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AN INTERVAL-VALUED PYTHAGOREAN TRAPEZOIDAL FUZZY MULTI-CRITERIA DECISION MAKING TECHNIQUE FOR PSYCHIATRIC DISORDER DIAGNOSIS

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Abstract. Despite the abundance of reports of mental disorders in medical diagnosis, few studies have employed standard techniques on representative patient groups. Psychiatric disorders are currently the most frequent cause of extended absences due to illness. Fuzzy Multi-Criteria Decision-Making (MCDM) techniques can be helpful in selecting appropriate treatments or interventions for patients with psychiatric disorders. These techniques enable decision-makers to consider multiple criteria that may have varying levels of importance or uncertainty. In this paper, interval-valued Pythagorean trapezoidal fuzzy numbers (IVPTrFN) are used to handle uncertainty or imprecision in a more complex manner. The Fuzzy Analytic Hierarchy Process (FAHP) is used to determine the symptom weight of a patient with a psychiatric disorder. Using this weight, the rank of the alternative disorder of a patient is determined by the Fuzzy Preference Ranking Organization Method for Enrichment Evaluation (FPROMETHEE) technique. Finally, sensitivity and comparative analyses are conducted to assess the reliability and consistency of the results.

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1. Introduction

Psychiatric disorders, also referred to as mental health disorders, are a broad category of illnesses that impact an individual's thoughts, feelings, actions, and general state of health [19]. Richard Mayou *et al.* [66] proposed that these illnesses often have a significant influence on a person's quality of life and can be just as crippling as physical ailments. Because mental illnesses have a substantial impact on a patient's general health and well-being, medical professionals must identify and diagnose them. Mood disorders like bipolar disorder

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and depression, anxiety disorders like post-traumatic stress disorder and generalised anxiety disorder, psychotic disorders like schizophrenia, and personality disorders like borderline personality disorder are all included in the broad category of psychiatric disorders [53]. These health conditions are frequently complicated, and objective medical testing is not the only method used to diagnose them. Psychiatric diseases, on the other hand, are usually diagnosed by clinical evaluation, which entails a thorough assessment of a patient's emotional and mental health. The International Classification of Diseases (ICD-10 or ICD-11) and the Diagnostic and Statistical Manual of Mental Disorders (DSM-5) are the main resources used by medical practitioners to categorize and diagnose psychiatric diseases. These classification schemes provide mental health practitioners with a consistent vocabulary and structure for identifying and categorising different illnesses according to specific standards and symptoms [35,52]. Sabry and Vohra [92] described the Role of Islam in the management of psychiatric disorders. Castrén [22] identified neurotrophins on psychiatric disorders and how important this is to the development of the adult brain's neural plasticity and network connections, and how new research connects these concepts to the diagnosis and treatment of mood disorders. Scangos *et al.* [95] proposed new and emerging approaches to treat psychiatric disorders. But as far as we are aware, no research has been done on the application of interval-valued Pythagorean trapezoidal based Multi-Criteria Decision-Making (MCDM) technique [41] for mental disorder patients. So, in this article, we address this problem and employ particular MCDM strategies, such as the Fuzzy Analytic Hierarchy Process (FAHP) [81] and the Fuzzy Preference Ranking Organization Method for Enrichment Evaluation (FPROMETHEE) [46], to recognize a patient's mental illness.

In-depth interviews with patients, discussions of their medical history, and behavioural and emotional observations are all parts of the diagnostic procedure for psychiatric diseases. To ensure an accurate diagnosis, collaboration with other medical specialists – such as psychologists and psychiatrists – is often essential. To aid in diagnosis, medical professionals may also utilise a range of psychological tests and screening instruments. It is crucial to stress that mental illnesses are more than just character defects or personal weaknesses. They can be caused by genetic, environmental, or psychological variables and have a biological and neurological foundation. To reduce suffering and improve a patient's overall health, an accurate diagnosis and appropriate treatment are essential. Furthermore, the connection between mental and physical health is becoming more widely acknowledged because untreated psychiatric diseases can either aggravate or cause a variety of physical illnesses.

An interval-valued Pythagorean trapezoidal fuzzy number (IVPTrFN) [90] is a type of fuzzy number that extends the concept of fuzzy numbers to handle uncertainty or imprecision in a more complex manner. It combines aspects of interval-valued fuzzy numbers [65], Pythagorean fuzzy numbers [68] and trapezoidal fuzzy numbers [90]. An interval-valued Pythagorean trapezoidal fuzzy number (IVPTrFN) combines aspects of these three types of fuzzy numbers to represent uncertainty and imprecision in a more sophisticated way, especially in decision-making or modelling where both the degree of membership and non-membership functions and the range of possible values are crucial. The IVPTRFN is an extension of the interval-valued intuitionistic trapezoidal fuzzy number (IVITrFN), where two membership functions satisfy the Pythagorean formula. The advantages of IVPTrFN in comparison with IVITrFN or any other fuzzy numbers are expressed in the following way:

(a) IVITrFN and TrIFN have very different abilities to represent uncertainty. The greatest membership degree and the least non-membership degree are expressed using intervals in the first case and specific real numbers in the second case.
(b) The main difference between these two fuzzy numbers is their sum of membership and non-membership functions with boundedness. The constraint condition of IVITrFN is $0 \leq \tilde{\mu}_I^U(x) + \tilde{\nu}_I^U(x) \leq 1$, whereas the constraint condition of IVPTrFN is $0 \leq (\tilde{\mu}_P^U(x))^2 + (\tilde{\nu}_P^U(x))^2 \leq 1$.
(c) The measure of uncertainty of the IVPTrFN is more rigid in its representation than IVITrFN because the space of membership grade of IVPTrFN is greater than the membership grade of IVITrFN.
(d) The IVITrFN must belong to IVPTrFN, while every IVPTrFN may not be in IVITrFN.
(e) Compared with IVPTrFN and IVITrFN, IVITrFN is less flexible in expressing membership and non-membership degrees, which means that the handling capacity of the IVPTrFN is greater than IVITrFN.

In this study, we demonstrate how the MCDM technique can be applied to select a disease disorder in a mental patient. It's important to note that while fuzzy MCDM techniques offer a systematic approach to decision-making, they rely on the accuracy of input data and the expertise of decision-makers. Additionally, the final decision should be made in consultation with healthcare professionals, taking into account the individual patient's needs and circumstances. The healthcare professionals provide their data in linguistic terms, and we determine the symptom weights using the Fuzzy AHP methodology. Based on the weights of the criteria, we rank the various types of psychiatric disorders of a patient using the fuzzy PROMETHEE method. The main contributions of our study are as follows:

(1) We introduce a novel technique for diagnosing psychiatric disorders, determining the specific disorder a patient is suffering from. Furthermore, we employ interval-valued Pythagorean trapezoidal fuzzy numbers (IVPTFN) to handle uncertainty in the diagnostic process.
(2) To identify the affected disorder, we apply two multi-criteria decision-making (MCDM) techniques: Fuzzy Analytic Hierarchy Process (FAHP) for determining symptom weights and Fuzzy Preference Ranking Organization Method for Enrichment Evaluations (FPROMETHEE) for optimizing the selection of the most likely disorder.
(3) We conduct a sensitivity analysis to verify the consistency and reliability of the proposed approach. Furthermore, we evaluate the flexibility of the model through a comparative analysis of various MCDM methods, including the PROMETHEE approach.
(4) We present a case study demonstrating the effectiveness of our proposed technique.

1.1. Motivation of this study

The motivation behind studying and treating patients with psychiatric disorders revolves around improving the lives of affected individuals, reducing suffering and societal impact, advancing scientific knowledge, and promoting a more compassionate and inclusive society that values mental health as an integral component of overall well-being. We demonstrate how to use mathematics to diagnose and treat patients with psychiatric disorders. Psychiatric diagnosis and treatment selection are inherently complex due to the presence of subjectivity, uncertainty, and overlapping symptoms in mental health disorders. Traditional diagnostic approaches often rely on qualitative assessments, leading to inconsistencies among clinicians. Furthermore, treatment selection is challenging due to individual variability in patient responses and the presence of comorbid conditions.

To address these challenges, fuzzy multi-criteria decision-making (MCDM) techniques provide a systematic and mathematically grounded approach to handling uncertainty in psychiatric evaluations. By allowing for partial truth values rather than rigid binary classifications, fuzzy MCDM better reflects the gradual and uncertain nature of psychiatric symptoms. This method enables more accurate diagnoses, personalized treatment recommendations and optimized clinical decision-making, ultimately leading to better patient outcomes and reducing the risk of misdiagnosis or ineffective treatment.

1.2. Research outline

Based on the motivation stated above, the primary outlines of our research are as follows:

(1) To select among the many disorders that have been significantly identified and are characterised by different overlapping symptoms when choosing a patient with a psychiatric disorder and determine the symptoms of some types of mental disorders and discuss them.
(2) To develop a comparative matrix based on criteria as symptoms and alternative disorder using Interval-valued Pythagorean trapezoidal fuzzy number (IVPTrFN) by the decision makers (DMs) to convey all the fuzziness, hesitation and uncertainty. Furthermore, aggregate all the ratings given by all decision-makers and convert them to a single matrix. Then, the data will be defuzzified to a crisp value to facilitate the solution.

(3) To determine the criteria weight, utilising the MCDM method FAHP. The weights indicate which symptoms are more serious and important than others. Additionally, obtain the rank of the alternative psychiatric disorders by the Fuzzy PROMETHEE method and determine the actual disorder the patient has.
(4) To perform sensitivity and comparison analysis in order to ensure that the outcomes are impartial and non-ambiguous. It demonstrates the internal consistency of the outcomes.

The paper is structured as follows: Section 1 contains the introduction and motivation of this study. The literature review on methodology and application is covered in Section 2. Sections 3 and 4 contain the preliminaries of mathematical tools and MCDM methodologies, respectively. Section 5 describes the symptoms of psychiatric disorders as criteria and the different types of psychiatric disorders as alternatives, respectively. Next, data collection and numerical description are covered in Sections 6 and 7, respectively. Sensitivity analysis and comparative study are provided in Section 8. Section 9 discusses the conclusions and future research scope based on the evaluated results.

2. Literature review

A literature review on medical diagnosis involves a comprehensive examination and analysis of existing research, scholarly articles, books, and other sources related to various aspects of medical diagnosis. This review aims to identify, summarise, evaluate and synthesise the available information on the topic.

2.1. Literature review on medical diagnosis

For the past several years, almost everyone has been insured by the health insurance network. Akdag *et al.* [5] improved the quality of evaluating hospital services. Peijia *et al.* [84] developed an intuitionistic fuzzy MCDM thermodynamic technique to support the hierarchical medical system. Ejegwa [33] described the approach of a max–min–max composite relation for Pythagorean fuzzy sets, improved upon it and applied it to medical diagnosis problems. After that, Naeem *et al.* [79] proposed smart health technologies that can offer efficient healthcare services like robotics in treatment and cure, artificial intelligence help for physicians, enormous data personalization of medicines, etc. Momena *et al.* [71] introduced the use of the generalised dual hesitant hexagonal fuzzy MCDM approach to prediagnose diseases based on symptoms. Joudar *et al.* [51] depicted an intelligent triage approach based on integrated fuzzy multi-criteria decision-making techniques for early identification of autism spectrum disorder (ASD). Perez-Aguilar *et al.* [85] estimated the priority level of interventions for Coronavirus disease 2019 (COVID-19) patients admitted to the emergency department.

To identify and categorise diseases or health conditions in a methodical and scientific manner, medical diagnosis [71] in research involves the use of a variety of techniques. Staab *et al.* [103] investigated the observation and diagnosis of psychiatric disorders in the primary stage, and Sun *et al.* [105] utilised artificial intelligence (AI) in the direct diagnosis and treatment of psychiatric patients. Furthermore, Chang *et al.* [24] detected and validated different types of crucial psychiatric disorders based on the frontal–posterior functional imbalance using deep learning technology. In the recent era, Stein *et al.* [104] analysed psychiatric disorders and their medical care through the paradigm shifts *vs.* incremental integration and Mullins *et al.* [77] examined the risk factors and suicide tendency among psychiatric disorder patients and the structure of their genes. Researchers use a range of techniques to gather data, analyze patterns and draw conclusions that help with disease understanding [95], prevention [106] and treatment [110]. Table 1 presents various methodologies used in medical diagnosis.

2.2. Literature on fuzzy numbers and their extensions

A brief disconcertion of the literature on fuzzy sets and their extensions is disclosed here. Lotfi A. Zadeh invented the fuzzy set [119] in 1965 and applied it in numerous fields for its capacity to capture ambiguity and hesitancy. Moslem [76] analysed the commuters' travel mode choice considering the Z-numbers in MCDM based Best Worst Method (BWM). Furthermore, Khan *et al.* [54] examined the safety of Dublin's bike-sharing process by using the intuitionistic fuzzy based aggregation operators. This study explores the Interval-valued

TABLE 1. Some studies on the topic of the medical diagnosis field.

Authors	Year	Optimization method	Application in medical diagnoses area
Parkash *et al.* [83]	2017	Max–min product	Fuzzy divergence measure on medical diagnosis
Guleria *et al.* [41]	2019	MCDM	Disease recognition
Zarandi *et al.* [120]	2019	MCDM	Diagnosis of depression
Dalsgaard *et al.* [27]	2020	STROBE	Mental disorders in childhood and adolescence
Shakeel *et al.* [97]	2020	Group decision making	Black supplier selection in industrial process
Zhang *et al.* [124]	2020	Mixed-effect model	Mental health problems during the COVID-19
Felipe *et al.* [37]	2021	Transcranial Direct Current Stimulation	Transcranial direct current stimulation in neurological and psychiatric disorders
Jalali *et al.* [50]	2021	MCDM	COVID-19 diagnosis
Molla *et al.* [68]	2021	MCDM	Medical diagnosis problem
Alqaysi *et al.* [12]	2022	MCDM	Hybrid diagnosis models for autism patients based on medical
Mengi *et al.* [67]	2022	MCDM	Socio-behavioral disorders diagnosis
Ahmad *et al.* [4]	2023	BWM, TOPSIS, WSM & MABAC	Psychological health of people
Alamoodi *et al.* [9]	2023	MCDM	Medical case studies of COVID-19
Albahri *et al.* [11]	2023	MCDM	Autism spectrum disorders
Munir *et al.* [78]	2023	MCDM	Disease diagnosis
Alamoodi *et al.* [10]	2024	FWZIC	Evaluate medical LLMs in the healthcare domain
Kumar [57]	2024	VIKOR	Rank the diseases among the vector-borne diseases
Lakshmi *et al.* [59]	2024	MCDM	Diagnose patient's medical condition

Pythagorean trapezoidal fuzzy numbers (IVPTrFNs) [96], an extension of intuitionistic fuzzy numbers [29] that provide a more flexible representation for handling uncertainty and ambiguity in decision-making and computational processes. IVPTrFNs allow for a richer expression of ambiguity and uncertainty [25, 94, 126], thereby improving decision-making in various uncertain scenarios. Furthermore, Wang *et al.* [112] employed Pythagorean fuzzy numbers to select sustainable food suppliers and Ahemad *et al.* [3] utilised interval-valued q-rung orthopair fuzzy numbers in a solid waste management system using MCDM methodologies.

As opposed to conventional fuzzy numbers, interval-valued Pythagorean trapezoidal fuzzy numbers (IVPTrFNs) [96] offer a more adaptable way to describe uncertainty. In situations where ambiguity prevails and the information supplied is imprecise or confusing, these statistics are especially helpful when modelling and making decisions. The IVPTrFN is an integrated form of interva [88], Pythagorean fuzzy number [33] and Trapezoidal fuzzy number [90]. Pythagorean fuzzy set used in financial analysis [14], industry [17], supply chain [49], engineering application [47], and so on. Ayyildiz *et al.* [16] proposed an interval-valued Pythagorean fuzzy AHP method-based supply chain performance evaluation by a new extension of the SCOR model. Rahman *et al.* [87] defined some interval-valued Pythagorean fuzzy Einstein weighted averaging aggregation operators and their applications. Touqeer *et al.* [108] proposed a chance-constraint programming technique to address interval-valued linear programming network problems by fuzzy Pythagorean constraints. This method is used to solve various network problems with objective functions that utilise interval-valued Pythagorean fuzzy numbers. Wang *et al.* [111] introduced the concept of interval-valued Pythagorean fuzzy information in three-way conflict analysis and prospect theory. Some recent applications of interval-valued Pythagorean trapezoidal fuzzy numbers in different areas are briefly discussed in Table 2.

2.3. Background of MCDM methodologies

A literature review on Multiple Criteria Decision Making (MCDM) techniques [83] offers valuable insights into the theoretical foundations, methodological developments and practical applications of various decision-making

TABLE 2. Literature review on interval-valued Pythagorean trapezoidal fuzzy numbers.

Authors	Year	Fuzzy environment	Application area
Ren *et al.* [89]	2016	Pythagorean fuzzy	AIIB advances in multiple areas after preliminary work like capital determination, loan application, and PFM confirmation
Roy *et al.* [90]	2016	Interval-valued Pythagorean Triangular Fuzzy	Selection of employment for job
Xuea *et al.* [114]	2016	Interval-valued intuitionistic fuzzy	Material selection with incomplete weight information
Zeng *et al.* [122]	2016	Interval type-2 trapezoidal fuzzy numbers	Real-world problem
Aghamohagheghia *et al.* [1]	2019	Interval type-2 trapezoidal fuzzy numbers	Application to a transport projects appraisal
Li *et al.* [61]	2019	Interval Pythagorean fuzzy set	Apply the proposed model to a real-world infrastructure project
Shakeel *et al.* [96]	2019	Interval-valued Pythagorean trapezoidal fuzzy numbers	Their application in group decision making
Yua *et al.* [118]	2019	Interval-valued Pythagorean fuzzy environment	Sustainable supplier selection approach
Zeng *et al.* [122]	2022	Interval type-2 trapezoidal fuzzy numbers	Selection of employment for job
Dorfeshan *et al.* [32]	2022	A Triangular Pythagorean fuzzy environment	Public-private partnership projects
Safaei *et al.* [93]	2022	Fuzzy-Logic	Evaluate Adult Obesity
Zhang *et al.* [123]	2022	Interval-Valued Pythagorean Hesitant Fuzzy Set	Application to multi-attribute group decision-making

methods across different domains. MCDM methods are used in various real-life applications, including education [65], industries [58], supply chain [72], computer sciences [109], biotechnologies [23], finance [14], medical diagnosis [73], transportation [30], energy production [102], urban planning [38], etc. Hussain *et al.* [48] analysed the educational challenges in ideological and political fields for achieving the Sustainable Development Goals (SDGs) and Badi *et al.* [18] studied the transport, logistics and supply chain system in free trade zones through MCDM methodologies. Debnath *et al.* [28] applied the MADM algorithm in a T-spherical fuzzy environment to select locations for hydrogen refuelling. Furthermore, Mondal *et al.* [74] employed the three-way decision-making methodology in a q-rung orthopair fuzzy environment, applied to the selection of sustainable transport for IoT service providers. The literature review on MCDM methodologies also covers Analytic Hierarchy Process (AHP) [64] and Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) [20] methods. AHP [17] is one of the popular and important MCDM methods used to determine the weights of the criteria and verify the consistency of the data sets. Further more, the AHP method integrated with spherical fuzzy numbers used by Gündoğdu *et al.* [42] in location selection for renewable energy power plants' applicability and validity, and Moslem [75] applied in solution of sustainable urban transport system. Similarly, PROMETHEE [81] is a widely used MCDM method designed to assist decision-makers in ranking and evaluating alternatives in complex decision-making situations involving multiple criteria or objectives. Khan *et al.* [55] determined the communication networks through computer analysis and examine the domination and double domination using the interval-valued T-spherical fuzzy concept and decision-making approach. Demir [31] applied MCDM based optimisation techniques in risk assessment and smart cities selections and Farooq *et al.* [34] evaluated the road safety and hazards through a fuzzy MCDM approach. A summary of recent works done on MCDM techniques based on AHP and PROMETHEE approaches is given in Table 3.

TABLE 3. Literature review on some recent studies in MCDM methodologies.

Authors	Year	Methodology	Uncertainty	Application area
Akram *et al.* [6]	2017	PROMETHEE	Pythagorean fuzzy N-Soft set	Selection of the best chemical ingredients in cloud seeding
Krishankumar *et al.* [56]	2017	PROMETHEE	Intuitionistic fuzzy environment	Supplier selection problem
Chen [117]	2018	PROMETHEE	Pythagorean fuzzy set	Selection problem of bridge construction methods
Hua *et al.* [46]	2029	PROMETHEE	Interval-valued Pythagorean fuzzy set	Sustainable supplier evaluation
Zavadskas *et al.* [121]	2019	PROMETHEE	Neutrosophic set	Evaluating hedonic shopping fares by one-on-one neuromarketing
Altun *et al.* [13]	2020	PROMETHEE	Neutrosophic sets	Renewable energy resource ranking problem
Zavadskas *et al.* [86]	2020	PROMETHEE	Hesitant fuzzy set	Green supplier selection
Goswami *et al.* [39]	2021	AHP & PROMETHEE	Triangular fuzzy set	Smartphone model selection
Molla *et al.* [68]	2021	Extended PROMETHEE	Pythagorean fuzzy sets	For medical diagnosis problems
Srivastava *et al.* [102]	2023	AHP & PROMETHEE	Crisp set	Cluster head selection
Torkzadeh *et al.* [107]	2023	AHP & PROMETHEE	An interval-valued Pythagorean fuzzy set	University selection
Ye *et al.* [116]	2023	PROMETHEE	Pythagorean fuzzy sets	For the selection of Cotton Woven Fabric
Oubahman *et al.* [81]	2024	AHP & PROMETHEE	Crisp set	Examining college applicants

3. Preliminaries of mathematical tools

Mathematical tools serve as fundamental elements in various subjects, helping in problem-solving, analysis, modelling, and decision-making. Preliminaries of mathematical tools encompass a wide array of foundational concepts and methods that form the basis for more advanced mathematical applications. Some key preliminaries are given below:

Lotfi A. Zadeh introduced the concept of a fuzzy set [119] in 1965 to address the uncertainty inherent in systems. In the fuzzy set, every element is assigned a degree of membership value, which represents the belongingness of the element in the set. Every element of a fuzzy set is represented as an ordered pair $(x, \mu_{\tilde{A}}(x))$ where 1st one element (x) itself and followed by its membership value $(\mu_{\tilde{A}}(x))$. Over time, the fuzzy set evolves and expands to incorporate an uncertainty-capturing capacity.

The intuitionistic fuzzy set developed by Atanassov [15] in 1986, where every element has two membership values, true membership value $(\mu_{\tilde{A}}(x))$ and false membership value $(\nu_{\tilde{A}}(x))$, represents the element's belongingness and non-belongingness, respectively. Another extension of the fuzzy set is the trapezoidal fuzzy set [125], where the membership function $(\mu_{\tilde{A}}(x))$ of the fuzzy set is a trapezoidal shape. Further, the intuitionistic fuzzy set $(\tilde{A})$, the membership function $(\mu_{\tilde{A}}(x))$ and non-membership function $(\nu_{\tilde{A}}(x))$ are satisfies the condition $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$ for all $x \in \tilde{A}$.

Another extension of the fuzzy set is the interval-valued intuitionistic fuzzy set ($\tilde{B}$) [126], where the membership ($\mu_{\tilde{B}}(y)$) and non-membership ($\nu_{\tilde{B}}(y)$) functions are interval-valued functions and satisfy the properties of the intuitionistic fuzzy set.

In 2013, Yager [115] proposed a new approach to the extension of fuzzy set, namely the Pythagorean fuzzy set where every element of the fuzzy set has two membership functions like intuitionistic fuzzy set [8], but the membership functions ($\mu_{\tilde{B}}(x), \nu_{\tilde{B}}(x)$) satisfy the boundary condition $0 \leq (\mu_{\tilde{B}}(x))^2 + (\nu_{\tilde{B}}(x))^2 \leq 1$ for all $x \in \tilde{B}$. The following section discusses various properties and operations of the Pythagorean fuzzy set.

3.1. Interval-valued Pythagorean fuzzy set (IVPFS)

This section discusses the interval-valued Pythagorean fuzzy set (IVPFS) [111] and its properties in detail. In IVPFS, there are two membership functions, which are interval-valued functions that satisfy the boundary condition of the Pythagorean fuzzy set. The interval-valued Pythagorean fuzzy set (IVPFS) is defined as follows:

Definition 1 (Interval-Valued Pythagorean Fuzzy Set (IVPFS) [111])**.** Let $\tilde{U}$ be a non-empty set of the universe. Then the interval-valued Pythagorean fuzzy set (IVPFS) in $\tilde{U}$ is defined as

$$\begin{aligned}\tilde{\mathfrak{P}} &= \left\{\left\langle t, \mu_{\tilde{\mathfrak{P}}}(t), \nu_{\tilde{\mathfrak{P}}}(t)\right\rangle : t \in \tilde{U}\right\} \\ &= \left\{\left\langle t, \left[\mu^l_{\tilde{\mathfrak{P}}}(t), \mu^u_{\tilde{\mathfrak{P}}}(t)\right], \left[\nu^l_{\tilde{\mathfrak{P}}}(t), \nu^u_{\tilde{\mathfrak{P}}}(t)\right]\right\rangle : t \in \tilde{U}\right\}\end{aligned} \tag{1}$$

where $\mu_{\tilde{\mathfrak{P}}}(t) = [\mu^l_{\tilde{\mathfrak{P}}}(t), \mu^u_{\tilde{\mathfrak{P}}}(t)]$ and $\nu_{\tilde{\mathfrak{P}}}(t) = [\nu^l_{\tilde{\mathfrak{P}}}(t), \nu^u_{\tilde{\mathfrak{P}}}(t)]$ are two membership functions of some interval values in $[0, 1]$, denoting the possible membership degree ($\mu_{\tilde{\mathfrak{P}}}(t)$) and non-membership degrees of the element ($\nu_{\tilde{\mathfrak{P}}}(t)$) for all $t \in \tilde{U}$ to the set $\tilde{\mathfrak{P}}$ and for $t \in \tilde{U}$ satisfy $0 \leq (\sup\{\mu_{\tilde{\mathfrak{P}}}(t)\})^2 + (\sup\{\nu_{\tilde{\mathfrak{P}}}(t)\})^2 \leq 1$.

In IVPFS ($\tilde{\mathfrak{P}}$), $\mu_{\tilde{\mathfrak{P}}}(t)$ and $\nu_{\tilde{\mathfrak{P}}}(t)$ are closed interval and their upper bounds and lower bounds are denoted by $\mu^u_{\tilde{\mathfrak{P}}}(t)$, $\nu^u_{\tilde{\mathfrak{P}}}(t)$, $\mu^l_{\tilde{\mathfrak{P}}}(t)$ and $\nu^l_{\tilde{\mathfrak{P}}}(t)$, respectively. Also, for each $t \in \tilde{U}$, $[\mu^l_{\tilde{\mathfrak{P}}}(t), \mu^u_{\tilde{\mathfrak{P}}}(t)]$ and $[\nu^l_{\tilde{\mathfrak{P}}}(t), \nu^u_{\tilde{\mathfrak{P}}}(t)]$ subset of $[0, 1]$ and $[\mu^l_{\tilde{\mathfrak{P}}}(t), \mu^u_{\tilde{\mathfrak{P}}}(t)] \in \mu_{\tilde{\mathfrak{P}}}(t)$ and $[\nu^l_{\tilde{\mathfrak{P}}}(t), \nu^u_{\tilde{\mathfrak{P}}}(t)] \in \nu_{\tilde{\mathfrak{P}}}(t)$ for all $t \in \tilde{U}$.

Definition 2 (Interval-Valued Pythagorean Fuzzy Number (IVPFN) [61])**.** Let the set of real numbers ($\mathbb{R}$) be a universal set of discourse. Then, interval-valued Pythagorean fuzzy number (IVPFN) $\tilde{\mathfrak{Q}}$ in $\mathbb{R}$ is defined on Definition 1 and satisfies the following conditions [99]:

(1) The membership ($\mu_{\tilde{\mathfrak{Q}}}(t)$) and non-membership ($\nu_{\tilde{\mathfrak{Q}}}(t)$) functions are convex.
(2) Support of $\tilde{\mathfrak{Q}}$ is bounded.
(3) The membership ($\mu_{\tilde{\mathfrak{Q}}}(t)$) and non-membership ($\nu_{\tilde{\mathfrak{Q}}}(t)$) functions are pice-wise continuous.

Remark 1. Let $\tilde{\mathfrak{B}}$ be an interval-valued Pythagorean fuzzy number (IVPFN) defined on a universal set ($\mathbb{R}$) and represented by $\tilde{\mathfrak{B}} = \{< t, \mu_{\tilde{\mathfrak{B}}}(t), \nu_{\tilde{\mathfrak{B}}}(t) >: t \in \mathbb{R}\}$. Then IVPFN $\tilde{\mathfrak{B}}$ transforms into different fuzzy numbers by satisfying following conditions:

(A) If the membership and non-membership functions satisfy $0 \leq \mu_{\tilde{\mathfrak{P}}}(t) + \nu_{\tilde{\mathfrak{P}}}(t) \leq 1$, interval-valued Pythagorean fuzzy number (IVPFN) is reduced to the interval-valued intuitionistic fuzzy number (IVIFN). The uncertainty space of the IVPFN is longer than the space of the IVIFN.
(B) For each element t, we can compute its hesitation interval ($\Pi_{\tilde{\mathfrak{B}}}(t)$) of the IVPFN, as follows:

$$\begin{aligned}\Pi_{\tilde{\mathfrak{B}}}(t) &= \left[\Pi^l_{\tilde{\mathfrak{B}}}(t), \Pi^u_{\tilde{\mathfrak{B}}}(t)\right] \\ &= \left[\left(1 - \left(\mu^u_{\tilde{\mathfrak{B}}}(t)\right)^2 - \left(\nu^u_{\tilde{\mathfrak{B}}}(t)\right)^2\right)^{1/2}, \left(1 - \left(\mu^l_{\tilde{\mathfrak{B}}}(t)\right)^2 - \left(\nu^l_{\tilde{\mathfrak{B}}}(t)\right)^2\right)^{1/2}\right].\end{aligned} \tag{2}$$

(C) If $\mu^l_{\tilde{\mathfrak{P}}}(t) = \mu^l_{\tilde{\mathfrak{P}}}(t)$ and $\nu^l_{\tilde{\mathfrak{P}}}(t) = \nu^l_{\tilde{\mathfrak{P}}}(t)$ for all $t \in \mathbb{R}$ then IVPFN is reduced to a Pythagorean fuzzy number (PFN).

Example 1. Let $\tilde{\mathfrak{P}}_1 = \{\langle 0.5, [0.4, 0.6], [0.3, 0.7]\rangle; 0.5 \in \mathbb{R}\}$ be an interval-valued Pythagorean fuzzy number (IVPFN) defined on $\mathbb{R}$. Then the membership interval $\mu_{\tilde{\mathfrak{P}}}(0.5) = [\mu^l_{\tilde{\mathfrak{P}}}(0.5), \mu^u_{\tilde{\mathfrak{P}}}(0.5)]$ is $[0.4, 0.6]$ and non-membership interval $\nu_{\tilde{\mathfrak{P}}}(0.5) = [\nu^l_{\tilde{\mathfrak{P}}}(0.5), \nu^u_{\tilde{\mathfrak{P}}}(0.5)]$ is $[0.3, 0.7]$. Further, $\sup(\mu_{\tilde{\mathfrak{P}}}(0.5)) + \sup(\nu_{\tilde{\mathfrak{P}}}(0.5)) = 0.6 + 0.7 \geq 1$ but $(\sup\{\mu_{\tilde{\mathfrak{P}}}(0.5)\})^2 + (\sup\{\nu_{\tilde{\mathfrak{P}}}(0.5)\})^2 = 0.6^2 + 0.7^2 = 0.85 \leq 1$ with $0.5 \in \mathbb{R}$.

3.2. Interval-valued Pythagorean trapezoidal fuzzy number (IVPTrFN)

This section discusses on the interval-valued Pythagorean trapezoidal fuzzy number (IVPTrFN) [96] and its properties in detail. The extension of the interval-valued Pythagorean fuzzy number (IVPFN) is IVPTrFN, where the membership functions are trapezoidal in shape.

Definition 3 (Interval-Valued Pythagorean Trapezoidal fuzzy number (IVPTrFN) [2])**.** Let the set of real numbers ($\mathbb{R}$) be a universal discourse set. The interval-valued Pythagorean Trapezoidal fuzzy number (IVPTrFN) ($\tilde{\mathfrak{M}}$) is defined on $\mathbb{R}$ as

$$\tilde{\mathfrak{M}} = \left\{\left\langle \xi, (\lambda_1, \lambda_2, \lambda_3, \lambda_4); \left[\mu^L_{\tilde{\mathfrak{M}}}, \mu^U_{\tilde{\mathfrak{M}}}\right], \left[\nu^L_{\tilde{\mathfrak{M}}}, \nu^U_{\tilde{\mathfrak{M}}}\right] \right\rangle; \xi \in \mathbb{R}\right\} \tag{3}$$

where $\lambda_1, \lambda_2, \lambda_3$ and λ_4 ($\lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \lambda_4$) are real numbers in $\mathbb{R}$ and $[\mu^L_{\tilde{\mathfrak{M}}}, \mu^U_{\tilde{\mathfrak{M}}}]$ and $[\nu^L_{\tilde{\mathfrak{M}}}, \nu^U_{\tilde{\mathfrak{M}}}]$ are two optimized interval value of membership and non-membership functions, respectively.

Definition 4. The interval-valued Pythagorean Trapezoidal Fuzzy Number (IVPTrFN) [96] ($\tilde{\mathfrak{M}}$) defined in Definition 3 has two interval-valued membership functions $[\mu^l_{\tilde{\mathfrak{M}}}(\xi), \mu^u_{\tilde{\mathfrak{M}}}(\xi)]$ and $[\nu^l_{\tilde{\mathfrak{M}}}(\xi), \nu^u_{\tilde{\mathfrak{M}}}(\xi)]$ which are trapezoidal in shape. The interval-valued membership function ($[\mu^l_{\tilde{\mathfrak{M}}}(\xi), \mu^u_{\tilde{\mathfrak{M}}}(\xi)]$) is mathematically expressed as

$$\mu^l_{\tilde{\mathfrak{M}}}(\xi) = \begin{cases} 0; & \text{if } \xi < \lambda_1 \\ \mu^L_{\tilde{\mathfrak{M}}}\frac{\xi-\lambda_1}{\lambda_2-\lambda_1}; & \text{if } \lambda_1 \leq \xi < \lambda_2 \\ \mu^L_{\tilde{\mathfrak{M}}}; & \text{if } \lambda_2 \leq \xi \leq \lambda_3 \\ \mu^L_{\tilde{\mathfrak{M}}}\frac{\lambda_4-\xi}{\lambda_4-\lambda_3}; & \text{if } \lambda_3 < \xi \leq \lambda_4 \\ 0; & \text{if } \lambda_4 < \xi \end{cases}, \quad \mu^u_{\tilde{\mathfrak{M}}}(\xi) = \begin{cases} 0; & \text{if } \xi < \lambda_1 \\ \mu^U_{\tilde{\mathfrak{M}}}\frac{\xi-\lambda_1}{\lambda_2-\lambda_1}; & \text{if } \lambda_1 \leq \xi < \lambda_2 \\ \mu^U_{\tilde{\mathfrak{M}}}; & \text{if } \lambda_2 \leq \xi \leq \lambda_3 \\ \mu^U_{\tilde{\mathfrak{M}}}\frac{\lambda_4-\xi}{\lambda_4-\lambda_3}; & \text{if } \lambda_3 < \xi \leq \lambda_4 \\ 0; & \text{if } \lambda_4 < \xi \end{cases} \tag{4}$$

and $\mu^l_{\tilde{\mathfrak{M}}}(\xi) \leq \mu^u_{\tilde{\mathfrak{M}}}(\xi)$ for all $\xi \in \mathbb{R}$. Similarly, the interval-valued non-membership function ($[\nu^l_{\tilde{\mathfrak{M}}}(\xi), \nu^u_{\tilde{\mathfrak{M}}}(\xi)]$) is mathematically expressed as

$$\nu^l_{\tilde{\mathfrak{M}}}(\xi) = \begin{cases} 1; & \text{if } \xi < \lambda_1 \\ \frac{\lambda_2-\xi+\nu^L_{\tilde{\mathfrak{M}}}(\xi-\lambda_1)}{\lambda_2-\lambda_1}; & \text{if } \lambda_1 \leq \xi < \lambda_2 \\ \nu^L_{\tilde{\mathfrak{M}}}; & \text{if } \lambda_2 \leq \xi \leq \lambda_3 \\ \frac{\xi-\lambda_3+\nu^L_{\tilde{\mathfrak{M}}}(\lambda_4-\xi)}{\lambda_4-\lambda_3}; & \text{if } \lambda_3 < \xi \leq \lambda_4 \\ 1; & \text{if } \lambda_4 < \xi \end{cases}, \quad \nu^u_{\tilde{\mathfrak{M}}}(\xi) = \begin{cases} 1; & \text{if } \xi < \lambda_1 \\ \frac{\lambda_2-\xi+\nu^U_{\tilde{\mathfrak{M}}}(\xi-\lambda_1)}{\lambda_2-\lambda_1}; & \text{if } \lambda_1 \leq \xi < \lambda_2 \\ \nu^U_{\tilde{\mathfrak{M}}}; & \text{if } \lambda_2 \leq \xi \leq \lambda_3 \\ \frac{\xi-\lambda_3+\nu^U_{\tilde{\mathfrak{M}}}(\lambda_4-\xi)}{\lambda_4-\lambda_3}; & \text{if } \lambda_3 < \xi \leq \lambda_4 \\ 1; & \text{if } \lambda_4 < \xi \end{cases} \tag{5}$$

and $\nu^l_{\tilde{\mathfrak{M}}}(\xi) \leq \nu^u_{\tilde{\mathfrak{M}}}(\xi)$ for all $\xi \in \mathbb{R}$. Further, the membership ($\mu_{\tilde{\mathfrak{M}}}(\xi)$) and non-membership ($\nu_{\tilde{\mathfrak{M}}}(\xi)$) functions satisfy the conditions

$$\begin{aligned} & 0 \leq \left(\sup\{\mu_{\tilde{\mathfrak{M}}}(\xi)\}\right)^2 + \left(\sup\{\nu_{\tilde{\mathfrak{M}}}(\xi)\}\right)^2 \leq 1 \quad \text{for all } \xi \in \mathbb{R} \\ \implies & 0 \leq \left(\mu^U_{\tilde{\mathfrak{M}}}\right)^2 + \left(\sup \nu^U_{\tilde{\mathfrak{M}}}\right)^2 \leq 1 \end{aligned} \tag{6}$$

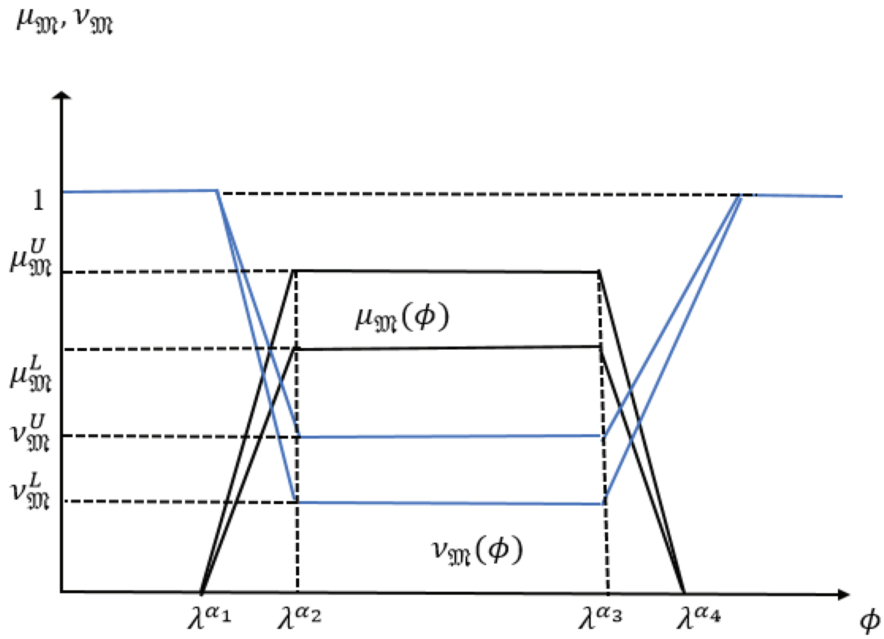


FIGURE 1. Geometrical representation of the membership ($[\mu^L_{\tilde{\mathfrak{M}}}, \mu^U_{\tilde{\mathfrak{M}}}]$) and non-membership ($[\nu^L_{\tilde{\mathfrak{M}}}, \nu^U_{\tilde{\mathfrak{M}}}]$) functions of IVPTrFN ($\tilde{\mathfrak{M}}$).

Figure 1 illustrates the Interval-Valued Pythagorean Trapezoidal Fuzzy Number (IVPTrFN) ($\tilde{\mathfrak{M}}$), where the membership function ($[\mu^L_{\tilde{\mathfrak{M}}}, \mu^U_{\tilde{\mathfrak{M}}}]$) is shown using black curve, and the non-membership function ($[\nu^L_{\tilde{\mathfrak{M}}}, \nu^U_{\tilde{\mathfrak{M}}}]$) is represented by blue curve.

Remark 2. Figure 1 represents the membership and non-membership functions of IVTrFN. The black colour line is the membership function and the blue colour line is the non-membership function of the interval-valued trapezoidal Pythagorean fuzzy number. Here $\mu^L_{\mathfrak{M}}$ and $\mu^U_{\mathfrak{M}}$ denote the lower bound and upper bound of membership function of IVPTrFN, respectively. In the same way, $\nu^L_{\mathfrak{M}}$ and $\nu^U_{\mathfrak{M}}$ denote the lower bound and upper bound of non-membership function of IVPTrFN, respectively.

Example 2. Consider three IVPTrFNs $\tilde{\mathfrak{P}}$, $\tilde{\mathfrak{Q}}$ and $\tilde{\mathfrak{R}}$ defined on the set of real numbers. The IVPTrFNs can be defined as $\tilde{\mathfrak{P}} = \{\langle \xi, (0.2, 0.3, 0.4, 0.5); [0.8, 0.9], [0.2, 0.3]\rangle; \xi \in \mathbb{R}\}$, $\tilde{\mathfrak{Q}} = \{\langle \xi, (0.4, 0.5, 0.6, 0.7); [0.7, 0.8], [0.1, 0.2]\rangle; \xi \in \mathbb{R}\}$ and $\tilde{\mathfrak{R}} = \{\langle \xi, (0.5, 0.6, 0.8, 0.9); [0.8, 0.9], [0.3, 0.4]\rangle; \xi \in \mathbb{R}\}$ where the membership intervals are $[0.8, 0.9]$, $[0.7, 0.8]$ and $[0.8, 0.9]$ and the non-membership intervals are $[0.2, 0.3]$, $[0.1, 0.2]$ and $[0.3, 0.4]$ of the IVPTrFNs $\tilde{\mathfrak{P}}$, $\tilde{\mathfrak{Q}}$ and $\tilde{\mathfrak{R}}$, respectively. Further, in IVPTrFN $\tilde{\mathfrak{P}}$, the $\sup\{\mu_{\tilde{\mathfrak{P}}}(\xi)\} = 0.9$ and $\sup\{\nu_{\tilde{\mathfrak{P}}}(\xi)\} = 0.3$, and then $(\sup\{\mu_{\tilde{\mathfrak{P}}}(\xi)\})^2 + (\sup\{\nu_{\tilde{\mathfrak{P}}}(\xi)\})^2 = (0.9)^2 + (0.3)^2 = 0.81 + 0.09 = 0.90 \leq 1$.

In order to perform an accurate comparison of interval-valued intuitionistic trapezoidal fuzzy numbers (IVITrFN) and interval-valued Pythagorean trapezoidal fuzzy numbers (IVPTrFN), it is necessary to first compare the various kinds of fuzzy sets by going over their definitions and fundamental characteristics.

The advantages of IVPTrFN lie in their ability to represent and manipulate uncertainty in a more nuanced and realistic way, providing a valuable tool for decision-making and modelling in uncertain environments.

Remark 3. Figure 2 illustrates the space of membership grades of different fuzzy numbers. In this figure, we illustrate the preciseness of the membership grades of the Intuitionistic fuzzy number, Pythagorean fuzzy

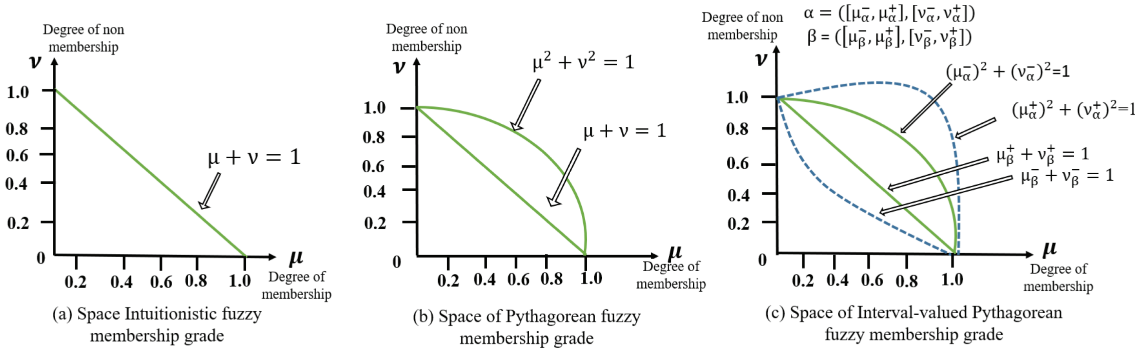


Figure 2. Comparison between IVPTrFN and different fuzzy numbers.

number and interval-valued Pythagorean trapezoidal fuzzy number, respectively. Additionally, it is observed that the specificity of the uncertainty of IVPTrFN is more precise than that of other fuzzy numbers.

3.3. Arithmetic operations on interval-valued Pythagorean trapezoidal fuzzy number (IVPTrFN)

The basic arithmetic operations [2] on interval-valued Pythagorean Trapezoidal fuzzy numbers (IVPTrFN) are defined in this section. The outputs of the arithmetic operations are also IVPTrFN. The arithmetic operations on IVPTrFNs are defined as follows:

Assume that the set of real numbers $(\mathbb{R}\nvdash)$ is the universal set of discourse. Further, assume two IVPTrFNs $\tilde{\mathfrak{A}}$ and $\tilde{\mathfrak{B}}$ define on $\mathbb{R}$ and δ be a scholar number. Two IVPTrFNs are defined as $\tilde{\mathfrak{A}} = \{\langle \xi, (\rho_1, \rho_2, \rho_3, \rho_4); [\mu^L_{\tilde{\mathfrak{A}}}, \mu^U_{\tilde{\mathfrak{A}}}], [\nu^L_{\tilde{\mathfrak{A}}}, \nu^U_{\tilde{\mathfrak{A}}}]\rangle; \xi \in \mathbb{R}\}$ and $\tilde{\mathfrak{B}} = \{\langle \xi, (\phi_1, \phi_2, \phi_3, \phi_4); [\mu^L_{\tilde{\mathfrak{B}}}, \mu^U_{\tilde{\mathfrak{B}}}], [\nu^L_{\tilde{\mathfrak{B}}}, \nu^U_{\tilde{\mathfrak{B}}}]\rangle; \xi \in \mathbb{R}\}$ and the arithmetic operations are defined as follows:

(A) Addition of two IVPTrFNs $\tilde{\mathfrak{A}}$ and $\tilde{\mathfrak{B}}$:

$$\begin{aligned}
\tilde{\mathfrak{A}} \oplus \tilde{\mathfrak{B}} &= \left\{\left\langle \xi, (\rho_1, \rho_2, \rho_3, \rho_4); \left[\mu^L_{\tilde{\mathfrak{A}}}, \mu^U_{\tilde{\mathfrak{A}}}\right], \left[\nu^L_{\tilde{\mathfrak{A}}}, \nu^U_{\tilde{\mathfrak{A}}}\right]\right\rangle; \xi \in \mathbb{R}\right\} \\
&\quad \oplus \left\{\left\langle \xi, (\phi_1, \phi_2, \phi_3, \phi_4); \left[\mu^L_{\tilde{\mathfrak{B}}}, \mu^U_{\tilde{\mathfrak{B}}}\right], \left[\nu^L_{\tilde{\mathfrak{B}}}, \nu^U_{\tilde{\mathfrak{B}}}\right]\right\rangle; \xi \in \mathbb{R}\right\} \\
&= \Bigg\{\Bigg\langle \xi, (\rho_1 + \phi_1, \rho_2 + \phi_2, \rho_3 + \phi_3, \rho_4 + \phi_4); \\
&\quad \left[\sqrt{\left(\mu^L_{\tilde{\mathfrak{A}}}\right)^2 + \left(\mu^L_{\tilde{\mathfrak{B}}}\right)^2 - \mu^L_{\tilde{\mathfrak{A}}} \times \mu^L_{\tilde{\mathfrak{B}}}}, \sqrt{\left(\mu^U_{\tilde{\mathfrak{A}}}\right)^2 + \left(\mu^U_{\tilde{\mathfrak{B}}}\right)^2 - \mu^U_{\tilde{\mathfrak{A}}} \times \mu^U_{\tilde{\mathfrak{B}}}}\right], \\
&\quad \left[\nu^L_{\tilde{\mathfrak{A}}} \times \nu^L_{\tilde{\mathfrak{B}}}, \nu^U_{\tilde{\mathfrak{A}}} \times \nu^U_{\tilde{\mathfrak{B}}}\right] \Bigg\rangle; \xi \in \mathbb{R}\Bigg\}.
\end{aligned} \tag{7}$$

(B) Scalar multiplication of IVPTrFNs $\tilde{\mathfrak{A}}$ by δ:

$$\begin{aligned}
\delta\tilde{\mathfrak{A}} = \delta \times \tilde{\mathfrak{A}} &= \delta \times \left\{\left\langle \xi, (\rho_1, \rho_2, \rho_3, \rho_4); \left[\mu^L_{\tilde{\mathfrak{A}}}, \mu^U_{\tilde{\mathfrak{A}}}\right], \left[\nu^L_{\tilde{\mathfrak{A}}}, \nu^U_{\tilde{\mathfrak{A}}}\right]\right\rangle; \xi \in \mathbb{R}\right\} \\
&= \left\{\left\langle \xi, (\delta\rho_1, \delta\rho_2, \delta\rho_3, \delta\rho_4); \left[\delta\mu^L_{\tilde{\mathfrak{A}}}, \delta\mu^U_{\tilde{\mathfrak{A}}}\right], \left[\delta\nu^L_{\tilde{\mathfrak{A}}}, \delta\nu^U_{\tilde{\mathfrak{A}}}\right]\right\rangle; \xi \in \mathbb{R}\right\}
\end{aligned} \tag{8}$$

where $\delta \geq 0$.

(C) Multiplication of two IVPTrFNs $\tilde{\mathfrak{A}}$ and $\tilde{\mathfrak{B}}$:

$$\begin{aligned}\tilde{\mathfrak{A}} \otimes \tilde{\mathfrak{B}} &= \{\langle \xi, (\rho_1, \rho_2, \rho_3, \rho_4); [\mu^L_{\tilde{\mathfrak{A}}}, \mu^U_{\tilde{\mathfrak{A}}}], [\nu^L_{\tilde{\mathfrak{A}}}, \nu^U_{\tilde{\mathfrak{A}}}]\rangle; \xi \in \mathbb{R}\} \\ &\quad \otimes \{\langle \xi, (\phi_1, \phi_2, \phi_3, \phi_4); [\mu^L_{\tilde{\mathfrak{B}}}, \mu^U_{\tilde{\mathfrak{B}}}], [\nu^L_{\tilde{\mathfrak{B}}}, \nu^U_{\tilde{\mathfrak{B}}}]\rangle; \xi \in \mathbb{R}\} \\ &= \Bigg\{\Bigg\langle \xi, (\rho_1\phi_1, \rho_2\phi_2, \rho_3\phi_3, \rho_4\phi_4); [\mu^L_{\tilde{\mathfrak{A}}} \times \mu^L_{\tilde{\mathfrak{B}}}, \mu^U_{\tilde{\mathfrak{A}}} \times \mu^U_{\tilde{\mathfrak{B}}}], \\ &\quad \left[\sqrt{\left(\nu^L_{\tilde{\mathfrak{A}}}\right)^2 + \left(\nu^L_{\tilde{\mathfrak{B}}}\right)^2 - \nu^L_{\tilde{\mathfrak{A}}} \times \nu^L_{\tilde{\mathfrak{B}}}}, \sqrt{\left(\nu^U_{\tilde{\mathfrak{A}}}\right)^2 + \left(\nu^U_{\tilde{\mathfrak{B}}}\right)^2 - \nu^U_{\tilde{\mathfrak{A}}} \times \nu^U_{\tilde{\mathfrak{B}}}}\right]\Bigg\rangle; \xi \in \mathbb{R}\Bigg\}\end{aligned} \tag{9}$$

(D) Scalar power of IVPTrFNs $\tilde{\mathfrak{A}}$ by δ:

$$\begin{aligned}\mathfrak{A}^\delta = \left(\tilde{\mathfrak{A}}\right)^\delta &= \{\langle \xi, (\rho_1, \rho_2, \rho_3, \rho_4); [\mu^L_{\tilde{\mathfrak{A}}}, \mu^U_{\tilde{\mathfrak{A}}}], [\nu^L_{\tilde{\mathfrak{A}}}, \nu^U_{\tilde{\mathfrak{A}}}]\rangle; \xi \in \mathbb{R}\}^\delta \\ &= \left\{\left\langle \xi, (\rho_1^\delta, \rho_2^\delta, \rho_3^\delta, \rho_4^\delta); \left[(\mu^L_{\tilde{\mathfrak{A}}})^\delta, (\mu^U_{\tilde{\mathfrak{A}}})^\delta\right], \left[(\nu^L_{\tilde{\mathfrak{A}}})^\delta, (\nu^U_{\tilde{\mathfrak{A}}})^\delta\right]\right\rangle; \xi \in \mathbb{R}\right\}\end{aligned} \tag{10}$$

where $\delta \geq 0$.

3.4. Defuzzification of IVPTrFN

Defuzzification [71] is a process of crispification from fuzzy numbers to define order relations. Since there is no order relation in the fuzzy number system, the defuzzification method assigns a crisp number for every fuzzy number and anyone can establish an order relation using the Euclidean metric or any other scale. Here, we propose a defuzzification method on the IVPTrFNs as follows:

Definition 5 (Defuzzification of IVPTrFN [65])**.** Let $\tilde{\mathfrak{M}}$ be an interval-valued Pythagorean Trapezoidal Fuzzy Number (IVPTrFN) defined on the set of real numbers ($\mathbb{R}$) and define as $\tilde{\mathfrak{M}} = \{\langle \xi, (\lambda_1, \lambda_2, \lambda_3, \lambda_4); [\mu^L_{\tilde{\mathfrak{M}}}, \mu^U_{\tilde{\mathfrak{M}}}], [\nu^L_{\tilde{\mathfrak{M}}}, \nu^U_{\tilde{\mathfrak{M}}}]\rangle; \xi \in \mathbb{R}\}$. Then, the defuzzification $\mathscr{D}(\tilde{\mathfrak{M}})$ of the IVPTrFN ($\tilde{\mathfrak{M}}$) is defined as

$$\mathscr{D}\left(\tilde{\mathfrak{M}}\right) = \left[\frac{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}{4} \times \left\{\frac{\mathscr{K}\left(\left(\mu^L_{\tilde{\mathfrak{M}}}\right)^2 - \left(\nu^L_{\tilde{\mathfrak{M}}}\right)^2\right) + \mathscr{L}\left(\left(\mu^U_{\tilde{\mathfrak{M}}}\right)^2 - \left(\nu^U_{\tilde{\mathfrak{M}}}\right)^2\right)}{\mathscr{K} + \mathscr{L}}\right\}\right] \tag{11}$$

where $\mathscr{K}$ and $\mathscr{L}$ are parameters with $\mathscr{K}, \mathscr{L} \in \mathbb{N}$.

Example 3. Let $\tilde{\mathfrak{P}} = \{\langle \xi, (0.3, 0.5, 0.7, 0.9); [0.9, 0.8], [0.15, 0.3]\rangle; \xi \in \mathbb{R}\}$ and $\tilde{\mathfrak{Q}} = \{\langle \xi, (4, 6, 9, 10); [0.85, 0.7], [0.1, 0.25]\rangle; \xi \in \mathbb{R}\}$ be any two interval-valued Pythagorean trapezoidal fuzzy numbers (IVPTrFNs). Then the defuzzified value $\mathscr{D}(-)$ of $\tilde{\mathfrak{P}}$ and $\tilde{\mathfrak{Q}}$ are

$$\begin{aligned}\mathscr{D}\left(\tilde{\mathfrak{P}}\right) &= \left[\frac{0.3 + 0.5 + 0.7 + 0.9}{4} \times \left\{\frac{\mathscr{K}(0.9^2 - 0.15^2) + \mathscr{L}(0.8^2 - 0.3^2)}{\mathscr{K} + \mathscr{L}}\right\}\right] \\ &= \begin{cases} 0.4013 & \text{when } \mathscr{K} = 1 \ \&\ \mathscr{L} = 1 \\ 0.4250 & \text{when } \mathscr{K} = 2 \ \&\ \mathscr{L} = 1 \\ 0.3775 & \text{when } \mathscr{K} = 1 \ \&\ \mathscr{L} = 2 \end{cases}\end{aligned}$$

and

$$\begin{aligned}\mathscr{D}\left(\tilde{\mathfrak{P}}\right) &= \left[\frac{4 + 6 + 9 + 10}{4} \times \left\{\frac{\mathscr{K}(0.85^2 - 0.1^2) + \mathscr{L}(0.7^2 - 0.25^2)}{\mathscr{K} + \mathscr{L}}\right\}\right] \\ &= \begin{cases} 4.1325 \text{ when } \mathscr{K} = 1 \ \&\ \mathscr{L} = 1 \\ 4.6313 \text{ when } \mathscr{K} = 2 \ \&\ \mathscr{L} = 1 \\ 3.7881 \text{ when } \mathscr{K} = 1 \ \&\ \mathscr{L} = 2. \end{cases}\end{aligned}$$

Remark 4. The defuzzification method of IVPTrFN is used in MCDM methods in the later section. For numerical computation, the defuzzification process is applied to simplify the decision matrices. All the theoretical and computational procedures are discussed in detail in the multi-criteria decision-making methods section.

4. Multi-criteria decision making (MCDM) methods

Multi-Criteria Decision Making (MCDM) methods [65] are employed when making decisions that involve multiple conflicting criteria or objectives. These methods [71] aim to provide a structured approach for evaluating and selecting the best alternative among a set of choices, considering various criteria with different importance or relevance. There are several MCDM methods, each with its own approach and mathematical principles. MCDM methods vary in complexity, mathematical rigour and applicability to different decision contexts. The choice of method depends on the nature of the decision problem, the availability of data, the decision-maker's preferences and the level of complexity one wishes to manage. Each method has its strengths and weaknesses and the selection should be based on the specific requirements and constraints of the decision situation. The main features of each of the analyzed MCDM methods are briefly described in this subsection, with an emphasis on special features that are essential for the analysis phase.

4.1. Fuzzy analytic hierarchy process (FAHP)

Thomas L. Saaty [91] developed the Analytic Hierarchy Process (AHP) in 1990, an MCDM method that uses pairwise comparison to perform the ranking. This strategy has a consistency ratio which is associated with it. Assuming that there are some criteria which are weighted by pairwise comparison using a linguistic terms conversion table. The interval-valued Pythagorean trapezoidal fuzzy numbers (IVPTrFN) with the AHP technique [65] capture the ambiguity of the problem because they allow the decision makers (DM) to compare factors while prioritising the linguistic terms. A 9-point scale [64] is typically used to express the pair-wise comparison matrix in traditional fuzzy AHP. The IVPTrFN is utilised in this work to express a pairwise comparison matrix. DMs can evaluate faithful results because of the fuzzy logic in the FAHP approach. The linguistic terms and IVPTrFN conversion table provide relative ratings from the DM's perspective in the comparison matrices.

Suppose that n number of factors are evaluated, and ratings in linguistic terms are given by k decision makers (DMs). Then, the FAHP method can be described in the following steps:

Step A. Identify the relevant factors for the study. Consider n factors based on a detailed literature review and decision-maker's (DM) opinions. Furthermore, data from k DMs is expressed in linguistic terms and then converted into IVPTrFNs. All comparison matrices are constructed as $n \times n$ order.

Step B. Given DMs' opinion in linguistic terms on different factors, construct the comparison matrices. Then, convert the comparison matrices into IVPTrFNs based on the conversion table. Assume that the d-th DM gives the comparison matrix $(\tilde{\mathcal{C}}_d)$ as

$$\tilde{\mathcal{C}}_d = \begin{bmatrix} \left(\tilde{\mathcal{S}}_{11}\right)_d & \left(\tilde{\mathcal{S}}_{12}\right)_d & \dots & \left(\tilde{\mathcal{S}}_{1n}\right)_d \\ \left(\tilde{\mathcal{S}}_{21}\right)_d & \left(\tilde{\mathcal{S}}_{22}\right)_d & \dots & \left(\tilde{\mathcal{S}}_{2n}\right)_d \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{\mathcal{S}}_{n1}\right)_d & \left(\tilde{\mathcal{S}}_{n2}\right)_d & \dots & \left(\tilde{\mathcal{S}}_{nn}\right)_d \end{bmatrix}_{n\times n} \tag{12}$$

i.e.,

$$\begin{aligned} \tilde{\mathcal{C}}_d = \left[\left(\tilde{\mathcal{S}}_{ab}\right)_d\right]_{n\times n} &= \left[\left(\left\langle(\lambda_1,\lambda_2,\lambda_3,\lambda_4);\left[\mu^L_{\tilde{\mathcal{S}}},\mu^U_{\tilde{\mathcal{S}}}\right],\left[\nu^L_{\tilde{\mathcal{S}}},\nu^U_{\tilde{\mathcal{S}}}\right]\right\rangle_{ab}\right)_d\right]_{n\times n} \\ &= \left[\left(\left\langle(\lambda_1,\lambda_2,\lambda_3,\lambda_4)_{ab};\left[\mu^L_{\tilde{\mathcal{S}}},\mu^U_{\tilde{\mathcal{S}}}\right]_{ab},\left[\nu^L_{\tilde{\mathcal{S}}},\nu^U_{\tilde{\mathcal{S}}}\right]_{ab}\right\rangle\right)_d\right]_{n\times n} \end{aligned} \tag{13}$$

where every entry of comparison matrix $(\tilde{\mathcal{C}}_d)$ is IVPTrFNs of $n \times n$ order. In equation (13), the ab-th coefficient is $(\tilde{\mathcal{S}}_{ab})_d$ given by d-th DM based on a-th factor on the basis of b-th factor, where $a, b = 1, 2, \dots, n$ and $d = 1, 2, \dots, k$.

Step C. Aggregate the comparison matrices $(\tilde{\mathcal{C}})$:
Aggregate k comparison matrices $(\tilde{\mathcal{C}}_d)$ given by DMs and construct a single comparison matrix $(\tilde{\mathcal{C}})$, where every entry $\langle(\lambda_1, \lambda_2, \lambda_3, \lambda_4); [\mu^L_{\tilde{\mathcal{S}}}, \mu^U_{\tilde{\mathcal{S}}}], [\nu^L_{\tilde{\mathcal{S}}}, \nu^U_{\tilde{\mathcal{S}}}]\rangle_{ab} = \langle((\lambda_1)_{ab}, (\lambda_2)_{ab}, (\lambda_3)_{ab}, (\lambda_4)_{ab}); [(\mu^L_{\tilde{\mathcal{S}}})_{ab}, (\mu^U_{\tilde{\mathcal{S}}})_{ab}], [(\nu^L_{\tilde{\mathcal{S}}})_{ab}, (\nu^U_{\tilde{\mathcal{S}}})_{ab}]\rangle$ is given by

$$\begin{cases} (\lambda_1)_{ab} = \min\limits_{d=1,2,\dots,k} ((\lambda_1)_{ab})_d \\ (\lambda_2)_{ab} = \sqrt[k]{\prod_{d=1}^{k} ((\lambda_2)_{ab})_d} \\ (\lambda_3)_{ab} = \sqrt[k]{\prod_{d=1}^{k} ((\lambda_3)_{ab})_d} \\ (\lambda_4)_{ab} = \max\limits_{d=1,2,\dots,k} ((\lambda_4)_{ab})_d \end{cases} \quad \& \quad \begin{cases} \left(\mu^L_{\tilde{\mathcal{S}}}\right)_{ab} = \max\limits_{d=1,2,\dots,k} \left(\left(\mu^L_{\tilde{\mathcal{S}}}\right)_{ab}\right)_d \\ \left(\mu^U_{\tilde{\mathcal{S}}}\right)_{ab} = \max\limits_{d=1,2,\dots,k} \left(\left(\mu^U_{\tilde{\mathcal{S}}}\right)_{ab}\right)_d \\ \left(\nu^L_{\tilde{\mathcal{S}}}\right)_{ab} = \min\limits_{d=1,2,\dots,k} \left(\left(\nu^L_{\tilde{\mathcal{S}}}\right)_{ab}\right)_d \\ \left(\nu^U_{\tilde{\mathcal{S}}}\right)_{ab} = \min\limits_{d=1,2,\dots,k} \left(\left(\nu^U_{\tilde{\mathcal{S}}}\right)_{ab}\right)_d \end{cases} \tag{14}$$

where $d = 1, 2, \dots, k$ and the single comparison matrix so formed as

$$\begin{aligned} \tilde{\mathcal{C}} = \left[\tilde{S}_{ab}\right]_{n\times n} &= \left[\left\langle(\lambda_1, \lambda_2, \lambda_3, \lambda_4); [\mu^L_{\tilde{\mathcal{S}}}, \mu^U_{\tilde{\mathcal{S}}}], [\nu^L_{\tilde{\mathcal{S}}}, \nu^U_{\tilde{\mathcal{S}}}]\right\rangle_{ab}\right]_{n\times n} \\ &= \left[\left\langle((\lambda_1)_{ab}, (\lambda_2)_{ab}, (\lambda_3)_{ab}, (\lambda_4)_{ab}); \left[(\mu^L_{\tilde{\mathcal{S}}})_{ab}, (\mu^U_{\tilde{\mathcal{S}}})_{ab}\right], \left[(\nu^L_{\tilde{\mathcal{S}}})_{ab}, (\nu^U_{\tilde{\mathcal{S}}})_{aba}\right]\right\rangle\right]_{n\times n} \end{aligned} \tag{15}$$

where $a, b = 1, 2, \dots, n$.

Step D. Evaluate defuzzified comparison matrix $(\mathcal{C})$:
Construct the defuzzified comparison matrix $(\mathcal{C})$ using the defuzzified formula in equation (11) from the aggregated comparison matrix determined in equation (15). The defuzzified comparison matrix takes the form

$$\mathcal{C} = \left[\mathscr{D}\left(\tilde{S}_{ab}\right)\right]_{n\times n} \tag{16}$$

where $a, b = 1, 2, \dots, n$.

Step E. Calculate the normalised defuzzified comparison matrix $(\mathcal{C}^n)$:
The normalised defuzzified comparison matrix $(\mathcal{C}_n)$ is formulated from the defuzzified comparison matrix $(\mathcal{C})$, as follows:

$$\mathcal{C}^n = [s_{ab}]_{n\times n} = \left[\frac{\mathscr{D}\left(\tilde{S}_{ab}\right)}{\sum_{b=1}^{n} \mathscr{D}\left(\tilde{S}_{ab}\right)}\right]_{n\times n} \tag{17}$$

where $a, b = 1, 2, \dots, n$.

Step F. Calculate factor weight (ρ_a):
Calculate the factor weight (ρ_a) of each factor using equation (18), as follows:

$$\rho_a = \frac{(\prod_{b=1}^{n} s_{ab})^{\frac{1}{n}}}{\sum_{a=1}^{n} (\prod_{b=1}^{n} s_{ab})^{\frac{1}{n}}} \tag{18}$$

where $a = 1, 2, \dots, n$.

Step G. Find out the weighted comparison matrix $(\mathcal{C}^w)$:
From defuzzified comparison matrix $(\mathcal{C})$ in equation (16) and factor weight (ρ_κ) in equation (18), evaluate the weighted comparison matrix $(\mathcal{C}^w)$, as follows:

$$\mathcal{C}^w = [t_{ab}]_{n\times n} = \left[\mathscr{D}\left(\tilde{S}_{ab}\right) \times \rho_a\right]_{n\times n} \tag{19}$$

where $a, b = 1, 2, \dots, n$.

TABLE 4. Number of factors (Λ) and RI value [53].

Λ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Step H. Calculate weighted sum of each factor:
Determine the weighted sum (W_a) of each factor using equation (20), as follows:

$$W_a = \frac{\sum_{b=1}^{n} t_{ab}}{\rho_a} \tag{20}$$

where $a, b = 1, 2, \ldots, n$.

Step I. Evaluate the maximum eigenvalue ($\lambda_{\max}$):
Calculate the maximum eigenvalue ($\lambda_{\max}$) as follows:

$$\lambda_{\max} = \frac{\sum_{a=1}^{n} W_a}{n} \tag{21}$$

for $a = 1, 2, \ldots, n$ where n is the number of factors.

Step J. Determine the Consistency Index (CI):
Calculate the Consistency Index (CI) for the comparison matrix as follows:

$$\mathrm{CI} = \frac{\lambda_{\max} - n}{n - 1} \tag{22}$$

where $\lambda_{\max}$ is the maximum eigenvalue of the comparison matrix and n is the number of factors.

Step K. Evaluate the Consistency Ratio (CR):
Find out the Consistency Ratio (CR) of the comparison matrix as follows:

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}} \tag{23}$$

where Random Index (RI) is given in Table 4.

The computational value of CR is acceptable if $\mathrm{CR} \leq 0.1$; otherwise, the comparison matrix is inconsistent and then the comparison matrix is reconstructed. The factor's weight (crisp weight) is evaluated by equation (20).

4.2. Preference ranking organization method for enrichment evaluation (PROMETHEE) method

The Preference Ranking Organization Method For Enrichment Evaluation (PROMETHEE) method was first introduced by Brans *et al.* [21] in 1982 and further extended by Behzadian *et al.* [20]. PROMETHEE includes the outranking techniques of PROMETHEE-I [113], which calculates entry and exit flows to achieve a partial ranking of options and PROMETHEE-II [101], which calculates net flows to achieve a full ranking of alternatives. The PROMETHEE approach [46] addresses numerous topics in the literature, including environmental management, hydrology and water management, business and financial management, chemistry, logistics and transportation, manufacturing, energy management and social sciences.

Consider a PROMETHEE-based MCDM problem in which n factors are associated with the collection of m alternatives. These alternatives are evaluated by a group of k decision experts (DMs) with respect to each factor. The performance rating of m alternatives with respect to n factors is employed to construct a decision matrix. Decision-makers provide data in linguistic terms, which are then converted into IVPTrFNs for numerical computation. The fuzzy PROMETHEE method is described in the following steps:

Step 1. Identification of factors and alternatives:
The factors and alternatives are selected through detailed literature studies and consideration of decision-makers' perspectives. Assume that the number of experts is denoted by k, the set of patients under investigation by m, and the set of criteria or parameters by n. The selection of appropriate and relevant linguistic terms is the most important factor for the next procedure. In this approach, a set of five linguistic terms is used to evaluate the considered actions or alternatives. The numeric values of these linguistic terms are given in the form of an interval-valued Pythagorean trapezoidal fuzzy number (IVPTrFN). These linguistic terms and their respective values are listed in the conversion table.

Step 2. Construct the decision matrix $(\tilde{\mathcal{D}}_d)$:
The decision matrix $(\tilde{\mathcal{D}}_d)$ is constructed by decision makers (DMs) in linguistic terms based on their experiences and further, it is converted into IVPTrFNs by a conversation table. The comparison matrix $(\tilde{\mathcal{D}}_d)$, provided by d-th DM, is

$$\tilde{\mathcal{D}}_d = \begin{bmatrix} \left(\tilde{\mathcal{T}}_{11}\right)_d & \left(\tilde{\mathcal{T}}_{12}\right)_d & \dots & \left(\tilde{\mathcal{T}}_{1n}\right)_d \\ \left(\tilde{\mathcal{T}}_{21}\right)_d & \left(\tilde{\mathcal{T}}_{22}\right)_d & \dots & \left(\tilde{\mathcal{T}}_{2n}\right)_d \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{\mathcal{T}}_{m1}\right)_d & \left(\tilde{\mathcal{T}}_{m2}\right)_d & \dots & \left(\tilde{\mathcal{T}}_{mn}\right)_d \end{bmatrix}_{m\times n} \tag{24}$$

i.e.,

$$\begin{aligned} \tilde{\mathcal{D}}_d = \left[\left(\tilde{\mathcal{T}}_{ra}\right)_d\right]_{m\times n} &= \left[\left(\left\langle(\lambda_1,\lambda_2,\lambda_3,\lambda_4);\left[\mu^L_{\tilde{\mathcal{T}}},\mu^U_{\tilde{\mathcal{T}}}\right],\left[\nu^L_{\tilde{\mathcal{T}}},\nu^U_{\tilde{\mathcal{T}}}\right]\right\rangle_{ra}\right)_d\right]_{m\times n} \\ &= \left[\left(\left\langle(\lambda_1,\lambda_2,\lambda_3,\lambda_4)_{ra};\left[\mu^L_{\tilde{\mathcal{T}}},\mu^U_{\tilde{\mathcal{T}}}\right]_{ra},\left[\nu^L_{\tilde{\mathcal{T}}},\nu^U_{\tilde{\mathcal{T}}}\right]_{ra}\right\rangle\right)_d\right]_{m\times n} \end{aligned} \tag{25}$$

where every entry of the decision matrix is an IVPTrFN. In equation (25), the ra-th coefficient denoted by $(\tilde{\mathcal{T}}_{ra})_d$ of decision matrix $(\tilde{\mathcal{D}}_d)$, represents the d-th decision maker's option on the r-th alternative with respect to the a-th factor, where $r = 1,2,\dots,m$, $a = 1,2,\dots,n$ and $d = 1,2,\dots,k$.

Step 3. Aggregated decision matrix $(\tilde{\mathcal{D}})$:
Aggregate each DM's assessment into one decision matrix by merging all k decision-makers' opinions. The aggregated decision matrix $(\tilde{\mathcal{D}})$ is derived from k individual decision matrices $(\tilde{\mathcal{D}}_d)$ by aggregating each entry $((\tilde{\mathcal{T}}_{ra})_d)$ using equation (14). The aggregated decision matrix $(\tilde{\mathcal{D}})$ is so formed as

$$\tilde{\mathcal{D}} = \left[\tilde{\mathcal{T}}_{ra}\right]_{m\times n} = \left[\left\langle(\lambda_1,\lambda_2,\lambda_3,\lambda_4);\left[\mu^L_{\tilde{\mathcal{T}}},\mu^U_{\tilde{\mathcal{T}}}\right],\left[\nu^L_{\tilde{\mathcal{T}}},\nu^U_{\tilde{\mathcal{T}}}\right]\right\rangle_{ra}\right]_{m\times n} \tag{26}$$

where $r = 1,2,\dots,m$ and $a = 1,2,\dots,n$.

Step 4. Calculate defuzzified aggregated decision matrix $(\mathcal{D})$:
Determine the defuzzified aggregated decision matrix $(\mathcal{D})$ using equation (11) from the aggregated decision matrix $(\tilde{\mathcal{D}})$ determined in equation (26). The defuzzified aggregated matrix $(\mathcal{D})$ takes the form

$$\mathcal{D} = [\mathcal{T}_{ra}]_{m\times n} = \left[\mathscr{D}\left(\tilde{\mathcal{T}}_{ra}\right)\right]_{m\times n} \tag{27}$$

where $r = 1,2,\dots,m$ and $a = 1,2,\dots,n$.

Step 5. Normalize the decision matrix $(\mathcal{D}^n)$:
Normalize the decision matrix $(\mathcal{D})$ to range $0-1$ and formulate normalized decision matrix $(\mathcal{D}^n)$ by using equation (28), as follows

$$\mathcal{D}^n = [\mathcal{U}^n_{ra}]_{m\times n} \tag{28}$$

where, $\mathcal{U}_{ra}^{n}$ is the ra-th normalized coefficient of the defuzzified aggregated decision matrix ($\mathcal{D}$) and is calculated as

$$\mathcal{U}_{ra}^{n} = \frac{\left(\mathcal{T}_{ra}^{w} - \min\limits_{r=1,2,\ldots,m}\{\mathcal{T}_{ra}^{w}\}\right)}{\left(\max\limits_{r=1,2,\ldots,m}\{\mathcal{T}_{ra}^{w}\} - \min\limits_{r=1,2,\ldots,m}\{\mathcal{T}_{ra}^{w}\}\right)} \tag{29}$$

when the factor a is a beneficial factor, and

$$\mathcal{U}_{ra}^{n} = \frac{\left(\max\limits_{r=1,2,\ldots,m}\{\mathcal{T}_{ra}^{w}\} - \mathcal{T}_{ra}^{w}\right)}{\left(\max\limits_{r=1,2,\ldots,m}\{\mathcal{T}_{ra}^{w}\} - \min\limits_{r=1,2,\ldots,m}\{\mathcal{T}_{ra}^{w}\}\right)} \tag{30}$$

when the factor a is a non-beneficial factor, where $r = 1, 2, \ldots, m$ and $a = 1, 2, \ldots, n$.

Step 6. Determine the evaluative difference ($\mathcal{V}_{(rs)a}$) of r-th alternative with respective to remaining alternatives, as follows:

$$\mathcal{V}_{(rs)a} = \mathcal{U}_{ra}^{n} - \mathcal{U}_{sa}^{n} \tag{31}$$

where $r, s = 1, 2, \ldots, m$ and $a = 1, 2, \ldots, n$.

Step 7. Evaluate the preference function ($\mathcal{P}_{(rs)a}$):
The preference function value ($\mathcal{P}_{(rs)a}$) calculated from the evaluative difference ($\mathcal{V}_{(rs)a}$) by equation (32) is as follows:

$$\mathcal{P}_{(rs)a} = \begin{cases} 0; & \text{if } \mathcal{V}_{(rs)a} \le 0 \text{ i.e., } \mathcal{U}_{ra}^{n} \le \mathcal{U}_{sa}^{n} \\ \mathcal{V}_{(rs)a}; & \text{if } \mathcal{V}_{(rs)a} > 0 \text{ i.e., } \mathcal{U}_{ra}^{n} > \mathcal{U}_{sa}^{n} \end{cases} \tag{32}$$

where $r, s = 1, 2, \ldots, m$ and $a = 1, 2, \ldots, n$.

Step 8. Determine weighted preference function ($\mathcal{P}_{(rs)a}^{w}$):
The weighted preference function ($\mathcal{P}_{(rs)a}^{w}$) is derived from the preference function ($\mathcal{P}_{(rs)a}$) and factor weights (W_a) evaluated from equation (20) in Section 4.1, as follows:

$$\mathcal{P}_{(rs)a}^{w} = W_a \times \mathcal{P}_{(rs)a} \tag{33}$$

where $r = 1, 2, \ldots, m$ and $a = 1, 2, \ldots, n$.

Step 9. Determine weighted aggregated preference function ($\mathcal{S}_{(rs)}^{w}$):
The weighted aggregated preference function ($\mathcal{S}_{(rs)}^{w}$) is derived from the weighted preference function ($\mathcal{P}_{(rs)a}^{w}$) using the following expression:

$$\mathcal{S}_{(rs)}^{w} = \frac{\sum_{a=1}^{n} \mathcal{P}_{(rs)}^{w}}{\sum_{a=1}^{n} W_a} = \sum_{a=1}^{n} \mathcal{P}_{(rs)}^{w} \left[\text{since } \sum_{a=1}^{n} W_a = 1\right] \tag{34}$$

where $r = 1, 2, \ldots, m$ and $a = 1, 2, \ldots, n$.

Step 10. Evaluate the leaving outranking flow ($\mathcal{L}_r$) and entering outranking flow ($\mathcal{E}_s^{w}$):
Calculate the leaving outranking flow ($\mathcal{L}_r$) from weighted aggregated preference function ($\mathcal{S}_{(rs)}^{w}$) as

$$\mathcal{L}_r = \sum_{s=1}^{n} \mathcal{S}_{(rs)}^{w} \text{ [when } r \ne s] \tag{35}$$

and calculate the entering outranking flow ($\mathcal{E}_s$) from weighted aggregated preference function ($\mathcal{S}_{(rs)}^{w}$) as

$$\mathcal{E}_s = \sum_{r=1}^{n} \mathcal{S}_{(rs)}^{w} \text{ [when } s \ne r] \tag{36}$$

where $r = 1, 2, \ldots, m$.

Step 11. Obtain the preference order:
In this step, ranking can be made either partially or completely. Partial ranking can be obtained using PROMETHEE-I. Then, the weighted similarity measure can be used to find out the alternatives and in case a complete ranking is needed, then the computation must proceed to one more step in PROMETHEE-II:

– When alternative r is preferred situation over alternative s, *i.e.*, $r\Phi s$:

$$\begin{aligned} r\Phi s \text{ when } \mathcal{L}_r > \mathcal{L}_s \text{ and } \mathcal{E}_r < \mathcal{E}_s; \\ \text{or, } \mathcal{L}_r > \mathcal{L}_s \text{ and } \mathcal{E}_r = \mathcal{E}_s; \\ \text{when } \mathcal{L}_r = \mathcal{L}_s \text{ and } \mathcal{E}_r < \mathcal{E}_s. \end{aligned} \tag{37}$$

– When both alternatives r and s are indifference situation, *i.e.*, $r\Upsilon s$:

$$r\Upsilon s \text{ when } \mathcal{L}_r = \mathcal{L}_s \text{ and } \mathcal{E}_r = \mathcal{E}_s. \tag{38}$$

– When both alternatives r and s are incomparable situation, *i.e.*, $r\Psi s$:

$$\begin{aligned} r\Psi s \text{ when } \mathcal{L}_r > \mathcal{L}_s \text{ and } \mathcal{E}_r > \mathcal{E}_s; \\ \text{or, } \mathcal{L}_r < \mathcal{L}_s \text{ and } \mathcal{E}_r < \mathcal{E}_s \end{aligned} \tag{39}$$

where $r, s = 1, 2, \ldots, m$.

Step 12. Complete ranking of alternatives (PROMETHEE-II):
The net outranking flow ($\mathcal{O}_r$) of the alternative r is obtained by taking the difference of two weighted similarity measures of the respective alternative, known as PROMETHEE-II, and is formulated as

$$\mathcal{O}_r = \mathcal{L}_r - \mathcal{E}_r \tag{40}$$

which provides the complete ranking $(\mathcal{L}_r, \mathcal{E}_r)$ of PROMETHEE-II as given in equation (41):

$$\begin{cases} r\Phi s & \text{when} \quad \mathcal{O}_r > \mathcal{O}_s \\ r\Upsilon s & \text{when} \quad \mathcal{O}_r = \mathcal{O}_s \end{cases} \tag{41}$$

where $r, s = 1, 2, \ldots, m$.

Thus, all alternatives are compared based on net outranking flow and a complete ranking is obtained without any incomparability of alternatives. The alternative with the greatest net flow is considered the most favourable or optimal solution.

First, we select common types of symptoms associated with different psychiatric disorders, and then we determine the weight of each symptom using fuzzy AHP and the rank of the alternative disorders using fuzzy PROMETHEE. Figure 3 represents the schema chart of the diagnosis of a patient with a psychiatric disorder.

4.3. Pseudo code depicting the empirical study

The decision-making framework is structured with n criteria and m alternatives. A panel of k decision-makers express their evaluations using linguistic terms, which are subsequently translated into comparison matrices of $n \times n$ order and decision matrices of $m \times n$ order. These linguistic assessments are then converted into interval-valued Pythagorean trapezoidal fuzzy numbers (IVPTrFN) to effectively handle uncertainty and vagueness. Subsequently, two multi-criteria decision-making (MCDM) techniques are applied to determine the criteria weights and rank the alternatives. The pseudo code of the proposed methodology is depicted as follows:

INPUT: k comparison matrices of $n \times n$ order and decision matrices of $m \times n$ order are collected
OUTPUT: Evaluate the criteria weight and rank the alternatives

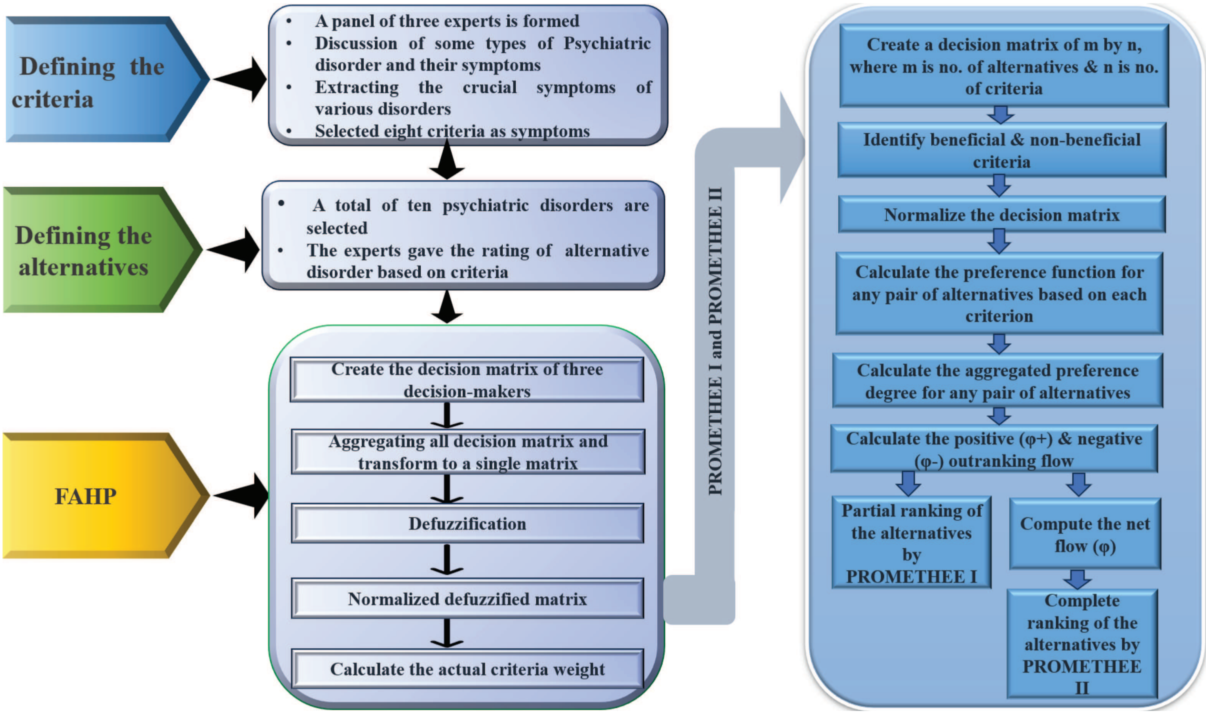


FIGURE 3. Flow diagram of diagnosis of a psychiatric disorder patient.

COMPUTE: Check the Consistency Ratio (CR) of the proposed model
INITIALIZE: IVPTrFN
OPERATION: FAHP and FPROMETHEE

```
MERGE k DMs' inputs in comparison and decision matrices
THEN normalize the comparison matrix

FOR FAHP
    FIND the weighted sum of each factor using the weighted comparison matrix
        THEN compute the maximum eigenvalue
    FIND the weight of the factor and consistency index
    IF Consistency Ratio (CR) ≤ 0.1
        THEN system is consistent and criteria weights are finalized
    OTHERWISE the comparison matrix is be restructured
END FOR

BEGIN PROMETHEE
    COMPUTE ranking of alternatives by the weighted pairwise comparison matrices
END PROMETHEE
```

5. Criteria and alternatives for psychiatric disorder diagnosis

In this section, we select the criteria, followed by the different types of psychiatric disorders.

5.1. Criteria selection for psychiatric disorder diagnosis

To achieve a thorough assessment, selecting criteria for evaluating psychiatric diseases is a complex task that requires consideration of several key aspects. The following are some key criteria that are frequently used when assessing mental illnesses. These criteria are established based on a review of psychiatric disorder literature and consultations with decision-makers.

(A) Disorganized thoughts or a diminished capacity for focus (C_1):
Lack of sleep, stress, anxiety, and some medications can cause concentration problems. Sometimes, the cause may be an underlying medical condition. The ability to focus can affect one's ability to carry out daily responsibilities and perform well at work or school [66]. Moreover, focus can be hindered by external stimuli such as clutter, noise or an uncomfortable workstation.

(B) Excessive fears or worries, or extreme feelings of guilt (C_2):
Excessive fear or anxiety, as well as extreme guilt, are common emotional experiences that can significantly impact a person's mental health and well-being [43]. These emotions may be a sign of several mental health issues, such as depressive disorders, anxiety disorders, and obsessive-compulsive disorders (OCD). Also, extreme feelings of guilt may stem from unresolved conflicts, past traumas, or unrealistic expectations of oneself.

(C) Severe fluctuations in mood of highs and lows (C_3):
Periods of high or euphoric mood (highs) and low moods or depression (lows) are referred to as extreme mood swings between highs and lows [36]. This pattern is indicative of bipolar disorder, a mood disease characterized by marked fluctuations in energy, activity and mood.

(D) Extreme tiredness, lack of energy, or difficulty falling asleep (C_4):
A persistent feeling of fatigue, tiredness or low energy is referred to as fatigue. It has two possible types: mental and physical. Most adults have dealt with it at some point in their lives [40]. Chronic exhaustion and sleep disorders may impair one's ability to focus, remember and make decisions. Due to shortened reaction times and diminished attention, sleep deprivation raises the possibility of accidents, including car crashes and workplace injuries.

(E) Detachment from reality (delusions), paranoia or hallucinations (C_5):
A person experiencing hallucinations may see, hear, or smell things that are not really there. Hallucinations are false or distorted sensory experiences that appear to be real [60]. These symptoms frequently cause the person experiencing them great distress and bewilderment, which can result in disruptive emotions, anxiety or despair.

(F) Significant alterations in eating behaviours (C_6):
Engaging in erratic eating habits, such as late-night snacking, mindless eating, or consuming excessive amounts of processed or unhealthy foods. It's essential to be aware of these changes, especially when they lead to drastic weight loss or gain, changes in energy levels, mood swings or health concerns [45].

(G) Problems with alcohol or drug use (C_7):
Professional assistance, such as addiction counselling, therapy, support groups, and medical treatment, is usually required to address alcohol or drug-related disorders[80]. Numerous physical health difficulties resulted from this, such as reduced immune systems, gastrointestinal diseases, respiratory problems, cardiovascular problems, liver disease and an increased chance of accidents or injuries.

(H) Insomnia (C_8):
Insomnia, a common sleep disorder, is defined as the inability to fall asleep, stay asleep or wake up from sleep despite having the opportunity to do so [44]. Cognitive abilities, such as attention, memory and problem-solving skills, can all be negatively impacted by insomnia, which can affect performance in the workplace, in learning environments and in day-to-day activities.

5.2. Different types of psychiatric disorder

Certainly, there are various types of psychiatric disorders that affect mental health and behaviour. Here is a list of some common psychiatric disorders categorized into different groups.

There are numerous psychiatric disorders, each with its own set of symptoms, causes, and treatments. These disorders are classified in the Diagnostic and Statistical Manual of Mental Disorders (DSM-5), which is commonly used by mental health professionals for diagnosis and treatment planning. Here are some of the major categories of psychiatric disorders:

(1) **Mood Disorders (A_1):**
Affective disorders or mood disorders are characterized by irregular mood swings and disruptive emotions that interfere with a person's daily functioning. Mood disorders feel like an emotional roller coaster, where a person may experience severe emotional highs and lows at different times [62]. A mood disorder may appear when such mood swings begin to interfere with daily life.

(2) **Anxiety Disorders (A_2):**
A class of mental health disorders known as anxiety disorders are characterized by excessive and persistent emotions of worry, fear, worry, or discomfort. These emotions are often strong and can make it challenging for a person to carry out their daily activities properly [70]. The severity of anxiety disorders, which are among the most common mental health conditions, can range from mild to severe.

(3) **Personality Disorders (A_3):**
A problematic way of thinking about oneself and reacting to others is a personality disorder, which is a mental health condition. People suffering from personality disorders often struggle with understanding emotions and have coping difficulties [100]. This means that connecting with other people is challenging for individuals, causing major problems and a negative impact on their family life, social relationships, work and school achievement, as well as their general quality of life.

(4) **Obsessive-Compulsive and Related Disorders (A_4):**
Obsessive-compulsive disorder (OCD) is characterized by a pattern of unwanted thoughts and worries (compulsions) that cause you to engage in repetitive actions (compulsions). These compulsions and obsessions severely disrupt daily life and cause extreme distress [60]. OCD often revolves around specific themes, such as an intense fear of contracting germs. We may obsessively wash our hands until they are quite sore to allay our fear of infection.

(5) **Trauma and Stressor-Related Disorders (A_5):**
Trauma and stress-related disorders are a range of emotional and behavioral issues that can be brought on by traumatic and stressful situations during childhood [26]. These painful and stressful events may involve physical or emotional violence or suffering, such as neglect, abuse, or family strife.

(6) **Eating Disorders (A_6):**
A mental illness that results in unhealthy eating patterns is called an eating disorder. Problems related to your eating habits, weight and body image, as well as thoughts about food, are among these conditions. Individuals with eating disorders often have distorted thoughts and beliefs about food, weight, and body image, which can perpetuate disordered eating behaviors [63]. Malnutrition and emotional distress associated with eating disorders can impair cognitive function, leading to difficulty concentrating, memory problems, and decreased productivity. Overall, eating disorders can have an important and long-lasting effect on a person's entire life.

(7) **Neuro developmental Disorders (A_7):**
Affected brain function and altered neurodevelopment lead to challenges with social, cognitive, and emotional functioning; these conditions are known as neurodevelopmental disorders or NDs. The two most prevalent NDs are attention-deficit/hyperactivity disorder (ADHD) and autism spectrum disorder (ASD) [98].

(8) Sleep Disorders (A_8):
Problems with the amount, quality and timing of sleep are referred to as sleep disorders (or sleep-wake disorders) and can cause distress, leading to interference with daily functioning. Sleep-wake disturbances are often associated with mental health problems such as anxiety, depression, or cognitive impairment, as well as physical illness [124]. Sleep-wake disorders come in many forms, the most common of which is insomnia. Other sleep-wake disorders include restless leg syndrome, narcolepsy, parasomnias, and obstructive sleep apnea.

(9) Mental disorder (A_9):
Mental disorders can significantly impair a person's ability to function in various areas of life, including work, school, relationships, and daily activities. This can result in difficulties maintaining employment, fulfilling social obligations, or managing household tasks [27]. The stigma surrounding mental illness and symptoms such as social anxiety or paranoia can lead to social withdrawal and isolation. Patients may feel misunderstood, ashamed, or embarrassed about their condition, leading them to avoid social interactions and activities [69]. Overall, mental disorders can diminish a person's quality of life by limiting their opportunities for personal growth, fulfillment and happiness. The chronic nature of many mental illnesses may also contribute to feelings of despair or resignation about the future.

(10) Dissociative identity disorder (A_{10}):
Dissociative Identity Disorder (DID), formerly known as Multiple Personality Disorder, is a complex mental health condition characterized by the presence of two or more distinct personality states or identity fragments within an individual. Each of these identities may have its own unique way of interacting with the world, memories, and behaviors. The transitions between these identities are often accompanied by memory gaps, and individuals with DID may experience difficulties in recalling personal information or events [37].

6. Model structure and data collection

This section provides a detailed description of the model formulation and data collection procedure. There are eight criteria considered for this study and ten alternatives are taken for optimal ranking. All criteria and alternatives are considered through a detailed literature review and consultation with decision-makers (DMs). A detailed discussion of the criteria is presented in Section 5.1 and a brief analysis of the alternatives is introduced in Section 5. Then, the comparison matrix of order 8×8 is constructed and the decision matrix of order 10×8 is built. All data were collected from the decision-makers (DMs) using linguistic terms, as presented in Tables 5 and 6 for the FAHP and FPROMETHEE methods, respectively. These linguistic evaluations were subsequently converted into interval-valued Pythagorean trapezoidal fuzzy numbers (IVPTrFNs) to capture the inherent uncertainty in the dataset, as discussed in detail in Section 3. Two fuzzy MCDM methodologies are employed to optimise the results, namely, the fuzzy AHP and fuzzy PROMETHEE methods, which are discussed in Section 4.

We collect data from three decision-makers (DMs) to diagnose a patient with psychiatric disorders. Three decision makers (DMs) are considered for this model as

DM_1: Medical officer with over 15 years of experience.
DM_2: Superintendent of a government medical college and hospital.
DM_3: Senior professor from the Psychology department in a research institute.

They provide their opinion through an unbiased and transparent process, as outlined in terms of linguistic variables in Tables 7 and 8, respectively. From Table 7, we obtain the comparison matrices in linguistic terms, which are then converted into IVPTrFNs. Similarly, from Table 8, we obtain the decision matrices in linguistic terms, which are also converted into IVPTrFNs. The fuzzy values and their defuzzified values for different parameters are shown in Tables 5 and 6, respectively.

TABLE 5. Conversion chart between linguistic term and IVTrPFN with its de-fuzzified values.

Linguistic term	Interval-Valued Trapezoidal Pythagorean Fuzzy Numbers (IVTrPFN)	De-fuzzified value		
		$\mathscr{K}=1$ & $\mathscr{L}=1$	$\mathscr{K}=2$ & $\mathscr{L}=1$	$\mathscr{K}=1$ & $\mathscr{L}=2$
Absolutely High I. (AHI)	$\langle(3,5,7,9);\{[0.85,0.95],[0.05,0.10]\}\rangle$	4.84	4.67	5.01
Essentially High I. (ESHI)	$\langle(2,4,6,8);\{[0.80,0.90],[0.10,0.15]\}\rangle$	3.54	3.41	3.68
Equally High I. (EQHI)	$\langle(1,3,5,7);\{[0.80,0.90],[0.05,0.10]\}\rangle$	2.88	2.77	2.98
Exactly Equal Importance (EEI)	$\langle(1,1,1,1);\{[0.85,0.95],[0.10,0.15]\}\rangle$	0.80	0.77	0.82
Equally Low Importance (EQLI)	$\left\langle(\frac{1}{7},\frac{1}{5},\frac{1}{3},1);\{[0.80,0.90],[0.10,0.15]\}\right\rangle$	0.30	0.29	0.31
Essentially Low I. (ESLI)	$\left\langle(\frac{1}{8},\frac{1}{6},\frac{1}{4},\frac{1}{2});\{[0.85,0.95],[0.05,0.10],\}\right\rangle$	0.21	0.20	0.22
Absolutely Low I. (ALI)	$\left\langle(\frac{1}{9},\frac{1}{7},\frac{1}{5},\frac{1}{3});\{[0.80,0.90],[0.10,0.15]\}\right\rangle$	0.14	0.13	0.14

TABLE 6. Linguistic term with equivalent IVTrPFN and de-fuzzified value for decision matrix.

Linguistic term	Interval-Valued Trapezoidal Pythagorean Fuzzy Numbers (IVTrPFN)	De-fuzzified value		
		$\mathscr{K}=1$ & $\mathscr{L}=1$	$\mathscr{K}=2$ & $\mathscr{L}=1$	$\mathscr{K}=1$ & $\mathscr{L}=2$
Very High (VH)	$\langle(7,8,9,10);\{[0.80,0.95],[0.10,0.20]\}\rangle$	6.34	6.01	6.67
High (H)	$\langle(6,7,8,9);\{[0.85,0.95],[0.15,0.25]\}\rangle$	5.78	5.60	5.95
Medium High (MH)	$\langle(5,6,7,8);\{[0.80,0.90],[0.05,0.15]\}\rangle$	4.63	4.47	4.79
Medium (M)	$\langle(4,5,6,7);\{[0.80,0.95],[0.05,0.15]\}\rangle$	4.17	3.95	4.40
Medium Little (ML)	$\langle(3,4,5,6);\{[0.85,0.95],[0.05,0.15]\}\rangle$	3.60	3.48	3.72
Little (L)	$\langle(2,3,4,5);\{[0.80,0.85],[0.10,0.20]\}\rangle$	2.30	2.27	2.33
Very Little (VL)	$\langle(1,2,3,4);\{[0.85,0.95],[0.05,0.20]\}\rangle$	1.98	1.92	2.04

The de-fuzzified values of IVTrPFN are evaluated by equation (11), where $\mathscr{K}$ and $\mathscr{L}$ are two positive variables ($\mathscr{K},\mathscr{L}\geq 1$). In Table 5, we consider ($\mathscr{K}=1\,\&\,\mathscr{L}=1$), ($\mathscr{K}=2\,\&\,\mathscr{L}=1$), ($\mathscr{K}=1\,\&\,\mathscr{L}=2$) and for further numerical computations, we only consider $\mathscr{K}=1$ and $\mathscr{L}=1$.

The defuzzified values of Interval-Valued Trapezoidal Pythagorean Fuzzy Numbers (IVTrPFN) are evaluated by equation (11), where $\mathscr{K}$ and $\mathscr{L}$ are two variables ($\mathscr{K},\mathscr{L}\geq 1$). In Table 6, we consider three cases: ($\mathscr{K}=1\,\&\,\mathscr{L}=1$), ($\mathscr{K}=2\,\&\,\mathscr{L}=1$) and ($\mathscr{K}=1\,\&\,\mathscr{L}=2$). For further numerical computation, we only consider $\mathscr{K}=1\,\&\,\mathscr{L}=1$.

The comparison matrix is constructed by three DMs in linguistic terms using Table 5 and presented in Table 7. The comparison matrix is used to calculate the weights of the factors and verify the consistency of the data set using the fuzzy AHP method discussed in Section 4.1. Furthermore, the decision matrix is constructed by three decision makers (DMs) in linguistic terms, as shown in Table 8, using Table 6. The decision matrix is utilized for ranking alternatives associated with factors weighted by using the fuzzy PROMETHEE method, theoretically discussed in Section 4.2. The numerical evaluation is shown in the later section.

7. Numerical illustration and discussion

This section represents the numerical illustration of the proposed medical disorder model. First, we calculate the factor weights and check the consistency of the comparison matrix using fuzzy AHP methodology in the IVPTrFN environment. The comparison matrix is depicted in Table 7 and the MCDM-based weighted cal-

TABLE 7. Comparison matrix in linguistic terms given by three decision makers (DMs).

	Criteria *vs.* Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
DM 1	Disorganized thoughts or a diminished capacity for focus (C_1)	EEI	AHI	EQLI	ESHI	EEI	EQHI	AHI	ALI
	Excessive fears or worries, or extreme feelings of guilt (C_2)	ALI	EEI	ESHI	EQLI	ALI	EEI	ESHI	ESHI
	Severe fluctuations in mood of highs and lows (C_3)	EQHI	ESLI	EEI	AHI	EQHI	EEI	EQLI	ALI
	Extreme tiredness, lack of energy, or difficulty falling asleep (C_4)	ESLI	EQHI	ALI	EEI	ESLI	AHI	ALI	AHI
	Detachment from reality (delusions), paranoia or hallucinations (C_5)	EEI	AHI	EQLI	ESHI	EEI	ESHI	EEI	EQHI
	Significant alterations in eating behaviors (C_6)	EQLI	EEI	EEI	ALI	ESLI	EEI	AHI	EEI
	Problems with alcohol or drug use (C_7)	ALI	ESLI	EQHI	AHI	EEI	ALI	EEI	ALI
	Insomnia (C_8)	AHI	ESLI	AHI	ALI	EQLI	EEI	AHI	EEI
	Criteria *vs.* Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
DM 2	Disorganized thoughts or a diminished capacity for focus (C_1)	AHI	EEI	EQLI	ESHI	EQHI	EEI	AHI	ALI
	Excessive fears or worries, or extreme feelings of guilt (C_2)	ALI	EEI	ESHI	ALI	EQLI	EEI	ALI	ESHI
	Severe fluctuations in mood of highs and lows (C_3)	EQHI	ESLI	EEI	EEI	ALI	EQHI	EQLI	EEI
	Extreme tiredness, lack of energy, or difficulty falling asleep (C_4)	ESLI	ALI	EQHI	AHI	EEI	ESLI	EQLI	ALI
	Detachment from reality (delusions), paranoia or hallucinations (C_5)	EEI	AHI	EQLI	ESHI	EEI	ESHI	EEI	EQHI
	Significant alterations in eating behaviors (C_6)	EQLI	EEI	EEI	ALI	ESLI	EEI	AHI	EEI
	Problems with alcohol or drug use (C_7)	ALI	ESLI	EQHI	AHI	EEI	ALI	EEI	ALI
	Insomnia (C_8)	AHI	ESLI	AHI	ALI	EQLI	EEI	AHI	EEI
	Criteria *vs.* Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
DM 3	Disorganized thoughts or a diminished capacity for focus (C_1)	EEI	AHI	EQLI	ESHI	EEI	EQHI	AHI	ALI
	Excessive fears or worries, or extreme feelings of guilt (C_2)	ALI	EEI	ESHI	EQLI	ALI	EEI	ESHI	ESHI
	Severe fluctuations in mood of highs and lows (C_3)	EQHI	ESLI	EEI	AHI	EQHI	EEI	EQLI	ALI
	Extreme tiredness, lack of energy, or difficulty falling asleep (C_4)	ESLI	EQHI	ALI	EEI	ESLI	AHI	ALI	AHI
	Detachment from reality (delusions), paranoia or hallucinations (C_5)	EEI	EQLI	EEI	ALI	ESLI	EEI	EEI	ALI
	Significant alterations in eating behaviors (C_6)	EEI	EEI	EQLI	ALI	EEI	ESLI	ALI	ALI
	Problems with alcohol or drug use (C_7)	ESLI	EQHI	ALI	EEI	AHI	ALI	EEI	AHI
	Insomnia (C_8)	ESLI	AHI	ALI	EEI	AHI	AHI	EQLI	EEI

culation methodology is discussed in Section 4.1. The weights of the factors are presented in Table 9. The Consistency Ratio (CR) of the consistency matrix is found less than 0.1 (CR $= -0.23$).

The weight of each factor, evaluated using the fuzzy AHP-based weighted method for psychiatric disorders, is presented in Table 9. From the result, the factor of Detachment from reality (delusions), paranoia or hallucinations (C_5) gets the maximum weight, followed by Disorganized thoughts or a diminished capacity for focus (C_1), Problems with alcohol or drug use (C_7), Excessive fears or worries, or extreme feelings of guilt (C_2), Severe fluctuations in mood of highs and lows (C_3), Extreme tiredness, lack of energy, or difficulty falling asleep (C_4) and Significant alterations in eating behaviours (C_6), respectively. The least weighted factor is Problems with alcohol or drug use (C_7) for this psychiatric disorder problem. The weights are further used in the fuzzy PROMETHEE method.

Here, we evaluate the ranking of alternatives, *i.e.*, different symptoms of psychiatric disorder, by the fuzzy PROMETHEE method. In this study, we consider ten symptoms as alternatives and rank them based on the decision matrix shown in Table 8 and the weight of the factors depicted in Table 9, respectively. The rank of the alternatives is calculated by the fuzzy PROMETHEE based MCDM technique under an IVPTrFN environment, which is discussed in Section 4.2. The ranking of the different symptoms is presented in Table 10 using the fuzzy PROMETHEE method.

Table 10 represents the alternative ranking with its associated data using the fuzzy PROMETHEE method and Figure 4 depicts it graphically. From the evaluated results, Neuro developmental Disorders (A_7) occupied the optimal rank and Anxiety Disorders (A_2) got the second optimal rank among the alternatives. Further, alternative Mental disorder (A_9), Mood Disorders (A_1), Trauma and Stressor-Related Disorders (A_5), Obsessive-Compulsive and Related Disorders (A_4), Personality Disorders (A_3), Eating Disorders (A_6) and Sleep Disorders (A_8) ranked 3rd, 4th, 5th, 6th, 7th, 8th and 9th rank among the alternatives, respectively. Lastly, the alternative

Table 8. Decision matrix in linguistic terms given by three decision makers (DMs).

	Alternative *vs.* Criteria	Confused thinking or reduced ability to concentrate (C_1)	Excessive fears or worries or extreme feelings of guilt (C_2)	Extreme mood changes of highs and lows (C_3)	Significant tiredness, low energy or problems sleeping (C_4)	Detachment from reality (delusions), paranoia or hallucinations (C_5)	Major changes in eating habits (C_6)	Problems with alcohol or drug use (C_7)	Insomnia (C_8)
DM 1	Mood Disorders (A_1)	VH	H	VH	L	M	ML	H	M
	Anxiety Disorders (A_2)	MH	VH	H	VH	H	L	VH	VH
	Personality Disorders (A_3)	H	MH	M	ML	M	ML	H	M
	Obsessive-Compulsive & Related Disorders (A_4)	H	M	MH	H	L	M	H	VH
	Trauma and Stressor-Related Disorders (A_5)	H	VH	M	MH	M	VL	H	H
	Eating Disorders (A_6)	H	L	H	M	ML	VH	M	M
	Neuro developmental Disorders (A_7)	VH	H	VH	M	VH	M	VH	VH
	Sleep Disorders (A_8)	M	M	MH	VH	ML	ML	H	H
	Mental disorder (A_9)	VH	VH	VH	H	M	M	H	VL
	Dissociative identity disorder (A_{10})	H	H	ML	VH	M	VL	H	MH
	Alternative *vs.* Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
DM 2	Mood Disorders (A_1)	VH	H	VH	L	M	ML	H	M
	Anxiety Disorders (A_2)	M	VH	MH	VH	VH	L	VH	VH
	Personality Disorders (A_3)	H	MH	M	M	ML	ML	VH	M
	Obsessive-Compulsive & Related Disorders (A_4)	MH	ML	H	H	ML	M	H	VH
	Trauma and Stressor-Related Disorders (A_5)	VH	H	M	MH	M	L	H	H
	Eating Disorders (A_6)	H	L	H	M	ML	VH	M	M
	Neuro developmental Disorders (A_7)	VH	H	H	ML	VH	M	VH	VH
	Sleep Disorders (A_8)	ML	M	H	H	ML	ML	VH	H
	Mental disorder (A_9)	H	VH	H	H	MH	M	H	VL
	Dissociative identity disorder (A_{10})	H	H	ML	VH	M	VL	H	MH
	Alternative *vs.* Criteria	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
DM 3	Mood Disorders (A_1)	H	VH	VH	L	M	ML	H	MH
	Anxiety Disorders (A_2)	M	VH	H	VH	H	L	VH	H
	Personality Disorders (A_3)	H	M	M	ML	MH	ML	H	MH
	Obsessive-Compulsive & Related Disorders (A_4)	H	MH	M	H	L	M	H	H
	Trauma and Stressor-Related Disorders (A_5)	VH	VH	M	MH	M	L	H	VH
	Eating Disorders (A_6)	H	ML	H	M	ML	VH	ML	MH
	Neuro developmental Disorders (A_7)	VH	H	H	MH	VH	M	VH	H
	Sleep Disorders (A_8)	MH	M	MH	H	M	ML	VH	H
	Mental disorder (A_9)	VH	H	VH	MH	M	ML	H	L
	Dissociative identity disorder (A_{10})	VH	H	M	VH	M	L	MH	H

Dissociative identity disorder (A_{10}) gets the least important symptoms of psychiatric disorder by the proposed model.

8. Sensitivity analysis and comparative analysis

In this section, we perform a sensitivity analysis and a comparative analysis. Performing a sensitivity analysis will help us to identify the crucial factors. Additionally, we compare our outcomes with several MCDM methodologies to verify the consistency of our approach.

Table 9. Representation of the criteria weight for multiple DMs' perspective using fuzzy AHP.

Criteria	Weight
Disorganized thoughts or a diminished capacity for focus (C_1)	0.1915
Excessive fears or worries, or extreme feelings of guilt (C_2)	0.1085
Severe fluctuations in mood of highs and lows (C_3)	0.1037
Extreme tiredness, lack of energy, or difficulty falling asleep (C_4)	0.0945
Detachment from reality (delusions), paranoia or hallucinations (C_5)	0.2259
Significant alterations in eating behaviours (C_6)	0.0774
Problems with alcohol or drug use (C_7)	0.0697
Insomnia (C_8)	0.1289

Table 10. Alternative ranking with associated data by fuzzy PROMETHEE method.

Alternative	Leaving flow ($\mathcal{L}_r$)	Entering flow ($\mathcal{E}_r$)	Outranking flow ($\mathcal{O}_r$)	Ranking
Mood Disorders (A_1)	0.1876	0.1575	0.03003	4
Anxiety Disorders (A_2)	0.2844	0.1212	0.1632	2
Personality Disorders (A_3)	0.0542	0.2023	−0.1481	7
Obsessive-Compulsive & Related Disorders (A_4)	0.1283	0.2019	−0.0736	6
Trauma and Stressor-Related Disorders (A_5)	0.1271	0.1723	−0.0452	5
Eating Disorders (A_6)	0.1165	0.2872	−0.1707	8
Neuro developmental Disorders (A_7)	0.4478	0.0544	0.3934	1
Sleep Disorders (A_8)	0.1011	0.2843	−0.1831	9
Mental disorder (A_9)	0.2429	0.1369	0.1061	3
Dissociative identity disorder (A_{10})	−0.1907	0.1907	−0.3814	10

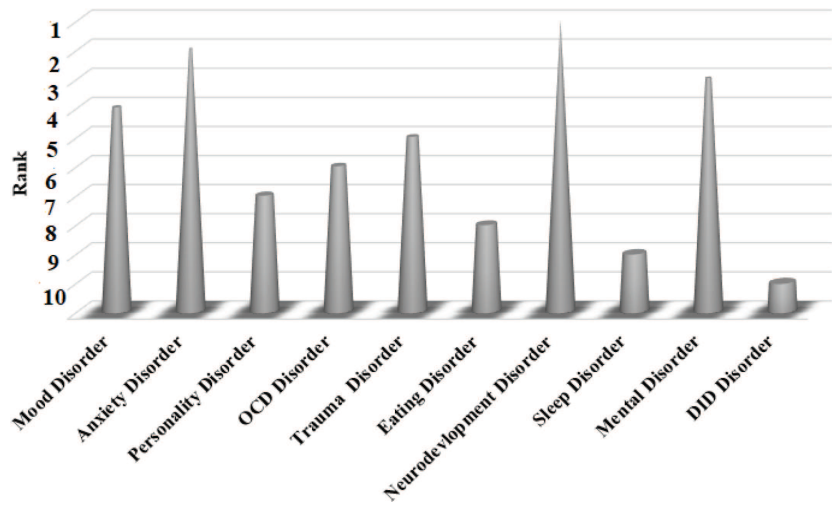


Figure 4. Ranking of psychiatric disorder using FPROMETHEE method by three DMs.

TABLE 11. Ranking between one DM and three DMs using MCDM method FPROMETHEE by removing the criteria Confused thinking (C_1) and Detachment from reality (C_5).

Alternative	One DM	Three DMs
Mood Disorders (A_1)	7	8
Anxiety Disorders (A_2)	1	1
Personality Disorders (A_3)	9	9
Obsessive-Compulsive & Related Disorders (A_4)	4	3
Trauma and Stressor-Related Disorders (A_5)	6	6
Eating Disorders (A_6)	8	7
Neuro developmental Disorders (A_7)	2	2
Sleep Disorders (A_8)	5	5
Mental disorder (A_9)	3	4
Dissociative identity disorder (A_{10})	10	10

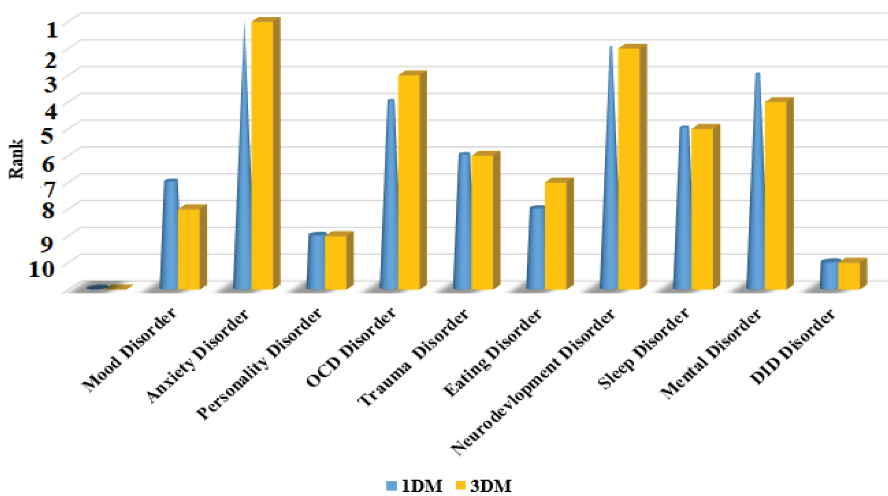


FIGURE 5. Ranking of different psychiatric disorders by one DM and three DMs.

8.1. Sensitivity analysis

We examine three different scenarios involving three patients V_i ($i = 1, 2, 3$) with mental disorders and determine the specific disorder each patient has.

Case 1 (Removing the criteria C_1 and C_5)

After diagnosing patient V_1, it is reported that there is no problem with the symptoms Disorganized thoughts or a diminished capacity for focus (C_1) and Detachment from reality (delusions), paranoia or hallucinations (C_5). Consequently, the ratings for C_1 and C_5 are eliminated. Given this instance, the ranking suggests that the patient has an anxiety problem.

The ranking of these alternative disorders changes slightly when the criteria C_1 and C_5 are removed, as shown in Table 11 and Figure 5. The alternative disorder $A_1, A_2, A_3, A_4, A_5, A_6, A_7$, A_8, A_9, A_{10} are ranked as 8, 1, 9, 3, 6, 7, 2, 5, 4, 10 for three DMs and 7, 1, 9, 4, 6, 8, 2, 5, 3, 10 for one DM, respectively.

TABLE 12. Ranking between one DM and three DMs using MCDM method PROMETHEE by removing the criteria excessive fears and worries (C_2) and Insomnia (C_8).

Alternative	One DM	Three DMs
Mood Disorders (A_1)	4	4
Anxiety Disorders (A_2)	3	2
Personality Disorders (A_3)	7	7
Obsessive-Compulsive & Related Disorders (A_4)	6	5
Trauma and Stressor-Related Disorders (A_5)	8	6
Eating Disorders (A_6)	5	8
Neuro developmental Disorders (A_7)	1	1
Sleep Disorders (A_8)	9	10
Mental disorder (A_9)	2	3
Dissociative identity disorder (A_{10})	10	9

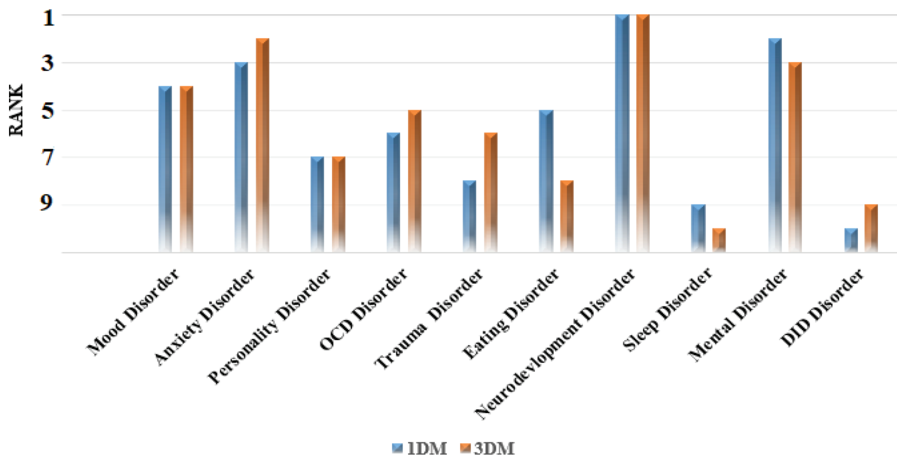


FIGURE 6. Ranking of psychiatric disorder by one DM and three DMs when C_2 and C_8 are removed.

Case 2 (Removing the criteria C_2 and C_8)

After diagnosing patient V_2, it is reported that there are fewer symptoms on average, including Excessive fears or worries or extreme feelings of guilt (C_2) and Insomnia (C_8). So, the ratings of C_2 and C_8 are removed. The ranking so obtained, considering this case, indicates that the patient is also suffering from A_7, *i.e.*, a neurodevelopment disorder.

In this case, Table 12 and Figure 6 show that A_7 is at 1st rank and remaining disorders such as $A_1, A_2, A_3, A_4, A_5, A_6, A_8, A_9, A_{10}$ are at rank 4, 3, 7, 6, 8, 5, 9, 2, 10, respectively for single DM and 4, 2, 7, 5, 6, 8, 10, 3, 9, respectively for three DMs.

Case 3 (Removing the criteria C_3 and C_6)

After diagnosing patient V_3, it is found that there are no issues with the symptoms of Severe fluctuations in mood of highs and lows (C_3) and Significant alterations in eating behaviours (C_6). Therefore, the ranking shows that the alternative disorder A_7 is ranked one. That means, the patient is suffering from a neurodevelopment disorder.

TABLE 13. Ranking between one DM and three DMs using MCDM method PROMETHEE by removing the criteria Extreme mood change (C_3) and Major changes in eating habits ($C6$).

Alternative	One DM	Three DMs
Mood Disorders (A_1)	5	5
Anxiety Disorders (A_2)	2	2
Personality Disorders (A_3)	7	6
Obsessive-Compulsive & Related Disorders (A_4)	6	7
Trauma and Stressor-Related Disorders (A_5)	4	3
Eating Disorders (A_6)	10	9
Neuro developmental Disorders (A_7)	1	1
Sleep Disorders (A_8)	8	8
Mental disorder (A_9)	3	4
Dissociative identity disorder (A_{10})	9	10

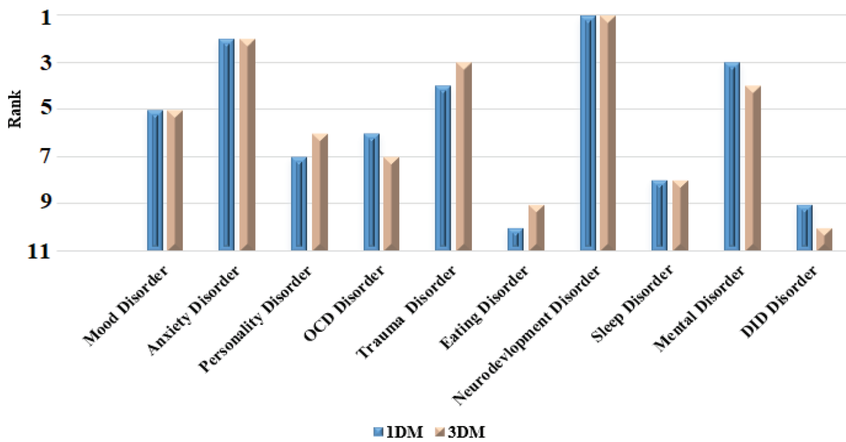


FIGURE 7. Ranking of psychiatric disorder by one DM and three DMs when C_3 and C_6 are removed.

Table 13 and Figure 7 depict that the alternative disorder A_7 is at rank 1. Also, the remaining disorder of alternatives $A_1, A_2, A_3, A_4, A_5, A_6, A_8, A_9, A_{10}$ are at rank 5, 2, 7, 6, 4, 10, 8, 3, 9, respectively for one DM and 5, 2, 6, 7, 3, 9, 8, 4, 10, respectively for three DMs.

Considering these three cases of individual disorder patients and transforming the linguistic rating to IVTrPFN, the rankings obtained are represented in Tables 11, 12 and 13 and Figures 5, 6 and 7, respectively. From the three sensitivity cases, it can be concluded that when factors are removed or weights are changed, the rank of the alternatives changes simultaneously.

8.2. Comparative analysis

To check the consistency and reliability of our proposed model, we conduct a comparative analysis of two sub-cases, one DM and three DMs data sets, using multiple MCDM based ranking methodologies. In addition to the PROMETHEE method, we use four other fuzzy MCDM techniques, including ELECTRE [7], TOPSIS [65], WASPAS [49] and CoCoSo [82] methods. Tables 14 and 15 present the ranks of alternative disorders in five different methods under the IVPTrFNs environment, using single DM and three DMs, respectively.

TABLE 14. Comparison of our proposed work with some existing methodologies based on one DM.

Alternative	ELECTRE	TOPSIS	WASPAS	CoCoSo	PROMETHEE
Mood Disorders (A_1)	4	4	4	4	4
Anxiety Disorders (A_2)	2	2	2	2	2
Personality Disorders (A_3)	10	10	10	9	7
Obsessive-Compulsive & Related D. (A_4)	3	7	6	5	6
Trauma and Stressor-Related D. (A_5)	7	5	5	6	5
Eating Disorders (A_6)	8	8	8	7	8
Neurodevelopmental Disorders (A_7)	1	1	1	1	1
Sleep Disorders (A_8)	9	9	9	10	9
Mental disorder (A_9)	6	3	3	3	3
Dissociative identity disorder (A_{10})	5	6	7	8	10

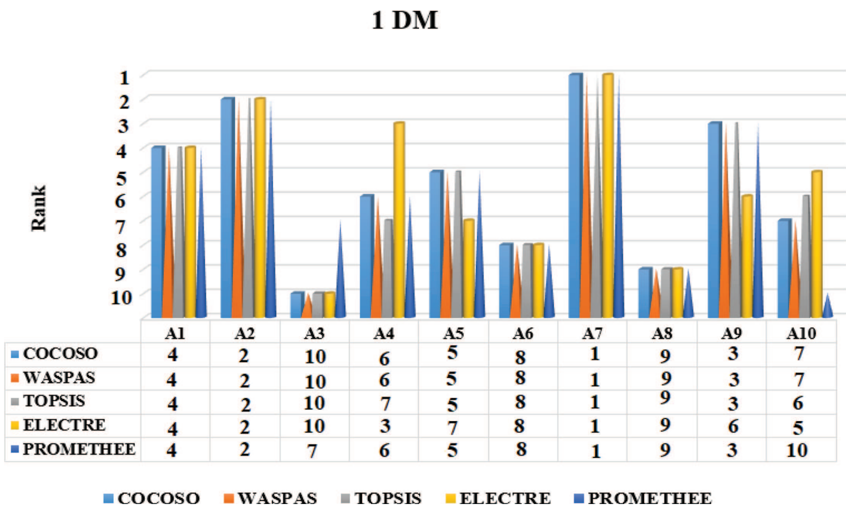


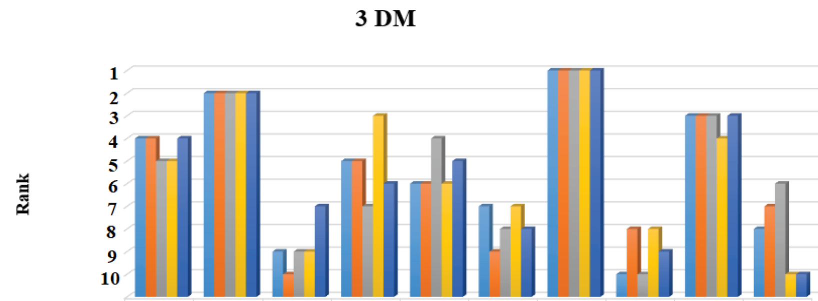
FIGURE 8. Ranking of different psychiatric disorders by 1 DM using MCDM methods.

Table 14 and Figure 8 depict that the disorder A_7 is at rank one for each MCDM technique and the ranks of the remaining disorders are slightly changed. For all five methods, A_1 is at rank 4, A_2 is at rank 2, A_6 is at rank 8, A_8 is at rank 9, and A_3 is at rank 10 for all four methods except the PROMETHEE method. In PROMETHHE, A_3 is at rank 7. Furthermore, A_4 ranks 6th by the WASPAS, COCOSO, and PROMETHEE methods, 7th by the TOPSIS method, and 3rd by the ELECTRE method. Then, A_5 is at rank 5 for all four methods except the ELECTRE method, for which A_5 is at rank 7. Furthermore, A_9 is ranked 3rd for all four methods, except for the ELECTRE method, for which A_9 is ranked 6. Additionally, A_{10} is ranked 7 for the WASPAS and CoCoSo methods, 6 for the TOPSIS method, 5 for the ELECTRE method and 10 for the PROMETHEE method, respectively. Therefore, based on the evaluation of a single decision-maker, the patient is determined to be suffering from disorder A_7, *i.e.*, neurodevelopment disorder.

Table 15 and Figure 9 depict that the disorder A_7 is at rank 1 for each MCDM technique and the ranks of the remaining disorders are slightly changed. For all five methods, A_2 is at rank 2. Furthermore, A_3 ranks 9th by the CoCoSo, TOPSIS, and ELECTRE methods, 10th by the WASPAS method and 7th by the PROMETHEE method. Then, A_4 is ranked 5th by the CoCoSo and WASPAS methods, 7th by the TOPSIS method, 3rd by

TABLE 15. A comparison of our proposed work with some existing methodologies based on three DMs.

Alternative	ELECTRE	TOPSIS	WASPAS	CoCoSo	PROMETHEE
Mood Disorders (A_1)	5	5	4	4	4
Anxiety Disorders (A_2)	2	2	2	2	2
Personality Disorders (A_3)	9	10	10	10	7
Obsessive-Compulsive & Related D. (A_4)	3	7	5	6	6
Trauma and Stressor-Related D. (A_5)	6	4	6	5	5
Eating Disorders (A_6)	7	8	9	8	8
Neuro developmental Disorders (A_7)	1	1	1	1	1
Sleep Disorders (A_8)	8	10	8	9	9
Mental disorder (A_9)	4	3	3	3	3
Dissociative identity disorder (A_{10})	10	6	7	7	10



	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
COCOSO	4	2	9	5	6	7	1	10	3	8
WASPAS	4	2	10	5	6	9	1	8	3	7
TOPSIS	5	2	9	7	4	8	1	10	3	6
ELECTRE	5	2	9	3	6	7	1	8	4	10
PROMETHEE	4	2	7	6	5	8	1	9	3	10

FIGURE 9. Ranking of different psychiatric disorders by three DMs using MCDM methodologies.

the ELECTRE method and 6th by the PROMETHEE method, respectively. After that, A_5 is ranked 6th by the CoCoSo, WASPAS, and ELECTRE methods, 4th by the TOPSIS method and 5th by the PROMETHEE method, respectively. Then, A_6 is ranked 7th by the CoCoSo and ELECTRE methods, 9th by the WASPAS method and 8th by the TOPSIS and PROMETHEE methods. Additionally, A_8 is ranked 10th by the COCOSO and TOPSIS methods, 8th by the WASPAS and ELECTREE methods and 9th by the PROMETHEE method. Then, A_9 is at rank 3rd by all four methods except the ELECTREE method, in which A_9 is at rank 4. Lastly, A_{10} is ranked 8th by the CoCoSo method, 7th by the WASPAS method, 6th by the TOPSIS method, and 10th by both the ELECTRE and WASPAS methods. Therefore, we can also say that the patient is suffering from the disorder A_7, *i.e.*, neurodevelopment disorder, according to the three decision-makers (DMs).

9. CONCLUSIONS AND FUTURE RESEARCH SCOPE

This section outlines the research implications of the proposed model and its potential real-life applications. Additionally, the limitations and future research scope are described for further studies.

9.1. Research implication

Diagnosing psychiatric disorders is a complex process that often involves integrating multiple sources of information and making decisions under uncertainty. The Interval-Valued Pythagorean Trapezoidal Fuzzy Multi-Criteria Decision Making (IVPTrF-MCDM) technique is a method used to handle uncertainty and ambiguity in decision-making processes. When applied to psychiatric diagnosis, this technique can have several practical implications:

(1) For diagnosing a psychiatric disorder in a patient, doctors can incorporate numerous criteria and information sources using the IVPTrF-MCDM approach. When compared to conventional approaches that depend solely on a single criterion or subjective judgement, this can result in more informed and accurate diagnostic judgements.
(2) By considering multiple criteria and factors simultaneously, the IVPTrF-MCDM technique can help clinicians tailor treatment plans to the specific needs of individual patients. This personalized approach may lead to better treatment outcomes and improved patient satisfaction.
(3) Traditional diagnostic methods may be susceptible to bias, such as clinician preferences or preconceptions. The IVPTrF-MCDM technique offers a systematic and objective approach to decision making, helping to minimize bias and ensure that diagnoses are based on relevant clinical evidence.
(4) When the IVPTrF-MCDM approach is used for psychiatric diagnosis, it can produce information and insights that support ongoing studies for mental healthcare quality improvement. Through a methodical assessment of the efficacy of various diagnostic criteria and decision-making methodologies, healthcare professionals can discern optimal practices and gradually enhance diagnostic procedures.
(5) Detecting various psychological issues in the initial stage can be helpful for the patient and get in good physical and mental condition. Furthermore, this model can be extended and applied to the diagnosis and prediction of various diseases, thereby contributing to a healthier environment.

Overall, the integration of interval-valued Pythagorean trapezoidal fuzzy numbers and MCDM techniques in the prediagnosis of psychiatric disorders holds promise for improving diagnostic accuracy, personalized treatment planning, decision support, resource allocation, and methodological advancement in mental healthcare research and practice.

9.2. Conclusions

Selecting a type of psychiatric disorder patient through the Interval-Valued Pythagorean Trapezoidal Fuzzy Multiple Criteria Decision-Making (MCDM) technique is a complex process that integrates the principles of fuzzy logic and decision-making under uncertainty. A more accurate and nuanced depiction of uncertainty in the diagnosis of psychiatric disorders is made possible by the application of Interval-Valued Pythagorean Trapezoidal Fuzzy MCDM methods. By recognizing the ambiguity and imprecision included in mental assessments, this approach offers a more adaptable framework for making decisions. This comprehensive approach helps in making a more informed decision, considering various aspects of the patient's condition. Additionally, the defuzzification technique employed in this paper is a crucial step in fuzzy logic systems, as it transforms fuzzy sets or fuzzy numbers into crisp values that can be easily interpreted and utilised for decision-making.

The results are calculated by the AHP and PROMETHEE methods in IVPTrFN environments. From the evaluated results using the fuzzy AHP method, the factor of Detachment from reality (delusions), paranoia, or hallucinations (C_5) receives the optimal weight and the weights of the other factors are shown in Table 9, respectively. After that, the ranking of the various symptoms of psychiatric disorder by the fuzzy PROMETHEE methods, the alternative Neuro developmental Disorders (A_7) occupied the 1st rank and the remaining alternatives ranks are shown in Table 10, respectively. Furthermore, sensitivity analysis and comparative analysis are conducted to verify the consistency, flexibility, and robustness of the results. Additionally, the research implications of the proposed study are drawn from the evaluated results.

9.3. Limitation and future research scope

This study has some limitations. The proposed method involves several parameters, and selecting the linguistic terms and their values appropriately can be challenging. The choice of parameters can significantly impact the results, and selecting values that accurately represent the decision-maker's preferences can be a subjective process. The criteria, which are selected by the decision maker, could be a limitation. The alternatives must be chosen based on the specified criteria. On the other hand, it is challenging for experts to evaluate the criteria weights and rank the alternatives using linguistic terms with their associated interval-valued Pythagorean trapezoidal fuzzy values.

This research can be extended by considering another type of fuzzy number associated with this method that involves more decision-makers. Additionally, a novel defuzzification approach can be used to address the issue. The proposed model can be applied in multiple decision-making optimization techniques according to the structural framework. Different MCDM techniques may also be incorporated to compare the final ranking with existing outcomes. However, alternative criteria and approaches can also be employed to address this issue, which may yield some intriguing results. Further, multiple sensitivity and comparative analysis cases can be conducted to examine the results.

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Conflicts of interest

All authors declare that there is no conflict of interest in this study and that they have no conflicts of interest among themselves.

Data availability statement

All the necessary data are cited in the article.

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