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JOINT OPTIMIZATION OF INVENTORY REPLENISHMENT AND TRANSPORTATION DECISIONS: MODELS AND SOLUTION ALGORITHMS

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Abstract. The joint optimization of transportation and inventory replenishment related decisions promises to yield significant cost savings coupled with higher customer satisfaction levels. This paper investigates a supply chain system consisting of a single supplier replenishing a single retailer, where the primary focus is on the retailer's decision-making process, aimed at determining the most efficient operational policy. This includes identifying the optimal replenishment quantity from the supplier and selecting the appropriate mix and size of the truck fleet under different situations. At first, the scenario whereby the retailer exclusively operates its limited fleet of trucks for inbound transportation is considered. An efficient solution procedure along with closed form expressions for the optimal ordering quantity and the number of trucks are devised. Subsequently, the problem is extended to incorporate environmental considerations under carbon tax and carbon cap policies. We propose a computationally efficient algorithm for generating the optimal operational policy following the carbon cap policy. Finally, to better resemble reality, the scope of the operational optimization model is extended *via* allowing the retailer the option to lease trucks from the external market. The conducted numerical experiments demonstrate that this flexibility can lead to significant cost reductions that are increasing with demand.

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1. Introduction

Inventory replenishment decisions are of vital importance for various supply chain (SC) operations. They encompass the determination of order quantities, as well as the timing and frequency of orders to maintain the right balance between cost, efficiency and customer responsiveness. In the logistics sector, transportation plays a pivotal role in ensuring the efficient movement of goods among the different SC stakeholders. In practical terms, inventory and transportation decisions are closely interconnected, and as pointed out by Li and Hai [1], making inventory and transportation decisions in isolation of each other may lead to suboptimal outcomes. Carter and Ferrin [2] noted that explicitly accounting for transportation costs greatly influences the optimum order lot sizing decisions. Therefore, when making transportation decisions pertaining to the sizes and types of trucks in the fleet, it is particularly important to integrate them into the inventory replenishment decision-making process. This integration is crucial for attaining the most efficient operational policy across the whole SC. To that end, this paper presents four optimization models, where all such models are based on the integration

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of inventory and transportation related decisions. A thorough literature review of mathematical models that jointly address transportation and inventory decisions for various SC configurations is provided in the work of Mosca *et al.* [3].

In reality, companies have various options for transporting their products, including exclusively relying on their own fleet of trucks, incorporating leased trucks into their operations, or entirely outsourcing their transportation operations to third-party logistics providers. It may also prove economically beneficial to transport a portion of the shipments using the company's own truck fleet while leasing additional trucks when the capacity of owned trucks is limited. Each of these transportation options has its merits, but integrating them with inventory replenishment decisions can enhance the operational efficiencies of the companies. One of the primary advantages of leasing trucks is the immediate cost savings realized when dealing with large replenishment quantities. Leasing also provides SC members with access to additional trucks without the substantial initial capital expenditure required to acquire new trucks. Vasiliauskas and Jakubauskas [4] indicated that outsourcing transportation activities offers benefits related to economies of scale, capital investment savings, and reduced financial risks. Li *et al.* [5] also mentioned that outsourcing transportation operations can enhance efficiency, achieve cost control, reliability, and speed. In this paper, we investigate the impact of leasing trucks from the market to transport a portion of the order quantity, whilst being integrated with inventory replenishment decisions.

Focusing solely on the potential operational and economic advantages associated with the integration of inventory and transportation decisions may seem attractive enough for SC members to adopt such an initiative. However, considering that storage and transportation activities are the primary sources of carbon emissions, it is essential to examine their environmental implications alongside the economic ones Indeed, SC activities including replenishment, production, transportation, and warehousing account for an average of 75% of carbon emissions within the industrial sector [6]. Furthermore, more than 90% of carbon emissions are attributed to the operations of SCs [7]. Consequently, SC members are facing increasing pressure to embrace sustainable practices and consistently reevaluate their SC operations to adhere to environmental regulations [8].

Several regulatory policies aimed at curbing carbon emissions have been put in place, including carbon tax, carbon cap, carbon offset, cap-and-trade mechanisms, and the adoption of eco-friendly technology practices. Under the carbon tax policy, a company is charged a fixed tax rate for every ton of carbon it emits. Conversely, the quantity-based carbon cap policy sets a limit on the allowed maximum carbon footprint. Such stringent environmental regulations are reshaping the traditional reliance on purely economic measures when optimizing SC operations. As such, this study introduces integrated economic and environmental models for the joint inventory replenishment and transportation problem. Given their proven effectiveness and current widespread acceptance, this work adopts carbon cap and carbon tax policies in the development of these integrated economic and environmental models.

The remaining sections of this paper are structured as follows. In the next section, we review relevant literature pertaining to the problems addressed in this paper. In the third section, we formulate the problem of jointly optimizing inventory and transportation decisions, which was originally introduced by Li and Hai [1], and derive closed form expressions for the optimal lot size and number of trucks. Sections 4 and 5 respectively present integrated economic and environmental models under carbon tax and carbon cap policies. Section 6 expands upon the model developed in the third section to include the option of leasing trucks from the external market. Finally, the last section concludes the paper and highlights potential avenues for future research.

2. Literature review

The economic ordering quantity (EOQ) model stands out as being the most popular deterministic inventory model. It assumes a per unit transportation cost, often referred to as the less-than-truckload (LTL) transportation mode, which is factored into the purchasing cost. In the context of LTL, it is assumed that a single truck has a sufficiently large capacity to accommodate any ordering quantity. Another transportation mode worth noting is the full truck load (FTL), where a fixed fee is charged per truck and is commonly used for transporting large quantities of goods. For a comprehensive examination of various transportation cost forms, one can refer to the review paper by Engebrethsen and Dauzère-Pérès [9].

Extensive research in the SC literature has explored the implications of incorporating transportation costs into inventory models. Several authors have emphasized the importance of explicitly incorporating transportation costs into inventory models, as they have an inevitable impact on overall operational costs and lot sizes. In their inventory model utilizing FTL transportation mode, Yıldırmaz *et al.* [10] concluded that disregarding transportation costs results in an average profit decrease of 2.25% and, in certain extreme cases, even up to 60%. Mendoza and Ventura [11] also conducted an analysis on the consequences of neglecting transportation costs and found an average monthly logistics cost increase of 14.7% and an 88.9% rise in transportation costs.

Inventory models considering FTL transportation costs were first studied by Lippman [12, 13], Iwaniec [14], and Aucamp [15]. Adelwahab and Sargious [16] and Swenseth and Godfrey [17] treated the integration of FTL and LTL transportation mode with inventory decisions as distinct problems and compared their cost performance. Rieksts and Ventura [18] developed a single-stage inventory model with two simultaneous modes of transportation, namely FTL and LTL. These studies integrated the per-truck cost into the combined inventory and transportation models while considering truck capacities.

Other related works include Zhao *et al.* [19] who presented a modified version of the EOQ model including both variable (LTL) and fixed (FTL) transportation costs. Inspired by a practical application, the model considers multiple uses of the vehicles with a cap on the maximum daily trips for each truck. For a company distributing coal to four of its subsidiaries, Zhao *et al.* [20] jointly optimized inventory and transportation decisions while utilizing self-owned or river-hired vessels. Both variable and fixed transportation costs are considered in the context of the Vendor Managed Inventory (VMI) concept, where a Markov Decision Process is adopted to formulate the ordering and delivery problems. In another work, Berman and Wang [21] developed a nonlinear model seeking to jointly minimize inventory and transportation costs for a two stage SC comprising a family of products that are shipped from a set of suppliers to a set of plants. The authors emphasized the importance of considering these two costs simultaneously and presented a heuristic as well as a branch-and-bound solution algorithm. Based on the EOQ model, Baboli *et al.* [22] proposed an algorithm for the determination of the optimal ordering policy for a two stage warehouse-retailer SC system, where the transportation cost from the warehouse to the retailer is accounted for. The authors considered the centralized case, where the two parties collaborate to minimize the chain wide total cost, and the decentralized case with each party acting independently to minimize their own costs. For a SC system comprising one producer and multiple retailers, Madadi *et al.* [23] explicitly considered the cost of transportation while setting the optimal inventory levels, and the optimal order and production lot sizes. Utilizing the FTL transportation mode, Ali and O'Connor [24] pointed out the computational intractability for the two-stage multi-period distribution system, and alternatively devised an efficient heuristic solution procedure capable of attaining near optimal solutions. The proposed approach establishes first the deployment schedule of the trucks to the downstream party followed by the resulting inventory levels. In lieu of minimizing the transportation cost, Grunewald *et al.* [25] devised a multi-period model that seeks to minimize the number of trucks used while taking into account inventory levels and limited trucks capacities. The model is developed for the case of single-sourced multiproducts having distinct dimensions with the demand varying deterministically from one period to the next.

It is noted that a wide range of transportation challenges can impact SC's effectiveness and efficiency, including route optimization, fleet size problem, and feet size and mix problem [26]. The route optimization problem entails finding the most efficient routes for transporting goods considering factors such as distance travelled, traffic congestion, and the types of vehicles used. Determining the appropriate number of vehicles for different shipments is a key concern in the feet size problem. The selection of truck types in addition to the fleet size is commonly referred to in the literature as the fleet mix and size problem. Each of the previously mentioned challenges has been thoroughly examined in the transportation literature [1]. Li and Hai [1] studied a joint inventory and transportation problem involving a truck capacity constraint. Their developed nonlinear mixed integer model simultaneously determines the reorder interval of the retailer and the number of trucks while minimizing the inventory replenishment cost and carbon emission cost. They devised an algorithm that solves

M sub-problems to determine the optimal operating policy, where M is the number of available trucks owned by the retailer.

Given that these SC activities are among the primary sources of carbon emissions, another inventory and transportation challenge pertains to reducing carbon footprint *via* embracing sustainable inventory and transportation practices The integration of sustainability considerations into SC modeling has experienced a significant upswing over the past two decades. The latest research on sustainable SC suggests that substantial reductions in carbon emissions can be achieved by improving the coordination of lot sizing and shipping decisions among SC members. However, most of these studies primarily focus on inventory decisions and do not consider the impact of transportation decisions. One pioneering work in this area is that of Chen *et al.* [27], who applied four carbon policies (carbon tax, carbon cap, carbon offset, and cap-and-trade) in the context of the EOQ model. Recent review papers on sustainable inventory models that incorporate carbon emissions considerations include the works of Das and Jharkharia [28], Shaharudin *et al.* [29], Chelly *et al.* [30], and Zhou *et al.* [6]. Particularly, Zhou *et al.* [6] focused their review on the carbon tax policy.

Compared to the above reviewed papers, this work introduces three noteworthy contributions. Firstly, the classical EOQ-based model is revisited to jointly optimize inventory and transportation decisions under the realistic setting of limited truck capacity, allowing for the extension of earlier works that merely relied on iterative solution algorithms. The closed-form expressions derived in this study stand out as a theoretical advancement as they significantly reduce computational complexity while providing structural insights into the optimal replenishment policy. Second, the authors explicitly model both fixed and variable transportation costs, with the variable component directly linked to more precise fuel consumption calculations. This goes beyond simplified cost representations commonly used in the literature and allows for a more accurate reflection of operational realities. Third, environmental concerns are accounted for *via* incorporating carbon tax and carbon cap regulatory policies into the optimization framework. To the authors' best knowledge, the closed-form characterization of optimal solutions under such environmental constraints has not been previously established in the relevant literature. Collectively, these contributions advance the theoretical foundation of integrated inventory-transportation models while providing practical tools for decision makers.

3. Economic model and a new solution procedure

In this section, we formulate the economic model for the joint inventory and transport capacity problem and introduce a new efficient solution procedure. The underlying assumptions of the model are basically identical to those of the EOQ model, except for the transportation cost structure. While the assumption of static demand may not be well-suited for a wide spectrum of real-life applications, there exist several situations where deterministic or near-deterministic demand is observed. Typically, large supermarket chains encounter stable demand for essential food products that constitute an inherent part of households' daily consumption. For instance, the weekly or monthly demand for items on the likes of rice, flour, sugar and cooking oil is generally predictable, with only minor variations over time, owing to the consistent dietary habits of consumers within the surrounding community. Bottled water consumption is another example where demand is steady and predictable, especially in countries with hot climate conditions. Hotels, restaurants, hospitals, and institutional buyers often place repetitive orders in almost identical quantities week after week. For such products, retailers can rely on deterministic demand models for planning inbound replenishment shipments and allocating truck fleets in advance. In such cases, minor variations in demand can be managed using average demand values to operationalize the models proposed in this work.

Although long-term demand averages remain stable for the examples discussed above, a key challenge lies in the anticipated short-term variability, as daily or weekly fluctuations may still arise due to promotions, marketing campaigns, unexpected orders, or weather-related factors. Another challenge concerns coordination with suppliers and logistics service providers. Even when demand is relatively stable, scheduling related constraints such as truck availability, driver shortages, or fuel price volatility can compromise the applicability of static models. Market uncertainty and competitors' actions also represent important challenges to be addressed. Even in otherwise stable product categories, sudden shifts in market conditions due to new entrants, aggressive price promotions by competitors, or regulatory interventions can significantly disrupt demand patterns. Nevertheless, despite these multiple challenges, the closed-form formulations developed hereafter still offer decision makers simple yet powerful guidelines that can serve as a benchmark or foundation for developing more robust planning tools adaptable to dynamic environments.

To formally define the problem, consider a retailer ordering a single product, facing a deterministic demand of D units per unit of time, from a supplier. The retailer incurs an ordering cost (A) each time it places an order of size Q units with the supplier. Additionally, there is an inventory holding cost (h) charged to the retailer per unit per unit of time. The retailer owns a fleet of M homogeneous trucks, each with a loading capacity of U units, to transport the ordered quantity from the supplier. In addition to the ordering and inventory holding costs, the retailer bears the transportation cost associated with utilizing its fleet of trucks. This transportation cost includes fixed costs (C_o^f), which are independent of the shipment quantity and involve factors such as truck depreciation, insurance, financing costs, maintenance and repairs, and driver wages. Furthermore, there are variable transportation costs (C_o^v), which are contingent on the quantity being transported. These costs are primarily influenced by various factors such as fuel consumption when the truck is at full capacity, fuel consumption when the truck is empty, the number of fully and partially loaded trucks, the quantity loaded on each truck, the distance covered, and the driving speed.

The lot size Q is shipped using $(m-1)$ fully loaded trucks and one partially filled truck, carrying $[Q-(m-1)U]$ units. Assuming a linear relationship between fuel consumption and truckload, the fuel consumption required to transport Q unit, can be written as:

$$L\left\{(m-1)\mathrm{FC}_f + \mathrm{FC}_e + \left(\frac{\mathrm{FC}_f - \mathrm{FC}_e}{U}\right)(Q-(m-1)U)\right\} + L\,m\,\mathrm{FC}_e \tag{1}$$

where:
L is the distance between the retailer and supplier in miles,
FC_e is the fuel consumption of an empty truck in gallons per mile,
FC_f is the fuel consumption of a full truck in gallons per mile.

The first term within the curly braces in equation (1) represents the fuel consumption for the $(m-1)$ full trucks. The subsequent two terms are derived from the assumption of a linear relationship between the fuel consumption and the quantity loaded onto the truck, as shown in Figure 1. The last term is the amount of fuel consumed by the empty m trucks on tier way back from the retailer to the supplier.

Denoting the fuel price per gallon as p, the fuel consumption cost is then expressed as follows:

$$pL\left(\frac{\mathrm{FC}_f - \mathrm{FC}_e}{U}\right)Q + 2\,pL\,\mathrm{FC}_e\,m. \tag{2}$$

As can be observed from equation (2), the fuel consumption cost is expressed as the sum of a variable cost and a fixed cost. Assuming that the driver's wage represents the major fixed cost component in comparison to other fixed cost elements (see, for instance, [31]), the variable and fixed transportation costs can be expressed as follows:

$$C_o^v = pL\left(\frac{\mathrm{FC}_f - \mathrm{FC}_e}{U}\right) \tag{3}$$

$$C_o^f = 2\,pL\,\mathrm{FC}_e + \mathrm{DW}\frac{2L}{v} \tag{4}$$

where DW is the driver wage in $ per hour and v is the truck speed in miles per hour.

The total transportation cost is then:

$$C_o^v\,Q + C_o^f m. \tag{5}$$

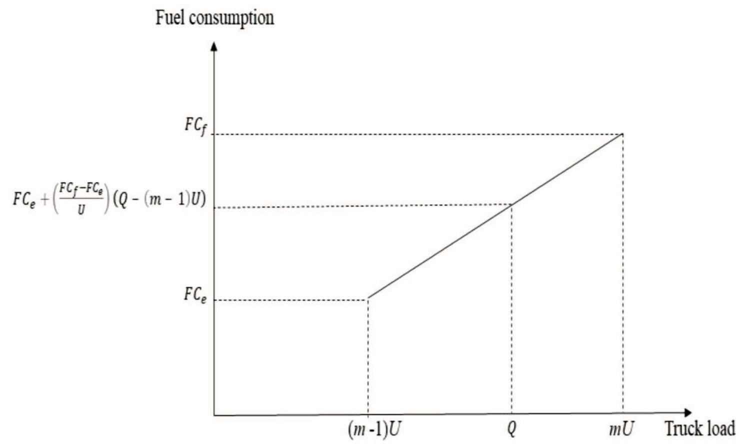

Figure 1. Derivation of fuel consumption as a function of the truck load.

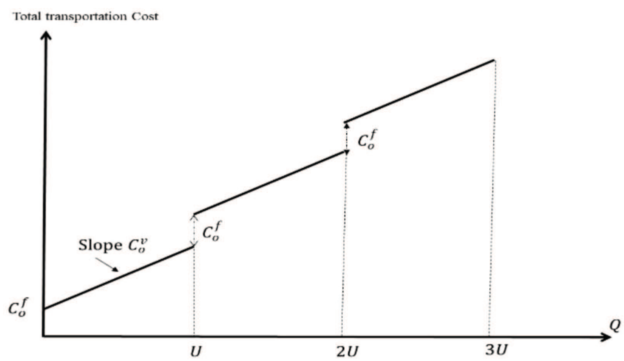

Figure 2. Staircase functional form of the total transportation cost.

Such cost function exhibits a distinctive staircase pattern, as illustrated in Figure 2. This functional form is commonly referred to in the literature as full truckload, piecewise, or multiple setup cost structure.

The retailer needs to jointly determine both the ordering quantity and the number of trucks required to transport the lot size Q, with the aim of minimizing its overall operational costs, $K(Q, m)$, which encompass ordering, inventory holding, and transportation costs. Therefore, the economic model for the joint inventory and transportation problem can be written as:

$$\text{Min } K_{ec}(Q,m) = AD/Q + 0.5hQ + D\big(C_o^v Q + C_o^f m\big)/Q \quad (6)$$

Subject to:

$$(m-1)U < Q \leq mU \quad (7)$$

$$m \leq M \quad (8)$$

m is integer.

Constraint (7) guarantees that the loading capacity of the m trucks is sufficient to transport the ordered quantity Q. Constraint (8) further ensures that the number of trucks utilized for transporting the lot size is always less or equal to the available number of owned trucks by the retailer.

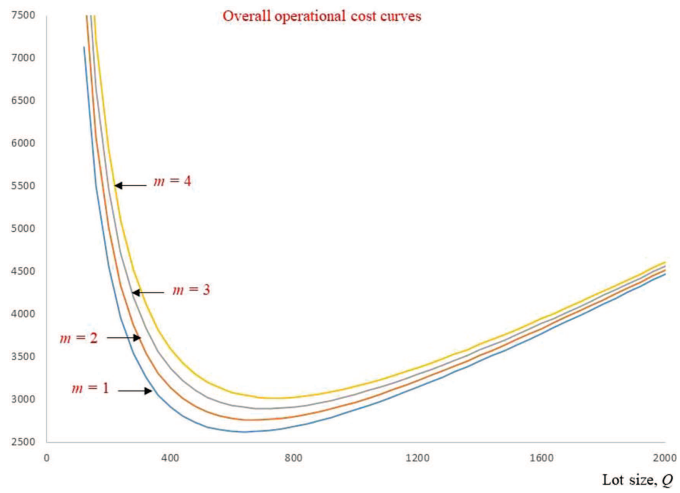

FIGURE 3. Overall operational costs for different m values.

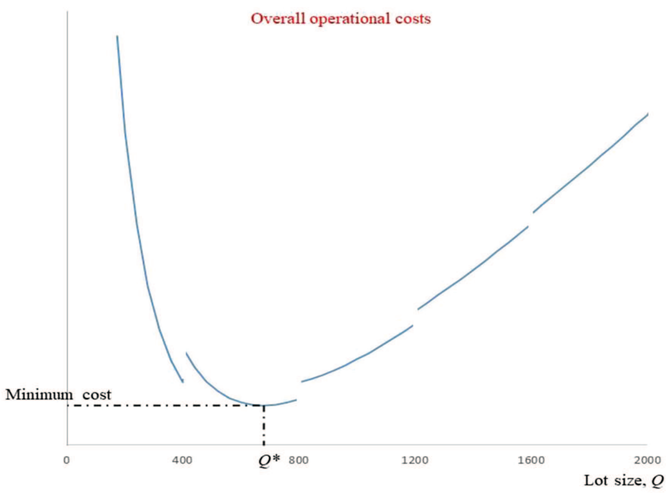

FIGURE 4. Variation of the overall operational costs as a function of the lot size.

As shown in Figure 4, the curve representing the overall operational costs is discontinuous at the truck loading capacities because of the staircase structure inherent in the total transportation cost. For a fixed m, the overall operational cost can be written as:

$$K_{ec}(Q, m) = D\frac{A + C_o^f m}{Q} + 0.5\, hQ + \mathrm{DC}_o^v. \tag{9}$$

For any positive Q, we have $K_{ec}(Q, m+1) - K_{ec}(Q, m) = \frac{DC_o^f}{Q} > 0$. Therefore, the curves for two consecutive m values do not intersect, as illustrated in Figure 3. Additionally, it is essential to note that only a segment of the curve $K_{ec}(Q, m)$ is feasible because of constraint (7). The feasible portions of each curve for different m values are visually represented in Figure 4.

The shipment size minimizing the overall operational costs in (9) for a given m is given by:

$$Q_{ec}(m) = \sqrt{\frac{2\left(A + C_o^f m\right)D}{h}}, \tag{10}$$

which can be rewritten as:

$$Q_{ec}(m) = Q_{eoq}\sqrt{1 + \frac{C_o^f}{A}m}, \tag{11}$$

where $Q_{eoq} = \sqrt{2AD/h}$.

Considering that the cost functions $K_{ec}(Q, m)$'s are non-intersecting convex functions, it is not difficult to demonstrate that if $Q_{ec}(m)$ is feasible, *i.e.* $(m-1)U < Q_{ec}(m) \leq mU$, then $K_{ec}(Q, m') > K_{ec}(Q_{ec}(m), m)$ for $m' > m$ and $Q > Q_{ec}(m)$ Nevertheless, there is a possibility that $K_{ec}(Q_{ec}(m), m)$ may exceed $K(Q, m')$ for $m' < m$ and $Q < Q_{ec}(m)$. Due to the convexity of $K_{ec}(Q, m)$ in (9), the shipment sizes that can result in overall operational costs lower than $K_{ec}(Q_{ec}(m), m)$ are the truck loading capacities, $(m'U, m' = 1, 2, m-1)$.

Using an objective function that includes a tax-based emission cost, Li and Hai [1] solved a similar problem, which they referred to as "joint inventory and transport capacity with carbon emission". In their solution approach, they employed an iterative algorithm that requires solving M sub-problems to ensure the attainment of the optimal solution. However, in the following theorem it is shown that the ordering quantity and the number of trucks can be determined using closed form expressions.

Theorem 1. (a) *$Q_{ec}(m)$ is feasible when the number of trucks is given by:*

$$m_{feas} = \left\lceil \frac{\alpha + \sqrt{\alpha^2 + 4\beta^2}}{2\beta^2} \right\rceil \tag{12}$$

where:
$\alpha = \frac{C_o^f}{A}$, $\beta = \frac{U}{Q_{eoq}}$, *and* $\lceil x \rceil$ *is the smallest integer larger than or equal to* x.

(b) *The number of FTL trucks with the least overall operational costs is:*

$$m_{\mathrm{FTL}} = \left\lceil 0.5\left(-1 + \sqrt{(1 + 4/\beta^2)}\right)\right\rceil. \tag{13}$$

(c) *The optimal number of trucks, m_{ec}, and ordering quantity, Q_{ec}, are given by:*

(1) *If $m_{\mathrm{FTL}} > M$, then*

$$m_{ec} = M \quad \text{and} \quad Q_{ec} = MU.$$

(2) *If $m_{feas} > M$ and $m_{\mathrm{FTL}} \leq M$, then*

$$m_{ec} = m_{\mathrm{FTL}} \quad \text{and} \quad Q_{ec} = m_{\mathrm{FTL}}U.$$

(3) *If $m_{feas} \leq M$, then*

$$\begin{aligned} &m_{ec} = m_{\mathrm{FTL}} \quad \text{and} \quad Q_{ec} = m_{\mathrm{FTL}}U \qquad \text{if} \quad K_{ec}(m_{\mathrm{FTL}}U, m_{\mathrm{FTL}}) < K_{ec}(Q_{ec}(m_{feas})m_{feas}), \quad \text{or} \\ &m_{ec} = m_{feas} \quad \text{and} \quad Q_{ec} = Q_{ec}(m_{feas}) \qquad \text{if} \quad K_{ec}(m_{\mathrm{FTL}}U, m_{\mathrm{FTL}}) > K_{ec}(Q_{ec}(m_{feas})m_{feas}), \end{aligned}$$

Proof. (a) First, note that when $Q_{ec}(m-1)$ is not feasible, $Q_{ec}(m-1) > (m-1)U$, then $Q_{ec}(m) > (m-1)U$. Therefore, the two inequalities in (7) reduce to $Q \leq mU$. Next, $Q_{ec}(m)$ is feasible when $Q_{ec}(m) \leq m$. Using (11), we have

$$Q_{eoq}\sqrt{1 + \frac{C_o^f}{A}m} \leq m, \quad \text{or}$$

$\beta^2 m - \alpha m - 1 \geq 0$, which holds true when:
$m \geq 0.5(-1 + \sqrt{1 + \frac{4}{\beta^2}})$. Given that m is integer, $Q_{ec}(m)$ is feasible when $= m_{feas}$.

(b) Given the convexity of $K_{ec}(Q, m)$ in Q, then for all $m \leq m_{feas}$, $Q_{ec}(m) = mU$. The overall operational cost at this full load quantities is:

$$K_{ec}(Um, m) = \frac{AD}{Um} + 0.5hUm + \left(\frac{C_o^f}{U} + C_o^v\right)D.$$

Using the first difference approach, the optimal number of trucks minimizing $K_{ec}(Um, m)$, is the first integer satisfying:

$$K_{ec}(Um, m) \leq K_{ec}(U(m+1), m+1) \text{ and}$$
$$K_{ec}(Um, m) \leq K_{ec}(U(m-1), m-1).$$

Substituting the expressions of the three cost functions in the last two inequalities and simplifying terms, we get

$$m(m-1) \leq \frac{2AD}{hU} \leq m(m+1).$$

It is easy to show that these two inequalities can be transformed into the closed-form expression, m_{FTL}, given in (13).

(c1) If $m_{\mathrm{FTL}} > M$, then $m_{ec} = M$ since $K_{ec}(Um, m)$, is decreasing with m for $m = 1, 2, \ldots, m_{\mathrm{FTL}}$.

(c2) If $m_{feas} > M$ and $m_{\mathrm{FTL}} \leq M$, then $m_{ec} = m_{\mathrm{FTL}}$ by the definition of m_{FTL}.

(c3) If $m_{feas} \leq M$, then it is necessary to compare the costs when ordering $Q_{ec}(m_{feas})$ and $m_{\mathrm{FTL}}U$ units (see Fig. 4). Therefore, the optimal ordering quantity is the quantity with the lower costs among these two options.

□

The following corollary provides an important structural property of the optimal solution.

Corollary. (a) *The retailer will transport the ordering quantity using FTL trucks in each of the following three cases:*

(1) $m_{\mathrm{FTL}} > M$,

(2) $m_{feas} > M$ *and* $m_{\mathrm{FTL}} \leq M$, *and*

(3) $m_{feas} \leq M$ *and* $K_{ec}(m_{\mathrm{FTL}}U, m_{\mathrm{FTL}}) < K_{ec}(Q_{ec}(m), m)$.

(b) *The retailer will ship the order quantity using* $(m_{feas} - 1)$ *FTL trucks and one LTL truck when:*

$$m_{feas} \leq M \quad \text{and} \quad K_{ec}(m_{\mathrm{FTL}}U, m_{\mathrm{FTL}}) > K_{ec}(Q_{ec}(m_{feas}), m_{feas}).$$

Proof. The proof follows as a consequence of the results of Theorem 1. □

As shown in Theorem 1, the optimal order quantity and the required number of trucks can be determined through closed-form expressions, depending on how the values of m_{feas} and m_{FTL} compare to the available number of trucks. Therefore, determining the optimal operational policy requires three comparison operations, as opposed to Li and Hai's algorithm, which exhibits a time complexity of $O(M)$.

Illustrative example

A retailer is packaging one of its best-selling products in stackable plastic crates that measure 24 inches in length, 16 inches in width, and 13 inches in height. The retailer operates its own fleet of 10 box trucks, each measuring $20' \times 8' \times 8'$, for transporting the product from the supplier to its premises. Considering the dimensions of both the crates and trucks, each truck has the capacity to accommodate approximately 400 crates. Hereafter, we assume that a unit of the product corresponds to a packaging unit, namely the crate with dimension $24'' \times 16'' \times 13''$. The demand rate for the product is equivalent to 5000 crates per year. Storing one full crate for one year costs the retailer \$2. Moreover, each order placed with the supplier results in a fixed

TABLE 1. Impact of EOQ parameters on shipment policy.

h	D	A = 50	100	250	500
2	1000	LTL	FTL	FTL	FTL
	2000	FTL	FTL	FTL	FTL
	5000	FTL	FTL	FTL	FTL
	10 000	FTL	FTL	FTL	FTL
5	1000	LTL	LTL	LTL	LTL
	2000	LTL	LTL	FTL	FTL
	5000	FTL	FTL	FTL	FTL
	10 000	FTL	FTL	FTL	FTL
10	1000	LTL	LTL	LTL	LTL
	2000	LTL	LTL	LTL	FTL
	5000	LTL	FTL	FTL	FTL
	10 000	FTL	FTL	FTL	FTL

cost of \$250 for the retailer. The fuel consumption for an empty and a fully loaded trucks is 0.072 and 0.092 gallons per mile, respectively [32]. The product is shipped over a distance of 100 miles between the supplier and retailer. The cost per gallon of the diesel fuel used by the truck is \$4.00, and the driver's wage is assumed to be the same across all trucks which is estimated at \$5 per hour. The average speed of the truck is 60 miles/hour.

After inputting the truck related data into equations (3) and (4), the fixed and variable transportation costs are \$84.43 per truck and \$0.02 per unit, respectively. Next, using Theorem 1, we get $m_{feas} = 5$ and $m_{\mathrm{FTL}} = 3$. Given that $m_{feas} < 10$, we need to compute the costs at $(m_{\mathrm{FTL}}U,\ m_{\mathrm{FTL}})$ and $(Q_{ec}(m_{feas}), m_{feas})$, where $Q_{ec}(m_{feas}) = 1833.2 < 2000$. The optimal operational policy is $Q_{ec} = 1200$, and $m_{ec} = 3$ since $K_{ec}(1833.2, 5) = 3766.5 > K_{ec}(1200, 3) = 3397.0$. Therefore, the optimal replenishment policy is to ship 1200 crates in three FTL trucks.

When the truck capacity is increased to 750 crates, the retailer must adopt a mixed shipment policy comprising one FTL truck and one LTL truck. In this case, the optimal order quantity is 1447 crates, shipped using two trucks with the second truck partially loaded. Furthermore, if the ordering cost and the demand rate are reduced to \$50 and 1000 units/year, respectively, the optimal policy is to ship 367 crates using a single LTL truck. By contrast, the numerical experiments presented below reveal that under high ordering costs and demand rates combined with low holding costs, the optimal policy utilizes solely FTL trucks.

The purpose of the following numerical experiments is to evaluate the impact of EOQ based parameters, namely ordering cost, holding cost, and demand rate, on the optimal mix of trucks to be used for shipment. To this end, we tested multiple values for each parameter. The holding cost was set at 2, 5, and 10; the ordering cost at 50, 100, 250, and 500; and the annual demand rate at 1000, 2000, 5000, and 10 000 units. These values were chosen to represent low, medium, and high levels for each parameter. Table 1 depicts the results of the numerical experiments conducted. The findings indicate that when ordering costs and demand rates are high, and holding costs are low, the optimal policy consistently relies exclusively on FTL trucks. This outcome is expected since high ordering costs and high demand rates naturally encourage larger order quantities in order to reduce the frequency of replenishments. At the same time, low holding costs make it economically feasible to retain higher levels of inventory. Together, these conditions drive the model toward shipment policies that favor larger lot sizes, which in turn are best accommodated through FTL shipments. Conversely, when holding costs increase or demand is reduced, the relative advantage of FTL shipments diminishes, and mixed policies that include LTL options may become more economically attractive. Thus, these experiments provide practical insights for managers by showing how variations in EOQ-related parameters can guide the choice of the most cost-effective shipment policy.

4. Integrated economic and environmental model under carbon tax policy

Before developing the integrated economic and environmental model under carbon tax regulation policy, we first establish a lower bound for the total carbon emissions generated by the inventory holding and transportation activities. To that end, we make use of the following environmental parameters:
$\hat{h}$: amount of carbon emitted while storing one unit of the product for one unit of time,
$\hat{C}_o^f = 2\mathrm{FE}\, L\, \mathrm{FC}_e$, fixed amount of carbon generated by one truck, where FE represents the carbon emissions released by the consumption of one gallon of fuel [1],
$\hat{C}_o^v = \mathrm{FE}\, L \left(\frac{\mathrm{FC}_f - \mathrm{FC}_e}{U}\right)$, carbon emissions released by each unit transported in the truck.

Using these carbon related parameters, the total carbon emissions are expressed as:

$$E(Q,m) = 0.5\,\hat{h}Q + D\frac{\hat{C}_o^v\, Q + \hat{C}_o^f m}{Q}\cdot \tag{14}$$

The joint inventory and transportation capacity problem with only environmental considerations can then be stated as:

$$E_{ev}^{\min} = \mathrm{Min}\left\{E(Q,m) = 0.5\hat{h}Q + \frac{D\hat{C}_o^f m}{Q} + D\hat{C}_o^v\right\} \tag{15}$$

Subject to: (7) and (8).

The solution to the environmental optimization problem establishes a lower bound for the carbon emissions as it represents the minimum amount of carbon that can be released by the inventory holding and transportation activities. Given the functional form of $E(Q,m)$, the solution procedure of the economic model, presented in Theorem 1, can be applied to find $E_{ev}^{\min}$.

The joint economic and environmental model under carbon tax regulation policy is a straightforward extension to the economic model. Specifically, by denoting tx as the tax rate to be charged for each ton of carbon released by the retailer, the objective function of the joint economic and environmental model under carbon tax policy is:

$$\begin{aligned} K_{tx}(Q,m) &= K_{ec}(Q,m) + tx\, E(Q,m) \\ &= \frac{AD}{Q} + 0.5\, hQ + \frac{DC_o^f m}{Q} + DC_o^v \\ &\quad + tx\left(0.5\hat{h}Q + \frac{D\hat{C}_o^f m}{Q} + D\hat{C}_o^v\right) \end{aligned}$$

which can be rewritten as:

$$K_{tx}(Q,m) = \frac{AD}{Q} + 0.5\left(h + tx\hat{h}\right)Q + \frac{D}{Q}\left(C_o^f + tx\hat{C}_o^f\right)m + D\left(C_o^v + tx\hat{C}_o^v\right). \tag{16}$$

The objective function in equation (16) shares the same functional form as the one in equation (9), though with distinct coefficients. Consequently, the solution procedure outlined in Theorem 1 to solve the economic model can also be applied to generate the optimal solution (K_{tx}, E_{tx}, Q_{tx}, and m_{tx}) for the joint economic and environmental model under a carbon tax policy.

Illustrative example

The carbon emission level released by one gallon of fuel depends on the type of fuel in use. For instance, the $CO2$ emissions per gallon is approximately 8.8 kg for gasoline and 10.1 kg for diesel [32]. As heavy-duty trucks commonly run on diesel, FE is thereby set at 0.0101 tons per gallon. Under the carbon tax policy, the imposed tax is determined on a per-ton basis of carbon emissions, with its magnitude significantly influenced by the rigor of government regulations on CO2 emissions. Such regulations vary widely from one country to another. For instance, in the United Kingdom, the tax is set at \$15.75 per ton of emitted CO2, while in Switzerland, it is \$68 per ton, and in Sweden, it reaches a considerably higher rate of \$168 per ton. In this illustrative example, the tax rate is set at \$50 per ton. The amount of carbon released during the storage of one unit of the product over one unit of time is 0.003 tons. The values for the remaining problem parameters are the same as those employed in the illustrative example presented in Section 3.

Using the carbon-related data, the fixed and variable carbon emissions are $\hat{C}_o^f = 145.44$ kg/truck and $\hat{C}_o^v = 0.0505$ kg/unit, respectively. The tax-dependent coefficients of the objective function in equation (16) are $h + tx\hat{h} = 2.15$, $C_o^f + tx\hat{C}_o^f = 91.702$, and $C_o^v + tx\hat{C}_o^v = 0.022525$. Employing these tax-based coefficients within the solution procedure in Theorem 1, the optimal operational policy is:

$$Q_{tx} = 1200, \text{ and } m_{tx} = 3 \text{ with a cost of } K_{tx} = 3590.6.$$

The optimal operational strategy for the joint economic and environmental model under carbon tax regulation is to procure 1200 units while utilizing three FTL trucks, which is also the optimal solution of the economic model. This results in an overall operational cost of \$3397.04 and an annual carbon footprint of 3.87 tons. In this case, a carbon tax rate of \$50 per ton is ineffective in curbing the retailer's carbon emissions, as it results in the same level of emissions as the economic model ($tx = 0$). By solving the environmental model in (15), it is determined that the minimum carbon footprint released by the retailer is 2.67 tons per year when shipping 400 units in a single FTL truck.

The retailer's carbon footprint remains the same at a level of 3.87 tons per year within the tax rates bracket of $[1, 201]$. The annual carbon footprint drops to 3.27 tons when the tax rates increase to the range $[202, 1937]$. It attains the minimum achievable level of 2.67 tons with carbon tax rates above \$1937. Faced with these elevated carbon tax rates, the retailer may consider adopting other carbon regulation policies, such as the implementation of a carbon cap initiative as discussed next.

5. Integrated economic and environmental model under carbon cap policy

The retailer's objective now is to establish an operational policy aimed at minimizing its overall operational costs without exceeding a carbon cap of C tons set by environmental legislators. In this case, the optimization problem to be solved is:

$$\text{Min } K_{cc}(Q,m) = \frac{AD}{Q} + 0.5\,hQ + \frac{DC_o^f m}{Q} + DC_o^v$$

Subject to:

$$\begin{aligned} &E(Q,m) = 0.5\,\hat{h}Q + \frac{D\hat{C}_o^f m}{Q} + D\hat{C}_o^v \le C \\ &(m-1)U < Q \le mU \\ &m \le M. \end{aligned} \tag{17}$$

Given that $E_{ev}^{\min}$ represents the minimum total carbon footprint that can be generated by the retailer, then the integrated economic and environmental model under carbon cap policy is feasible only when $C \ge E_{ev}^{\min}$.

Moreover, the optimal solution to the economic model (Q_{ec}, m_{ec}) is a feasible and optimal solution to the integrated economic and environmental model under carbon cap policy when $C \ge E(Q_{ec}, m_{ec})$. In the following, we assume that $E_{ev}^{\min} < C < E(Q_{ec}, m_{ec})$ to ensure the feasibility of the environmental model and prevent the occurrence of a trivial solution (Q_{ec}, m_{ec}). For a given number of trucks, the theorem below provides closed form expressions for determining the optimal shipment quantity.

Theorem 2. *The optimal shipment quantity, $Q_{cc}(m)$, to the integrated economic and environmental model under carbon cap policy is given by:*

$$Q_{cc}(m) = \begin{cases} Q_{ec}(m) & \text{if } Q^l_{cc}(m) \leq Q_{ec}(m) \leq Q^u_{cc}(m) \\ Q^l_{cc}(m) & \text{if } Q_{ec}(m) \leq Q^l_{cc}(m) \\ Q^u_{cc}(m) & \text{if } Q_{ec}(m) \geq Q^u_{cc}(m) \end{cases} \tag{18}$$

where

$$Q^l_{cc}(m) = \max\left[(m-1)U,\ \frac{\left(C - D\hat{C}^v_o\right) - \sqrt{\left(C - D\hat{C}^v_o\right)^2 - 2\hat{h}D\hat{C}^f_o m}}{\hat{h}}\right] \tag{19}$$

$$Q^u_{cc}(m) = \min\left[mU,\ \frac{\left(C - D\hat{C}^v_o\right) + \sqrt{\left(C - D\hat{C}^v_o\right)^2 - 2\hat{h}D\hat{C}^f_o m}}{\hat{h}}\right]. \tag{20}$$

Proof. The carbon emissions constraint stated in equation (17) can be easily converted to the following quadratic inequality:

$$0.5\hat{h}Q^2 - \left(C - D\hat{C}^v_o\right)Q + DC^f_o m \leq C.$$

This inequality is satisfied when

$$\frac{\left(C - D\hat{C}^v_o\right) - \sqrt{\left(C - D\hat{C}^v_o\right)^2 - 2\hat{h}D\hat{C}^f_o m}}{\hat{h}} \leq Q \leq \frac{\left(C - D\hat{C}^v_o\right) + \sqrt{\left(C - D\hat{C}^v_o\right)^2 - 2\hat{h}D\hat{C}^f_o m}}{\hat{h}}.$$

To further ensure that $(m-1)U < Q \leq mU$, it is necessary to have $Q^l_{cc}(m) \leq Q \leq Q^u_{cc}(m)$, where $Q^l_{cc}(m)$ and $Q^u_{cc}(m)$ are given by (19) and (20), respectively. It then follows from the convexity of $K_{cc}(Q, m)$ that the optimal shipment quantity is given by (18). It is important to note that the boundaries for the shipping quantity in equations (19) and (20) are not defined when $(C - D\hat{C}^v_o)^2 < 2\hat{h}D\hat{C}^f_o m$. Consequently, the maximum number of trucks that can employed for transporting Q is the largest number $m^{\max}$ satisfying $(C - D\hat{C}^v_o)^2 \geq 2\hat{h}D\hat{C}^f_o m^{\max}$. The upper bound for the optimal number of trucks is then $m^{\max} = \left\lfloor (C - D\hat{C}^v_o)^2 / (2\hat{h}D\hat{C}^f_o) \right\rfloor$, where $\lfloor x \rfloor$ is the largest integer smaller than or equal to x. With the assumption that $E^{\min}_{ev} < C$, then it is not difficult to show that $(C - D\hat{C}^v_o)^2 > 2\hat{h}D\hat{C}^f_o m'$, where m' denotes the number of trucks resulting with $E^{\min}_{ev}$. As a result, the boundary conditions for the shipping quantities in equations (19) and (20) are feasible. □

The procedural steps to determine the optimal solution for the integrated economic and environmental model under carbon cap policy are outlined below:

Step 0: Set $m = 1$, $m^{\max} = \lfloor (C - D\hat{C}^v_o)^2 / (2\hat{h}D\hat{C}^f_o) \rfloor$, and $K_{cc} = +\infty$.
Step 1: If $m > \text{Min}(M, m^{\max})$, Go to step 6
Step 2: Compute:
$Q_{ec}(m) = \sqrt{\frac{2(A + C^f_o m)D}{h}}$
$Q^l_{cc}(m)$ and $Q^u_{cc}(m)$ from (19) and (20) respectively.
Step 3: If $Q^l_{cc}(m) < Q_{ec}(m) \leq Q^u_{cc}(m)$, Do:
Compute $K_{cc}(Q_{ec}(m), m)$ using (9)
If $K_{cc}(Q_{ec}(m), m) < K_{cc}$, Do:

$K_{cc} = K_{cc}(Q_{ec}(m), m)$
$Q_{cc} = Q_{ec}(m)$
$m_{cc} = m$
Go to Step 6.

Step 4: If $Q_{ec}(m) > Q^u_{cc}(m)$, Do:
Set $Q_{cc}(m) = Q^u_{cc}(m)$
Compute $K_{cc}(Q_{cc}(m), m)$ using (9)
If $K_{cc}(Q_{cc}(m), m) < K_{cc}$, Do:
$K_{cc} = K_{cc}(Q_{cc}(m), m)$
$Q_{cc} = Q_{cc}(m)$
$m_{cc} = m$
$m \to m + 1$
Go to Step 1.
If $K_{cc}(Q_{cc}(m), m) \geq K_{cc}$, Do:
$m \to m + 1$
Go to Step 1.

Step 5: If $Q_{ec}(m) < Q^l_{cc}(m)$, Do:
Set $Q_{cc}(m) = Q^l_{cc}(m)$
Compute $K_{cc}(Q_{cc}(m), m)$ using (9)
If $K_{cc}(Q_{cc}(m), m) < K_{cc}$, Do:
$K_{cc} = K_{cc}(Q_{cc}(m), m)$
$Q_{cc} = Q_{cc}(m)$
$m_{cc} = m$
$m \to m + 1$
Go to Step 1.
If $K_{cc}(Q_{cc}(m), m) \geq K_{cc}$, Do:
$m \to m + 1$
Go to Step 1.

Step 6: Stop.

The algorithm terminates based on one of the following three conditions. Firstly, it stops upon finding the first feasible $Q_{ec}(m)$. Secondly, it ends after being executed M times without reaching a feasible $Q_{ec}(m)$. The last condition prompting the algorithm termination is when m surpasses $m^{\max}$ for the first time.

Illustrative example

Continuing with the same illustrative example presented earlier, assume now that the environmental authority caps the retailer's annual carbon emissions at 3.3 tons. The results of the different steps of the algorithm are detailed next.

$$m^{\max} = 2.$$

First iteration: $m = 1$.

$Q^l_{cc}(1) = 276.2, Q^u_{cc}(1) = 400$, and $Q_{ec}(1) = 1293.1$. Therefore $Q_{cc}(1) = 400$ and $K_{cc}(400, 1) = 4680.4 < +\infty$.
$K_{cc} = 4680.4$, $Q_{cc} = 400$, and $m_{cc} = 1$.

Second iteration: $m = 2$.

$Q^l_{cc}(2) = 766.2, Q^u_{cc}(2) = 800$, and $Q_{ec}(2) = 1447.2$. Therefore $Q_{cc}(2) = 800$ and $K_{cc}(800, 2) = 3517.9 < 4680.38$.
$K_{cc} = 3517.9, Q_{cc} = 800$, and $m_{cc} = 2$.

Given than $m^{\max} = 2$, the algorithm terminates at the second iteration with the optimal solution to transport 800 units in two FTL trucks at an overall operational cost of \$3517.9. Using equation (14), the resulting annual

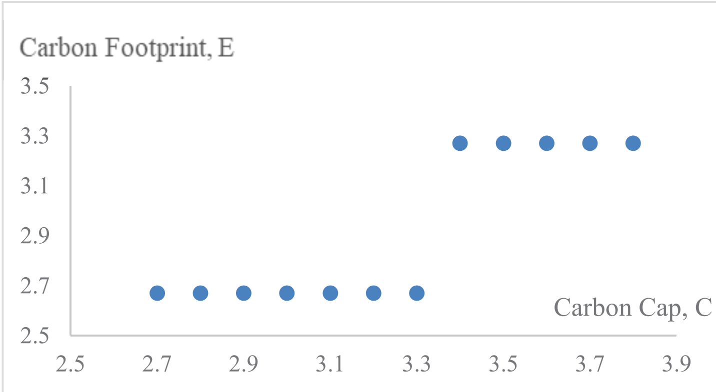

FIGURE 5. Carbon footprint variation as a function of the carbon cap.

carbon emissions amount to 3.27 tons. In comparison to the economic model results, the imposition of a carbon cap restriction led to an increase in the overall operational cost from $3397.0 to $3517.9, representing a rise of 3.56%. However, the amount of emitted carbon reduced from 3.87 tons to 3.27 tons, or an equivalent reduction of 15.5%. Therefore, the marginal increase in operational cost is significantly dominated by the substantial decrease in carbon emissions. It can, therefore, be inferred that the retailer can effectively reduce its carbon footprint without adversely impacting the operational costs *via* adjusting shipment quantities and the size of its truck fleets. Similar result was noted in the inventory literature but in a distinct context [27].

As per the results of this illustrative example as well as the example presented in the previous section, the carbon cap policy with a cap of 3.3 tons proved to be more environmentally effective than the carbon tax policy with a tax rate of $50 per ton of emitted carbon. As previously noted, following the carbon tax policy the carbon footprint remained constant at 3.87 tons per year within the tax rate interval of $[1, 201]$. It decreased to 3.27 tons when the tax rate rose to the range $[202, 1937]$, and only reached the minimum achievable level of 2.67 tons when the tax rate exceeded $1937, which could prove to be prohibitively costly. By contrast, as shown in Figure 5, under the carbon cap policy the level of carbon emissions ranged between 2.67 and 3.27 tons when the cap varied between 2.7 and 38 tons. These findings confirm the superior environmental effectiveness of the carbon cap policy compared to the carbon tax policy, as the cap ensures compliance with emission targets rather than relying solely on tax-based economic incentives.

6. Economic model with owned and leased trucks

For a better resemblance to many real life scenarios, the retailer is now exploring the possibility of transporting another highly demanded product using its owned fleet of trucks in addition to leasing trucks from the market. The retailer's goal is to determine a more cost-effective and flexible fleet mix of owned and leased trucks, rather than relying exclusively on its limited in-house trucks. Indeed, the leasing option offers the retailer the advantages to access additional trucks when needed without investing in acquiring new trucks This option holds particularly appealing for the retailer when dealing with fluctuating transportation needs, as leasing contracts can be customized to suit specific durations.

It is assumed that both fixed and variable costs for leased trucks are higher than those of the owned trucks. This assumption is reasonable as, in addition to the standard fixed costs, such as depreciation, insurance, financing costs, maintenance and repairs, and driver wages, leasing a truck incurs a fixed leasing fee. Moreover, if the leasing contract includes mileage restrictions, a mileage fee is charged to the retailer. In this case, mileage fee must be added to the variable transportation costs. Without loss of generality, we also assume that both owned and leased trucks are of the same type and have identical loading capacities. Finally, we further assume that there is no restriction on the number of trucks to be leased from the market.

The following additional notation is introduced:

C_l^v variable transportation cost per unit transported in a leased truck,
C_l^f fixed transportation cost per leased truck,
Q_o quantity transported by the owned trucks,
Q_l quantity transported by the leased trucks,
m_o number of owned trucks in operation
m_l number of leased trucks.

The economic model with owned and leased trucks is mathematically stated as follows:

$$\text{Min } K_{eol}(Q, m_o, m_l) = AD/Q + 0.5\,hQ + D\Big(C_o^v\, Q_o + C_o^f m_o + C_l^v Q_l + C_l^f m_l\Big)/Q \tag{21}$$

Subject to:

$$Q_o + Q_l = Q \tag{22}$$

$$(m_o - 1)U < Q_o \leq m_o U \tag{23}$$

$$(m_l - 1)U < Q_l \leq m_l U \tag{24}$$

$$m_o \leq M \tag{25}$$

m_o and m_l are integers.

The following lemma provides an important result pertaining to the structure of the optimal solution to the economic model with owned and leased trucks.

Lemma. *In an optimal solution of the economic model with owned and leased trucks, leased trucks should not be utilized when the loading capacity of the owned trucks is sufficient to transport the ordering quantity.*

Proof. Let's assume that in the optimal solution we have $Q_o < MU$ and $Q_l > 0$.

Consider the case when $0 < Q_l < MU - Q_o$. Then, adding Q_l to Q_o, does not change the ordering quantity, but it leads to cost saving. This is because it is more cost-effective to transport Q_l in owned trucks instead of leased trucks. Consequently, the current solution is not optimal. Similarly, it can be proven that when $Q_l > MU - Q_o > 0$, the solution is also suboptimal. □

Based on the outcome of the lemma, we have $Q_l = 0$ and $Q = Q_o$ when $m_o \leq M$. Moreover, in this case, the overall operational cost is:

$$K_{eol}(Q, m_o, 0) = \big(A + C_o^f m_o\big)D/Q + 0.5\,hQ + DC_o^v, \tag{26}$$

and the optimal ordering quantity and number of trucks can be determined using the results of Theorem 1.

On the other hand, when $m_o = M$, the quantity to be shipped by leased trucks is $Q_l = Q - MU$ and the overall operational costs become:

$$K_{eol}(Q, M, m_l) = \frac{AD}{Q} + 0.5\,hQ + D\Big(C_o^v\, MU + C_o^f M + C_l^v(Q - MU) + C_l^f m_l\Big)/Q. \tag{27}$$

In such case, the optimal ordering quantity is:

$$Q_{eol}(M, m_l) = \sqrt{\frac{2D\Big(A + C_o^f M + C_l^f m_l - (C_l^v - C_o^v)MU\Big)}{h}}, \tag{28}$$

and should satisfy:

$$MU + (m_l - 1)U < Q_{eol}(M, m_l) \leq MU + m_l U. \tag{29}$$

Note that once the term under the root of equation (28) is negative when $m_l = 1$, then leased trucks should not be utilized and the optimal ordering quantity is MU.

Building on the preceding analysis and results, we propose the following algorithm to generate the optimal operating policy for the economic model with owned and leased trucks. The proposed algorithm consists of two phases. In the first phase, it solves the economic model using only owned trucks. In case a feasible $Q_{eol}(m, 0)$ cannot be found (that is $m_{feas} > M$ where m_{feas} is given by (12)), the algorithm proceeds to its second phase. During this phase, it utilizes M owned truck and m_l leased trucks until a feasible $Q_{eol}(M, m_l)$ is obtained.

Phase I

Find m_{feas} and m_{FTL} using (12) and (13), respectively.

- If $m_{feas} \leq M$, then set:
- $m_o = m_{\text{FTL}}$ and $Q_o = m_{\text{FTL}}U$ and $m_l = 0$ if $K_{eol}(m_{\text{FTL}}U, m_{\text{FTL}}, 0) < K_{eol}(Q_{ec}(m_{feas}), m_{feas}, 0)$, where $Q_{ec}(m_{feas})$ is given by (11), or
- $m_o = m_{feas}$ and $Q_{ec} = Q_{ec}(m_{feas})$ and $m_l = 0$ if $K_{eol}(m_{\text{FTL}}U, m_{\text{FTL}}) > K_{eol}(Q_{ec}(m_{feas}), m_{feas})$.
- Stop
- If $m_{feas} > M$,
- If $m_{\text{FTL}} \leq M$, then
  set $K_{eol} = K_{eol}(m_{\text{FTL}}U, m_{\text{FTL}})$,
  $m_o = m_{\text{FTL}}$ and $Q_o = m_{\text{FTL}}U$ and $m_l = 0$
  Go To the second phase
- If $m_{\text{FTL}} > M$, then
  set $K_{eol} = K_{eol}(MU, M)$,
  $m_o = M$ and $Q_o = MU$ and $m_l = 0$
  Go To the second phase.

Phase II

Step 5: Set $m = 1$, and go to Step 6.
Step 6: Compute
  $Q_{eol}(M, m)$ using (28)
Step 7: If $MU + (m - 1)U < Q_{eol}(M, m) \leq MU + mU$, Do:
  Compute $K_{eol}(Q_{eol}(M, m), M, m)$ using (27)
    If $K_{eol}(Q_{eol}(M, m), M, m) < K_{eol}$, Do:
      $K_{eol} = K_{eol}(Q_{eol}(M, m), M, m)$
      $Q_{eol} = Q_{eol}(M, m)$
      $m_o = M$ and $m_l = m$
      $m \to m + 1$
      Go to Step 9.
    If $K_{eol}(Q_{eol}(M, m), M, m) \geq K_{eol}$ Go to Step 9.
Step 8: If $Q_{eol}(M, m) > MU + mU$, Do:
    Set $Q_{eol}(M, m) = MU + mU$
  Compute $K_{eol}(Q_{eol}(M, m), M, m)$ using (27)
    If $K_{eol}(Q_{eol}(M, m), M, m) < K_{eol}$, Do:
      $K_{eol} = K_{eol}(Q_{eol}(M, m), M, m)$
      $Q_{eol} = Q_{eol}(M, m)$
      $m_o = M$ and $m_l = m$
      $m \to m + 1$
      Go to Step 6.

If $K_{eol}(Q_{eol}(M,m),M,m) \geq K_{eol}$, Do:
$m \rightarrow m+1$
Go to Step 6.

Step 9: Stop.

Illustrative example

The retailer is managing the inventory of a product with a holding cost rate of \$0.5 per unit per unit of time, an ordering cost of \$500 per order, and an annual demand of 10 000 units. Simultaneously, it is looking to determine the optimal transportation policy for this product. The retailer has access to only seven trucks from its own fleet of trucks, each with a loading capacity of 400 units. The fixed and variable transportation costs are \$84.43 per truck and \$0.02 per unit, respectively. Additionally, the retailer is considering leasing trucks with the same loading capacity from the market, but with fixed and variable transportation costs of \$95.22 per truck and \$0.03 per unit, respectively. The results of the various iterations of the above algorithm are summarized below.

First Phase:

Given that $m_{\text{FTL}} = 7$ and $m_{feas} = 26$, the outputs of the first phase are $Q_{eol}(m,0) = 2800$ units, $m_o = 7$ trucks, $m_l = 0$, and $K_{eol}(2800,7,0) = \$4796.5$.

Second Phase:

First iteration: $m_l = 1$.

$Q_{eol}(7,1) = 6806.6 > 3200$. Therefore $Q_{eol}(7,1) = 3200$ and $K_{eol}(3200,7,1) = 4719.5 < 4796.5$
$K_{eol} = 4719.5, Q_{eol} = 3200, m_0 = 7$, and $m_l = 1$.

Second iteration: $m_l = 2$.

$Q_{eol}(7,2) = 7080.8 > 3600$. Therefore $Q_{eol}(7,2) = 3600$ and $K_{eol}(3600,7,2) = 4681.8 < 4719.5$
$K_{eol} = 4681.8, Q_{eol} = 3600, m_0 = 7$, and $m_l = 2$.

Third iteration: $m_l = 3$.

$Q_{eol}(7,3) = 7344.8 > 4000$. Therefore $Q_{eol}(7,3) = 4000$ and $K_{eol}(4000,7,3) = 4671.7 < 4681.8$
$K_{eol} = 4671.7, Q_{eol} = 4000, m_0 = 7$, and $m_l = 3$.

Fourth iteration: $m_l = 4$.

$Q_{eol}(7,4) = 7599.7 > 4400$. Therefore $Q_{eol}(7,4) = 4400$ and $K_{eol}(4400,7,4) = 4681.6 > 4671.7$
$K_{eol} = 4671.7, Q_{eol} = 4000, m_0 = 7$, and $m_l = 3$.

All the values of $Q_{eol}(M,m_l)$ from the fifth to the twentieth iteration were found to be infeasible and did not yield lower overall operational costs compared to the value obtained in third iteration, $K_{eol} = 4671.7$.

Twenty-first iteration:

$Q_{eol}(7,21) = 11068.2 < 11200$.
Therefore $Q_{eol}(7,21) = 11068.2$ and $K_{eol}(11068.2,7,21) = 5834.1 > 4671.7$
$K_{eol} = 4671.7, Q_{eol} = 4000, m_0 = 7$, and $m_l = 3$.

The optimal solution is to order 4000 units, with 2800 units transported by 7 owned FTL trucks, and the remaining 1200 units are shipped using 3 leased FTL trucks, resulting in a total operational cost of \$4671.7. When comparing this cost to the overall operational cost of the truck-constrained problem solely utilizing owned trucks, the retailer can realize a cost reduction of \$4796.5 – \$4671.7 = \$124.8 by opting to lease trucks instead. In case the ordering cost increases to \$1000, the cost savings realized by opting to lease trucks also increases to \$908. Likewise, if only demand is increased to 20 000, a significantly greater cost savings of \$2869 can be achieved. These examples, confirm the economic significance of maintaining a diverse fleet of trucks rather than solely relying on owned trucks.

7. Conclusion

This paper tackles the joint optimization of inventory and transportation decisions within the EOQ framework using homogeneous fleet of trucks. For the case when the transport capacity is limited due to the retailer relying exclusively on its own fleet of trucks, a simple and efficient iterative solution procedure is proposed. Closed-form expressions are then derived to determine both the optimal order quantity and the number of trucks, thereby reducing the computational complexity compared to earlier iterative algorithm.

The obtained computational results demonstrate that when the ordering cost and demand rate are high and the holding cost is low, the optimal solution calls for the use of FTL shipments only. This outcome is expected for large ordering quantities, and its validity is confirmed *via* conducting a broader set of computational experiments across different parameter ranges. These extended experiments consistently reinforced the insight that FTL shipments dominate under such cost and demand conditions.

We also investigated the impact of various environmental regulations by embedding carbon tax and carbon cap policies into the optimization framework. The numerical findings suggest that while carbon taxes may have a limited effect unless tax rates are very high, the carbon cap policy consistently proves to be more environmentally effective. Notably, this result is reinforced by an expanded set of scenarios, thereby supporting the generalizability of the conclusions drawn. Nonetheless, it is noted that a full analytical proof of this finding is complex due to the staircase structure of the cost function and the nonlinear nature of the carbon cap constraint, and the authors suggest this as an avenue for future research.

Finally, we extended the purely economic model to account for the scenario where the retailer may lease additional trucks from the external market. The results show that such flexibility leads to significant cost reductions, particularly at higher demand levels, confirming the economic significance of maintaining a mixed fleet rather than relying solely on owned trucks.

An interesting extension to the research presented in this paper involves analyzing the cost alongside environmental impacts of leasing trucks. While leasing trucks leads to increased fixed and variable transportation costs, it may also contribute to carbon reductions, as leasing can provide access to newer and more fuel-efficient trucks. One may also explore the same problem settings addressed herein but in the presence of a heterogeneous fleet of trucks and/or stochastic, in lieu of deterministic, demand. Another potential venue for future research is to explore the option of outsourcing a portion or the whole transportation process, as opposed to leasing trucks, through partnerships with highly skilled third-party logistics providers. Additionally, extending the analysis to tackle the multi-product case in the context of a more involved supply chain network comprising multi stages and multi firms at each stage presents a promising avenue for future research (see for instance [33]).

Data availability statement

All data supporting the findings of this study are available within the paper.

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