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ANALYSING THE IMPACT OF CARBON REGULATION POLICIES ON THE CLEAN ENERGY ADVERTISEMENT INPUT DECISION

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Abstract. This study introduces a novel ceiling constraint on clean energy advertisement input and examines its interaction with two carbon regulation policies – carbon tax (CT) and carbon allowance mechanism (CAM) – in shaping operational decisions. The key findings are as follows: (1) Under a low ceiling, both policies lead to identical advertisement input, yet CT results in lower profits for chain members. Under a moderate or high ceiling, CAM induces higher advertisement input than CT. (2) Regardless of the ceiling level, CAM consistently leads to higher power demand, higher total emissions, and lower conventional power prices compared to CT. (3) When the ceiling constraint is binding, raising it reduces conventional power prices, power demand, and emissions under both policies, while increasing the equilibrium values for other decision variables. Theoretical and managerial contributions: Theoretically, this study advances low-carbon operations modeling by incorporating a regulatory ceiling on advertisement input, offering a refined framework for evaluating carbon policies. It further identifies critical threshold levels (*e.g.*, of the ceiling) that dictate the relative efficacy of CT *versus* CAM in promoting green input and shaping profitability. From a managerial perspective, the findings offer clear guidance; policymakers can use them to design balanced regulations, while power generators can better select operational strategies under different policy regimes.

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1. Introduction

In recent years, global carbon emissions have reached 36.8 billion tons, with the power industry alone accounting for as much as 14.7 billion tons – far surpassing the industrial and transportation sectors – thus constituting the largest source of global carbon emissions. Statistics show that over the past three decades, emissions from the power sector have represented about 40% of the global total[1,2]. The International Energy Agency's Net Zero Emissions Program further underscores the urgency, stating that the power sector must achieve net-zero

Keywords. Carbon tax, carbon allowance mechanism, clean energy advertisement input, ceiling regulation.

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[1] https://www.statista.com/statistics/276629/global-co2-emissions.
[2] https://www.statista.com/statistics/1423179/global-ghg-emissions-by-sector-annual/.

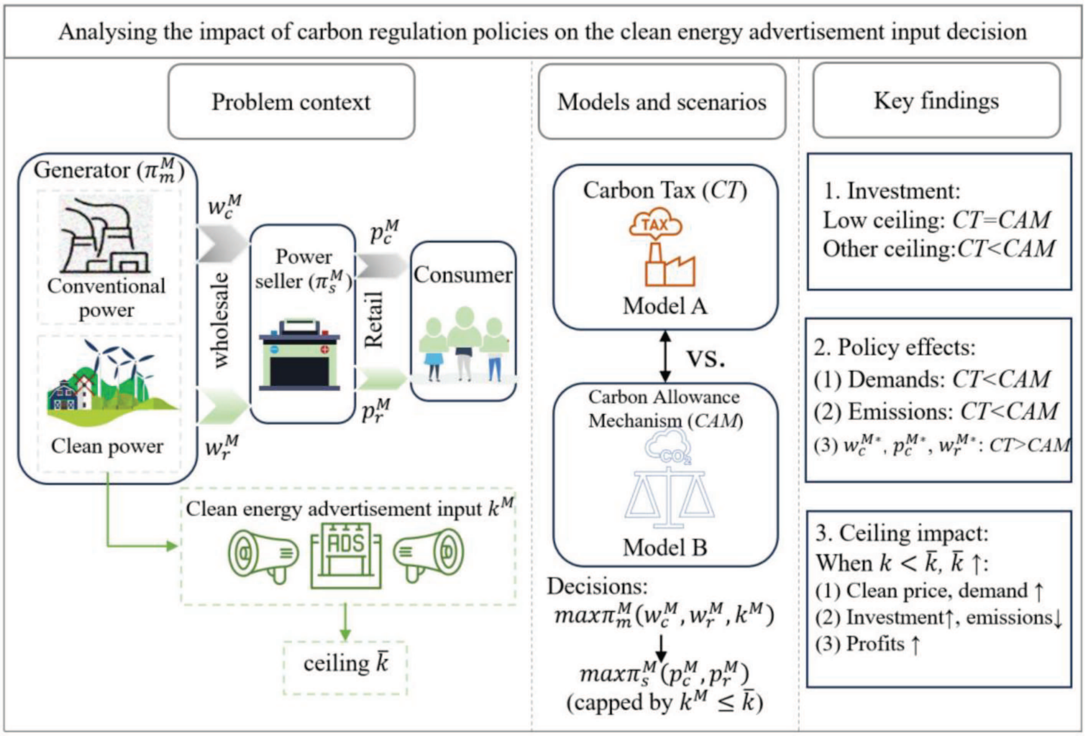


Figure 1. Research framework.

emissions by 2040[3]. Hence, increasing the share of clean power in power generation is particularly crucial and urgent for reducing carbon emissions.

Economies have sought to incentivize clean energy inputs through targeted carbon regulation policies, enabling power enterprises to reduce emissions and meet their decarbonization targets. According to the World Bank's Carbon Pricing Status and Trends 2022 report, among the 68 carbon pricing instruments currently in operation, 36 are carbon taxes, and 32 are carbon allowance mechanisms[4]. Countries such as Finland and Denmark introduced carbon taxes as early as 1992, while Switzerland and South Korea launched their carbon allowance mechanisms in 2008 and 2015, respectively [1]. A Carbon Tax (CT) is a policy that imposes a tax on enterprises based on the amount of carbon they emit, aiming to achieve emission reductions by internalizing the cost of pollution – effectively enforcing the polluter pays principle through price regulation [2]. In contrast, the Carbon Allowance Mechanism (CAM) operates by setting a unit carbon allowance for each unit of conventional power generated, based on industry benchmarks. Enterprises that keep their carbon emissions below this allowance can benefit by selling surplus carbon allowances in the carbon market; conversely, those exceeding the allowance must purchase additional allowances from the market [3].

Under both policies, penalizing or incentivizing power enterprises helps reduce carbon emissions by reducing conventional power production and increasing the share of clean power. The level of clean energy input in the global power sector has risen substantially. For instance, solar power generation exceeded 1000 TWh for the first time in 2021, and wind power approached 2000 TWh; together, zero-carbon clean energy sources met nearly 40% of global power demand[5]. However, current clean power generation remains insufficient to fulfil the ambitious

[3] https://www.in-en.com/article/html/energy-2327814.shtml.
[4] http://www.tanjiaoyi.com/article-47034-1.html.
[5] http://www.doc88.com/p-87087511804687.html.

commitments made by various economies. There is still an urgent need to further increase the share of clean power in the electricity mix (Fig. 1).

By actively promoting the low-carbon performance of clean power to consumers and enhancing their advertising level on clean energy, power enterprises can play a positive role in promoting the zero-carbon transition of the power sector. For example, China holds an Energy Conservation Awareness Week and a National Low-Carbon Day every year, through which the public and organizations are widely educated on energy saving and carbon reduction, thereby encouraging green, low-carbon development and climate action[6]. While enterprises promote the low-carbon and environmentally-friendly attributes of clean energy through promotional activities, the associated publicity costs (referred to in this study as advertisement input) inevitably have an impact. Excessive advertisement input can reduce enterprise profits and lead to inefficient use of resources and capital. Therefore, this study considers a ceiling constraint on advertisement input to balance economic and environmental objectives while better satisfying the power demand of consumers.

The implementation of carbon regulation policies and the ceiling constraint on clean energy advertisement input carry significant implications for power enterprises. In particular, it remains unclear whether the power sector should choose the CAM or the CT, nor is it evident what level of clean energy advertisement input would be most beneficial. Therefore, this study addresses the following three research questions:

(1) Which policy – CT or CAM – is more conducive to increasing clean energy advertisement input and reducing carbon emissions?
(2) How does the ceiling constraint affect the generator's pricing, profit, and input decisions?
(3) How do the cost of clean energy advertisement input, unit carbon allowance, and unit carbon price influence the enterprise's decision-making?

Accordingly, this research constructs a power supply chain in which the generator acts as the dominant player and the power seller serves as the subordinate player. Two models are established based on the two carbon regulatory policies, namely CT and CAM. In both models, the generator is required by carbon regulation policies to invest in clean energy advertisement for clean power production, subject to the ceiling constraint. The generator sells clean power and conventional power to the seller at different wholesale prices, and the seller in turn sells clean power and conventional power to the seller at different wholesale prices, and the seller then sells them to consumers at different retail prices.

To address these questions, we develop game-theoretic models incorporating a novel regulatory ceiling on clean energy advertisement input. Our analysis yields several critical and actionable insights: First, we identify a threshold effect of the ceiling – only under a stringent ceiling do CT and CAM result in identical advertisement input; otherwise, CAM stimulates significantly higher clean energy advertisement input. Second, we reveal a fundamental policy trade-off: while CAM boosts clean power demand and corporate profits (particularly under a low ceiling), it also leads to higher total emissions compared to CT, which excels in emission reduction but tends to suppress profitability in most cases. Third, we demonstrate that adjusting the ceiling can serve as a powerful complementary tool to carbon policies, enabling policymakers to fine-tune outcomes toward specific economic or environmental objectives. These findings not only clarify the comparative efficacy of CT *versus* CAM under real-world constraints but also provide a refined framework for simultaneously managing economic incentives and environmental targets in the power sector.

Building on these insights, this paper makes two key contributions to the literature on low-carbon operations and policy design. First, it advances modelling frameworks by incorporating a regulatory ceiling on advertisement input – a realistic yet underexplored constraint – offering a more nuanced tool to evaluate carbon policies. Secondly, it provides clear, evidence-based guidance for policymakers on selecting and calibrating policies (CT/CAM) and the ceiling based on prioritized objectives (*e.g.*, emission reduction *vs.* green input); and for power enterprises on optimizing pricing and input strategies under different regulatory scenarios.

The remainder of this paper is organized as follows. Section 2 conducts a literature review, divided into two main streams: carbon regulation policies and low-carbon input behaviors, to position our research within the

[6] https://www.gov.cn/zhengce/zhengceku/202306/content_6888459.htm.

existing academic discourse. Section 3 details the model formulation and solution. It begins with the problem description and framework, followed by the formal specification of demand, cost, emission, and profit functions under two carbon policies, and then derives the equilibrium solutions, separately for the Carbon Tax Model (Model A) and the Carbon Allowance Mechanism Model (Model B). Section 4 presents a comparative analysis of the equilibrium outcomes from the two models, addressing our core research questions. Section 5 performs a sensitivity analysis on key parameters to examine their influence on the system. Section 6 provides numerical analyses to illustrate the theoretical results, exploring the impact of the advertisement input ceiling and the unit carbon price, and culminates in a discussion of policy implications. Finally, Section 7 concludes the paper by summarizing the key findings, presenting actionable proposals, and suggesting avenues for future research.

2. Literature review

This study is primarily related to two streams of research: carbon regulation policies and low carbon input behaviour. A detailed review of the relevant literature is provided below.

2.1. Carbon regulation policies

Carbon regulation policies refer to various measures implemented by governments to incentivize enterprises, industries, or countries to reduce carbon emissions. Among these, CT and the CAM are two of the most typical policy instruments. This section, therefore, reviews the existing research on CT and CAM.

First, the CAM has been extensively studied, with particular attention to its impacts on corporate carbon emissions, economic profitability, and environmental sustainability. In a pivotal study, Tong *et al.* [4] examined the joint effects of the CAM and consumers' low-carbon preferences on the manufacturing supply chain. They found that carbon allowances, carbon prices, and consumer preferences for low-carbon products are key determinants of manufacturers and retailers' strategic behaviour. Regarding clean technology input under the CAM, Dong and Wu [5] examined how enterprises respond to a stringent CAM and improvements in clean technology efficiency. They found that regardless of how low the carbon emission ceiling is set, enterprises that already possess clean technology – or those using conventional technology with high emission intensity – are unlikely to adopt new clean technology. Lin *et al.* [6] proposed an integrated energy management and dispatch system combining carbon capture technology, CAM, and energy storage, demonstrating that this approach achieves high operational stability and significantly reduces total costs. In another significant contribution, Stuhlmacher *et al.* [7] undertook a comprehensive spatial economic assessment of the European Union's CAM, projecting that EU carbon emissions would peak at the beginning of the mechanism's second phase. Additionally, Chen *et al.* [8] conducted an in-depth analysis of the CAM's influence on the deployment of low-carbon technologies by power generators. Their research indicated that, in terms of low-carbon technology input, a regulation based on unit carbon allowances consistently outperforms one based on total carbon allowances.

Second, numerous scholars have also examined the impact of CT. Chen *et al.* [9] investigated the impact of CT on low-carbon technology input by power enterprises, finding that CT does not always promote such input – especially when the unit carbon emissions from conventional energy production are relatively high. Xia *et al.* [10] studied how a progressive CT incentivizes manufacturers to invest in carbon reduction technology, arguing that not only can a progressive CT encourage such input, but a higher CT rate exerts a stronger incentive effect. Much of the literature on CT also considers carbon subsidy policies. Yu [11] analyzes the choice between CT and carbon subsidy policy among domestic and international interest groups such as industry associations, public finance groups, and environmental organizations. Kök *et al.* [12] comprehensively assessed the effectiveness of direct subsidies (*e.g.*, tax credits) and indirect subsidies (*e.g.*, CT) in enhancing clean energy input and reducing carbon emissions in power enterprises, and the research results indicated that indirect subsidies, such as CT, can lead to both lower carbon emissions and higher clean energy input. Jin *et al.* [13] empirically conducted the optimal CT rate for achieving government objectives, finding that when two competing enterprises produce conventional products and green products respectively, the government can increase revenue, reduce expenditures, and lower carbon emissions by setting a higher CT rate.

Furthermore, similar to this study, several scholars have compared the impact of CT and CAM. Hu *et al.* [1] compared the advantages and disadvantages of CT and CAM for the manufacturing industry in terms of manufacturer profits, social welfare, and consumer surplus. The results indicate that CAM outperforms CT in most scenarios, and that CAM only becomes ineffective when the carbon allowance is exceedingly high. Pun and Ghamat [14] further found that for industries with large market scales, CAM is more beneficial than CT for manufacturers, consumers, and social welfare. Tan *et al.* [15] discovered that in advancing the diffusion of green technology, CAM is more efficient than CT and helps reduce emission reduction costs. Hussain and Lee [16] developed a duopoly game model to compare the strengths and weaknesses of carbon-neutral tools under different scenarios. Their results show that CT and CAM play a crucial role in achieving carbon neutrality, and that different combinations of these tools reduce carbon emissions to varying degrees. In addition, Sun and Yang [17] compared the emission reduction performance of two competing manufacturers under CT and CAM. They concluded that CAM leads to greater emission reduction than CT when manufacturers focus on improving the efficiency of their carbon reduction measures. Chen *et al.* [18] compared the clean innovation effect of CT and CAM and its impact on corporate profits. Their results indicate that CT tends to reduce corporate profits, whereas CAM is more effective in reducing emissions and stimulating clean innovation, though its impact on profits remains uncertain. Akulke and Aydin [19] compared equipment selection and scheduling problems in multi-energy microgrids under CT and CAM, indicating that for both policies, incorporating seasonal data and the annual average carbon price in an integrated model leads to more efficient equipment selection.

In summary, while extensive research has been conducted on carbon regulation policies, few studies have considered clean energy input within the power industry. Moreover, the relevant literature does not consider the role of a ceiling constraint on clean energy advertisement input. Without such a ceiling, the profit and capital distribution of power enterprises could be adversely affected. Therefore, although investigating clean energy advertisement input is crucial, none of the reviewed studies address this specific topic. Accordingly, this study reviews the literature on low-carbon input – including low-carbon products, low-carbon technologies, and clean energy – to bridge this gap.

2.2. Low-carbon input behaviour

Low-carbon input refers to a series of inputs made by enterprises to reduce carbon emissions, such as low-carbon products, low-carbon technologies, and clean energy. Accordingly, this section reviews the literature along these three dimensions.

First, regarding research on low-carbon products, Shuai *et al.* [20] used a logistic regression model to analyse consumers' willingness to pay for such products, revealing significant differences among different consumer groups. Furthermore, several scholars have investigated the low-carbon product issues under carbon regulation policies. Meng *et al.* [21] explored the production choices of two competing enterprises under CT, and found that in a Nash game, both enterprises would opt for either a low-carbon product or an ordinary product, depending stochastically on an intermediate carbon tax rate. Duan *et al.* [22] developed a comprehensive supply chain model encompassing manufacturers, retailers, and demand markets. They assessed the impact of CAM, CT, and a subsidy policy for low-carbon products on the supply chain equilibrium. The findings revealed that an escalation in the carbon price could significantly enhance the profitability of the entire supply chain system – but only when coupled with a subsidy policy for low-carbon products.

Second, in the research on low-carbon technologies, Zhang *et al.* [23] examined the influence of consumer perceptions on enterprises' adoption of low-carbon technology; their findings suggest a nuanced relationship: enterprises are less likely to adopt low-carbon technology only when consumer perceptions vary moderately. In contrast, both corporate profits and social welfare could be maximized when the difference in consumer perceptions is either large or small. Chen *et al.* [24] investigated low-carbon technology input strategies of power enterprises under different power structures, finding that when the generator and the seller are relatively balanced in power, the generator is more willing to invest in low-carbon technology. Furthermore, several scholars have analysed low-carbon technology input under carbon regulation policies. Wang *et al.* [25] explored the influence of CT on the diffusion of low-carbon technologies among enterprises, showing that CT can moderately enhance both innovation and sharing of low-carbon technologies. Building on this, Ding and Hu [26] delved into the effects of green technology innovation on the environmental efficiency of Chinese enterprises under a dual policy framework combining CT and a carbon subsidy. Their research indicated that when low-carbon regulation enhances enterprises' proclivity for green technology innovation, the combined impact of CT and subsidy policies on green efficiency becomes more pronounced. Additionally, Vandana and Cerchione [27] evaluated the impact of CAM and CT on the profitability, emission reduction, and green technology input of the supply chain. They found that under CAM, green technology input is lower under a specific condition, whereas under CT, greater green technology input occurs under another specific condition.

Finally, regarding research on clean energy inputs, many scholars have analysed such inputs under CAM. Amiri-Pebdani *et al.* [28] analyzed clean energy input and energy pricing under CAM, finding that different scenarios affect how CAM influences clean energy output and power generation differently. Overall, however, CAM can increase the share of clean energy generation and reduce energy prices. Chen *et al.* [29] further investigated the effects of different market types on clean energy inputs under CAM, indicating that the benchmarking method consistently yields better outcomes than the grandfathering method in promoting clean energy inputs, both in competitive and cooperative markets. Several scholars also analysed clean energy inputs under CT. Chien *et al.* [30] analysed the impact of CT on low-carbon energy transition, concluding that higher carbon taxes favor such transition in ASEAN countries. Dogan *et al.* [31] empirically assessed the influence of energy and environmental taxes on clean energy advancement, revealing that such taxes tend to hinder clean energy deployment in EU nations. In addition, some research compares input levels under different carbon regulation policies. Zhu *et al.* [32] studied the impact of clean power standards and clean energy certificate policies on enterprise input, showing that under the clean energy certificate policy, quotas play an important role in driving clean energy input. Yan *et al.* [33] analysed the impact of CAM and clean energy portfolio standards on clean energy input in power enterprises, finding that a mix of the two mechanisms most effectively promotes such inputs. Meng and Yu [34] explored the impacts of clean energy portfolio standards and CT on power enterprises, noting that higher allowance ratios and carbon tax prices discourage power generation by thermal enterprises. Furthermore, He *et al.* [35] compared the effectiveness and efficiency of CAM and CT in power sector generation expansion planning, concluding that including both policies in planning models helps better assess their effectiveness regarding clean energy input. Xiao *et al.* [36] evaluated the impacts of CT, CAM, and clean energy portfolio standards on multiple energy, environmental, and climate goals, finding that these policies all possess self-stabilizing functions, though policy failure can weaken this stabilizing effect and reduce welfare.

In summary, while many scholars have analysed the low-carbon input behaviour of power enterprises under the carbon regulation policies, comparative studies on the effects of CT *versus* CAM remain limited. To date, only a few works – such as Vandana and Cerchione [27], which contrasted green technology input under the two mechanisms, and He *et al.* [35] and Xiao *et al.* [36], which compared the effectiveness of CT and CAM on clean energy inputs – have undertaken such comparative analyses. Notably, none of these studies incorporates the role of clean energy advertisement input into their frameworks, nor do they consider that such input may be subject to a ceiling constraint. Therefore, this study not only compares the impacts of CT and CAM on input, pricing, emission reductions, and the profitability of the generator and seller, but also further explores how a ceiling on clean energy advertisement input influences these outcomes. Moreover, it aims to identify optimal strategy choices that can simultaneously enhance the profit of power enterprises, environmental benefits, and consumer rights.

Table 1 summarizes and illustrates the major similarities and differences between the reviewed studies and the current study.

TABLE 1. Synthesis and comparison of the existing literature.

Literature	Clean energy input	CT	CAM	Clean energy advertisement input ceiling	Power industry
Amiri-Pebdani *et al.* [28]	✓		✓		✓
Vandana and Cerchione [27]		✓	✓		
Yan *et al.* [33]	✓		✓		✓
Chien *et al.* [30]	✓	✓			
Pun and Ghamat [14]		✓	✓		✓
Xiao *et al.* [36]	✓	✓	✓		✓
Dogan *et al.* [31]	✓	✓			
Meng and Yu [34]	✓	✓			✓
He *et al.* [35]	✓	✓	✓		✓
Akulke and Aydin [19]	✓	✓	✓		✓
This study	✓	✓	✓	✓	✓

3. Model formulation and solution

3.1. Problem description and notations

This study examines a simplified power supply chain consisting of a generator and a single seller. The generator engages in clean energy advertisement activities subject to a ceiling constraint under varying carbon regulation policies, and is therefore bound by both carbon policy requirements and the ceiling regulation.

Accordingly, the generator sells conventional and clean power to the seller at wholesale prices, and the seller then sells both types of power to consumers at retail prices to satisfy the power demand.

3.2. Model formulation

3.2.1. Demand functions

Following the power demand function specified by Chen *et al.* [29], we posit that power demand is influenced by both the power price and the clean energy advertisement input. The demand for clean power and conventional power is given respectively by:

$$q_r^M = a - p_r^M + \theta p_c^M + bk^M, \tag{1}$$

$$q_c^M = a - p_c^M + \theta p_r^M - bk^M. \tag{2}$$

Here, a denotes the potential market demand, p_c^M is the conventional power retail price, and p_r^M is the clean power retail price. The parameter θ $(0 < \theta < 1)$ denotes the cross-price competition factor between conventional and clean power; a larger θ indicates stronger competition between the two types of power. The variable k^M denotes the clean energy advertisement input, and b $(0 < b < 1)$ denotes the preference factor for clean energy advertisement. A larger b implies a greater influence of clean energy advertisement input, reflecting that when consumers have a higher preference for clean power, their willingness to purchase it increases.

3.2.2. Cost functions

Since the generator produces both conventional and clean power, its cost structure reflects different energy inputs. Following the input cost function set by Menanteau *et al.* [37], we define the total conventional power production cost and total clean energy advertisement input cost function as, respectively:

$$G(q_c) = \frac{1}{2}c\left(q_c^M\right)^2, \tag{3}$$

$$G(k) = \frac{1}{2} d\left(k^M\right)^2. \tag{4}$$

Here, c denotes the conventional power unit production cost; a larger c indicates a higher conventional power production cost, which reduces the generator's incentive to invest in conventional energy. The parameter d denotes the clean energy advertisement input cost factor; a larger d indicates a higher clean power promotion cost, thereby lowering the generator's willingness to undertake such advertisement activities.

3.2.3. Carbon emission function

Since clean power generation yields little to no carbon emissions, we assume its per unit emission is zero, while conventional power generation emits e units per unit of output. Therefore, following Chen *et al.* [8], the total carbon emissions function is given by:

$$T = eq_c^M. \tag{5}$$

3.2.4. Carbon regulation policies

The government implements carbon regulation policies to incentivize generators to increase clean power production and reduce carbon emissions. These policies can be divided into CT and CAM. This study utilizes two corresponding models, denoted by superscripts A and B, respectively, with the set of models represented as $M = \{A, B\}$. Under CT, the government collects a unit carbon tax t_1 per unit of carbon emissions from the generator. Under CAM, the generator can trade unit carbon allowances at a unit price t_2, with a benchmark allowance of e_0 per unit of conventional power output, thereby fulfilling the emission reduction requirements proposed by the government. To facilitate clearer analysis and comparison, we assume that the unit carbon tax price t_1 and the unit carbon trading price t_2 are equal, and refer to both as the unit carbon price t.

3.2.5. Enterprises' profit policies

As the producer of power, the generator bears the cost associated with carbon emissions, which can be mitigated by increasing clean power production. However, given the intermittent nature of clean energy, excessive clean energy advertisement input may induce a large amount of clean power, which is unfavourable to power supply and security, thereby damaging the generator's interest and occasioning a waste of funds. To address this, we assume that the government stipulates a ceiling of $\bar{k}$on clean energy advertisement input exhibits, under which the generator must operate.

When the government imposes CT on the generator, the generator's profit function is

$$\left.\begin{array}{l} \pi_m^A = w_r^A q_c^A + w_c^A q_c^A - \frac{1}{2} d\left(k^A\right)^2 - \frac{1}{2} c\left(q_c^A\right)^2 - eq_c^A t \\ \text{s.t. } k^A \leq \bar{k} \end{array}\right\}. \tag{6}$$

Here, $w_r^A\ q_r^A$ denotes the gain from selling clean power, $w_c^A\ q_c^A$ the gain from selling conventional power, $d(k^A)^2/2$ the total cost of clean energy advertisement input, $c(q_c^A)^2/2$ the total production cost of conventional power, and $eq_c^A\ t$ the cost of carbon emissions under CT.

When the government imposes the CAM on the generator, the generator's profit function is

$$\left.\begin{array}{l} \pi_m^B = w_r^B q_r^B + w_c^B q_c^B - \frac{1}{2} d\left(k^B\right)^2 - \frac{1}{2} c\left(q_c^B\right)^2 + e_0 q_r^B t - (e - e_0) q_c^B t \\ \text{s.t. } k^B \leq \bar{k} \end{array}\right\}. \tag{7}$$

In equation (7), the first four items have the same meaning as in Model A. The term $e_0 q_r^B t$ indicates the carbon trading income generated from clean power, while $(e - e_0) q_c^B t$ denotes the carbon cost or income associated with conventional power. Specifically, if $e > e_0$, this item represents the cost of carbon emission; if $e < e_0$, it represents the income of carbon reduction.

The seller's profit function is identical under both CT and CAM:

$$\pi_s^M = \left(p_r^M - w_r^M\right)q_r^M + \left(p_c^M - w_c^M\right)q_c^M. \tag{8}$$

In equation (8), $(p_r^M - w_r^M)q_r^M$ denotes the gain from selling clean power, and $(p_c^M - w_c^M)q_c^M$ denotes the gain from selling conventional power.

3.3. Model solution

In this section, we analyze two models of clean energy advertisement input by the generator under the ceiling constraint, employing CT and CAM policies, respectively. First, we examine Model A, where the generator invests in clean energy advertisement under CT and the ceiling constraint; the carbon tax cost incurred is eq_c^At. Second, we examine Model B, where the generator invests under CAM and the ceiling constraint. In this case, the generator earns carbon trading revenue $e_0q_r^Bt$ from selling clean power and incurs a cost (or receives a benefit) from conventional power emissions, expressed as $(e-e_0)q_c^Bt$. Since both models are subject to a ceiling constraint, we introduce the superscript j to denote the constraint scenario: $j=1$ corresponds to the case where the generator is subject to the ceiling constraint, and $j=2$ corresponds to the case where it is not.

The decision sequence for both models is expressed as:

$$\max\ \pi_m^M\left(w_c^M, w_r^M, k^M\right) \longrightarrow \max\ \pi_s^M\left(p_c^M, p_r^M\right),$$

where $M \in \{A, B\}$ denotes the model type. All equilibrium solutions are marked with an asterisk (*). As each model yields two distinct scenarios based on the ceiling constraint, the optimal solutions are denoted accordingly with superscripts. For example, the equilibrium wholesale price for conventional power in Model A is denoted as $w_c^{A1^*}$ under the constrained scenario and $w_c^{A2^*}$ under the unconstrained scenario. A similar notation applies to Model B. The detailed solution process is as follows.

3.3.1. Carbon tax model (Model A)

For Model A, we employ the backward induction method. First, we derive the first-order conditions of the power seller's profit function in equation (8) with respect to the retail prices of conventional and clean power, obtaining the reaction functions:

$$p_r^A\left(w_r^A, k^A\right) = \frac{a + bk^A + a\theta - bk\theta + w_r^A - \theta^2 w_r^A}{2(1-\theta^2)}, \tag{9}$$

$$p_c^A\left(w_c^A, k^A\right) = \frac{a(1+\theta) - bk^A(1-\theta) + (1-\theta^2)w_c^A}{2(1-\theta^2)}. \tag{10}$$

Substituting these into the generator's profit function in equation (6) and introducing the Lagrange multiplier λ, we construct the Lagrangian:

$$L\left(w_c^A, w_r^A, k^A, \lambda\right) = \left\{\left\{\begin{array}{c} 2abck^A - a^2c - b^2c\left(k^A\right)^2 - 4aet + 4bek^At \\ +\left(a(4-2c\theta) - 4et\theta + 2bk^A(2+c\theta)\right)w_r^A \\ -\left(4+c\theta^2\right)\left(w_r^A\right)^2 - (4+c)\left(w_c^A\right)^2 - 4d\left(k^A\right)^2 \\ +2w_c^A\left(a(2+c) - (2+c)bk^A + 2et + (4+c)\theta w_r^A\right) \end{array}\right\}/8 + \lambda\left(\bar{k} - k^A\right)\right\}. \tag{11}$$

The KKT conditions are derived from the first-order derivatives:

$$\begin{cases} c2a + ac - (2+c)bk^A + 2et - (4+c)w_c^A + (4+c)\theta w_r^A = 0 \\ 2a + 2bk^A - ac\theta + bck^A\theta - 2et\lambda + (4+c)\theta w_c^A - \left(4+c\theta^2\right)w_r^A = 0 \\ abc - b^2ck^A - 4dk^A + 2bet - b(2+c)w_c^A + b(2+c\theta)w_r^A - 4\lambda = 0 \\ \lambda\left(k^A - \bar{k}\right) = 0 \\ \lambda^A \geq 0. \end{cases} \tag{12}$$

Solving equation (12) yields two scenarios:

Scenario A1

When $\lambda > 0$, by complementary slackness, $k^{A1^*} = \bar{k}$. Thus, the optimal clean energy advertisement input is

$$k^{A1*} = \bar{k}. \tag{13}$$

Substituting $k^{A1^*} = \bar{k}$ into equation (11) and solving yields:

$$w_c^{A1*} = \frac{a(4+c(2-\theta))(1+\theta)-(1-\theta)\big(b\bar{k}(4+c(2+\theta))-4et(1+\theta)\big)}{2(4+c)(1-\theta)(1+\theta)}, \tag{14}$$

$$w_r^{A1*} = \frac{a(1+\theta)+b\bar{k}(1-\theta)}{2(1-\theta^2)}, \tag{15}$$

$$\lambda = \frac{abc(1+\theta)-4(4+c)d\bar{k}(1+\theta)+4bet(1+\theta)+b^2\bar{k}(8+c-c\theta)}{4(4+c)(1+\theta)}. \tag{16}$$

The condition $\lambda > 0$ implies $\bar{k} < \bar{k_1}$, where

$$\bar{k}_1 = \frac{b(ac+4et)(1+\theta)}{4(4+c)d(1+\theta)-b^2(8+c(1-\theta))}. \tag{17}$$

Substituting equations (13)–(15) into the reaction functions (9) and (10) gives:

$$p_c^{A1*} = \frac{a(12+c(4-\theta))(1+\theta)-(1-\theta)\big(b\bar{k}(12+c(4+\theta))-4et(1+\theta)\big)}{4(4+c)(1-\theta^2)}, \tag{18}$$

$$p_r^{A1*} = \frac{3\big(a(1+\theta)+b\bar{k}(1-\theta)\big)}{4(1-\theta^2)}. \tag{19}$$

Substituting equations (13)–(15), and equations (18) and (19) into equations (1), (2), and (5) yield:

$$q_c^{A1^*} = \frac{a - b\bar{k} - et}{4 + c}, \tag{20}$$

$$q_r^{A1^*} = \frac{4et\theta + b\bar{k}(4 + c(1 - \theta)) + a(4 + c(1 + \theta))}{4(4 + c)}, \tag{21}$$

$$T^{A1^*} = \frac{e(a - b\bar{k} - et)}{4 + c}. \tag{22}$$

Finally, substituting into the profit functions (6) and (8) gives:

$$\pi_m^{A1^*} = \frac{\left\{\begin{matrix} a^2(1+\theta)(8+c+c\theta) - 2a\big(bc\bar{k} - 4et\big)\big(-1+\theta^2\big) - (1-\theta) \\ (4((4+c)d\bar{k}^2 - e^2t^2)(1+\theta) - b^2\bar{k}^2(8+c(1-\theta)) - 8be\bar{k}t(1+\theta))\end{matrix}\right\}}{(8(4+c)(1-\theta)(1+\theta))}, \tag{23}$$

$$\pi_s^{A1^*} = \frac{\left\{\begin{matrix} 2a\big(bc(8+c)\bar{k} - 16et\big)\big(1-\theta^2\big) + a^2(1+\theta)\big(32+8c(1+\theta)+c^2(1+\theta)\big) \\ +(1-\theta)(b^2\bar{k}^2\big(32+8c(1-\theta)+c^2(1-\theta)\big) + 16et(2b\bar{k}+et)(1+\theta)) \end{matrix}\right\}}{16(4+c)^2(1-\theta^2)}. \tag{24}$$

Scenario A2

When $\lambda = 0$, by complementary slackness, $k^{A2^*} \geq \bar{k}$. Substituting $\lambda = 0$ into the KKT conditions (12) and solving gives:

$$w_c^{A2^*} = \frac{\left\{\begin{matrix} 2(et(1-\theta)\big(4d(1+\theta) - 3b^2\big) \\ -a(b^2(2+c(1-\theta)) - d(4+c(2-\theta))(1+\theta)))\end{matrix}\right\}}{(1-\theta)(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))}, \tag{25}$$

$$w_r^{A2^*} = \frac{2b^2(et(1-\theta)-2a)+2a(4+c)d(1+\theta)}{(1-\theta)(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))}, \tag{26}$$

$$k^{A2^*} = \frac{b(ac+4et)(1+\theta)}{4(4+c)d(1+\theta)-b^2(8+c(1-\theta))}. \tag{27}$$

The condition $k^{A2^*} \geq \bar{k}$ implies $\bar{k} \geq \bar{k_1}$. Substituting equations (25)–(27) into the reaction functions (9) and (10) yields:

$$p_c^{A2^*} = \frac{\begin{Bmatrix} et(1-\theta)\big(4d(1+\theta)-5b^2\big) \\ +a(b^2(6+2c(1-\theta)) \\ -d(12+c(4-\theta))(1+\theta)) \end{Bmatrix}}{(1-\theta)(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))}, \tag{28}$$

$$p_r^{A2^*} = \frac{3b^2(et(1-\theta)-2a)+3a(4+c)d(1+\theta)}{(1-\theta)(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))}. \tag{29}$$

Substituting equations (25)–(29) into equations (1), (2), and (5) give:

$$q_c^{A2^*} = \frac{2a(2d(1+\theta)-b^2)+et(b^2(1-\theta)-4d(1+\theta))}{4(4+c)d(1+\theta)-b^2(8+c(1-\theta))}, \tag{30}$$

$$q_r^{A2^*} = \frac{b^2(et(1-\theta)-2a)+4det\theta(1+\theta)+ad(1+\theta)(4+c(1+\theta))}{4(4+c)d(1+\theta)-b^2(8+c(1-\theta))}, \tag{31}$$

$$T^{A2^*} = \frac{2ae(2d(1+\theta)-b^2)+e^2t(b^2(1-\theta)-4d(1+\theta))}{4(4+c)d(1+\theta)-b^2(8+c(1-\theta))}. \tag{32}$$

Finally, substituting into the profit functions (6) and (8) yields:

$$\pi_m^{A2^*} = \frac{\begin{Bmatrix} 4aet(1-\theta)\big(b^2-2d(1+\theta)\big)+e^2t^2(1-\theta)(+4d(1+\theta) \\ -b^2(1-\theta))+a^2(4b^2-d(1+\theta)(8+c(1+\theta))) \end{Bmatrix}}{2(1-\theta)(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))}, \tag{33}$$

$$\pi_s^{A2^*} = \frac{\begin{Bmatrix} 2aet(1-\theta)\Big(b^2d(1+\theta)(16+c+c\theta)-4(b^4+4d^2(1+\theta)^2)\Big) \\ +2e^2t^2(1-\theta)\Big(b^4(1-\theta)+8d^2(1+\theta)^2-4b^2d\big(1-\theta^2\big)\Big) \\ +a^2(4b^2(2b^2-d(1+\theta)(8+c(1+\theta)))+d^2(1+\theta)^2(32+8c(1+\theta)+c^2(1+\theta))) \end{Bmatrix}}{(1-\theta)(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2}. \tag{34}$$

Based on the above derivations, we summarize the equilibrium results for Model A in Proposition 1.

Proposition 1. *Under Model A, two scenarios emerge:*

When $\bar{k} < \bar{k_1}$, the optimal solutions are: wholesale prices $w_c^{A1^*}$ and $w_r^{A1^*}$, retail prices $p_c^{A1^*}$ and $p_r^{A1^*}$, clean energy advertisement input k^{A1^*}, demands $q_c^{A1^*}$ and $q_r^{A1^*}$, total carbon emissions T^{A1^*}, and profits $\pi_m^{A1^*}$ and $\pi_s^{A1^*}$. The explicit expressions are given in equations (13)–(15) and equations (18)–(24).

When $\bar{k} \geq \bar{k_1}$, the optimal solutions are: wholesale prices $w_c^{A2^*}$ and $w_r^{A2^*}$, retail prices $p_c^{A2^*}$ and $p_r^{A2^*}$, clean energy advertisement input k^{A2^*}, demands $q_c^{A2^*}$ and $q_r^{A2^*}$, total carbon emissions T^{A2^*}, and profits $\pi_m^{A2^*}$ and $\pi_s^{A2^*}$. The explicit expressions are given in equations (25)–(34).

3.3.2. Carbon allowance model (Model B)

Following a solution procedure analogous to Model A, we employ the backward induction method for Model B. First, we derive the first-order conditions of the power seller's profit function (8) with respect to the retail prices of conventional and clean power. This yields the following reaction functions, which are structurally identical to those in Model A but with the model-specific superscript B:

$$\mathrm{p}_r^B\left(w_r^B, k^B\right) = \frac{a + bk^B + a\theta - bk\theta + w_r^B - \theta^2 w_r^B}{2(1-\theta^2)}, \tag{35}$$

$$\mathrm{p}_c^B\left(w_c^B, k^B\right) = \frac{a(1+\theta) - bk^B(1-\theta) + (1-\theta^2)w_c^B}{2(1-\theta^2)}. \tag{36}$$

Substituting these into the generator's profit function (7) and introducing the Lagrange multiplier μ, we construct the Lagrangian:

$$L\left(w_c^B, w_r^B, k^B, \mu\right) = \left\{\begin{array}{c} 2abck^B - a^2c - b^2c\left(k^B\right)^2 - 4d\left(k^B\right)^2 \\ -4aet + 4bek^Bt - (4+c)\left(w_c^B\right)^2 \\ +4aw - r^B + 4bk^Bw - r^B + 4te - 0\left(2a - (1-\theta)(w_c^B + w_r^B)\right) \\ -2ac\theta w_r^B + 2bck^B\theta w_r^B - 4et\theta w_r^B - \left(4 + c\theta^2\right)\left(w_r^B\right)^2 \\ +2w_c^B\left(a(2+c) - (2+c)bk^B + 2et + (4+c)\theta w_r^B\right) \end{array}\right\} + 8\mu\left(\bar{k} - k^B\right). \tag{37}$$

The corresponding KKT conditions are derived from the first-order derivatives:

$$\begin{cases} (a(2+c) - (2+c)bk^B + 2et - 2t(1-\theta)e_0 - (4+c)w_c^B + (4+c)\theta w_r^B) = 0 \\ (2a + 2bk^B - ac\theta + bck^B\theta - 2et\theta - 2t(1-\theta)e_0 + (4+c)\theta w_c^B - (4 - c\theta^2)w_r^B) = 0 \\ (abc - b^2ck^B - 4dk^B + 2bet - b(2+c)w_c^B + b(2+c\theta)w_r^B) - 4\mu = 0 \\ \mu\left(k^B - \bar{k}\right) = 0 \\ \mu \geq 0. \end{cases} \tag{38}$$

Solving equation (38) yields two scenarios:

Scenario B1

When $\mu > 0$, by complementary slackness, $k^{B1^*} = \bar{k}$. Substituting this into equation (38) and solving yields the optimal clean energy advertisement input:

$$k^{B1^*} = \bar{k}, \tag{39}$$

$$w_c^{B1^*} = \frac{\left\{\begin{array}{c} a(4 + c(2-\theta))(1+\theta) - t(4+c\theta)(1-\theta^2)e_0 \\ +(1-\theta)(4et(1+\theta) - b\bar{k}(4 + c(2+\theta))) \end{array}\right\}}{2(4+c)(1-\theta^2)}, \tag{40}$$

$$w_r^{B1^*} = \frac{a(1+\theta) + b\bar{k}(1-\theta) - t(1-\theta^2)e_0}{2(1-\theta^2)}, \tag{41}$$

$$\mu = \frac{abc(1+\theta) - 4(4+c)d\bar{k}(1+\theta) + 4bet(1+\theta) + b^2\bar{k}(8 + c - c\theta) + bct(1-\theta^2)e_0}{4(4+c)(1+\theta)}. \tag{42}$$

The feasibility condition $\mu > 0$ implies $\bar{k} < \bar{k_2}$, where the threshold is:

$$\bar{k}_2 = \frac{b(1+\theta)(ac + 4et + ct(1-\theta)e_0)}{4(4+c)d(1+\theta) - b^2(8 + c(1-\theta))}. \tag{43}$$

Substituting equations (39)–(41) into the reaction functions (35) and (36) gives the optimal retail prices and subsequent equilibrium quantities and profits, as detailed in equations (44)–(50).

$$p_c^{B1^*} = \frac{\left\{\begin{array}{c} (1-\theta)\left(4et(1+\theta) - b\bar{k}(12 + c(4+\theta))\right) \\ +a(12 + c(4-\theta))(1+\theta) - t(4+c\theta)(1-\theta^2)e_0 \end{array}\right\}}{4(4+c)(1-\theta^2)}, \tag{44}$$

$$p_r^{B1^*} = \frac{3\big(a(1+\theta) + b\bar{k}(1-\theta)\big) - t(1-\theta^2)e_0}{4(1-\theta^2)}. \tag{45}$$

$$q_c^{B1^*} = \frac{a - b\bar{k} - et + t(1-\theta)e_0}{4+c}, \tag{46}$$

$$q_r^{B1^*} = \frac{\begin{array}{c} b\bar{k}(4+c(1-\theta)) + 4et\theta + a(4+c(1+\theta)) \\ +t(1-\theta)(4+c(1+\theta))e_0 \end{array}}{4(4+c)}, \tag{47}$$

$$T^{B1^*} = \frac{e(a - b\bar{k} - et + t(1-\theta)e_0)}{4+c}, \tag{48}$$

$$\pi_m^{B1^*} = \frac{\left\{\begin{array}{c} a^2(1+\theta)(8+c(1+\theta)) + 2a\big(bc\bar{k} - 4et\big)\big(1-\theta^2\big) + t^2(1-\theta)^2(1+\theta)(8+c+c\theta)e_0^2 \\ -(1-\theta)\big(b^2\bar{k}^2(8+c(1-\theta)) + 8be\bar{k}t(1+\theta) - 4\big((4+c)d\bar{k}^2 - e^2t^2\big)(1+\theta)\big) \\ +2t(1-\theta^2)((bc\bar{k} - 4et)(1-\theta) + a(8+c(1+\theta)))e_0 \end{array}\right\}}{8(4+c)(1-\theta^2)}, \tag{49}$$

$$\pi_s^{B1^*} = \frac{\left\{\begin{array}{c} t^2(1-\theta)^2(1+\theta)(32+8c(1+\theta)+c^2(1+\theta))e_0^2 \\ +2a\big(bc(8+c)\bar{k} - 16et\big)\big(1-\theta^2\big) + a^2(1+\theta)\big(32+8c(1+\theta)+c^2(1+\theta)\big) \\ +(1-\theta)\big(b^2\bar{k}^2\big(32+8c(1-\theta)+c^2(1-\theta)\big) - 32be\bar{k}t(1+\theta) - 16e^2t^2(1+\theta)\big) \\ +2t\big(1-\theta^2\big)\big(\big(bc(8+c)\bar{k} - 16et\big)(1-\theta) + a\big(32+8c(1+\theta)+c^2(1+\theta)\big)\big)e_0 \end{array}\right\}}{16(4+c)^2(1-\theta^2)}. \tag{50}$$

Scenario B2

When $\mu = 0$, by complementary slackness, $k^{B2^*} \geq \bar{k}$. Substituting $\mu = 0$ into equation (38) and solving yields the unconstrained optimal solutions, provided in equations (51)–(60). The feasibility condition $k^{B2^*} \geq \bar{k}$ implies $\bar{k} \geq \bar{k}_2$.

$$w_c^{B2^*} = \frac{\left\{\begin{array}{c} 2a\big(d(4+c(2-\theta))(1+\theta) - b^2(2+c(1-\theta))\big) \\ -2et(1-\theta)\big(3b^2 - 4d(1+\theta)\big) \\ +t(1-\theta)\big(b^2(4-c(1-\theta)) - 2d(1+\theta)(4+c\theta)\big)e_0 \end{array}\right\}}{(1-\theta)(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))}, \tag{51}$$

$$w_r^{B2^*} = \frac{\left\{\begin{array}{c} 2a(4+c)d(1+\theta) - 2b^2(2a - et(1-\theta)) \\ +t(1-\theta)(2(4+c)d(1+\theta) - b^2(4+c(1-\theta)))e_0 \end{array}\right\}}{(1-\theta)(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))}, \tag{52}$$

$$k^{B2^*} = \frac{b(1+\theta)(ac + 4et + ct(1-\theta)e_0)}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))}. \tag{53}$$

$$p_c^{B2^*} = \frac{\left\{\begin{array}{c} et(1-\theta)\big(4d(1+\theta) - 5b^2\big) \\ +t(1-\theta)\big(b^2(2-c(1-\theta)) - d(1+\theta)(4+c\theta)\big)e_0 \\ -a\big(b^2(6+2c(1-\theta)) - d(12+c(4-\theta))(1+\theta)\big) \end{array}\right\}}{(1-\theta)(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))}, \tag{54}$$

$$p_r^{B2^*} = \frac{\left\{\begin{array}{c} 3a(4+c)d(1+\theta) - 3b^2(2a - et(1-\theta)) \\ +t(1-\theta)(b^2(2+c(1-\theta)) - (4+c)d(1+\theta))e_0 \end{array}\right\}}{(1-\theta)(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))}. \tag{55}$$

$$q_c^{B2^*} = \frac{\left\{\begin{array}{c} 4d(1+\theta\theta)(a - et) - b^2(2a - et(1-\theta)) \\ -2t(1-\theta)(b^2 - 2d(1+\theta))e_0 \end{array}\right\}}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))}, \tag{56}$$

$$q_r^{B2^*} = \frac{\left\{\begin{array}{c} d(1+\theta)(4et\theta + a(4+c(1+\theta))) - b^2(2a+et(-1+\theta)) \\ +t(1-\theta)(d(1+\theta)(4+c(1+\theta)) - 2b^2)e_0 \end{array}\right\}}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))}, \tag{57}$$

$$T^{B2^*} = \frac{\left\{\begin{array}{c} 4de(1+\theta)(a-et) - eb^2(2a - et(1-\theta)) \\ -2et(1-\theta)(b^2 - 2d(1+\theta))e_0 \end{array}\right\}}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))}, \tag{58}$$

$$\pi_m^{B2^*} = \frac{\left\{\begin{array}{c} 4aet(1-\theta)\big(b^2 - 2d(1+\theta)\big) + e^2t^2(1-\theta)\big(4d(1+\theta) - b^2(1-\theta)\big) \\ 2t(1-\theta)\big(2et(1-\theta)\big(b^2-2d(1+\theta)\big) + a\big(d(1+\theta)(8+c(1+\theta)) - 4b^2\big)\big)e_0 \\ -a^2\big(4b^2 - d(1+\theta)(8+c(1+\theta))\big) + t^2(1-\theta)^2(d(1+\theta)(8+c(1+\theta)) - 4b^2)e_0^2 \end{array}\right\}}{2(1-\theta)(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))}, \tag{59}$$

$$\pi_s^{B2^*} = \frac{\left\{\begin{array}{c} 2aet(1-\theta)\Big(b^2d(1+\theta)(16+c+c\theta) - 4b^4 - 16d^2(1+\theta)^2\Big) \\ +2e^2t^2(1-\theta)\Big(b^4(1-\theta) + 8d^2(1+\theta)^2 - 4b^2d\big(1-\theta^2\big)\Big) \\ +a^2\Big(8b^4 - 4b^2d(1+\theta)(8+c+c\theta) + d^2(1+\theta)^2\big(32+8c(1+\theta)+c^2(1+\theta)\big)\Big) \\ +t^2(1-\theta)^2(8b^4 - 4b^2d(1+\theta)(8+c+c\theta) + d^2(1+\theta)^2(32+8c(1+\theta)+c^2(1+\theta)))e_0^2 \\ +2t(1-\theta)(et(1-\theta)(b^2d(1+\theta)(16+c+c\theta) - 4b^4 - 16d^2(1+\theta)^2) \\ +a(8b^4 - 4b^2d(1+\theta)(8+c+c\theta) + d^2(1+\theta)^2(32+8c(1+\theta)+c^2(1+\theta))))e_0 \end{array}\right\}}{(1-\theta)(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2}. \tag{60}$$

We summarize the equilibrium results for Model B in Proposition 2.

Proposition 2. *Under Model B, two scenarios emerge:*

When $\bar{k} < \bar{k_2}$, the optimal solutions are: wholesale prices $w_c^{B1^*}$ and $w_r^{B1^*}$, retail prices $p_c^{B1^*}$ and $p_r^{B1^*}$, clean energy advertisement input k^{B1^*}, demands $q_c^{B1^*}$ and $q_r^{B1^*}$, total carbon emissions T^{B1^*}, and profits $\pi_m^{B1^*}$ and $\pi_s^{B1^*}$. The explicit expressions are given in equations (39)–(41) and equations (44)–(50).

When $\bar{k} \geq \bar{k_2}$, the optimal solutions are: wholesale prices $w_c^{B2^*}$ and $w_r^{B2^*}$, retail prices $p_c^{B2^*}$ and $p_r^{B2^*}$, clean energy advertisement input k^{B2^*}, demands $q_c^{B2^*}$ and $q_r^{B2^*}$, total carbon emissions T^{B2^*}, and profits $\pi_m^{B2^*}$ and $\pi_s^{B2^*}$. The explicit expressions are given in equations (51)–(60).

To ensure the Hessian matrix is negative definite and all equilibrium solutions under models A and B are positive, we assume $a > b\bar{k} + et$, $d > b^2/(2(1+\theta))$ and $e_{01} < e_0 < \min\{e_{0n}\}, n = \{2,3,\ldots,7\}$. The specific expressions are provided in Appendix A.

4. Comparative analysis

This section compares the equilibrium outcomes – including power prices, clean energy advertisement inputs, power demands, total carbon emissions, and enterprises' profits – under the two policy models. Since $\bar{k_1} < \bar{k_2}$, and following the approach of Zhang *et al.* [38], we partition the analysis into the following three scenarios based on the level of the ceiling $\bar{k}$:

(1) Low ceiling ($\bar{k} < \bar{k_1}$): the generator is constrained by the ceiling regulation in both Model A and Model B.
(2) Moderate ceiling ($\bar{k_1} \leq \bar{k} < \bar{k_2}$): the generator is not constrained in Model A, but is constrained in Model B.
(3) High ceiling ($\bar{k} \geq \bar{k_2}$): the generator is not constrained in either Model A or Model B.

Propositions 3–6 present the detailed comparative results under the two models; the corresponding proofs are provided in Appendix B.

Proposition 3. *Comparing the wholesale and retail prices of conventional power and clean power under Models A and B yields the following results:*

(i) *when* $\bar{k} < \bar{k_1}$, $w_c^{A^*} > w_c^{B^*}$, $w_r^{A^*} > w_r^{B^*}$, $p_c^{A^*} > p_c B^*$, $p_r^{A^*} > p_r^{B^*}$;

(ii) *when* $\bar{k_1} \leq \bar{k} < \bar{k_2}$, $w_c^{A^*} > w_c^{B^*}, w_r^{A^*} > w_r^{B^*}, p_c^{A^*} > p_c^{B^*}; \begin{cases} p_r^{A^*} < p_r^{B^*}, \text{ if } a < a_1, \\ p_r^{A^*} \geq p_r^{B^*}, \text{ if } a \geq a_1. \end{cases}$

(iii) *when* $\bar{k} \geq \bar{k_2}$, $w_c^{A^*} > w_c^{B^*}, w_r^{A^*} > w_r^{B^*}, p_c^{A^*} > p_c^{B^*}, p_r^{A^*} > p_r^{B^*}$.

Proposition 3 compares wholesale and retail power prices under Models A and B. The results indicate that (1) Under a low or high ceiling ($\bar{k} < \bar{k_1}$ or $\bar{k} \geq \bar{k_2}$), both wholesale and retail prices for conventional and clean power are higher in Model A than in Model B. This implies that whether both models are constrained by the ceiling or neither is constrained, the outcome remains consistent: CT acts as a punitive policy that encourages power enterprises to set higher prices. (2) Under a moderate ceiling ($\bar{k_1} \leq \bar{k} < \bar{k_2}$), the comparison of wholesale prices for both types of power and the retail price of conventional power follows the same pattern as in the low/high ceiling cases. However, the retail price of conventional power depends on power potential demand a: when $a < a_1$, Model B exhibits a higher power retail price for clean energy; when $a > a_1$, Model A exhibits a higher power retail price for clean energy. The reason for this dependence may be explained as follows. Under a moderate ceiling, clean energy advertisement input and clean power demand are higher in Model B than in Model A ($k^{A^*} < k^{B^*}$ and $q_r^{A^*} < q_r^{B^*}$). If a is small, the seller in Model B faces a market where clean power occupies a larger share of demand; to extract higher profit from a relatively limited consumer base, the seller has an incentive to raise the clean power retail price. If a is large, the seller is more inclined to expand market share, and thus tends to set a lower retail price for clean power to attract more consumers and boost overall profit.

Proposition 4. *Comparing the clean energy advertisement inputs under Models A and B yields:*

(i) *when* $\bar{k} < \bar{k_1}, k^{A^*} = k^{B^*}$;

(ii) *when* $\bar{k_1} \leq \bar{k} < \bar{k_2}, k^{A^*} < k_{B^*}$;

(iii) *when* $\bar{k} \geq \bar{k_2}, k^{A^*} < k_{B^*}$.

The results indicate that (1) when $\bar{k}$ is low ($\bar{k} < \bar{k_1}$), both Model A and Model B exhibit identical clean energy advertisement inputs. When k is low, both Model A and Model B are constrained by the ceiling and operate at k; hence, they exhibit identical such inputs. (2) When $\bar{k}$ is moderate or high ($\bar{k} \geq \bar{k_1}$), Model B generates higher clean energy advertisement input than Model A. Under a moderate ceiling, where Model A is unconstrained, and Model B is constrained, Model A exhibits a lower input than Model B. This indicates that CT promotes the clean energy advertisement input only when the ceiling is binding; once the constraint is relaxed, CT actually reduces such input. When both models are unconstrained, Model B still produces greater input, suggesting that CAM is consistently more effective than CT in stimulating the generator's clean energy advertisement input. Thus, whether the generator faces a binding ceiling or not, CAM is more effective in promoting clean energy advertisement input. From the perspective of encouraging such input, policymakers would benefit from favoring CAM over CT.

Proposition 5. *Comparing the demands for conventional and clean power, and total carbon emissions under Models A and B yields:*

(i) *when* $\bar{k} < \bar{k_1}, q_c^{A^*} < q_c^{B^*}, q_r^{A^*} < q_r^{B^*}, T^{A^*} < T^{B^*}$;

(ii) *when* $\bar{k_1} \leq \bar{k} < \bar{k_2}, q_c^{A^*} < q_c^{B^*}, q_r^{A^*} < q_r^{B^*}, T^{A^*} < T^{B^*}$;

(iii) *when* $\bar{k} \geq \bar{k_2}, q_c^{A^*} < q_c^{B^*}, q_r^{A^*} < q_r^{B^*}, T^{A^*} < T^{B^*}$.

Proposition 5 compares conventional and clean power demands and total carbon emissions under the two models. The findings reveal that, regardless of the ceiling level, both types of demands and total carbon emissions are consistently higher under Model B than under Model A. This pattern may be rationalized as follows: when the ceiling is binding ($\bar{k} < \bar{k_1}$), clean energy advertisement inputs are equal in both models, but wholesale and retail prices for both clean and conventional power are lower under Model B. Lower prices make consumers more willing to purchase power under Model B. When the ceiling is moderate or high ($\bar{k} \geq \bar{k_1}$), not only

TABLE 2. Comparative results of profits under the two models.

Case	Condition	$\pi_m^{A^*}$ *vs.* $\pi_m^{B^*}$	$\pi_s^{A^*}$ *vs.* $\pi_s^{B^*}$
$\bar{k} < \bar{k_1}$	None	$\pi_m^{A^*} < \pi_m^{B^*}$	$\pi_s^{A^*} < \pi_s^{B^*}$
$\bar{k_1} < \bar{k} < \bar{k_2}$	$e_0 < \breve{e}_0$ $e_0 \geq \breve{e}_0$	$\pi_m^{A^*} > \pi_m^{B^*}$ $\pi_m^{A^*} \leq \pi_m^{B^*}$	$\pi_s^{A^*} < \pi_s^{B^*}$
$\bar{k} \geq \bar{k_2}$	$a < a_2$ $a \geq a_2$	$\pi_m^{A^*} < \pi_m^{B^*}$	$\pi_s^{A^*} > \pi_s^{B^*}$ $\pi_s^{A^*} \leq \pi_s^{B^*}$

are retail prices for both types of power generally lower in Model B, but clean energy advertisement input is also higher. These factors together further stimulate power demand under Model B. Furthermore, conventional power demand is higher in Model B, the generator produces more conventional power, which in turn leads to higher total carbon emissions under CAM than under CT.

Combining Propositions 4 and 5 reveals that higher clean energy advertisement input and clean power demand in Model B do not guarantee lower total carbon emissions, which depend critically on conventional power demand. This conclusion aligns with real-life observations: for example, despite an increase in India's clean power capacity by August 2023, carbon emissions continued to rise due to a rapidly growing share of coal-based generation, which still accounted for about 70% of the country's power supply[7]. Therefore, if the government aims to reduce carbon emissions, it must not only encourage clean energy input but also actively restrict conventional power production. Policy design should therefore consider the generator's overall energy-input mix, not merely its clean-energy investment.

Proposition 6. *Comparing the profits of the generator and seller under Models A and B yields the following expressions (summarized in Tab. 2):*

(i) *when* $\bar{k} < \bar{k_1}, \pi_m^{A^*} < \pi_m^{B^*}, \pi_s^{A^*} < \pi_s^{B^*}$;

(ii) *when* $\bar{k_1} \leq \bar{k} < \bar{k_2}, \pi_s^{A^*} < \pi_s^{B^*}; \begin{cases} \pi_m^{A^*} > \pi_m^{B^*} \text{ if } e_0 < \breve{e}_0, \\ \pi_m^{A^*} \leq \pi_m^{B^*} \text{ if } e_0 \geq \breve{e}_0. \end{cases}$

(iii) *when* $\bar{k} \geq \bar{k_2}, \pi_m^{A^*} < \pi_m^{B^*}; \begin{cases} \pi_s^{A^*} > \pi_s^{B^*} \text{ if } a < a_2, \\ \pi_s^{A^*} \leq \pi_s^{B^*}, \text{ if } a \geq a_2. \end{cases}$

Proposition 6 compares the profits of the generator and seller under Models A and B. The results indicate that:

(1) When $\bar{k}$ is low ($\bar{k} < \bar{k_1}$), both the generator and the seller earn higher profits under Model B. This follows from the fact that Model B exhibits lower wholesale and retail prices with higher power demand when the ceiling is low, which jointly boost the profits of both chain members.

(2) When $\bar{k}$ is moderate ($\bar{k_1} \leq \bar{k} < \bar{k_2}$), the seller is consistently more profitable under Model B, while the profit of the generator comparison depends on the unit of carbon allowance e_0. If $e_0 < \breve{e}_0$, the generator earns more under Model A. A smaller e_0 indicates a higher unit abatement cost for the generator under CAM; combined with higher total emissions in Model B, this raises the conventional power abatement cost and lowers the revenue from surplus allowances, reducing the generator's profit under CAM. Conversely, if $e_0 > \breve{e}_0$, the generator profits more under Model B. A larger e_0 enables the generator to benefit substantially from selling excess carbon allowances. For example, in September 2023, Jiangxi Datang International Fuzhou Power Generation Co., Ltd. sold 1.86 million tonnes of carbon allowances in the carbon market, earning about RMB 140 million[8].

[7] https://news.bjx.com.cn/html/20230914/1331832.shtml.

[8] https://news.bjx.com.cn/html/20230926/1334254.shtml.

TABLE 3. Optimal strategies under different perspectives.

Perspective	Indicator	Conditions for the optimality of Model A	Conditions for the optimality of Model B
Environment	Clean energy advertisement inputs	$\bar{k} < \bar{k_1}$	$\bar{k} \geq \bar{k_1}$
	Total carbon emissions	Always better	–
Consumer	Power price	–	(1) $\bar{k} < \bar{k_1}$ (2) $\bar{k_1} \leq \bar{k} < \bar{k_2}$ and $a > a_1$ (3) $\bar{k} \geq \bar{k_2}$
	Power demand	–	Always better
Enterprises	Enterprises' profit	–	(1) $\bar{k} < \bar{k_1}$ (2) $\bar{k_1} \leq \bar{k} < \bar{k_2}$ and $e_0 > \breve{e}_0$ (3) $\bar{k} \geq \bar{k_2}$ and $a > a_2$

(3) When $\bar{k}$ is high ($\bar{k} \geq \bar{k_2}$), the generator is always more profitable under Model B. When k is high, the generator can produce more clean power. Under CT, this merely reduces its carbon emission cost, while under CAM, it additionally gains revenue by selling carbon allowances generated from clean power. Therefore, the generator's profit in Model B is higher. However, the seller's profit comparison depends on potential power demand a. If $a < a_2$, the seller is more profitable under Model A. Although the demand is slightly lower in Model A, the seller faces higher wholesale prices for both types of power, allowing it to extract a higher profit. If $a > a_2$, the seller is more profitable under Model B, because the higher demand under CAM outweighs the price effect.

Therefore, the generator can confidently choose CAM in two broad scenarios, *i.e.*, when the ceiling is either low or high, or when the unit carbon allowance is sufficiently large.

From Propositions 3–6, we can derive Corollary 1.

Corollary 1. (1) *Environmental perspective: Total carbon emissions are always lower under Model A (CT). Clean energy advertisement input and clean power demand are never lower in Model B (CAM).*

(2) *Consumer perspective: Demands for both conventional and clean power are consistently higher under Model B; the prices of both power types are often lower under Model B than under Model A.*

(3) *Enterprise's perspective: Only under Model B can both the generator and the seller simultaneously have a chance to achieve higher profits.*

These observations are summarized in Table 3.

From Corollary 1 and Table 3, neither Model A nor Model B can simultaneously optimize the interests of the environment, consumers, and enterprises. (1) From the environmental perspective, while promoting the energy transition and increasing the share of clean power is beneficial – an objective for which Model B appears better suited – this study shows that higher clean energy advertisement input does not necessarily reduce carbon emissions. In fact, total emissions under Model A are consistently lower than under Model B. (2) From the consumers' perspective, Model A does not consistently lower the power prices or reduce demand. In contrast, Model B consistently delivers lower prices and higher demand for both types of power, regardless of the ceiling on clean energy advertisement input. (3) From the enterprise perspective, Model A cannot guarantee greater profits for both the generator and the seller at the same time. Under certain conditions, however, Model B does enable both enterprises to achieve greater profits simultaneously.

Based on the findings, it is evident that Model A is more effective in reducing total carbon emissions, whereas Model B exhibits greater potential for achieving optimal results in regard to the clean energy advertisement input, power demands, power prices, and corporate profitability. Therefore, the government can guide strategy selection and policy formulation according to current goals: (1) When the present goal is to reduce total carbon emissions, the government should implement the CT policy. (2) When the goal is to promote energy transition, the government should set a ceiling above a certain threshold ($\bar{k} \geq \bar{k_1}$) and implement CAM for generators. (3) When the target is to stimulate market power consumption, the government should implement CAM and adjust the ceiling based on the magnitude of potential power demand. (4) When the target is to enhance corporate profitability, the government should develop CAM and determine the ceiling based on the unit of carbon allowance and the power potential demand.

5. Sensitive analysis

Propositions 7–9 examine the impact of the ceiling ($\bar{k}$), the unit carbon allowance (e_0), and the clean energy advertisement input cost factor (d) on the equilibrium solutions under both models. Detailed proofs provided in Appendix C.

Proposition 7. *As the ceiling ($\bar{k}$) increases,*

(i) $\frac{\partial w_r^{M1^*}}{\partial \bar{k}} > 0, \frac{\partial p_r^{M1^*}}{\partial \bar{k}} > 0, \frac{\partial k^{M1^*}}{\partial \bar{k}} > 0, \frac{\partial T^{M1^*}}{\partial \bar{k}} < 0, \frac{\partial q_r^{M1^*}}{\partial \bar{k}} > 0, \frac{\partial \pi_m^{M1^*}}{\partial \bar{k}} > 0, \frac{\partial \pi_s^{M1^*}}{\partial \bar{k}} > 0;$

(ii) $\frac{\partial w_c^{M1^*}}{\partial \bar{k}} < 0, \frac{\partial p_c^{M1^*}}{\partial \bar{k}} < 0, \frac{\partial q_c^{M1^*}}{\partial \bar{k}} < 0,$ where $M = \{A, B\}$.

Proposition 7 investigates the effect of $\bar{k}$ on all equilibrium solutions under scenarios A1 and B1. The results indicate that as $\bar{k}$ rises, the wholesale and retail prices of clean power, clean energy advertisement input, clean power demand, and the profits of both the generator and seller increase. The wholesale and retail prices of conventional power, total carbon emissions, and conventional power demand decrease. The mechanism behind these effects is straightforward. A higher $\bar{k}$ set by the government encourages the generator to invest more in clean energy advertisement. The resulting increase in advertisement cost and in the share of clean power induces the generator to raise the wholesale price of clean power, which boosts its profit. At the same time, the increased consumer preference for clean power allows the seller to raise retail prices, thereby increasing the seller's profit. The decline in conventional power demand reduces total carbon emissions, and the associated reduction in carbon emission costs further contributes to the growth of power enterprises' profits. Therefore, by appropriately raising the ceiling on clean energy advertisement input, the government can effectively increase the proportion of clean power and reduce carbon emissions.

Proposition 8. *As the unit carbon allowance (e_0) increases,*

(i) $\frac{\partial B_c^{Bj*}}{\partial e_0} < 0, \frac{\partial B_r^{Bj*}}{\partial e_0} < 0, \frac{\partial p_c^{Bj*}}{\partial e_0} < 0, \frac{\partial p_r^{B1^*}}{\partial e_0} < 0, \frac{\partial k^{B1^*}}{\partial e_0} = 0, \frac{\partial k^{B2^*}}{\partial e_0} > 0,$
$\frac{\partial q_c^{Bj*}}{\partial e_0} > 0, \frac{\partial T^{Bj*}}{\partial e_0} > 0, \frac{\partial q_r^{Bj*}}{\partial e_0} > 0, \frac{\partial \pi_m^{B1^*}}{\partial e_0} > 0, \frac{\partial \pi_s^{B1^*}}{\partial e_0} > 0;$

(ii) $\begin{cases} \frac{\partial p_r^{Bj*}}{\partial e_0} > 0, & \text{if } c > c_1 \text{ and } d < \frac{b^2(1-\theta)}{1+\theta} \\ \frac{\partial p_r^{Bj*}}{\partial e_0} \leq 0, & \text{otherwise} \end{cases},$

(iii) $\begin{cases} \frac{\partial \pi_m^{Bj*}}{\partial e_0} < 0, & \text{if } c > c_1 \text{ and } d < \frac{b^2(1-\theta)}{1+\theta} \\ \frac{\partial \pi_m^{Bj*}}{\partial e_0} \geq 0, & \text{otherwise} \end{cases}, \begin{cases} \frac{\partial \pi_s^{Bj*}}{\partial e_0} < 0, & \text{if } e < \bar{\bar{e}}_0 \text{ and } t > t_2 \\ \frac{\partial \pi_s^{Bj*}}{\partial e_0} \geq 0, & \text{otherwise} \end{cases}.$ Where $j = \{1, 2\}$.

Proposition 8 investigates the effect of e_0 on all equilibrium solutions under scenarios B1 and B2, and the results show:

(1) In Scenario B1, as e_0 increases, wholesale and retail prices for conventional and clean power decrease, while clean energy advertisement input remains unchanged, demands for both types of power rise, total carbon emissions increase, and the profits of both the generator and the seller improve. In this case, because the

generator remains under the ceiling constraint, clean energy advertisement input is unaffected by changes in e_0. The higher e_0 reduces the generator's abatement cost and increases the carbon allowance revenue obtained through clean power sales. These effects incentivize the generator to reduce wholesale prices for both power types, which in turn leads the seller to reduce retail prices. Lower prices increase demand for both conventional and clean power; the resulting increase in conventional power demand drives up total carbon emissions. Despite lower prices, both the generator and the seller benefit from expanded power demands.

(2) In Scenario B2, as e_0 increases, wholesale and retail prices for conventional power and the wholesale power price for clean power decrease, while clean energy advertisement input, conventional power demand, and total carbon emissions increase. The key difference between Scenarios B2 and B1 lies in the response to the clean energy advertisement input. Here, the ceiling constraint is not binding, so a higher e_0 prompts the generator to invest more in clean energy advertisement, aiming to boost the share of clean power and thereby obtain more surplus carbon allowances to sell. The effects on the clean power retail price and the seller's profit are more complex, as detailed in points (2) and (3) below.

(3) In Scenario B2, the impact on the clean power retail price depends on cost conditions: When the unit production cost of conventional power c is relatively high and the clean energy advertisement input cost factor d is relatively low ($i.e., c > c_1$ and $d < b^2(1-\theta)/(1+\theta)$), the clean power retail price increases as e_0 increases. Otherwise, the retail price of clean power decreases as e_0 increases. The intuition is that under high c and low d, an increase in e_0 spurs the generator to substantially raise clean energy advertisement input. Because the resulting boost in clean power share strengthens the seller's market position, the seller has an incentive to set a higher retail price for clean power.

(4) In Scenario B2, the profit responses of the generator and seller depend on the levels of e_0 and the carbon price t: If e_0 is relatively low and t is relatively high ($i.e., e_0 < \bar{e_0}$ and $t > t_1$, or $e_0 < \bar{\bar{e_0}}$ and $t > t_2$), the profit of both the generator and the seller decrease as e_0 increases. Otherwise, profits increase with higher e_0. When e_0 is low and t is high, the generator faces higher abatement costs and earns less revenue from carbon allowances generated by clean power. This squeezes the generator's profit, which in turn lowers the seller's profit. Exceedingly high e_0 can weaken emission reduction incentives. The generator facing a high e_0 has less motive to reduce conventional power production. This aligns with real world experience: during the 2008 financial crisis, many high emitting EU enterprises slowed production, leaving large volumes of surplus allowances unused. In that year alone, ten enterprises received extra allowances worth €500 million, yet they did not deploy those allowances to lower actual emissions[9].

Proposition 9. *As the clean energy advertisement input cost factor (d) increases,*

(i) $\frac{\partial W_c^{M2^*}}{\partial d} > 0, \frac{\partial p_c^{M2^*}}{\partial d} > 0, \frac{\partial q_c^{M2^*}}{\partial d} > 0, \frac{\partial T_c^{M2^*}}{\partial d} > 0;$

(ii) $\frac{\partial W_r^{M2^*}}{\partial d} < 0, \frac{\partial p_r^{M2^*}}{\partial d} < 0, \frac{\partial k_c^{M2^*}}{\partial d} < 0, \frac{\partial \pi_m^{M2^*}}{\partial d} < 0, \frac{\partial \pi_s^{M2^*}}{\partial d} < 0, \text{where } M = \{A, B\}.$

Proposition 9 investigates the effect of d on all equilibrium solutions under scenarios A2 and B2. The study reveals that an increase in d raises the wholesale and retail prices, as well as total carbon emissions and demand for conventional power in both scenarios, lowers the prices and demand of clean energy, reduces clean energy advertisement input, and decreases the profits of both the generator and the seller. The intuition is straightforward: a higher d makes clean energy promotion more costly. The generator responds by cutting back on clean energy advertisement input and shifting production toward conventional power to meet demand. The divergent demand trends for the two power types prompt different pricing strategies: Higher conventional power demand pushes the generator to raise its wholesale price to maintain profit, and lower clean power demand prompts the generator to lower the clean power wholesale price to remain competitive. The seller, in turn, sets retail prices following the generator's wholesale decisions: A higher wholesale price for conventional power leads

[9] http://www.tanpaifang.com/tanzhibiao/201306/2021485_2.html.

TABLE 4. Parameters and their values.

Parameters	a	b	c	d	e	θ	e_0	t	$\bar{k}$
Values	20	0.8	0.3	1	3	0.3	3.5	3	$[0.1, 4]$
Units	GW	–	\$/GW	\$/GW^2	kg/GW	–	kg/GW	\$/kg	GW

the seller to raise its retail price to preserve margin, a lower wholesale price for clean power allows the seller to lower the retail price to attract more consumers. For enterprises, the increase in d hurts the profits of both the generator and the seller. For the government, a higher d undermines clean power production, consumer interests, and environmental benefits. Thus, both the government and enterprises should find ways to reduce this cost factor.

6. Numerical analysis

To further illustrate the impacts of the ceiling constraint and carbon regulation policies on the operational decisions of power enterprises, this section applies numerical analyses. The analysis is divided into two parts: (1) the impact of the clean energy advertisement input ceiling ($\bar{k}$), and (2) the impact of the unit carbon price (t). Throughout this section, $M = \{A, B\}$ and $i = \{c, r\}$.

6.1. The impact of the clean energy advertisement input ceiling

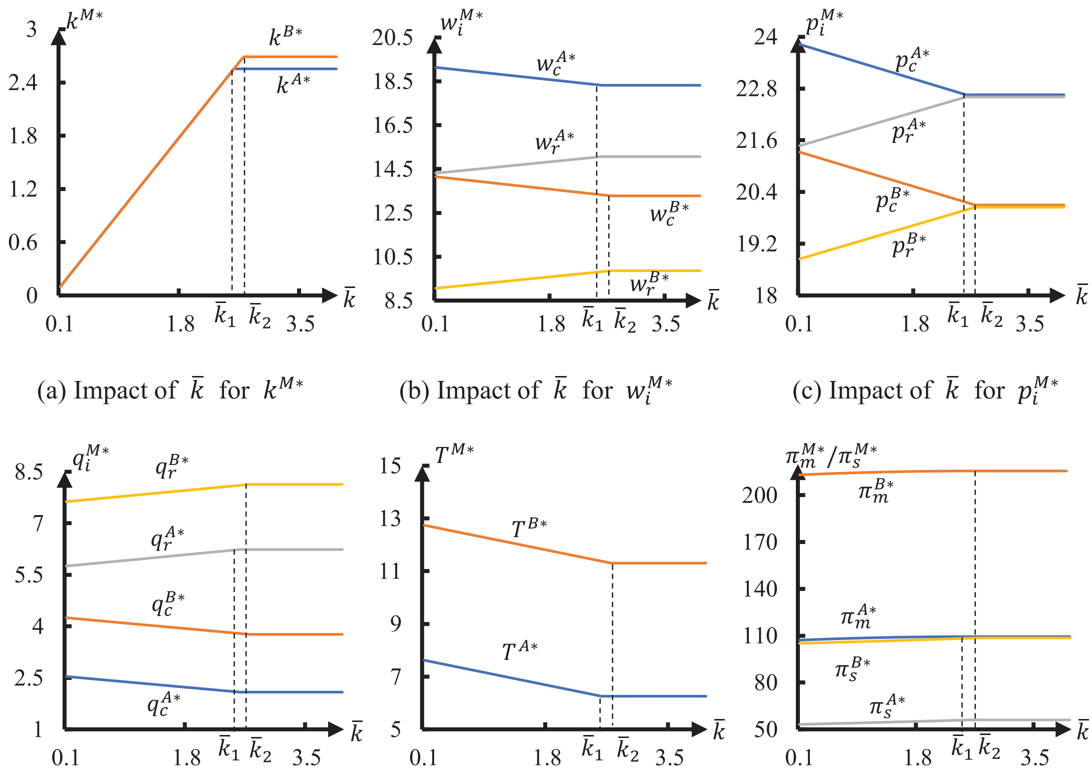
The following exogenous parameter values are adopted: $a = 20$ (GW), $b = 0.8$, $c = 0.3$ (\$/GW), $d = 1$ (\$/GW^2), $e = 3$ (kg/GW), $\theta = 0.3$, $e_0 = 3.5$ (kg/GW), and $t = 3$ (\$/kg).

Based on the assumptions and parameter values above, the threshold levels of the ceiling are calculated as $\bar{k_1} \approx 2.55$ (GW) and $\bar{k_2} \approx 2.69$ (GW). To examine how $\bar{k}$ affects each equilibrium solution across different ranges, we vary $\bar{k}$ over the interval $[0.1, 4]$ and plot the results in Figures 2a–2f. The parameters and their values are summarized in Table 4.

Figures 2a–2f investigates the effects of the ceiling $\bar{k}$ on equilibrium solutions, leading to the following conclusions:

(1) Comparison between Model A and Model B: Model B consistently yields higher clean energy advertisement input, higher demands for both conventional and clean power, higher total carbon emissions, and higher profits for the generator and seller. At the same time, Model B exhibits lower wholesale and retail prices for both types of power. Under the chosen parameter values, $a > \{a_1, a_2\}$ and $e_0 > \breve{e_0}$, these numerical analyses fully support the theoretical findings of Propositions 3–6. (2) Impact of raising the ceiling $\bar{k}$: When $\bar{k} < \{\bar{k_1}, \bar{k_2}\}$, an increase in $\bar{k}$ leads to a decline in wholesale and retail prices, demand of conventional power, and total carbon emissions under both models; an increase in clean energy advertisement input, wholesale and retail prices, and demand for clean energy, and the profits of the generator and seller. These patterns confirm the analytical results of Proposition 7.

Model B (CAM) exhibits higher clean energy advertisement input, greater power demand, and higher enterprise profits, together with lower power prices, suggesting that CAM is more conducive to energy transition, power consumption, and corporate benefit than CT. However, Model A (CT) consistently results in lower total carbon emissions. Raising the ceiling $\bar{k}$ can promote clean energy advertisement input, lower conventional power prices, reduce total carbon emissions, and raise corporate profits. Therefore, the government should choose between CT and CAM based on whether the primary objective is emission reduction (favor CT) or stimulating clean energy investment and market activity (favor CAM). Set the ceiling $\bar{k}$ at a reasonably high level to harness the benefits described above.



(a) Impact of $\bar{k}$ for k^{M*} (b) Impact of $\bar{k}$ for w_i^{M*} (c) Impact of $\bar{k}$ for p_i^{M*}

(d) Impact of $\bar{k}$ for q_i^{M*} (e) Impact of $\bar{k}$ for w_i^{M*} (f) Impact of $\bar{k}$ for π_m^{M*} and π_s^{M*}

FIGURE 2. Effect of $\bar{k}$ on all equilibrium solutions.

TABLE 5. Parameters and their values.

Parameters	a	b	c	d	e	θ	e_0	t	$\bar{k}$
Values	20	0.8	0.3	1	3	0.3	3.5	[1.5, 2.3]	2.2
Units	GW	–	\$/GW	\$/GW^2	kg/GW	–	kg/GW	\$/kg	GW

6.2. The impact of the unit carbon price

The following exogenous parameter values are used: $a = 20$ (GW), $b = 0.8$, $c = 0.3$ (\$/GW), $d = 1$ (\$/GW^2), $e = 3$ (kg/GW), $\theta = 0.3$, $e_0 = 3.5$ (kg/GW), and $\bar{k} = 2.2$ (GW).

Based on the earlier assumptions and these parameter values, we have $\bar{k} > \{\bar{k_1}, \bar{k_2}\}$. To examine how t affects all equilibrium solutions in Scenarios A2 and B2, we vary $t \in [1.5, 2.3]$ and present the results in Figures 3a–3f, respectively. The parameters and their values are summarized in Table 5.

Figures 3a–3f investigates the impact of t on each equilibrium solution under scenarios A2 and B2. The main findings are as follows:

(1) Comparison between Model A (CT) and Model B (CAM): Relative to Model A, Model B consistently shows higher clean energy advertisement input, demands for both conventional and clean power, total carbon emissions, and the profits of the generator and the seller. In contrast, Model B exhibits lower wholesale and

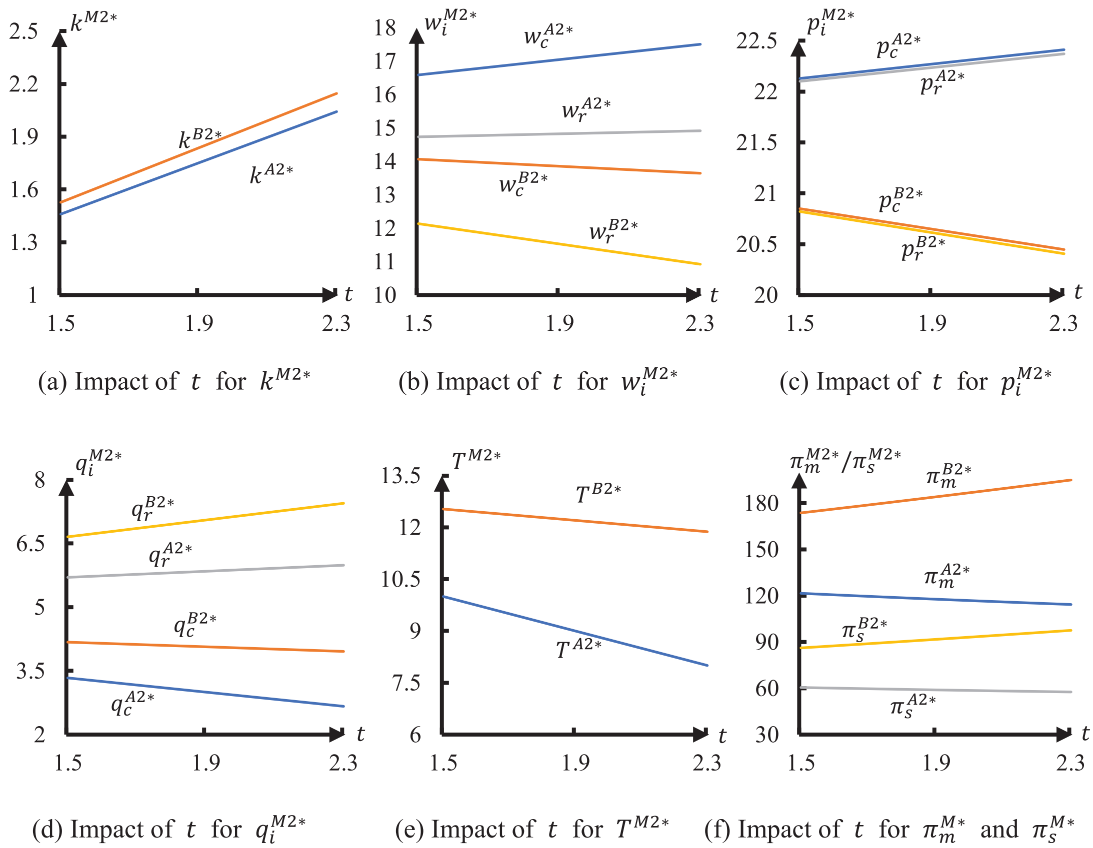


FIGURE 3. Effect of t on all equilibrium solutions of Scenarios A2 and B2.

retail prices for both types of power. These results align with the numerical patterns in Figure 2 and further validate Propositions 3–6. (2) Effect of raising t under Model A (CT): As t increases, clean energy advertisement input, wholesale and retail prices of both power types, and clean power demand increase, conventional power demand, total carbon emissions, and the profits of the generator and seller decrease. Under Model A, a higher t acts as a penalty on emissions. The generator responds by increasing clean energy advertisement input to increase the share of clean power, thereby cutting conventional power production and emissions. Meanwhile, the higher carbon cost pushes the generator and the seller to raise power prices. For instance, Singapore-the first Southeast Asian country to impose CT-expects its carbon prices to rise from 25 S$/tonne in 2024 to 80 S$/tonne by 2030, which is anticipated to drive up power prices[10]. Therefore, the added carbon cost reduces enterprise profits. (3) Under Model B, as t increases, clean energy advertisement input, clean power demand, and profits of both the generator and seller increase, while wholesale and retail prices of both conventional and clean power, conventional power demand, and total carbon emissions all decrease. This pattern can be explained as follows: under CAM, with the parameter condition $e < e_0$, the generator earns revenue not only from selling surplus carbon allowances from emission reduction efforts, but also from selling carbon allowances generated by clean power production. The higher carbon price t therefore enhances the value of these allowance sales, which in turn raises the profits of the generator and the seller and provides a stronger incentive to invest in clean energy advertisement.

[10] http://www.cnenergynews.cn/zhiku/2022/03/02/detail_20220302118532.html.

It can be observed that in both models, higher t effectively increases clean energy advertisement input, reduces total carbon emissions, and enhances clean power demand. Consequently, the government is well-positioned to establish an appropriate carbon pricing mechanism that reflects prevailing circumstances, thereby stimulating enterprises to increase their clean energy advertisement input while concurrently mitigating overall carbon emissions.

6.3. Policy implications

Based on the numerical results in Sections 6.1 and 6.2, this subsection provides policy implications for carbon regulation and ceiling regulation design.

(1) Policy selection: CT *vs.* CAM.

Overall, CAM (Model B) is more favorable to both consumers and enterprises, but not necessarily beneficial from an environmental perspective. First, compared to CT, CAM allows consumers to obtain higher power demand at lower power retail prices ($p_c^{B^*} < p_c^{A^*}, p_r^{B^*} < p_r^{A^*}, q_c^{B^*} > q_c^{A^*}, q_r^{B^*} > q_r^{A^*}$), and enables both the generator and the seller to achieve higher profits ($\pi_m^{B^*} > \pi_m^{A^*}$ and $\pi_s^{B^*} > \pi_s^{A^*}$). If policymakers aim to promote market consumption and economic development, they can implement CAM to achieve this effect. Second, neither CT nor CAM can be entirely beneficial from an environmental standpoint. Under a low ceiling, the CT is relatively better, as it can produce lower carbon emissions with the same level of clean energy advertisement input ($k^{A^*} = k^{B^*}$ and $T^{A^*} < T^{B^*}$). However, under a high ceiling, both CT and CAM have their respective advantages and disadvantages: CT can reduce carbon emissions ($T^{A^*} < T^{B^*}$), while CAM can promote greater clean energy advertisement input ($k^{B^*} > k^{A^*}$). If the government intends to promote sustainable environmental development, it can determine the specific policy to implement by assessing the current ceiling level and setting clear emission reduction or input targets. For instance, CT can be implemented under a low ceiling; CT can also be adopted when emission reduction targets are urgent, whereas CAM can be chosen when promoting investment is a priority. In summary, under realistic parameter settings, it is impossible to find a policy that simultaneously benefits consumers, the environment, and enterprises. The government must select the policy based on current development priorities, referring to the recommendations above.

(2) Setting the ceiling regulation.

Under a low ceiling ($\bar{k} < \bar{k_1}$), increasing $\bar{k}$ benefits both the environment and enterprises, but not necessarily consumers. From an environmental perspective, a rise in $\bar{k}$ encourages enterprises to invest more in clean energy advertisement and reduces carbon emissions. From an enterprise perspective, an increase in $\bar{k}$ also boosts profits for both the generator and the seller. However, from a consumer perspective, although a higher $\bar{k}$ helps consumers obtain conventional power at lower prices and increases their clean power demand, it also leads to higher clean power prices and reduces conventional power demand, which is unfavorable for consumers with low-carbon preferences. Under a moderate ceiling ($\bar{k_1} \leq \bar{k} < \bar{k_2}$), an increase in $\bar{k}$ has no impact on CT (Model A) but enhances environmental and enterprise benefits under CAM (Model B), with uncertain effects on consumers. Under a high ceiling ($\bar{k} \geq \bar{k_2}$), an increase in $\bar{k}$ has no impact on either CT or CAM.

Therefore, when setting the ceiling regulation, the government should avoid setting an excessively high input ceiling, as enterprises may abandon input altogether if the constraint is unreachable, yielding no benefits for consumers or the environment. It is recommended that the government set a moderately low initial ceiling based on actual conditions and gradually raise it over time to ensure environmental and enterprise benefits. If consumer interests are eroded as a result, measures such as consumption subsidies or agreements with enterprises to share part of the increased benefits can be considered, thereby fostering sustainable and positive interactions among the consumer market, economic development, and environmental protection.

(3) Effects of carbon price.

As the carbon price t increases, environmental benefits improve, and both enterprise and consumer benefits under CAM can also be enhanced. First, an increase in t has a clear positive impact on the environment, continuously incentivizing clean energy advertisement input and reducing carbon emissions. Second, under CAM, an increase in t not only allows both supply chain enterprises to achieve higher profits but also enables consumers to obtain higher clean power demand at lower retail prices while reducing their conventional energy demand, thereby curbing the use of conventional power at the source. However, under CT, an increase in t reduces enterprise profits and forces consumers to purchase electricity at higher prices, effectively transferring the emission reduction and input costs from enterprises to consumers. Therefore, to achieve comprehensive improvements in consumer, environmental, and enterprise benefits while increasing t, CAM should be implemented. If the government chooses to implement CT, policy adjustments can be made based on current development goals: if the government aims to promote market consumption, enhance enterprise profitability, or strengthen environmental sustainability, it may consider reducing t Tor transitioning from the CT to the CAM.

7. Conclusions and proposals

The principal findings of this study are summarized as follows:

(1) When the ceiling is low, clean energy advertisement input is identical under Model A and Model B; however, both the generator and the seller earn lower profits under Model A. When the ceiling is moderate or high, clean energy advertisement input is higher under Model B. In this case, the size of the seller's profit is influenced by the unit carbon allowance and the potential market demand: The seller's profit under Model B is larger when either the unit carbon allowance or the potential market demand is larger. (2) Regardless of the ceiling, compared with Model A, Model B consistently shows higher demand for both conventional and clean power, as well as higher total carbon emissions, while the wholesale and retail prices for conventional power and the wholesale price for clean power are always lower in Model B. The retail price of clean power depends on the potential market demand, and it is higher under Model A when the potential market demand is higher. (3) When the generator is subject to a ceiling constraint, a rise in the ceiling leads to decreases in wholesale and retail prices, conventional power demand, and total carbon emissions under both models. Conversely, wholesale and retail prices, clean power demand, clean energy advertisement input, and profits of the generator and seller all increase as the ceiling increases. (4) When the generator is unconstrained by the ceiling, an increase in the cost factor of clean energy advertisement inputs results in higher wholesale and retail prices, conventional power demand, and total carbon emissions. Meanwhile, wholesale and retail prices and demand for clean power, the clean energy advertisement input, and the profits of the generator and the seller all decrease.

Based on these conclusions, the following recommendations are proposed. The government can select and design carbon regulation policies according to current objectives, and set the ceiling on clean energy advertisement input based on the actual scenario: When the primary target is to reduce total carbon emissions, the government should implement the CT policy. When the target is to promote energy transition, the government should set the ceiling above a certain threshold and implement the CAM for generators. When the target is to stimulate power market consumption, the government should implement CAM and adjust the ceiling in accordance with the level of potential power demand. When the target is to support economic development and enhance corporate profitability, the government should adopt CAM and set the ceiling based on the unit of carbon allowance and potential power demand.

Nomenclature

Parameters	Descriptions
a	Potential market power demand, $a > 0$
θ	Cross-price competition factor, $0 < \theta < 1$
b	Clean energy advertisement preference factor, $0 < b < 1$
d	Clean energy advertisement input cost factor, $d > 0$
e	Unit carbon emissions, $e > 0$
e_0	Unit carbon allowance, $e_0 > 0$
t	Unit carbon price, $t > 0$

c	Unit production cost of conventional power, $c > 0$
$\bar{k}$	The ceiling of clean energy advertisement input (It is briefly referred to as the ceiling throughout the paper), $\bar{k} > 0$
Variables	Descriptions
p_r^M	Clean power retail price
p_c^M	Conventional power retail price
w_r^M	Clean power wholesale price
w_c^M	Conventional power wholesale price
k^M	Clean energy advertisement input
q_r^M	Demand for clean power
q_c^M	Demand for conventional power
T^M	Total carbon emissions
π_m^M	Profit of power generator
π_s^M	Profit of power seller
Superscript	Descriptions
$M = \{A, B\}$	Superscript A denotes Model A, and B denotes Model B
$*$	Equilibrium solution
Others	Descriptions
CT	Carbon tax
CAM	Carbon allowance mechanism

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this study.

Data availability statement

The research data associated with this article are included in the article.

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Appendix A. List of values for e_{0n}

The specific expressions for the unit carbon allowance values e_{0n} are as follows:

$$e_{01} = \frac{2a\left(b^2 - 2d(1+\theta)\right) + et\left(4d(1+\theta) - b^2(1-\theta)\right)}{2t(1-\theta)(2d(1+\theta) - b^2)},$$

$$e_{02} = \frac{4ab^2 - 2b^2et(1-\theta) - 2a(4+c)d(1+\theta)}{t(1-\theta)(b^2(4+c(1-\theta)) - 2(4+c)d(1+\theta))},$$

$$e_{03} = \frac{2b\bar{k} + et(1+\theta)}{t - t\theta^2},$$

$$e_{04} = \frac{a(4+c(2-\theta))(1+\theta)+(1-\theta)(4et(1+\theta)-b\bar{k}(4+c(2+\theta)))}{t(4+c\theta)(1-\theta^2)},$$

$$e_{05} = \frac{2(et(1-\theta)(4d(1+\theta)-3b^2)+a(d(4+c(2-\theta))(1+\theta))-b^2(2+c(1-\theta)))}{t(1-\theta)(2d(1+\theta)(4+c\theta)-b^2(4-c+c\theta))},$$

$$e_{06} = \frac{et(1-\theta)(4d(1+\theta)-5b^2)+a(b^2(6+2c(1-\theta))-d(12+c(4-\theta))(1+\theta))}{t(1-\theta)(d(1+\theta)(4+c\theta)-b^2(2-c+c\theta))},$$

$$e_{07} = \frac{3b^2et(1-\theta)+3a(4+c)d(1+\theta)-6ab^2}{t(1-\theta)((4+c)d(1+\theta)-b^2(2+c(1-\theta)))}.$$

Appendix B. Comparative analysis of Model A and Model B

Comparing the critical values from Model A and Model B yields:

$$\bar{k}_1-\bar{k}_2=-\frac{\left(bcte_0\left(1-\theta^2\right)\right)}{4(4+c)d(1+\theta)-b^2(8+c(1-\theta))}<0.$$

Therefore, $\bar{k_1} < \bar{k_2}$. This relationship defines three distinct regimes for comparing the equilibrium solutions of the two models: (1) When k is low ($\bar{k} < \bar{k_1}$), we compare Scenario A1 with Scenario B1. (2) When k is moderate ($\bar{k_1} \leq \bar{k} < \bar{k_2}$), we compare Scenario A2 with Scenario B1. (3) When k is high ($\bar{k} \geq \bar{k_2}$), we compare Scenario A2 with Scenario B2.

Proof of Proposition 3. We compare the optimal conventional power whole price, clean power whole price, conventional power retail price and clean power retail price between Model A and Model B.

(1) When $\bar{k} < \bar{k_1}$, there are

$$w_c^{A^*}-w_c^{B^*}=w_c^{A1^*}-w_c^{B1^*}=\frac{t(4+c\theta)e_0}{2(4+c)}>0.$$
$$w_r^{A^*}-w_r^{B^*}=w_r^{A1^*}-w_r^{B1^*}=\frac{te_0}{2}>0.$$
$$p_c^{A^*}-p_c^{B^*}=p_c^{A1^*}-p_c^{B1^*}=\frac{t(4+c\theta)e_0}{4(4+c)}>0.$$
$$p_r^{A^*}-p_r^{B^*}=p_r^{A1^*}-p_r^{B1^*}=\frac{te_0}{4}>0.$$

(2) When $\bar{k_1} \leq \bar{k} < \bar{k_2}$, there are

$$w_c^{A^*}-w_c^{B^*}=w_c^{A2^*}-w_c^{B1^*}=\frac{t(4+c\theta)e_0}{2(4+c)}-\frac{\left\{\begin{array}{l} b(4+c(2+\theta))(abc(1+\theta)+4bet(1+\theta) \\ -4(4+c)d\bar{k}(1+\theta)+b^2\bar{k}(8+c-c\theta)) \end{array}\right\}}{2(4+c)(1+\theta)(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))}>0.$$

$$w_r^{A^*}-w_r^{B^*}=w_r^{A2^*}-w_r^{B1^*}=\frac{1}{2}\left(\frac{b^2(ac+4et)1+\theta-b\bar{k}(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))}{(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))(1+\theta)}+te_0\right)>0.$$

$$p_c^{A^*}-p_c^{B^*}=p_c^{A2^*}-p_c^{B1^*}=\frac{t(4+c\theta)e_0}{4(4+c)}-\frac{\left\{\begin{array}{l} b(12+c(4+\theta))(abc(1+\theta)+4bet(1+\theta) \\ -4(4+c)d\bar{k}(1+\theta)+b^2\bar{k}(8+c-c\theta)) \end{array}\right\}}{4(4+c)(1+\theta)(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))}>0.$$

$$p_r^{A^*}-p_r^{B^*}=p_r^{A2^*}-p_r^{B1^*}=\frac{1}{4}\left(3b\left(\frac{b(ac+4et)}{4(4+c)d(1+\theta)-b^2(8+c(1-\theta))}-\frac{\bar{k}}{1+\theta}\right)+te_0\right).$$

If $a < a_1$, then $p_r^{A^*}-p_r^{B^*} < 0$, if $a \geq a_1$, then $p_r^{A^*}-p_r^{B^*} \geq 0$, where

$$a_1=\frac{\begin{array}{c}12b(4+c)d\bar{k}(1+\theta)-3b^3\bar{k}(8+c(1-\theta)) \\ -b^2t(12e-e_0(8+c(1-\theta)))(1+\theta)-4(4+c)dte_0(1+\theta)^2\end{array}}{3b^2c(1+\theta)}.$$

(3) When $\bar{k} \geq \bar{k_2}$, there are

$$w_c^{A^*} - w_c^{B^*} = w_c^{A2^*} - w_c^{B2^*} = \frac{t(2d(1+\theta)(4+c\theta) - b^2(4-c+c\theta))e_0}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))} > 0.$$

$$w_r^{A^*} - w_r^{B^*} = w_r^{A2^*} - w_r^{B2^*} = \frac{t(2d(1+\theta)(4+c\theta) - b^2(4-c+c\theta))e_0}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))} > 0.$$

$$p_c^{A^*} - p_c^{B^*} = p_c^{A2^*} - p_c^{B2^*} = \frac{t(d(1+\theta)(4+c\theta) - b^2(2-c(1-\theta)))e_0}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))} > 0.$$

$$p_r^{A^*} - p_r^{B^*} = p_r^{A2^*} - p_r^{B2^*} = \frac{t\big((4+c)d(1+\theta) - b^2(2+c(1-\theta))\big)e_0}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))} > 0.$$

□

Proof of Proposition 4. We compare the optimal clean energy advertisement input between Model A and Model B.

(1) When $\bar{k} < \bar{k_1}$, there are

$$k^{A^*} - k^{B^*} = k^{A1^*} - k^{B1^*} = 0.$$

(2) When $\bar{k_1} \leq \bar{k} < \bar{k_2}$, there are

$$k^{A^*} - k^{B^*} = k^{A2^*} - k^{B1^*} = \frac{b(ac+4et)(1+\theta)}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))} - \bar{k} < 0.$$

(3) When $\bar{k} \geq \bar{k_2}$, there are

$$k^{A^*} - k^{B^*} = k^{A2^*} - k^{B2^*} = -\frac{bct(1-\theta^2)e_0}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))} < 0.$$

□

Proof of Proposition 5. We compare the optimal demand for conventional power, demand for clean power, total carbon emissions between Model A and Model B.

(1) When $\bar{k} < \bar{k_1}$, there are

$$q_c^{A^*} - q_c^{B^*} = q_c^{A1^*} - q_c^{B1^*} = -\frac{t(1-\theta)e_0}{4+c} < 0.$$

$$q_r^{A^*} - q_r^{B^*} = q_r^{A1^*} - q_r^{B1^*} = -\frac{t(1-\theta)(4+c+c\theta)e_0}{4(4+c)} < 0.$$

$$T^{A^*} - T^{B^*} = T^{A1^*} - T^{B1^*} = -\frac{et(1-\theta)e_0}{4+c} < 0.$$

(2) When $\bar{k_1} \leq \bar{k} < \bar{k_2}$, there are

$$q_c^{A^*} - q_c^{B^*} = q_c^{A2^*} - q_c^{B1^*} = \frac{\begin{array}{c} b(4(4+c)d\bar{k}(1+\theta) - b^2\bar{k}(8+c(1-\theta)) \\ -(4bet+abc)(1+\theta)) \end{array}}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))(4+c)} - \frac{t(1-\theta)e_0}{(4+c)} < 0.$$

$$q_r^{A^*} - q_r^{B^*} = q_r^{A2^*} - q_r^{B1^*} = \frac{\left\{\begin{array}{c} b(4+c(1-\theta))(abc(1+\theta) - 4(4+c)d\bar{k}(1+\theta) \\ +4bet(1+\theta) + b^2\bar{k}(8+c-c\theta)) - t(1-\theta)(4+c+c\theta) \\ e_0(4(4+c)d(1+\theta) - b^2(8+c(1-\theta))) \end{array}\right\}}{(4+c)(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))} < 0.$$

$$T^{A^*} - T^{B^*} = T^{A2^*} - T^{B1^*} = \frac{\begin{array}{c} be(4(4+c)d\bar{k}(1+\theta) - b^2\bar{k}(8+c(1-\theta)) \\ -abc(1+\theta) - 4bet(1+\theta)) \end{array}}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))(4+c)} - \frac{te(1-\theta)e_0}{4+c} < 0.$$

(3) When $\bar{k} \geq \bar{k_2}$, there are

$$q_c^{A^*} - q_c^{B^*} = q_c^{A2^*} - q_c^{B2^*} = -\frac{2t(1-\theta)(2d(1+\theta)-b^2)e_0}{4(4+c)d(1+\theta)-b^2(8+c(1-\theta))} < 0.$$

$$q_r^{A^*} - q_r^{B^*} = q_r^{A2^*} - q_r^{B2^*} = -\frac{t(1-\theta)(d(1+\theta)(4+c+c\theta)-2b^2)e_0}{4(4+c)d(1+\theta)-b^2(8+c(1-\theta))} < 0.$$

$$T^{A^*} - T^{B^*} = T^{A2^*} - T^{B2^*} = -\frac{2te(1-\theta)(2d(1+\theta)-b^2)e_0}{4(4+c)d(1+\theta)-b^2(8+c(1-\theta))} < 0.$$

□

Proof of Proposition 6. We compare the optimal profit of power generator and profit of power seller between Model A and Model B.

(1) When $\bar{k} < \bar{k_1}$, there are

$$\pi_m^{A^*} - \pi_m^{B^*} = \pi_m^{A1^*} - \pi_m^{B1^*} = -\frac{\begin{matrix} te_0(2\big(bc\bar{k} - 4et\big)(1-\theta) + 2a(8+c+c\theta) \\ +t(1-\theta)(8+c+c\theta)e_0) \end{matrix}}{8(4+c)} < 0,$$

$$\pi_s^{A^*} - \pi_s^{B^*} = \pi_s^{A1^*} - \pi_s^{B1^*} = -\frac{te_0\left\{\begin{matrix} 2\big(bc(8+c)\bar{k} - 16et\big)(1-\theta) \\ +2a\big(32+8c(1+\theta)+c^2(1+\theta)\big) \\ +t(1-\theta)(32+8c(1+\theta)+c^2(1+\theta))e_0 \end{matrix}\right\}}{16(4+c)^2} < 0.$$

(2) When $\bar{k_1} \leq \bar{k} < \bar{k_2}$, there are

$$\pi_m^{A^*} - \pi_m^{B^*} = \pi_m^{A2^*} - \pi_m^{B1^*} = \frac{\left\{\begin{matrix} \big(abc(1+\theta) - 4(4+c)d\bar{k}(1+\theta) + 4bet(1+\theta) + b^2\bar{k}(8+c-c\theta)\big)^2 \\ -te_0(2\big(bc\bar{k} - 4et\big)(1-\theta) + 2a(8+c+c\theta) \\ +t(1-\theta)(8+c+c\theta)e_0)\big(4(4+c)d(1+\theta) - b^2(8+c(1-\theta))\big) \end{matrix}\right\}}{8(4+c)(1+\theta)(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))},$$

if $e_0 < \breve{e_0}$, then $\pi_m^{A^*} > \pi_m^{B^*}$, $e_0 \geq \breve{e_0}, \pi_m^{A^*} \leq \pi_m^{B^*}$, where

$$\breve{e_0} = \frac{(1-\theta)\big(bc\bar{k} - 4et\big) - 8a - ca(1+\theta)}{t(1-\theta)(8+c+c\theta)} - 2\sqrt{\frac{\left\{\begin{matrix} (4+c)(4aet(8+c(1+\theta))(1-\theta^2)(b^2-2d(1+\theta)) \\ +a^2(1+\theta)(8+c+c\theta)\big(d(1+\theta)(8+c+c\theta) - 4b^2\big) \\ -(1-\theta)(16b(4+c)de\bar{k}t(1+\theta)^2 - 4b^4\bar{k}^2(8+c-c\theta) \\ +4b^3e\bar{k}t(8+c+8\theta-c\theta^2) + 4d(1+\theta)^2(4e^2t^2(1-\theta) \\ +(4+c)d\bar{k}^2(8+c+c\theta)) + b^2(1+\theta)(16e^2t^2\theta \\ -d\bar{k}^2(128+32c+c^2(1-\theta^2)))) \end{matrix}\right\}}{t^2(-1+\theta)^2(1+\theta)(8+c+c\theta)^2(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))}},$$

$$\pi_s^{A^*} - \pi_s^{B^*} = \pi_s^{A2^*} - \pi_s^{B1^*}$$

$$= \frac{\left\{\begin{matrix} b\big(abc(1+\theta) - 4(4+c)d\bar{k}(1+\theta) + 4bet(1+\theta) + b^2\bar{k}(8+c-c\theta)\big) \\ (128(4+c)det(1+\theta)^2 - b^3\bar{k}(8+c(1-\theta))(32+c(8+c)(1-\theta)) \\ -4b^2et\big(32-c^2(1-\theta)\big)(1+\theta) + 4b(4+c)d\bar{k}(32+c(8+c)(1-\theta))(1+\theta) \\ +ac(1+\theta)(8(4+c)(8+c)d(1+\theta) - b^2(96+c(24+c)+c(8+c)\theta))) \\ -te_0(2(bc(8+c)\bar{k} - 16et)(1-\theta) + 2a(32+c(8+c)(1+\theta)) \\ +t(1-\theta)(32+c(8+c)(1+\theta))e_0) \end{matrix}\right\}}{16(1+\theta)(4+c)^2(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2} < 0.$$

(3) When $\bar{k} \geq \bar{k_2}$, there are

$$\pi_m^{A^*} - \pi_m^{B^*} = \pi_m^{A2^*} - \pi_m^{B2^*} = -\frac{te_0\left\{\begin{array}{c} 4et(1-\theta)\big(b^2 - 2d(1+\theta)\big) \\ +2a\big(d(1+\theta)(8+c+c\theta) - 4b^2\big) \\ +t(1-\theta)(d(1+\theta)(8+c+c\theta) - 4b^2)e_0 \end{array}\right\}}{2(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))} < 0.$$

$$\pi_s^{A^*} - \pi_s^{B^*} = \pi_s^{A2^*} - \pi_s^{B2^*} = \frac{\left\{\begin{array}{c} te_0(2et(1-\theta)(4b^4 + 16d^2(1+\theta)^2 - b^2d(1+\theta)(16+c+c\theta)) \\ -2a(8b^4 - 4b^2d(1+\theta)(8+c+c\theta) \\ +d^2(1+\theta)^2(32 + 8c(1+\theta) + c^2(1+\theta))) \\ -t(1-\theta)(8b^4 - 4b^2d(1+\theta)(8+c+c\theta) \\ +d^2(1+\theta)^2(32+8c(1+\theta) + c^2(1+\theta)))e_0) \end{array}\right\}}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2},$$

if $a < a_2$, then $\pi_s^{A^*} > \pi_s^{B^*}, a \geq a_2, \pi_s^{A^*} \leq \pi_s^{B^*}$, where

$$a_2 = \frac{et(1-\theta)\big(4b^4 + 16d^2(1+\theta)^2 - b^2d(1+\theta)(16+c+c\theta)\big)}{8b^4 - 4b^2d(1+\theta)(8+c+c\theta) + d^2(1+\theta)^2(32+8c(1+\theta)+c^2(1+\theta))} - \frac{e_0t(1-\theta)}{2}.$$

□

Appendix C. Sensitivity analysis proofs

This appendix provides the detailed proofs for the sensitivity analysis Propositions 7, 8, and 9 by deriving the first-order partial derivatives of the equilibrium solutions with respect to key parameters.

Proof of Proposition 7. Since the ceiling k only directly affects constrained scenarios, we analyze its impact on the A1 and B1 scenarios.

Impact on Model A1 equilibrium:

$$\frac{\partial w_c^{A1^*}}{\partial \bar{k}} = -\frac{b(4+c(2+\theta))}{2(4+c)(1+\theta)} < 0, \frac{\partial w_r^{A1^*}}{\partial \bar{k}} = \frac{b(1-\theta)}{2-2\theta^2} > 0,$$

$$\frac{\partial p_c^{A1^*}}{\partial \bar{k}} = -\frac{b(1-\theta)(12+c(4+\theta))}{4(4+c)(1-\theta^2)} < 0, \frac{\partial p_r^{A1^*}}{\partial \bar{k}} = \frac{3(b(1-\theta))}{4(1-\theta^2)} > 0,$$

$$\frac{\partial k^{A1^*}}{\partial \bar{k}} = 1 > 0, \frac{\partial q_c^{A1^*}}{\partial \bar{k}} = -\frac{b}{4+c} < 0, \frac{\partial q_r^{A1^*}}{\partial \bar{k}} = \frac{b(4+c-c\theta)}{4(4+c)} > 0,$$

$$\frac{\partial \pi_m^{A1^*}}{\partial \bar{k}} = \frac{\begin{array}{c} 2abc(1-\theta^2) + (1-\theta)(2b^2\bar{k}(8+c(1-\theta)) \\ +8(4+c)d\bar{k}(1+\theta) - 8bet(1+\theta)) \end{array}}{8(4+c)(1-\theta)(1+\theta)} > 0,$$

$$\frac{\partial \pi_s^{A1^*}}{\partial \bar{k}} = \frac{\begin{array}{c} 2abc(8+c)(1-\theta^2) + (1-\theta)(2b^2\bar{k}(32+8c(1-\theta) \\ +c^2(1-\theta)) - 32bet(1+\theta)) \end{array}}{16(4+c)^2(1-\theta^2)} > 0.$$

Impact on Model B1 equilibrium:

$$\frac{\partial w_c^{B1^*}}{\partial \bar{k}} = -\frac{b(1-\theta)(4+c(2+\theta))}{2(4+c)(1-\theta^2)} < 0, \frac{\partial w_r^{B1^*}}{\partial \bar{k}} = \frac{b(1-\theta)}{2(1-\theta^2)} > 0,$$

$$\frac{\partial p_c^{B1^*}}{\partial \bar{k}} = -\frac{b(1-\theta)(12+c(4+\theta))}{4(4+c)(1-\theta^2)} < 0, \frac{\partial p_r^{B1^*}}{\partial \bar{k}} = \frac{3(b-b\theta)}{4(1-\theta^2)} > 0,$$

$$\frac{\partial k^{B1^*}}{\partial \bar{k}} = 1 > 0, \frac{\partial q_c^{B1^*}}{\partial \bar{k}} = -\frac{b}{4+c} < 0, \frac{\partial q_r^{B1^*}}{\partial \bar{k}} = \frac{b(4+c-c\theta)}{4(4+c)} > 0,$$

$$\frac{\partial \pi_m^{B1^*}}{\partial \bar{k}} = \frac{\left\{\begin{array}{c} 2abc\big(1-\theta^2\big) - 8(4+c)d\bar{k}(1+\theta) + 8bet(1+\theta)) \\ +(1-\theta)(2b^2\bar{k}(8+c(1-\theta)) + 2bct(1-\theta)(1-\theta^2)e_0 \end{array}\right\}}{8(4+c)(1-\theta^2)} > 0,$$

$$\frac{\partial \pi_s^{B1^*}}{\partial \bar{k}} = \frac{\left\{\begin{array}{c} t^2(1-\theta)^2(1+\theta)(32+8c(1+\theta)+c^2(1+\theta))e_0^2 \\ +2a\big(bc(8+c)\bar{k} - 16et\big)\big(1-\theta^2\big) + a^2(1+\theta)\big(32+8c(1+\theta)+c^2(1+\theta)\big) \\ +(1-\theta)\big(b^2\bar{k}^2\big(32+8c(1-\theta)+c^2(1-\theta)\big) + 32be\bar{k}t(1+\theta) + 16e^2t^2(1+\theta)\big) \\ +2t\big(1-\theta^2\big)\big(\big(bc(8+c)\bar{k} - 16et\big)(1-\theta) + a\big(32+8c(1+\theta)+c^2(1+\theta)\big)\big)e_0 \end{array}\right\}}{16(4+c)^2(1-\theta^2)} > 0.$$

□

Proof of Proposition 8. The parameter e_0 only exists in Model B. Therefore, we analyze its impact on the B1 and B2 scenarios.

Impact on Model B1 equilibrium:

$$\frac{\partial w_c^{B1^*}}{\partial e_0} = -\frac{t(4+c\theta)}{2(4+c)} < 0, \frac{\partial w_r^{B1^*}}{\partial e_0} = -\frac{t(1-\theta^2)}{2(1-\theta^2)} < 0, \frac{\partial p_c^{B1^*}}{\partial e_0} = -\frac{t(4+c\theta)}{4(4+c)} < 0,$$

$$\frac{\partial p_r^{B1^*}}{\partial e_0} = -\frac{t(1-\theta^2)}{4(1-\theta^2)} < 0, \frac{\partial k^{B1^*}}{\partial e_0} = 0, \frac{\partial q_c^{B1^*}}{\partial e_0} = \frac{t-t\theta}{4+c} > 0, \frac{\partial q_r^{B1^*}}{\partial e_0} = \frac{t(1-\theta)(4+c+c\theta)}{4(4+c)} > 0,$$

$$\frac{\partial \pi_m^{B1^*}}{\partial e_0} = \frac{2t(1-\theta^2)((bc\bar{k} - 4et)(1-\theta) + a(8+c+c\theta)) + 2t^2(1-\theta)^2(1+\theta)(8+c+c\theta)e_0}{8(4+c)(1-\theta^2)} > 0,$$

$$\frac{\partial \pi_s^{B1^*}}{\partial e_0} = \frac{\left\{\begin{array}{c} 2t^2(1-\theta)^2(1+\theta)\big(32+8c(1+\theta)+c^2(1+\theta)\big)e_0 \\ +2t(1-\theta^2)((bc(8+c)\bar{k} - 16et)(1-\theta) + a(32+8c(1+\theta)+c^2(1+\theta))) \end{array}\right\}}{16(4+c)^2(1-\theta^2)} > 0.$$

Impact on Model B2 equilibrium:

$$\frac{\partial w_c^{B2^*}}{\partial e_0} = -\frac{t(2d(1+\theta)(4+c\theta) - b^2(4-c+c\theta))}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))} < 0,$$

$$\frac{\partial w_r^{B2^*}}{\partial e_0} = -\frac{t(2(4+c)d(1+\theta) - b^2(4+c(1-\theta)))}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))} < 0,$$

$$\frac{\partial p_c^{B2^*}}{\partial e_0} = -\frac{t(d(1+\theta)(4+c\theta) - b^2(2-c(1-\theta)))}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))} < 0,$$

$$\frac{\partial k^{B2^*}}{\partial e_0} = \frac{bct(1-\theta)(1+\theta)}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))} > 0,$$

$$\frac{\partial q_c^{B2^*}}{\partial e_0} = \frac{2t(1-\theta)(2d(1+\theta) - b^2)}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))} > 0,$$

$$\frac{\partial q_r^{B2^*}}{\partial e_0} = \frac{t(1-\theta)(d(1+\theta)(4+c+c\theta) - 2b^2)}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))} > 0,$$

$$\frac{\partial p_r^{B2^*}}{\partial e_0} = -\frac{t((4+c)d(1+\theta) - b^2(2+c(1-\theta)))}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))},$$

when $c > c_1$ and $d < \frac{b^2(1-\theta)}{1+\theta}$, $\frac{\partial p_r^{B2^*}}{\partial e_0} > 0$, otherwise, $\frac{\partial p_r^{B2^*}}{\partial e_0} \leq 0$. Where

$$c_1 = \frac{2(b^2 - 2d(1+\theta))}{d(1+\theta) - b^2(1-\theta)},$$

$$\frac{\partial \pi_m^{B2^*}}{\partial e_0} = \frac{\begin{array}{c} t(a\big(d(1+\theta)(8+c+c\theta) - 4b^2\big) + t(1-\theta)(2b^2(e-2e_0) \\ +d(1+\theta)(e_0(8+c+c\theta) - 4e))) \end{array}}{4(4+c)d(1+\theta) - b^2(8+c(1-\theta))},$$

when $e_0 < \bar{e_0}$ and $t > t_1, \frac{\partial \pi_m^{B2^*}}{\partial e_0} < 0$, otherwise, $\frac{\partial \pi_m^{B2^*}}{\partial e_0} \geq 0$. Where

$$\bar{e_0} = \frac{2e(b^2 - 2d(1+\theta))}{4b^2 - d(1+\theta)(8+c+c\theta)}, \quad t_1 = \frac{a(d(1+\theta)(8+c+c\theta) - 4b^2)}{(1-\theta)(d(1+\theta)(4e - e_0(8+c+c\theta)) - 2b^2(e-2e_0))}.$$

$$\frac{\partial \pi_s^{B2^*}}{\partial e_0} = \frac{\left\{\begin{array}{c} 2t(et(1-\theta)(b^2d(1+\theta)(16+c+c\theta) - 4b^4 - 16d^2(1+\theta)^2) \\ +a\big(8b^4 - 4b^2d(1+\theta)(8+c+c\theta) + d^2(1+\theta)^2\big(32+8c(1+\theta)+c^2(1+\theta)\big)\big) \\ +te_0(1-\theta)(8b^4 - 4b^2d(1+\theta)(8+c+c\theta) + d^2(1+\theta)^2(32+8c(1+\theta)+c^2(1+\theta)))) \end{array}\right\}}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2},$$

when $e_0 < \bar{\bar{e_0}}$ and $t > t_2$, $\frac{\partial \pi_s^{B2^*}}{\partial e_0} < 0$, otherwise, $\frac{\partial \pi_s^{B2^*}}{\partial e_0} \geq 0$. Where

$$\bar{\bar{e_0}} = \frac{e(4b^4 + 16d^2(1+\theta)^2 - b^2d(1+\theta)(16+c+c\theta))}{8b^4 - 4b^2d(1+\theta)(8+c+c\theta) + d^2(1+\theta)^2(32+8c(1+\theta)+c^2(1+\theta))},$$

$$t_2 = \frac{a(8b^4 - 4b^2d(1+\theta)(8+c+c\theta) + d^2(1+\theta)^2(32+8c(1+\theta)+c^2(1+\theta)))}{\left\{\begin{array}{c} (1-\theta)(4b^4(e-2e_0) + b^2d(1+\theta)(4e_0(8+c+c\theta) - e(16+c+c\theta)) + \\ d^2(1+\theta)^2(16e - e_0(32+8c(1+\theta)+c^2(1+\theta)))) \end{array}\right\}}.$$

□

Proof of Proposition 9. This proposition analyzes the impact ofthe clean energy advertisement input cost factor (d) on the unconstrained scenarios (A2 and B2).

Impact on Model A2 equilibrium:

$$\frac{\partial w_c^{A2^*}}{\partial d} = \frac{2b^2(ac+4et)(1+\theta)(4+c(2+\theta))}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2} > 0,$$

$$\frac{\partial w_r^{A2^*}}{\partial d} = -\frac{2b^2(4+c)(ac+4et)(1+\theta)}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial p_c^{A2^*}}{\partial d} = \frac{b^2(ac+4et)(1+\theta)(12+c(4+\theta))}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2} > 0,$$

$$\frac{\partial p_r^{A2^*}}{\partial d} = -\frac{3b^2(4+c)(ac+4et)(1+\theta)}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial k^{A2^*}}{\partial d} = -\frac{4b(4+c)(ac+4et)(1+\theta)^2}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial (q_c^{A2^*} + q_r^{A2^*})}{\partial d} = -\frac{b^2c(ac+4et)(1-\theta)(1+\theta)^2}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial q_c^{A2^*}}{\partial d} = \frac{4b^2(ac+4et)(1+\theta)^2}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2} > 0$$

$$\frac{\partial q_r^{A2^*}}{\partial d} = -\frac{b^2(ac+4et)(4+c(1-\theta))(1+\theta)^2}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial \pi_m^{A2^*}}{\partial d} = -\frac{b^2(ac+4et)^2(1+\theta)^2}{2(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial \pi_s^{A2^*}}{\partial d} = -\frac{\begin{array}{c} 2b^2(ac+4et)(1+\theta)^2(b^2cet(1-\theta) - 2ab^2c \\ +ac(8+c)d(1+\theta) + 16det(1+\theta)) \end{array}}{(4(4+c)d(1+\theta) - b^2(8+c(1-\theta)))^3} < 0.$$

Impact on Model B2 equilibrium:

$$\frac{\partial w_c^{B2^*}}{\partial d} = \frac{2b^2(1+\theta)(4+c(2+\theta))(ac+4et+ct(1-\theta)e_0)}{(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2} > 0,$$

$$\frac{\partial w_r^{B2^*}}{\partial d} = -\frac{2b^2(4+c)(1+\theta)(ac+4et+ct(1-\theta)e_0)}{(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial p_c^{B2^*}}{\partial d} = \frac{b^2(1+\theta)(12+c(4+\theta))(ac+4et+ct(1-\theta)e_0)}{(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2} > 0,$$

$$\frac{\partial p_r^{B2^*}}{\partial d} = -\frac{3b^2(4+c)(1+\theta)(ac+4et+ct(1-\theta)e_0)}{(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial k^{B2^*}}{\partial d} = -\frac{4b(4+c)(1+\theta)^2(ac+4et+ct(1-\theta)e_0)}{(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial q_c^{B2^*}}{\partial d} = \frac{4b^2(1+\theta)^2(ac+4et+ct(1-\theta)e_0)}{(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2} > 0,$$

$$\frac{\partial q_r^{B2^*}}{\partial d} = -\frac{b^2(4+c(1-\theta))(1+\theta)^2(ac+4et+ct(1-\theta)e_0)}{(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial(q_c^{B2^*}+q_r^{B2^*})}{\partial d} = -\frac{b^2c(1-\theta)(1+\theta)^2(ac+4et+ct(1-\theta)e_0)}{(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial \pi_m^{B2^*}}{\partial d} = -\frac{b^2(1+\theta)^2(ac+4et+ct(1-\theta)e_0)^2}{2(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2} < 0,$$

$$\frac{\partial \pi_m^{B2^*}}{\partial d} = -\frac{b^2(1+\theta)^2(ac+4et+ct(1-\theta)e_0)^2}{2(4(4+c)d(1+\theta)-b^2(8+c(1-\theta)))^2} < 0.$$

□