
- Stocking the inventory in production process and reworking process causes carbon emissions. Thus, emitted unit of carbon because of carrying inventory is EC where

$$\begin{aligned}\mathrm{EC} = e_{\mathrm{CC}}\Bigg\{&\frac{(1-\lambda)P-D(\varphi,s)}{\theta}\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)+\frac{((1-\lambda)P-D(\varphi,s))}{\theta}\\
&\times\left(\frac{1}{\theta}\left(1-e^{-\theta(t_2-t_1)}\right)+\frac{1}{\theta}\left(e^{-\theta t_2}-e^{-\theta t_1}\right)\right)+\frac{(R-D(\varphi,s))}{\theta}\\
&\times\left(t_2-t_1+\frac{1}{\theta}\left(e^{-\theta(t_2-t_1)}-1\right)\right)+\frac{D(\varphi,s)}{\theta}\left(\frac{1}{\theta}\left(e^{\theta(T-t_2)}-1\right)-(T-t_2)\right)\\
&+\frac{\lambda P}{\theta}\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)+\frac{R}{\theta}\left(\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)-(t_2-t_1)\right)\Bigg\}.\end{aligned}$$

- Because of physical presence of stock, deterioration process occurs which results carbon emissions. Thus, emitted unit of carbon because of deterioration process is ED where

$$\mathrm{ED} = e_{\mathrm{DP}}\Bigg\{((1-\lambda)P-D(\varphi,s))\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)+((1-\lambda)P-D(\varphi,s))$$

$$\times\left(\frac{1}{\theta}\left(1-e^{-\theta(t_2-t_1)}\right)+\frac{1}{\theta}\left(e^{-\theta t_2}-e^{-\theta t_1}\right)\right)+(R-D(\varphi,s))\left(t_2-t_1+\frac{1}{\theta}\left(e^{-\theta(t_2-t_1)}-1\right)\right)$$
$$+D(\varphi,s)\left(\frac{1}{\theta}\left(e^{\theta(T-t_2)}-1\right)-(T-t_2)\right)+\lambda P\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)$$
$$+R\left(\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)-(t_2-t_1)\right)\Bigg\}.$$

In the current model, carbon tax policy is considered. Further, production firm makes investment in green technology to control the emitted unit of carbon. Thus, carbon tax is

$$\mathrm{CT}=\sigma\Bigg[e_{\mathrm{ST}}+e_T\rho Q+e_{\mathrm{PP}}Pt_1+e_{\mathrm{RP}}R(t_2-t_1)+e_{\mathrm{CC}}\Bigg\{\frac{(1-\lambda)P-D(\varphi,s)}{\theta}\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)$$
$$+\frac{((1-\lambda)P-D(\varphi,s))}{\theta}\left(\frac{1}{\theta}\left(1-e^{-\theta(t_2-t_1)}\right)+\frac{1}{\theta}\left(e^{-\theta t_2}-e^{-\theta t_1}\right)\right)+\frac{(R-D(\varphi,s))}{\theta}$$
$$\times\left(t_2-t_1+\frac{1}{\theta}\left(e^{-\theta(t_2-t_1)}-1\right)\right)+\frac{D(\varphi,s)}{\theta}\left(\frac{1}{\theta}\left(e^{\theta(T-t_2)}-1\right)-(T-t_2)\right)$$
$$+\frac{\lambda P}{\theta}\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)+\frac{R}{\theta}\left(\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)-(t_2-t_1)\right)\Bigg\}$$
$$\times e_{\mathrm{DP}}\Bigg\{((1-\lambda)P-D(\varphi,s))\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)+((1-\lambda)P-D(\varphi,s))$$
$$\times\left(\frac{1}{\theta}\left(1-e^{-\theta(t_2-t_1)}\right)+\frac{1}{\theta}\left(e^{-\theta t_2}-e^{-\theta t_1}\right)\right)+(R-D(\varphi,s))\left(t_2-t_1+\frac{1}{\theta}\left(e^{-\theta(t_2-t_1)}-1\right)\right)$$
$$+D(\varphi,s)\left(\frac{1}{\theta}\left(e^{\theta(T-t_2)}-1\right)-(T-t_2)\right)+\lambda P\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)$$
$$+R\left(\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)-(t_2-t_1)\right)\Bigg\}\Bigg]\left(1-\tau\left(1-e^{-\beta\emptyset}\right)\right).$$

Finally, we have

$$\text{Average Total profit}=\frac{1}{T}[\text{revenue}-\text{setup cost}-\text{production cost}-\text{outsourcing cost}-\text{reworking cost}$$
$$-\text{holding cost}-\text{deterioration cost}-\text{green investment}-\text{carbon tax}].$$

Therefore,

$$\mathrm{TP}(s,\varphi,P,Q)=\frac{1}{T}\Bigg[sD(\varphi,s)T-S_c-M(P)Pt_1-(1+a_1)K-(1+a_2)M(P)\rho Q-\rho QC_T-C_RR(t_2-t_1)-\varphi$$
$$-C_H\Bigg\{\frac{(1-\lambda)P-D(\varphi,s)}{\theta}\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)+\frac{((1-\lambda)P-D(\varphi,s))}{\theta}$$
$$\times\left(\frac{1}{\theta}\left(1-e^{-\theta(t_2-t_1)}\right)+\frac{1}{\theta}\left(e^{-\theta t_2}-e^{-\theta t_1}\right)\right)+\frac{(R-D(\varphi,s))}{\theta}\left(t_2-t_1+\frac{1}{\theta}\left(e^{-\theta(t_2-t_1)}-1\right)\right)$$
$$+\frac{D(\varphi,s)}{\theta}\left(\frac{1}{\theta}\left(e^{\theta(T-t_2)}-1\right)-(T-t_2)\right)+\frac{\lambda P}{\theta}\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)$$
$$+\frac{R}{\theta}\left(\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)-(t_2-t_1)\right)\Bigg\}-C_D\Bigg\{((1-\lambda)P-D(\varphi,s))\left(t_1+\frac{1}{\theta}\left(e^{-\theta t_1}-1\right)\right)$$
$$+((1-\lambda)P-D(\varphi,s))\left(\frac{1}{\theta}\left(1-e^{-\theta(t_2-t_1)}\right)+\frac{1}{\theta}\left(e^{-\theta t_2}-e^{-\theta t_1}\right)\right)+(R-D(\varphi,s))$$

$$\times\left(t_2-t_1+\frac{1}{\theta}\left(e^{-\theta(t_2-t_1)}-1\right)\right)+D(\varphi,s)\left(\frac{1}{\theta}\left(e^{\theta(T-t_2)}-1\right)-(T-t_2)\right)$$
$$+\lambda P\left(t_1+\frac{1}{\theta}(e^{-\theta t_1}-1)\right)+R\left(\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)-(t_2-t_1)\right)\Bigg\}-\sigma\Bigg[e_{\mathrm{ST}}+e_T\rho Q+e_{\mathrm{PP}}Pt_1$$
$$+e_{\mathrm{RP}}R(t_2-t_1)+e_{\mathrm{CC}}\Bigg\{\frac{(1-\lambda)P-D(\varphi,s)}{\theta}\left(t_1+\frac{1}{\theta}(e^{-\theta t_1}-1)\right)+\frac{((1-\lambda)P-D(\varphi,s))}{\theta}$$
$$\times\left(\frac{1}{\theta}\left(1-e^{-\theta(t_2-t_1)}\right)+\frac{1}{\theta}(e^{-\theta t_2}-e^{-\theta t_1})\right)+\frac{(R-D(\varphi,s))}{\theta}\left(t_2-t_1+\frac{1}{\theta}\left(e^{-\theta(t_2-t_1)}-1\right)\right)$$
$$+\frac{D(\varphi,s)}{\theta}\left(\frac{1}{\theta}\left(e^{\theta(T-t_2)}-1\right)-(T-t_2)\right)+\frac{\lambda P}{\theta}\left(t_1+\frac{1}{\theta}(e^{-\theta t_1}-1)\right)$$
$$+\frac{R}{\theta}\left(\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)-(t_2-t_1)\right)\Bigg\}+e_{\mathrm{DP}}\Bigg\{((1-\lambda)P-D(\varphi,s))\left(t_1+\frac{1}{\theta}(e^{-\theta t_1}-1)\right)$$
$$+((1-\lambda)P-D(\varphi,s))\left(\frac{1}{\theta}\left(1-e^{-\theta(t_2-t_1)}\right)+\frac{1}{\theta}(e^{-\theta t_2}-e^{-\theta t_1})\right)+(R-D(\varphi,s))$$
$$\times\left(t_2-t_1+\frac{1}{\theta}\left(e^{-\theta(t_2-t_1)}-1\right)\right)+D(\varphi,s)\left(\frac{1}{\theta}\left(e^{\theta(T-t_2)}-1\right)-(T-t_2)\right)$$
$$+\lambda P\left(t_1+\frac{1}{\theta}(e^{-\theta t_1}-1)\right)+R\left(\frac{1}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)-(t_2-t_1)\right)\Bigg\}\Bigg]\Bigg]\left(1-\tau\left(1-e^{-\beta\emptyset}\right)\right). \tag{17}$$

Objective of the problem

$$\begin{aligned}&\operatorname{Max}\mathrm{TP}(s,\varphi,P,Q)\\&s>0,\quad \varphi>0,\quad P>0,\quad Q>0.\end{aligned}$$

5. Solution methodology

Classical optimization technique cannot be applied directly to obtain the optimal solution of the inventory problem. Various researchers such as Treanţă [50], Treanţă *et al.* [52], Guo *et al.* [20], Ciontescu and Treanţă [10], and Treanţă and Ciontescu [51] explored various optimization techniques to obtain the optimal solution of the problem. Here, closed form solution is not possible because of non-linear expression in decision variables. Consider s,φ,P and Q are non-zero real numbers and such that there exist unique values of s,φ,P and Q for which optimal value of the profit function exists and satisfies the first order derivative conditions. These are as follows:

$$\frac{\partial \mathrm{TP}}{\partial s}=0, \tag{18}$$
$$\frac{\partial \mathrm{TP}}{\partial \varphi}=0, \tag{19}$$
$$\frac{\partial \mathrm{TP}}{\partial P}=0, \tag{20}$$
$$\text{and}\quad \frac{\partial \mathrm{TP}}{\partial Q}=0. \tag{21}$$

Search Algorithm

Step 1. Fix $\varphi=P=Q=0$, and find the value of s^* from equation (18).
Step 2. Fix $P=Q=0$, and used the value of s^* obtained in step-1 and find the value of φ^* from equation (19).

Step 3. Fix $Q = 0$, and used the value of s^*, φ^* obtained in step-1 and step-2 respectively and find the value of P^* from equation (20).

Step 4. Used the value of s^*, φ^*, P^* obtained in step-1, step-2 and step-3 respectively and find the value of Q from equation (21).

Step 5. Repeat the step-1 to step-4 till the revised values of decision variables are not same to the two decimal places to the previous values.

Step 6. Find value of different principal minors of the Hessian Matrix at $(s^*, \varphi^*, P^*, Q^*)$

$$\begin{bmatrix} \frac{\partial^2 \mathrm{TP}}{\partial s^2} & \frac{\partial^2 \mathrm{TP}}{\partial s \partial \varphi} & \frac{\partial^2 \mathrm{TP}}{\partial s \partial P} & \frac{\partial^2 \mathrm{TP}}{\partial s \partial Q} \\ \frac{\partial^2 \mathrm{TP}}{\partial \varphi \partial s} & \frac{\partial^2 \mathrm{TP}}{\partial \varphi^2} & \frac{\partial^2 \mathrm{TP}}{\partial \varphi \partial P} & \frac{\partial^2 \mathrm{TP}}{\partial \varphi \partial Q} \\ \frac{\partial^2 \mathrm{TP}}{\partial P \partial s} & \frac{\partial^2 \mathrm{TP}}{\partial P \partial \varphi} & \frac{\partial^2 \mathrm{TP}}{\partial P^2} & \frac{\partial^2 \mathrm{TP}}{\partial P \partial Q} \\ \frac{\partial^2 \mathrm{TP}}{\partial Q \partial s} & \frac{\partial^2 \mathrm{TP}}{\partial Q \partial \varphi} & \frac{\partial^2 \mathrm{TP}}{\partial Q \partial P} & \frac{\partial^2 \mathrm{TP}}{\partial Q^2} \end{bmatrix}.$$

Step 7. Get the value of TP at $(s^*, \varphi^*, P^*, Q^*)$. This is the optimal solution of the manufacturing firm.

6. Numerical analysis

To validate the developed model, data has been collected from an electronic industry ABC (The name is given ABC as per the agreement with the company as company do not want to publish the details) produces high-tech products. As production process is not perfect, so company outsourced certain fraction of products to satisfy the demand during downtime of production process. Because of environment friendly nature, manufacturing industry investing in green technology and ready to pay carbon tax on the emitted of carbon. Further, to reduce the waste from the production system, remanufacturing is considered by them. Above-mentioned industry provided the following data for better research insights.

$R = 330$ (units/year), $\lambda = 0.09$, $K = 4998$ (units/setup), $\beta_1 = 290$, $\beta_2 = 11710$, $\beta_3 = 0.009$, $\rho = 0.05$, $D_0 = 46$, $\alpha_1 = 0.6$, $\alpha_2 = 0.2$, $s_{\max} = 9000$ (\$/unit), $s_{\min} = 5000$ (\$/unit), $\theta = 0.15$, $a_1 = -0.3$, $a_2 = 0.3$, $C_R = 48$ (\$/unit), $\beta = 0.002$, $C_H = 15$ (\$/unit), $C_D = 4.7$ (\$/unit), $e_{\mathrm{ST}} = 100$ (Kg/unit), $e_{\mathrm{PP}} = 5$ (Kg/unit), $e_{\mathrm{RP}} = 20$ (Kg/unit), $e_{\mathrm{CC}} = 0.6$ (Kg/unit), $e_{\mathrm{DP}} = 0.3$ (Kg/unit), $e_T = 0.1$ (Kg/unit), $\sigma = 5.29$ (\$/Kg).

On applying the said algorithm to obtain the optimal solution, required result is as follows:

Table 2. Optimal solution.

P^* (unit/year)	Q^* (unit/)	s^* (\$/unit)	φ^* (\$/year)	TP* (\$/year)
3672.34	7581.34	8976.81	473.17	1.09486×10^7

Remark. No work is reported in the literature focusing the issues of environment, society, and economy simultaneously as focused in the current work. So, it is not possible to compare the current work quantitively with the work reported in the literature. By relaxing some of the assumptions in the current work, it is observed that current work and some previously quoted works in literature have same base model.

(1) If we ignore the assumption of deterioration and consider only selling price dependent demand and more important on neglecting the environment issue, then the base of current model reduces to Bachar *et al.* [3].
(2) If we consider perfect production system, no deterioration, time dependent demand, no reworking, and no outsourcing, then the base of current model reduces to Vandana *et al.* [53].

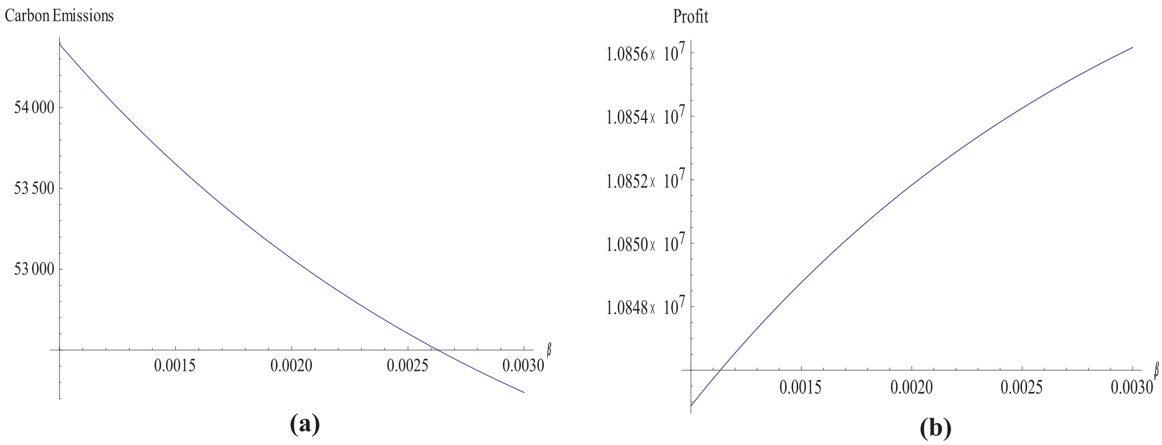

FIGURE 2. (a) Effect of β on level of carbon emissions. (b) Effect of β on profit of inventory system.

(3) If we consider no deterioration, no smart production system, and no environment issue, then the base of current model reduces to Bachar *et al.* [2].

(4) If we consider no deterioration, constant demand, no smart production, and no environment issue, then the base of current model reduces to Alkahtani [1].

6.1. Sensitivity analysis

Objective of this section is to explore the effect of key parameters on the decision-making process of inventory problem.

6.1.1. Effect of efficiency coefficient of carbon reduction technology

Figures 2a and 2b show how the profitability of the inventory system and the quantity of carbon emissions caused by various operational activities linked with the production system are affected by the efficiency coefficient of carbon reduction technology. It is observed that an increase in β leads to a reduction in carbon emissions, which leads to a reduction in carbon tax. With the appropriate selection of carbon reduction technology, the system's overall profit rises and the goal of sustainability can be reached.

6.1.2. Effect of tax imposed by regulatory bodies

Figures 3a and 3b show how the tax levied by the government governed regulatory bodies affected the total tax paid by the manufacturer and the system's overall profit. A higher tax rate lower down the inventory system's profit. As a result, the company moves towards the technology, which lowers carbon emissions from the operational activities associated with the system.

6.1.3. Effect of scaling parameters of demand

The effects of elasticity parameter of investment green technology on system's demand and profit are depicted in Figures 4a and 4b. It demonstrates that demand and, thus system profit rise as the elasticity coefficient increases. The demand and profit of the inventory system are seen to be sharply skewed. The effects of selling price elasticity on demand and profit are depicted in Figures 4c and 4d. Demand and profit are both benefited by the increase in selling price elasticity. According to Figures 4a–4d, investment elasticity in green technology has more effect on demand and profit in comparison to selling price elasticity. This is because buyers prioritize environmental sustainability. These results indicate that if the decision-maker make suitable decision based on the effect of elasticity parameter of green investment, and price elasticity parameter then the profit of the system increases.

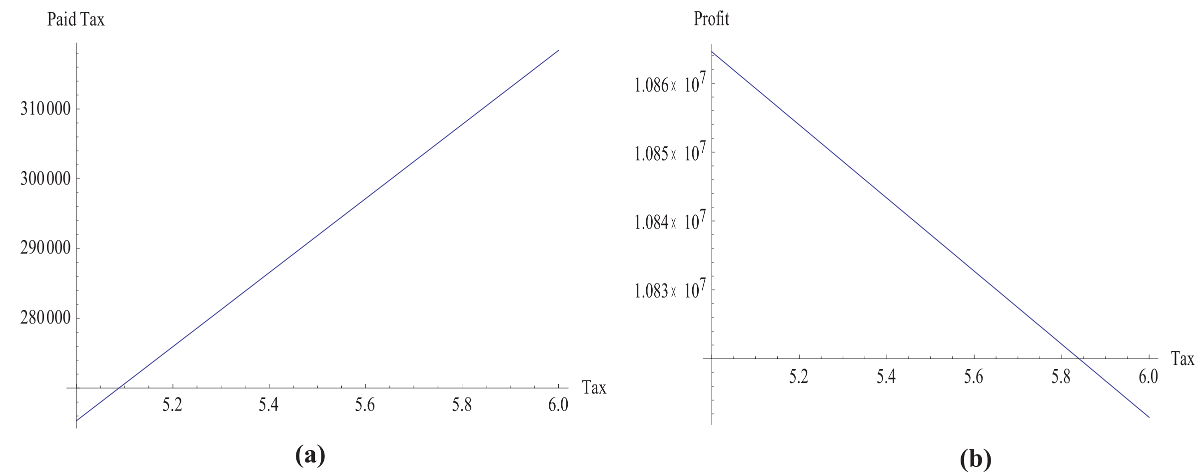

FIGURE 3. (a) Effect of tax on amount of tax paid. (b) Effect of tax on profit of inventory system.

6.1.4. Effect of different components of production cost

The impact of various components on the system's production costs and profit is shown in Figures 5a–5f. It is observed that production cost is positively correlated with different components, and profit is negatively correlated with them. Share decline is observed in profit due to increase in b_1 in comparison to other components. Production cost is also highly sensible with respect to b_1 also. Production cost and profit are least affected by b_2 out of three components. Results indicate that selection of vendor to get the raw material is very material to earn more profit for the system.

6.1.5. Effect of outsourced quantity

Figures 6a and 6b are reflecting the effect of ρ on the outsourced quantity and on the profit. It is observed that as the value of ρ increases, outsourced quantity increases whereas total profit of the system decreases. For the decision maker, it is important to balance between the inhouse production quantity and the outsourced quantity to earn more profit for the system.

6.1.6. Sensitivity Analysis with respect to various costs

(1) According to Table 3, production rate, production lot, and profit all decline due to rise in holding cost. The optimal selling price and investment in green technology show very little variation due to the variation in holding cost
(2) According to Table 3, the system's overall profit declines as the cost of deterioration rises. The optimal values of production rate, production lot, selling price, and green investment did not change significantly over time. Deterioration cost consequently has a detrimental effect on the system's profit.
(3) From Table 3, it can be shown that when the system's rework cost increases, the inventory system's profit declines. Due to the change in the reworking cost, no discernible change in other variables is seen.
(4) Table 3 shows that as setup cost rises, system's overall profit declines as a result. Hence, setup cost of the production system has a negative correlation with the system's profit.

6.1.7. Effect of deterioration rate

The impact of deterioration rate on the optimal solution to the inventory problem is seen in Table 4. It has been noted that when the rate of deterioration rises, the production rate, production lot, and selling price of the product rise to satisfy the demand. The profitability of the manufacturing system declines because of this deterioration. The increase in degradation results in a high shift in profit.

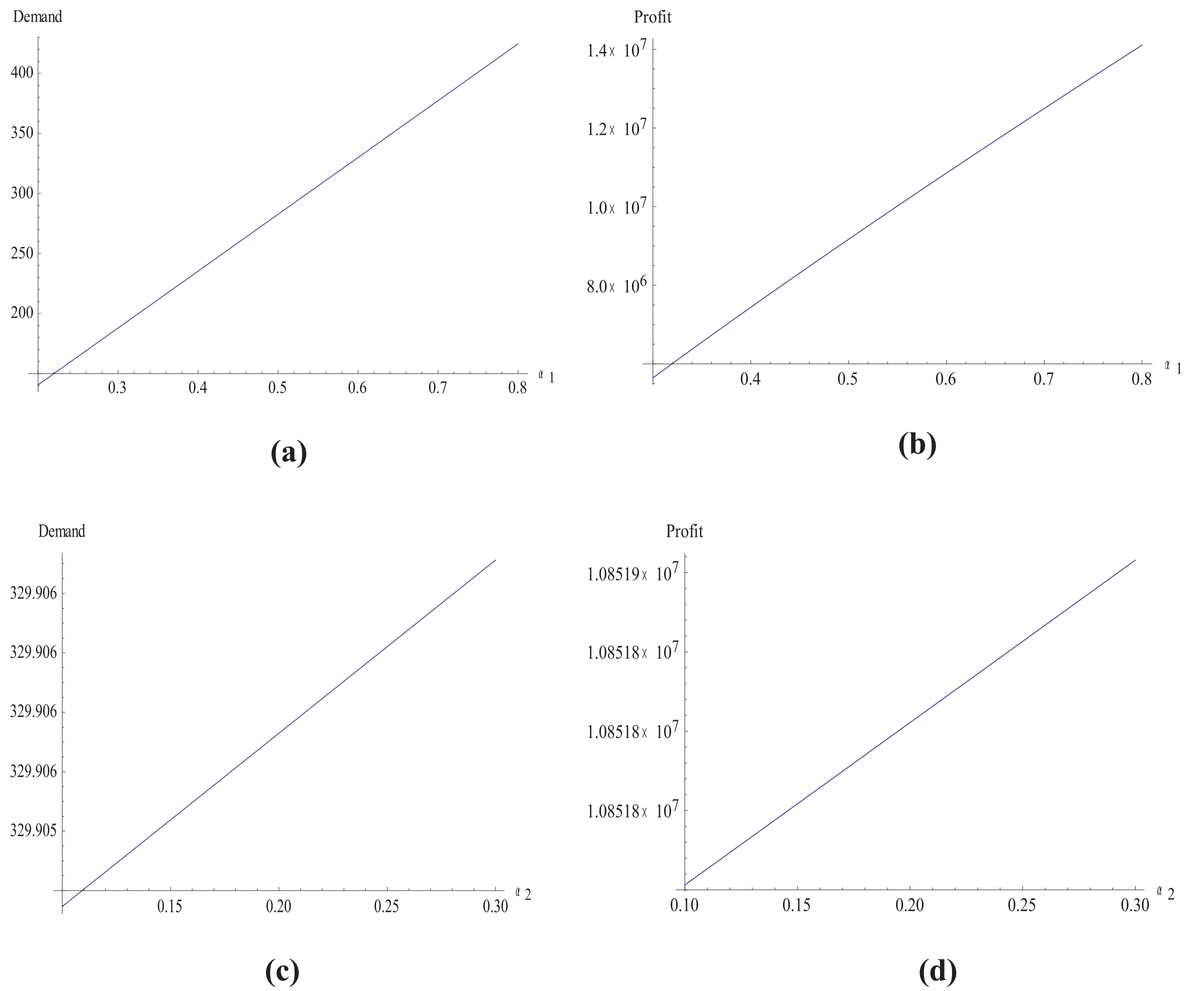

FIGURE 4. (a) Effect of green technology investment elasticity on demand. (b) Effect of green technology investment elasticity on profit. (c) Effect of selling price elasticity on demand. (d) Effect of selling price elasticity on profit.

6.1.8. Effect of fixed component and variable component of outsourcing cost

Table 5 shows the impact of outsourcing fixed cost and variable components on the optimal solution to the inventory problem. The system's profit is adversely affected by both components of outsourcing cost. However, compared to the fixed component, the variable component of outsourcing expenses has a more detrimental effect.

7. MANAGERIAL INSIGHTS

Major recommendations to improve the customers' satisfaction and to achieve the objective of environment sustainability are as follows:

(1) The manufacturing industry management must strike a balance between the quantity of product that is outsourced and client satisfaction. Lower profit for the system is the outcome of increased outsourcing volume. Therefore, the manager only outsourced the necessary quantity to maintain market reputation.

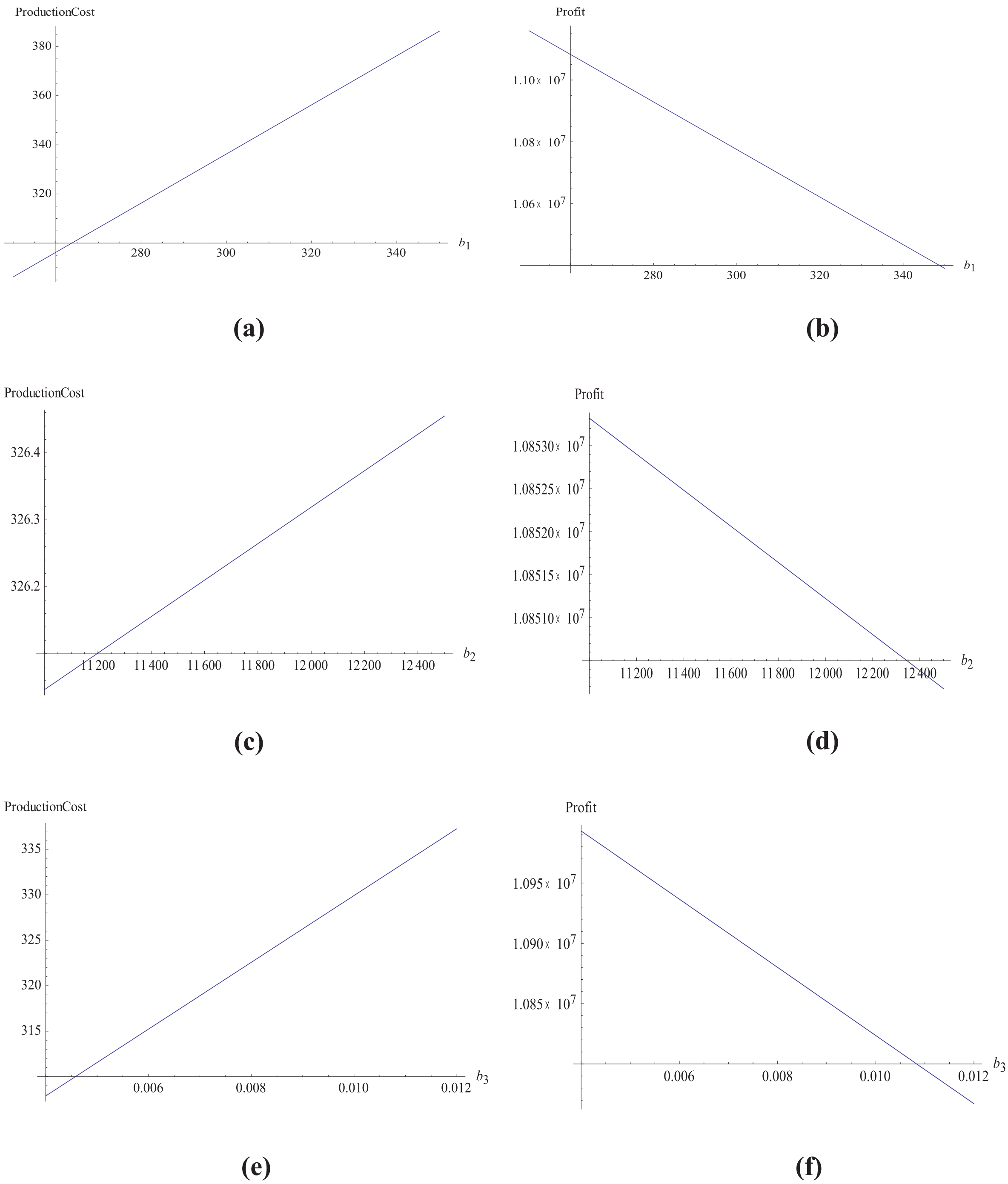

FIGURE 5. (a) Effect of b_1 on production cost. (b) Effect of b_1 on profit. (c) Effect of b_2 on production cost. (d) Effect of b_2 on profit. (e) Effect of b_3 on demand. (f) Effect of b_3 on profit.

TABLE 3. Sensitivity analysis with respect to cost parameters.

Parameter	% Change	P^*	Q^*	s^*	φ^*	TP*
C_h	−40	3930.30	7781.59	8996.96	473.332	1.09679×10^7
	−20	3930.30	7781.59	8996.96	473.332	1.09509×10^7
	20	3672.34	7581.35	8976.81	473.174	1.09060×10^7
	40	3672.34	7581.35	8976.81	473.174	1.08632×10^7
C_d	−40	3672.34	7581.35	8976.81	473.174	1.09527×10^7
	−20	3672.34	7581.35	8976.34	473.174	1.09507×10^7
	20	3672.34	7581.35	8976.81	473.174	1.09467×10^7
	40	3672.34	7581.35	8976.81	473.174	1.09447×10^7
C_R	−40	3930.30	7781.59	8996.96	473.332	1.09603×10^7
	−20	3672.34	7581.35	8976.81	473.174	1.09541×10^7
	20	3672.34	7581.35	8976.81	473.174	1.09433×10^7
	40	3672.34	7581.35	8976.81	473.174	1.09379×10^7
S_c	−40	3672.34	7581.35	8976.81	473.174	1.09521×10^7
	−20	3672.34	7581.35	8976.34	473.174	1.09504×10^7
	20	3672.34	7581.35	8976.81	473.174	1.09470×10^7
	40	3672.34	7581.35	8976.81	473.174	1.09453×10^7

TABLE 4. Sensitivity analysis with deterioration.

Parameter	% Change	P^*	Q^*	s^*	φ^*	TP*
θ	−40	3366.64	7340.36	8969.67	473.076	1.10185×10^7
	−20	3679.96	7581.66	8981.30	473.277	1.09618×10^7
	20	3930.30	7781.59	8996.96	473.332	1.08016×10^7
	40	3997.68	7802.02	8999.08	473.349	1.07004×10^7

TABLE 5. Sensitivity analysis with respect to different components of outsourcing cost.

Parameter	% Change	P^*	Q^*	s^*	φ^*	TP*
a_1	−40	3672.34	7581.35	8976.81	473.174	1.09499×10^7
	−20	3672.34	7581.35	8976.81	473.174	1.09490×10^7
	20	3672.34	7581.35	8976.81	473.174	1.09480×10^7
	40	3672.34	7581.35	8976.81	473.174	1.09476×10^7
a_2	−40	3930.30	7781.59	8976.81	473.332	1.09589×10^7
	−20	3930.30	7781.59	8976.81	473.332	1.09501×10^7
	20	3672.34	7581.35	8976.81	473.332	1.09428×10^7
	40	3672.34	7581.35	8976.81	473.332	1.09367×10^7

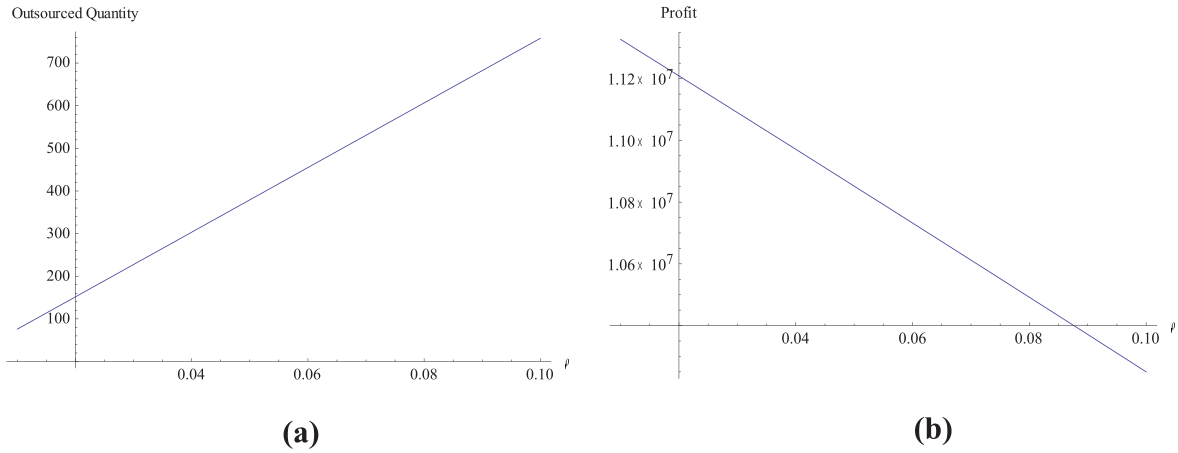

FIGURE 6. (a) ρ *vs.* outsourced quantity. (b) ρ *vs.* profit.

(2) To slow down the rate of deterioration, management in the manufacturing sectors must invest in preservation technologies or other tools. The objectives of financial and environmental sustainability are aided by this.
(3) The manager of the manufacturing sector learns more about smart production systems for the automatic screening process and to cut production costs as much as feasible. Because the variable component of the outsourcing cost is production-dependent, this has a detrimental impact on the system's profit.
(4) Managers of manufacturing companies must look into effective strategies to reduce carbon emissions utilizing green technologies. This is both economically feasible and helpful in reducing environmental carbon emissions.
(5) A manufacturing company's manager must concentrate on a system improvement program since the manufacturing of defective items and the costs associated with reworking them both have an adverse effect on the environment.

8. Concluding remarks, research limitations, and future extension

Due to the general trend toward environmental protection, manufacturing companies are now giving carbon emission-related issues their highest priority. The main source of greenhouse gas emissions is manufacturing. Additionally, it is impossible to avoid imperfect production process. To achieve the goal of a cleaner manufacturing system as a result is extremely difficult for the manufacturer. In the situation of imperfect manufacturing process, the manufacturer faces a significant task in ensuring consumer satisfaction. Therefore, the goal of the current study is to obtain the solution of all these practical challenges in the manufacturing system. In this work, a smart manufacturing system is considered assuming imperfect production environments. The idea behind product outsourcing is to meet consumer demand as the production system in not perfect. Reworking is intended to make the production process more orderly. Selling price and financial investment in green technology have been reported to be related to consumer demand for smart products. The model also takes a carbon tax into account to reduce carbon emissions.

The quantitative research demonstrates that investing in efficient green technology reduces carbon emissions while also boosting system profitability. With more customers becoming environmentally conscious, this also serves to raise demand. Controlling the selling price boosts both the demand and the manufacturers' profit. The outcome demonstrates that outsourcing is not economically feasible but is beneficial for customer satisfaction, particularly in situations when manufacturing procedures are substandard. As the variable component of outsourcing cost depends on manufacturing cost, the study contends that manufacturers must devise new strategies to lower manufacturing costs. The manufacturing company's earnings is negatively impacted by the deterioration rate, reworking cost, and carrying cost

Research limitations, and future extension

There are various research limitations of the current study. One of the limitations is that deterioration rate has been considered constant. Therefore, current study can be extended by considering variable deterioration rate which can be control with the help of preservation technology. Nowadays, industries are moving from offline system to online system because of technical development and the COVID-19 situation which is not discussed in current research work. Therefore, in future, demand can be considered a function of the selling price of offline and online channels [46]. Further, in current research effect of promotional actions for smart products can be discussed [28]. Practical utility of the proposed model can be enhanced by considering the effect of learning and forgetting in the setup cost (Yadav *et al.*, [56]). Other mechanisms, such as carbon caps, carbon cap-and-trade, etc., might be taken into consideration to reduce carbon emissions to make the model sustainable (Yadav *et al.*, [55]).

Data availability statement

No new data/codes were created or analyzed in this study.

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