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A REVIEW PAPER ON THE ECONOMIC LOT SCHEDULING PROBLEM WITH THE COMMON CYCLE APPROACH

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Abstract. Production scheduling is a critical component of modern manufacturing, as it ensures the efficient use of limited and often expensive production resources. The integration of lot sizing and scheduling has therefore attracted significant scholarly attention, offering opportunities to improve capacity utilization and inventory management. This paper reviews and synthesizes the literature on the Economic Lot Scheduling Problem (ELSP), with particular emphasis on studies adopting the Common Cycle (CC) approach. The reviewed works are systematically classified into key categories, including defective production processes, shortage policies, maintenance activities, deteriorating items, objective functions, and modeling constraints. This taxonomy not only organizes the diverse body of research but also highlights thematic trends and methodological contributions across more than three decades of ELSP studies.

Mathematics Subject Classification. 90B30, 90B05.

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1. Introduction

In the late twentieth century, the US Small Business Administration reported that inventory costs account for 45% to 90% of total business expenses [125]. This striking figure highlights the central role of inventory in organizations where limited resources and capital are closely tied to stock management. Poor inventory estimation or excessive stockholding can lead to depreciation, obsolescence, and product deterioration, ultimately increasing operational costs. As a result, optimizing production and inventory levels is crucial for reducing costs and enhancing overall business efficiency.

One of the most influential milestones in inventory theory is the Economic Order Quantity (EOQ) model, first introduced by Ford Whitman Harris in 1913 [131]. His article, "*How Many Parts to Make at Once*", marked the beginning of a century-long research stream on economic lot sizing. Although Harris's work was

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initially overlooked and often mistakenly attributed to Wilson, later studies by Erlenkotter [46–48] clarified Harris's contribution and impact. The EOQ model quickly became one of the most cited and applied concepts in operations management, forming the cornerstone of economic production and ordering decisions.

The basic EOQ model assumes instantaneous replenishment, which is unrealistic for manufacturing contexts. To address this, in 1918, Taft introduced the Economic Production Quantity (EPQ) model, incorporating gradual production and delivery rates. His formulation allowed production batches to be aligned with both machine capacity and customer demand, significantly reducing production and holding costs [119].

In subsequent decades, attention shifted toward more complex settings. In the 1950s, Eilon [42] and Rogers [127] extended EOQ/EPQ concepts to multi-item systems, giving rise to the Economic Lot Scheduling Problem (ELSP). In the context of ELSP models, demand and production rates are known, the sequence of producing them is uncertain, and the focus is on decisions about the cycle scheduling, which is repeated periodically. Due to the complexity and non-linearity of the problem, no specific algorithm is proposed to obtain the optimal solution. Gallego and Shaw [54] showed that the ELSP problem is an NP-Hard (non-deterministic polynomial), which means that solving this problem is impossible in polynomial time without removal of certain conditions and assumptions. Elmaghraby [45] reviewed the literature on ELSP problems until 1976 and highlighted the different approaches to solving this problem. Then, Lopez and Kingsman [93] compared different methods for ELSP until 1990, in terms of different criteria. After that, Silver *et al.* [136] performed another review of the literature on ELSP. They showed that in the literature about ELSP, a number of innovative approaches to problem-solving were developed. Overall, there are five general approaches to solving the ELSP:

(1) *Common Cycle* (CC) or *Rotational Cycle* (RC): The cycle of all items is considered the same. Therefore, the optimal common cycle is easily obtained by this method. The advantage of this approach is to find a feasible solution quickly [68].
(2) *Basic Period* (BP): cycle lengths are integer multiples of a base period. Identical lot sizes are associated with items and finding the feasible solution has complexities [12, 77].
(3) *Time-varying lot size* (TV-LS): Lot sizes vary dynamically. This approach is more complex and time-consuming than two other approaches, but it finds a better solution to the problem [40, 166].
(4) *Independent Solution (IS) and miscellaneous heuristics*: Analytical relaxations providing bounds. A well-known approach of this kind is independent solution (IS) approach that solves the problem analytically by relaxing synchronization constraint. Consequently, the solution obtained by IS provides a lower bound of optimal solution (in some cases the solution is infeasible due to the relaxing constraint) [19].
(5) *Hybrid/other approaches* that combine problem-specific heuristics and metaheuristics.

Among the various approaches to ELSP, the Common Cycle (CC) policy has emerged as one of the most widely adopted due to its simplicity, feasibility, and strong industrial applicability. While the CC approach may not always deliver the theoretical optimum, it offers practically implementable solutions and enables coordinated scheduling in real-world production environments. Its relevance is evident across diverse industries such as pharmaceuticals (batch production of drugs), automotive (parts scheduling), food processing (multi-product production lines), and electronics (component manufacturing) – all of which involve a single facility managing multiple products under capacity constraints.

Empirical evidence further underscores the prominence of CC in ELSP research. Chan *et al.* [19] analyzed 105 papers published between 1997 and 2012 and reported that CC was the most frequently employed policy, appearing in 41% of studies (46 articles). Similarly, Santander-Mercado and Jubiz-Díaz [130] reviewed 126 ELSP papers from 1958 to 2015 and found that approximately 32% (40 papers) utilized the CC approach. Expanding on these efforts, Beck and Glock [8] conducted a comprehensive content analysis of 242 ELSP papers published up to 2019. Their coding-based methodology categorized research streams across scheduling policies, solution methods, assumptions, and extensions, providing a broad overview of the field's evolution. However, their review did not delve into detailed assessments of individual scheduling policies. In particular, the CC approach – despite its widespread use and practical relevance – remains underexplored in terms of its solution methodologies, modeling frameworks, and managerial implications.

Then, Martinelli *et al.* [96] a systematic literature review on single-machine sequencing problems in make-to-order (MTO) environments, emphasizing Industry 4.0 contexts, sequencing complexity, and heuristic *vs.* exact solution procedures. That study focused primarily on order sequencing challenges such as setup times, due dates, processing times, and uncertainties that are typical in MTO environments. Recently, Ying *et al.* [165] conducted a large-scale systematic review of the Single-Machine Scheduling Problem (SMSP) using citation-based techniques, namely Main Path Analysis (MPA) and Cluster Analysis (CA). By examining 2904 articles published since 1987, their study identified the main developmental trajectories of SMSPs, highlighted the most influential works in different periods, and mapped out key thematic clusters such as deterioration and learning effects, constraint-based scheduling, maintenance-related extensions, advanced heuristic methods, and agent-based approaches. Their contribution lies in providing, for the first time, a comprehensive knowledge-diffusion perspective on SMSPs, outlining how the field has evolved historically and suggesting directions for future research.

1.1. Scope of the review

This study focuses specifically on Economic Lot Scheduling Problem (ELSP) research that applies to the Common Cycle (CC) policy in a single-machine environment. The CC approach is one of the most widely used and practically implemented scheduling strategies in the ELSP literature, accounting for a significant share of published studies (41% in [19]; 32% in [130]). By narrowing the scope to this homogeneous framework, the review achieves three goals:

(i) It allows a clear classification of modeling assumptions, problem environments, and solution techniques.
(ii) It facilitates consistent methodological comparisons through a shared cycle-time structure; and
(iii) It generates robust managerial insights relevant to industries where feasible cyclic schedules are essential.

Restricting the focus to single-machine systems further reflects the original and most extensively studied ELSP configuration, which also serves as the theoretical foundation for many later extensions. This choice enhances methodological clarity, improves comparability across studies, and allows for a more in-depth analysis without the additional complexity of multi-machine settings. Importantly, this focus does not suggest that the CC policy or single-machine systems are superior to other approaches but rather represents a deliberate decision to enable analytical depth.

Several surveys have provided broader perspectives on ELSP and related scheduling problems. Santander-Mercado and Jubiz-Díaz [130] reviewed 126 papers (1958–2015), categorizing them by scheduling approaches and solution methods, and reported that about 32% (40 papers) employed the CC approach. Beck and Glock [8] conducted a large-scale content analysis of 242 ELSP papers published up to 2019, classifying research streams by policies, solution methods, assumptions, and extensions, but without a focused assessment of specific scheduling strategies such as CC. More recently, Ying *et al.* [165] analyzed nearly 3000 single-machine scheduling studies using citation-based methods, mapping developmental trajectories and thematic clusters in the broader scheduling field. However, their work did not explore the methodological and managerial implications of CC-based ELSP models in depth. Similarly, reviews on sequencing in make-to-order (MTO) environments have highlighted sequencing complexity under Industry 4.0 contexts but paid little attention to the integration of lot-sizing and scheduling decisions that define ELSP.

Against this background, our review fills an important gap by systematically analyzing 141 CC-based single-machine ELSP studies published between 1990 and August 2025. The list of the 141 selected papers is shown in Appendix A. These papers are categorized according to solution methods, problem environments, objective functions, and modeling structures. In addition, the review provides statistical insights into publication venues, contributing countries, and authors. In doing so, this study not only complements existing surveys but also offers a deeper, policy-focused perspective on the most widely applied and practically relevant scheduling strategy in ELSP research. For completeness, a brief theoretical formulation of the classic ELSP model under the CC approach is provided in Appendix B.

2. Review methodology

Lopez and Kingsman [93] reviewed the literature on the ELSP. They analyzed solution methodologies used by published articles in the field until 1990. To ensure comprehensiveness, transparency, and reproducibility, this review was designed and conducted in two phases. At the end of these phases, articles meeting the eligibility criteria were coded and analyzed. The scope of this study is limited to peer-reviewed journal articles in English, published between 1990 and August 2025, that examine the Economic Lot Scheduling Problem (ELSP) under a single-machine environment with the Common Cycle (CC) policy.

Inclusion criteria

- Peer-reviewed journal article, English language, published 1990–Aug 2025.
- Explicit focus on ELSP with CC policy (as the main or part of a hybrid approach).
- Contribution falls into one of four categories:
  (1) main CC-based modeling approach,
  (2) hybrid models combining CC with other policies/methods,
  (3) analytical derivation of upper/lower bounds, or
  (4) solving special cases.

Exclusion criteria

- Conference papers, theses, or non-peer-reviewed work.
- Non-English publications.
- Multi-machine models without a distinct CC component.
- Studies focusing exclusively on other policies (BP, TV-LS, etc.).

2.1. Initial phase: Core databases

Previous surveys often restricted their search to the keywords "*ELSP*" or "*economic lot scheduling problem*." However, some relevant papers are indexed under alternative terms such as *lot sizing*, *EPQ*, or *EMQ*, particularly when combined with the single-machine setting. To avoid this labeling bias, we expanded the search strategy.

Searches were carried out in Scopus and Web of Science (WoS), using the following four sets of keywords (in title, abstract, or keywords):

- Set 1: {Economic lot scheduling problem} OR {ELSP},
- Set 2: {Lot sizing AND single machine},
- Set 3: {Economic production quantity AND Single machine} OR {EPQ AND Single machine}, and
- Set 4: {Economic manufacturing quantity AND Single machine} OR {EMQ AND Single machine}.

Raw retrievals are summarized in Table 1 (before merging and deduplication): WoS = 396, Scopus = 459. After merging both databases and removing duplicates, 656 unique records remained. Each record was screened at the title/abstract level, and articles explicitly adopting the CC policy were identified. This process yielded 117 CC-ELSP papers in the first phase.

2.2. Secondary phase: Complementary search

Since some relevant CC-based ELSP papers are not indexed in Scopus or WoS, a complementary search was conducted in Google Scholar. To improve precision, the search was limited to article titles and restricted to the years 1990–2025. Eight sets of keywords were used:

- Set 5: {lot scheduling AND common cycle} OR {lot sizing AND common cycle},
- Set 6: {lot scheduling AND single machine} OR {lot sizing AND single machine},
- Set 7: {Production AND single machine} OR {Manufacturing AND single machine}, and
- Set 8: {Production AND common cycle} or {Manufacturing AND common cycle}.

TABLE 1. Number of articles extracted from databases based on selected keywords.

Set	Scopus	WoS	Google Scholar (In title)
{Economic lot scheduling problem} or {ELSP}	287	241	
{Lot sizing & single machine}	132	118	
{Economic production quantity & Single machine} or {EPQ & Single machine}	39	33	
{Economic manufacturing quantity & Single machine} or {EMQ & Single machine}	1	4	
{lot scheduling & common cycle} or {lot sizing & common cycle}			16
{lot scheduling & single machine} or {lot sizing & single machine}			55
{Production & single machine} or {Manufacturing & single machine}			205
{Production & common cycle} or {Manufacturing & common cycle}			22
Total	459	396	298

Then all the obtained articles are merged, and duplicates are removed, non-English papers, and conference papers were removed, the rest of the papers are all examined one by one, and articles related to the CC approach are separated. The total number of articles from this phase is 298, and details are given in Table 1.

In the secondary phase, a total of 298 articles were retrieved from Google Scholar. Among these, the records that were not already indexed in WoS or Scopus were carefully reviewed at the title, abstract, and problem-definition levels. Through this process, we identified 24 additional journal papers explicitly addressing ELSP with the CC policy that had not been captured in Phase 1. These were added to the previously identified 117 papers from Phase 1, resulting in a final dataset of 141 CC-based single-machine ELSP articles included in this review.

3. Review statistics

This section presents descriptive statistics of reviewed articles, including information about publication years, journals, contributing authors, citations, and countries of origin. It also highlights the studies that have had the greatest performance and impact in the field.

To begin with, it is essential to examine the temporal distribution of publications. Figure 1 shows the number of ELSP studies adopting the CC approach published each year between 1990 and 2025.

As illustrated in Figure 1, research activity in this area has been relatively scattered but persistent over the past three decades, with fluctuations in publication intensity. The early years of the 1990s witnessed a modest but steady flow of contributions, usually between 2 and 5 articles per year. During the 2000s, the publication trend remained moderate, rarely exceeding 6 papers per year.

A significant increase can be observed in the mid-2010s, particularly in 2015, which marked the highest peak with 10 publications in a single year. This peak indicates a renewed research interest in CC-based ELSP models, likely influenced by the expansion of metaheuristic approaches and the integration of modern production constraints (*e.g.*, sustainability, just-in-time, and capacity considerations). The upward trend continued in the following years, with 8 articles in 2016 and between 4 and 6 papers annually up to 2021.

In total, 48 of the 141 reviewed papers (34%) were published in the most recent decade (2015–2025). This concentration of publications demonstrates that ELSP with the CC approach has remained an active and relevant research area in recent years, particularly in light of new production paradigms such as Industry 4.0, sustainability-driven scheduling, and the rise of hybrid/metaheuristic solution techniques. The continued publication of papers in 2022–2025 (albeit at a slightly lower annual rate) further underscores the ongoing attention scholars are paying to this problem.

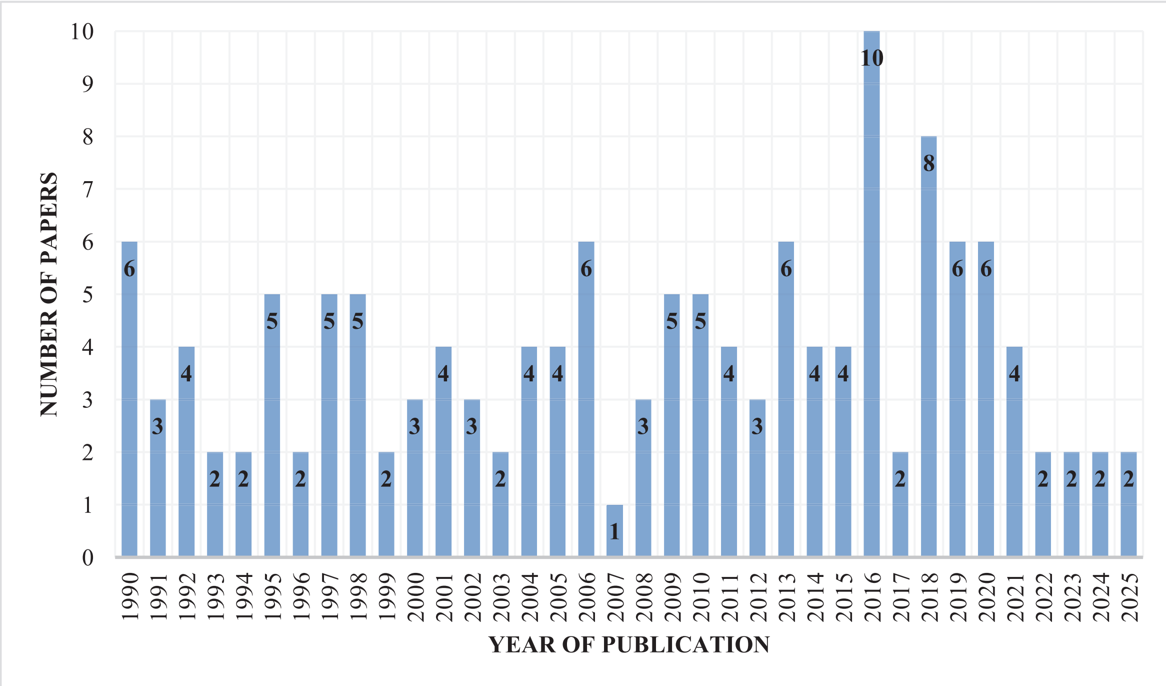

FIGURE 1. Publications on ELSP by year.

In summary, the time-based analysis of publications reveals three key observations:

(1) Steady foundation: The 1990s and 2000s laid a steady groundwork of 2–5 papers annually.
(2) Research peak: A surge of interest in the mid-2010s, culminating in a record number of contributions in 2015.
(3) Sustained relevance: More than one-third of all contributions were published in the last decade, reflecting the problem's adaptability to modern research challenges.

3.1. Journals statistics

Figure 2 illustrates the distribution of reviewed articles across journals. To ensure clarity, only journals that published more than three papers on ELSP with the CC approach are reported. As shown, 14 journals account for 86 papers, representing about 61% of the total reviewed literature. This concentration highlights that a relatively small set of high-impact journals has served as the primary outlet for research on CC-based ELSP models.

The International Journal of Production Economics (18 articles) and the International Journal of Production Research (15 articles) stand out as the most influential publication venues in this domain. Together, these two journals alone account for nearly 23% of all reviewed papers, reflecting their central role in disseminating research at the interface of production planning, scheduling, and inventory management.

The next tier of outlets includes IIE Transactions and the European Journal of Operational Research (8 papers each), followed by Scientia Iranica, Production Planning and Control, and the International Journal of Industrial Engineering, each with 5 publications. These journals demonstrate a sustained interest in applied scheduling and lot-sizing research, often linking methodological innovations with practical manufacturing contexts.

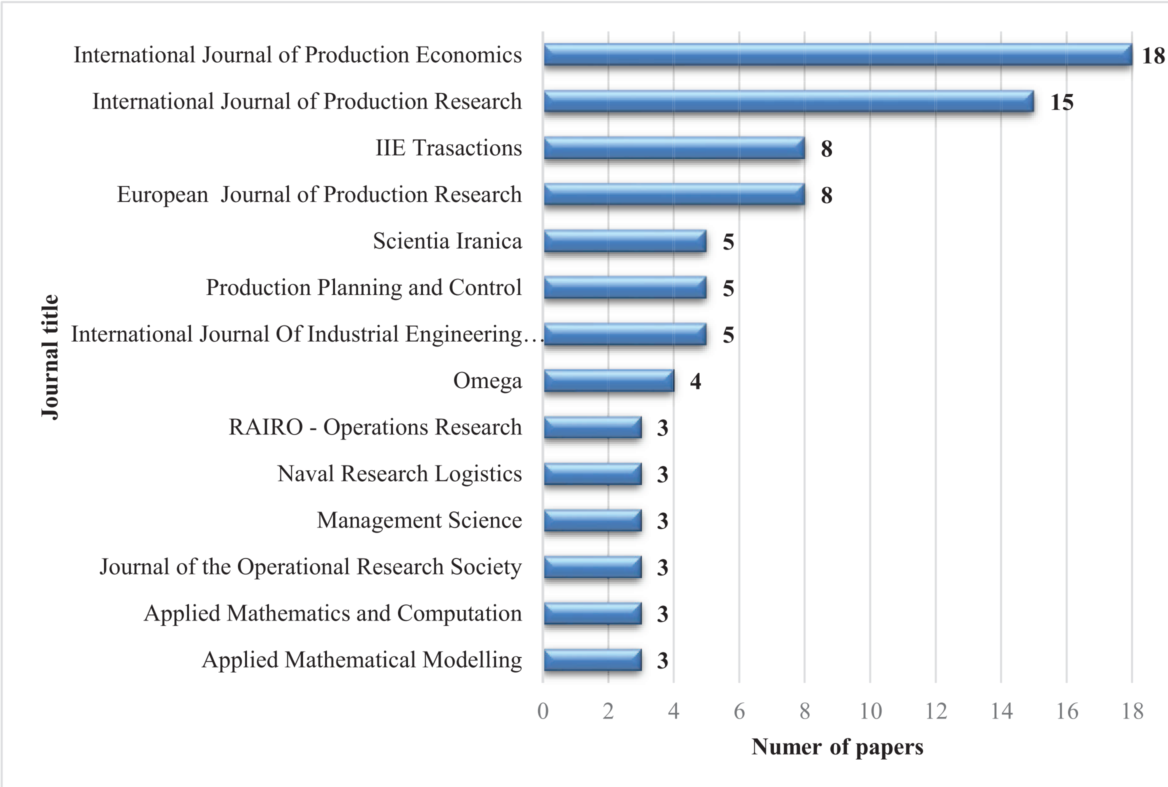

Figure 2. Distribution of articles in different journals.

In addition, specialized outlets such as Omega, RAIRO-Operations Research, Naval Research Logistics, and Management Science have also contributed multiple high-quality papers, reinforcing the cross-disciplinary importance of ELSP research. Journals with a stronger emphasis on mathematical and computational approaches, including Applied Mathematics and Computation and Applied Mathematical Modelling, are also represented, confirming the methodological diversity of the field.

3.2. Authors statistics

In this study, all 141 selected articles were carefully re-examined to identify their contributing authors. The number of publications and collaborations per author were then aggregated to determine the most active researchers in the field. Figure 3 presents the top authors with more than four publications on ELSP using the CC approach, ranked according to their number of contributions.

As shown in Figure 3, Taleizadeh, A.A. is the most prolific author in this domain, with 18 publications, followed by Nobil, A.H. with 13 papers and Cárdenas-Barrón, L.E. with 11 papers. Other highly active contributors include Chiu, Y.S.P. (10 papers) and Moon, I. (9 papers). These scholars represent leading figures in the study of lot-sizing and scheduling problems, often combining CC-based ELSP models with extensions such as shortage policies, sustainability concerns, and metaheuristic solution approaches.

Authors such as Chiu, S.W. (8 papers), Yano, C.A. (6 papers), and Wee, H.M. (6 papers) also play a significant role, frequently bridging theoretical contributions with applications in supply chain and production planning. The list is complemented by Niaki, S.T.A., Sedigh, A.H.A., and Banerjee, A., each with 5 contributions, who

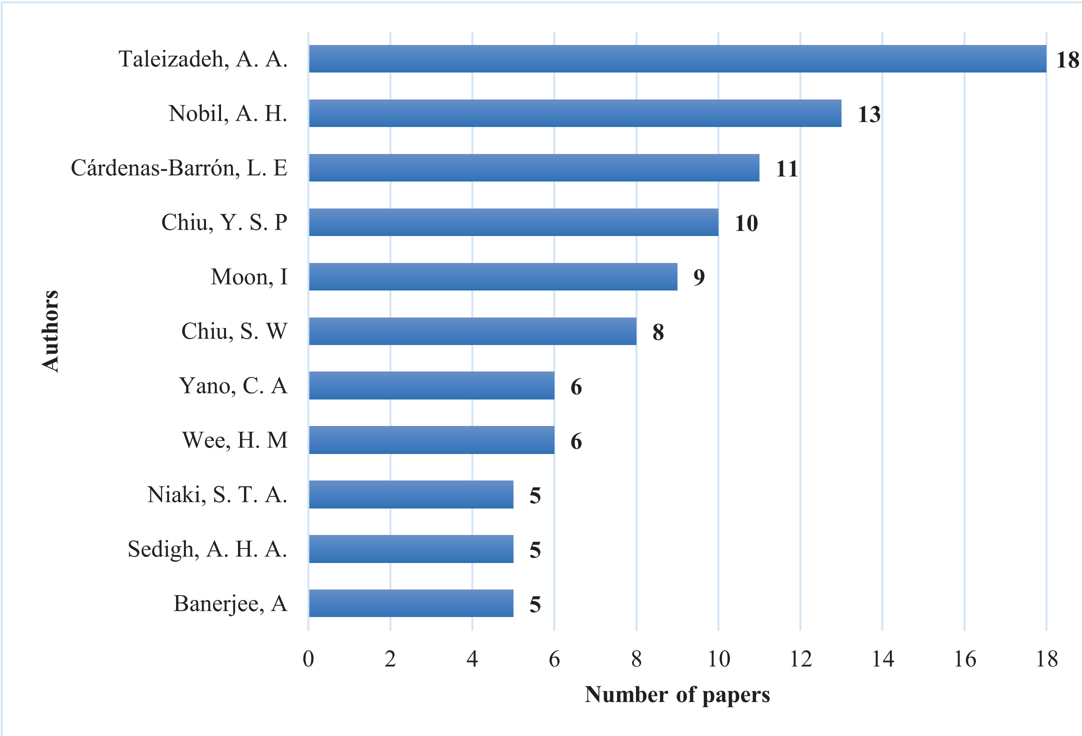

Figure 3. Number of publications by top 10 authors.

add to the methodological diversity of the field through hybrid modeling, applied optimization, and analytical case studies.

In total, these top 11 authors account for 96 papers, representing a substantial share of the literature on ELSP with the CC approach. Their collective contributions underscore the fact that a relatively small group of researchers has shaped much of the development and visibility of CC-based ELSP research over the last three decades. Moreover, strong collaborations among these authors – particularly across Iran, Mexico, Taiwan, South Korea, and the United States – reflect the international and cooperative nature of this research stream.

3.3. Countries statistics

Figure 4 presents the top 12 countries where researchers have contributed to ELSP studies using the CC approach. This information was obtained from the affiliations of the authors: first, each author of the 141 selected papers was identified, and then the corresponding country of research was determined based on institutional affiliation.

As shown in Figure 4, the United States ranks first, with 75 author affiliations, highlighting its long-standing leadership in the field of operations research and production planning. Iran (67 affiliations) and Taiwan (52 affiliations) follow closely, reflecting the strong presence of active research groups in these countries. These three nations together account for nearly 55% of all affiliations, demonstrating their central role in advancing CC-based ELSP research.

The data confirms that ELSP with the CC approach has been investigated by a truly international community. Notably, the concentration of research efforts in Asia (Iran, Taiwan, South Korea, India, Singapore, Malaysia) demonstrates the growing importance of production planning and lot-sizing in fast-developing economies with

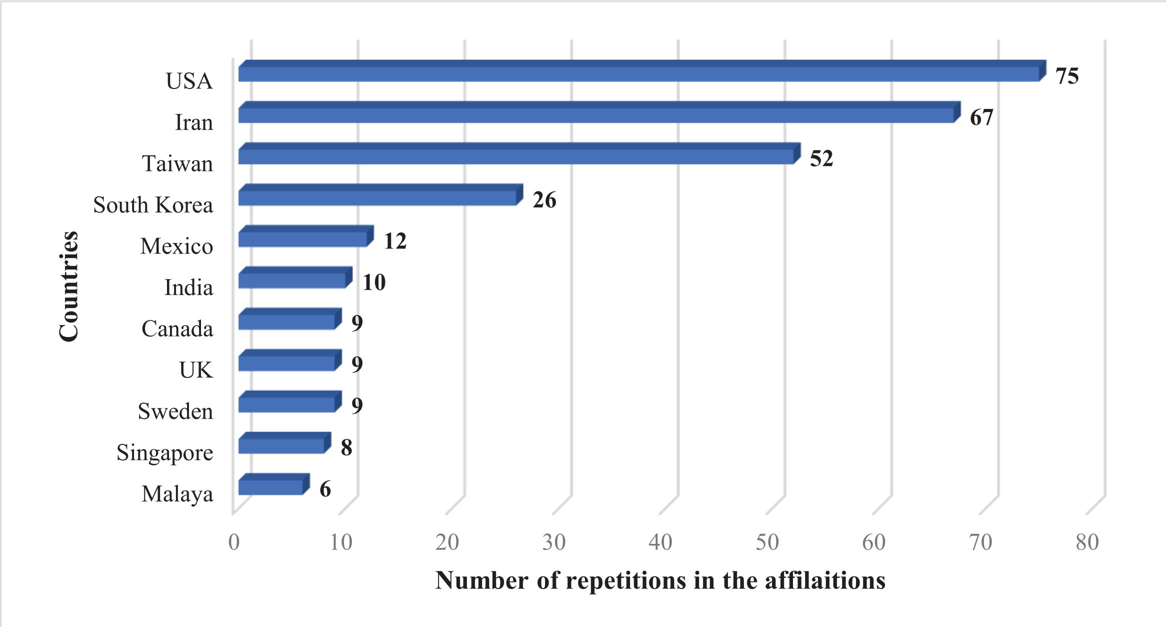

Figure 4. Number of publications by top 10 countries.

strong manufacturing sectors. Meanwhile, the USA and European countries (UK, Sweden) continue to provide substantial theoretical and methodological contributions.

In summary, while the USA remains the leading contributor, the active roles of Iran and Taiwan highlight the global and collaborative nature of ELSP research. This distribution also reflects the balance between established centers of operations research in North America and Europe and emerging hubs of applied research in Asia and Latin America.

3.4. Citation analysis

One of the key indicators of an article's impact on the scientific community is the number of citations it receives. To capture this dimension, citation data for all 141 selected papers were extracted from Google Scholar (as of August 24, 2025). Collectively, these articles have received 5145 citations, underscoring the relevance and influence of ELSP research with the CC approach.

Figure 5 highlights the top 10 most cited articles in this domain. The most cited contribution is the paper by Tang and Teunter [152], with more than 170 citations, followed closely by Ben-Daya and Hariga [10] and Teunter *et al.* [153], each exceeding 150 citations. These works represent milestone studies that introduced new modeling insights and solution methods, establishing a strong foundation for subsequent research.

Other highly cited contributions include Silver [135] and Taleizadeh *et al.* [148], both surpassing 140 citations, further highlighting their enduring influence. Studies such as Federgruen and Katalan [50] and Carreno [16] also rank among the most recognized works, demonstrating the breadth of influential contributions across different decades. More recent entries, including Tiwari *et al.* [155], illustrate how newer approaches continue to attract scholarly attention, especially when addressing modern challenges such as metaheuristic optimization or industry-oriented constraints.

Overall, the citation distribution confirms that while a relatively small subset of studies has achieved particularly high visibility and long-term impact, the broader body of literature continues to be cited actively, reflecting the ongoing relevance of CC-based ELSP models in both methodological and applied research streams.

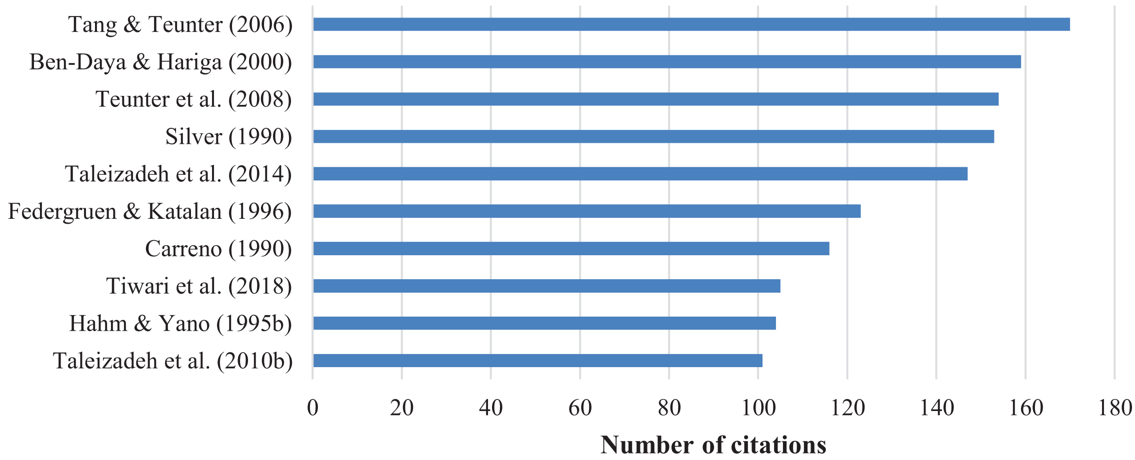

FIGURE 5. Top 10 highly cited.

4. Classification of the papers

In this section, we present and analyze the classification of the 141 reviewed studies. It is worth noting that seven of these works [52, 56, 85, 99, 100, 112, 160] primarily sought to extend or refine earlier studies, building upon previously established models.

Although the focus of this review is confined to single-machine ELSP models with the CC approach, a small number of exceptional papers have investigated alternative production layouts. These include: (a) job shop environments [71, 122, 157], (b) flow shop settings [2, 43], and (c) group technology (GT) systems [91, 105, 134]. These contributions are cited here only to illustrate the diversity of contexts in which the CC approach has occasionally been applied. Nevertheless, they do not alter the scope of this review, which remains strictly centered on the single-machine ELSP framework.

In this section, we present the taxonomical classification of the 141 reviewed studies. The classification criteria were selected based on two considerations:

(1) dimensions that frequently appear in the ELSP literature, and
(2) core modeling aspects that fundamentally influence the structure and solution of CC-based ELSP models.

Accordingly, the reviewed studies are classified along the following dimensions:

– Shortage policies (no shortage, full backordering, lost sales, partial backordering);
– Defective production processes (scrap, imperfect quality, reworkable items, etc.);
– Maintenance activities (preventive and corrective maintenance);
– Deteriorating items (fixed lifetime *vs.* inventory-dependent deterioration);
– Objective functions (cost minimization *vs.* profit maximization);
– Constraints and problem conditions (*e.g.*, setup times, warehouse space, budget, service levels);
– Decision variables (continuous *vs.* integer); and
– Input data characteristics (deterministic *vs.* stochastic).

It should be clarified that the production environment (*e.g.*, job shop, flow shop, group technology) is not part of the classification criteria in this review. These environments were only mentioned briefly to acknowledge a few exceptional studies outside the single-machine setting. The focus of our taxonomy and analysis remains strictly on single-machine ELSP models with the CC approach.

***Job shop*:** This layout groups similar machinery together. Using this layout, items travel in the system based on their production sequence from one shop to another. Ouenniche and Boctor [122] minimized system costs by addressing sequencing and ELSP in a multi-item production system for the finite horizon in a job shop environment using a mixed integer non-linear programming (MINLP) model. Hennet [71] investigated ELSP employing a CC approach for a multi-stage production planning in a job-shop. Torabi *et al.* [157] applied a CC approach to address a multi-product ELSP, considering a flexible job shop and finite horizon using the MINLP model.

***Flow shop*:** This layout complies with the operations. Therefore, this layout enables the system to maintain a continuous flow of items through the system. Akrami *et al.* [2] addressed a multi-product ELSP for flexible flow shops considering finite horizon; production steps are in series and a finite buffer between steps. El-Najdawi [43] presented a heuristic algorithm developed based on CC approach for the ELSP in a multi-stage multi-product flow shop environment.

***Group technology (GT)*:** This method is a combination of flow shop and job shop in which items are grouped based on similarities to be produced in a pseudo flow shop (*i.e.*, flow shops formed for a group of items). Kuo and Inman [91] solved group a technology ELSP (GT-ELSP) employing a heuristic. After that, Moon *et al.* [105] extended former heuristic by considering modified cycle length for feasible solutions and provided a hybrid genetic algorithm for GT-ELSP. Shirodkar *et al.* [134] developed a heuristic for GT-ELSP with a time-varying lot size approach. The solutions of heuristic compared with the lower and upper bounds obtained by an infeasible independent solution (IS) and CC, respectively.

In addition, studies of Hahm and Yano [62], Hahm and Yano [63], Hennet [71], Jensen and Khouja [80] and Chiu *et al.* [28–30], addressed assembly and production for parts of a product. Chiu *et al.* [29] addressed a vendor–buyer inventory system consists of multiple repairable items sharing common parts. To optimize the system, they adopted two stages of production: one for common parts and another for end products. Hahm and Yano [62] scheduled various assembly components produced by a third party coupled with delivery of items to the customer. Then, Hahm and Yano [63] presented a heuristic to obtain “just-in-time” scheduling for production of multiple items and their delivery to a subsequent facility in each cycle. Jensen and Khouja [80] presented an efficient algorithm that obtains the optimal solution for Hahm and Yano [62].

4.1. Shortage

Shortage policies represent a critical dimension of ELSP modeling, as they define how unsatisfied demand is treated within the system. Incorporating shortage policies enables models to reflect realistic conditions in which customer demand cannot always be met, whereas omitting them implies an idealized setting with consistently high service levels.

When inventory is insufficient to meet demand, organizations incur shortage-related costs. In the reviewed literature, ELSP studies adopting the CC approach can be grouped into four categories: (a) no shortage, (b) full backordering, (c) lost sales, and (d) partial backordering. The overall distribution of studies across these categories is shown in Figure 6, while their detailed taxonomical structure is illustrated in Figure 7. Notably, some studies present models that consider both permissible and non-permissible shortages. In such cases, the permissible shortage policy is treated as the primary category, as it represents the more general formulation.

4.1.1. Without shortage

These studies represent cases in which shortages are not permitted, and inventory levels are planned to fully satisfy customer demand at all times. Among the 141 reviewed papers, 75% (88 studies) fall into this category, highlighting that the majority of CC-based ELSP research assumes a non-shortage environment.

4.1.2. Fully-backordering

This category of studies examines situations where shortages are allowed and customers are willing to wait for replenishment. In such cases, a backorder occurs: demand is placed, but inventory is insufficient, so unmet orders are queued until production catches up. Once items are manufactured, previously backordered demands

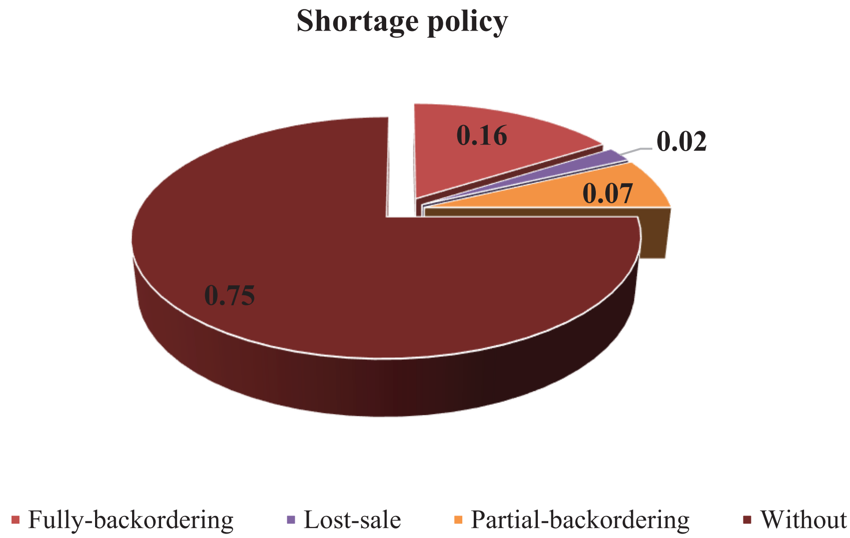

FIGURE 6. Distribution of studies by type of shortage policy.

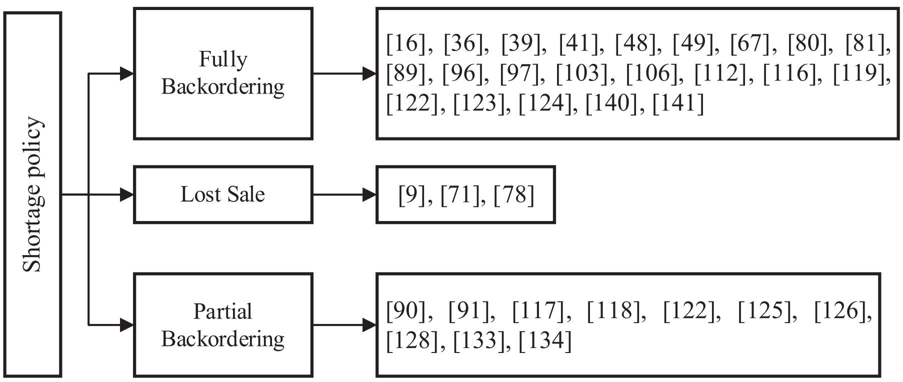

FIGURE 7. Tree structure of studies by type of shortage policy.

are satisfied first, followed by new requests. These systems are particularly relevant in markets where customers accept waiting – most often in monopolistic or low-competition environments.

However, this policy entails several drawbacks, including:

(a) interruptions in production flow,
(b) potential damage to a company's reputation,
(c) higher procurement costs for urgent raw material supply, and
(d) penalties or costs associated with delayed deliveries.

As shown in Figure 6, studies addressing backordering policies represent approximately 16% of the total literature (22 papers). A more detailed classification of these works is provided in Figure 7. For example, Gallego [52] analyzed rotation schedules in ELSP under machine capacity constraints, establishing tighter lower bounds and extending results to systems with permissible backorders. Gupta [61] examined ELSP with the CC policy for multiple items, deriving both the optimal lot size and shortage level. Elhafsi and Bai [44] considered variable production rates controlled at the beginning and during the cycle in CC-based ELSP systems with shortages. More recently, Pasandideh *et al.* [125] modeled ELSP with shortages and non-conforming items, where defective items are either scrapped or reworked depending on the severity of the defect.

4.1.3. Lost sale

The lost sales phenomenon arises under two conditions: (a) customer demand exceeds available inventory, and (b) customers, unable or unwilling to wait, choose substitute products. In such cases, the unmet demand is permanently lost, and the resulting lost sales costs are incorporated into inventory deficiency analysis. This policy introduces both tangible costs (*e.g.*, direct loss of revenue) and intangible costs, including (a) damage to the company's credibility and (b) erosion of customer loyalty.

Studies adopting the lost sales policy are summarized in Figure 7. For example, Bollapragada and Rao [11] developed an ELSP model integrating resource allocation, lot sizing, and lost sales by analyzing product assignments to production lines with varying costs and capabilities. Similarly, Khoury *et al.* [86] applied the CC approach to address ELSP under limited capacity conditions, explicitly accounting for demand lost due to insufficient inventory.

4.1.4. Partial backordering

Under the partial backordering policy, a proportion of the unmet demand is eventually satisfied, while the remainder is permanently lost. Specifically, if a shortage of size S occurs, then αS units (where $0 \leq \alpha \leq 1$) are backordered and fulfilled after some delay, whereas $(1-\alpha)S$ units are lost sales. This hybrid treatment of shortages reflects scenarios where some customers are willing to wait, while others switch to substitutes. The studies adopting this policy are illustrated in Figure 7. It should be noted that papers not shown in this figure do not incorporate shortage considerations.

Several studies have investigated ELSP under partial backordering conditions. For example, Taleizadeh *et al.* [141] proposed a model for a manufacturing system with scrap, service-level requirements, and partial backordering, in which shortages are addressed through a combination of lost sales and backorders. Later, Taleizadeh *et al.* [146] developed an exact algorithm for ELSP that incorporated rework, budget constraints, service levels, and partial backordering. Building on this line of research, Taleizadeh and Wee [139] analyzed a multi-item, capacity-constrained, imperfect production system with immediate rework and partial backordering, aiming to minimize total costs by optimizing cycle length and backorder quantities.

4.2. Defective manufacturing process

Defective production is a crucial aspect of system modeling, as real-world manufacturing processes are seldom flawless. Incorporating elements such as scrap, rework, or imperfect-quality items allows models to capture realistic production environments, whereas excluding them assumes idealized processes without defects.

As illustrated in the tree structure of studies on defective items, 82 out of 141 papers (approximately 58%) addressed issues related to defective production. For example, Giri *et al.* [57] examined an ELSP model that incorporated capacity constraints and quality-related costs arising from potential non-conforming items (Fig. 8).

Moon *et al.* [104] investigated defective production in ELSP by assuming that the process begins in an in-control state, producing conforming items. After a certain number of units, the system transitions to an out-of-control state, generating defective products. The transition time was modeled as a random variable. Such defective production may arise due to operator errors, poor-quality raw materials, or process instability. Earlier, Moon [100] developed an ELSP model that incorporated investments aimed at reducing setup costs and improving quality.

In defective production systems, a key issue concerns inspection policies. Most studies employed 100% inspection to detect defective items. Ma *et al.* [94] compared three scenarios: (a) no screening, (b) in-process inspection, and (c) post-production 100% screening. Other studies – such as Ben-Daya and Hariga [10], Moon *et al.* [104], Giri *et al.* [57], and Parveen and Rao [124] – adopted inspection-and-restoration approaches, in which inspections were performed at fixed intervals to detect shifts from the in-control to the out-of-control state. For example,

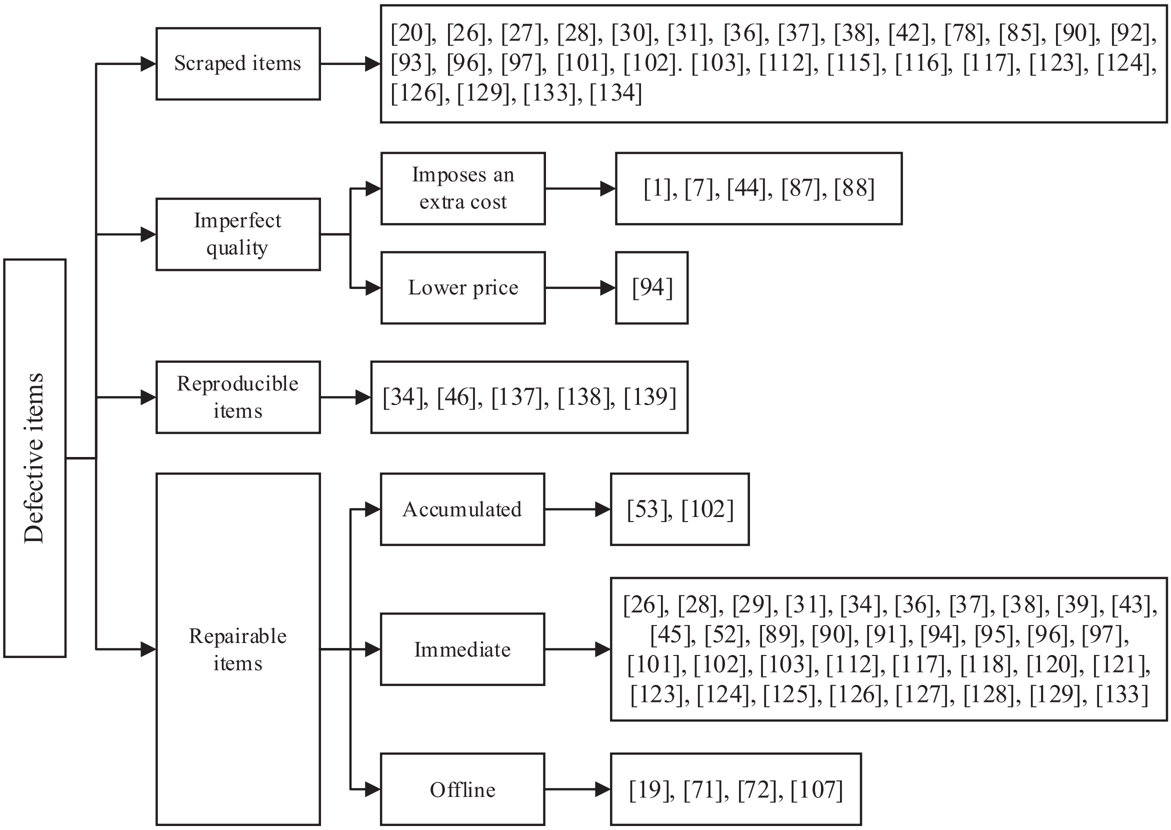

FIGURE 8. Tree structure of studies by type of defective items.

Ben-Daya and Hariga [10] analyzed ELSP under deteriorating processes, optimizing the number of inspections during each cycle.

Defective items in the literature are categorized as follows:

- ***Scrapped/Disposed/Rejected items*:** Items with no further use, disposed of or recycled at a cost. Taleizadeh *et al.* [141] modeled scrap under both uniform and normal defective-rate distributions. Nobil *et al.* [111] studied a buyer–vendor inventory system considering scenarios with a single vendor (lost sales allowed, batch deliveries) and multiple vendors with a joint shipment policy.
- ***Imperfect quality items*:** Items that are neither scrapped nor suitable for rework. This category considers two phenomena:
  - (I) ***Imposes an extra cost*:** Some studies only account for additional system costs caused by imperfect items. For instance, Moon *et al.* [103] examined the effect of stabilization periods on ELSP by defining target yield rates. Adhisatya *et al.* [1] investigated two imperfect key modules, each with probabilities of shifting to out-of-control states.
  - (II) ***Lower price*:** Items with minor defects can be sold at reduced prices. Nobil and Taleizadeh [109] studied a multi-product system in which defectives were either reworked or auctioned without rework.
- ***Reproducible items*:** Items that can be recycled and remanufactured. Tang and Teunter [152] analyzed ELSP with hybrid production lines handling both new and returned products, applying a CC approach with two lots per item. Later, Teunter *et al.* [153] extended this work to systems with separate dedicated lines for production and remanufacturing.

– ***Repairable/re-workable items*:** Items identified as nonconforming during inspection but made acceptable after additional processing. Three rework policies are identified:
  (I) ***Immediate rework*:** Rework occurs immediately after production, at equal or higher rates than normal production. Taleizadeh *et al.* [142] studied random defectives under service-level constraints, while Nobil *et al.* [114] addressed multi-item imperfect systems with non-zero setup times and rework/scrap options.
  (II) ***Accumulated rework*:** Defectives are collected across M cycles and reworked in the $(M+1)$-th cycle. Haji and Haji [64] modeled this scenario, while Nobil *et al.* [120] proposed an ELSP with scrap and reworkable items across multiple products, comparing policies of (a) warehousing defectives for later rework *vs.* (b) repairing items within the same cycle.
  (III) ***Reworked off-line*:** Rework does not impact the main production process (time is ignored). This may occur when rework is outsourced, performed in another stage, or occurs at very high rates. Khouja [85] analyzed such a model under imperfect quality, assuming all defective items were reworked off-line without affecting machine utilization.

4.3. Maintenance activities

Maintenance policies are an important consideration in ELSP modeling, as machine reliability and downtime directly affect production scheduling and overall efficiency. Incorporating maintenance, whether preventive or corrective, adds realism to models; while omitting it implicitly assumes uninterrupted machine availability.

The reviewed literature highlights two main types of maintenance policies:

– Preventive maintenance (PM): PM involves planned; routine maintenance aimed at preserving machine performance and avoiding unexpected breakdowns. This requires proper scheduling of maintenance activities along with records of inspections and service history. Only three studies applied PM in the context of ELSP, namely Ben-Daya and Hariga [10], Taleizadeh *et al.* [148], and Taleizadeh [138]. For instance, Taleizadeh *et al.* [148] examined a defective ELSP with shortages, incorporating permissible interruptions in the production process for PM activities, while Taleizadeh [138] analyzed the combined effects of PM and partial backordering.
– Corrective maintenance (CM): CM focuses on addressing machinery failures after they occur by identifying, isolating, and repairing the issue so that the system returns to acceptable operating conditions. Giri *et al.* [57] considered CM within an ELSP framework, also including PM as part of their model. They assumed a normally distributed time for the process to shift from the in-control to the out-of-control state, with inspections scheduled at the end of each cycle to determine the need for corrective action.

Overall, only 4 out of 141 reviewed papers (approximately 3%) incorporated maintenance policies in their ELSP models, underscoring that this remains a relatively underexplored but practically significant dimension of research.

4.4. Deteriorating items

Deterioration refers to the decline in the usability or value of items over time, and it is particularly critical for perishable goods such as food, pharmaceuticals, and fashion products. Models that ignore deterioration implicitly assume that items have an unlimited lifetime, which is rarely the case in real-world settings.

Traditional inventory models generally assume that product quality remains intact until delivery. In reality, however, items often lose value due to deterioration, physical deformities, discoloration, or even shifts in consumer preferences that render products obsolete [4]. Products with a finite or limited shelf life – such as dairy, meat, and fruit – are typically classified as deteriorating items, since their availability decreases or evaporates over time regardless of demand. Early studies, such as Whitin [162], highlighted the obsolescence of fashion items, while Chare and Schrader [22] provided the first formal model of decaying inventory using a negative exponential function.

In the reviewed literature, deteriorating items are analyzed in two main categories:

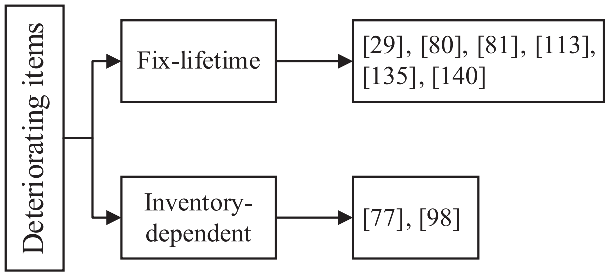

FIGURE 9. Tree structure of studies by type of deteriorating policy.

(1) Fixed lifetime: Items that have a predetermined usable lifespan (*e.g.*, one week). Chowdhury and Sarker [38] proposed an optimal schedule and ordering policy that accounts for shelf-life constraints in the production of a family of items on a single machine, adjusting either the production rate, cycle length, or both. Sharma [133] incorporated shortages and provided mathematical formulations to optimize cycle length under deterioration. As another example, Koulamas [90] studied tools with finite lifespans that required periodic replacement by robots.
(2) Inventory-dependent deterioration rate: Items that deteriorate continuously over time. Lin *et al.* [92] modeled ELSP with products subject to significant deterioration, while Nobil *et al.* [114] introduced a single-machine inventory model with exponential deterioration rates. They derived both upper and lower bounds for the optimal cycle length and solved the model using bisection and Newton–Raphson methods.

As illustrated in Figure 9, only about 6% (8 papers) of the reviewed studies addressed deterioration in single-machine ELSP systems, highlighting it as an important but relatively underexplored area of research.

4.5. Objective function

The choice of objective function determines the optimization goal of ELSP models. While the majority of studies focus on minimizing total cost, a smaller number aim to maximize profit or consider multi-objective trade-offs. Limiting analysis to cost minimization may restrict the range of managerial insights, whereas incorporating alternative objectives can reveal broader strategic implications.

Among the 141 reviewed papers, each employed a single objective function in its modeling. The dominant approach is cost minimization, while only three studies explicitly sought to maximize profit: Hariga [69], Kim and Goyal [87], and Shafiee-Gol *et al.* [132]. For instance, Hariga [69] examined insufficient production capacity and introduced collaboration with external suppliers to meet excess demand, maximizing annual profit margins by optimizing the number of setups and external orders in two EPOQ models. Kim and Goyal [87] analyzed a composite sourcing policy for a multi-product ELSP, explicitly accounting for raw material costs and supplier selection to maximize profit. More recently, Shafiee-Gol *et al.* [132] studied ELSP with defective items and rework, incorporating pricing decisions as a key factor. They assumed that item prices followed a random distribution and customer demand was a linearly decreasing function of price. By demonstrating that the profit function was concave, they derived optimal solutions for both production cycle length and product prices.

4.6. Constraints and problem conditions

Constraints represent the practical limitations and organizational conditions that shape real-world ELSP systems. Incorporating constraints makes models more realistic and directly applicable to industrial contexts, while omitting them improves analytical tractability but reduces realism.

The constraints highlighted in Figure 10 were chosen because they are the most frequently addressed categories in the reviewed literature and represent fundamental operational limitations encountered in production and inventory systems. Specifically, the surveyed studies most often considered constraints such as setup time,

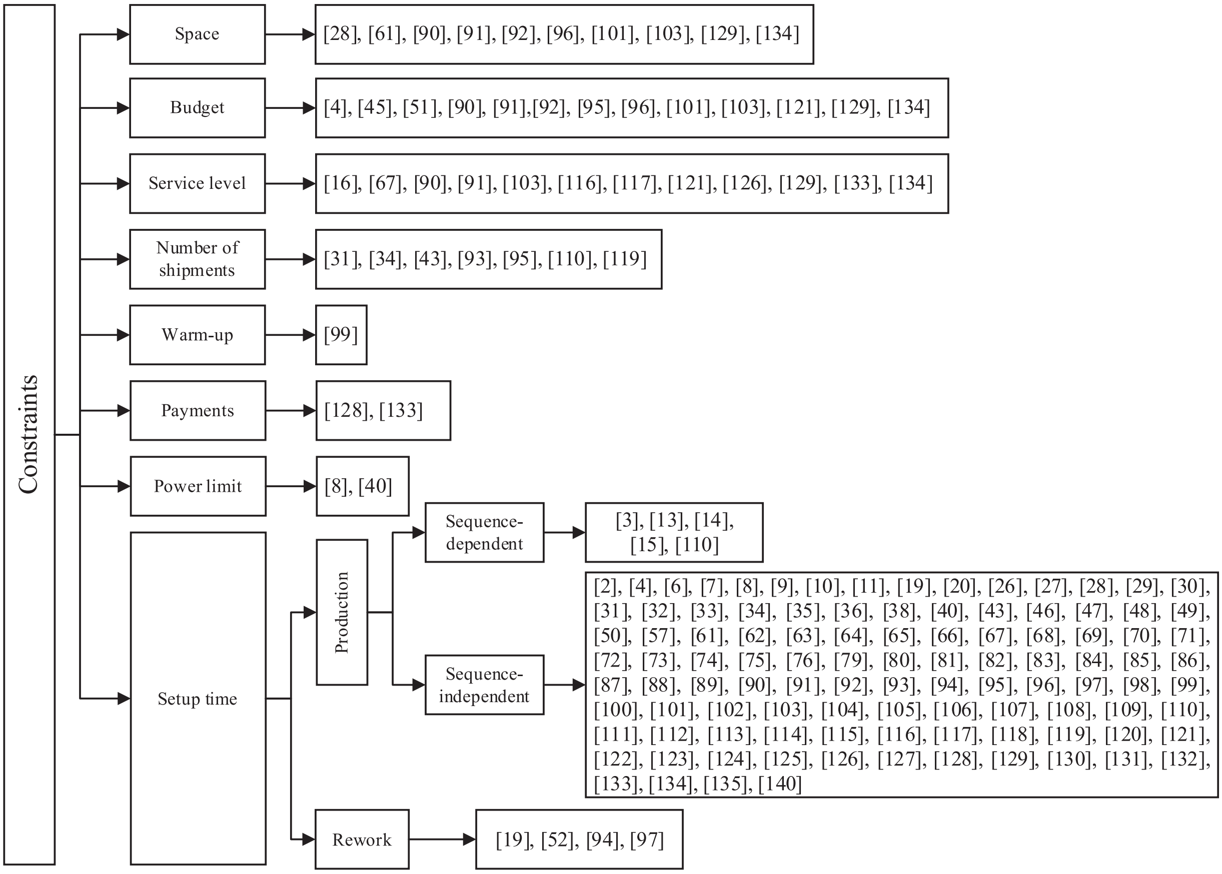

FIGURE 10. Tree structure of studies by type of constraints.

number of shipments, budget restrictions, warehouse space, and service level requirements. In addition, several papers examined other important factors, including upper and lower bounds for production ratios (*e.g.*, [83,101]), setup cost bounds (*e.g.*, [56]), product–machine allocation (*e.g.*, [110]), insufficient capacity (*e.g.*, [86]), safety stock policies (*e.g.*, [164]), and sequence-dependent setups (*e.g.*, [63, 80]).

Several examples illustrate the impact of these constraints on ELSP modeling. Vemuganti *et al.* [159] studied warehouse space and budget restrictions in a multi-machine common cycle system and analyzed their effect on lot size, cycle length, and total cost. Houshyar [76] examined a single-machine system with warehouse capacity limits, proposing an optimal solution for a special case and a heuristic for feasible schedules. Choi and Nobel [37] incorporated unit load sizes, material handling equipment, and warehouse capacity into a nonlinear integer programming model solved heuristically. Taleizadeh *et al.* [144] and Nobil *et al.* [113] addressed shipment constraints in multi-echelon supply chains with discrete deliveries. Jodlbauer and Reitner [81] considered stochastic demand with safety stock and service-level constraints, proposing algorithms for optimizing cycle length under different service-level and cost trade-offs.

Thus, the classification presented in Figure 10 reflects both the most common constraints reported in literature and those with the greatest relevance for linking theoretical models to practical production planning and scheduling environments.

4.6.1. Setup times

One of the most common constraints considered in ELSP models is machine setup time prior to production. Setup times are generally divided into two main categories:

(a) setup time for normal production, and
(b) setup time for the rework process.

Within normal production, setup times can be further classified as either:

- Sequence-independent setup times, where the setup duration remains constant regardless of the production sequence; and
- Sequence-dependent setup times, where the setup duration varies according to the order in which items are produced.

A number of studies have addressed sequence-independent setup times. For example, Haji and Mansuri [65] modeled sequence-independent setup times in ELSP and derived the optimal cycle length using a closed-form solution procedure. Dobson and Yano [41] examined a batch production system with constant demand and supply, where items flowed in small batches along a production line with potential inventory buffers at intermediate or terminal stages. Federgruen and Katalan [50] investigated a random ELSP with cyclical base-stock policies under significant consumption uncertainty, incorporating sequence-independent setup times, processing times, or their combinations. Haji *et al.* [66] extended this line of work by analyzing sequence-independent setup times for the rework process in imperfect production systems, where defective products were reprocessed to meet acceptable quality standards. Later, Nobil *et al.* [112] revisited the study of Haji *et al.* [66], identifying conditions under which shortages could be avoided during setup times.

Other research has focused on sequence-dependent setup times. Alle *et al.* [3] formulated and solved an ELSP with performance decay under sequence-dependent setups, while Brander and Forsberg [15] proposed a heuristic for disassembly operations with sequence-dependent setup times, optimizing disassembly frequencies and the corresponding cyclic schedule.

4.6.2. Decision variables

Decision variables represent the controllable elements of ELSP models, including factors such as cycle length, lot size, or sequencing of items. Models that rely solely on continuous variables are generally simpler to formulate and solve, while incorporating integer or binary variables makes the models more realistic but also significantly increases their computational complexity.

In the reviewed literature, decision variables can be grouped into two categories:

(1) Continuous variables: *e.g.*, cycle length, production quantity per period, shortage quantity per period, and product price.
(2) Integer or binary variables: *e.g.*, allocation of items to machines, production sequence per machine, number of shipments, maximum batch size, maximum warehouse capacity, number of cycles, and number of inspections per period.

Accordingly, ELSP studies can be divided into two modeling approaches:

- Continuous nonlinear programming, where the formulation involves one or more continuous variables.
- Mixed-integer nonlinear programming (MINLP), where both continuous and discrete variables are included in the model.

The distribution of studies across these two approaches is presented in Figure 11. Continuous nonlinear models capture the core trade-offs of cycle length, production, and shortages, whereas MINLP models provide a richer representation of practical decision-making by also incorporating discrete choices such as setup sequences, shipments, and inspections.

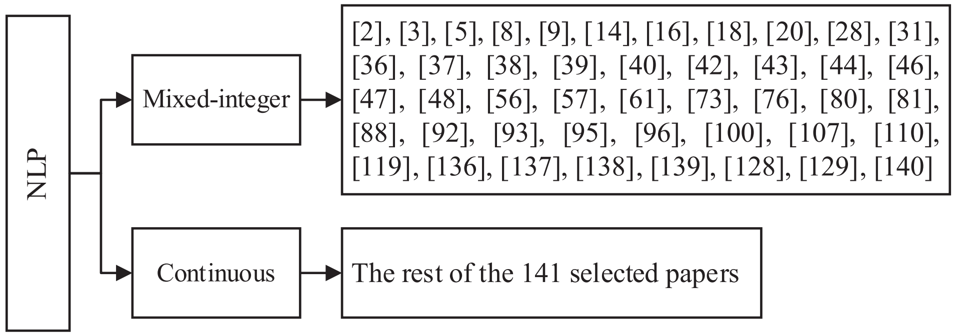

FIGURE 11. Tree structure of studies by type of problem.

4.6.3. Input parameters

Input parameter assumptions are a fundamental aspect of ELSP modelling, as they directly influence both the realism and the robustness of solutions. Deterministic models assume that all input data (*e.g.*, demand, production rate, setup time) are perfectly known in advance. By contrast, stochastic models capture uncertainty by assigning probability distributions to one or more parameters, thus better reflecting real-world manufacturing environments. A third stream of research applies fuzzy logic, which handles vagueness or imprecision in input data when exact values or distributions are difficult to specify.

Based on the reviewed studies, ELSP models can therefore be classified into three categories according to their input data assumptions (Fig. 12):

(a) **Deterministic models**: These assume complete and certain knowledge of all parameters. Deterministic modelling dominates the literature because of its relative tractability. Within this stream, some studies focus on exact optimization (closed form or mathematical programming), while others propose heuristics to generate feasible solutions in practical settings.

(b) **Stochastic models:** These account for randomness in one or more parameters such as yield rate, demand rate, production time, setup time, product deterioration, or process failure. For example, Yano [164] studied a multi-item system with random yield rates, developing an iterative scheme to obtain optimal cycle lengths and yield adjustments. Federgruen and Katalan [50] incorporated random demand, production, and setup times in a stochastic ELSP. Ben-Daya and Hariga [10] modeled process reliability where the shift from in-control to out-of-control follows an exponential distribution. Haji and Haji [64] addressed random rework rates for imperfect products, while Chiu *et al.* [26] considered product damage governed by a continuous uniform distribution.

(c) **Fuzzy models**: These approaches recognize that certain parameters may not be known precisely and cannot always be described by standard probability distributions. Instead, fuzzy sets capture vagueness in inputs such as demand, processing times, or deterioration rates. For instance, Sangari *et al.* [129] applied fuzzy logic to ELSP to handle imprecise demand function.

5. GAP ANALYSIS AND PRACTICAL CONTRIBUTIONS

A key objective of this review is not only to classify existing studies but also to identify the research gaps and assess their relevance to real-world scheduling systems. Although the Common Cycle (CC) approach provides analytical tractability and has been widely applied in the ELSP literature, several areas remain underdeveloped:

(I) Integration with modern production paradigms – Few studies explicitly link CC-based ELSP models with Industry 4.0 concepts such as cyber-physical systems, real-time data, and digital twins. Bridging this gap could enhance the applicability of ELSP models to smart factories.

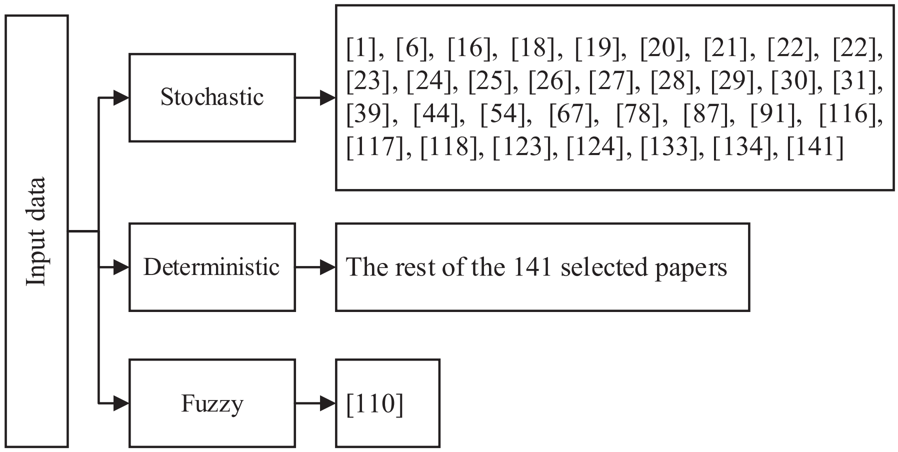

FIGURE 12. Tree structure of studies by type of input data.

(II) Limited consideration of stochastic and fuzzy environments – While most models assume deterministic inputs, real-world production systems face uncertainty in demand, processing times, and quality levels. Expanding stochastic and fuzzy-based ELSP models would improve their robustness for managerial decision-making.

(III) Underrepresentation of maintenance and deterioration – Only a small share of studies (≈3% on maintenance, ≈6% on deterioration) incorporate machine reliability and product perishability. Yet, these are critical issues in industries such as food, pharmaceuticals, and electronics.

(IV) Multi-objective decision-making – The dominance of cost-minimization objectives (≈98% of papers) limits managerial insights. Profit maximization, service-level trade-offs, and sustainability-driven objectives remain relatively unexplored.

(V) Empirical validation – Most CC-ELSP models remain theoretical, with limited empirical or case-based validation in industrial contexts. Applying these models to real scheduling data in sectors such as automotive, pharmaceuticals, and food processing could strengthen their practical impact.

Despite these gaps, CC-based ELSP studies contribute significantly to scheduling real systems. Their main practical contributions include:

- Providing simple and implementable policies for coordinating multi-product production cycles.
- Offering analytical and heuristic tools that reduce computational complexity for planners.
- Addressing extensions relevant to practice, such as shortages, quality defects, rework, and shipment policies.
- Highlighting how constraints like capacity, budget, and warehouse space affect scheduling decisions.

Overall, the reviewed body of literature demonstrates that CC-based ELSP models are not only academically relevant but also provide valuable insights for practitioners managing scheduling in constrained, multi-product production environments. At the same time, the identified gaps offer fertile ground for future research that can bring ELSP modeling closer to the realities of modern manufacturing.

6. Conclusion

This review has systematically examined 141 journal articles on the Economic Lot Scheduling Problem (ELSP) under the Common Cycle (CC) approach, published between 1990 and August 2025. By applying a structured taxonomy, the studies were categorized across key modeling dimensions, including shortage policies, defective production, maintenance, deterioration, objective functions, constraints, decision variables, and input assumptions. Bibliometric statistics further revealed the main contributing journals, countries, and authors, providing a comprehensive overview of the research landscape.

The principal contributions of this study are threefold. First, it offers an updated and detailed literature review of CC-based ELSP models, extending prior surveys by covering the most recent decade of research. Second, it introduces a refined taxonomy that captures both classical and emerging themes, thereby enhancing the analytical clarity of the field. Third, the paper highlights critical gaps in the literature, including limited treatment of stochastic and fuzzy environments, underrepresentation of maintenance and deterioration, and the scarcity of empirical validation.

From a managerial perspective, the CC approach remains highly relevant for real-world scheduling because it provides simple, implementable, and coordinated production policies that are especially suitable for multi-product environments constrained by capacity and resources. Industries such as pharmaceuticals, food processing, automotive, and electronics can benefit from the insights offered by CC-based ELSP models, particularly when addressing quality defects, shortages, and shipment policies.

Looking ahead, future research should focus on bridging theoretical models with practical applications. Promising directions include linking ELSP with Industry 4.0 technologies, incorporating sustainability and carbon considerations, adopting multi-objective optimization frameworks, and validating models with industrial case studies and empirical data. Addressing these gaps will strengthen the practical applicability of CC-based ELSP research and ensure its continued relevance in modern production environments.

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Appendix A. List of the 141 selected papers

ID	Paper
[1]	Adhisatya *et al.* [1]
[2]	Akrami *et al.* [2]
[3]	Alle *et al.* [3]
[4]	Banerjee *et al.* [6]
[5]	Banerjee [5]
[6]	Ben-Daya and Hariga [10]
[7]	Beck and Glock [7]
[8]	Beck *et al.* [9]
[9]	Bollapragada and Rao [11]
[10]	Bourland and Yano [13]
[11]	Bourland *et al.* [14]
[12]	Brander and Forsberg [15]
[13]	Casas-Liza *et al.* [17]
[14]	Carreno [16]
[15]	Chan and Song [18]
[16]	Chan *et al.* [20]
[17]	Chang and Yao [21]
[18]	Chiu *et al.* [23]
[19]	Chiu *et al.* [24]
[20]	Chiu *et al.* [25]
[21]	Chiu *et al.* [26]
[22]	Chiu *et al.* [27]
[23]	Chiu *et al.* [28]
[24]	Chiu *et al.* [29]
[25]	Chiu *et al.* [30]
[26]	Chiu *et al.* [31]
[27]	Chiu *et al.* [32]
[28]	Chiu *et al.* [33]
[29]	Chiu *et al.* [34]
[30]	Chiu *et al.* [36]
[31]	Chiu *et al.* [35]
[32]	Choi and Noble [37]
[33]	Chowdhury and Sarker [38]
[34]	Das Roy and Sana [39]
[35]	Dobson and Yano [41]
[36]	Elhafsi and Bai [44]
[37]	El-Najdawi [43]
[38]	Eynan [49]
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Appendix B. Classic ELSP Model with the CC Approach

The classic ELSP under the Common Cycle (CC) approach assumes that all items share the same production period T. In this formulation, n denotes the number of items, A_i represents the setup cost for item i, h_i is the holding cost per unit per unit of time for item i, p_i is the production rate, d_i is the demand rate, c_i denotes the production cost per unit, and T refers to the common cycle length. The total cost (TC) is the sum of setup, holding, and production costs:

$$\mathrm{TC} = \sum_{i=1}^{n} \left(\frac{A_i}{T} + \frac{h_i d_i}{2} \left(1 - \frac{d_i}{p_i}\right) T + d_i c_i \right). \tag{B.1}$$

Johnson and Montgomery [82] showed that this function is convex in T. Thus, the optimal cycle length can be obtained by differentiation:

$$T^* = \sqrt{\frac{2\sum_{i=1}^{n} A_i}{\sum_{i=1}^{n} \left(h_i d_i \left(1 - \frac{d_i}{p_i}\right)\right)}}. \tag{B.2}$$

When shortage costs (π_i) are incorporated into the classic model, the total cost (TC) function is extended to include terms for the shortage quantity per item. The resulting cost function remains convex, and the optimal cycle length (T^*)

and optimal shortage levels (S_i^*) can be obtained analytically using standard calculus, as shown in equations (B.3)–(B.5).

$$\mathrm{TC}=\sum_{i=1}^{n}\left(\frac{A_i}{T}+\frac{h_i d_i}{2}\left(1-\frac{d_i}{p_i}\right)T+\frac{(h_i+\pi_i)\,p_i}{2d_iT\,(p_i-d_i)}\left(\frac{S_i^2}{T}\right)+\frac{\pi_i}{T}\left(\frac{S_i}{T}\right)-h_iS_i+d_ic_i\right) \tag{B.3}$$

$$T^*=\sqrt{\frac{2\sum_{i=1}^{n}A_i-\sum_{i=1}^{n}\frac{\pi_i^2 d_i}{(h_i+\pi_i)}\left(1-\frac{d_i}{p_i}\right)}{\sum_{i=1}^{n}h_id_i-\sum_{i=1}^{n}\frac{h_i^2 d_i}{(h_i+\pi_i)}\left(1-\frac{d_i}{p_i}\right)}} \tag{B.4}$$

$$S_i^*=\sqrt{\frac{(h_iT^*+\pi_i)\,d_i}{(h_i+\pi_i)}\left(1-\frac{d_i}{p_i}\right)}. \tag{B.5}$$

When a portion of items produced is defective and cannot be reworked, the model is extended to incorporate both the proportion of rejected items (α_i) and the cost of discarding them (v_i). The corresponding total cost function remains convex (Eq. (B.6)), and the optimal cycle length can again be derived analytically (Eq. (B.7)).

$$\mathrm{TC}=\sum_{i=1}^{n}\left(\frac{A_i}{T}+\frac{h_id_i}{2}\left(1-\frac{d_i}{(1-\alpha_i)\,p_i}\right)T+\frac{d_i\,(c_i+\alpha_iv_i)}{1-\alpha_i}\right) \tag{B.6}$$

$$T^*=\sqrt{\frac{2\sum_{i=1}^{n}A_i}{\sum_{i=1}^{n}\left(h_id_i\left(1-\frac{d_i}{(1-\alpha_i)p_i}\right)\right)}}. \tag{B.7}$$

When defective items can be reworked, the model is further extended by incorporating the rework rate (β_i), rework cost (y_i), and rework rate (r_i). In this case, defective items are assumed to be reprocessed immediately after the regular production cycle. The total cost function is reformulated to include these terms (Eq. (B.8)), and the corresponding optimal cycle length is derived in closed form (Eq. (B.9)). These extensions demonstrate how the classic CC-based ELSP can integrate rework considerations without losing its convexity properties, thus allowing analytical solutions for optimal cycle length to be maintained.

$$\mathrm{TC}=\sum_{i=1}^{n}\left(\frac{A_i}{T}+\frac{h_id_i}{2}\left(\left(1-\alpha_i-\beta_i-\frac{d_i}{p_i}\right)(1+\beta_i)+\left(1-\frac{d_i}{r_ip_i}\right)\beta_i^2\right)T+\frac{d_i\,(c_i+\alpha_iv_i+\beta_ir_i)}{1-\alpha_i}\right) \tag{B.8}$$

$$T^*=\sqrt{\frac{2\sum_{i=1}^{n}A_i}{\sum_{i=1}^{n}\left(h_id_i\left(\left(1-\alpha_i-\beta_i-\frac{d_i}{p_i}\right)(1+\beta_i)+\left(1-\frac{d_i}{r_ip_i}\right)\beta_i^2\right)\right)}}. \tag{B.9}$$