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REMANUFACTURING AUTHORIZATION AND BLOCKCHAIN PLATFORM JOINING STRATEGIES FOR A COMPETITIVE SUPPLY CHAIN

Ronghua Lu* and Yisheng Wu

Abstract. Authorized remanufacturing has become a potential strategy for original equipment manufacturers (OEMs) facing competition from third-party remanufacturers (TPRs). However, the lack of transparency in the remanufacturing process reduces consumers' willingness-to-pay (WTP) for remanufactured products. Blockchain technology, with its traceability and tamper-proof features, enhances the transparency of remanufacturing, thereby increasing consumers' WTP for remanufactured products. Yet, the use of blockchain may also raise concerns about privacy leakage among consumers. Based on this, this paper constructs a supply chain game-theoretic model involving an OEM, a TPR, and a retailer to investigate the interplay between remanufacturing authorization strategies and blockchain adoption strategies. Four game-theoretic models are established, considering whether the TPR joins the retailer's blockchain platform and which authorization mode is selected. The results show that the OEM's choice of remanufacturing authorization strategy is jointly influenced by the cost advantage of remanufacturing, the fixed authorization fee, and the cost of blockchain adoption. For the TPR, joining the retailer's blockchain platform is beneficial under both authorization strategies when the unit variable cost of blockchain adoption and consumers' privacy concerns are low.

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1. Introduction

Sustainable development emphasizes both corporate growth and environmental protection [19, 34, 40, 41]. Remanufacturing, which is dedicated to restoring used products to a condition similar to their original state, provides a viable approach for sustainable development, offering significant economic and environmental benefits [11, 39]. Compared to new products, remanufactured products can save up to 50% in costs, 60% in energy, and 70% in materials and cut emissions by up to 80% [13, 48]. The profit potential of remanufacturing has attracted many TPRs [14, 27]. However, TPR-produced remanufactured items might pose competitive threats to new products and erode their market share [6]. In the face of competition from TPRs, OEMs are confronted with the choice of whether to engage in remanufacturing themselves or collaborate with TPRs [45]. On one hand, if an OEM lacks proprietary remanufacturing technology or considers the inherently lower profitability of remanufactured products compared to new ones, it may authorize remanufacturing activities to TPRs [3]. On the other hand, to avoid intellectual property disputes and brand issues, authorizing remanufacturing becomes a feasible option for companies engaging in this practice [25, 59]. As a result, authorized remanufacturing is increasingly

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School of Management Engineering, Nanjing Institute of Technology, Nanjing 211167, P.R. China.
*Corresponding author: lrhlrx@163.com

favored by businesses. For example, Apple entrusts its remanufacturing operations to Foxconn, who sells the remanufactured products through its exclusive channels [59]. In reality, there are different strategies of remanufacturing authorization, including the fixed authorization fee or the unit authorization fee strategy [20]. For example, since 2015, AISIN Group has authorized Huadu to remanufacture and sell transmissions by charging a unit authorization fee [13]. Different remanufacturing authorization strategies have distinct mechanisms through which they affect firms' profits [30]. Therefore, investigating how to adopt appropriate authorization strategies to balance the competitive and cooperative dynamics between the OEM and the TPR is of significant practical relevance.

Although remanufacturing offers numerous advantages, the consumer market for remanufactured products has not developed as smoothly as anticipated. The global remanufacturing market is valued at over $100 billion annually, yet the market share of remanufactured products is only about one-tenth of that [48]. A significant part of the reason lies in the lack of transparency in the production process of remanufactured items, inadequate promotion, and consumers' doubts about the quality of remanufactured products [1], leading to lower WTP for them [2]. The application of blockchain technology makes all aspects of remanufacturing secure, transparent, and accurate, enhancing consumers' trust in remanufactured products, thereby ensuring better acceptance when purchasing these items [4, 10]. For instance, PaPa Second-hand Goods Trading Platform uses blockchain technology to provide quality inspection information for second-hand mobile phones and other products, reducing consumers' concerns about quality issues [52]. However, adopting blockchain still faces challenges such as cost, privacy breaches, and legal issues [18]. Sharing one's own data on the blockchain may place a company at a competitive disadvantage [9]. In practice, remanufacturing enterprises might choose to cooperate with retail companies to sell their remanufactured products through these retailers. Brands like HP and Canon sell their remanufactured printers *via* platforms like Tmall [16], and remanufactured toner cartridges from various brands are sold through Amazon [44]. At the same time, with increasing competition in the retail sector, many retail companies opt to establish blockchain platforms to enhance consumer trust in products and increase sales. Companies such as JD.com, Walmart, and Amazon have all built blockchain traceability platforms [16, 21, 23]. However, not all suppliers will join the blockchain platforms of retailers [7]. Under what conditions will the TPR join the retailer's blockchain platform? Specifically, when the TPR joins the retailer's blockchain platform, it will transform the remanufactured product market, consequently affecting the remanufacturing authorization fees under the unit licensing fee strategy. This then raises the critical question: Which authorization fee strategy (unit *versus* fixed) should the OEM select to maximize its advantages? These issues warrant further investigation.

In summary, although blockchain technology can enhance consumer trust in remanufactured products, it may also introduce risks such as the potential leakage of privacy information. Secondly, when the OEM and TPR cooperate under an authorization contract, the authorization fee under the unit remanufacturing authorization strategy varies with the market share of remanufactured products. In this context, blockchain technology will influence the authorization fee through its effect on the market share of remanufactured products. Therefore, this paper specifically addresses three key research questions:

(1) What impact does blockchain technology have on the market share of remanufactured products?
(2) Which authorization strategy (unit *versus* fixed) optimizes benefits for the OEM?
(3) How do different authorization strategies affect the TPR's decisions to join the retailer's blockchain platform?

In view of this, this paper establishes a supply chain comprising an OEM, a TPR, and a retailer, where the OEM manufactures new products while authorizing the TPR to produce remanufactured products, with both products sold through the retailer. Based on this, considering consumer preferences and privacy concerns, we analyze the decisions regarding remanufacturing authorization methods and blockchain platform adoption, first examining the OEM's authorization strategy and then investigating the necessary conditions for the TPR to join the retailer's blockchain platform. The main research findings are summarized as follows: For Question (1), we find that the impact of blockchain technology on the market share of remanufactured products is determined by the combined effect of three key factors: the unit cost of joining the blockchain platform, the consumer privacy concern cost, and the value discount multiplier. For Question (2), we conclude that the OEM's authorization strategy selection exhibits differential decision-making patterns based on the TPR's blockchain joining status. When the TPR abstains from joining the retailer's blockchain platform, the OEM's strategy determination primarily depends on its production cost and fixed authorization fee. However, when the TPR joins the blockchain platform, the decision framework expands to incorporate three additional critical factors – the unit cost of joining blockchain platform, the consumer privacy concern cost, and the value discount multiplier. For Question (3), our research indicates that the TPR's decision to join the retailer's blockchain platform is independent of the OEM's authorization strategies but exhibits significant correlation with the OEM's production cost of new products.

The principal contribution of this research can be summarized as follows: Through a systematic investigation of the tripartite interaction among blockchain technology, remanufacturing authorization strategies, and consumer behavior, this study elucidates the causal mechanisms linking technological innovation, institutional design, and user behavior. It provides a comprehensive theoretical framework for optimizing remanufacturing supply chain systems through blockchain-enabled restructuring. The literature most closely related to our work includes several key studies. First, Zhang *et al.* [56] primarily examined how blockchain technology influences the choice between remanufacturing outsourcing and authorization models. Similarly, Yang *et al.* [52] investigated blockchain's impact on cooperation and competition modes between an OEM and a TPR. However, these studies were limited to examining unit authorization strategy, whereas our research systematically analyzes various remanufacturing authorization strategies, specifically comparing unit *versus* fixed authorization strategy in the blockchain-enabled environment. Second, while Liu *et al.* [30] explored remanufacturing authorization strategy selection under conditions where blockchain affects consumer search behavior, our study makes a distinct contribution by examining how blockchain adoption simultaneously alleviates consumer concerns about remanufactured product quality and increases privacy risks, thereby presenting the OEM with more complex authorization strategy decisions. Third, existing studies (such as [30, 52, 56]) have predominantly focused on OEM-TPR dyadic relationships. Our research extends this theoretical framework by incorporating a retailer as a decision maker, enabling simultaneous analysis of the OEM's authorization strategy selection and the TPR's decisions regarding joining the retailer-owned blockchain platform.

2. Literature review

The main areas of relevant literature related to the subject of this paper include: External competition in remanufacturing, remanufacturing authorization strategies, and the application of blockchain in remanufacturing.

2.1. External competition in remanufacturing

Research on competition between OEMs and TPRs primarily began with Majumder and Groenevelt [33], who found that the profit of the OEM increases with the rise in remanufacturing cost, indicating that if the cost saving from remanufacturing is not sufficiently high, remanufactured products will not have a market. In recent studies, Yang *et al.* [51] discovered that higher anticipated regret sensitivity could partially alleviate competition between new and remanufactured products. Sun and Liu [45] found that the cap-and-trade mechanism can promote the OEM entry into remanufacturing. Qian *et al.* [39] investigated the impact of digital advertising strategies on the competition between manufacturers and remanufacturers. Du *et al.* [12] incorporated compatible manufacturers into the competitive framework between OEMs and TPRs, and found that a compatible manufacturer can successfully enter the market by adopting a low-price strategy. Li *et al.* [25] and Liu *et al.* [30] examined the effect of authorization on the competitive strategies of the OEM and TPR. Based on the above studies, our paper primarily focuses on the competitive and cooperative strategies among the OEM, TPR, and retailer when the OEM authorizes remanufacturing to the TPR and both new and remanufactured products are sold through the retailer.

2.2. Remanufacturing authorization strategies

A comparative analysis was conducted between the fixed and unit authorization fee strategies for remanufacturing authorization. Hong *et al.* [20] focused on the scenario where both the OEM and the TPR recover used products for remanufacturing, and concluded that the optimal authorization strategy for the OEM depends on the threshold of the fixed authorization fee. Zhao *et al.* [57] compared the pricing, service, and recovery strategies under OEM remanufacturing *versus* retailer remanufacturing under technology authorization by the OEM, finding that the remanufacturing model where the retailer pays the fixed technology authorization fee is optimal. Building upon this, Liu *et al.* [27] considered the perspective that patent authorization can improve the quality of remanufactured products and studied the optimal patent authorization strategy for the OEM. Lin *et al.* [26] found that in the context of the dynamics of remanufacturing, in-house strategy is better than outsourcing strategy. Esenduran *et al.* [13] compared laissez-faire and mandatory authorization for third-party remanufacturing and found that the choice of which policy to use depends largely on the remanufacturing cost. However, the aforementioned studies did not consider the impact of blockchain technology on remanufacturing authorization strategies. Although Yang *et al.* [52] simultaneously considered the impact of the authorization mechanism and blockchain on the choice of remanufacturing models, their focus was solely on the effect of the fixed authorization fee on the competition and cooperation between the OEM and TPR. Liu *et al.* [30] also investigated the interaction between authorization strategies and blockchain adoption in a competitive remanufacturing supply chain, but their focus was primarily on how blockchain affects consumer search behavior. In contrast, our study examines the interplay between remanufacturing authorization and blockchain under the conditions where blockchain enhances consumers' WTP for remanufactured products but also introduces privacy leakage risks.

2.3. Blockchain adoption in remanufacturing

Regarding the implementation of blockchain technology in remanufacturing business, Tozanlı *et al.* [47] proposed a new method for obtaining optimal incentives for trade-in products based on a blockchain-supported disassembly ordering system. Considering the influence of online platforms, Ma *et al.* [31] and Ma and Hu [31] used differential game models to explore the impact of blockchain adoption on the operations of closed-loop supply chains and platform sales model choices. Xu *et al.* [50] assuming that the adoption of blockchain can reduce remanufacturing cost, investigated the influence of carbon cap-and-trade regulation on the manufacturer's blockchain adoption strategies. Zhang *et al.* [56] mainly studied the impact of blockchain technology on the selection between outsourcing or authorizing remanufacturing models. Closely related to our study, several works focus on the issue of competition between the OEM and TPR under the premise that blockchain adoption can alleviate consumer distrust of remanufactured products. Niu *et al.* [36] considering consumer risk aversion and distrust in remanufactured product quality, studied the blockchain adoption strategies of the supplier and manufacturer. Yang *et al.* [52] investigated the choice between cooperative and competitive models for the OEM and TPR when the use of blockchain can eliminate consumers' concerns about the quality of remanufactured products. Wang *et al.* [48] taking into account varying consumer preferences for new and remanufactured products under different blockchain adoption strategies, established a reverse supply chain game model involving the OEM and recycler, studying optimal blockchain adoption strategies under different levels of quality distrust and unit blockchain costs. Gong *et al.* [17] researched the optimal blockchain adoption and distribution channel choice strategies for the OEM, concluding that, whether through direct sales or retail, the OEM will adopt blockchain technology only when the value discount for remanufactured products produced by the OEM is high and the unit cost of adopting blockchain technology is low. Liu *et al.* [30] investigated the interaction between licensing strategies and blockchain adoption in a competitive remanufacturing supply chain consisting of a manufacturer and a remanufacturer by considering consumer search behavior. While the aforementioned studies mainly examined scenarios where the TPR joins the OEM's blockchain platform or builds his own blockchain platform, whereas this paper focuses on examining whether TPR joins the retailer's blockchain platform under remanufacturing authorization strategies.

Table 1. Comparison of our paper with related literature.

Literature	Competition between OEMs and TPRs	Consumer preference	Fixed authorization fee strategy	Unit authorization fee strategy	Blockchain adoption in remanufacturing	Blockchain platform built by retailers
Yang *et al.* [51]; Sun and Liu [45]	√	√				
Li *et al.* [25]	√	√		√		
Hong *et al.* [20]	√		√	√		
Zhao *et al.* [57]; Liu *et al.* [27]		√	√	√		
Esenduran *et al.* [13]	√	√		√		
Yang *et al.* [52]; Zhang *et al.* [56]	√	√		√	√	
Xu *et al.* [50]					√	
Niu *et al.* [36]		√			√	
Wang *et al.* [48]		√			√	
Gong *et al.* [17]	√	√			√	
Liu *et al.* [30]	√	√	√	√	√	
This paper	√	√	√	√	√	√

Table 1 summarizes the differences between our paper and some important related papers.

3. Problem description and assumptions

Assume that the remanufacturing supply chain consists of an OEM, a TPR, and a retailer. The OEM only produces new products, and authorizes the TPR to manufacture remanufactured products while charging a certain authorization fee. The OEM has two remanufacturing authorization strategies available: one is the fixed authorization fee strategy, where the TPR pays a fixed fee F to the OEM, regardless of the quantity of remanufactured products; the other is the unit authorization fee strategy, where the TPR pays a unit fee f to the OEM for each remanufactured product sold. New and remanufactured products compete in the same market and are sold through the retailer. Facing the competitive threat from new products, if the TPR uses blockchain technology to disclose information about remanufactured products, it will provide more information to consumers, making them more willing to purchase remanufactured products. Assume that the retailer has already established a blockchain platform, and the TPR needs to decide whether to join the retailer's blockchain platform.

According to the above assumptions, based on the OEM's remanufacturing authorization strategy choice and the TPR's decision to join the blockchain platform, there are four models. Use superscript ij to denote a specific model, where $i = \{F, R\}$ represents the OEM's fixed authorization fee strategy and unit authorization fee strategy, respectively; and $j = \{N, B\}$ represents whether the TPR does not join and joins the retailer's blockchain platform. Use subscript $k = \{M, T, R\}$ to indicate the OEM, TPR, and retailer, respectively. Figure 1

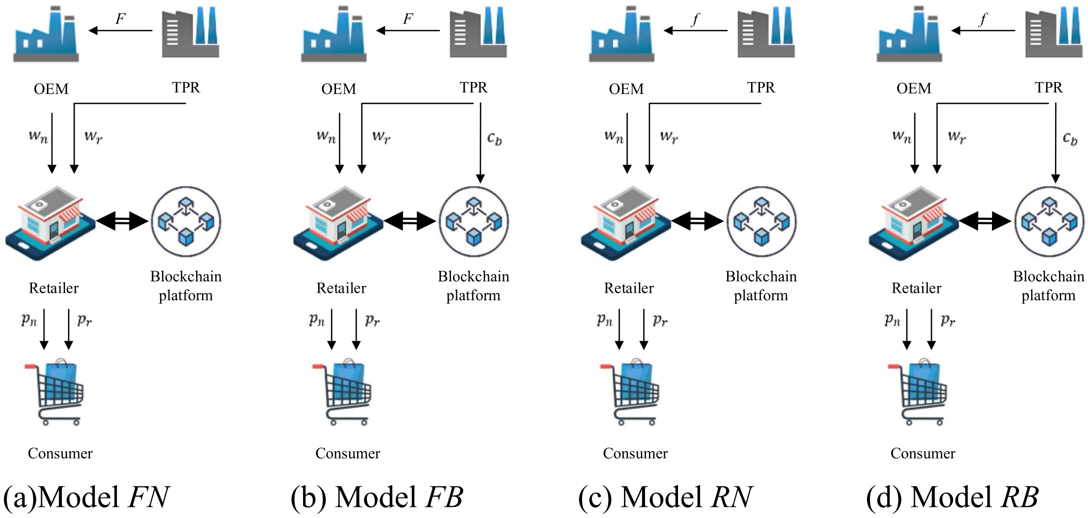


FIGURE 1. Supply chain structure.

illustrates the supply chain structure under different authorization strategies and blockchain platform joining strategies.

To ensure that the model settings are reasonable and easy to handle, this paper makes the following assumptions regarding the authorization strategy for remanufactured products, the strategy for joining blockchain platforms, consumer preferences, pricing strategies, cost structures, and the sequence of decision-making.

Remanufacturing authorization strategy. As previously mentioned, when the OEM authorizes the TPR to conduct remanufacturing, there are two strategies: the fixed authorization fee strategy and the unit authorization fee strategy. The amounts of the fixed authorization fee F and the unit authorization fee f are determined by the OEM, and it is assumed that the unit authorization fee f is an endogenous variable [22]. In the unit authorization fee strategy, the OEM will determine the optimal unit authorization fee f.

Blockchain platform joining strategy. This article primarily considers the scenario where the retailer has already established a blockchain platform. To establish consumer trust, OEMs proactively disclose comprehensive product specifications, quality certifications, after-sales service information, etc. Consequently, the market information transparency of new products is relatively high. Since this paper primarily focuses on the blockchain platform joining strategy of the remanufacturer, the scenario of the OEM joining the blockchain platform is not considered. Liu *et al.* [29] made similar assumption in their study. Compared with new products, the opaque information of remanufactured products makes it difficult for consumers to accurately assess their actual quality, leading to a common perception that their quality is inferior to that of new products [43]. Consequently, in the scenario where the retailer has already established a blockchain platform, the TPR considers whether to disclose the true quality of its products through this blockchain platform to gain more consumer trust and enhance purchasing willingness [5, 7, 31, 35]. If the TPR joins this blockchain platform, there will be certain costs involved [8, 24], such as operational costs [7, 53], changes or standardization of business processes, and higher energy consumption [7]. We assume the unit cost is c_b. To focus on reflecting the TPR's blockchain platform joining strategy, consistent with the research by Zhang *et al.* [54], Niu *et al.* [35] and Chen *et al.* [7], it is assumed that the costs for the retailer to establish and operate the blockchain platform are zero.

Consumer preference. Assuming the market size is 1, consumers' WTP for a new product, denoted as θ, is uniformly distributed within the interval $[0, 1]$ [15, 25]. Due to the "remanufactured" label on remanufactured products, consumers perceive their quality to be lower than that of new products [3, 48, 51]. Therefore, when the TPR does not participate in the retailer's blockchain platform (Model *FN* and Model *RN*), the WTP for remanufactured products is less than that for new products, represented by a discount $\delta\theta$, where $\delta \in (0,1)$ indicates the value discount consumers are willing to accept for remanufactured products. Thus, the utility for consumers purchasing new and remanufactured products can be expressed as $u_n = \theta - p_n$ and $u_r = \delta\theta - p_r$, respectively, where p_n and p_r represent the selling prices of new and remanufactured products, respectively. According to utility maximization theory, when $u_n > u_r$ (*i.e.*, $\theta > \frac{p_n - p_r}{1-\delta}$), consumers purchase new products; when $0 < u_r < u_n$ (*i.e.*, $\frac{p_r}{\delta} < \theta < \frac{p_n - p_r}{1-\delta}$), consumers purchase remanufactured products. Thus, the demand functions for new and remanufactured products are $q_n = \int_{\frac{p_n - p_r}{1-\delta}}^{1} \mathrm{d}\theta = 1 - \frac{p_n - p_r}{1-\delta}$ and $q_r = \int_{\frac{p_r}{\delta}}^{\frac{p_n - p_r}{1-\delta}} \mathrm{d}\theta = \frac{p_n - p_r}{1-\delta} - \frac{p_r}{\delta}$, respectively. Consequently, the inverse demand functions are $p_n = 1 - q_n - \delta q_r$ and $p_r = \delta(1 - q_n - q_r)$, where q_n and q_r represent the sales quantities of new and remanufactured products, respectively [17, 49].

When the TPR joins the retailer's blockchain platform (Model *FB* and Model *RB*), consumers gain access to truthful information about remanufactured products, which increases their WTP but still keeps it below that of new products, *i.e.*, $\lambda\delta\theta(\lambda > 1, \lambda\delta < 1)$, where λ represents the multiplier on the value discount consumers are willing to pay for remanufactured products [17, 36, 48, 49, 52]. Moreover, when consumers purchase remanufactured products supported by blockchain technology, there might be concerns over privacy issues [38]. Assume the cost of consumer privacy concern is t, with higher values of t indicating greater concern over privacy and information security [28, 42, 54, 55]. Under these assumptions, the utilities for consumers purchasing new and remanufactured products become $u_n = \theta - p_n$ and $u_r = \lambda\delta\theta - p_r - t$. The utility maximization conditions change to when $u_n > u_r$ (*i.e.*, $\theta > \frac{p_n - p_r - t}{1-\lambda\delta}$), consumers choose new products; when $0 < u_r < u_n$ (*i.e.*, $\frac{p_r + t}{\lambda\delta} < \theta < \frac{p_n - p_r - t}{1-\lambda\delta}$), consumers choose remanufactured products. The corresponding demand functions for new and remanufactured products are $q_n = \int_{\frac{p_n - p_r - t}{1-\lambda\delta}}^{1} \mathrm{d}\theta = 1 - \frac{p_n - p_r - t}{1-\lambda\delta}$ and $q_r = \int_{\frac{p_r + t}{\lambda\delta}}^{\frac{p_n - p_r - t}{1-\lambda\delta}} \mathrm{d}\theta = \frac{p_n - p_r - t}{1-\lambda\delta} - \frac{p_r + t}{\lambda\delta}$. Thus, the inverse demand functions become $p_n = 1 - q_n - \lambda\delta q_r$ and $p_r = \lambda\delta(1 - q_n - q_r) - t$.

Pricing strategy and cost structure. The OEM and TPR provide the retailer with wholesale prices of new and remanufactured products, denoted as w_n and w_r, respectively. The retailer determines the selling prices of new and remanufactured products, which are p_n and p_r, respectively. The unit production cost of new products is c_n, and the unit remanufacturing cost of remanufactured products is c_r, with $c_r < \min(\delta c_n, \delta\lambda c_n - c_b - t)$ [17, 46, 52, 58]. This assumption guarantees the economic viability of remanufacturing while enabling the TPR to maintain cost competitiveness in remanufacturing operations.

Sequence of decisions. Assume that the OEM, TPR, and retailer engage in a Stackelberg game, where the OEM acts as the leader [17, 30]. In the first stage, the OEM determines the appropriate remanufacturing authorization strategy and the optimal wholesale price of new products. In the second stage, the TPR decides whether to join the retailer's blockchain platform and sets the optimal wholesale price of remanufactured products. In the third stage, the retailer establishes the optimal sales quantities and selling prices of new and remanufactured products. It is assumed that the OEM, TPR, and retailer are all risk-neutral and aim to maximize their own profits [13, 48].

Table 2 summarizes the parameters and decision variables used in this paper.

4. Model formulation and solution

4.1. Fixed authorization fee strategy

4.1.1. The TPR does not join the retailer's blockchain platform (Model FN)

Under Model *FN*, the TPR pays the OEM a fixed authorization fee F that is independent of the remanufactured quantity, while choosing not to join the retailer's blockchain platform. First, the OEM sets the optimal wholesale price of new products w_n to maximize its own profit; subsequently, the TPR sets the optimal wholesale price of remanufactured products w_r to maximize its own profit; finally, the retailer determines the optimal

TABLE 2. Parameters and decision variables.

Symbols	Definitions
Parameters	
c_n	Unit production cost of new products
c_r	Unit production cost of remanufactured products
c_b	Unit variable cost incurred by the TPR after joining the retailer's blockchain platform
t	Cost associated with consumers' privacy concerns
F	Fixed authorization fee that the OEM charges to the TPR under a fixed authorization fee strategy
θ	Consumers' WTP for new products
δ	Value discount that consumers are willing to pay for remanufactured products when the TPR does not join the retailer's blockchain platform
λ	Value discount multiplier that consumers are willing to pay for remanufactured products when the TPR joins the retailer's blockchain platform
p_n	Retail price of new products
p_r	Retail price of remanufactured products
Decision variables	
q_n	Sales quantity of new products
q_r	Sales quantity of remanufactured products
w_n	Wholesale price of new products
w_r	Wholesale price of remanufactured products
f	Unit authorization fee that the OEM charges to the TPR under the unit authorization fee strategy
Other symbols	
π_k^{ij}	Profit of supply chain member k under model *ij*. Here, $i = \{F, R\}$ represents the OEM's fixed authorization fee strategy and unit authorization fee strategy, respectively; $j = \{N, B\}$ represents whether the TPR does not join or joins the retailer's blockchain platform, respectively; $k = \{M, T, R\}$ denotes the OEM, TPR, and retailer, respectively

sales quantities of new and remanufactured products, q_n and q_r, to maximize its own profit. The profit functions of the OEM, TPR, and retailer are given by equations (1), (2), and (3), respectively.

$$\pi_M^{FN} = (w_n - c_n)q_n + F \tag{1}$$
$$\pi_T^{FN} = (w_r - c_r)q_r - F \tag{2}$$
$$\pi_R^{FN} = (p_n - w_n)q_n + (p_r - w_r)q_r. \tag{3}$$

According to backward induction, the optimal strategies and maximum profits for the OEM, TPR, and retailer under Model *FN* are obtained, as shown in Table 3.

To ensure that Model *FN* is meaningful, it must be guaranteed that $\beta_1 > 0$ and $\beta_2 > 0$.

4.1.2. The TPR joins the retailer's blockchain platform (Model FB)

Under Model *FB*, the TPR pays the OEM a fixed authorization fee F that is independent of the remanufacturing quantity, while choosing to join the retailer's blockchain platform. First, the OEM sets the optimal wholesale price of new products w_n to maximize its own profit; subsequently, TPR sets the optimal wholesale price of remanufactured products w_r to maximize its own profit; finally, the retailer determines the optimal sales quantities of new and remanufactured products, q_n and q_r, to maximize its own profit. The profit functions of the OEM, TPR, and retailer are given by equations (4), (5), and (6), respectively.

$$\pi_M^{FB} = (w_n - c_n)q_n + F \tag{4}$$

TABLE 3. Equilibrium solutions of Model *FN*.

Decision variables	Equilibrium solutions
w_n^{FN}	$\dfrac{2(1-\delta)+(2-\delta)c_n+c_r}{2(2-\delta)}$
w_r^{FN}	$\dfrac{2\delta(1-\delta)+\delta(2-\delta)c_n+(4-\delta)c_r}{4(2-\delta)}$
p_n^{FN}	$\dfrac{2(3-2\delta)+(2-\delta)c_n+c_r}{4(2-\delta)}$
p_r^{FN}	$\dfrac{2\delta(5-3\delta)+\delta(2-\delta)c_n+(4-\delta)c_r}{8(2-\delta)}$
q_n^{FN}	$\dfrac{\beta_1}{8(1-\delta)}$
q_r^{FN}	$\dfrac{\beta_2}{8\delta(1-\delta)(2-\delta)}$
π_M^{FN}	$\dfrac{\beta_1^2}{16(1-\delta)(2-\delta)}+F$
π_T^{FN}	$\dfrac{\beta_2^2}{32\delta(1-\delta)(2-\delta)^2}-F$
π_R^{FN}	$\dfrac{\beta_1^2}{32(1-\delta)(2-\delta)}-\dfrac{\beta_2^2}{64\delta(1-\delta)(2-\delta)^2}+\dfrac{(\delta\beta_1+\beta_2)(\delta-c_r)}{16\delta(1-\delta)(2-\delta)}$

$$\pi_T^{FB}=(w_r-c_r-c_b)q_r-F \tag{5}$$

$$\pi_R^{FB}=(p_n-w_n)q_n+(p_r-w_r)q_r. \tag{6}$$

According to backward induction, the optimal strategies and maximum profits for the OEM, TPR, and retailer under Model *FB* are obtained, as shown in Table 4.

To ensure that Model *FB* is meaningful, it must be guaranteed that $\beta_3>0$ and $\beta_4>0$.

4.2. Unit authorization fee strategy

4.2.1. The TPR does not join the retailer's blockchain platform (Model RN)

Under Model *RN*, the TPR pays the OEM a unit authorization fee f for each remanufactured product sold, while electing not to join the retailer's blockchain platform. First, the OEM determines the optimal wholesale price of new products w_n and the optimal unit authorization fee of remanufactured products f to maximize its own profit; subsequently, the TPR sets the optimal wholesale price of remanufactured products w_r to maximize its own profit; finally, the retailer establishes the optimal sales quantities of new and remanufactured products, q_n and q_r, to maximize its own profit. The profit functions of the OEM, TPR, and the retailer are presented in equations (7), (8), and (9), respectively.

$$\pi_M^{RN}=(w_n-c_n)q_n+fq_r \tag{7}$$

$$\pi_T^{RN}=(w_r-c_r-f)q_r \tag{8}$$

$$\pi_R^{RN}=(p_n-w_n)q_n+(p_r-w_r)q_r. \tag{9}$$

According to backward induction, the optimal strategies and maximum profits for the OEM, TPR, and retailer under Model *RN* are obtained, as shown in Table 5.

Table 4. Equilibrium solutions of Model *FB*.

Decision variables	Equilibrium solutions
w_n^{FB}	$\dfrac{2(1-\delta\lambda)+(2-\delta\lambda)c_n+c_r+c_b+t}{2(2-\delta\lambda)}$
w_r^{FB}	$\dfrac{2\delta\lambda(1-\delta\lambda)+\delta\lambda(2-\delta\lambda)c_n+(4-\delta\lambda)(c_b+c_r)-(4-3\delta\lambda)t}{4(2-\delta\lambda)}$
p_n^{FB}	$\dfrac{2(3-2\delta\lambda)+(2-\delta\lambda)c_n+c_r+c_b+t}{4(2-\delta\lambda)}$
p_r^{FB}	$\dfrac{2\delta\lambda(5-3\delta\lambda)+\delta\lambda(2-\delta\lambda)c_n+(4-\delta\lambda)(c_r+c_b)-(12-7\delta\lambda)t}{8(2-\delta\lambda)}$
q_n^{FB}	$\dfrac{\beta_3}{8(1-\delta\lambda)}$
q_r^{FB}	$\dfrac{\beta_4}{8\delta\lambda(1-\delta\lambda)(2-\delta\lambda)}$
π_M^{FB}	$\dfrac{\beta_3^2}{16(1-\delta\lambda)(2-\delta\lambda)}+F$
π_T^{FB}	$\dfrac{\beta_4^2}{32\delta\lambda(1-\delta\lambda)(2-\delta\lambda)^2}-F$
π_R^{FB}	$\dfrac{\beta_3^2}{32(1-\delta\lambda)(2-\delta\lambda)}-\dfrac{\beta_4^2}{64\delta\lambda(1-\delta\lambda)(2-\delta\lambda)^2}+\dfrac{(\delta\lambda\beta_3+\beta_4)(\delta\lambda-c_r-c_b-t)}{16\delta\lambda(1-\delta\lambda)(2-\delta\lambda)}$

To ensure that Model *RN* is meaningful, it must be guaranteed that $\beta_1>0$ and $\delta c_n - c_r > 0$.

4.2.2. The TPR joins the retailer's blockchain platform (Model RB)

Under Model *RB*, the TPR pays the OEM a unit authorization fee f for each remanufactured product sold, while electing to join the retailer's blockchain platform. First, the OEM determines the optimal wholesale price of new products w_n and the remanufacturing unit authorization fee f to maximize its own profit. Then, the TPR sets the optimal wholesale price of remanufactured products w_r to maximize its own profit. Finally, the retailer decides on the optimal sales quantities of new and remanufactured products, q_n and q_r, to maximize its own profit. The profit functions of the OEM, TPR, and retailer are given by equations (10), (11), and (12), respectively.

$$\pi_M^{RB} = (w_n - c_n)q_n + fq_r \tag{10}$$

$$\pi_T^{RB} = (w_r - c_r - c_b - f)q_r \tag{11}$$

$$\pi_R^{RB} = (p_n - w_n)q_n + (p_r - w_r)q_r. \tag{12}$$

According to backward induction, the optimal strategies and maximum profits of the OEM, TPR, and retailer under Model *RB* are obtained as shown in Table 6.

To ensure that Model *RB* is meaningful, it must be guaranteed that $\beta_3>0$ and $\delta\lambda c_n - c_r - c_b - t > 0$.

5. Result analysis

This section first conducts a comparative analysis of equilibrium strategies under different remanufacturing authorization and blockchain platform joining strategies; it then analyzes the impact of key parameters on the equilibrium strategies and the profits of the OEM, TPR, and retailer.

TABLE 5. Equilibrium solutions of Model *RN*.

Decision variables	Equilibrium solutions
w_n^{RN}	$\frac{1+c_n}{2}$
f^{RN}	$\frac{\delta - c_r}{2}$
w_r^{RN}	$\frac{2\delta + \delta c_n + c_r}{4}$
p_n^{RN}	$\frac{3+c_n}{4}$
p_r^{RN}	$\frac{6\delta + \delta c_n + c_r}{8}$
q_n^{RN}	$\frac{\beta_1}{8(1-\delta)}$
q_r^{RN}	$\frac{\delta c_n - c_r}{8\delta(1-\delta)}$
π_M^{RN}	$\frac{2\delta(1-\delta)(1-c_n)^2 + (\delta c_n - c_r)^2}{16\delta(1-\delta)}$
π_T^{RN}	$\frac{(\delta c_n - c_r)^2}{32\delta(1-\delta)}$
π_R^{RN}	$\frac{4\delta(1-\delta)(1-c_n)^2 + (\delta c_n - c_r)^2}{64\delta(1-\delta)}$

5.1. Comparative analysis

This section primarily conducts a comparative analysis of the equilibrium strategies under Model *Fj* and Model *Rj* ($j = N, B$), Model *FN* and Model *FB*, as well as Model *RN* and Model *RB*. Comparative analysis reveals that the consumer privacy concern cost t and the unit variable cost c_b incurred after the TPR joins the retailer's blockchain platform exert essentially consistent impacts on equilibrium strategies across all models. Consequently, the comparative analyses between Model *FN* and Model *FB* as well as Model *RN* and Model *RB*, focus exclusively on examining the influence of c_b on equilibrium strategies.

5.1.1. *Analysis of remanufacturing authorization strategies (Model Fj and Model Rj ($j = N, B$))*

This subsection analyzes the equilibrium strategies and profits under two scenarios – when the TPR does not join *versus* when it joins the retailer's blockchain platform – to identify optimal remanufacturing authorization strategies. Theorem 1 provides a comparative analysis of the wholesale prices, retail prices, and sales quantities of new and remanufactured products under Model *Fj* and Model *Rj* ($j = N, B$).

Theorem 1. *Regardless of whether the TPR joins the retailer's blockchain platform, the following relationships hold:* $w_n^{Fj^*} < w_n^{Rj^*}$, $w_r^{Fj^*} < w_r^{Rj^*}$, $p_n^{Fj^*} < p_n^{Rj^*}$, $p_r^{Fj^*} < p_r^{Rj^*}$, $q_n^{Fj^*} = q_n^{Rj^*}$, *and* $q_r^{Fj^*} > q_r^{Rj^*}$.

From Theorem 1, it can be observed that regardless of whether the TPR joins the retailer's blockchain platform or not, under the unit authorization fee strategy, the wholesale and retail prices of both new and remanufactured products are respectively higher than those under the fixed authorization fee strategy. Moreover, the differences in wholesale prices between the two authorization fee strategies are equal for both new and remanufactured products, and the differences in retail prices are also equal for both types of products.

TABLE 6. Equilibrium solutions of Model *RB*.

Decision variables	Equilibrium solutions
w_n^{RB}	$\frac{1+c_n}{2}$
f^{RB}	$\frac{\delta\lambda - c_r - c_b - t}{2}$
w_r^{RB}	$\frac{2\delta\lambda + \delta\lambda c_n + c_r + c_b - 3t}{4}$
p_n^{RB}	$\frac{3+c_n}{4}$
p_r^{RB}	$\frac{6\delta\lambda + \delta\lambda c_n + c_r + c_b - 7t}{8}$
q_n^{RB}	$\frac{\beta_3}{8(1-\delta\lambda)}$
q_r^{RB}	$\frac{\delta\lambda c_n - c_r - c_b - t}{8\delta\lambda(1-\delta\lambda)}$
π_M^{RB}	$\frac{2(1-\delta\lambda)\delta\lambda(1-c_n)^2 + (\delta\lambda c_n - c_r - c_b - t)^2}{16\delta\lambda(1-\delta\lambda)}$
π_T^{RB}	$\frac{(\delta\lambda c_n - c_r - c_b - t)^2}{32\delta\lambda(1-\delta\lambda)}$
π_R^{RB}	$\frac{4(1-\delta\lambda)\delta\lambda(1-c_n)^2 + (\delta\lambda c_n - c_r - c_b - t)^2}{64\delta\lambda(1-\delta\lambda)}$

Furthermore, the sales quantity of new products is the same under both the fixed and unit authorization fee strategies. However, the sales quantity of remanufactured products is greater under the fixed authorization fee strategy compared to the unit authorization fee strategy. This indicates that the sales quantity of new products is unaffected by the remanufacturing authorization strategy, but the pricing strategies for both new and remanufactured products, as well as the sales quantity strategy for remanufactured products, are influenced by the authorization strategy. Compared to the unit authorization fee strategy, the fixed authorization fee strategy reduces both the wholesale and retail prices of new and remanufactured products while increasing the market share of remanufactured products. Moreover, since $w_n^{FN^*} - c_n < w_n^{RN^*} - c_n$, the unit authorization fee strategy can enhance the OEM's marginal profit from producing new products.

Further analysis of the wholesale and retail prices of new and remanufactured products under Model *Fj* and Model *Rj* leads to Corollary 1.

Corollary 1. *Under the unit authorization fee strategy compared to the fixed authorization fee strategy, the retailer's marginal profits for both new and remanufactured products decrease, and the magnitude of reduction is identical for both product categories. When the TPR does not join the retailer's blockchain platform, the marginal profit reduction is* $\frac{\delta - c_r}{4(2-\delta)}$*; whereas when the TPR joins the platform, the reduction becomes* $\frac{\delta\lambda - c_r - c_b - t}{4(2-\delta\lambda)}$.

The following section provides a comparative analysis of the profits for the OEM, TPR, and retailer under Model *Fj* and Model *Rj* ($j = N, B$). The analysis conclusions are presented in Theorem 2.

Theorem 2. (1) *When the TPR does not join the retailer's blockchain platform: if* $F \geq F_1$*, then* $\pi_M^{FN^*} \geq \pi_M^{RN^*}$*; otherwise,* $\pi_M^{FN^*} < \pi_M^{RN^*}$*. If* $F \leq F_2$*, then* $\pi_T^{FN^*} \geq \pi_T^{RN^*}$*; otherwise,* $\pi_T^{FN^*} < \pi_T^{RN^*}$.

(2) *When the TPR joins the retailer's blockchain platform: if* $F \geq F_3$*, then* $\pi_M^{FB^*} \geq \pi_M^{RB^*}$*; otherwise,* $\pi_M^{FB^*} < \pi_M^{RB^*}$*. If* $F \leq F_4$*, then* $\pi_T^{FB^*} \geq \pi_T^{RB^*}$*; otherwise,* $\pi_T^{FB^*} < \pi_T^{RB^*}$*.*

(3) *Regardless of whether the TPR joins the retailer's blockchain platform,* $\pi_R^{Fj^*} > \pi_R^{Rj^*}$*.*

Theorem 2 elucidates which authorization strategy the OEM prefers, primarily depending on the relationship between the fixed authorization fee F it charges to the TPR and the critical value F_1 or F_3. When the TPR does not join the retailer's blockchain platform, if $F \geq F_1$, the OEM prefers the fixed authorization fee strategy; otherwise, the OEM prefers the unit authorization fee strategy. For the TPR, if $F \leq F_2$, it prefers the fixed authorization fee strategy; otherwise, it prefers the unit authorization fee strategy.

When the TPR joins the retailer's blockchain platform, if $F \geq F_3$, the OEM prefers the fixed authorization fee strategy; otherwise, the OEM prefers the unit authorization fee strategy. For the TPR, if $F \leq F_4$, it prefers the fixed authorization fee strategy; otherwise, it prefers the unit authorization fee strategy.

The retailer's profit remains unaffected by the fixed authorization fee F. Crucially, under the fixed authorization fee strategy, the retailer consistently achieves higher profits compared to the unit authorization fee strategy – regardless of the TPR's decision to join the blockchain platform. This explains the retailer's inherent preference for OEMs to adopt fixed authorization fee strategy. The primary reason for this preference is that under the fixed authorization fee strategy, the OEM cannot influence the retailer through the fixed authorization fee, granting the retailer greater decision-making autonomy. Under the unit authorization fee strategy, the OEM can affect the retailer's decisions *via* the unit authorization fee f. As established in Corollary 1, the retailer experiences reduced marginal profits on both new and remanufactured products, coupled with declining sales of remanufactured products, which collectively erode its overall profitability. In contrast, the OEM gains increased profit by exercising wholesale price premiums while maintaining stable new product sales quantity.

5.1.2. Comparative analysis of different equilibrium strategies under the fixed authorization fee strategy (Model FN and Model FB)

This subsection conducts a comparative analysis of equilibrium strategies under the fixed authorization fee strategy, examining both scenarios where the TPR abstains from and joins the retailer's blockchain platform, with the objective of determining the TPR's optimal blockchain adoption strategy. First, we compare the wholesale prices, retail prices, and sales quantities of new and remanufactured products under Model *FN* and Model *FB*. The conclusions are presented in Theorem 3. Second, we compare the profits of the OEM, TPR, and retailer, with the findings summarized in Theorem 4.

Theorem 3. *Under the fixed authorization fee strategy:*

(1) *When* $c_b \leq c_{b1}^F$*,* $w_n^{FB^*} \leq w_n^{FN^*}$ *and* $p_n^{FB^*} \leq p_n^{FN^*}$*; otherwise,* $w_n^{FB^*} > w_n^{FN^*}$ *and* $p_n^{FB^*} > p_n^{FN^*}$*.*

(2) *When* $c_b \leq c_{b2}^F$*,* $w_r^{FB^*} \leq w_r^{FN^*}$*; otherwise,* $w_r^{FB^*} > w_r^{FN^*}$*. When* $c_b \leq c_{b3}^F$*,* $p_r^{FB^*} \leq p_r^{FN^*}$*; otherwise* $p_r^{FB^*} > p_r^{FN^*}$*.*

(3) *When* $c_b \leq c_{b4}^F$*,* $q_n^{FB^*} \leq q_n^{FN^*}$*; otherwise,* $q_n^{FB^*} > q_n^{FN^*}$*. When* $c_b \leq c_{b5}^F$*,* $q_r^{FB^*} \geq q_r^{FN^*}$*; otherwise,* $q_r^{FB^*} < q_r^{FN^*}$*.*

According to Theorem 3, under the fixed authorization fee strategy, when the unit variable cost c_b is relatively small, the wholesale and retail prices of new products are lower when the TPR joins the platform compared to when it does not. As c_b increases, when $c_b > c_{b1}^F$, the wholesale and retail prices of new products under the joining strategy will be higher than those under the non-joining strategy.

When c_b is relatively small, the wholesale and retail prices of remanufactured products under platform joining will be lower than those under non-joining. As c_b increases, the joining strategy induces gradual price inflation for remanufactured products. When $c_b > c_{b2}^F$, the wholesale price of remanufactured products under the joining strategy will be higher than those under the non-joining strategy; when $c_b > c_{b3}^F$, the retail price of remanufactured products under the joining strategy will be higher than those under the non-joining strategy.

When c_b is relatively small, the sales quantity of new products when the TPR joins the platform is less than when it does not join, while the sales quantity of remanufactured products is higher when joining compared to not joining. As c_b increases, when $c_b > c_{b4}^F$, the sales quantity of new products when joining will be greater than when not joining; when $c_b > c_{b5}^F$, the sales quantity of remanufactured products when joining will be lower than when not joining.

The observed pattern emerges because the TPR's blockchain joining enhances supply chain transparency, enabling consumers to authenticate remanufactured product attributes. This verification effect reduces information asymmetry and systematically increases consumers' WTP for remanufactured products. When the blockchain joining cost c_b is relatively low, the TPR's marginal cost increase remains minimal. Under these conditions, the TPR reduces its wholesale price to induce larger retailer order quantity for remanufactured products, thereby expanding market share through increased sales quantity. Concurrently, to counteract competitive pressure from the TPR's platform joining, the OEM lowers new product wholesale price to mitigate demand erosion and protect its market position. However, when c_b becomes sufficiently high, the TPR is compelled to raise the wholesale price of remanufactured products to offset its increased operational cost, ultimately leading to reduced sales quantity. Under such circumstances, the OEM gains a competitive advantage as more consumers switch to new products, prompting it to increase new product wholesale price to maximize its profit.

Theorem 4. *Under the fixed authorization fee strategy:*

(1) *When* $c_b \geq c_{b6}^F$, $\pi_M^{FB^*} \geq \pi_M^{FN^*}$*; otherwise,* $\pi_M^{FB^*} < \pi_M^{FN^*}$.
(2) *When* $c_b \leq c_{b7}^F$, $\pi_T^{FB^*} \geq \pi_T^{FN^*}$*; otherwise,* $\pi_T^{FB^*} < \pi_T^{FN^*}$.
(3) *When* $c_b \leq c_{b8}^F$, $\pi_R^{FB^*} \geq \pi_R^{FN^*}$*; otherwise,* $\pi_R^{FB^*} < \pi_R^{FN^*}$.

According to Theorem 4, under the fixed authorization fee strategy, the value of c_b determines whether the TPR is willing to join the retailer's blockchain platform. When c_b is relatively small, joining the platform is more beneficial for the TPR and the retailer, while not joining is more advantageous for the OEM. As c_b increases: When $c_b > c_{b7}^F$, the TPR will be unwilling to join the retailer's blockchain platform; when $c_b > c_{b6}^F$, the OEM prefers that the TPR joins the retailer's blockchain platform; when $c_b > c_{b8}^F$, the retailer prefers that the TPR does not join the platform.

5.1.3. Comparative analysis of different equilibrium strategies under the unit authorization fee strategy (Model RN and Model RB)

This subsection conducts a comparative analysis of equilibrium strategies under the unit authorization fee strategy, examining both scenarios where the TPR abstains from and joins the retailer's blockchain platform, with the objective of determining the TPR's optimal blockchain adoption strategy. First, we compare the wholesale prices, retail prices, and sales quantities of new and remanufactured products under Model *RN* and Model *RB*. The conclusions are presented in Theorem 5. Second, we compare the profits of the OEM, TPR, and retailer. The findings are summarized in Theorem 6.

Theorem 5. *Under the unit authorization fee strategy:*

(1) $w_n^{RB^*} = w_n^{RN^*}$ *and* $p_n^{RB^*} = p_n^{RN^*}$.
(2) *When* $c_b \leq c_{b1}^R$, $f^{RB^*} \geq f^{RN^*}$*; otherwise,* $f^{RB^*} < f^{RN^*}$.
(3) *When* $c_b \leq c_{b2}^R$, $w_r^{RB^*} \leq w_r^{RN^*}$*; otherwise,* $w_r^{RB^*} > w_r^{RN^*}$. *When* $c_b \leq c_{b3}^R$, $p_r^{RB^*} \leq p_r^{RN^*}$*; otherwise,* $p_r^{RB^*} > p_r^{RN^*}$.
(4) *When* $c_b \leq c_{b4}^R$, $q_n^{RB^*} \leq q_n^{RN^*}$*; otherwise,* $q_n^{RB^*} > q_n^{RN^*}$. *When* $c_b \leq c_{b5}^R$, $q_r^{RB^*} \geq q_r^{RN^*}$*; otherwise,* $q_r^{RB^*} < q_r^{RN^*}$.

According to Theorem 5, under the unit authorization fee strategy, the wholesale and retail prices of new products are not affected by whether TPR joins the retailer's blockchain platform. When the unit variable cost c_b is relatively small, the unit authorization fee charged by the OEM to the TPR under the joining strategy will be higher. As c_b increases, when $c_b > c_{b1}^R$, the unit authorization fee under the joining strategy will be lower than that under the non-joining strategy. When c_b is relatively small, both the wholesale and retail prices of remanufactured products under the joining strategy will be lower than those under the non-joining strategy. As c_b increases, when $c_b > c_{b2}^R$, the wholesale price of remanufactured products under the joining strategy will be higher than that under the non-joining strategy; when $c_b > c_{b3}^R$, the retail price of remanufactured products under the joining strategy will be higher than that under the non-joining strategy. When c_b is relatively small, the sales quantity of new products under the joining strategy will be less than that under the non-joining strategy, while the sales quantity of remanufactured products will be greater. As c_b increases, when $c_b > c_{b4}^R$, the sales quantity of new products under the joining strategy will be greater than that under the non-joining strategy; when $c_b > c_{b5}^R$, the sales quantity of remanufactured products under the joining strategy will be less.

This finding indicates that, regardless of the TPR's strategic choice, the OEM maintains a consistent wholesale price for new products, and the retailer subsequently adopts a follow-the-leader strategy by keeping the retail price of new products unchanged. When c_b is relatively low, the joining strategy enables the TPR to reduce the wholesale price for remanufactured products. This subsequently drives down the retail price and increases the sales volume, ultimately boosting the TPR's profitability. The reduced retail price of remanufactured products attracts more consumers to switch from new products, thereby decreasing new product sales. To offset this revenue loss, the OEM will impose higher unit authorization fee on the TPR. Conversely, when c_b exceeds a critical threshold, platform adoption leads to prohibitively high operational cost for the TPR. This cost pressure forces the TPR to increase the wholesale price, which consequently raises the retail price and reduces demand for remanufactured products while stimulating new product sales. As the OEM gains greater profits from enhanced new product sales, it has an incentive to maintain supply chain stability by offering the TPR reduced unit authorization fee.

Theorem 6. *Under the unit authorization strategy: when* $c_b \le c_{b6}^R$, $\pi_M^{RB^*} \ge \pi_M^{RN^*}$, $\pi_T^{RB^*} \ge \pi_T^{RN^*}$, *and* $\pi_R^{RB^*} \ge \pi_R^{RN^*}$*; otherwise,* $\pi_M^{RB^*} < \pi_M^{RN^*}$, $\pi_T^{RB^*} < \pi_T^{RN^*}$, *and* $\pi_R^{RB^*} < \pi_R^{RN^*}$.

According to Theorem 6, under the unit authorization fee strategy, when the unit variable cost c_b is relatively small, the joining strategy is more beneficial for the OEM, TPR, and retailer. Moreover, all three parties have a consistent critical value for the unit variable cost c_b regarding both the joining and non-joining strategies, which is c_{b6}^R.

5.2. Sensitivity analysis

In this section, we analyze the impact of key parameters on the equilibrium strategies and the profits of the OEM, TPR, and retailer. First, we select parameters that reflect consumer influence, such as the value discount multiplier λ that consumers are willing to pay for remanufactured products when the TPR joins the retailer's blockchain platform, and the consumer privacy concern cost t. Second, we choose parameters that reflect the efforts of the supply chain members such as the unit variable cost c_b incurred after the TPR joins the retailer's blockchain platform.

Theorem 7 primarily reflects the impact of the value discount multiplier λ on the equilibrium strategies and the profits of the OEM, TPR, and retailer in Model *FB* and Model *RB*.

Theorem 7. (1) $\frac{\partial w_n^{FB^*}}{\partial \lambda} < 0$. *When* $c_b \ge 2 - c_r - t - (2-\delta\lambda)^2(1+\frac{c_n}{2}))$, $\frac{\partial w_r^{FB^*}}{\partial \lambda} \ge 0$*; otherwise,* $\frac{\partial w_r^{FB^*}}{\partial \lambda} < 0$. $\frac{\partial p_n^{FB^*}}{\partial \lambda} < 0$. $\frac{\partial p_r^{FB^*}}{\partial \lambda} > 0$. $\frac{\partial q_n^{FB^*}}{\partial \lambda} < 0$. $\frac{\partial q_r^{FB^*}}{\partial \lambda} > 0$. $\frac{\partial \pi_M^{FB^*}}{\partial \lambda} < 0$. *When* $\delta\lambda \le \frac{2}{3}$*, or when* $\delta\lambda > \frac{2}{3}$ *and* $c_b < \frac{\delta\lambda(2-\delta\lambda)^2 c_n - 2\delta\lambda(1-\delta\lambda)(3\delta\lambda-2)}{(3\delta\lambda-2)(4-5\delta\lambda+2\delta^2\lambda^2)} - c_r - t$, $\frac{\partial \pi_T^{FB^*}}{\partial \lambda} > 0$*; when* $\delta\lambda > \frac{2}{3}$ *and* $c_b \ge \frac{\delta\lambda(2-\delta\lambda)^2 c_n - 2\delta\lambda(1-\delta\lambda)(3\delta\lambda-2)}{(3\delta\lambda-2)(4-5\delta\lambda+2\delta^2\lambda^2)} - c_r - t$, $\frac{\partial \pi_T^{FB^*}}{\partial \lambda} \le 0$. $\frac{\partial \pi_R^{FB^*}}{\partial \lambda} > 0$.

(2) $\frac{\partial w_n^{RB^*}}{\partial \lambda} = 0$, $\frac{\partial f^{RB^*}}{\partial \lambda} > 0$, $\frac{\partial w_r^{RB^*}}{\partial \lambda} > 0$, $\frac{\partial p_n^{RB^*}}{\partial \lambda} = 0$, $\frac{\partial p_r^{RB^*}}{\partial \lambda} > 0$, $\frac{\partial q_n^{RB^*}}{\partial \lambda} < 0$, $\frac{\partial q_r^{RB^*}}{\partial \lambda} > 0$, $\frac{\partial \pi_M^{RB^*}}{\partial \lambda} > 0$, $\frac{\partial \pi_T^{RB^*}}{\partial \lambda} > 0$, *and* $\frac{\partial \pi_R^{RB^*}}{\partial \lambda} > 0$.

Theorem 7(1) establishes that under the fixed authorization fee strategy (Model *FB*), an increase in the value discount multiplier λ leads to higher remanufactured product sales but lower new product sales. To stimulate demand for new products, both the OEM and retailer will reduce new product wholesale and retail prices. When the unit variable cost c_b is relatively high, the TPR will increase the wholesale price of remanufactured products. This leads to a reduction in the OEM's profit, an increase in the retailer's profit, and the TPR's profit will vary depending on the value discount $\delta\lambda$ and the unit variable cost c_b. If the value discount $\delta\lambda$ is less than 2/3, or greater than 2/3 with a relatively small unit variable cost c_b, the TPR's profit will increase as the value discount multiplier λ increases; conversely, it will decrease as the value discount multiplier λ increases. This analysis reveals that while greater consumer valuation of remanufactured products (higher $\delta\lambda$) benefits the retailers and harms the OEM, its impact on the TPR depends critically on the joint consideration of the value discount and the unit variable cost.

According to Theorem 7(2), when the OEM adopts the unit authorization fee strategy (Model *RB*), the value discount multiplier λ increases, indicating a higher acceptance of remanufactured products by consumers, which in turn leads to an increase in the wholesale price, retail price, and sales quantity of remanufactured products, thereby increasing the profits of the TPR and retailer. Since the wholesale and retail prices of new products are not affected by the value discount multiplier λ, and the unit authorization fee charged by the OEM to the TPR increases, the profit of the OEM grows. Therefore, an increase in the value discount multiplier is beneficial to all members of the supply chain.

Theorem 8 reflects the impact of the consumer privacy concern cost t on the wholesale prices, retail prices, sales quantities, and profits of supply chain members for both new and remanufactured products in Model *FB* and Model *RB*.

Theorem 8. (1) $\frac{\partial w_n^{FB^*}}{\partial t} > 0$, $\frac{\partial w_r^{FB^*}}{\partial t} < 0$, $\frac{\partial p_n^{FB^*}}{\partial t} > 0$, $\frac{\partial p_r^{FB^*}}{\partial t} < 0$, $\frac{\partial q_n^{FB^*}}{\partial t} > 0$, $\frac{\partial q_r^{FB^*}}{\partial t} < 0$, $\frac{\partial \pi_M^{FB^*}}{\partial t} > 0$, $\frac{\partial \pi_T^{FB^*}}{\partial t} < 0$ *and* $\frac{\partial \pi_R^{FB^*}}{\partial t} < 0$.
(2) $\frac{\partial w_n^{RB^*}}{\partial t} = 0$, $\frac{\partial f^{RB^*}}{\partial t} < 0$, $\frac{\partial w_r^{RB^*}}{\partial t} < 0$, $\frac{\partial p_n^{RB^*}}{\partial t} = 0$, $\frac{\partial p_r^{RB^*}}{\partial t} < 0$, $\frac{\partial q_n^{RB^*}}{\partial t} > 0$, $\frac{\partial q_r^{RB^*}}{\partial t} < 0$, $\frac{\partial \pi_M^{RB^*}}{\partial t} < 0$, $\frac{\partial \pi_T^{RB^*}}{\partial t} < 0$, *and* $\frac{\partial \pi_R^{RB^*}}{\partial t} < 0$.

Theorem 8(1) demonstrates that under the fixed authorization fee strategy (Model *FB*), higher consumer privacy concern cost intensifies skepticism toward the TPR's blockchain joining, triggering a shift to new product purchases that boosts new product sales. In response, the OEM capitalizes on this demand increase by raising its wholesale price to maximize profits, while the TPR is forced to reduce its wholesale price to mitigate market share erosion. However, the retailer experiences net profit decline as gains from new product sales fail to offset losses in the remanufactured segment, resulting in profit reduction that monotonically increases with privacy concern cost. Consequently, rising privacy concern cost creates asymmetric impacts across the supply chain: they adversely affect both the TPR (through price compression and demand loss) and the retailer (through profit imbalance), while paradoxically benefiting the OEM through enhanced pricing power and demand substitution effects.

According to Theorem 8(2), when the OEM adopts the unit authorization fee strategy (Model *RB*), the impact of the consumer privacy concern cost on the wholesale price, retail price, sales quantity of remanufactured products, as well as the profits of the TPR and retailer is similar to that under the fixed authorization fee strategy (Model *FB*). What differs is that the wholesale and retail prices of new products remain unchanged. Therefore, the profit the OEM gains from new product sales stays the same. However, as the consumer privacy concern cost increases, the unit authorization fee the OEM charges the TPR decreases, and the sales quantity of remanufactured products declines. Consequently, the overall profit of the OEM will decrease. Therefore, it demonstrates that increased consumer privacy concern cost adversely affects all supply chain members.

Theorem 9 reflects the impact of the unit variable cost c_b, incurred by the TPR joining the retailer's blockchain platform, on the wholesale prices, retail prices, sales quantities, and profits of supply chain members for both new and remanufactured products in Model *FB* and Model *RB*.

Theorem 9. (1) $\frac{\partial w_n^{FB^*}}{\partial c_b} > 0$, $\frac{\partial w_r^{FB^*}}{\partial c_b} > 0$, $\frac{\partial p_n^{FB^*}}{\partial c_b} > 0$, $\frac{\partial p_r^{FB^*}}{\partial c_b} > 0$, $\frac{\partial q_n^{FB^*}}{\partial c_b} > 0$, $\frac{\partial q_r^{FB^*}}{\partial c_b} < 0$, $\frac{\partial \pi_M^{FB^*}}{\partial c_b} > 0$, $\frac{\partial \pi_T^{FB^*}}{\partial c_b} < 0$, *and* $\frac{\partial \pi_R^{FB^*}}{\partial c_b} < 0$.

(2) $\frac{\partial w_n^{RB^*}}{\partial c_b} = 0$, $\frac{\partial f^{RB^*}}{\partial c_b} < 0$, $\frac{\partial w_r^{RB^*}}{\partial c_b} > 0$, $\frac{\partial p_n^{RB^*}}{\partial c_b} = 0$, $\frac{\partial p_r^{RB^*}}{\partial c_b} > 0$, $\frac{\partial q_n^{RB^*}}{\partial c_b} > 0$, $\frac{\partial q_r^{RB^*}}{\partial c_b} < 0$, $\frac{\partial \pi_M^{RB^*}}{\partial c_b} < 0$, $\frac{\partial \pi_T^{RB^*}}{\partial c_b} < 0$, *and* $\frac{\partial \pi_R^{RB^*}}{\partial c_b} < 0$.

Theorem 9(1) demonstrates that under the fixed authorization fee strategy (Model FB), the TPR elevates the wholesale price of remanufactured products to offset rising unit variable cost c_b. This wholesale price increase propagates through the supply chain: the retailer correspondingly raises the retail price of remanufactured products, depressing demand. The resulting demand substitution effect drives consumers toward new products, enabling the OEM to simultaneously benefit from both increased sales quantity and a higher wholesale price for new products. Although the retailer gains profit from expanded new product sales, this benefit is outweighed by losses in the remanufactured segment, yielding monotonically decreasing net profit as the unit variable cost c_b increases. Consequently, blockchain-induced cost escalations create fundamentally asymmetric outcomes – harming the TPR through demand erosion, reducing the retailer's profit through negative net effect, while uniquely benefiting the OEM through dual revenue channels: premium pricing and demand substitution.

According to Theorem 9(2), when the OEM adopts the unit authorization fee strategy (Model RB), the impact of the increased unit variable cost c_b on the wholesale price, retail price, and sales quantity of remanufactured products – as well as on the profits of the TPR and the retailer – is similar to that under the fixed authorization fee strategy (Model *FB*). The key difference is that the wholesale and retail prices of new products remain unchanged. Similar to the effect of consumer privacy concern cost on the OEM's profit, the OEM's profit in this case decreases as the unit variable cost c_b increases. Consequently, increased blockchain operational cost generates Pareto-inferior outcomes, adversely affecting all supply chain members unlike the asymmetric effects seen in Model *FB*.

6. Numerical analysis

This section focuses on analyzing the effects of λ, c_b, and t on the OEM's remanufacturing authorization strategy and the TPR's blockchain platform joining strategy. Referring to the parameter values used by Örsdemir *et al.* [37] and Gong *et al.* [17], we let the baseline parameters $\delta = 4$, $\lambda = 1.1$, $c_n = 0.3$ or 0.7, $c_r = 0.06$, $c_b = 0.01$, $t = 0.02$, and $F = 0.002$.

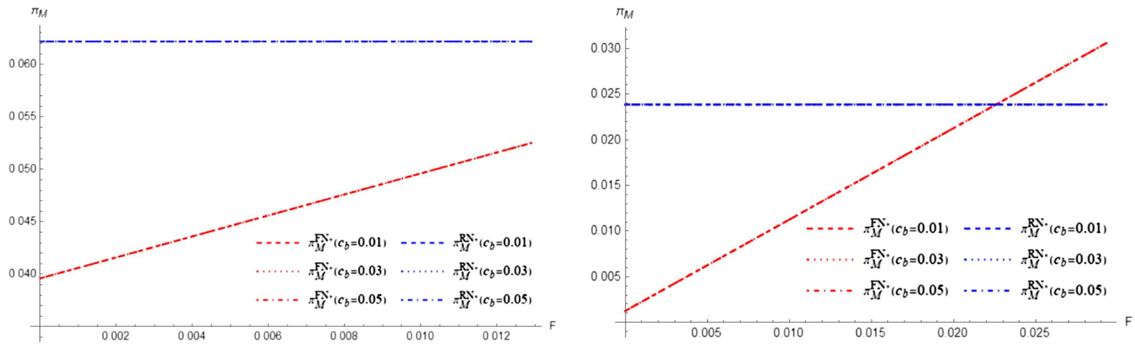
6.1. The OEM's remanufacturing authorization strategy

We first analyze the OEM's remanufacturing authorization strategy when the TPR does not join the retailer's blockchain platform. Next, we analyze the remanufacturing authorization strategy of the OEM when the TPR joins the retailer's blockchain platform. Finally, we analyze whether a win–win–win scenario may arise for the OEM, TPR, and retailer under Model *FN* and Model *RN* as well as Model *FB* and Model *RB*, and the conditions necessary to achieve such an outcome.

6.1.1. The OEM's remanufacturing authorization strategy when the TPR does not join the retailer's blockchain platform

Figure 2 shows the variation in the OEM's profit with changes in F under Model *FN* and Model *RN*, for different values of c_b, when c_n is set to 0.3 and 0.7, respectively.

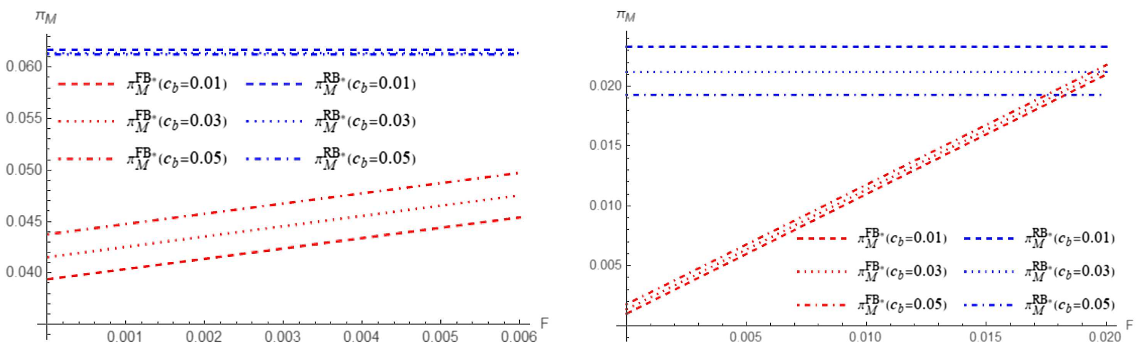
From Figure 2, it can be observed that, since the TPR does not join the retailer's blockchain platform, the value of c_b has no influence on the OEM's profit or its remanufacturing authorization strategy. Similarly, parameters λ and t do not influence the OEM's remanufacturing authorization strategy. However, comparing Figures 2a and 2b reveals that the unit production cost of new products c_n affects the OEM's remanufacturing authorization strategy. When c_n is relatively low, the OEM's profit under Model *RN* is always higher than under Model *FN*, leading the OEM to consistently choose the unit authorization fee strategy. Conversely, when c_n is relatively high, as F increases, the OEM shifts from the unit authorization fee strategy to the fixed authorization fee strategy.



(a) Variation of $\pi_M^{iN*}(i = F, R)$ with respect to F for different c_b when $c_n = 0.3$

(b) Variation of $\pi_M^{iN*}(i = F, R)$ with respect to F for different c_b when $c_n = 0.7$

FIGURE 2. Variations in OEM's profit with changes in F under Model *FN* and Model *RN* for different values of c_n and c_b.



(a) Variation of $\pi_M^{iB*}(i = F, R)$ with respect to F for different values of c_b when $c_n =$ 0.3

(b) Variation of $\pi_M^{iB*}(i = F, R)$ with respect to F for different values of c_b when $c_n =$ 0.7

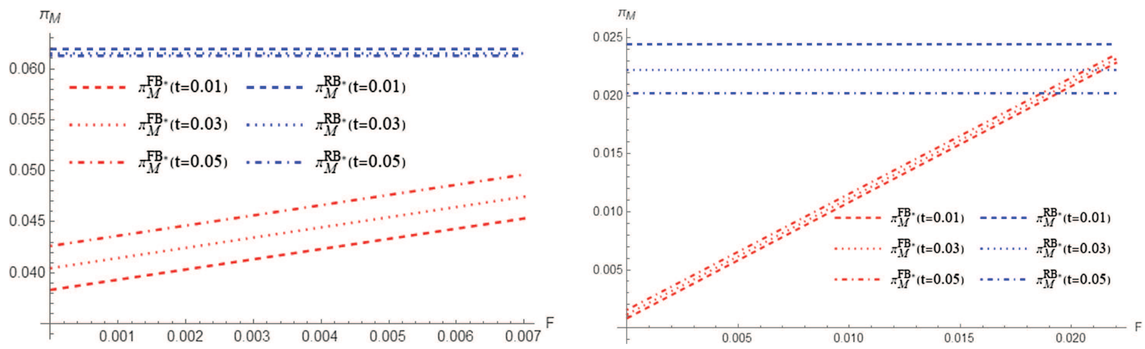
FIGURE 3. Variation in OEM's profit with changes in F under Model *FB* and Model *RB* for different values of c_n and c_b.

6.1.2. The OEM's remanufacturing authorization strategy when the TPR joins the retailer's blockchain platform

Figures 3, 4, and 5 respectively show the variation in OEM's profit with changes in F under Model FB and Model RB when c_n is set to 0.3 and 0.7, and for different values of c_b, t, and λ.

From Figures 3 to 5, it can be observed that when the TPR joins the retailer's blockchain platform: under Model *FB*, the OEM's profit increases with the increase of c_b, t, and F, but decreases with the increase of λ; under Model *RB*, the OEM's profit decreases with the increase of c_b and t, but increases with the increase of λ, and is independent of F.

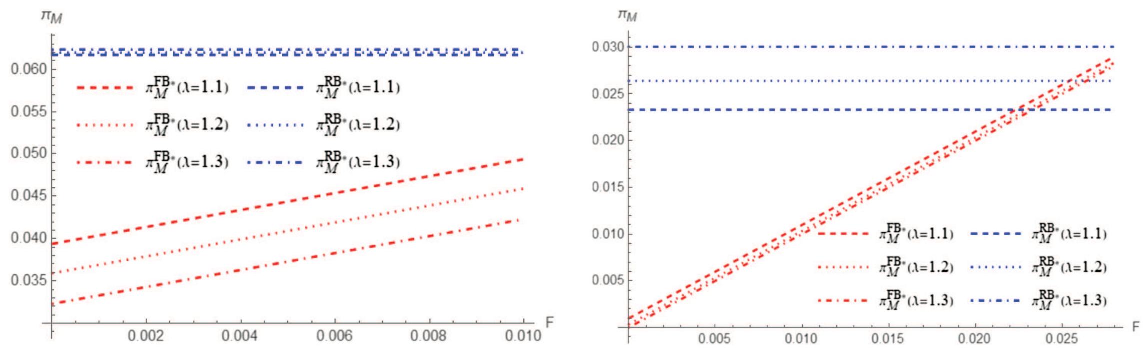
From Figures 3a to 5a, it is evident that when c_n is relatively low, regardless of the values of c_b, t, λ and F, the OEM's profit under Model *RB* is always higher than under Model *FB*. Therefore, the OEM will consistently choose the unit authorization fee strategy.



(a) Variation of $\pi_M^{iB*}(i=F,R)$ with respect to F for different values of t when $c_n = 0.3$

(b) Variation of $\pi_M^{iB*}(i=F,R)$ with respect to F for different values of t when $c_n = 0.7$

FIGURE 4. Variation in OEM's profit with changes in F under Model FB and Model RB for different values of c_n and t.



(a) Variation of $\pi_M^{iB*}(i=F,R)$ with respect to F for different values of λ when $c_n = 0.3$

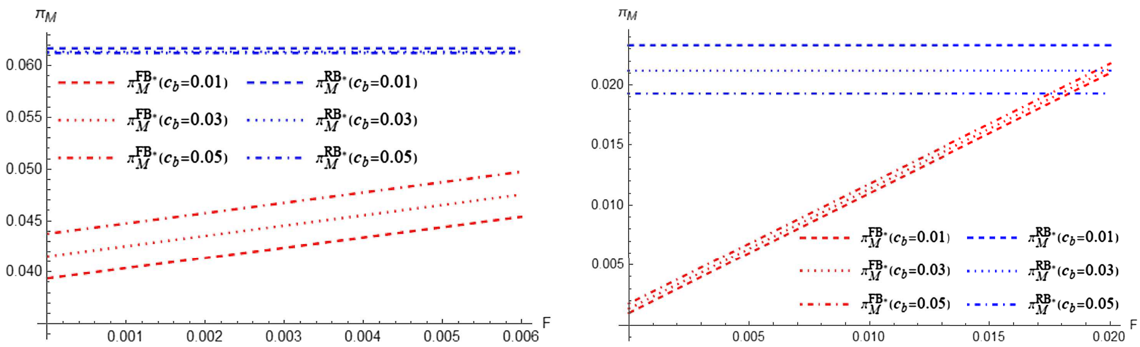
(b) Variation of $\pi_M^{iB*}(i=F,R)$ with respect to F for different values of λ when $c_n = 0.7$

FIGURE 5. Variation in OEM's profit with changes in F under Model FB and Model RB for different values of c_n and λ.

From Figures 3b to 5b, it is clear that when c_n is relatively high, as c_b, t, and F increase and λ decreases, the OEM shifts from choosing the unit authorization fee strategy to the fixed authorization fee strategy.

6.1.3. The conditions for achieving a win–win-win situation for the OEM, TPR, and retailer

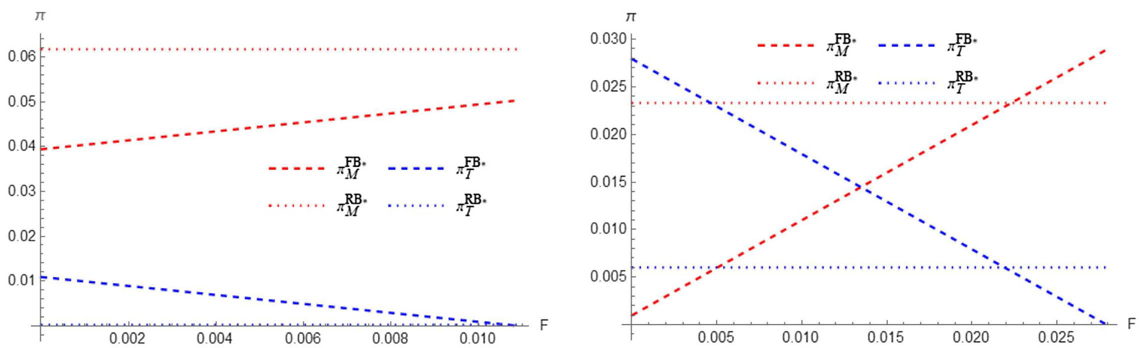
Figure 6 illustrates the variation in profits of the OEM and TPR with respect to changes in F under Model FN and Model RN when c_n takes different values. When $c_n = 0.3$, $F_1 = 0.0226$ and $F_2 = 0.0231$. However, to ensure that the TPR's profit is greater than zero, it must be guaranteed that $0 \leq F < 0.0129$. Therefore, when the TPR does not join the retailer's blockchain platform, the OEM's profit under the unit authorization fee strategy is always higher than under the fixed authorization fee strategy. Consequently, the OEM will always choose the unit authorization fee strategy. The TPR generally prefers the OEM to provide the fixed authorization fee



(a) Variation of $\pi_M^{iB*}(i=F,R)$ with respect to F for different values of c_b when $c_n=0.3$

(b) Variation of $\pi_M^{iB*}(i=F,R)$ with respect to F for different values of c_b when $c_n=0.7$

FIGURE 6. Variation in profits of the OEM and TPR with respect to changes in F under Model *FN* and Model *RN* when c_n takes different values.



(a) Variation of π_M^{iB*} and $\pi_T^{iB*}(i=F,R)$ with respect to changes in F when $c_n=0.3$

(b) Variation of π_M^{iB*} and $\pi_T^{iB*}(i=F,R)$ with respect to changes in F when $c_n=0.7$

FIGURE 7. Variation in profits of the OEM and TPR with respect to changes in F under Model *FB* and Model *RB* when c_n takes different values.

strategy, except within the narrow range of $0.0125 < F < 0.0129$, where the unit authorization fee strategy becomes more advantageous.

When $c_n = 0.7$, $F_1 = 0.0226$ and $F_2 = 0.0231$. Similarly, to ensure that the TPR's profit is greater than zero, it must be guaranteed that $0 \leq F < 0.0294$. Hence, when the TPR does not join the retailer's blockchain platform, the OEM will choose the unit authorization fee strategy when $0 \leq F < 0.0226$, and the fixed authorization fee strategy when $0.0226 < F < 0.0294$. The TPR prefers the OEM to provide the fixed authorization fee strategy when $0 \leq F < 0.0231$, and the unit authorization fee strategy when $0.0231 < F < 0.0294$. Therefore, when $0.0226 < F < 0.0231$, both the OEM and TPR are inclined to adopt the fixed authorization fee strategy.

Figure 7 illustrates the variation in profits for the OEM and TPR with respect to changes in F under Model *FB* and Model *RB* when c_n takes different values. From Figure 7, it can be observed that when $c_n = 0.3$, $F_3 = 0.0223$, and $F_4 = 0.0107$. However, to ensure that the TPR's profit is greater than zero, it must be guaranteed that $0 \leq F < 0.0109$. Therefore, when the TPR joins the retailer's blockchain platform, the OEM's profit under the unit authorization fee strategy is always higher than under the fixed authorization fee strategy.

Consequently, the OEM will always choose the unit authorization fee strategy. The TPR generally prefers the OEM to provide the fixed authorization fee strategy, except within the narrow range of $0.0107 < F < 0.0109$, where the unit authorization fee strategy becomes more advantageous.

When $c_n = 0.7$, $F_3 = 0.0223$, and $F_4 = 0.0219$. Similarly, to ensure that the TPR's profit is greater than zero, it must be guaranteed that $0 \leq F < 0.0279$. Therefore, when the TPR joins the retailer's blockchain platform, the OEM will choose the unit authorization fee strategy when $0 \leq F < 0.0223$, and the fixed authorization fee strategy when $0.0223 < F < 0.0279$. The TPR prefers the OEM to provide the fixed authorization fee strategy when $0 \leq F < 0.0219$, and the unit authorization fee strategy when $0.0219 < F < 0.0279$. Therefore, when $0.019 < F < 0.0223$, both the OEM and TPR are inclined to adopt the unit authorization fee strategy.

A synthesis of Figures 6 and 7 indicates that when the unit production cost of new products is relatively low, both the OEM and TPR will select the unit authorization fee strategy and achieve mutual benefits only when the fixed authorization fee falls within a very narrow range, regardless of the TPR's participation in the retailer's blockchain platform. Conversely, when the unit production cost is relatively high, mutual benefits can be realized across a substantially wider range of fixed authorization fee values. The specific strategy selection depends on blockchain adoption: when the TPR does not join the platform, both parties choose the fixed authorization fee strategy, whereas when joining, they adopt the unit authorization fee strategy. As established in Theorem 2(3), the retailer consistently prefers the OEM to implement the fixed authorization fee strategy. This preference structure implies that no scenario exists where all three parties benefit simultaneously when the unit production cost is low. The only potential for tripartite benefits occurs exclusively under conditions of high unit production cost combined with the TPR's decision not to join the blockchain platform.

6.2. The TPR's strategy for joining the blockchain platform

In this section, we first analyze the TPR's strategy for joining the blockchain platform under the fixed authorization fee strategy, and then examine the TPR's strategy for joining the blockchain platform under the unit authorization fee strategy. We vary the values of c_b and t from 0 to 0.03. Unless otherwise specified, the other parameters remain set as $\delta = 0.4$, $\lambda = 1.1$, $c_n = 0.3$ or 0.7, $c_r = 0.06$, and $F = 0.002$.

6.2.1. The TPR's strategy for joining the blockchain platform under the fixed authorization fee strategy

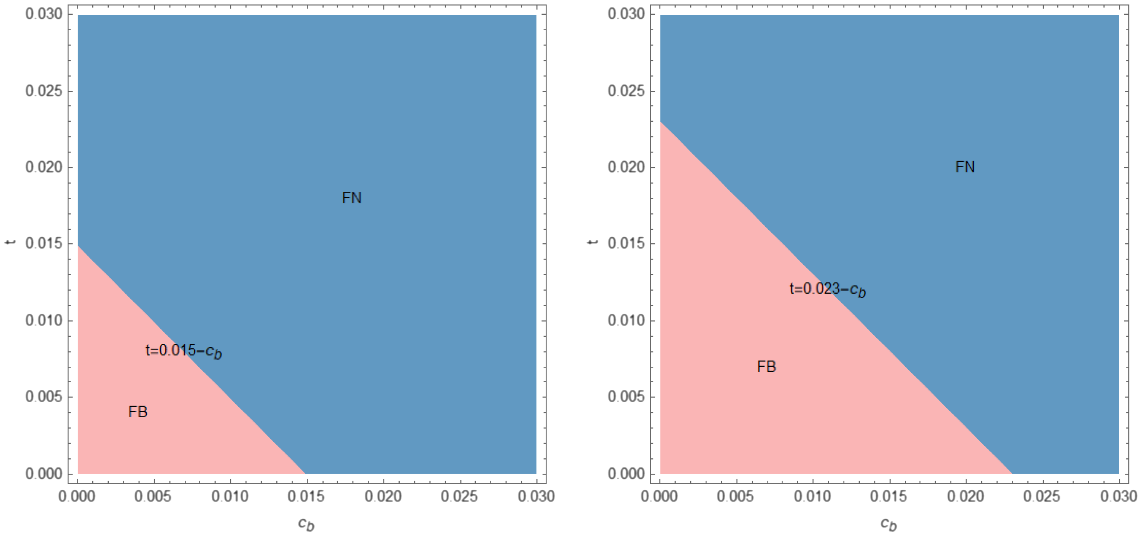
Figures 8–10 demonstrate the blockchain platform joining strategies of the TPR under different values of c_n and λ in the fixed authorization fee scenario.

From Figures 8 to 10, it can be observed that under the fixed authorization fee strategy, when c_b and t take relatively small values, the TPR will choose to join the retailer's blockchain platform. The effects of c_b and t on the TPR's decision to join the retailer's blockchain platform are consistent. When the range of variation for c_b and t remains unchanged, as c_n and λ increase, the likelihood of the TPR choosing to join the retailer's blockchain platform will also increase. When both c_n and λ are sufficiently large, joining the retailer's blockchain platform is always beneficial for the TPR.

6.2.2. The TPR's strategy for joining the blockchain platform under the unit authorization fee strategy

Figures 11–13 illustrate the blockchain platform joining strategies of the TPR under varying values of c_n and λ in the unit authorization fee scenario.

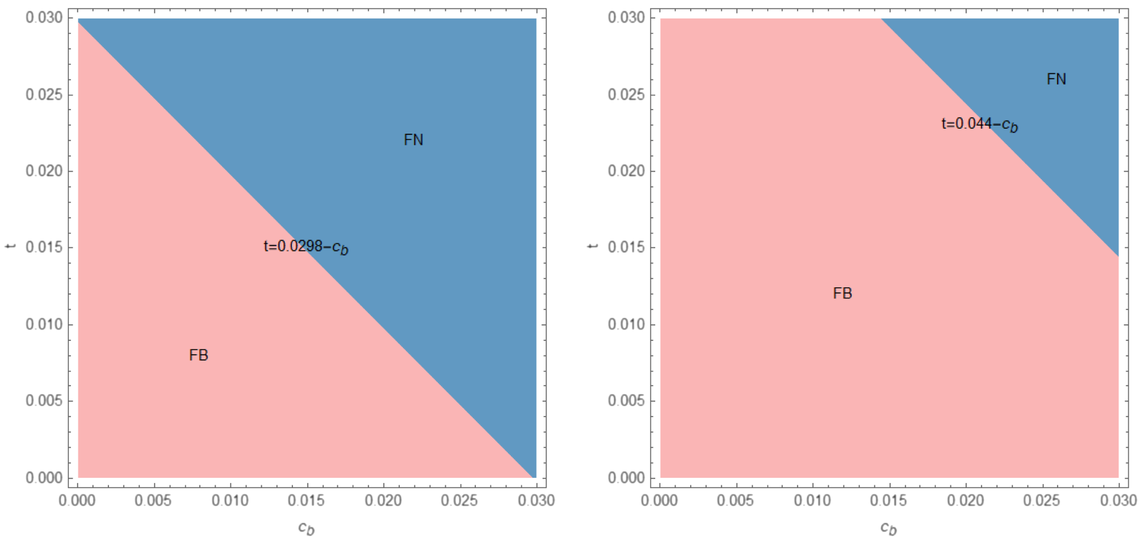
As demonstrated in Figures 10–12, the TPR's blockchain platform joining strategy under the unit authorization fee strategy remains largely consistent with that under the fixed licensing fee strategy. Specifically, when c_b and t are relatively small, the TPR will choose to join the retailer's blockchain platform. While maintaining the same range of variation for c_b and t, the likelihood of the TPR joining the retailer's blockchain platform increases progressively as c_n and λ rise. If c_n and λ are sufficiently large, joining the retailer's blockchain platform is always advantageous for the TPR.



(a) The TPR's strategy for joining the blockchain platform when $c_n = 0.3$

(b) The TPR's strategy for joining the blockchain platform when $c_n = 0.7$

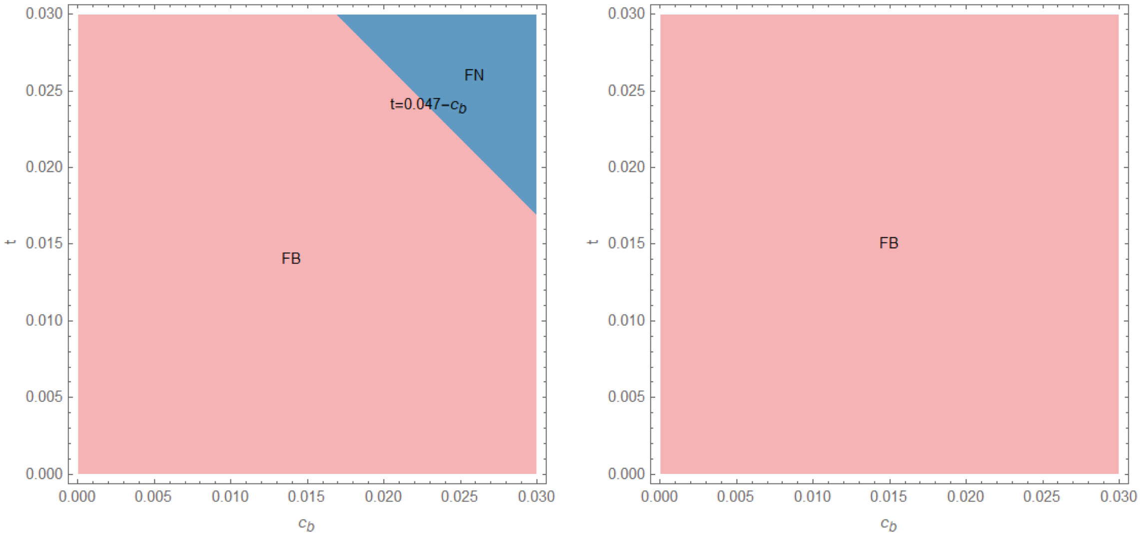
FIGURE 8. The blockchain platform joining strategy of the TPR under different values of c_n in the fixed authorization fee scenario.



(a) The TPR's strategy for joining the blockchain platform when $\lambda = 1.2$

(b) The TPR's strategy for joining the blockchain platform when $\lambda = 1.3$

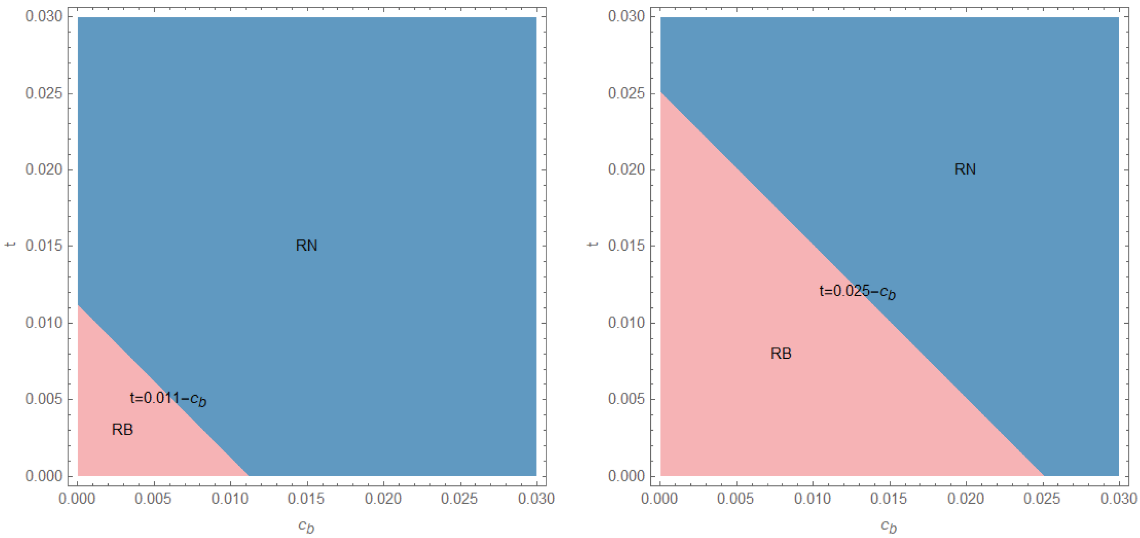
FIGURE 9. The blockchain platform joining strategy of the TPR under different values of λ in the fixed authorization fee scenario ($c_n = 0.3$).



(a) The TPR's strategy for joining the blockchain platform when $\lambda = 1.2$

(b) The TPR's strategy for joining the blockchain platform when $\lambda = 1.3$

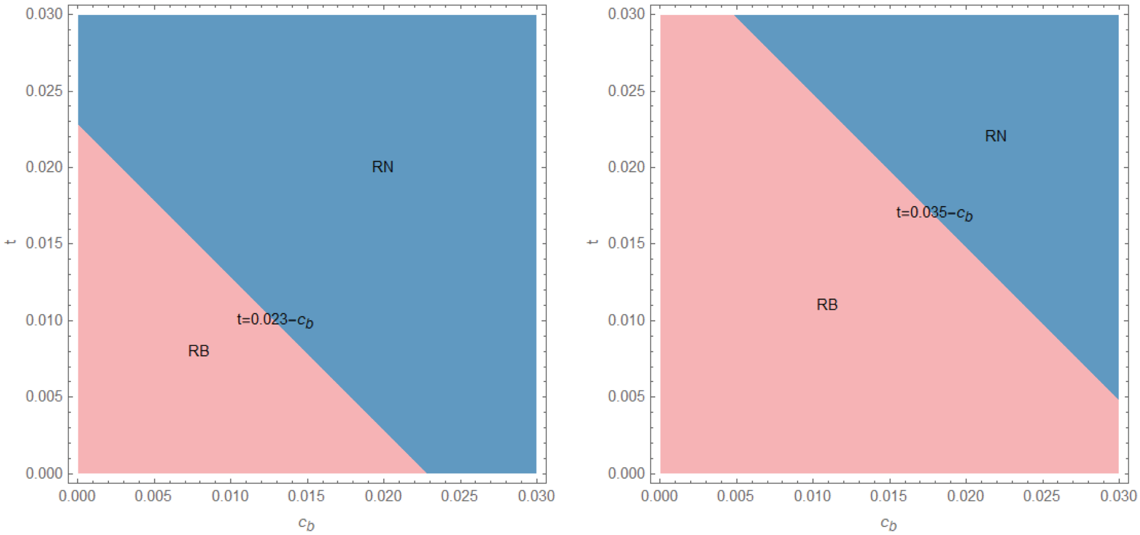
FIGURE 10. The blockchain platform joining strategy of the TPR under different values of λ in the fixed authorization fee scenario ($c_n = 0.7$).



(a) The TPR's strategy for joining the blockchain platform when $c_n = 0.3$

(b) The TPR's strategy for joining the blockchain platform when $c_n = 0.7$

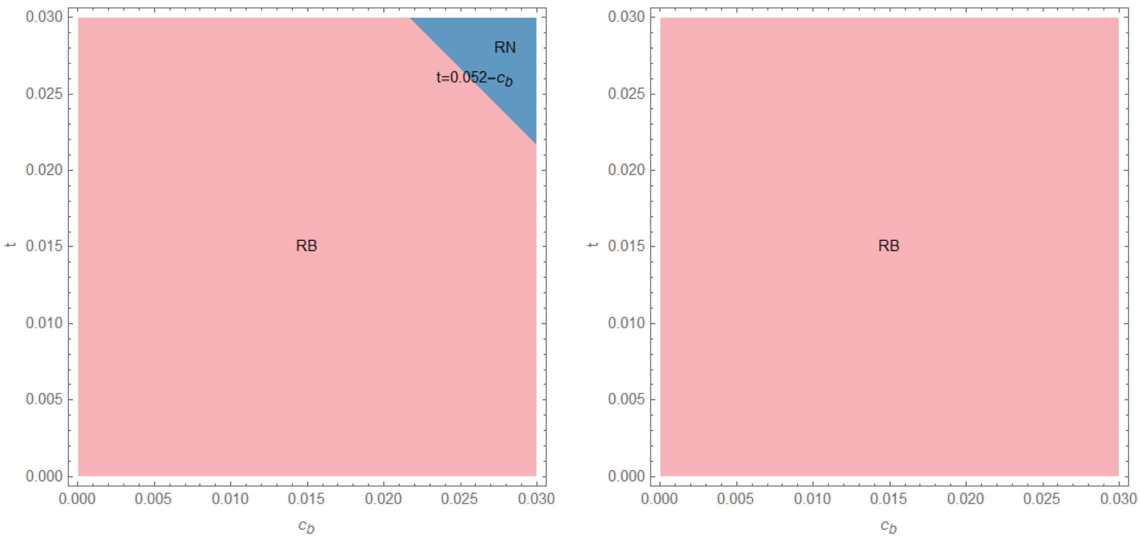
FIGURE 11. The blockchain platform joining strategy of the TPR under different values of c_n in the unit authorization fee scenario.



(a) The TPR's strategy for joining the blockchain platform when $\lambda = 1.2$

(b) The TPR's strategy for joining the blockchain platform when $\lambda = 1.3$

FIGURE 12. The blockchain platform joining strategy of the TPR under different values of λ in the unit authorization fee scenario ($c_n = 0.3$).



(a) The TPR's strategy for joining the blockchain platform when $\lambda = 1.2$

(b) The TPR's strategy for joining the blockchain platform when $\lambda = 1.3$

FIGURE 13. The blockchain platform joining strategy of the TPR under different values of λ in the unit authorization fee scenario ($c_n = 0.7$).

7. Conclusions

Authorized remanufacturing has emerged as a viable option for OEMs when facing competition from TPRs. Different authorization strategies have varying impacts on both OEMs and TPRs. Furthermore, the lack of transparency in the remanufactured product production process reduces consumers' purchase willingness. Blockchain technology's traceability can alleviate consumers' concerns about remanufactured product quality. However, implementing blockchain technology incurs certain costs and may raise consumer concerns about privacy leakage. Based on these considerations, this study constructs four decision models to examine whether the OEM should adopt a fixed authorization fee or unit authorization fee strategy when authorizing the TPR remanufacturing, and whether the TPR should join the retailer's blockchain platform. The models provide equilibrium outcomes for the OEM, TPR, and retailer under different scenarios. Through comparative analysis, sensitivity analysis, and numerical case studies of equilibrium strategies under various remanufacturing authorization approaches and blockchain joining strategies, the following conclusions are drawn.

7.1. Main conclusions

(1) **The OEM's remanufacturing authorization strategy.** When the TPR does not join the retailer's blockchain platform, the OEM may adopt the fixed authorization fee strategy only if both the unit production cost of new products and the fixed authorization fee are sufficiently high; otherwise, the unit authorization fee strategy will be implemented. When the TPR joins the retailer's blockchain platform, the OEM might choose the fixed authorization fee strategy exclusively under conditions where the unit production cost of new products, the unit variable cost of blockchain platform joining, and the consumer privacy concern cost are significantly high, while the value discount multiplier remains relatively low; failing these conditions, the OEM will opt for the unit authorization fee strategy.

(2) **The TPR's blockchain platform joining strategy.** It remains largely consistent across both authorization strategies: the TPR will choose to join the retailer's blockchain platform when the unit variable cost and consumer privacy concern cost associated with joining the retailer's blockchain platform are relatively low; otherwise, it will opt not to join. Notably, when both the production cost of new products and consumers' value discount multiplier are sufficiently high, TPR can derive greater benefits from joining the retailer's blockchain platform, even when facing relatively high unit variable cost and consumer privacy concern cost.

(3) **The win–win or tripartite–win scenario for supply chain members.** When the unit production cost of new products is relatively low, both the OEM and TPR may simultaneously adopt the unit authorization fee strategy to achieve a win–win outcome, regardless of whether the TPR joins the retailer's blockchain platform – though the probability of such win–win realization remains comparatively small. However, when facing higher unit production cost of new products, a win–win scenario emerges under different conditions: when the TPR abstains from blockchain joining, both parties may achieve mutual benefits through concurrent adoption of the fixed authorization fee strategy; when the TPR joins the platform, the win–win outcome becomes attainable through mutual selection of the unit authorization fee strategy. Notably, the probability of achieving win–win outcomes increases significantly when the unit production cost of new products is elevated. Crucially, a tripartite–win scenario involving the OEM, TPR and retailer can only materialize under the specific condition of high unit production cost of new products coupled with the TPR's non-joining the blockchain platform.

7.2. Managerial implications

Based on the obtained research findings, we further propose the following managerial implications, which can facilitate the development of action plans for different stakeholders.

Supply chain enterprises. For OEMs, the selection of remanufacturing authorization strategies is influenced by factors including the unit production cost of new products, fixed authorization fee, and post-blockchain-joining cost. Therefore, prior to finalizing authorization strategies, OEMs should conduct comprehensive analyses of their production cost structures, potential unit variable costs arising from TPRs' blockchain platform joining, value discount multipliers, and consumer privacy concern costs. For TPRs, when remanufacturing demonstrates significant cost advantages and blockchain adoption substantially enhances consumer trust indices, proactive blockchain network joining is recommended despite relatively high joining costs or elevated consumer privacy concern costs. Conversely, absent these conditions, TPRs should perform rigorous analyses of both joining costs and consumer privacy concern costs before making joining decisions.

Consumers. Consumers exhibit a marked preference for OEMs' adoption of fixed authorization fee strategies due to the resultant price reductions for both new and remanufactured products. Lower blockchain adoption costs significantly enhance consumer benefits and remanufactured product purchase propensity, whereas heightened privacy concerns correlate strongly with new product preference. Consequently, blockchain technology's dual attributes of operational security and economic efficiency emerge as critical factors influencing consumer acceptance of remanufactured products.

Government-enterprise cooperation. To enhance the market share of remanufactured products, regulators should incentivize OEMs to adopt fixed authorization fee strategies through policy incentives or subsidies. When high blockchain implementation costs or significant consumer privacy concerns deter TPRs from joining blockchain networks, regulators can provide remanufacturing subsidies to incentivize their participation. Consequently, from both supply chain development and consumer protection perspectives, regulators ought to intensify policy support, reinforce blockchain security infrastructure, and accelerate the large-scale application of blockchain technology.

7.3. Limitations and future research

This study has several limitations that point to a few potential research directions. Firstly, the current framework exclusively examines the scenario where an OEM solely manufactures new products while authorizing a TPR to handle remanufacturing, whereas future studies could extend to situations where the OEM engages in both new and remanufactured product production concurrently with remanufacturing authorization. Secondly, while our analysis focuses on the TPR joining the retailer-led blockchain platform, subsequent research should investigate alternative configurations including the TPR participating in OEM-operated blockchain networks or establishing independent blockchain infrastructures. Finally, the single-retailer distribution model adopted herein warrants expansion to examine how multi-retailer systems or hybrid distribution structures would impact blockchain adoption strategies, particularly regarding platform selection and coordination mechanisms in such multi-actor ecosystems.

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Data availability statement

The research data associated with this article are included in the article.

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Appendix A. Proofs

Proof of equilibrium solutions of Model FN in Table 3. Given that the profit functions of the OEM, TPR, and retailer are represented by equations (1), (2), and (3), respectively. According to backward induction, first, substituting $p_n = 1 - q_n - \delta q_r$ and $p_r = \delta(1 - q_n - q_r)$ into equation (3) yields: $\pi_R^{FN} = (1 - q_n - \delta q_r - w_n)q_n + (\delta(1 - q_n - q_r) - w_r)q_r$. The Hessian matrix of π_R^{FN} with respect to q_n^{FN} and q_r^{FN} is obtained as $H = \begin{bmatrix} -2 & -2\delta \\ -2\delta & -2\delta \end{bmatrix}$, and $\det(H) = 4\delta(1-\delta) > 0$. Therefore, the profit function of the retailer has a unique optimal solution. Taking the first partial derivatives of π_R^{FN} with respect to q_n^{FN} and q_r^{FN} and setting them equal to zero, we obtain $q_n^{FN^*}(w_n, w_r) = \frac{1-\delta-w_n+w_r}{2(1-\delta)}$ and $q_r^{FN^*}(w_n, w_r) = \frac{\delta w_n - w_r}{2\delta(1-\delta)}$. Subsequently, substituting $q_r^{FN^*}(w_n, w_r) = \frac{\delta w_n - w_r}{2\delta(1-\delta)}$ into equation (2) yields $\pi_T^{FN} = \frac{(\delta w_n - w_r)(w_r - c_r)}{2\delta(1-\delta)} - F$. Taking the second derivative of π_T^{FN} with respect to w_r^{FN}, we obtain $\frac{d^2\pi_T^{FN}}{d(w_r^{FN})^2} = -\frac{1}{\delta(1-\delta)} < 0$. Therefore, the profit function of the TPR has a unique optimal solution. Taking the first derivative of π_T^{FN} with respect to w_r^{FN} and setting it equal to zero gives $w_r^{FN^*}(w_n) = \frac{\delta w_n + c_r}{2}$. Substituting $w_r^{FN^*}(w_n) = \frac{\delta w_n + c_r}{2}$ into $q_n^{FN^*}(w_n, w_r)$ and $q_r^{FN^*}(w_n, w_r)$ respectively, we get $q_n^{FN^*}(w_n) = \frac{2(1-\delta)-(2-\delta)w_n+c_r}{4(1-\delta)}$ and $q_r^{FN^*}(w_n) = \frac{\delta w_n - c_r}{4\delta(1-\delta)}$. Next, substituting $q_n^{FN^*}(w_n) = \frac{2(1-\delta)-(2-\delta)w_n+c_r}{4(1-\delta)}$ into equation (1) yields $\pi_M^{FN} = \frac{(w_n - c_n)(2(1-\delta)-(2-\delta)w_n+c_r)}{4(1-\delta)} + F$. Taking the second derivative of π_M^{FN} with respect to w_n^{FN}, we obtain $\frac{d^2\pi_M^{FN}}{d(w_n^{FN})^2} = -\frac{2-\delta}{2(1-\delta)} < 0$. Therefore, the profit function of the OEM has a unique optimal solution. Taking the first derivative of π_M^{FN} with respect to w_n^{FN} and setting it equal to zero gives $w_n^{FN^*} = \frac{2(1-\delta)+(2-\delta)c_n+c_r}{2(2-\delta)}$. Finally, first substituting $w_n^{FN^*}$ into $w_r^{FN^*}(w_n)$, $q_n^{FN^*}(w_n)$, and $q_r^{FN^*}(w_n)$, we obtain $w_r^{FN^*} = \frac{2\delta(1-\delta)+\delta(2-\delta)c_n+(4-\delta)c_r}{4(2-\delta)}$, $q_n^{FN^*} = \frac{\beta_1}{8(1-\delta)}$, and $q_r^{FN^*} = \frac{\beta_2}{8\delta(1-\delta)(2-\delta)}$; then substituting $q_n^{FN^*}$ and $q_r^{FN^*}$ into $p_n = 1 - q_n - \delta q_r$ and $p_r = \delta(1 - q_n - q_r)$, we get $p_n^{FN^*} = \frac{2(3-2\delta)+(2-\delta)c_n+c_r}{4(2-\delta)}$ and $p_r^{FN^*} = \frac{2\delta(5-3\delta)+\delta(2-\delta)c_n+(4-\delta)c_r}{8(2-\delta)}$; and lastly, substituting $w_n^{FN^*}$, $w_r^{FN^*}$, $q_n^{FN^*}$, $q_r^{FN^*}$, $p_n^{FN^*}$, and $p_r^{FN^*}$ into equations (1), (2), and (3), respectively, we obtain $\pi_M^{FN^*} = \frac{\beta_1^2}{16(1-\delta)(2-\delta)} + F$, $\pi_T^{FN^*} = \frac{\beta_2^2}{32\delta(1-\delta)(2-\delta)^2} - F$, and $\pi_R^{FN^*} = \frac{\beta_1^2}{32(1-\delta)(2-\delta)} - \frac{\beta_2^2}{64\delta(1-\delta)(2-\delta)^2} + \frac{(\delta\beta_1+\beta_2)(\delta-c_r)}{16\delta(1-\delta)(2-\delta)}$. □

Proof of equilibrium solutions of Model FB in Table 4. Given that the profit functions of the OEM, TPR, and the retailer are presented in equations (4), (5), and (6), respectively. According to backward induction, first, substituting $p_n = 1 - q_n - \lambda\delta q_r$ and $p_r = \lambda\delta(1 - q_n - q_r) - t$ into equation (6) yields $\pi_R^{FB} = (1 - q_n - \lambda\delta q_r - w_n)q_n + (\lambda\delta(1 - q_n - q_r) - t - w_r)q_r$. Taking the Hessian matrix of π_R^{FB} with respect to q_n^{FB} and q_r^{FB}, we obtain $H = \begin{bmatrix} -2 & -2\delta\lambda \\ -2\delta\lambda & -2\delta\lambda \end{bmatrix}$ and $\det(H) = 4\delta\lambda(1 - \delta\lambda) > 0$. Therefore, the profit function of the retailer has a unique optimal solution. Taking the first partial derivatives of π_R^{FB} with respect to q_n^{FB} and q_r^{FB} and setting them equal to zero, we obtain $q_n^{FB^*}(w_n, w_r) = \frac{1-\delta\lambda-w_n+w_r+t}{2(1-\delta\lambda)}$ and $q_r^{FB^*}(w_n, w_r) = \frac{\delta\lambda w_n - w_r - t}{2\delta\lambda(1-\delta\lambda)}$. Subsequently, substituting $q_r^{FB^*}(w_n, w_r) = \frac{\delta\lambda w_n - w_r - t}{2\delta\lambda(1-\delta\lambda)}$ into equation (5) yields $\pi_T^{FN} = \frac{(\delta\lambda w_n - w_r - t)(w_r - c_r - c_b)}{2\delta\lambda(1-\delta\lambda)} - F$. Taking the second derivative of π_T^{FB} with respect to w_r^{FB}, we obtain $\frac{d^2\pi_T^{FB}}{d(w_r^{FB})^2} = -\frac{1}{\delta\lambda(1-\delta\lambda)} < 0$. Therefore, the profit function of the TPR has a unique optimal solution. Taking the first derivative of π_T^{FB} with respect to w_r^{FB} and setting it equal to zero gives $w_r^{FB^*}(w_n) = \frac{\delta\lambda w_n + c_r + c_b - t}{2}$. Substituting $w_r^{FB^*}(w_n) = \frac{\delta\lambda w_n + c_r + c_b - t}{2}$ into $q_n^{FB^*}(w_n, w_r)$ and $q_r^{FB^*}(w_n, w_r)$ respectively, we get $q_n^{FB^*}(w_n) = \frac{2(1-\delta\lambda)-(2-\delta\lambda)w_n+c_r+c_b+t}{4(1-\delta\lambda)}$ and $q_r^{FB^*}(w_n) = \frac{\delta\lambda w_n - c_r - c_b - t}{4\delta\lambda(1-\delta\lambda)}$. Next, substituting $q_n^{FB^*}(w_n) = \frac{2(1-\delta\lambda)-(2-\delta\lambda)w_n+c_r+c_b+t}{4(1-\delta\lambda)}$ into equation (4) yields $\pi_M^{FB} = \frac{(w_n-c_n)(2(1-\delta\lambda)-(2-\delta\lambda)w_n+c_r+c_b+t)}{4(1-\delta\lambda)} + F$. Taking the second derivative of π_M^{FB} with respect to w_n^{FB}, we obtain $\frac{d^2\pi_M^{FB}}{d(w_n^{FB})^2} = -\frac{2-\delta\lambda}{2(1-\delta\lambda)} < 0$. Therefore, the profit function of the OEM has a unique optimal solution. Taking the first derivative of π_M^{FB} with respect to w_n^{FB} and setting it equal to zero gives $w_n^{FB^*} = \frac{2(1-\delta\lambda)+(2-\delta\lambda)c_n+c_r+c_b+t}{2(2-\delta\lambda)}$. Finally, first substituting $w_n^{FB^*}$ into $w_r^{FB^*}(w_n)$, $q_n^{FB^*}(w_n)$, and $q_r^{FB^*}(w_n)$, we obtain $w_r^{FB^*} = \frac{2\delta\lambda(1-\delta\lambda)+\delta\lambda(2-\delta\lambda)c_n+(4-\delta\lambda)(c_b+c_r)-(4-3\delta\lambda)t}{4(2-\delta\lambda)}$, $q_n^{FB^*} = \frac{\beta_3}{8(1-\delta\lambda)}$, and $q_r^{FB^*} = \frac{\beta_4}{8\delta\lambda(1-\delta\lambda)(2-\delta\lambda)}$; then substituting $q_n^{FB^*}$ and $q_r^{FB^*}$ into $p_n = 1 - q_n - \lambda\delta q_r$ and $p_r = \lambda\delta(1 - q_n - q_r) - t$ we get $p_n^{FB^*} = \frac{2(3-2\delta\lambda)+(2-\delta\lambda)c_n+c_r+c_b+t}{4(2-\delta\lambda)}$ and $p_r^{FB^*} = \frac{2\delta\lambda(5-3\delta\lambda)+\delta\lambda(2-\delta\lambda)c_n+(4-\delta\lambda)(c_r+c_b)-(12-7\delta\lambda)t}{8(2-\delta\lambda)}$; Lastly, substituting $w_n^{FB^*}$, $w_r^{FB^*}$, $q_n^{FB^*}$, $q_r^{FB^*}$, $p_n^{FB^*}$, and $p_r^{FB^*}$ into equations (4), (5), and (6), respectively, we obtain $\pi_M^{FB^*} = \frac{\beta_3^2}{16(1-\delta\lambda)(2-\delta\lambda)} + F$, $\pi_T^{FB^*} = \frac{\beta_4^2}{32\delta\lambda(1-\delta\lambda)(2-\delta\lambda)^2} - F$, and $\pi_R^{FB^*} = \frac{\beta_3^2}{32(1-\delta\lambda)(2-\delta\lambda)} - \frac{\beta_4^2}{64\delta\lambda(1-\delta\lambda)(2-\delta\lambda)^2} + \frac{(\delta\lambda\beta_3+\beta_4)(\delta\lambda-c_r-c_b-t)}{16\delta\lambda(1-\delta\lambda)(2-\delta\lambda)}$. □

Proof of Equilibrium solutions of Model RN in Table 5. Given that the profit functions of the OEM, TPR, and retailer are presented in equations (7), (8), and (9), respectively. According to backward induction, first, substituting $p_n = 1 - q_n - \delta q_r$ and $p_r = \delta(1 - q_n - q_r)$ into equation (9) yields $\pi_R^{RN} = (1 - q_n - \delta q_r - w_n)q_n + (\delta(1 - q_n - q_r) - w_r)q_r$. Taking the Hessian matrix of π_R^{RN} with respect to q_n^{RN} and q_r^{RN}, we obtain $H = \begin{bmatrix} -2 & -2\delta \\ -2\delta & -2\delta \end{bmatrix}$ and $\det(H) = 4\delta(1 - \delta) > 0$. Therefore, the profit function of the retailer has a unique optimal solution. Taking the first partial derivatives of π_R^{RN} with respect to q_n^{RN} and q_r^{RN} and setting them equal to zero, we get $q_n^{RN^*}(w_n, w_r) = \frac{1-\delta-w_n+w_r}{2(1-\delta)}$ and $q_r^{RN^*}(w_n, w_r) = \frac{\delta w_n - w_r}{2\delta(1-\delta)}$. Subsequently, substituting $q_r^{RN^*}(w_n, w_r) = \frac{\delta w_n - w_r}{2\delta(1-\delta)}$ into equation (8) yields $\pi_T^{RN} = \frac{(\delta w_n - w_r)(w_r - c_r - f)}{2\delta(1-\delta)}$. Taking the second derivative of π_T^{RN} with respect to w_r^{RN}, we obtain $\frac{d^2\pi_T^{RN}}{d(w_r^{RN})^2} = -\frac{1}{\delta(1-\delta)} < 0$. Therefore, the profit function of TPR has a unique optimal solution. Taking the first derivative of π_T^{RN} with respect to w_r^{RN} and setting it equal to zero gives $w_r^{RN^*}(w_n, f) = \frac{\delta w_n + c_r + f}{2}$. Substituting $w_r^{RN^*}(w_n, f) = \frac{\delta w_n + c_r + f}{2}$ into $q_n^{RN^*}(w_n, w_r)$ and $q_r^{RN^*}(w_n, w_r)$ respectively, we get $q_n^{RN^*}(w_n, f) = \frac{2(1-\delta)-(2-\delta)w_n+c_r+f}{4(1-\delta)}$ and $q_r^{RN^*}(w_n, f) = \frac{\delta w_n - c_r - f}{4\delta(1-\delta)}$. Next, substituting $q_n^{RN^*}(w_n, f) = \frac{2(1-\delta)-(2-\delta)w_n+c_r+f}{4(1-\delta)}$ and $q_r^{RN^*}(w_n, f) = \frac{\delta w_n - c_r - f}{4\delta(1-\delta)}$ into equation (7) yields $\pi_M^{RN} = \frac{(w_n-c_n)(2(1-\delta)-(2-\delta)w_n+c_r+f)}{4(1-\delta)} + \frac{(\delta w_n - c_r - f)f}{4\delta(1-\delta)}$. Taking the Hessian matrix of π_M^{RN} with respect to w_n^{RN} and f^{RN}, we obtain $H = \begin{bmatrix} -\frac{2-\delta}{2(1-\delta)} & \frac{1}{2(1-\delta)} \\ \frac{1}{2(1-\delta)} & -\frac{1}{2\delta(1-\delta)} \end{bmatrix}$ and $\det(H) = \frac{1}{2\delta(1-\delta)} > 0$. Therefore, the profit function of the OEM has a unique optimal solution. Taking the first partial derivatives of π_M^{RN} with respect to w_n^{RN} and f^{RN} and setting them equal to zero gives $w_n^{RN^*} = \frac{1+c_n}{2}$ and $f^{RN^*} = \frac{\delta - c_r}{2}$. Finally, first substituting $w_n^{RN^*}$ and f^{RN^*} into $w_r^{RN^*}(w_n, f)$, $q_n^{RN^*}(w_n, f)$, and $q_r^{RN^*}(w_n, f)$, we gain $w_r^{RN^*} = \frac{2\delta + \delta c_n + c_r}{4}$, $q_n^{RN^*} = \frac{\beta_1}{8(1-\delta)}$, and $q_r^{RN^*} = \frac{\delta c_n - c_r}{8\delta(1-\delta)}$; then substituting $q_n^{RN^*}$ and $q_r^{RN^*}$ into $p_n = 1 - q_n - \delta q_r$ and $p_r = \delta(1 - q_n - q_r)$, we get $p_n^{RN^*} = \frac{3+c_n}{4}$ and $p_r^{RN^*} = \frac{6\delta + \delta c_n + c_r}{8}$; lastly, substituting $w_n^{RN^*}$, f^{RN^*}, $w_r^{RN^*}$, $q_n^{RN^*}$, $q_r^{RN^*}$, $p_n^{RN^*}$, and $p_r^{RN^*}$ into equations (7), (8), and (9), respectively, we obtain $\pi_M^{RN^*} = \frac{2\delta(1-\delta)(1-c_n)^2 + (\delta c_n - c_r)^2}{16\delta(1-\delta)}$, $\pi_T^{RN^*} = \frac{(\delta c_n - c_r)^2}{32\delta(1-\delta)}$, and $\pi_R^{RN^*} = \frac{4\delta(1-\delta)(1-c_n)^2 + (\delta c_n - c_r)^2}{64\delta(1-\delta)}$. □

Proof of equilibrium solutions of Model RB in Table 6. Given that the profit functions of the OEM, TPR, and retailer are represented by equations (10), (11), and (12) respectively. According to backward induction, first substitute $p_n = 1 - q_n - \lambda\delta q_r$ and $p_r = \lambda\delta(1 - q_n - q_r) - t$ into equation (12) to obtain $\pi_R^{RB} = (1 - q_n - \lambda\delta q_r - w_n)q_n + (\lambda\delta(1 - q_n - q_r) - t - w_r)q_r$. By calculating the Hessian matrix of π_R^{RB} with respect to q_n^{RB} and q_r^{RB}, we get $H = \begin{bmatrix} -2 & -2\delta\lambda \\ -2\delta\lambda & -2\delta\lambda \end{bmatrix}$ and $\det(H) = 4\delta\lambda(1 - \delta\lambda) > 0$. Therefore, the profit function of the retailer has a unique optimal solution. By taking the first-order partial derivatives of π_R^{RB} with respect to q_n^{RB} and q_r^{RB} and setting them equal to zero, we obtain $q_n^{RB^*}(w_n, w_r) = \frac{1 - \delta\lambda - w_n + w_r + t}{2(1-\delta\lambda)}$ and $q_r^{RB^*}(w_n, w_r) = \frac{\delta\lambda w_n - w_r - t}{2\delta\lambda(1-\delta\lambda)}$. Secondly, substituting $q_r^{RB^*}(w_n, w_r) = \frac{\delta\lambda w_n - w_r - t}{2\delta\lambda(1-\delta\lambda)}$ into equation (11), we obtain $\pi_T^{RB} = \frac{(\delta\lambda w_n - w_r - t)(w_r - c_r - c_b - f)}{2\delta\lambda(1-\delta\lambda)}$. Taking the second-order derivative of π_T^{RB} with respect to w_r^{RB}, we get $\frac{d^2\pi_T^{RB}}{d(w_r^{RB})^2} = -\frac{1}{\delta\lambda(1-\delta\lambda)} < 0$. Therefore, the profit function of the TPR has a unique optimal solution. Taking the first-order derivative of π_T^{RB} with respect to w_r^{RB} and setting it equal to zero, we obtain $w_r^{RB^*}(w_n, f) = \frac{\delta\lambda w_n + c_r + c_b + f - t}{2}$. Substituting $w_r^{RB^*}(w_n, f) = \frac{\delta\lambda w_n + c_r + c_b + f - t}{2}$ into $q_n^{RB^*}(w_n, w_r)$ and $q_r^{RB^*}(w_n, w_r)$ respectively, yields $q_n^{RB^*}(w_n, f) = \frac{2(1-\delta\lambda) - (2-\delta\lambda)w_n + c_r + c_b + f + t}{4(1-\delta\lambda)}$ and $q_r^{RB^*}(w_n, f) = \frac{\delta\lambda w_n - c_r - c_b - f - t}{4\delta\lambda(1-\delta\lambda)}$. Then, Substituting $q_n^{RB^*}(w_n, f) = \frac{2(1-\delta\lambda) - (2-\delta\lambda)w_n + c_r + c_b + f + t}{4(1-\delta\lambda)}$ and $q_r^{RB^*}(w_n, f) = \frac{\delta\lambda w_n - c_r - c_b - f - t}{4\delta\lambda(1-\delta\lambda)}$ into equation (10), we obtain $\pi_M^{RB} = \frac{(w_n - c_n)(2(1-\delta\lambda) - (2-\delta\lambda)w_n + c_r + c_b + f + t)}{4(1-\delta\lambda)} + \frac{(\delta\lambda w_n - c_r - c_b - f - t)f}{4\delta\lambda(1-\delta\lambda)}$. Calculating the Hessian matrix of π_M^{RB} with respect to w_n^{RB} and f^{RB}, we get $H = \begin{bmatrix} -\frac{2-\delta\lambda}{2(1-\delta\lambda)} & \frac{1}{2(1-\delta\lambda)} \\ \frac{1}{2(1-\delta\lambda)} & -\frac{1}{2\delta\lambda(1-\delta\lambda)} \end{bmatrix}$ and $\det(H) = \frac{1}{2\delta\lambda(1-\delta\lambda)} > 0$. Therefore, the profit function of the OEM has a unique optimal solution. Taking the first-order partial derivatives of π_M^{RB} with respect to w_n^{RB} and f^{RB} and setting them equal to zero, we obtain $w_n^{RB^*} = \frac{1+c_n}{2}$ and $f^{RB^*} = \frac{\delta\lambda - c_r - c_b - t}{2}$. Finally, first substitute $w_n^{RB^*}$ and f^{RB^*} into $w_r^{RB^*}(w_n, f)$, $q_n^{RB^*}(w_n, f)$, and $q_r^{RB^*}(w_n, f)$, respectively, to obtain $w_r^{RB^*} = \frac{2\delta\lambda + \delta\lambda c_n + c_r + c_b - 3t}{4}$, $q_n^{RB^*} = \frac{\beta_3}{8(1-\delta\lambda)}$, and $q_r^{RB^*} = \frac{\delta\lambda c_n - c_r - c_b - t}{8\delta\lambda(1-\delta\lambda)}$; then, substituting $q_n^{RB^*}$ and $q_r^{RB^*}$ into $p_n = 1 - q_n - \lambda\delta q_r$ and $p_r = \lambda\delta(1 - q_n - q_r) - t$, we get $p_n^{RB^*} = \frac{3+c_n}{4}$ and $p_r^{RB^*} = \frac{6\delta\lambda + \delta\lambda c_n + c_r + c_b - 7t}{8}$; lastly, substituting $w_n^{RB^*}$, f^{RB^*}, $w_r^{RB^*}$, $q_n^{RB^*}$, $q_r^{RB^*}$, $p_n^{RB^*}$, and $p_r^{RB^*}$ into equations (10), (11), and (12), respectively, we obtain $\pi_M^{RB^*} = \frac{2(1-\delta\lambda)\delta\lambda(1-c_n)^2 + (\delta\lambda c_n - c_r - c_b - t)^2}{16\lambda\delta(1-\delta\lambda)}$, $\pi_T^{RB^*} = \frac{(\delta\lambda c_n - c_r - c_b - t)^2}{32\delta\lambda(1-\delta\lambda)}$, and $\pi_R^{RB^*} = \frac{4(1-\delta\lambda)\delta\lambda(1-c_n)^2 + (\delta\lambda c_n - c_r - c_b - t)^2}{64\delta\lambda(1-\delta\lambda)}$. □

Proof of Theorem 1. Since $c_r < \delta c_n$, it follows $c_r < \delta$. Therefore, $w_n^{FN^*} - w_n^{RN^*} = -\frac{\delta - c_r}{2(2-\delta)} < 0$, which implies $w_n^{FN^*} < w_n^{RN^*}$. $w_r^{FN^*} - w_r^{RN^*} = -\frac{\delta - c_r}{2(2-\delta)} < 0$, therefore $w_r^{FN^*} < w_r^{RN^*}$. $p_n^{FN^*} - p_n^{RN^*} = -\frac{\delta - c_r}{4(2-\delta)} < 0$, therefore $p_n^{FN^*} < p_n^{RN^*}$. $p_r^{FN^*} - p_r^{RN^*} = -\frac{\delta - c_r}{4(2-\delta)} < 0$, therefore $p_r^{FN^*} < p_r^{RN^*}$. $q_n^{FN^*} - q_n^{RN^*} = 0$, therefore $q_n^{FN^*} = q_n^{RN^*}$. $q_r^{FN^*} - q_r^{RN^*} = \frac{\delta - c_r}{4\delta(2-\delta)} > 0$, therefore $q_r^{FN^*} > q_r^{RN^*}$.

Since $c_r < \delta\lambda c_n - c_b - t$, it follows $c_r < \delta\lambda - c_b - t$. Therefore, $w_n^{FB^*} - w_n^{RB^*} = -\frac{\delta\lambda - c_r - c_b - t}{2(2-\delta\lambda)} < 0$, which implies $w_n^{FB^*} < w_n^{RB^*}$. $w_r^{FB^*} - w_r^{RB^*} = -\frac{\delta\lambda - c_r - c_b - t}{2(2-\delta\lambda)} < 0$, therefore $w_r^{FB^*} < w_r^{RB^*}$. $p_n^{FB^*} - p_n^{RB^*} = -\frac{\delta\lambda - c_r - c_b - t}{4(2-\delta\lambda)} < 0$, therefore $p_n^{FB^*} < p_n^{RB^*}$. $p_r^{FB^*} - p_r^{RB^*} = -\frac{\delta\lambda - c_r - c_b - t}{4(2-\delta\lambda)} < 0$, therefore $p_r^{FB^*} < p_r^{RB^*}$. $q_n^{FB^*} - q_n^{RB^*} = 0$, therefore $q_n^{FB^*} = q_n^{RB^*}$. $q_r^{FB^*} - q_r^{RB^*} = \frac{\delta\lambda - c_r - c_b - t}{4\delta\lambda(2-\delta\lambda)} > 0$, therefore $q_r^{FB^*} > q_r^{RB^*}$. □

Proof of Corollary 1.

$$\left(p_n^{FN^*} - w_n^{FN^*}\right) - \left(p_n^{RN^*} - w_n^{RN^*}\right) = \left(p_r^{FN^*} - w_r^{FN^*}\right) - \left(p_r^{RN^*} - w_r^{RN^*}\right) = \frac{\delta - c_r}{4(2-\delta)}$$

$$\left(p_n^{FB^*} - w_n^{FB^*}\right) - \left(p_n^{RB^*} - w_n^{RB^*}\right) = \left(p_r^{FB^*} - w_r^{FB^*}\right) - \left(p_r^{RB^*} - w_r^{RB^*}\right) = \frac{\delta\lambda - c_r - c_b - t}{4(2-\delta\lambda)}.$$

□

Proof of Theorem 2. (1) $\pi_M^{FN^*} - \pi_M^{RN^*} = F - \frac{(\delta - c_r)^2}{8\delta(2-\delta)}$, therefore, when $F \geq F_1$, $\pi_M^{FN^*} \geq \pi_M^{RN^*}$; when $F < F_1$, $\pi_M^{FN^*} < \pi_M^{RN^*}$. $\pi_T^{FN^*} - \pi_T^{RN^*} = \frac{(\delta - c_r)(\delta(1-\delta) + \delta(2-\delta)c_n - (3-2\delta)c_r)}{8\delta(2-\delta)^2} - F$, therefore, when $F \leq F_2$, $\pi_T^{FN^*} \geq \pi_T^{RN^*}$; when $F > F_2$, $\pi_T^{FN^*} < \pi_T^{RN^*}$. $\pi_R^{FN^*} - \pi_R^{RN^*} = \frac{(\delta - c_r)(\delta(2-\delta)(1-c_n) + (3-\delta)(\delta - c_r))}{16\delta(2-\delta)^2} > 0$, therefore $\pi_R^{FN^*} > \pi_R^{RN^*}$.

(2) $\pi_M^{FB^*} - \pi_M^{RB^*} = F - \frac{(\delta\lambda - c_r - c_b - t)^2}{8\delta\lambda(2-\delta\lambda)}$, therefore, when $F \geq F_3$, $\pi_M^{FB^*} \geq \pi_M^{RB^*}$; when $F < F_3$, $\pi_M^{FB^*} < \pi_M^{RB^*}$. $\pi_T^{FB^*} - \pi_T^{RB^*} = \frac{(\delta\lambda - c_r - c_b - t)\left(\delta\lambda(1-\delta\lambda) + \delta\lambda(2-\delta\lambda)c_n - (3-2\delta\lambda)(c_r + c_b + t)\right)}{8\delta\lambda(2-\delta\lambda)^2} - F$, therefore, when $F \leq F_4$, $\pi_T^{FB^*} \geq \pi_T^{RB^*}$; when $F > F_4$, $\pi_T^{FB^*} < \pi_T^{RB^*}$.

(3) $\pi_R^{FN^*} - \pi_R^{RN^*} = \frac{(\delta - c_r)(\delta(2-\delta)(1-c_n) + (3-\delta)(\delta - c_r))}{16\delta(2-\delta)^2} > 0$, therefore $\pi_R^{FN^*} > \pi_R^{RN^*}$; $\pi_R^{FB^*} - \pi_R^{RB^*} = \frac{(\delta\lambda - c_r - c_b - t)(\delta\lambda(2-\delta\lambda)(1-c_n) + (3-\delta\lambda)(\delta\lambda - c_r - c_b - t))}{16\delta\lambda(2-\delta\lambda)^2} > 0$, therefore $\pi_R^{FB^*} > \pi_R^{RB^*}$. Thus, it can be concluded that $\pi_R^{Fj^*} > \pi_R^{Rj^*}$.

□

Proof of Theorem 3. (1) $w_n^{FB^*} - w_n^{FN^*} = \frac{(2-\delta)(c_b + t) - \delta(\lambda - 1)(2 - c_r)}{2(2-\delta)(2-\delta\lambda)}$ and $p_n^{FB^*} - p_n^{FN^*} = \frac{(2-\delta)(c_b + t) - \delta(\lambda - 1)(2 - c_r)}{4(2-\delta)(2-\delta\lambda)}$, therefore, when $c_b \leq c_{b1}^F$, $w_n^{FB^*} \leq w_n^{FN^*}$ and $p_n^{FB^*} \leq p_n^{FN^*}$; otherwise, $w_n^{FB^*} > w_n^{FN^*}$ and $p_n^{FB^*} > p_n^{FN^*}$.

(2) $w_r^{FB^*} - w_r^{FN^*} = \frac{2\delta(\lambda-1)(2(1-\delta) - \delta\lambda(2-\delta)) + \delta(\lambda-1)(2-\delta)(2-\delta\lambda)c_n + 2\delta(\lambda-1)c_r + (2-\delta)(4-\delta\lambda)c_b - (2-\delta)(4-3\delta\lambda)t}{4(2-\delta)(2-\delta\lambda)}$, therefore, when $c_b \leq c_{b2}^F$, $w_r^{FB^*} \leq w_r^{FN^*}$; otherwise, $w_r^{FB^*} > w_r^{FN^*}$.
$p_r^{FB^*} - p_r^{FN^*} = \frac{2\delta(\lambda-1)(2(5-3\delta) - 3\delta\lambda(2-\delta)) + \delta(2-\delta)(\lambda-1)(2-\delta\lambda)c_n + 2\delta(\lambda-1)c_r + (2-\delta)(4-\delta\lambda)c_b - (2-\delta)(12-7\delta\lambda)t}{8(2-\delta)(2-\delta\lambda)}$, therefore, when $c_b \leq c_{b3}^F$, $p_r^{FB^*} \leq p_r^{FN^*}$; otherwise, $p_r^{FB^*} > p_r^{FN^*}$.

(3) $q_n^{FB^*} - q_n^{FN^*} = \frac{(1-\delta)(c_b + t) - \delta(\lambda - 1)(c_n - c_r)}{8(1-\delta)(1-\delta\lambda)}$, therefore, when $c_b \geq c_{b4}^F$, $q_n^{FB^*} \geq q_n^{FN^*}$; otherwise, $q_n^{FB^*} < q_n^{FN^*}$. $q_r^{FB^*} - q_r^{FN^*} = \frac{2\delta^2\lambda(1-\delta)(1-\delta\lambda)(\lambda-1) + \delta^2\lambda(2-\delta)(2-\delta\lambda)(\lambda-1)c_n - (1-\delta)(2-\delta)(4-3\delta\lambda)(t + c_b) + (\lambda-1)(4(1-\delta)(2-\delta) - \delta\lambda(4-3\delta)(3-\delta-\delta\lambda))c_r}{8\delta\lambda(1-\delta)(1-\delta\lambda)(2-\delta)(2-\delta\lambda)}$, therefore, when $c_b \leq c_{b5}^F$, $q_r^{FB^*} \geq q_r^{FN^*}$; otherwise, $q_r^{FB^*} < q_r^{FN^*}$.

□

Proof of Theorem 4. (1) $\pi_M^{FB^*} - \pi_M^{FN^*} = \frac{(1-\delta)(2-\delta)\beta_3^2 - (1-\delta\lambda)(2-\delta\lambda)\beta_1^2}{16(1-\delta)(2-\delta)(1-\delta\lambda)(2-\delta\lambda)}$, therefore, when $c_b \geq c_{b6}^F$, $\pi_M^{FB^*} \geq \pi_M^{FN^*}$; otherwise, $\pi_M^{FB^*} < \pi_M^{FN^*}$.

(2) $\pi_T^{FB^*} - \pi_T^{FN^*} = \frac{(1-\delta)(2-\delta)^2\beta_4^2 - \lambda(1-\delta\lambda)(2-\delta\lambda)^2\beta_2^2}{32\delta\lambda(1-\delta)(2-\delta)^2(1-\delta\lambda)(2-\delta\lambda)^2}$, when $c_b \leq c_{b7}^F$, $\pi_T^{FB^*} \geq \pi_T^{FN^*}$; otherwise, $\pi_T^{FB^*} < \pi_T^{FN^*}$.

(3) $\pi_R^{FB^*} - \pi_R^{FN^*} = \frac{\beta_3^2}{32(1-\delta\lambda)(2-\delta\lambda)} - \frac{\beta_4^2}{64\delta\lambda(1-\delta\lambda)(2-\delta\lambda)^2} + \frac{(\delta\lambda\beta_3 + \beta_4)(\delta\lambda - c_r - c_b - t)}{16\delta\lambda(1-\delta\lambda)(2-\delta\lambda)} - \frac{\beta_1^2}{32(1-\delta)(2-\delta)} + \frac{\beta_2^2}{64\delta(1-\delta)(2-\delta)^2} - \frac{(\delta\beta_1 + \beta_2)(\delta - c_r)}{16\delta(1-\delta)(2-\delta)}$, when $c_b \leq c_{b8}^F$, $\pi_R^{FB^*} \geq \pi_R^{FN^*}$; otherwise, $\pi_R^{FB^*} < \pi_R^{FN^*}$.

□

Proof of Theorem 5. (1) $w_n^{RB^*} - w_n^{RN^*} = 0$, therefore, $w_n^{RB^*} = w_n^{RN^*}$. $p_n^{RB^*} - p_n^{RN^*} = 0$, therefore, $p_n^{RB^*} = p_n^{RN^*}$.

(2) $f^{RB^*} - f^{RN^*} = \frac{1}{2}(\delta(\lambda - 1) - c_b - t)$, therefore, when $c_b \leq c_{b1}^R$, $f^{RB^*} \geq f^{RN^*}$; otherwise, $f^{RB^*} < f^{RN^*}$.

(3) $w_r^{RB^*} - w_r^{RN^*} = \frac{1}{4}(\delta(\lambda-1)(2+c_n)+c_b-3t)$, therefore, when $c_b \le c_{b2}^R$, $w_r^{RB^*} \le w_r^{RN^*}$; otherwise, $w_r^{RB^*} > w_r^{RN^*}$.
$p_r^{RB^*} - p_r^{RN^*} = \frac{1}{8}(\delta(\lambda-1)(6+c_n)+c_b-7t)$, when $c_b \le c_{b3}^R$, $p_r^{RB^*} \le p_r^{RN^*}$; otherwise, $p_r^{RB^*} > p_r^{RN^*}$.

(4) $q_n^{RB^*} - q_n^{RN^*} = \frac{(1-\delta)(c_b+t)-\delta(\lambda-1)(c_n-c_r)}{8(1-\delta)(1-\delta\lambda)}$, when $c_b \le c_{b4}^R$, $q_n^{RB^*} \le q_n^{RN^*}$, otherwise, $q_n^{RB^*} > q_n^{RN^*}$.
$q_r^{RB^*} - q_r^{RN^*} = \frac{\delta\lambda c_n-(t+c_b+c_r)}{8\delta\lambda(1-\delta\lambda)} - \frac{\delta c_n - c_r}{8(1-\delta)\delta}$, when $c_b \le c_{b5}^R$, $q_r^{RB^*} \ge q_r^{RN^*}$; otherwise, $q_r^{RB^*} < q_r^{RN^*}$.

□

Proof of Theorem 6. $\pi_M^{RB^*} - \pi_M^{RN^*} = \frac{(1-\delta)(\delta\lambda c_n-c_r-c_b-t)^2-\lambda(1-\delta\lambda)(\delta c_n-c_r)^2}{16\delta\lambda(1-\delta)(1-\delta\lambda)}$, therefore, when $c_b \le c_{b6}^R$, $\pi_M^{RB^*} \ge \pi_M^{RN^*}$; otherwise, $\pi_M^{RB^*} < \pi_M^{RN^*}$.

$\pi_T^{RB^*} - \pi_T^{RN^*} = \frac{(1-\delta)(\delta\lambda c_n-c_r-c_b-t)^2-\lambda(1-\delta\lambda)(\delta c_n-c_r)^2}{32\delta\lambda(1-\delta)(1-\delta\lambda)}$, therefore, when $c_b \le c_{b6}^R$, $\pi_T^{RB^*} \ge \pi_T^{RN^*}$; otherwise, $\pi_T^{RB^*} < \pi_T^{RN^*}$.

$\pi_R^{RB^*} - \pi_R^{RN^*} = \frac{(1-\delta)(\delta\lambda c_n-c_r-c_b-t)^2-\lambda(1-\delta\lambda)(\delta c_n-c_r)^2}{64\delta\lambda(1-\delta)(1-\delta\lambda)}$, therefore, when $c_b \le c_{b6}^R$, $\pi_R^{RB^*} \ge \pi_R^{RN^*}$; otherwise, $\pi_R^{RB^*} < \pi_R^{RN^*}$. □

Proof of Theorem 7. (1) $\frac{\partial w_n^{FB^*}}{\partial\lambda} = -\frac{\delta(2-c_r-c_b-t)}{2(2-\delta\lambda)^2} < 0$. $\frac{\partial w_r^{FB^*}}{\partial\lambda} = \frac{\delta\left((2-\delta\lambda)^2(2+c_n)-2(2-c_r-c_b-t)\right)}{4(2-\delta\lambda)^2}$, therefore, when $c_b \ge 2-c_r-t-(2-\delta\lambda)^2(1+\frac{c_n}{2}))$, $\frac{\partial w_r^{FB^*}}{\partial\lambda} \ge 0$; otherwise, $\frac{\partial w_r^{FB^*}}{\partial\lambda} < 0$. $\frac{\partial p_n^{FB^*}}{\partial\lambda} = -\frac{\delta(2-c_r-c_b-t)}{4(2-\delta\lambda)^2} < 0$. $\frac{\partial p_r^{FB^*}}{\partial\lambda} = \frac{\delta((2-\delta\lambda)^2c_n+2(c_r+c_b+t)+2((10-3\delta\lambda)(1-\delta\lambda)+\delta\lambda))}{8(2-\delta\lambda)^2} > 0$. $\frac{\partial q_n^{FB^*}}{\partial\lambda} = -\frac{\delta(c_n-c_r-c_b-t)}{8(1-\delta\lambda)^2} < 0$. $\frac{\partial q_r^{FB^*}}{\partial\lambda} = \frac{2\delta^2\lambda^2(1-\delta\lambda)^2+\delta^2\lambda^2(2-\delta\lambda)^2c_n+(8-24\delta\lambda+21\delta^2\lambda^2-6\delta^3\lambda^3)(c_r+c_b+t)}{8\delta\lambda^2(1-\delta\lambda)^2(2-\delta\lambda)^2} > \frac{2\delta^2\lambda^2(1-\delta\lambda)^2+(\delta^2\lambda^2(2-\delta\lambda)^2+(8-24\delta\lambda+21\delta^2\lambda^2-6\delta^3\lambda^3))(c_r+c_b+t)}{8\delta\lambda^2(1-\delta\lambda)^2(2-\delta\lambda)^2} = \frac{2\delta^2\lambda^2(1-\delta\lambda)^2+(\delta^2\lambda^2(2-\delta\lambda)^2+(1-\delta\lambda)^2(8(1-\delta\lambda)+\delta^2\lambda^2)))(c_r+c_b+t)}{8\delta\lambda^2(1-\delta\lambda)^2(2-\delta\lambda)^2} > 0$. $\frac{\partial \pi_M^{FB^*}}{\partial\lambda} = -\frac{\delta\beta_3(2(1-\delta\lambda)+(2-\delta\lambda)c_n-(3-2\delta\lambda)(c_r+c_b+t))}{16(1-\delta\lambda)^2(2-\delta\lambda)^2} < 0$. $\frac{\partial \pi_T^{FB^*}}{\partial\lambda} = \frac{\beta_4(2\delta\lambda(1-\delta\lambda)(2-3\delta\lambda)+\delta\lambda(2-\delta\lambda)^2c_n+(2-3\delta\lambda)(4-5\delta\lambda+2\delta^2\lambda^2)(c_r+c_b+t))}{32\delta\lambda^2(2-\delta\lambda)^3(1-\delta\lambda)^2}$, when $\delta\lambda \le \frac{2}{3}$, or when $\delta\lambda > \frac{2}{3}$ and $c_b < \frac{\delta\lambda(2-\delta\lambda)^2c_n-2\delta\lambda(1-\delta\lambda)(3\delta\lambda-2)}{(3\delta\lambda-2)(4-5\delta\lambda+2\delta^2\lambda^2)} - c_r - t$, $\frac{\partial \pi_T^{FB^*}}{\partial\lambda} > 0$; when $\delta\lambda > \frac{2}{3}$ and $c_b \ge \frac{\delta\lambda(2-\delta\lambda)^2c_n-2\delta\lambda(1-\delta\lambda)(3\delta\lambda-2)}{(3\delta\lambda-2)(4-5\delta\lambda+2\delta^2\lambda^2)} - c_r - t$, $\frac{\partial \pi_T^{FB^*}}{\partial\lambda} \le 0$.

$$\frac{\partial \pi_R^{FB^*}}{\partial\lambda} = \frac{\delta^2\lambda^2(2-\delta\lambda)^3c_n^2 - 8\delta^2\lambda^2(2-\delta\lambda)(1-\delta\lambda)^2c_n - 2\delta^2\lambda^2(2-\delta\lambda)\left(2-\delta^2\lambda^2\right)c_n(c_r+c_b+t) + 4\delta^2\lambda^2(1-\delta\lambda)^2(10-3\delta\lambda)(1-c_r-c_b-t) - \left(32-112\delta\lambda+134\delta^2\lambda^2-65\delta^3\lambda^3+10\delta^4\lambda^4\right)(c_r+c_b+t)^2}{64\delta\lambda^2(2-\delta\lambda)^3(1-\delta\lambda)^2} > 0.$$

(2) $\frac{\partial w_n^{RB^*}}{\partial\lambda} = 0$, $\frac{\partial f^{RB^*}}{\partial\lambda} = \frac{\delta}{2} > 0$, $\frac{\partial w_r^{RB^*}}{\partial\lambda} = \frac{\delta(2+c_n)}{4} > 0$, $\frac{\partial p_n^{RB^*}}{\partial\lambda} = 0$, $\frac{\partial p_r^{RB^*}}{\partial\lambda} = \frac{\delta(6+c_n)}{8} > 0$, $\frac{\partial q_n^{RB^*}}{\partial\lambda} = -\frac{\delta(c_n-c_r-c_b-t)}{8(1-\delta\lambda)^2} < 0$, $\frac{\partial q_r^{RB^*}}{\partial\lambda} = \frac{\delta\lambda(\delta\lambda c_n-c_r-c_b-t)+(1-\delta\lambda)(c_r+c_b+t)}{8\delta\lambda^2(1-\delta\lambda)^2} > 0$, $\frac{\partial \pi_M^{RB^*}}{\partial\lambda} = \frac{(\delta\lambda c_n-c_r-c_b-t)(\delta\lambda(c_n-c_r-c_b-t)+(1-\delta\lambda)(c_r+c_b+t))}{16\delta\lambda^2(1-\delta\lambda)^2} > 0$, $\frac{\partial \pi_T^{RB^*}}{\partial\lambda} = \frac{(\delta\lambda c_n-c_r-c_b-t)(\delta\lambda(c_n-c_r-c_b-t)+(1-\delta\lambda)(c_r+c_b+t))}{32\delta\lambda^2(1-\delta\lambda)^2} > 0$, and $\frac{\partial \pi_R^{RB^*}}{\partial\lambda} = \frac{(\delta\lambda c_n-c_r-c_b-t)(\delta\lambda(c_n-c_r-c_b-t)+(1-\delta\lambda)(c_r+c_b+t))}{64\delta\lambda^2(1-\delta\lambda)^2} > 0$.

□

Proof of Theorem 8. (1) $\frac{\partial w_n^{FB^*}}{\partial t} = \frac{1}{2(2-\delta\lambda)} > 0$, $\frac{\partial w_r^{FB^*}}{\partial t} = -\frac{4-3\delta\lambda}{4(2-\delta\lambda)} < 0$, $\frac{\partial p_n^{FB^*}}{\partial t} = \frac{1}{4(2-\delta\lambda)} > 0$, $\frac{\partial p_r^{FB^*}}{\partial t} = -\frac{12-7\delta\lambda}{8(2-\delta\lambda)} < 0$, $\frac{\partial q_n^{FB^*}}{\partial t} = \frac{1}{8(1-\delta\lambda)} > 0$, $\frac{\partial q_r^{FB^*}}{\partial t} = -\frac{4-3\delta\lambda}{8\delta\lambda(1-\delta\lambda)(2-\delta\lambda)} < 0$, $\frac{\partial \pi_M^{FB^*}}{\partial t} = \frac{\beta_3}{8(1-\delta\lambda)(2-\delta\lambda)} > 0$, $\frac{\partial \pi_T^{FB^*}}{\partial t} = -\frac{(4-3\delta\lambda)\beta_4}{16\delta\lambda(1-\delta\lambda)(2-\delta\lambda)^2} < 0$, and $\frac{\partial \pi_R^{FB^*}}{\partial t} = -\frac{\delta\lambda\beta_4+8(1-\delta\lambda)(2-\delta\lambda)(\delta\lambda-c_r-c_b-t)}{32\delta\lambda(1-\delta\lambda)(2-\delta\lambda)^2} < 0$.

(2) $\frac{\partial w_n^{RB^*}}{\partial t} = 0$, $\frac{\partial f^{RB^*}}{\partial t} = -\frac{1}{2} < 0$, $\frac{\partial w_r^{RB^*}}{\partial t} = -\frac{3}{4} < 0$, $\frac{\partial p_n^{RB^*}}{\partial t} = 0$, $\frac{\partial p_r^{RB^*}}{\partial t} = -\frac{7}{8} < 0$, $\frac{\partial q_n^{RB^*}}{\partial t} = \frac{1}{8(1-\delta\lambda)} > 0$, $\frac{\partial q_r^{RB^*}}{\partial t} = -\frac{1}{8\delta\lambda(1-\delta\lambda)} < 0$, $\frac{\partial \pi_M^{RB^*}}{\partial t} = -\frac{\delta\lambda c_n-c_r-c_b-t}{8\delta\lambda(1-\delta\lambda)} < 0$, $\frac{\partial \pi_T^{RB^*}}{\partial t} = -\frac{\delta\lambda c_n-c_r-c_b-t}{16\delta\lambda(1-\delta\lambda)} < 0$, and $\frac{\partial \pi_R^{RB^*}}{\partial t} = -\frac{\delta\lambda c_n-c_r-c_b-t}{32\delta\lambda(1-\delta\lambda)} < 0$.

□

Proof of Theorem 9. (1) $\frac{\partial w_n^{FB^*}}{\partial c_b} = \frac{1}{2(2-\delta\lambda)} > 0$, $\frac{\partial w_r^{FB^*}}{\partial c_b} = \frac{4-\delta\lambda}{4(2-\delta\lambda)} > 0$, $\frac{\partial p_n^{FB^*}}{\partial c_b} = \frac{1}{4(2-\delta\lambda)} > 0$, $\frac{\partial p_r^{FB^*}}{\partial c_b} = \frac{4-\delta\lambda}{8(2-\delta\lambda)} > 0$, $\frac{\partial q_n^{FB^*}}{\partial c_b} = \frac{1}{8(1-\delta\lambda)} > 0$, $\frac{\partial q_r^{FB^*}}{\partial c_b} = -\frac{4-3\delta\lambda}{8\delta\lambda(1-\delta\lambda)(2-\delta\lambda)} < 0$, $\frac{\partial \pi_M^{FB^*}}{\partial c_b} = \frac{\beta_3}{8(1-\delta\lambda)(2-\delta\lambda)} > 0$, $\frac{\partial \pi_T^{FB^*}}{\partial c_b} = -\frac{(4-3\delta\lambda)\beta_4}{16\delta\lambda(1-\delta\lambda)(2-\delta\lambda)^2} < 0$, and $\frac{\partial \pi_R^{FB^*}}{\partial c_b} = -\frac{\delta\lambda\beta_4+8(1-\delta\lambda)(2-\delta\lambda)(\delta\lambda-c_r-c_b-t)}{32\delta\lambda(1-\delta\lambda)(2-\delta\lambda)^2} < 0$.

(2) $\frac{\partial w_n^{RB^*}}{\partial c_b} = 0$, $\frac{\partial f^{RB^*}}{\partial c_b} = -\frac{1}{2} < 0$, $\frac{\partial w_r^{RB^*}}{\partial c_b} = \frac{1}{4} > 0$, $\frac{\partial p_n^{RB^*}}{\partial c_b} = 0$, $\frac{\partial p_r^{RB^*}}{\partial c_b} = \frac{1}{8} > 0$, $\frac{\partial q_n^{RB^*}}{\partial c_b} = \frac{1}{8(1-\delta\lambda)} > 0$, $\frac{\partial q_r^{RB^*}}{\partial c_b} = -\frac{1}{8\delta\lambda(1-\delta\lambda)} < 0$, $\frac{\partial \pi_M^{RB^*}}{\partial c_b} = -\frac{\delta\lambda c_n-c_r-c_b-t}{8\delta\lambda(1-\delta\lambda)} < 0$, $\frac{\partial \pi_T^{RB^*}}{\partial c_b} = -\frac{\delta\lambda c_n-c_r-c_b-t}{16\delta\lambda(1-\delta\lambda)} < 0$, and $\frac{\partial \pi_R^{RB^*}}{\partial c_b} = -\frac{\delta\lambda c_n-c_r-c_b-t}{32\delta\lambda(1-\delta\lambda)} < 0$.

□

Appendix B. Threshold values

The notations used in this paper are as follows:

$\beta_1 = 2(1-\delta)-(2-\delta)c_n+c_r$, $\beta_2 = 2\delta(1-\delta)+\delta(2-\delta)c_n-(4-3\delta)c_r$, $\beta_3 = 2(1-\delta\lambda)-(2-\delta\lambda)c_n+c_r+c_b+t$, and $\beta_4 = 2\delta\lambda(1-\delta\lambda)+\delta\lambda(2-\delta\lambda)c_n-(4-3\delta\lambda)(c_r+c_b+t)$.

$F_1 = \frac{(\delta-c_r)^2}{8\delta(2-\delta)}$, $F_2 = \frac{(\delta-c_r)(\delta(1-\delta)+\delta(2-\delta)c_n-(3-2\delta)c_r)}{8\delta(2-\delta)^2}$, $F_3 = \frac{(\delta\lambda-c_r-c_b-t)^2}{8\delta\lambda(2-\delta\lambda)}$, $F_4 = \frac{(\delta\lambda-c_r-c_b-t)\left(\delta\lambda(1-\delta\lambda)+\delta\lambda(2-\delta\lambda)c_n-(3-2\delta\lambda)(c_r+c_b+t)\right)}{8\delta\lambda(2-\delta\lambda)^2}$.

$c_{b1}^F = \frac{\delta(\lambda-1)(2-c_r)}{2-\delta} - t$, $c_{b2}^F = \frac{(2-\delta)(4-3\delta\lambda)t-2\delta(\lambda-1)(2(1-\delta)-\delta\lambda(2-\delta))-\delta(2-\delta)(\lambda-1)(2-\delta\lambda)c_n-2\delta(\lambda-1)c_r}{(2-\delta)(4-\delta\lambda)}$, $c_{b3}^F = \frac{(2-\delta)(12-7\delta\lambda)t-2\delta(\lambda-1)(2(5-3\delta)-3\delta\lambda(2-\delta))-\delta(2-\delta)(\lambda-1)(2-\delta\lambda)c_n-2\delta(\lambda-1)c_r}{(2-\delta)(4-\delta\lambda)}$, $c_{b4}^F = \frac{\delta(\lambda-1)(c_n-c_r)}{1-\delta} - t$, $c_{b5}^F = \frac{2\delta^2\lambda(1-\delta)(1-\delta\lambda)(\lambda-1)+\delta^2\lambda(2-\delta)(2-\delta\lambda)(\lambda-1)c_n+(\lambda-1)(4(1-\delta)(2-\delta)-\delta\lambda(4-3\delta)(3-\delta-\delta\lambda))c_r}{(1-\delta)(2-\delta)(4-3\delta\lambda)} - t$, $c_{b6}^F = \sqrt{\frac{(1-\delta\lambda)(2-\delta\lambda)}{(1-\delta)(2-\delta)}}\beta_1 + (2-\delta\lambda)c_n - 2(1-\delta\lambda) - c_r - t$, $c_{b7}^F = \frac{2\delta\lambda(1-\delta\lambda)+\delta\lambda(2-\delta\lambda)c_n}{4-3\delta\lambda} - \sqrt{\frac{\lambda(1-\delta\lambda)}{(1-\delta)}\frac{(2-\delta\lambda)}{(2-\delta)(4-3\delta\lambda)}}\beta_2 - c_r - t$, c_{b8}^F is the root of $\pi_R^{FB^*} = \pi_R^{FN^*}$.

$c_{b1}^R = \delta(\lambda-1)-t$, $c_{b2}^R = 3t-\delta(\lambda-1)(2+c_n)$, $c_{b3}^R = 7t-\delta(\lambda-1)(6+c_n)$, $c_{b4}^R = c_{b4}^F = \frac{\delta(\lambda-1)(c_n-c_r)}{1-\delta} - t$, $c_{b5}^R = \frac{\delta^2\lambda(\lambda-1)c_n-(1-\delta-\lambda(1-\delta\lambda))c_r}{1-\delta} - t$, $c_{b6}^R = \delta\lambda c_n - c_r - \sqrt{\frac{\lambda(1-\delta\lambda)}{1-\delta}}(\delta c_n - c_r) - t$.