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A BI-OBJECTIVE INVENTORY OPTIMIZATION IN FORWARD AND REVERSE LOGISTIC SUPPLY CHAINS WITH SHORTAGES

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Abstract. Nowadays, remanufacturing is a sustainable and cost-effective process that restores used products or components to their original performance standards, often making them as good as new. This article focuses on the process of remanufacturing used products, emphasizing their restoration to the original functionality and performance standards and finding a cost-effective solution. It considers various aspects of the remanufacturing process, including collection, inspection, repair, and reassembly, while highlighting the environmental benefits associated with this sustainable practice. We have presented a detailed analysis of all cost components and carbon emissions associated with each process in the system, including costs incurred at the primary manufacturer, primary retailer, collection center, and other relevant stages. The main aim of this article is to optimize total system cost and carbon emissions associated with each process. To get the model optimum, we have solved the bi-objective problem by non-dominated sorting genetic algorithm (NSGA-II), which ensures an optimal balance between the two objectives. The major novelties of this work include imperfect screening, quadratic demand, and unequal shipment. For model validation, a numerical example has been analyzed on the basis of a case study, which results in a set of Pareto optimal solutions for the problem. A sensitivity analysis has been presented to evaluate the impact of varying parameters on the outcomes. The findings of this study reveal that it is possible to achieve up to a 65.21% reduction in costs through the proposed approach.

Mathematics Subject Classification. 90B05.

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1. Introduction

Currently, eco-friendly businesses are the main focus of every company to reduce environmental effects. Reuse, recycling, remanufacturing, and reworking are effective strategies for the company, as they contribute to developing a sustainable environmental supply chain. Companies are trying to establish a sustainable supply chain by considering forward, backward, and closed-loop supply chains. Various types of products fall under the closed loop, *e.g.*, electronic products, glass products, automobile parts, batteries, tires, textiles, etc. Manufacturers prioritize enhancing the residual value of end-of-use products, deciding whether to outsource to retailers or third-party service providers or retain it exclusively.

Keywords. Closed loop supply chain (CLSC), shortage, quadratic demand, recycling center, optimization, Pareto front.

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In modern supply chains, several practical challenges limit the effectiveness of classical closed-loop supply chain (CLSC) models. In real-world supply chains, customer demand is rarely constant over time. Instead, it tends to increase as the production system and market gradually adapt to each other. Over a time period, the production system learns market behavior, promotional strategies take effect, and customer awareness grows, all of which contribute to a rising demand rate. A time-dependent quadratic demand function provides a flexible and realistic way to capture this increasing behavior, as it allows demand to grow at both linear and accelerating rates. Incorporating such non-linear but increasing demand into CLSC models enables more accurate planning and cost estimation compared to the traditional assumption of constant demand.

Another critical concern is the issue of imperfect quality in production processes. Defective items inevitably arise and must be identified through screening. Unlike traditional approaches that assume a fixed screening rate, practical systems often experience variable screening rates due to machine efficiency, operator performance, or product heterogeneity. This variability significantly influences the availability of usable products and the overall system cost, making it essential to account for such uncertainty in supply chain models.

Furthermore, Realistic supply chain operations often involve unequal shipment quantities rather than fixed lot sizes. In the presence of imperfect quality and variable screening, the effective shipment quantities are naturally unequal, as they directly depend on the proportion of usable items obtained after inspection. The objective of the study is to develop a CLSC model that:

(i) Incorporates time-dependent quadratic demand to reflect the increasing nature of real-world customer demand.
(ii) Accounts for imperfect production quality with variable screening rates.
(iii) Integrates an unequal shipment policy driven by the availability of perfectly screened items.
(iv) Considers carbon emissions generated at each process, thereby aligning supply chain decisions with environmental sustainability goals.

The contributions of this work are as follows: we first construct a forward and reverse logistics model to evaluate both the total cost and the carbon emissions generated across the entire closed-loop supply chain. In the forward process, a supply chain model is developed by incorporating a quadratic time-dependent demand function together with variable screening rates, capturing the dynamic nature of market demand and the variability in product quality. To better reflect practical distribution, an unequal shipment policy is introduced for delivering perfectly screened products to customers. Furthermore, the proportion of major defective products is emphasized as a key factor in the reverse supply chain, since it directly affects the repair and remanufacturing processes, thereby influencing overall system cost and performance.

The structure of this article is as follows: Section 1 gives a brief introduction, motivation, objective and contribution, while Section 2 shows how our work fits into existing research. Section 3 includes the assumptions, notations, and formulation of the mathematical model. Section 4 gives the case study and numerical illustrations, Section 5 discusses the solution methodology and results and Section 6 gives the sensitivity analysis over some model parameters. Graphical illustrations are done in Section 7 utilizing the numerical outputs obtained at the sensitivity analysis. Section 8 concludes the managerial insights, and Section 9 discusses the conclusion, followed by a scope of future work.

2. Literature survey

As the literature on CLSC is already extensive, it is not practical to present a detailed review of all existing studies here. However, we limit our review to studies that emphasize the specific aspects of the problem under consideration in this research.

In 2018, Su *et al.* [29] analyzed a sustainable closed-loop supply chain model and found that manufacturers prioritize the third-party service providers over retailers when both perform equally in collecting the used product. In 2023, Maheshwari *et al.* [22] studied a sustainable supply chain model for forward, reverse, and closed loop supply chains to develop resource-efficient rework and remanufacturing models.

Considering the forward as well as the reverse flow of the supply chain, Chung *et al.* [5] proposed a profit-maximizing inventory system in linking the joint profit of the supplier, the manufacturer, the third-party recycle dealer, and the retailer through a contractual framework. Becerra *et al.* [3] invented a multi-objective mixed integer non-linear programming (MO-MINLP) model to minimize the economic and social costs, reduce carbon emissions, and maximize the social impact of SC operations, focusing on (i) LIT (location, inventory, and transportation) decisions in an integrated manner; (ii) the three sustainability aspects, *i.e.*, economic, environmental, and social (iii) a closed-loop supply chain (CLSC) structure. By evaluating the cost-saving performances with full, partial, or no recovery information, Yang *et al.* [31] investigated a periodic inventory model on centering the recovery information of containers. Kausar *et al.* [20] introduced a sustainable inventory management policy that incorporates the learning effect and carbon emission, highlighting the environmental benefits of remanufacturing discarded goods. Mitra [25] addressed the inventory management issue in CLSC by developing deterministic and stochastic models for a two-echelon system with correlated demands and returns under generalized cost structures, revealing that higher return rates and correlations reduce net demand variability but do not always result in cost savings. Using the system-dynamics approach, Poles and Cheong [26] invented an inventory control model for a remanufacturing process in a CLSC, concluding that reducing residence time and increasing service agreements can enhance inventory management efficiency for companies engaged in remanufacturing. Babaeinesami *et al.* [1] developed a cost-minimizing CLSc model based on a distribution network that meets customer needs. They applied the NSGA-II and ϵ-constraint methods, demonstrating that NSGA-II provides superior results compared to the ϵ-constraint approach. Motivated by challenges in the automotive industry, Yildizbasi *et al.* [32] proposed a mixed-integer CLSC model. They designed a CLSC network that captures the overall flow dynamics of the system.

Many researchers have developed inventory management models by considering shortages and backorders. Recently, Dolai and Mondal [7] presented a profit-maximizing inventory model by considering shortage and a time-dependent demand function to optimize the production rate and screening rate. In 2016, Hsu and Hsu [16] developed an economic production quantity (EPQ) model that considers different scenarios, including known or random defective rates and the withdrawal of faulty items, to determine optimal production lot size and backorder quantity for a manufacturer under an imperfect production process. Rout *et al.* [27] proposed an EPQ model that considered shortages, imperfect production, inspection error, and rework, with the first case addressing the crisp model, whereas the second considers a type-2 fuzzy approach. In this model, they developed a cost-minimizing model that incorporates inspection errors, backlogged shortages, and reworked items. Ghomi-Avili [11] designed a multi-objective model for closed-loop supply chain network design (CLSCND) with price-dependent demand, considering random disruptions, shortages, resilience strategies, transportation methods, and three objective functions for unfulfilled demand.

Demand is a critical component in supply chain management, and it is influenced by various factors such as time, price, consumer behavior, and external market conditions. Demand often varies with time due to seasonality, trends, and product life-cycle stages. Also, demand is sensitive to price changes, as reflected by the law of demand. In 2002, Khanra and Chaudhuri [21] discussed an order-level inventory problem for deterioration items with continuous, quadratic time-dependent demand. Singh *et al.* [28] explored a production inventory model for a consistent deterioration rate in board and assembly units to minimize the total cycle cost by optimizing production time, considering quadratic time capacity and total production rates. Barman and Mahata [2] presented an integrated two-echelon supply chain inventory model with a single manufacturer and multiple retailers, where each retailer's demand depends on the product's price. Chen and Chang [4] addressed the problem of determining optimal retail price, replenishment cycle, and shipments for deteriorating items in a one-manufacturer and multi-retailer channel setting. Mateen *et al.* [23] analyzed the interaction between a vendor and multiple retailers in a vendor-managed inventory system under stochastic demand. Giri *et al.* [12] studied a two-level supply chain model of single manufacturers and multiple retailers considering price-dependent demand and normally distributed lead time for centralized and decentralized scenarios.

The primary sources of carbon emissions in a production system include shipping, production, and inventory management. In 2020, Mishra *et al.* [24] investigated a closed-loop supply chain focusing on carbon emissions and waste control with the main objectives being the minimization of total cost along with shipment number, container size, and cycle length. For the closed-loop supply chain of perishable products, Esmaeilian *et al.* [9] developed a mathematical model to minimize the total carbon emissions and maximize the social benefits. Considering remanufacturing and refurbishing, Jauhari *et al.* [18] studied a closed-loop supply chain model with an imperfect manufacturing process, employing a cap and trade policy and green technology investment to reduce emissions. Ullah [30] discussed the effect of the remanufacturing rate on the transportation distance of the reverse supply chain. Yu *et al.* [33] used non-cooperative and cooperative games to exhibit a systematic analysis aimed at enhancing cooperation and coordination in a three-echelon closed-loop supply chain under differentiated carbon tax regulation. Focusing on environmental sustainability, Jauhari *et al.* [19] developed a profit-maximizing closed-loop supply chain model incorporating circular index, green investment factor, hybrid production system, and delay-in-payment mechanism. Guo *et al.* [13] studied a remanufacturing closed-loop supply chain model that accounts for uncertain market demand, recycled product quality, and the effect of carbon taxes and subsidies. To address uncertainty and solve the discrete optimization problem, a fuzzy optimization constraint programming approach is used along with Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) to obtain approximate optimal solutions.

The literature survey shows that no one has solved the closed-loop supply chain (CLSC) model with a variable screening rate for the time-dependent quadratic demand function, as for a nonlinear demand function, a variable screening rate allows adjustment to operations in response to changing demand patterns. Labor shortages and machinery faults are significant issues that disturb the screening process. In this situation, shortages may arise, and to avoid shortages, increasing screening rates is needed. Therefore, considering unequal shipment size, we have developed a closed-loop supply chain with variable screening rates for the quadratic type of demand function.

However, we have shown some major literature reviews along the relevant domain, and it is shown in Table 1.

From the above study, it is observed that very few or no articles have been found on CLSC with variable screening rates and unequal shipment sizes. Moreover, no researcher has emphasized the importance of the proportion of major defective products. Thus, in this proposed study, we have shown that shortages of secondary products can arise in the reverse flow of the supply chain.

3. Formulation of forward and reverse flow of supply chain

3.1. Assumption and notation

The following assumptions and notations are considered to develop our model.

3.1.1. Assumptions

(1) A single item with an imperfect production model has been considered.
(2) A constant production rate is considered.
(3) A time-dependent demand function is considered, which is of the form $D(t) = a_1 + a_2 t + a_3 t^2$, where $a_1 > 0, a_2 > 0, a_3 > 0$.
(4) Two types of screening rates have been taken.
(5) Carbon emissions are considered.
(6) Unequal shipment size is taken here.
(7) Rework of imperfect screening items is allowed.
(8) Shortages and back-orders are allowed.
(9) We assume that the rate of repairable inventory (R_r) is greater than the rate of remanufacturing inventory (P_r).
(10) The screening rate (Y) at the primary manufacturer is dependent on the production rate (P) *i.e.*, $Y = bP; \quad 0 < b < 1$.

3.1.2. Notations

TABLE 1. Major literature review in the relevant field.

Author(s) with citation	Model used	Nature of demand function	Screening rate	Shipment size	Backorder	Carbon emission
Maheswari *et al.* [22]	Three layer supply chain model	Constant	No	Equal	No	No
Kausar *et al.* [20]	CLSC	Constant	No	Equal	No	Yes
Gautam *et al.* [10]	Profit maximizing supply chain model	Constant	No	Fixed	No	Yes
Jauhari & Wangsa [17]	CLSC	Constant	No	Equal	Fully	Yes
Dolai & Mondal [7]	EPQ	Time dependent	Variable	No	Fully	No
Elfarouk *et al.* [8]	CLSC	Constant	No	Equal	No	Yes
Hasan *et al.* [14]	CLSC	Quadratic	No	Fixed	Partial	Yes
Babaeinesami *et al.* [1]	CLSC	Deterministic	No	No	No	Yes
Yildizabasi *et al.* [32]	CLSC	Deterministic	No	No	No	No
Guo *et al.* [13]	CLSC	Uncertain	No	No	No	Yes
Jauhari *et al.* [19]	CLSC	Circularity index and green investment factor	Constant	Equal	No	Yes
This article	Forward and reverse supply chain model	Quadratic	Two different type	Unequal	Partial	Yes

Symbols	Description
Parameters	
P	Production rate at the primary manufacturer (units per unit time)
D_s	Demand rate at the secondary retailer (units per unit time)
P_r	Remanufacturing rate at the secondary manufacturer (units per unit time)
R	Rework rate at the primary manufacturer (units per unit time)
R_r	Repairable rate at the repairing center (units per unit time)
C_p	production cost at the primary manufacturer (per unit)
C_s	Set up cost at the primary manufacturer
C_{msh}	Shortage cost at the primary manufacturer (per unit)
C_r	Rework cost at the primary manufacturer (per unit)
C_b	Back-order cost at the primary manufacturer (per unit)
C_{rp}	Purchase cost of the retailer (per unit)
C_o	Ordering cost of the retailer (per unit)
C_{up}	Collection cost of the used inventory (per unit)
C_{rr}	Repairable cost (per unit)
C_{sm}	Secondary manufacturing cost (per unit)

C_{ssh}	Shortage cost at the secondary retailer (per unit)
C_{bs}	Back-order cost at the secondary retailer (per unit)
C_{sr}	Screening cost at the collection center (per unit)
H_m	Holding cost at the primary manufacturer (per unit)
H_s	Holding cost at the primary retailer (per unit)
H_c	Holding cost at the collection center (per unit)
H_r	Holding cost at the repairable center (per unit)
H_{sm}	Holding cost at the secondary manufacturer (per unit)
l	Shipment increasing rate ($l > 1$)
n	Number of shipments to the primary retailer (a finite number)
C_{ssd}	Salvage cost at the scrap dealer (per unit)
C_{scrsd}	Screening cost at the scrap dealer (per unit)
C_{srs}	Salvage cost at the secondary retailer (per unit)
γ	Proportion of major defective product
α	Proportion of secondary manufacturable product
η	Fraction of delivered inventory to the primary retailer
a	Imperfect fraction of the screening rate
b	Screening fraction of the production rate
C_{srp}	Purchase cost at the secondary retailer per unit
e_{S_P}	Emission related to production set-up
e_{S_R}	Emission related to rework set-up
e_p	Emission related to production
e_r	Emission related to rework
e_w	Emission related to holding items at the primary manufacturer
$e_{S_{R_r}}$	Emission related to the repairable set-up
e_{R_r}	Emission related to the repairable items
e_{RC}	Emission related to holding items in the repairable center
$e_{S_{S_M}}$	Emission related to secondary manufacturing set-up
e_{S_M}	Emission related to secondary manufacturing items
e_{SM}	Emission related to the holding items in the secondary manufacturing center
e_{P_r}	Emission related to the holding items at the primary retailer
e_{C_c}	Emission related to the holding items at the collection center
$e_{S_{S_d}}$	Emission related to the salvage item at the scrap dealer
$e_{S_{S_r}}$	Emission related to the salvage item at the secondary retailer
Dependent variable	
T_2	Time at which screening stops
T_3	Time at which rework stops
T	Cycle length
T_1'	Repairable run time
T_2'	Secondary manufacturing run time
$C(Y, Y_1)$	Screening cost at the manufacturer
$C(Y_2)$	Screening cost the secondary retailer
q	First lot size quantity
Y	Screening rate at the manufacturer
Decision variable	
T_1	Production run time at the manufacturer
Y_1	Increased screening rate at the manufacturer
Y_2	Screening rate at the secondary retailer

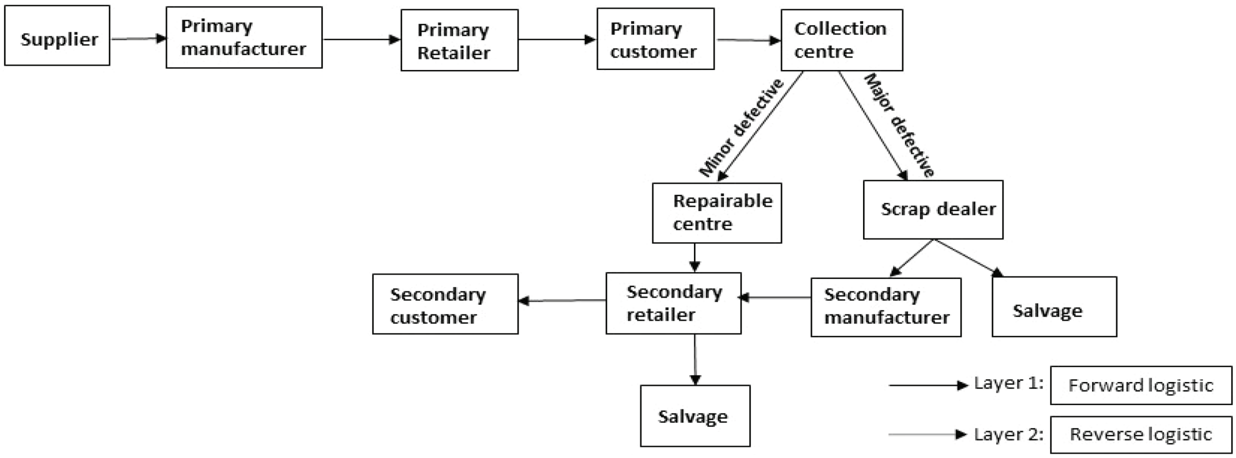

Figure 1. Forward and reverse supply chain.

3.2. Model formulation

In this paper, we develop a two-layer supply chain model for a single type of item in an imperfect production system. The two layers of the supply chain are the forward supply chain (FSC) and the reverse supply chain (RSC). FSC consists of the supplier, primary manufacturer, primary retailer, and primary customer. In the RSC, there is a collection center, repairable center, scrap dealer, secondary manufacturer, secondary retailer, and secondary customer. Our model takes into account the sale of remanufacturable products in a secondary market. As the life span of the used product is less than that of a new product, the market value of the used product is less than that of the new product. The collection center plays an important role in building up the RSC. The collection center collects all used products and bifurcates them into two divisions, major defective and minor defective. The minor defective products are sent to the repairable center for repairing products to sell in the secondary market. The major faulty products are sent to the scrap dealer. The scrap dealer identifies the products as remanufacturable and salvaged items. After the secondary manufacturer remanufactures the products, they sell them to the secondary market. Figure 1 illustrates the structure of the closed-loop supply chain (CLSC), demonstrating the flow of returned products through remanufacturing and recycling.

3.2.1. Primary manufacturer

Here, we discuss an imperfect production system that incorporates the screening of produced items with a variable screening rate. We divide the whole cycle at the primary manufacturer into four phases: production phase, screening phase, rework phase, and demand depletion phase. Figures 2 and 3 depict the inventory dynamics of serviceable and reworkable items over an entire cycle. In the production phase, the primary manufacturer starts production at $t = 0$ with production rate P and stops at T_1. To deliver a non-defective product to the customers, the screening process is essential in the production plant. We consider the screening process to be along with production, and the screening rate (Y) is less than the production rate (P). Due to various reasons, such as labor shortages and machine faults, the screening rate is variable, so after the screening process, the number of perfect items at time t is $Y(1-a)$. Therefore, the unsorted item increases with rate $P - Y$ and the perfect item rate becomes $Y(1-a) - D(t)$. We consider the amount of perfect inventory to be less than the demand rate, *i.e.*, $Y(1-a) < D(t)$; therefore, shortages occur at a rate $D(t) - Y(1-a)$. After completion of production, the manufacturer increases the screening rate (from Y to Y_1) to solve the shortage issue along with the market demand. Rework is needed on the screened defective products. Hence, rework starts at T_2 and continues up to T_3. Only demand fulfillment occurs in the demand depletion phase (from T_3 to T).

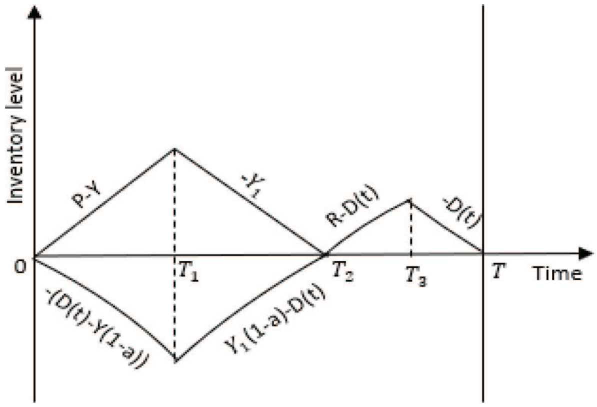

FIGURE 2. Inventory illustration at the primary manufacturer.

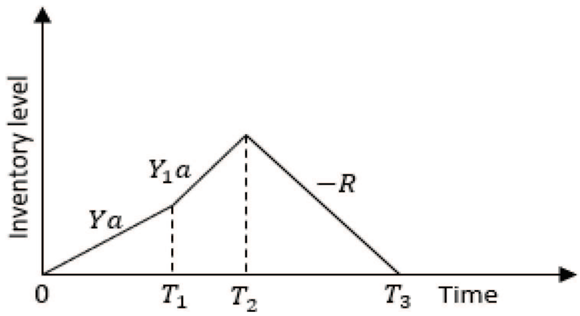

FIGURE 3. Structure of reworkable inventory over time.

The following governing differential equations describe the instantaneous states of the serviceable inventory level in different situations:

$$\frac{\mathrm{d}I_1}{\mathrm{d}t} = P - Y, \qquad 0 \le t \le T_1, \quad I_1(0) = 0 \tag{1}$$

$$\frac{\mathrm{d}I_2}{\mathrm{d}t} = -Y_1, \qquad T_1 \le t \le T_2, \quad I_2(T_2) = 0 \tag{2}$$

$$\frac{\mathrm{d}I_3}{\mathrm{d}t} = -(a_1 + a_2t + a_3t^2 - Y(1-a)), \qquad 0 \le t \le T_1, \quad I_3(0) = 0 \tag{3}$$

$$\frac{\mathrm{d}I_4}{\mathrm{d}t} = Y_1(1-a) - (a_1 + a_2t + a_3t^2), \qquad T_1 \le t \le T_2, \quad I_4(T_2) = 0 \tag{4}$$

$$\frac{\mathrm{d}I_5}{\mathrm{d}t} = R - (a_1 + a_2t + a_3t^2), \qquad T_2 \le t \le T_3, \quad I_5(T_2) = 0 \tag{5}$$

$$\frac{\mathrm{d}I_6}{\mathrm{d}t} = -(a_1 + a_2t + a_3t^2), \qquad T_3 \le t \le T, \quad I_6(T) = 0. \tag{6}$$

Similarly, the differential equations describe the instantaneous states of the reworkable inventory level as follows:

$$\frac{\mathrm{d}I_7}{\mathrm{d}t} = Ya, \qquad 0 \le t \le T_1, \quad I_7(0) = 0 \tag{7}$$

$$\frac{\mathrm{d}I_8}{\mathrm{d}t} = Y_1a, \qquad T_1 \le t \le T_2, \quad I_8(T_1) = I_7(T_1) \tag{8}$$

$$\frac{\mathrm{d}I_9}{\mathrm{d}t} = -R, \qquad T_2 \le t \le T_3, \quad I_9(T_3) = 0. \tag{9}$$

Using the boundary conditions, we solve the above first-order differential equations to get the following set of solutions:

$$I_1(t) = (P - Y)t, \qquad 0 \le t \le T_1 \tag{10}$$
$$I_2(t) = Y_1(T_2 - t), \qquad T_1 \le t \le T_2 \tag{11}$$
$$I_3(t) = Y(1-a)t - \left(a_1 t + \frac{a_2 t^2}{2} + \frac{a_3 t^3}{3}\right), \qquad 0 \le t \le T_1 \tag{12}$$
$$I_4(t) = Y_1(1-a)(t - T_2) + a_1(T_2 - t) + \frac{a_2}{2}(T_2^2 - t^2) + \frac{a_3}{3}(T_2^3 - t^3), \qquad T_1 \le t \le T_2 \tag{13}$$
$$I_5(t) = R(t - T_2) + a_1(T_2 - t) + \frac{a_2}{2}(T_2^2 - t^2) + \frac{a_3}{3}(T_2^3 - t^3), \qquad T_2 \le t \le T_3 \tag{14}$$
$$I_6(t) = a_1(T - t) + \frac{a_2}{2}(T^2 - t^2) + \frac{a_3}{3}(T^3 - t^3), \qquad T_3 \le t \le T \tag{15}$$
$$I_7(t) = Yat, \qquad 0 \le t \le T_1 \tag{16}$$
$$I_8(t) = Y_1 a(t - T_1) + YaT_1, \qquad T_1 \le t \le T_2 \tag{17}$$
$$I_9(t) = R(T_3 - t), \qquad T_2 \le t \le T_3. \tag{18}$$

Using $I_8(T_2) = I_9(T_2)$, $I_1(T_1) = I_2(T_1)$ and $I_5(T_3) = I_6(T_3)$, the following relations can be obtained

$$T_3 = T_2 + \frac{1}{R}[YaT_1 + Y_1 a(T_2 - T_1)] \tag{19}$$
$$T_2 = \frac{1}{Y_1}[(P - Y)T_1 + Y_1 T_1] \tag{20}$$
$$\begin{aligned} a_1 T + \frac{a_2}{2}T^2 + \frac{a_3}{3}T^3 &= a_1 T_3 + \frac{a_2}{2}T_3^2 + \frac{a_3}{3}T_3^3 + R(T_3 - T_2) + a_1(T_2 - T_3) \\ &\quad + \frac{a_2}{2}(T_2^2 - T_3^2) + \frac{a_3}{3}(T_2^3 - T_3^3). \end{aligned} \tag{21}$$

Generation of cost components

At the primary manufacturer, there are various types of costs arise *e.g.*, production cost, rework cost, set-up cost, shortage cost, holding cost, screening cost and backorder cost.

$$\text{Production cost per cycle} = C_p P T_1 \tag{22}$$
$$\text{Rework cost per cycle} = C_r R(T_3 - T_2) \tag{23}$$
$$\text{Set-up cost per cycle} = C_s \tag{24}$$
$$\text{Shortage cost per cycle} = C_{msh}\left(\frac{a_1}{2}T_1^2 + \frac{a_2}{6}T_1^3 + \frac{a_3}{12}T_1^4 - Y(1-a)\frac{T_1^2}{2}\right) \tag{25}$$

$$\begin{aligned} \text{Holding cost per cycle} &= H_m\left[\int_0^{T_1} I_1(t)\,\mathrm{d}t + \int_{T_1}^{T_2} I_2(t)\,\mathrm{d}t + \int_{T_2}^{T_3} I_5(t)\,\mathrm{d}t + \int_{T_3}^{T} I_6(t)\,\mathrm{d}t\right] \\ &= H_m\Bigg[(P - Y)\frac{T_1^2}{2} + Y_1\left\{T_2(T_2 - T_1) - \frac{T_2^2 - T_1^2}{2}\right\} + R\left\{\frac{T_3^2 - T_2^2}{2} - T_2(T_3 - T_2)\right\} \\ &\quad + a_1\left\{T_2(T_3 - T_2) - \frac{T_3^2 - T_2^2}{2}\right\} + \frac{a_2}{2}\left\{T_2^2(T_3 - T_2) - \frac{T_3^3 - T_2^3}{3}\right\} \\ &\quad + \frac{a_3}{3}\left\{T_2^3(T_3 - T_2) - \frac{T_3^4 - T_2^4}{4}\right\} + a_1\left\{T(T - T_3) - \frac{T^2 - T_3^2}{2}\right\} \end{aligned}$$

$$+\frac{a_2}{2}\left\{T^2(T-T_3)-\frac{T^3-T_3^3}{3}\right\}+\frac{a_3}{3}\left\{T^3(T-T_3)-\frac{T^4-T_3^4}{4}\right\}\Bigg] \tag{26}$$

$$\text{Screening cost per cycle} = YT_1C_1e^{C_2Y}+Y_1(T_2-T_1)C_3e^{C_4Y_1} \tag{27}$$

$$\begin{aligned}\text{Backorder cost per cycle} &= C_b\int_{T_1}^{T_2} I_4(t)\,\mathrm{d}t\\ &= C_b\Bigg[Y_1(1-a)\left\{\frac{T_2^2-T_1^2}{2}-T_2(T_2-T_1)\right\}+a_1\left\{T_2(T_2-T_1)-\frac{T_2^2-T_1^2}{2}\right\}\\ &\quad+\frac{a_2}{2}\left\{T_2^2(T_2-T_1)-\frac{T_2^3-T_1^3}{3}\right\}+\frac{a_3}{3}\left\{T_2^3(T_2-T_1)-\frac{T_2^4-T_1^4}{4}\right\}\Bigg].\end{aligned} \tag{28}$$

Using equations (2)–(28), we obtain the total cost per cycle at the primary manufacturer

$$\begin{aligned}\mathrm{TC_{PM}} &= \text{Production cost}+\text{Rework cost}+\text{Set-up cost}+\text{Shortage cost}+\text{Holding cost}\\ &\quad+\text{Screening cost}+\text{Backorder cost}\\ &= C_pPT_1+C_rR(T_3-T_2)+C_s+C_{msh}\left(\frac{a_1}{2}T_1^2+\frac{a_2}{6}T_1^3+\frac{a_3}{12}T_1^4-Y(1-a)\frac{T_1^2}{2}\right)\\ &\quad+H_m\Bigg[(P-Y)\frac{T_1^2}{2}+Y_1\left\{T_2(T_2-T_1)-\frac{T_2^2-T_1^2}{2}\right\}+R\left\{\frac{T_3^2-T_2^2}{2}-T_2(T_3-T_2)\right\}\\ &\quad+a_1\left\{T_2(T_3-T_2)-\frac{T_3^2-T_2^2}{2}\right\}+\frac{a_2}{2}\left\{T_2^2(T_3-T_2)-\frac{T_3^3-T_2^3}{3}\right\}\\ &\quad+\frac{a_3}{3}\left\{T_2^3(T_3-T_2)-\frac{T_3^4-T_2^4}{4}\right\}+a_1\left\{T(T-T_3)-\frac{T^2-T_3^2}{2}\right\}\\ &\quad+\frac{a_2}{2}\left\{T^2(T-T_3)-\frac{T^3-T_3^3}{3}\right\}+\frac{a_3}{3}\left\{T^3(T-T_3)-\frac{T^4-T_3^4}{4}\right\}\Bigg]\\ &\quad+YT_1C_1e^{C_2Y}+Y_1(T_2-T_1)C_3e^{C_4Y_1}\\ &\quad+C_b\Bigg[Y_1(1-a)\left\{\frac{T_2^2-T_1^2}{2}-T_2(T_2-T_1)\right\}+a_1\left\{T_2(T_2-T_1)-\frac{T_2^2-T_1^2}{2}\right\}\\ &\quad+\frac{a_2}{2}\left\{T_2^2(T_2-T_1)-\frac{T_2^3-T_1^3}{3}\right\}+\frac{a_3}{3}\left\{T_2^3(T_2-T_1)-\frac{T_2^4-T_1^4}{4}\right\}\Bigg].\end{aligned} \tag{29}$$

3.2.2. Primary retailer

Due to shortages, the primary manufacturer cannot transport inventory in an equal shipment to the primary retailer. To reduce the shortages and maintain goodwill, the manufacturer ships inventory in n unequal shipments [15], in which the first lot is of size q, the second lot of size ql,....., nth lot of size ql^{n-1} [4]. As a result, the production batch size delivered to the retailer by the manufacturer is as follows (Fig. 4):

$$q+ql+ql^2+...+ql^{n-1}=q\frac{l^n-1}{l-1}\cdot \tag{30}$$

We denote the amount of received inventory by U, *i.e.*, $U=a_1(T-T_2)+\frac{a_2}{2}(T^2-T_2^2)+\frac{a_3}{3}(T^3-T_2^3)+Y(1-a)T_1+Y_1(1-a)(T_2-T_1)$.
Therefore, the size of the first lot is

$$q=U\frac{l-1}{l^n-1}\cdot \tag{31}$$

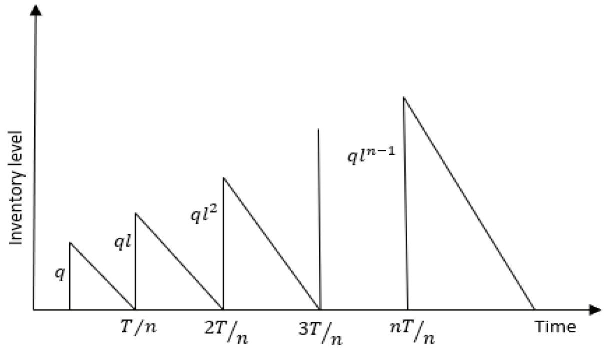

FIGURE 4. Inventory illustration at the primary retailer.

Generation of cost components

$$\text{Purchase cost per cycle} = C_{rp}\left(q\frac{l^n-1}{l-1}\right) \tag{32}$$

$$\text{Ordering cost per cycle} = C_o \tag{33}$$

$$\text{Holding cost per cycle} = H_s\left[\frac{qT}{2n}\left\{\frac{1-l^n}{(1-l)^2} - \frac{nl^n}{1-l}\right\}\right]. \tag{34}$$

Using equations (32)–(34), we obtain the total cost per cycle at the primary retailer

$$\begin{aligned} \mathrm{TC}_{\mathrm{PR}} &= \text{Purchase cost} + \text{Ordering cost} + \text{Holding cost} \\ &= C_{rp}\left(q\frac{l^n-1}{l-1}\right) + C_o + H_s\left[\frac{qT}{2n}\left\{\frac{1-l^n}{(1-l)^2} - \frac{nl^n}{1-l}\right\}\right]. \end{aligned} \tag{35}$$

3.2.3. Collection center

The collection center collects all the returned products used by the end customers to reduce the environmental impact of waste accumulation. It plays a key role in the recycling and reverse logistics processes. The collection center collects η fraction of delivered product, *i.e.*, ηU for the next cycle. When used products are returned, the collection center inspects and categorizes them based on the severity of defects. The minor defective products are sent to the repairable center, and the major defective products to the scrap dealer.

Generation of cost components

$$\text{Collection cost per cycle} = C_{uc}\eta U \tag{36}$$

$$\text{Screening cost per cycle} = C_{sr}\eta U \tag{37}$$

$$\text{Holding cost per cycle} = H_c\frac{(\eta U)^2}{2}\cdot \tag{38}$$

Using equations (36)–(38), we obtain the total cost per cycle at the collection center

$$\begin{aligned} \mathrm{TC}_{CC} &= \text{Collection cost} + \text{Screening cost} + \text{Holding cost} \\ &= C_{uc}\eta U + C_{sr}\eta U + H_c\frac{(\eta U)^2}{2}\cdot \end{aligned} \tag{39}$$

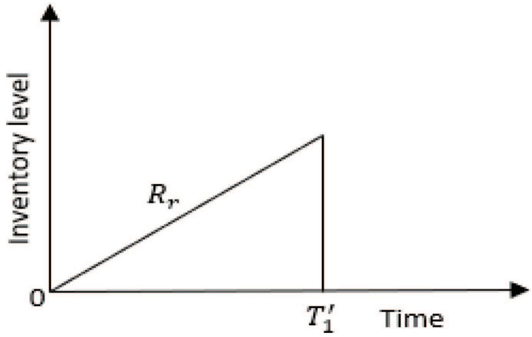

FIGURE 5. Repairable inventory at the repairable center.

3.2.4. Repairable center

The repairable center collects $(1-\gamma)\eta U$ units of minor defective used products from the collection center and repairs at a rate of R_r [5]. Therefore, the total time required at the repairable center for repairing the defective product is (Fig. 5)

$$T_1' = \frac{(1-\gamma)\eta U}{R_r}\cdot \tag{40}$$

The differential equation represents the amount of repairable inventory is given by:

$$\frac{\mathrm{d}I_{10}}{\mathrm{d}t} = R_r, \quad 0 \leq t \leq T_1', \quad I_{10}(0) = 0. \tag{41}$$

Using the boundary condition, the solution of the above equation can be represented by:

$$I_{10}(t) = R_r t, \quad 0 \leq t \leq T_1'. \tag{42}$$

Generation of cost components

$$\text{Repairable cost per cycle} = C_{rr} R_r T_1' \tag{43}$$

$$\text{Holding cost per cycle} = H_r R_r \frac{T_1'^2}{2}\cdot \tag{44}$$

Using equations (43) and (44), we obtain the total cost per cycle at the repairable center

$$\begin{aligned} \mathrm{TC}_{RC} &= \text{Repairable cost} + \text{Holding cost} \\ &= C_{rr} R_r T_1' + H_r R_r \frac{T_1'^2}{2}\cdot \end{aligned} \tag{45}$$

3.2.5. Scrap dealer

The scrap dealer screens the major defective items, *i.e.*, $\gamma\eta U$ units received from the collection center and discards $(1-\alpha)\gamma\eta U$ units of products.

Generation of cost components

$$\text{Screening cost per cycle} = C_{scrsd} \gamma\eta U \tag{46}$$

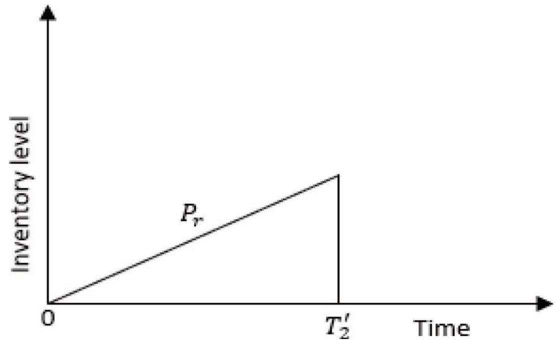

FIGURE 6. Structure of remanufacturing inventory.

$$\text{Salvage cost per cycle} = C_{ssd}(1-\alpha)\gamma\eta U. \tag{47}$$

Therefore, the total cost per cycle at the scrap dealer is given by

$$\begin{aligned}\mathrm{TC}_{SD} &= \text{Screening cost} + \text{Salvage cost}\\ &= C_{scrsd}\gamma\eta U + C_{ssd}(1-\alpha)\gamma\eta U.\end{aligned} \tag{48}$$

3.2.6. Secondary manufacturing center

The secondary manufacturer receives $\alpha\gamma\eta U$ units of products from the scrap dealer and remanufactures them at a rate of P_r [6]. Therefore, the time required for the remanufacturing at the secondary manufacturing center is obtained as (Fig. 6)

$$T_2' = \frac{\alpha\gamma\eta U}{P_r}\cdot \tag{49}$$

Generation of cost components

$$\text{Secondary manufacturing cost per cycle} = C_{sm}P_rT_2' \tag{50}$$

$$\text{Holding cost per cycle} = H_{sm}P_r\frac{T_2'^2}{2}\cdot \tag{51}$$

Thus, the total cost per cycle at the secondary manufacturing center is given by

$$\begin{aligned}\mathrm{TC}_{SM} &= \text{Secondary manufacturing cost} + \text{Holding cost}\\ &= C_{sm}P_rT_2' + H_{sm}P_r\frac{T_2'^2}{2}\cdot\end{aligned} \tag{52}$$

3.2.7. Secondary retailer

Secondary retailers wait until they receive remanufactured inventory from the repairable center or the secondary manufacturing center. Consequently, shortage arises due to unfulfilled customer demand. There are two cases depending on the value of α, γ, P_r and R_r.

In some cases, $T_1' < T_2'$, particularly when γ and α are small, making $\gamma\alpha$ small enough to be dominated by the factor $\frac{1-\gamma}{R_r}$. In other cases, $T_1' > T_2'$, which may occur when γ is close to 1, reducing $1-\gamma$ significantly, while $\gamma\alpha$ remains relatively large due to α. Therefore, demand fulfillment and the partial backlogging process starts at time T_1' or T_2', depending on the value of α and γ.

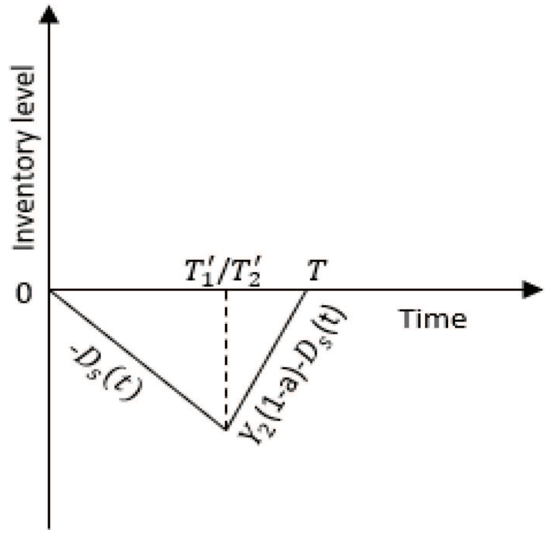

FIGURE 7. Inventory illustration at the secondary retailer.

The instantaneous states of the serviceable inventory at the secondary retailer is represented by the following equations:

$$\frac{\mathrm{d}I_{11}}{\mathrm{d}t} = -D_s(t), \qquad 0 \le t \le T_1'/T_2', \quad I_{11}(0) = 0 \tag{53}$$

$$\frac{\mathrm{d}I_{12}}{\mathrm{d}t} = Y_2(1-a) - D_s(t), \qquad T_1'/T_2' \le t \le T, \quad I_{12}(T) = 0. \tag{54}$$

Using the boundary conditions, we obtain the solution of the above differential equations:

$$I_{11}(t) = -D_s(t)t, \quad 0 \le t \le T_1'/T_2' \tag{55}$$

$$I_{12}(t) = \Big(Y_2(1-a) - D_s(t)\Big)(t-T). \tag{56}$$

From the above figure (Fig. 7), it is clear that the secondary retailer initiates the process of meeting customer demand at either T_1' or T_2'. So, there are two cases depending on T_1' and T_2'.

Generation of cost components

Case I: When $T_1' < T_2'$

$$\text{Shortage cost per cycle} = C_{ssh}D_s(t)\frac{T_1'^2}{2} \tag{57}$$

$$\text{Backorder cost per cycle} = C_{bs}\Big[Y_2(1-a) - D_s(t)\Big]\left[\frac{T^2 - T_1'^2}{2} - T(T-T_1')\right] \tag{58}$$

$$\text{Salvage cost per cycle} = C_{srs}Y_2a(T-T_1') \tag{59}$$

$$\text{Screening cost per cycle} = C_5e^{C_6Y_2}Y_2(T-T_1'). \tag{60}$$

Therefore, the total cost per cycle at the secondary retailer is given by:

$$\mathrm{TC}_{SR} = \text{Shortage cost} + \text{Backorder cost} + \text{Salvage cost} + \text{Screening cost}$$

$$
\begin{aligned}
&= C_{ssh} D_s(t) \frac{T_1'^2}{2} + C_{bs}\Big[Y_2(1-a) - D_s(t)\Big]\left[\frac{T^2 - T_1'^2}{2} - T(T - T_1')\right] \\
&\quad + C_{srs} Y_2 a (T - T_1') + C_5 e^{C_6 Y_2} Y_2 (T - T_1'). \qquad (61)
\end{aligned}
$$

Case II: When $T_2' < T_1'$

$$\text{Shortage cost per cycle} = C_{ssh} D_s(t) \frac{T_2'^2}{2} \qquad (62)$$

$$\text{Backorder cost per cycle} = C_{bs}\Big[Y_2(1-a) - D_s(t)\Big]\left[\frac{T^2 - T_2'^2}{2} - T(T - T_2')\right] \qquad (63)$$

$$\text{Salvage cost per cycle} = C_{srs} Y_2 a (T - T_2') \qquad (64)$$

$$\text{Screening cost per cycle} = C_5 e^{C_6 Y_2} Y_2 (T - T_2'). \qquad (65)$$

Therefore, the total cost per cycle at the secondary retailer is given by:

$$
\begin{aligned}
\mathrm{TC}_{SR} &= \text{Shortage cost} + \text{Backorder cost} + \text{Salvage cost} + \text{Screening cost} \\
&= C_{ssh} D_s(t) \frac{T_2'^2}{2} + C_{bs}\Big[Y_2(1-a) - D_s(t)\Big]\left[\frac{T^2 - T_2'^2}{2} - T(T - T_2')\right] \\
&\quad + C_{srs} Y_2 a (T - T_2') + C_5 e^{C_6 Y_2} Y_2 (T - T_2'). \qquad (66)
\end{aligned}
$$

3.3. Calculation of total cost across the forward and reverse supply chain

Using equations (29)–(61), the total inventory cost in a forward and reverse supply chain flow (in case $T_1' < T_2'$) is obtained as

$$
\begin{aligned}
\mathrm{TC} &= \mathrm{TC}_{\mathrm{PM}} + \mathrm{TC}_{\mathrm{PR}} + \mathrm{TC}_{CC} + \mathrm{TC}_{RC} + \mathrm{TC}_{SD} + \mathrm{TC}_{SM} + \mathrm{TC}_{SR} \\
&= C_p P T_1 + C_r R (T_3 - T_2) + C_s + C_{msh}\left(\frac{a_1}{2} T_1^2 + \frac{a_2}{6} T_1^3 + \frac{a_3}{12} T_1^4 - Y(1-a)\frac{T_1^2}{2}\right) \\
&\quad + H_m\Bigg[(P - Y)\frac{T_1^2}{2} + Y_1\left\{T_2(T_2 - T_1) - \frac{T_2^2 - T_1^2}{2}\right\} + R\left\{\frac{T_3^2 - T_2^2}{2} - T_2(T_3 - T_2)\right\} \\
&\quad + a_1\left\{T_2(T_3 - T_2) - \frac{T_3^2 - T_2^2}{2}\right\} + \frac{a_2}{2}\left\{T_2^2(T_3 - T_2) - \frac{T_3^3 - T_2^3}{3}\right\} \\
&\quad + \frac{a_3}{3}\left\{T_2^3(T_3 - T_2) - \frac{T_3^4 - T_2^4}{4}\right\} + a_1\left\{T(T - T_3) - \frac{T^2 - T_3^2}{2}\right\} \\
&\quad + \frac{a_2}{2}\left\{T^2(T - T_3) - \frac{T^3 - T_3^3}{3}\right\} + \frac{a_3}{3}\left\{T^3(T - T_3) - \frac{T^4 - T_3^4}{4}\right\}\Bigg] \\
&\quad + Y T_1 C_1 e^{C_2 Y} + Y_1 (T_2 - T_1) C_3 e^{C_4 Y_1} \\
&\quad + C_b\Bigg[Y_1(1-a)\left\{\frac{T_2^2 - T_1^2}{2} - T_2(T_2 - T_1)\right\} + a_1\left\{T_2(T_2 - T_1) - \frac{T_2^2 - T_1^2}{2}\right\} \\
&\quad + \frac{a_2}{2}\left\{T_2^2(T_2 - T_1) - \frac{T_2^3 - T_1^3}{3}\right\} + \frac{a_3}{3}\left\{T_2^3(T_2 - T_1) - \frac{T_2^4 - T_1^4}{4}\right\}\Bigg] \\
&\quad + C_{rp}\left(q\frac{l^n - 1}{l - 1}\right) + C_o + H_s\left[\frac{qT}{2n}\left\{\frac{1 - l^n}{(1-l)^2} - \frac{n l^n}{1 - l}\right\}\right]
\end{aligned}
$$

$$+ C_{uc}\eta U + C_{sr}\eta U + H_c\frac{(\eta U)^2}{2} + C_{rr}R_rT_1' + H_rR_r\frac{T_1'^2}{2}$$
$$+ C_{scrsd}\gamma\eta U + C_{ssd}(1-\alpha)\gamma\eta U + C_{sm}P_rT_2' + H_{sm}P_r\frac{T_2'^2}{2} + C_{ssh}D_s(t)\frac{T_1'^2}{2}$$
$$+ C_{bs}\left[Y_2(1-a) - D_s(t)\right]\left[\frac{T^2 - T_1'^2}{2} - T(T-T_1')\right] + C_{srs}Y_2a(T-T_1') + C_5e^{C_6Y_2}Y_2(T-T_1'). \quad (67)$$

3.4. Calculation of carbon emission across the supply chain

Carbon emissions in supply chains refer to the greenhouse gases (primarily CO_2) generated by activities involved in producing, reworking, holding and delivering goods. These emissions can arise at various stages, including raw material extraction, manufacturing, warehousing and logistics. As global concerns about climate change intensifies, companies face increasing pressure to minimize their carbon footprints throughout the supply chain. Reducing supply chain emissions involves improving energy efficiency, adopting renewable energy sources and implementing sustainable practices.
Emission related to the production and rework setup $= e_{S_P} + e_{S_R}$
Emission related to production $= e_pPT_1$
Emission related to rework $= e_rR(T_3 - T_1)$
Emission related to holding items at the primary manufacturer =

$$e_w\left[\int_0^{T_1} I_1(t)\,\mathrm{d}t + \int_{T_1}^{T_2} I_2(t)\,\mathrm{d}t + \int_{T_2}^{T_3} I_5(t)\,\mathrm{d}t + \int_{T_3}^{T} I_6(t)\,\mathrm{d}t\right]$$

Emission related to the holding items at the primary retailer $= e_{P_r}\left[\frac{qT}{2n}\left\{\frac{1-l^n}{(1-l)^2} - \frac{nl^n}{1-l}\right\}\right]$
Emission related to the holding items at the collection center $= e_{C_c}\frac{(\eta U)^2}{2}$
Emission related to the repairable setup $= e_{S_{R_r}}$
Emission related to the repairable items $= e_{R_r}R_rT_1'$
Emission related to holding items at the repairable center $= e_{RC}R_r\frac{T_1'^2}{2}$ Emission related to the secondary manufacturing setup $= e_{S_{S_M}}$
Emission related to the secondary manufacturing items $= e_{S_M}P_rT_2'$
Emission related to the holding items at the secondary manufacturing center $= e_{SM}P_r\frac{T_2'^2}{2}$
Emission related to salvage items at the scrap dealer $= e_{S_{s_d}}(1-\alpha)\gamma\eta U$
Emission related to salvage items at the secondary retailer $= e_{S_{s_r}}Y_2a(T - T_1')$, when $T_1' < T_2'$.

Therefore, the total carbon emission from the forward and reverse supply chain is obtained as

$$\mathrm{TCE} = e_{S_P} + e_{S_R} + e_pPT_1 + e_rR(T_3 - T_1) + e_w\left[(P-Y)\frac{T_1^2}{2} + Y_1\left\{T_2(T_2 - T_1) - \frac{T_2^2 - T_1^2}{2}\right\}\right.$$
$$+ R\left\{\frac{T_3^2 - T_2^2}{2} - T_2(T_3 - T_2)\right\} + a_1\left\{T_2(T_3 - T_2) - \frac{T_3^2 - T_2^2}{2}\right\}$$
$$+ \frac{a_2}{2}\left\{T_2^2(T_3 - T_2) - \frac{T_3^3 - T_2^3}{3}\right\} + \frac{a_3}{3}\left\{T_2^3(T_3 - T_2) - \frac{T_3^4 - T_2^4}{4}\right\}$$
$$+ a_1\left\{T(T - T_3) - \frac{T^2 - T_3^2}{2}\right\} + \frac{a_2}{2}\left\{T^2(T - T_3) - \frac{T^3 - T_3^3}{3}\right\}$$
$$\left. + \frac{a_3}{3}\left\{T^3(T - T_3) - \frac{T^4 - T_3^4}{4}\right\}\right] + e_{P_r}\left[\frac{qT}{2n}\left\{\frac{1-l^n}{(1-l)^2} - \frac{nl^n}{1-l}\right\}\right]$$

$$+ e_{C_c}\frac{(\eta U)^2}{2} + e_{S_{R_r}} + e_{R_r}R_rT_1' + e_{RC}R_r\frac{T_1'^2}{2} + e_{S_{S_M}} + e_{S_M}P_rT_2' + e_{SM}P_r\frac{T_2'^2}{2}$$
$$e_{S_{s_d}}(1-\alpha)\gamma\eta U + e_{S_{s_r}}Y_2a(T-T_1'), \quad \text{when} \quad T_1' < T_2'. \tag{68}$$

Therefore, our objective of the proposed model is given by

$$\begin{cases} & \text{Minimize } \left\{\frac{1}{T}\text{TC}, \frac{1}{T}\text{TCE}\right\} \\ \text{Subject to} & \quad Y < Y_1 \\ & Y(1-a)T_1 < a_1T_1 + \frac{a_2}{2}T_1^2 + \frac{a_3}{3}T_1^3 \\ & Y_1(1-a)(T_2-T_1) > a_1(T_2-T_1) + \frac{a_2}{2}(T_2^2-T_1^2) + \frac{a_3}{3}(T_2^3-T_1^3) \\ & Y_2(1-a) > D_s(t). \end{cases} \tag{69}$$

4. Case study and numerical illustration

The demand for batteries, particularly lithium-ion batteries used in electric vehicles (EVs), has increased significantly in recent years. As the number of EVs on the road continues to grow, battery remanufacturing has become essential for enhancing sustainability and reducing costs. However, due to high demand, manufacturers often face supply shortages. Once production halts, they can shift their focus entirely to screening and reworking defective units to address backlog shortages. Additionally, when demand is time-dependent, estimating the optimal screening rate becomes increasingly challenging. During our research, we identified several decision-making challenges raised by the manager, which were integrated into our research questions:

(a) What will be the optimal production run time that will minimize the overall cost of the system?
(b) What will be the optimal increasing screening rate that helps to reduce shortages?
(c) What will be the optimal screening rate at the secondary retailer to reduce shortages?
(d) Who devotes less time to produce secondary products: the repairable center or secondary manufacturer?

The data for numerical illustration is shown in Table 2.

5. Solution methodology

NSGA-II is a popular multi-objective optimization technique. It operates based on evolutionary principles, aiming to find a set of optimal solutions (Pareto-optimal solutions) for problems with multiple conflicting objectives. NSGA-II begins with the generation of an initial population of solutions, where each solution is represented as a vector of decision variables. Each solution is then evaluated with respect to two objectives, namely total cost and carbon emissions. Fast non-dominated sorting is applied to classify solutions into different Pareto fronts based on dominance relations. Within each front, a crowding distance measure is assigned to preserve diversity and ensure a well-spread distribution of solutions. Pareto solutions are selected using binary tournament selection, considering both rank and crowding distance. New offspring solutions are then generated through genetic operators: crossover, which combines the characteristics of two parents, and mutation, which introduces random variations. The parent and offspring populations are merged, and elitism is applied to retain the best-ranked solutions for the next generation. This iterative process continues until a termination criterion, such as a maximum number of generations, is satisfied. The final result is a diverse set of non-dominated solutions that approximate the Pareto front, illustrating the trade-off between minimizing cost and minimizing carbon emissions. Below is a step-by-step breakdown of the solution process:

Table 2. Numerical data from case study.

Parameter	Numerical value	Parameter	Numerical value
P	3000 units/month	D_s	2500 units/month
P_r	5000 units/month	R	7500 units/month
R_r	7000 units/month	C_p	30 /unit
C_s	76000 /set-up	C_{msh}	5 /unit
C_r	5 /unit	C_b	3 /unit
C_{rp}	45 /unit	C_o	1000/order
C_{up}	3 /unit	C_{rr}	8 /unit
C_{sm}	12 /unit	C_{ssh}	4 /unit
C_{bs}	2 /unit	C_{sr}	0.1 /unit
H_m	40 units/month	H_s	35 units/month
H_c	43 units/month	H_r	45 units/month
H_{sm}	39 units/month	l	1.06
n	6	C_{ssd}	5 /unit
C_{scrsd}	0.1/unit	C_{srs}	5 /unit
γ	0.48	α	0.6
η	0.3	a	0.1
b	0.02	C_{srp}	15 /unit
e_{S_P}	2.7 ton CO_2/set-up	e_{S_R}	2.9 ton CO_2/set-up
e_P	0.6 ton CO_2/unit	e_w	55 ton CO_2/unit/unit time
$e_{S_{R_r}}$	2.8 ton CO_2/set-up	e_{R_r}	0.8 ton CO_2/unit
e_{RC}	52 ton CO_2/unit/unit time	$e_{S_{S_M}}$	2.5 ton CO_2/set-up
e_{S_M}	0.9 ton CO_2/unit	e_{SM}	50 ton CO_2/unit/unit time
e_{P_r}	45 ton CO_2/unit/unit time	e_{C_c}	48 ton CO_2/unit/unit time
$e_{S_{s_d}}$	0.7 ton CO_2/unit	$e_{S_{s_r}}$	0.7 ton CO_2/unit

Algorithm 1. NSGA-II Algorithm [6].

Solution representation, $t := 0$ (generation counter), Maximum allowed generation $= T$;
Initialize random population ($P(t)$);
Evaluate ($P(t)$) and assign rank using dominance depth method and diversity using crowding distance operator to $P(t)$;
while $t \leq T$ **do**
    $M(t) :=$ selection ($P(t)$);
    $Q(t) :=$ variation ($M(t)$);
    Evaluate $Q(t)$;
    Merge population $\hat{P}(t) = (P(t) \bigcup Q(t))$;
    Assign rank using dominance depth method and diversity using crowding distance operator
    to $\hat{P}(t)$;
    $P(t+1) :=$ survivor ($\hat{P}(t)$);
    $t := t + 1$;
end while

Improvement of solutions across generations is achieved through elitism, where the best non-dominated solutions are preserved when forming the next population. The merging of parent and offspring populations, followed by sorting and diversity maintenance, allows the algorithm to gradually refine the population. Over successive iterations, this process drives the solutions closer to the true Pareto front, ultimately providing a diverse set of non-dominated solutions that represent the trade-off between minimizing total cost and minimizing carbon emissions.

TABLE 3. Parameter values of NSGA II.

Population	200
Maximum generation	100
Crossover probability	1.0
Mutation probability	0.33
Distribution index for crossover	20
Distribution index for mutation	100

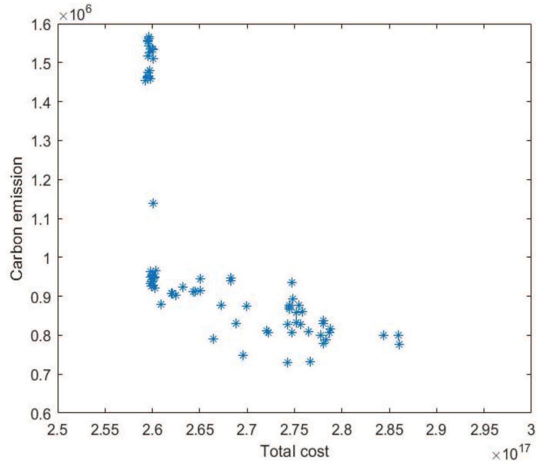

FIGURE 8. Generation 20.

5.1. Results and discussion

The experiments have been conducted using MATLAB R2021a software with the implementation of NSGA II algorithm. The values of the relevant parameters have been shown in Table 3. To analyze the improvement in solution quality, the Pareto fronts for five generations have been shown, which highlights the changes in the objective space and the movement towards better convergence. Figures 8–12 illustrates the Pareto fronts generated over five generations of the optimization process. These fronts represent the progression of non-dominated solutions across the evolutionary algorithm. In the twentieth generation, the Pareto front is widely dispersed, with solutions far from optimality. By the 100 generation, the front shows a significant improvement, with solutions better aligned along the true Pareto front.

Table 4 summarizes the results of the optimization process conducted over 100 generations, repeated 10 times. The table presents the values of the two objective functions, total cost per unit time and total carbon emission per unit time, along with the corresponding decision variables for each execution. In each run, the minimum value from the population of 200 individuals is selected as it represents the best solution for that specific generation, ensuring that the results reflect the optimal performance achieved during the optimization process. These results demonstrate the performance of the algorithm and its ability to generate reliable and consistent solutions across multiple runs. The variation in values across the 10 executions is minimal, highlighting the robustness of the optimization process.

For each generation, the minimum value from the population is recorded during each run. The final optimal value is computed as the average of these minimum values over all 10 runs, providing a reliable estimate of the solution's quality. Therefore, the optimal total cost per unit time is obtained as $TC^* = 2.59 \times 10^{17}$, and the

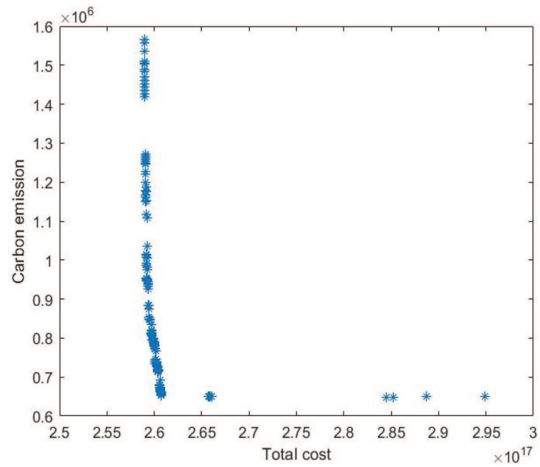

FIGURE 9. Generation 40.

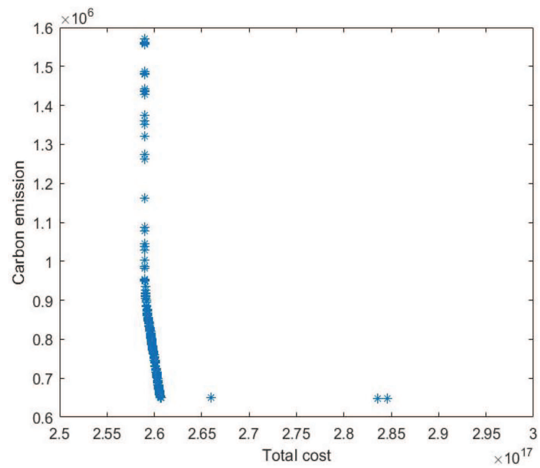

FIGURE 10. Generation 60.

TABLE 4. Results of 100 Generation across 10 runs.

Execution No.	T_1^*	Y_1^*	Y_2^*	Total cost (TC*)	Total carbon emission (TCE*)
1	0.154	1000.003	2777.78	2.59×10^{17}	648 452.6
2	0.154	1000.011	2777.78	2.59×10^{17}	648 485.3
3	0.154	1000.007	2777.78	2.59×10^{17}	648 425.2
4	0.154	1000.007	2777.78	2.59×10^{17}	648 457.2
5	0.154	1000.004	2777.78	2.59×10^{17}	648 415.4
6	0.154	1000.018	2777.78	2.59×10^{17}	648 459.4
7	0.154	1000.018	2777.78	2.59×10^{17}	648 555.7
8	0.154	1000.003	2777.78	2.59×10^{17}	648 511.1
9	0.154	1000.005	2777.78	2.59×10^{17}	648 531.8
10	0.154	1000	2777.78	2.59×10^{17}	648 465.2

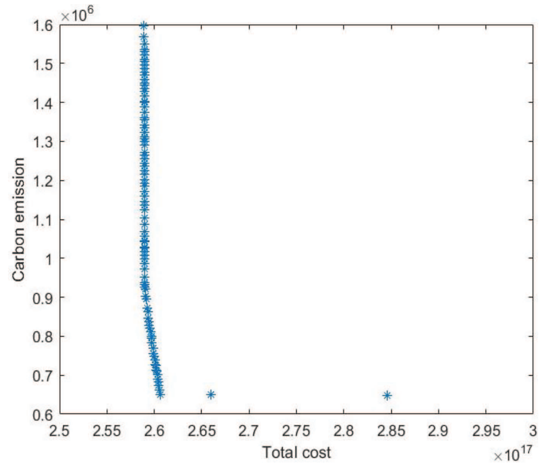

FIGURE 11. Generation 80.

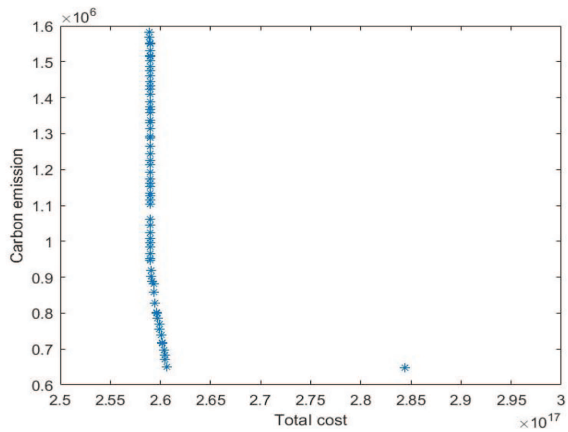

FIGURE 12. Generation 100.

optimal total carbon emission per unit time is $\mathrm{TCE}^* = 648475.9$ followed by $T_1^* = 0.154$ month, $Y_1^* = 1000.008$ and $Y_2^* = 2777.78$.

6. Sensitivity analysis

In this section, we have demonstrated the effect of parameter changes on total cost and carbon emissions. The evaluation is carried out by varying the parameter values by +20%, +10%, −10%, and −20%. To determine the optimal values of total cost (TC^0) and carbon emission (CE^0), each parameter is adjusted individually while keeping the remaining parameters constant. The percentage increase or decrease in the total cost and carbon emission is calculated by $\mathrm{RCTC} = \frac{\mathrm{TC}^0 - \mathrm{TC}^*}{\mathrm{TC}^*} \times 100\%$ and $\mathrm{RCCE} = \frac{\mathrm{TCE}^0 - \mathrm{TCE}^*}{\mathrm{TCE}^*} \times 100\%$. The results of the corresponding sensitivity analysis are given in the following Table 5.

Table 5 describes the sensitivity analysis of the proposed model in the successive changes +20%, +10%, −10%, and −20% of the parameters γ, α, l, η and a by applying the NSGA II algorithm. The relative changes in total cost (RCTC) and carbon emission (RCTCE) offer a comprehensive understanding of the system's behavior under different scenarios. The parameter γ exhibits a marginal influence on both RCTC and RCTCE. From the

TABLE 5. Variation of total cost and carbon emission with parameter changes.

Parameter	% change	Total cost (TC^0)	Carbon emission (TCE^0)	RCTC (%)	RCTCE (%)
γ	+20	2.60×10^{17}	648 496.28	+0.386	+0.003
	+10	2.59×10^{17}	648 472.42	0	−0.001
	−10	2.58×10^{17}	648 499.92	−0.386	+0.004
	−20	2.58×10^{17}	648 585.96	−0.386	+0.017
α	+20	2.59×10^{17}	648 550.74	0	+0.011
	+10	2.59×10^{17}	648 497.56	0	+0.003
	−10	2.59×10^{17}	648 503.8	0	+0.004
	−20	2.59×10^{17}	648 562.84	0	+0.013
l	+20	2.59×10^{17}	649 356.62	0	+0.0136
	+10	2.59×10^{17}	648 932.82	0	+0.070
	−10	2.59×10^{17}	647 981.94	0	−0.076
	−20	2.59×10^{17}	647 426.7	0	−0.162
η	+20	2.58×10^{17}	924 662.32	−0.386	+42.59
	+10	2.58×10^{17}	780 282.16	−0.386	+20.32
	−10	2.59×10^{17}	529 304.42	0	−18.38
	−20	2.60×10^{17}	422 516.28	+0.386	−34.84
a	+20	7.40×10^{17}	613 348.46	+186.10	−5.417
	+10	4.37×10^{17}	630 653.98	+68.72	−2.748
	−10	1.53×10^{17}	667 022.13	−40.93	+2.86
	−20	9.01×10^{16}	686 084.54	−65.21	+5.799
P_r	+20	2.59×10^{17}	648 482.2	0	+0.0009
	+10	2.59×10^{17}	648 500.2	0	+0.0037
	−10	2.59×10^{17}	648 455.1	0	−0.0032
	−20	2.59×10^{17}	648 423.9	0	−0.008
R_r	+20	2.60×10^{17}	648 451	+0.3861	−0.0038
	+10	2.59×10^{17}	648 494.8	0	+0.0029
	−10	2.58×10^{17}	648 494.6	−0.3861	+0.0029
	−20	2.57×10^{17}	648 611.1	−0.7722	+0.021
R	+20	2.59×10^{17}	649 141.9	0	+0.1027
	+10	2.59×10^{17}	648 760.6	0	+0.0439
	−10	2.59×10^{17}	648 205	0	−0.0418
	−20	2.59×10^{17}	647 982.21	0	−0.076
b	+20	2.59×10^{17}	780 088.6	0	+20.2957
	+10	2.59×10^{17}	714 385.9	0	+10.1638
	−10	2.59×10^{17}	583 099.9	0	−10.0815
	−20	2.59×10^{17}	517 873.1	0	−20.1399

variation of γ we can conclude that the system is relatively insensitive to changes in γ, indicating that it has a limited role in the overall optimization of cost and emissions.

The parameter η has a profound and asymmetric impact on carbon emissions. A +20% increase in η significantly raises carbon emissions by +42.39% while reducing total cost by −0.386%. Conversely, a −20% change decreases carbon emissions by −34.84% and increases total cost by +0.386%. This demonstrates that η plays a critical role in balancing cost and environmental objectives. Proper adjustment of η can lead to significant improvements in emissions at a manageable cost trade-off.

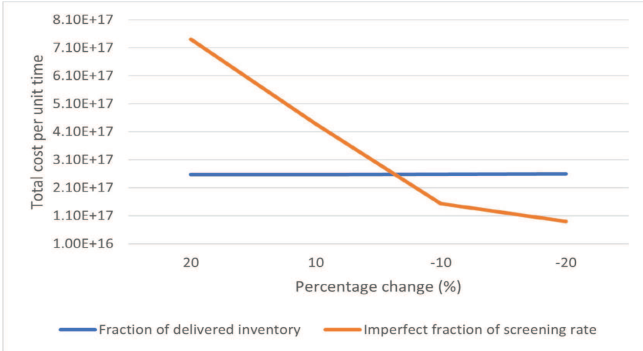

FIGURE 13. Total cost under % change of η and a.

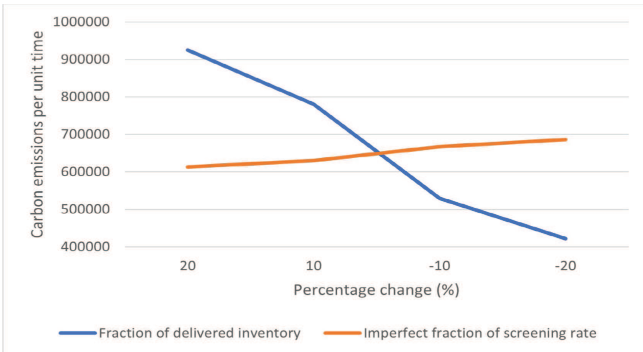

FIGURE 14. Carbon emission under % change of η and a.

However, the parameter a strongly affects both total cost and carbon emissions. A +20% increase in a results in a sharp rise in total cost (+186.10%) and a decrease in carbon emissions (−5.417%). Conversely, a −20% change significantly reduces total cost (−65.21%) and increases carbon emissions (+5.799). These results indicate that a is a highly sensitive parameter with the potential to drive large shifts in system dynamics. This sensitivity underscores the need for careful selection of a to balance cost and emissions.

From Table 5, it can be observed that the variation in parameter R_r retains the conflicting nature of the objective functions. In contrast, the effect of parameter b has a more significant impact, as its increasing trend does not influence the total cost but leads to higher carbon emissions, with a similar trend observed in the reverse flow.

7. Graphical illustration

In this section, we draw some figures based on the data from sensitivity analysis to illustrate the proposed model. Figure 13 reveals the total cost under % change of highly sensitive parameters η and a. The total cost gets the maximum value at the −20% change and minimum value at the +20% change under the fraction of inventory delivered (η) to the primary retailer. However, it becomes maximum at the change +20% and minimum at the change −20% under the imperfect fraction of the screening rate (a).

Similarly, Figure 14 shows the change in carbon emissions under the % change of η and a. Similar views have been raised as in the case of the total cost. The maximum value of carbon emission is achieved at the +20% change and assumes a minimum at the −20% change under the fraction of inventory delivered (η) to the

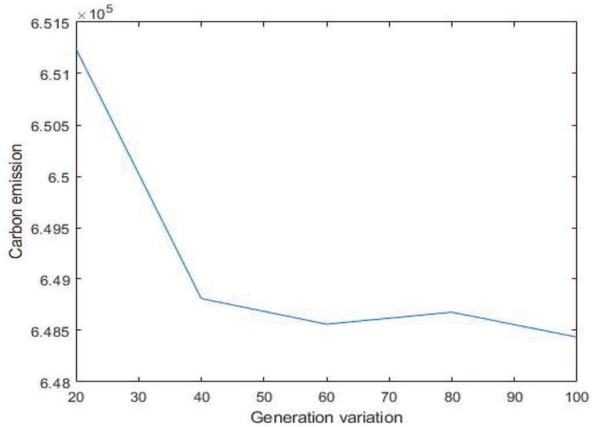

FIGURE 15. Variation of carbon emission under different generations.

primary retailer. Similarly, it becomes maximum with the change −20% and minimum with the change +20% under the imperfect fraction of the screening rate (a).

The Figure 15 shows the variation in carbon emissions over multiple generations during the optimization process. As the number of generation increases, carbon emissions exhibit a decreasing trend, indicating the convergence of the optimization process toward more environmentally sustainable solutions.

8. Managerial insights

Nowadays, reusing used products plays an important role in reducing waste and conserving natural resources. By extending the lifecycle of products through reuse, we can decrease the demand for new raw materials and energy consumption. Indeed, people prioritize buying low-priced products often aim to save money and manage their budgets effectively. A recent study reveals that many researchers are working on forward and reverse supply chains, but they are not giving priority to the imperfect screening process. Therefore, we have developed a forward and reverse supply chain with imperfect screening and unequal shipment size. In this model, the following managerial insights may be developed over here:

(i) A reduced imperfection in screening ensures defective or substandard products are detected and removed early in the supply chain. Therefore, it helps to minimize the carrying costs associated with defective products.
(ii) Improved screening ensures that only high-quality items are stored in inventory, optimizing space and resources.
(iii) Recognizing the primary defects in collected used inventory helps prioritize repairable components and salvageable materials and also helps to maximize the recovery of value from used products, leading to long-term benefits for both businesses and the environment.
(iv) The collection of used products is an essential step in achieving sustainability, reducing costs, and driving innovation. It not only benefits businesses through resource optimization and revenue generation but also contributes to environmental preservation and social responsibility.
(v) The improvement of the Pareto front across generations demonstrates that running NSGA-II for a higher number of iterations leads to more reliable and well-distributed trade-off solutions.
(vi) From the sensitivity analysis table, it can be observed that the imperfect fraction of the screening rate plays a significant role in system performance. An increase in the imperfect fraction leads to a rise in the average system cost, while simultaneously causing a reduction in carbon emissions, and the reverse trend is observed when the imperfect fraction decreases.

9. Conclusion

This study develops a forward and reverse logistics supply chain model where the used inventory can be further used as new inventory. By integrating a variable screening rate with a time-dependent quadratic demand function for the production of new inventory, our research endeavors to develop a closed-loop supply chain (CLSC) model that more realistically captures the dynamics of production, screening, and recovery processes in practice. This integration not only enhances the accuracy of cost estimation and policy formulation but also supports sustainable supply chain management by effectively balancing new production with product returns and reusability. In addition to these aspects, our study also emphasizes the role of unequal shipment sizes and the significance of the proportion of major defective products, both of which are critical for improving the practical applicability and robustness of the proposed model. As a result, we have developed a forward and reverse logistic supply chain model assuming two types of screening rates, unequal shipment and rework of identified imperfect products. For model validation, a case study has been performed and numerical experimentation has been done with the help of NSGA II algorithm. To focus on the novelty, the numerical results have been taken for five generations. Also, we have plotted the Pareto front for five generations to observe the progression of the optimization process. The results show that as the number of generations increases, the Pareto front aligns more closely with the true Pareto front, demonstrating the convergence of the algorithm. For optimality, the solutions from the last generation have been utilized, as they represent the trade-offs between objectives. Therefore, the optimal solution is getting from the "Generation 100" and the optimal production run time is $T_1^* = 0.154$ month, the optimal increased screening rate $Y_1^* = 1000.008$ and the optimal screening rate at the primary retailer is $Y_2^* = 2777.778$ respectively. To observe the effect of parameters, a sensitivity analysis has occurred with respect to the parameters γ, α, l, η, and a.

Moreover, the present model is analyzed using hypothetical data, which may limit its practical applicability, incorporating real datasets could provide more realistic insights.

Scope of future work

Transportation plays an important role in developing a sustainable supply chain. This model can be extended by incorporating transportation in the future.

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Conflict of Interest

The authors declare that there is no conflict of interest regrading the publication of this article.

Data availability statement

No new data is generated for this research.

Author contribution statement

Mou Jana: Writing the original draft, software use, data generation. ***Debjani Chakraborty:*** Conceptualization, supervision, Writing – review and editing. ***Adrijit Goswami:*** Graphical illustration, sensitivity analysis, Writing – review and editing.

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