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A NOVEL NON-PARAMETER FILLED FUNCTION METHOD FOR UNCONSTRAINED GLOBAL OPTIMIZATION

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Abstract. The filled function is considered to be an effective method for solving global optimization problems. It obtains the global optimal solution of the optimization problem by alternating the two stages of minimization and filling. Firstly, this paper proposes a new parameter-free and continuously differentiable filled function, which does not contain exponential terms and logarithmic terms, and overcomes some shortcomings in the form of the original parameter filled function. Secondly, it is proved that the proposed function satisfies the filled property and some good analytical properties, and the corresponding filled function algorithm is given. Experiments are carried out on some benchmark functions and the application of earthwork allocation problem. Finally, the numerical results show the effectiveness and stability of the algorithm.

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1. Introduction

With the rapid development of information technology, the global optimization method is widely used in the fields of artificial intelligence, financial management and engineering design, which is a very meaningful research direction in the field of optimization. The global optimization method mainly studies the global optimality conditions and optimization algorithms of various nonlinear optimization problems. In the latest developments in the fields of artificial intelligence, electronic information and social sciences, many key issues need to determine the global optimal solution. However, the traditional nonlinear programming method usually can only find the local minimum. For non-convex objective functions with multiple local minima, it is a very difficult task to find the global minimum. Therefore, the research of global optimization problem not only has great attraction, but also faces great challenges. It has become a research focus in the field of optimization, and has been favored by researchers in the past two or three decades.

At present, the existing global optimization methods are mainly divided into two categories: stochastic algorithms [1–3] based on biology or statistics and deterministic algorithms [4–28] based on the analytical properties of the objective function. Solving global optimization problems with multiple non-global minima mainly faces two major difficulties: one is how to jump out of the current local minima to find a better point, and the other is how to determine the currently found point as a global minimum. The filled function method is an effective

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deterministic global optimization method to solve such problems. Ge [5] first proposed the filled function algorithm for solving unconstrained global optimization problems. The filled function method is used to solve the global optimization problem, which is divided into two stages: the minimization stage and the filling stage. These two stages are carried out alternately to approach the global minimum point of the objective function. In the minimization stage, starting from the selected initial point, any effective local search method can be used to find the local minimum point of the objective function. In the filling stage, the filled function is constructed based on the current local minimum point and the objective function, and the point in the neighborhood of the current local minimum point is used as the initial point to minimize the filled function, so as to find a feasible point better than the current local minimum point, and then the feasible point is used as the initial point to minimize the objective function. The minimization stage and the filling stage are repeated in turn until the minimization of the filled function cannot give a better solution. The last local minimum is identified as the global minimum of the objective function. This overcomes the defect that the general optimization algorithm is easy to fall into "premature".

The first generation filled function given by Ge is as follows:

$$P(x, x_k^*, f(x), r, \rho) = \frac{1}{r + f(x)} \exp\left(-\frac{\|x - x_k^*\|^2}{\rho^2}\right),$$

where r and ρ are two parameters. When these two parameters are not properly selected, the convergence and stability of the algorithm will be affected, and parameter adjustment will also increase the amount of calculation in practical applications. Moreover, Ge's function contains an exponential term $\exp(-\frac{\|x-x_k^*\|^2}{\rho^2})$. In the case of a large domain or a small parameter ρ, the image of the filled function is almost flat, and the algorithm will end at a false saddle point. In order to overcome the difficulty of parameter adjustment, the single parameter filled function given by Ge and Qin [6] is as follows:

$$P(x, x_k^*, A) = -[f(x) - f(x_k^*)] \exp\left(A\|x - x_k^*\|^2\right).$$

The reduction of parameters will increase the computational efficiency, but it still contains the exponential term. The existence of the index term will lead to numerical overflow.

After that, a one-parameter filled function without exponential term and logarithmic term is given in [7], the form is as follows:

$$P(x, x_k^*) = \frac{1}{\arctan(f(x) - f(x_k^*))} - a\|x - x_k^*\|^p,$$

where p a positive integer. However, the function is discontinuous at $f(x) = f(x_k^*)$, and the local search method based on gradient information cannot be used here.

Because of the defect of parameter selection, one parameter-free continuous differentiable filled function is given in [8]:

$$P(x, x_k^*) = \frac{1}{1 + \|x - x_k^*\|^2} \varphi(f(x) - f(x_k^*)),$$

$$\varphi(t) = \begin{cases} \frac{\pi}{2}, & t \geq 0, \\ \frac{\pi}{2} - \arctan(t^2), & t < 0. \end{cases}$$

After, scholars have proposed more continuous differentiable filled functions that don't contain exponential-logarithmic terms and don't contain parameters, such as the filled function in [9], *i.e.*

$$P(x, x_k^*) = \frac{1}{1 + \|x - x_k^*\|^2} g(f(x) - f(x_k^*)),$$

where $g(t) = \begin{cases} 1, & t \geq 0, \\ 1 - \frac{t^2}{6}, & t < 0, \end{cases}$ and the filled function in [10], *i.e.*

$$P(x, x_k^*) = \frac{1}{1 + \|x - x_k^*\|} g(f(x) - f(x_k^*)),$$
$$g(t) = \begin{cases} 1, & t \geq 0, \\ t^3 + 1, & t < 0. \end{cases}$$

The authors of [11] propose a class of one-parameter filled functions with the same local minima as the objective function for global optimization problems:

$$P(x, x_k^*, r) = -\ln(\|x - x_k^*\| + 1) + g_r(f(x) - f(x_k^*)),$$
$$g_r(t) = \begin{cases} 1, & t \geq 0, \\ 1 - r\varphi(t), & t < 0. \end{cases}$$

The function $\varphi(t) : R \to R$ is a continuously differentiable function and satisfies the following conditions: (c1) $\varphi(0) = 0$ and $\varphi'(0) = 0$; (c2) $\varphi'(t) < 0$ for all $t \in (-\infty, 0)$. This represents a highly promising class of functions. Based on the bezier curve, the authors in [12] first introduce a univariate function $v(t)$, and then define a new filled function with respect to $v(t)$, which does not contain exponential-logarithmic terms and is numerically stable.

In the field of global optimization of unconstrained continuous functions, the filled function method has been widely studied as an effective global optimization method. According to the form and properties of the function, the filled function can be divided into the following categories: parameter filled function [13, 14], parameter-free filled function [15, 16], continuous differentiable filled function [17, 18], filled function with the same local minimum point as the objective function [19], etc. Among them, the continuous differentiability makes the filled function make full use of the efficient local optimization algorithm, while effectively avoiding the introduction of additional local optimal solutions. In order to improve the performance of the algorithm, scholars have been committed to combining the filled function with other optimization strategies in recent years. For example, the authors in [20] try to combine the filled function with the adaptive strategy of the initial point and the narrow valley widening strategy to further enhance the ability to escape the local minimum. In the process of the evolution of the filled function form, the definition of the filled function is gradually improved. Zhang *et al.* [21] improved the definition of the filled function. A new definition of filled function is given in [22], which is the most widely used definition at present. Gao *et al.* [23] also proposed a new definition of the filled function and constructed a filled function with the same minimum point as the objective function, which avoids the minimization of the objective function and the filled function alternately and greatly improves the computational efficiency. At the same time, the filled function method has also been extended from the initial solution of unconstrained global optimization problems to the solution of integer programming problems [24,25], constrained optimization problems [26, 27], nonlinear equations [28] and other problems, and its application scope has been expanding. For instance, the study in [29] proposes an innovative filled function and evaluates its deterministic efficiency in solving the facility layout/location problem. Although the filled function algorithm has made great progress, it is still limited to local minima when dealing with multimodal global optimization problems.

Based on the above content, this paper proposes a continuous differentiable parameter-free filled function. The organization of this paper is as follows: Section 2 describes the basic definition of the filled function and some assumptions. Section 3 constructs a novel continuous differentiable parameter-free filled function and proves that it satisfies both the filled properties and additional analytical properties. Section 4 gives the specific steps of the algorithm. Section 5 includes numerical experiments using the proposed algorithm for optimization problems. In the sixth section, the proposed algorithm is applied to the earthwork allocation problem. Section 7 presents the conclusion of the paper.

2. Preliminaries

In this paper, we consider the following unconstrained global optimization problem:

$$\begin{aligned} &\min f(x) \\ &\text{s.t. } x \in R^n. \end{aligned} \tag{1}$$

where $f(x) : R^n \to R$ is a continuously differentiable function. This section introduces some assumptions and definitions to ensure the existence of the global optimal solution of the Problem (1) and the implementation of the proposed algorithm.

Assumption 2.1. *$f(x)$ is coercive function on R^n, i.e. $f(x) \to +\infty$ as $\|x\| \to +\infty$.*

Actually, Assumption 2.1 implies that there exists a compact set $X \subset R^n$ whose interior contains all global minimizers of $f(x)$. Therefore the Problem (1) is equivalent to the following problem:

$$\begin{aligned} &\min f(x) \\ &\text{s.t. } x \in X. \end{aligned} \tag{2}$$

Here, X is a compact set which can be expressed as $X = \{x_i \in R | t_i \le x_i \le \omega_i, i = 1, 2 \cdots, n\}$.

Assumption 2.2. *$f(x)$ has only a finite number of minimums in X.*

Assumption 2.3. *The value of $f(x)$ for any x inside X is smaller than the value of $f(x)$ for x on the boundary of X.*

The original definition of the filled function proposed by Ge [5] is as follows:

Definition 2.1. Let x_k^* is a local minimizer. A function $P(x, x_k^*)$ is called a filled function of $f(x)$ at the current minimizer x_k^* if $P(x, x_k^*)$ has the following properties:

(1) x_k^* is a maximizer of $P(x, x_k^*)$ and the whole basin B_k^* of $f(x)$ at x_k^* becomes a part of a hill of $P(x, x_k^*)$;
(2) $P(x, x_k^*)$ has no minimizers or saddle points in any higher basin of $f(x)$ than B_k^*;
(3) If $f(x)$ has a lower basin than B_k^*, then there is a point x' in such a basin that minimizes $P(x, x_k^*)$ on the line through x and x_k^*.

For the definitions of basin and hill, see [5]. However, the third item in Definition 2.1 is difficult to be realized, and the point lower than the current local minimizer of the objective function may not be found by using the local search methods. On the basis of Definition 2.1, a new definition of the filled function is given in [22], *i.e.*:

Definition 2.2. A function $P(x, x_k^*)$ is called a filled function of $f(x)$ at a local minimizer x_k^* if $P(x, x_k^*)$ has the following properties:

(1) x_k^* is a strict local maximizer of $P(x, x_k^*)$;
(2) $P(x, x_k^*)$ has no stationary point in the region $X_1 = \{f(x) \ge f(x_k^*), x \in X \backslash \{x_k^*\}\}$;
(3) If x_k^* is not a global minimizer of $f(x)$, then $P(x, x_k^*)$ will have a minimizer in the region $X_2 = \{f(x) < f(x_k^*), x \in X\}$.

Definition 2.2 makes it easier to construct filled functions and find better local minimizer. The definition of the filled function shows that if $P(x, x_k^*)$ is a filled function satisfying Definition 2.2 and x_k^* is not a global minimizer of $f(x)$, then in the process of minimizing $P(x, x_k^*)$ by using local search method, a point $\widetilde{x_k^*} \in X_2$ can be found, satisfying $f(\widetilde{x_k^*}) < f(x_k^*)$. Then, taking $\widetilde{x_k^*}$ as the initial point, using the local search method to minimize $f(x)$, we can jump out of the current local minimizer x_k^* and find a better local minimizer of $f(x)$.

3. A non-parameter filled function and its properties

Based on the Definition 2.2, this paper constructs a non-parameter filled functions of $f(x)$ at the local minimizer x_k^*, its form as follows:

$$P(x, x_k^*) = \frac{1}{1 + \|x - x_k^*\|^2} - \arctan\Big((\min[0, f(x) - f(x_k^*)])^2\Big). \tag{3}$$

In formula (3), $f(x)$ is the objective function, x_k^* is the current local minimizer of $f(x)$, $\|\cdot\|$ represents the Euclidean distance. It's easy to verify that the function (3) is continuously differentiable. The following theorems show that, under Assumptions 2.1–2.3, function (3) meets the qualification to be called a filled function.

Lemma 3.1. *The function $y = -\arctan(t)$ is continuously differentiable and monotonically decreasing on $[0, +\infty)$. For $\forall x \neq x_k^* \in X$, the following inequality holds:*

$$0 = -\arctan(0) > -\arctan\left[f(x) - f(x_k^*)\right]^2.$$

Proof. For $\forall t \in (-\infty, +\infty)$, we have

$$(-\arctan(t))' = -\frac{1}{1 + t^2} < 0,$$

therefore, $y = -\arctan(t)$ is a monotone decreasing function on $(-\infty, +\infty)$. Combining with $0 < [f(x) - f(x_k^*)]^2$, we can get

$$0 = -\arctan(0) > -\arctan\left[f(x) - f(x_k^*)\right]^2.$$

□

Theorem 3.1. *Suppose that x_k^* is a local minimizer of $f(x)$, then x_k^* is a strict local maximizer of $P(x, x_k^*)$.*

Proof. Since x_k^* is a local minimizer of $f(x)$, there exists a neighbourhood $N(x_k^*, \delta)$ of x_k^* with $\delta > 0$, such that $f(x) \geq f(x_k^*)$ for $\forall x \in N(x_k^*, \delta) \cap X$. For any $x \neq x_k^* \in N(x_k^*, \delta) \cap X$, one has

$$P(x, x_k^*) = \frac{1}{1 + \|x - x_k^*\|^2} < \frac{1}{1 + \|x_k^* - x_k^*\|^2} = P(x_k^*, x_k^*).$$

Thus, x_k^* is a strict local maximizer of $P(x, x_k^*)$. □

Theorem 3.2. *Suppose that x_k^* is a local minimizer of a continuously differentiable function $f(x)$ and one point $x \in X_1 = \{f(x) \geq f(x_k^*), x \in X \backslash \{x_k^*\}\}$, then x is not a stationary point of $P(x, x_k^*)$.*

Proof. Let $x \in X_1$, *i.e.* $f(x) \geq f(x_k^*)$ and $x \neq x_k^*$. By the form of the filled function in equation (3), we have

$$P(x, x_k^*) = \frac{1}{1 + \|x - x_k^*\|^2},$$

$$\nabla P(x, x_k^*) = -\frac{2\|x - x_k^*\|}{\left(1 + \|x - x_k^*\|^2\right)^2} \cdot \frac{x - x_k^*}{\|x - x_k^*\|} = -\frac{2(x - x_k^*)}{\left(1 + \|x - x_k^*\|^2\right)^2} \neq 0,$$

then, for $\forall x \in X_1$, $d = (x - x_k^*) \in X_1$, we can still get

$$(x - x_k^*)^T \nabla P(x, x_k^*) = -\frac{2(x - x_k^*)^2}{\left(1 + \|x - x_k^*\|^2\right)^2} < 0.$$

Thus, $P(x, x_k^*)$ has no stationary points in set X_1. For $\forall x \in X_1$, $d = (x - x_k^*) \in X_1$ is the descend direction of function $P(x, x_k^*)$ at x. □

Theorem 3.3. *Let x_k^* is a local non-global minimum point of $f(x)$, then the filled function has a minimum point on X_2. Moreover, if there exists $x_q \in X_2$ such that*

$$\nabla f(x_q)^T (x_q - x_k^*) > \eta > 0,$$

here

$$\eta = \frac{(x_q - x_k^*)^2 \left(1 + (f(x_q) - f(x_k^*))^4\right)}{f(x_k^*) - f(x_q)} > 0.$$

Therefore, we have $\nabla P(x, x_k^*)^T d^* > 0$, $d^* = x_q - x_k^*$.

Proof. Assume that $x_j^* \notin X_2$ is a minimizer of $P(x, x_k^*)$, we can obtain $x_j^* \in X_1$ and $\nabla P({x_j}^*, x_k^*) = -\frac{2({x_j}^* - x_k^*)}{(1+\|{x_j}^* - x_k^*\|^2)^2} = 0$, so there is $x_j^* = x_k^* = 0$. According to Theorem 3.1, if x_k^* is a strict local maximum point of $P(x, x_k^*)$, then there exists a contradiction denoted by $x_j^* \neq x_k^*$. Additionally, if x_j^* is a saddle point, this contradicts Theorem 3.2. Assumption 2.3 indicates that the truth of $X_2 \subseteq \operatorname{int} X$, consequently, $x_j^* \in X_2 \subseteq \operatorname{int} X$ holds.

Now we will prove that a feasible direction d^* is an ascent direction for some points in X_2. Let $x_q \in X_2$. By the filled function form in equation (3). We obtain

$$P(x, x_k^*) = \frac{1}{1 + \|x - x_k^*\|^2} - \arctan\left[f(x) - f(x_k^*)\right]^2,$$

where the gradient of the function $P(x, x_k^*)$ at point x_q is as follows:

$$\nabla P(x_q, x_k^*) = -\frac{2(x_q - x_k^*)}{\left(1 + \|x_q - x_k^*\|^2\right)^2} - \frac{2(f(x_q) - f(x_k^*))}{1 + (f(x_q) - f(x_k^*))^4} \cdot \nabla f(x_q).$$

Hence,

$$\begin{aligned}
\nabla P(x_q, x_k^*)^T d^* &= -\frac{2(x_q - x_k^*)^T d^*}{\left(1 + \|x_q - x_k^*\|^2\right)^2} - \frac{2(f(x_q) - f(x_k^*))}{1 + (f(x_q) - f(x_k^*))^4} \cdot \nabla f(x_q)^T d^*, \\
\nabla P(x_q, x_k^*)^T (x_q - x_k^*) &= -\frac{2(x_q - x_k^*)^T (x_q - x_k^*)}{\left(1 + \|x_q - x_k^*\|^2\right)^2} - \frac{2(f(x_q) - f(x_k^*))}{1 + (f(x_q) - f(x_k^*))^4} \cdot \nabla f(x_q)^T \cdot (x_q - x_k^*) \\
&= -\frac{2(x_q - x_k^*)^2}{\left(1 + \|x_q - x_k^*\|^2\right)^2} - \frac{2(f(x_q) - f(x_k^*))}{1 + (f(x_q) - f(x_k^*))^4} \cdot \nabla f(x_q)^T \cdot (x_q - x_k^*),
\end{aligned}$$

because $x_q \in X_2$, $f(x_q) < f(x_k^*)$. Thus, we have

$$= -\frac{2(x_q - x_k^*)^2}{\left(1 + \|x_q - x_k^*\|^2\right)^2} + \frac{2(f(x_k^*) - f(x_q))}{1 + (f(x_q) - f(x_k^*))^4} \cdot \nabla f(x_q)^T \cdot (x_q - x_k^*).$$

From the assumption of the theorem, $\nabla f(x_q)^T (x_q - x_k^*) > 0$. Hence, we obtain

$$\nabla P(x_q, x_k^*)^T (x_q - x_k^*) > -2(x_q - x_k^*)^2 + \frac{2(f(x_k^*) - f(x_q))}{1 + (f(x_q) - f(x_k^*))^4} \cdot \nabla f(x_q)^T \cdot (x_q - x_k^*).$$

Because

$$\nabla f(x_q)^T(x_q - x_k^*) \geq \frac{(x_q - x_k^*)^2\left(1 + (f(x_q) - f(x_k^*))^4\right)}{f(x_k^*) - f(x_q)} > 0,$$

$$\nabla P(x_q, x_k^*)^T(x_q - x_k^*) > -2(x_q - x_k^*)^2 + \frac{2(f(x_k^*) - f(x_q))(x_q - x_k^*)^2\left(1 + (f(x_q) - f(x_k^*))^4\right)}{\left(1 + (f(x_q) - f(x_k^*))^4\right)(f(x_k^*) - f(x_q))},$$

$$\nabla P(x_q, x_k^*)^T(x_q - x_k^*) > -2(x_q - x_k^*)^2 + 2(x_q - x_k^*)^2 = 0.$$

Thus, $\nabla P(x, x_k^*)^T d^* > 0$. This proves the theorem.

The aforementioned three theorems demonstrate that the function $P(x, x_k^*)$ satisfies the three filled properties stipulated in Definition 2.2. Furthermore, according to Definition 2.2, within the region X_1, all feasible directions d of the filled function $P(x, x_k^*)$ are descending. Theorem 3.3 states that, somewhere in X_2, d^* will be an ascent direction of filled function $P(x, x_k^*)$. Assuming both $P(x, x_k^*)$ and $f(x)$ are continuously differentiable, it follows that the filled function $P(x, x_k^*)$ possesses a stationary point within the region X_2.

The following theorems are additional properties of $P(x, x_k^*)$. □

Theorem 3.4. *Assume that $x_1, x_2 \in X_1$ and $\|x_2 - x_k^*\| > \|x_1 - x_k^*\|$, then $P(x_2, x_k^*) < P(x_1, x_k^*)$.*

Proof. Since $x_1, x_2 \in X_1$, then

$$P(x_1, x_k^*) = \frac{1}{1 + \|x_1 - x_k^*\|^2},$$
$$P(x_2, x_k^*) = \frac{1}{1 + \|x_2 - x_k^*\|^2}.$$

but $\|x_2 - x_k^*\| > \|x_1 - x_k^*\|$, $P(x_k^*, x_k^*) = 1$, which means that $\|x_2 - x_k^*\|^2 > \|x_1 - x_k^*\|^2$, hence

$$P(x_2, x_k^*) < P(x_1, x_k^*) < P(x_k^*, x_k^*).$$

□

Based on Theorem 3.4, within the region X_1, the value of the filled function decreases as the distance from the current local minimum point increases, thereby aiding in escaping from the current local minimum and continuing the search for a lower optimum.

Theorem 3.5. *Let $x_1, x_2 \in X_2$, $f(x_1) \geq f(x_2)$ and $\|x_2 - x_k^*\| > \|x_1 - x_k^*\|$, then $P(x_2, x_k^*) < P(x_1, x_k^*)$.*

Proof. Since $x_1, x_2 \in X_2$, $f(x_1) \geq f(x_2)$, then $(f(x_1) - f(x_k^*))^2 \leq (f(x_2) - f(x_k^*))^2$, and

$$\begin{aligned} &P(x_2, x_k^*) - P(x_1, x_k^*) \\ &\quad = \frac{1}{1 + \|x_2 - x_k^*\|^2} - \arctan\left(f(x_2) - f(x_k^*)\right)^2 - \left(\frac{1}{1 + \|x_1 - x_k^*\|^2} - \arctan\left(f(x_1) - f(x_k^*)\right)^2\right) \\ &\quad = \frac{1}{1 + \|x_2 - x_k^*\|^2} - \frac{1}{1 + \|x_1 - x_k^*\|^2} + \arctan\left(f(x_1) - f(x_k^*)\right)^2 - \arctan\left(f(x_2) - f(x_k^*)\right)^2 \\ &\quad < 0. \end{aligned}$$

Thus, $P(x_2, x_k^*) < P(x_1, x_k^*)$. □

Theorem 3.6. *Assume that $x_1 \in X_1, x_2 \in X_2$ and $\|x_2 - x_k^*\| > \|x_1 - x_k^*\|$, then $P(x_2, x_k^*) < P(x_1, x_k^*)$.*

Proof. Since $x_1 \in X_1, x_2 \in X_2$, then $f(x_1) \geq f(x_k^*), f(x_2) < f(x_k^*)$. By the filled function form in equation (3), we obtain

$$P(x_1, x_k^*) = \frac{1}{1 + \|x_1 - x_k^*\|^2},$$
$$P(x_2, x_k^*) = \frac{1}{1 + \|x_2 - x_k^*\|^2} - \arctan\left(f(x_2) - f(x_k^*)\right)^2,$$

also $\|x_2 - x_k^*\| > \|x_1 - x_k^*\|$, which means that

$$\frac{1}{1 + \|x_2 - x_k^*\|^2} < \frac{1}{1 + \|x_1 - x_k^*\|^2},$$

thus

$$\begin{aligned} P(x_2, x_k^*) - P(x_1, x_k^*) &= \frac{1}{1 + \|x_2 - x_k^*\|^2} - \arctan\left(f(x_2) - f(x_k^*)\right)^2 - \frac{1}{1 + \|x_1 - x_k^*\|^2} \\ &< 0. \end{aligned}$$

Hence, $P(x_2, x_k^*) < P(x_1, x_k^*)$. □

Theorems 3.1–3.3 indicates that as x diverges from x_k^* the value of the filled function decreases, and points satisfying $f(x) < f(x_k^*)$ can be obtained in the region X_2. During the process of minimizing the filled function, Theorems 3.4–3.6 guarantees that the minimization sequence will converge to a minimum point with a lower objective function value than the current one.

4. The filled function algorithm

In the previous section, through theoretical analysis, it was not only proven that the parameter-free filled function constructed in this paper satisfies Definition 2.2, but also possesses some additional properties. Subsequently, the corresponding filled function algorithm will be constructed utilizing this new filled function. A new filled function algorithm global minimizer of $f(x)$ is proposed as follows:

Algorithm 4.1. Step 0. Choose an initial point $x_k^0 \in X$ and a small positive number $0 < \delta \leq 1$. Make $e_1, e_2, \cdots, e_{2n}$ as the positive and negative coordinate directions, where n is the dimension of the optimization problem. The loop continuation criterion for the algorithm is that $j \leq m, m = 2n$. Let $k = 1$, representing the number of iterations.

Step 1. Apply a local search algorithm to minimize $f(x)$ starting from an initial point $x_k^0 \in X$ and obtain a minimizer x_k^* of $f(x)$.

Step 2. Construct the filled function at the minimizer x_k^*

$$P(x, x_k^*) = \frac{1}{1 + \|x - x_k^*\|^2} - \arctan\Big(\left(\min[0, f(x) - f(x_k^*)]\right)^2\Big).$$

Set j=1, go to step 3.

Step 3. If $j \leq m$, go to step 4; otherwise, the algorithm stops and x_k^* is taken as a global solution of $f(x)$.

Step 4. Let $x = x_k^* + \delta e_j$, where δ is a step size. If $x \in X$, then go to step 5; otherwise, $j = j + 1$, go to step 3.

Step 5. If $f(x) < f(x_k^*)$, then let $k = k + 1$, $x_k^0 = x$, go to step 1; otherwise, go to step 6.

Step 6. Apply a local search algorithm to minimize $P(x, x_k^*)$ starting from x and obtain a minimizer $\widetilde{x_k^*}$, go to step 7.

Step 7. If $f(\widetilde{x_k^*}) > f(x_k^*)$, then let $j = j + 1$, go to step 3; otherwise, go to step 8.

Step 8. Let $k = k + 1$, $x_k^0 = \widetilde{x_k^*}$, go to step 1.

Some explanations about the filled function Algorithm 4.1 can be given as follows:

(1) Based on the aforementioned framework of the filled function algorithm, any effective local search method can be utilized to minimize both the objective function and the filled function. In this paper, the quasi-Newton method is employed to minimize both $f(x)$ and $P(x, x_k^*)$.
(2) In Step 0, n denotes the dimension of the problem, $e_j = (0, \cdots, \underset{j}{1}, \cdots 0), j = 1, 2, \cdots, n$ represents the positive directions and $e_j = -e_{j-n}, j = n+1, \cdots, 2n$ represents the negative directions. Each e_j is a unit vector. Furthermore, you can also choose more search directions to expand the search scope, such as $e = (1, 1), e = (-1, -1)$, but this means higher computational costs.
(3) The first property stated in Definition 2.2 indicates that x_k^* is a local maximum point of the filled function, and therefore cannot be directly utilized as the initial point for minimizing the filled function. For a sufficiently small real number δ and a unit vector e_j along an axial direction, let $x = x_k^* + \delta e_j$ serve as the new initial point for minimizing the filled function.
(4) Step 5 indicates that it does not require to obtain the local minimizers of the filled function $P(x, x_k^*)$; as long as a point $x = x_k^* + \delta e_j$ satisfying $f(x) < f(x_k^*)$ is found, Algorithm 4.1 proceeds to Step 1 to initiate a new iteration.

5. Numerical examples

In this section, in order to verify the feasibility and effectiveness of the proposed algorithm, we use filled functions to solve Problems 5.1–5.7. These functions are widely used test functions, where Problems 5.1–5.5 are two-dimensional, Problems 5.6 and 5.7 are n-dimensional. Compared with the latest algorithms [16] and Liu's algorithm [10]. The algorithm is implemented in Matlab R2022A. Numerical results show the effectiveness of the proposed method. All the results obtained by the new filled function algorithm are shown in Tables 1–8, and are compared with the algorithms mentioned above. For these tables, we use symbols to abbreviate the expression. The meanings of these symbols are as follows:

k	:	The number of iterations.
x_k^*	:	The local minimum point of the k-th iteration.
$f(x_k^*)$	:	The value of local minimum point x_k under f.
x^*	:	The global minimum point of $f(x)$.
$f(x^*)$	:	The global optimal value of $f(x)$.
N	:	The number of function evaluations of $f(x)$ and $P(x, x_k^*)$ before termination.

Problem 5.1. 3-hump back came1 function

$$\begin{aligned} \min\ & f(x) = 2x_1^2 - 1.05x_1^4 + \frac{1}{6}x_1^6 - x_1x_2 + x_2^2, \\ \text{s.t.}\ & -3 \le x_1 \le 3, \\ & -3 \le x_2 \le 3. \end{aligned}$$

This example has three local minimum points within the feasible region, among which $x^* = (0, 0)^T$ with $f(x^*) = 0$. Starting with the initial point $(-1, 1)$, the proposed filled function algorithm succeeds in identifying the global optimal solution. The computational results are summarized in (Tab. 1).

Problem 5.2. 6-hump back camel function

$$\begin{aligned} \min\ & f(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 - x_1x_2 - 4x_2^2 + 4x_2^4, \\ \text{s.t.}\ & -3 \le x_1 \le 3, \\ & -3 \le x_2 \le 3. \end{aligned}$$

TABLE 1. The calculation result of Problem 5.1.

k	Our algorithm		Algorithm in [10]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	(2.243e−8, 8.6013e−8)	6.4883e−15	(−0.2565, −0.4722)	0.2289
2			(−0.4064e−7, −0.3367e−7)	3.0680e−15

TABLE 2. The calculation result of Problem 5.2.

k	Our algorithm		Algorithm in [16]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	(0.0821, 0.7082)	−0.0313	(−1.6071, −0.5686)	2.1403
2	(0.0898, 0.7127)	−1.0316	(0.0898, 0.7127)	−1.0316

TABLE 3. The calculation result of Problem 5.3.

k	Our algorithm		Algorithm in [16]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	(−1.0000, 2.0000e−7)	1.0000	(−1.0000, −0.0000)	2.1403
2	(6.4868e−8, −1.1513e−8)	1.6964e−14	(−0.1039e−6, 0.0005e−6)	4.3211e−14

It has six local minimizers and two of which are global, *i.e.* its global optimal solution is $x^* = (-0.0898, -0.7127)^T$ or $(0.0898, 0.7127)^T$, global optimal value is $f(x^*) = -1.0316$. Starting from the initial point $(-1, 0.5)$, the filled function algorithm constructed in this paper successfully identified the global optimal solution to the problem. The specific solution results are presented in (Tab. 2).

Problem 5.3. Treccani function

$$\begin{aligned} \min\ & f(x) = x_1^4 + 4x_1^3 + 4x_1^2 + x_2^2, \\ \text{s.t.}\ & -3 \le x_1 \le 3, \\ & -3 \le x_2 \le 3. \end{aligned}$$

This example has two local minimum points within the feasible region, among which $x^* = (0, 0)^T$ or $(-2, 0)^T$ with $f(x^*) = 0$. Starting with the initial point $(-1, -2)$, the proposed filled function algorithm succeeds in identifying the global optimal solution. The computational results are summarized in (Tab. 3).

Problem 5.4. Rastrigin function

$$\begin{aligned} \min\ & f(x) = x_1^2 + x_2^2 - \cos(18x_1) - \cos(18x_2), \\ \text{s.t.}\ & -1 \le x_1 \le 1, \\ & -1 \le x_2 \le 1. \end{aligned}$$

This example approximately contains 50 local minima within the feasible region, which $x^* = (0, 0)^T$, with $f(x^*) = -2$. Starting with the initial point $(0.5, 0.5)$, the proposed filled function algorithm succeeds in identifying the global optimal solution. The computational results are summarized in (Tab. 4).

TABLE 4. The calculation result of Problem 5.4.

k	Our algorithm		Algorithm in [16]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	(0.3429, 0.3439)	−1.7578	(1.0408, 1.0408)	0.1798
2	(0.0000, 0.3469)	−1.8769	(1.0408, 0.0000)	−0.9101
3	(−2.2360e−15, −2.6220e−9)	−2.0000	(0.0728e−9, −0.1013e−9)	−2.0000

TABLE 5. The calculation result of Problem 5.5.

c	k	Our algorithm		Algorithm in [10]	
		x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
0.2	1	(1.0860, 1.0269)	4.8917	(8.8344, −3.3354)	9.4093
	2	(0.4919, 0.2703)	8.1752e−15	(1.8784, −0.3458)	1.9805e−15
0.5	1	(0.0420, −0.0948)	0.5175	(3.9733, −1.7709)	3.3347
	2	(0.9432, −0.1746)	8.3538e−16	(1.4013, 0)	0.2264
	3			(1.0000, 0)	0
0.05	1	(1.5975, −0.2874)	1.3604e−14	(4.0604, −2.1833)	7.2212
	2			(1.8513, −0.4021)	1.1414e−12

Problem 5.5. 2-dimensional function

$$\begin{aligned} \min\ & f(x) = [1 - 2x_2 + c\sin(4\pi x_2) - x_1]^2 + [x_2 - 0.5\sin(2\pi x_1)]^2, \\ \text{s.t.}\ & -10 \le x_1 \le 10, \\ & -10 \le x_2 \le 10. \end{aligned}$$

In this problem, the value of c is taken as 0.5, 0.05, and 0.2, respectively. The number of local minimizers depends on the value of c, its global optimal value $f(x^*) = 0$ for c takes any value. When $c = 0.5$ and $c = 0.2, 0.05$, respectively, the filled function algorithm constructed in this paper successfully identifies approximate global optimal solutions for the problem, with initial points set at $(0, 0)$ and $(1, 1)$. The specific computational results are presented in (Tab. 5).

Problem 5.6. n-dimensional function

$$\begin{aligned} \min\ & \frac{\pi}{n}\Big[10\sin^2(\pi x_1) + g(x) + (x_n - 1)^2\Big], \\ \text{s.t.}\ & -10 \le x_i \le 10, i = 1, 2, \cdots, n. \end{aligned}$$

This is a high-dimensional problem, in which $g(x) = \sum_{i=1}^{n-1}[(x_i - 1)^2(1 + 10\sin^2(\pi x_{i+1}))]$. This problem contains approximately 10^n local minima within the feasible region, where $x^* = (1, 1, \cdots, 1)^T, f(x^*) = 0$. When $n = 3, 5, 10$ are taken respectively, the filled function algorithm constructed in this paper successfully finds the global optimal solution of the problem with $(-3, -3, -3)$, $(2, 3, 2, 1, -2)$ and $(3, 6, 5, 5, 5, \cdots, 9)$ as the initial points respectively. The specific solution results are as follows (Tabs. 6–8).

Problem 5.7. n-dimensional rastrigin function

$$\begin{aligned} \min\ & f(x) = 10n + \sum_{i=1}^{n}\big(x_i^2 - 10\cos(2\pi x_i)\big), \\ \text{s.t.}\ & -5.12 \le x_i \le 5.12, \qquad i = 1, 2, \cdots, n. \end{aligned}$$

TABLE 6. The calculation result of Problem 5.6 when $n = 3$.

k	Our algorithm		Algorithm in [10]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	$(-0.9797, -2.9897, -2.9975)$	37.6032	$(-0.5665, -4.3568, -2.8141)$	147.6024
2	$(1.0000, 1.0000, 1.0000)$	1.0392e−14	$(1.9900, 1.0000, 1.0000)$	1.0367
3			$(1.0000, 1.0000, 1.0000)$	1.2257e−13

TABLE 7. The calculation result of Problem 5.6 when $n = 5$.

k	Our algorithm		Algorithm in [10]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	$(1.9895, 2.9794, 1.9974, 1.0000, 1.0000)$	3.7362	$(4.5092, 4.9502, -0.8846, -3.0831, 5.4416)$	165.8637
2	$(1.0000, 1.0000, 1.0000, 1.0000, 1.000)$	2.2153e−13	$(1.9900, 1.0000, 1.0000, 1.0000, 1.0000)$	0.6220
3			$(1.0000, 1.0000, 1.0000, 1.0000, 1.0000)$	8.6588e−13

TABLE 8. The calculation result of Problem 5.6 when $n = 10$.

k	Our algorithm		Algorithm in [10]	
	x_k^*	$f(x_k^*)$	x_k^*	$f(x_k^*)$
1	$(1.0095, 0.4583, 1.1101, 3.3406, 3.9862, 3.9955, 3.9879, 3.9939, 3.9931, 6.9828)$	27.4404	$(6.2612, 7.8479, 2.3701, 0.8488, 8.7385, 0.0580, 0.4644, 7.5710, 7.2606, 2.2505)$	359.2339
2	$(1.0006, 1.1237, 0.6874, 0.9455, 1.1969, 0.8760, 3.5298, 3.9907, 3.9978, 6.9858)$	19.1386	$(1.0013, 0.5326, -0.9803, 1.0066, 1.0152, 0.9937, 0.5966, 1.0862, 0.9954, -0.1773)$	1.8292
3	$(0.9886, 1.3851, 1.0373, 0.9900, 0.8513, 1.3041, 1.0852, 0.8539, 1.0057, 2.0251)$	0.5042	$(1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000)$	1.4103e−13
4	$(1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000)$	1.5113e−13		

This is an n-dimensional multimodal and non-convex function. Its feasible region contains multiple local minima. For different dimensions n, its global minimum is $x^* = (0, 0, \cdots, 0), f(x^*) = 0$. We solve this problem for $n = 10, 20, 30, 50$. For $n = 10, f(x^*) = 1.7200\text{e}{-9}$; For $n = 20, f(x^*) = 1.9189\text{e}{-9}$; For $n = 30, f(x^*) = 1.9310\text{e}{-9}$; For $n = 50, f(x^*) = 2.0230\text{e}{-9}$.

From Tables 1 to 8, it can be observed that the filled function algorithm constructed in this paper, as well as the filled function algorithms presented in [10, 16], are all capable of finding the global optimal solution or an approximate global optimal solution to the problem, and the accuracy of the optimal solution is similar, which shows that our algorithm is feasible. In order to further illustrate the effectiveness of the algorithm, the numerical results of this paper are compared with the results in [10, 16] (see Tab. 9). It can be seen that the proposed algorithm can find the global minimum value of the objective function in a small number of iterations, and the number of iterations is better than or not inferior to the other two algorithms. For the high-dimensional

TABLE 9. Compared with [16] and [10].

Pro	n	Our algorithm		Algorithm in [16]		Algorithm in [10]	
		k	N	k	N	k	N
1	2	1	357	–	–	2	378
2	2	2	673	2	180	2	277
3	2	2	366	2	591	2	259
4	2	3	619	3	453	3	553
5($c = 0.2$)	2	2	1449	3	2631	2	392
5($c = 0.5$)	2	2	878	2	150	3	470
5($c = 0.05$)	2	1	725	5	3129	2	493
6	3	2	3098	4	1036	3	962
	5	3	3535	6	3582	2	2287
	7	2	3997	3	4554	2	2250
	10	4	10 316	12	29 487	5	12 795
7	10	12	17 550	–	–	–	–
	20	19	20 980	24	24 099	–	–
	30	33	54 626	35	44 668	–	–
	50	52	602 665	–	–	–	–

TABLE 10. Compared with [12].

Pro	n	Our algorithm			Algorithm in [12]		
		f_{best}	f_{mean}	*sr*	f_{best}	f_{mean}	*sr*
1	2	1.4368e−14	1.6579e−14	1	3.0312e−16	3.211e−14	1
2	2	−1.0316	−1.0316	1	−1.0316	−1.0316	0.9
3	2	2.1285e−14	2.0745e−12	1	6.5032e−18	2.5313e−14	1
4	2	−2.0000	−2.0000	0.8	–	–	–
5($c = 0.2$)	2	1.1272e−15	5.0978e−13	1	7.3507e−16	0.035469	1
5($c = 0.5$)	2	6.4105e−16	3.3974e−14	1	1.4692e−15	0.0019613	1
5($c = 0.05$)	2	4.3852e−14	8.8348e−14	0.7	1.33e−15	1.9309e−15	0.3
6	3	1.8216e−14	1.7499e−13	1	–	–	–
	5	4.5190e−13	2.0466e−7	1	–	–	–
	10	1.5174e−13	8.3078e−10	0.8	–	–	–
7	10	8.9327e−10	2.4981e−9	1	–	–	–
	20	1.7196e−9	1.8944e−9	1	–	–	–
	30	1.6637e−10	1.9128e−9	0.7	–	–	–

problem of Problem 5.7, it can also be successfully optimized. From the perspective of the number of function evaluations (N), our algorithm shows advantages over the algorithm in [16] in most cases, and is superior to the algorithm in [10] in a few cases. From the overall performance point of view, our algorithm is feasible and effective.

For each test function, the results are averaged in 10 independent runs at random. Table 10 shows the comparison results. The symbolic meanings used in the table are as follows: f_{best}: the best function value for 10 runs, f_{mean}: the average function value, sr: success rate. The algorithm in [12] is numerically stable and has a high success rate in ten random tests. In the first five test functions, the algorithm in this paper can achieve the same accuracy as that in [12], and the success rate is also similar. Even in the fifth test $c = 0.05$, there is a higher success rate. When the problem dimension increases, the success rate decreases because the search space

increases exponentially, which increases the difficulty of the local search algorithm, but it can also achieve a success rate of 70%. It can be considered that our algorithm is stable.

6. An application of the filled function algorithm

With the rapid advancement of urbanization in China, how to balance soil and water conservation in urban expansion has become an important issue that needs to be solved urgently in urban planning and construction. Soil and water conservation projects cover site leveling, embankment, embankment and underground engineering, etc., and these projects must consider the allocation of earthwork. The problem of earthwork allocation can usually be regarded as a transportation problem in management operations research. Taking a residential project in Yangzhou City mentioned in [30] as an example, an appropriate model is established and mathematical methods are used to solve the optimal scheme of earthwork allocation to achieve the purpose of minimizing the allocation cost.

The cost of earthwork allocation is mainly composed of comprehensive unit price per unit volume, transportation distance and earthwork allocation amount. The symbols and meanings of each parameter used in the model establishment are as follows: W_i: Excavated earthwork volume in excavation area i, T_i: The amount of earthwork required for filling area i, X_{ij}: Amount of earthwork transported from zone i to zone j, C_{ij}^w, C_{ij}^y and C_{ij}^t represent the excavation unit price, transportation unit price and filling unit price from excavation area i to filling area j respectively. C_{ij}: Comprehensive unit price of earthwork from excavation area i to filling area j, S_{ij}: The transportation distance from excavation area i to filling area j.

The empirical formula of the comprehensive unit price of earthwork allocation in [31] is $C_{ij} = C_{ij}^w + P_{ij}(C_{ij}^y \times S_{ij} + C_{ij}^t)$. However, in the identified items, because C_{ij} is a fixed value function independent of traffic volume, and its value is given by the project team, C_{ij} can be regarded as a constant.

The earthwork construction of a residential project in Yangzhou is divided into building area, road square area and greening area [30]. Among them, the building area includes foundation excavation section W_1/T_1, site leveling section W_2/T_2, site dredging section W_3/T_3; the road square is divided into pipeline construction section W_4/T_4, site leveling section W_5/T_5, site backfilling section W_6/T_6; the greening area is divided into greening soil section W_7/T_7, green space excavation section W_8/T_8, green space backfill section W_9/T_9. The earthwork volume of excavation and backfilling of this project is shown in Table 11 (Unit: $10^4\,\mathrm{m}^3$). Table 12 (Unit: kilometers) shows the transport distance between different sections.

According to the above content, the linear programming function model of total freight is established as follows:

$$f(X_{ij}) = \sum_{i=1}^{9}\sum_{j=1}^{9} C_{ij}X_{ij}S_{ij},$$

$$\begin{cases} \sum_{j=1}^{9} X_{ij} = W_i, & i = 1, 2, \cdots, 9; \\ \sum_{i=1}^{9} X_{ij} = T_j, & j = 1, 2, \cdots, 9; \\ X_{ij} \geq 0, & i = 1, 2, \cdots, 9; j = 1, 2, \cdots, 9. \end{cases}$$

Because in the determined project, C_{ij} is a constant, the penalty function method is used to transform it into an unconstrained optimization problem:

$$g(X_{ij}) = f(X_{ij})/C_{ij} + kp(X_{ij}) \tag{4}$$

where k is the penalty factor and $p(X_{ij})$ is a penalty function. Let the constraint function:

$$h_i(X_{ij}) = \sum_{j=1}^{9} X_{ij} - W_i, \qquad i = 1, 2, \cdots, 9;$$

TABLE 11. Excavation and filling volumes by construction section [30].

Section	Volume	Section	Volume	Section	Volume
W_1/T_1	7.10/0.00	W_4/T_4	0.52/0.52	W_7/T_7	0.00/1.25
W_2/T_2	0.00/0.00	W_5/T_5	0.00/0.45	W_8/T_8	0.00/6.03
W_3/T_3	1.91/0.00	W_6/T_6	0.00/2.96	W_9/T_9	1.68/0.00

TABLE 12. Transportation distance between different sections S_{ij} [30].

Origin (i)	Destination (j)								
	1	2	3	4	5	6	7	8	9
1	0.00	0.35	0.36	1.43	1.48	1.70	1.21	1.30	1.35
2		0.00	0.27	1.34	1.37	1.58	1.24	1.37	1.44
3			0.00	0.99	1.10	1.39	1.27	1.40	1.55
4				0.00	0.60	0.68	1.10	1.16	1.22
5					0.00	0.50	1.11	1.26	1.33
6						0.00	1.07	1.26	1.36
7							0.00	0.62	0.71
8								0.00	0.53
9									0.00

$$h_{9+j}(X_{ij}) = \sum_{i=1}^{9} X_{ij} - T_j, \qquad j = 1, 2, \cdots, 9.$$

that is $p(X_{ij}) = \sum_{t=1}^{18} h_t^2(X_{ij})$. The implicit condition is $X_{ij} \geq 0$.

Because the excavation and filling in the same section does not involve the amount of transportation deployment, so $X_{ii} = 0$; if the excavation amount W_i in area i is 0.00, then all X_{ij} transferred from area i is 0. Similarly, if the filling amount T_i in area j is 0.00, then all X_{ij} transferred to area j is 0.00. Then the problem (4) is simplified, and the problem (4) is solved by the filled function algorithm in this paper:

$$(X_{15}, X_{16}, X_{17}, X_{18}, X_{36}, X_{37}, X_{38}, X_{96}, X_{97}, X_{98}) = (0.45, 1.7, 0.69, 4.26, 0.84, 0.4, 0.67, 0.42, 0.16, 1.1).$$

If we assume that the comprehensive unit price of transportation is 1 CNY/(m^3·km), the comprehensive freight cost obtained by our algorithm is 13.81. Compared with the result of 13.95 obtained by the table operation method, our algorithm is better, but compared with the result 13.75 obtained in [30], our algorithm is slightly inferior. In general, our algorithm can be used to solve the problem of earthwork allocation.

7. CONCLUSION

The filled function method is a deterministic method for solving global optimization. It has been proved to be an effective method for solving the global optimal solution of unconstrained optimization problems. In this paper, a new continuous differentiable parameter-free filled functions for solving global optimization problems is constructed. The filled function has no exponential term and logarithmic term, which overcomes the defect of numerical overflow. At the same time, it also avoids the difficulty of parameter adjustment. It is proved that the method conforms to the basic properties of the definition of filled function and some additional good analytical properties. The proposed algorithm is tested on some benchmark functions and applied to the earthwork allocation problem. Numerical experiments show that the algorithm is feasible and effective.

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Data availability statement

The research data associated with this article are included in the article.

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