RAIRO-Oper. Res. 60 (2026) 1357–1391
https://doi.org/10.1051/ro/2026029

RAIRO Operations Research
www.rairo-ro.org

A GAME THEORETICAL STUDY ON LEAD TIME, PRICING AND INVENTORY DECISIONS WITH PRICE AND INVENTORY DEPENDENT DEMAND

Ata Allah Taleizadeh[1,2], Aida Hami-Dindar[3], Leopoldo Eduardo Cárdenas-Barrón[4,*] and Mahsa Noori-daryan[5]

Abstract. Nowadays, determining optimal lead time, inventory, and pricing decisions has become a critical challenge in competitive markets, particularly when demand exhibits sensitivity to both price and product availability. This study develops a game-theoretic model in which demand depends jointly on price and inventory levels for complementary products. The analysis considers both single-firm and duopoly settings under alternative competitive regimes, including Nash, Stackelberg, and cooperative strategies. The results show that cooperative behavior yields the highest profits, while pricing and inventory decisions exert stronger impacts on performance than lead-time adjustments. From a managerial perspective, the findings provide guidance on when firms facing complementary demand should compete or cooperate, how coordination can reduce lead times and holding costs, and how sensitivity insights support more effective pricing and inventory decisions.

Mathematics Subject Classification. 90B05.

Received December 4, 2024. Accepted March 10, 2026.

1. Introduction

Fast and reliable service has become a critical challenge for firms operating in competitive markets. At the same time, pricing and inventory decisions play a central role in shaping customer demand and firms' profitability. In many inventory systems, these decisions are closely interrelated, yet they are often analyzed separately in the literature. This study is motivated by the need to better understand how pricing, inventory, and delivery lead-time decisions interact and jointly affect firm performance.

To address this issue, we analyze firms' behavior under two different scenarios. In the first scenario, a single firm determines its pricing, inventory, and lead-time decisions to satisfy customer demand. In the second scenario, the model is extended to a duopoly setting in which two firms supplying complementary and interdependent

Keywords. Pricing, lead time, game theory, Nash equilibrium, Stackelberg, cooperative.

[1] School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.
[2] Department of Industrial Engineering, Istinye University, Istanbul, Turkey.
[3] Department of Industrial Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran.
[4] Tecnologico de Monterrey, School of Engineering and Sciences, Ave. Eugenio Garza Sada 2501, Monterrey, N. L.64849, México.
[*]Corresponding author: lecarden@tec.mx

products interact strategically. In this context, market demand depends on both price and inventory decisions, reflecting the joint consumption nature of complementary products.

By integrating pricing, inventory, and lead-time decisions within a unified analytical framework, this study contributes to the growing literature on coordinated decision-making in inventory systems. The decision variables considered in the model are price, lead time, and inventory level, and optimization methods are employed to determine their optimal values. The proposed framework enables a systematic comparison of firm behavior under different competitive and cooperative settings, while maintaining analytical tractability through closed-form solutions.

In markets where firms supply fully complementary components, pricing, inventory availability, and delivery lead time are inherently interdependent in practice because they jointly shape service reliability and customers' ability to complete consumption. Although the products are technologically complementary, the supplying firms are often independent entities that may compete on price and responsiveness while benefiting from the joint consumption relationship. Accordingly, different strategic interaction patterns can arise: (i) simultaneous decision-making when firms have comparable market power (modeled as Nash competition), (ii) leader–follower behavior when one firm has a structural advantage such as brand dominance, scale, or better demand information (modeled as Stackelberg competition), and (iii) coordination through alliances, long-term contracts, or joint planning agreements when firms can internalize complementarities and pursue joint profit improvement (modeled *via* a cooperative/centralized benchmark). This practical context motivates the unified game-theoretic framework developed in this study.

The rest of this article is organized as follows. Section 2 outlines related literature. Section 3 presents a problem description. Section 4 formulates the profit function of an inventory model for two scenarios: Single firm and two competing firms. Section 5 optimizes the inventory model. Section 6 solves several numerical examples and provides a sensitivity analysis. Finally, Section 7 gives some conclusions and future research directions.

2. Literature review

Many factors affect the profitability of companies, but when the issue of competition is raised among firms, establishing jointly the inventory, lead time and pricing become a challenge task. This paper considers jointly three streams of research which are lead time, inventory and pricing. Recently, studies have been done in these fields. Some of this research has focused on one element lead-time, inventory, or pricing. Other studies have focused on decisions regarding two of these elements.

Section 2 is organized around the main research streams relevant to this study, namely lead time, inventory, and pricing decisions. For each stream, we briefly summarize the key developments and conceptual insights, and then position the present research by highlighting how it extends these streams through their joint integration in a game-theoretic framework with fully complementary products. The focus is on synthesizing the evolution of each research stream and identifying the remaining gaps that motivate the proposed modeling framework.

2.1. Lead time in inventory systems

Lead time is considered a very important variable in inventory control systems and many studies have been conducted to realize the effect of lead time which some of these studies explore the impact of lead time reduction [26, 38]. For example, Leng and Parlar [21] studied reducing lead time in a two-level supply chain including a retailer and a manufacturer. They divided lead time into three components: setup time, production time and transport time. They analyzed the problem of reducing lead time using Nash equilibrium and Stackelberg. Then, they proposed a profit-sharing contract in order to accomplish supply chain coordination and showed that two members of the supply chain reach to their maximum profit under this contract. Treville *et al.* [44] developed a model for the optimization of production in which lead time is an endogenous decision. They calculated the cost differential necessary to compensate for the risk exposure due to lead time. More recent studies also emphasize the strategic role of lead time in competitive environments. For example, Benioudakis *et al.* [2] examined lead-time quotations in unobservable make-to-order systems with strategic customers, showing how firms optimally choose both price and lead time under risk aversion and load control considerations. Zhai *et al.* [55] developed three models with different structures in order to reduce lead time. They determined optimal policies of decision using Stackelberg and cost-sharing approaches.

Ray *et al.* [29] considered a company that sells products with a constant demand. The company manages the inventory of raw materials using a (Q, r) policy. The replenishment lead time is random. They studied the effect of various types of investment schemes on reducing lead time. Lee [20] considered an inventory system with infinite horizon for an $(S-1, S)$ inventory model with Poisson demand and random lead time in order to reduce replenishment lead time in which the firm in charge of inventory pays periodic payments to the supplier. In this inventory model, it is assumed that the utility function of supplier follows an exponential distribution and the lead time follows a normal distribution. Also, a contract is designed in which supplier receives a fixed amount plus incentive payments for the replenishment orders supplied during the lead time. They concluded that reducing lead time leads to a significant reduction in costs. Chaharsooghi and Heydari [4] investigated the effect of statistical characteristics of lead time in a supply chain under uncertainty condition for both uniform and normal distributions and presented solution methods for the selection of supplier, transport method and production program. Cobb *et al.* [8] addressed the problem of modeling lead time demand in a continuous review system. They estimated a distribution, composed of a mixture of polynomial distributions obtained from experimental data of demand with the aim to create an accurate probability density function for demand lead time.

In recent years, researchers have turned to bring together two important research streams: lead time management and the effect of inventory on the market demand (inventory billboard effect) [36,37,39]. In make-to-order settings, Zhai and Cheng [54] analyze lead-time quotation together with coordination instruments, highlighting how centralized benchmarks serve as natural comparators to decentralized equilibria, an approach we also adopt in our cooperative (centralized) case. Wu *et al.* [49] analyzed lead time and inventory of production firms using Nash equilibrium. They showed that an increase in lead times leads to an increase in inventory levels. Furthermore, a considerable amount of research has been carried out on the problem related to simultaneously determining optimal lead times and prices. For example, Boyaci and Ray [3] developed a model that incorporates delivery time and pricing decisions considering capacity requirements and costs for two substitutable products: regular and express. The goal of Boyaci and Ray [3]'s model is to specify the delivery time of the express product and the price of two products, taking into account the effect of the delivery time reduction on capacity requirements and costs. They concluded that prices can decrease when the firm incurs capacity related costs.

Pekgün *et al.* [28] proposed a model in which the demand is linear with price and lead time dependent. They considered that the pricing and lead time decisions are made by the marketing and production departments, respectively. In the decentralized case, the firm's profits and prices are lower, lead times are longer and the total demand created is larger as compared to the centralized case. They determined that using a transfer price with bonus payments can coordinate the decisions of the firms. Studied the newsvendor problem with endogenous adjustment of price and lead time. They considered that the demand within the sales season and the lead time needed for customization are uncertain. The selling price and the lead time influences the market demand. Their model determines the optimal selling price, lead time and order quantity jointly.

Zhu [56] considered a decentralized supply chain composed of a supplier and a retailer with price and lead time sensitive demand. He studied the behavior of members of the supply chain using Stackelberg game. He also investigated the effects of production capacity on profits. Studied a two-level supply chain with one supplier and one producer. They supposed that the producer observes a Poisson demand process where the arrival rate is dependent on selling price, delivery time and delivery reliability. They concluded that coordination of the supply chain for most cases is achieved using an all-unit quantity discount strategy. Xiao *et al.* [52] examined a make-to-order duopoly system in order to make decision on lead time, price and structure of channel decision using game theory. They found that lead time increases in a decentralized supply chain and each of two producers may select different channel structures under symmetric duopoly. They also found that the effect of decentralization on lead time is heavily dependent on the game-theoretic approach applied. Decision timing under competition is also critical, as Karray *et al.* [15] show that when strategic variables (*e.g.*, price or marketing effort) are selected in different sequences, equilibrium outcomes change markedly. This aligns with our game-theoretic analysis across simultaneous-move (Nash) and leader–follower (Stackelberg) regimes. Overall, the lead-time literature has evolved along three main directions: operational models treating lead time as an exogenous or stochastic parameter, strategic models endogenizing lead time through investment or contractual mechanisms and competitive models integrating lead time with pricing decisions under decentralized control. However, these streams have largely developed independently and predominantly focus on single products or substitutable goods. The interaction between endogenous lead time, inventory-dependent demand, and complementary-product competition under alternative game-theoretic regimes remains insufficiently explored. This study contributes to the lead-time stream by embedding lead time as a strategic decision variable within a unified pricing–inventory framework for fully complementary products.

2.2. Inventory control in inventory systems

Inventory level is considered a very significant variable in inventory control systems and several research works have been done in this area. Classical inventory models suppose that the market demand is independent of inventory levels [40–42]. However, many marketing researchers and practitioners have realized that an increase in inventory (or shelf space), induces more customers to buy. For example, Min and Zhou [23] derived an inventory model for perishable items with sales rate depending on the stocks. The uncovered demand is partially backordered and the backordered demand ratio is dependent on the negative inventory level within the stock-out period. The inventory model imposes a ceiling on the amount of on-display stocks due to the fact that too much inventory makes a negative impression on the purchaser. Wang and Gerchak [45] studied a wholesaler who has demand rates which are dependent of the display space that is dedicated to that item by themselves and their competitors. They proposed two ways to envision and model the demand and market split. One considers that demand depends on aggregated inventory exhibited and the other assumes that demand is a function of the individual display level at retailer.

Roy and Chaudhuri [30] developed two inventory–production models for demand rate depending on the instantaneous inventory level in which production rate is dependent on both demand and the level of inventory. Chen *et al.* [6] studied the issue of coordination of a distribution system considering vendor-managed-inventory and consignment arrangements. They studied a profit maximization problem in cooperative and non-cooperative situations and they concluded that decentralized non-cooperative competition has a tendency to lead to increase prices and reduce inventory, which in combination lead to lower profits.

Devangan *et al.* [9] proposed an inventory model for a supply chain considering that retailer demand is affected by the amount of inventory displayed on the shelf. They assumed that inventory displayed influences the demand positively. Basically, they designed an individually rational buyback contract that completely coordinates the supply chain. Wu *et al.* [48] examined the benefits of supply lead time reduction facilitated by RFID adoption. They also studied whether companies can be better when they have a lead time as short as possible, when there is the inventory billboard effect. Saha *et al.* [31] examine a strategic inventory–pricing framework for substitutable goods within a two-manufacturer and common retailer structure. They incorporate the notion of strategic inventory to influence market outcomes and pricing decisions, highlighting how inventory levels are used as a competitive lever. While their model advances understanding of inventory–price interactions under substitutability, it does not endogenize lead time decisions nor analyze game-theoretic regimes (*e.g.*, centralized benchmark *vs.* decentralized Nash/Stackelberg). Our work extends this line by embedding lead time into the joint optimization framework and allowing direct comparison across competitive and cooperative settings. Edalatpour *et al.* [11] further integrate pricing and inventory decisions for deteriorating complementary products under sustainability considerations. Their framework accounts for economic, environmental, and social constraints, using heuristic algorithms to derive solutions. While their study contributes to the literature on joint pricing-inventory models, it does not incorporate endogenous lead time or game-theoretic structures, which distinguishes the present work. Kausar *et al.* [16] investigated inventory and pricing decisions in a closed-loop supply chain with remanufacturing. Their results highlight how sustainable practices, including recycling and remanufacturing, interact with joint pricing–inventory decisions to enhance overall profitability. Li *et al.* [22] solved an inventory management problem with dual channels, which is administered by a supplier. They considered that demand of dual channels is inventory-level-dependent. In addition, there are some studies that simultaneously study the impact of the inventory and price on the profitability of companies. For example, Xiang *et al.* [50] developed a dynamic cooperative replenishment model under uncertainty, emphasizing the benefits of collaboration in inventory control for large-scale industries such as shipbuilding. For multi-item settings with product interdependencies, Edalatpour and Mirzapour Al-e-Hashem [10] study simultaneous pricing and inventory control for substitutes and complements, documenting how cross-effects reshape optimal policies. Our model extends this line by jointly integrating lead time with pricing-inventory decisions in a complementary-product context. Chen and Hu [5] formulated a joint pricing and inventory model with price adjustment costs and deterministic demand which is dependent on the price. The inventory model was derived for a single product over a finite planning horizon. Their inventory model also determines jointly the ordering quantity and a price. Unlike the present study, prior works have not simultaneously incorporated the inventory effect into the joint determination of lead time and pricing decisions. Moreover, existing models seldom capture the specific features of complementary demand, nor do they provide a unified game-theoretic framework that integrates pricing and lead-time choices. This paper addresses these gaps by explicitly modeling the interdependence of inventory, price, and lead time in the context of complementary products under competitive and cooperative games. Such limitations reinforce the novelty of our work, which explicitly incorporates the inventory effect together with lead time and pricing when analyzing fully complementary products in game-theoretic environments.

2.3. Pricing in inventory systems

Pricing is another influential factor in the decision-making process that has a significant impact in inventory systems. A remarkable amount of studies have been carried out on the optimal pricing decisions for a single product or substitutable products. For example, Transchel *et al.* [43] considered a production company that manufactures two products on a common resource with the single production capacity. The comparison of centralized and decentralized planning reveals that favorable profit is obtained through coordination of pricing and capacity decisions. Kumar *et al.* [18] proposed a mathematical model for a three-level synchronized and non-synchronized supply chain considering the exponential price-dependent demand. The model takes into account ordering/setup, carrying and transportation costs. They demonstrated the optimality of inventory decisions with and without coordination. Sivashankari *et al.* [33] examine complementary products in an imperfect production system with reworking and scrap, under a Bertrand price-dependent demand. Their model jointly optimizes price and lot size and shows that reworking defective items increases profitability. While relevant to complementary product settings, their framework does not incorporate lead time, inventory-based demand effects, or game-theoretic competition, which distinguishes the present work. Guan and Zhao [12] examined multi-retailer inventory system in order to optimize inventory and pricing decisions simultaneously with the aim to maximize the profit. In this inventory, each retailer has a random demand and uses the (r, Q) policy to control inventory. Pando *et al.* [27] derived a profit maximization model in which they considered non-linear holding cost and stock dependent demand. They provided the necessary and sufficient condition for profitability of the system.

Chen *et al.* [7] studied a decision making model for two firms competing in a Stackelberg game. They have done an equilibrium analysis for both the centralized and decentralized schemes with and without cooperation. They found that non-cooperative approach leads to set a higher revenue-sharing percentage and lower cost for the retailer, and a higher retail price and less display space for the manufacturer. Consequently, the non-cooperative approach provides less profit for the manufacturer. Chen and Hu [5] considered the periodic-review inventory system in which the demand in each period is uncertain and rises with respect to the inventory level. Avinadav *et al.* [1] proposed two inventory models to determine optimal price, order quantity and replenishment period when the demand function is separable into components of price and inventory age. In addition to classical models, recent research integrates sustainability concerns into pricing–inventory decisions. Jauhari *et al.* [14] studied joint pricing and green inventory policies under carbon tax regulation, showing that environmental investments significantly reshape firms' pricing and inventory strategies. Heydari and Noruzinasab [13] presented an incentive policy to coordinate ordering, lead time and pricing in a two-level supply chain. The model has a stochastic demand depending on price and lead time. They showed that coordinated decision making reduces price and lead time to retailer, and this increases order size. Kumar *et al.* [19] proposed dynamic pricing strategies under auto-correlated stochastic demand using exponential smoothing. Their study illustrates how advanced demand forecasting methods can improve the efficiency of inventory and pricing decisions.

The notion of complementary products emerges when consumers may have to buy more than one product at the same time to achieve the complete utility of both products [53]. Complementary products have been considered in the recent years and therefore, few researches have considered the pricing decision for the complementary products [25, 34, 46, 53]. Beyond classical pricing models, recent studies explicitly address complementary products. Mondal *et al.* [24] examine pricing and bundling for complementary items in a two-stage supply chain, showing how joint policies interact with remanufacturing and green innovation. Their evidence reinforces the need for integrated pricing–inventory choices when complementarities drive demand, precisely the setting considered in our work. Soon [34] studied Nash equilibrium in pricing models in which the demand function is defined by a nonlinear complementarity problem (NCP) and the restriction of pricing includes complementary conditions. He compared the deterministic model with the probabilistic pricing model. Yue *et al* [53], presented a model to achieve the optimal strategies for a company making decisions considering information asymmetry. They assumed that the consumers have necessity to purchase two complementary products as a mixed bundle, supplied by two different companies when the market demand is dependent on the pricing strategy of both companies. They followed a Bertrand game to analyze their model. Mukhopadhyay *et al.* [25] proposed a duopoly market where two distinct companies provide complementary products in a leader–follower approach. They concluded that if the follower company unconditionally shares the information, then this would benefit the leader company but affects to the follower company and the whole system. A recently published study by Kim *et al.* [17] investigates a multi-item inventory model with a leading product pricing mechanism to analyze complementarity relationships among items. Their framework reveals how pricing the lead product influences demand across the bundle and how inventory levels interact in multi-product settings. While their work advances our understanding of multi-item dependency and pricing, it does not endogenize lead time nor explore game-theoretic interaction. We build on this line by simultaneously modeling pricing, inventory, and lead time under complementary demand and comparing Nash, Stackelberg, and centralized benchmarks. Wei *et al.* [46] analyzed the pricing decisions for two complementary products for a supply chain with two producers and one common retailer. However, these studies did not examine the products which are simultaneously complementary and dependent (or fully complementary). Furthermore, to the best of our knowledge, no research has considered the pricing problem of complementary products by including the lead time and inventory adjustment in order to obtain the more profits. Yet, the integration of pricing, lead time, and inventory decisions for fully complementary products under Nash, Stackelberg, and cooperative games remains unexplored. Addressing this gap, the present study develops a unified framework that integrates pricing, lead time, and inventory for complementary products under Nash, Stackelberg, and cooperative games.

2.4. Research gap

After reviewing the literature across lead time, inventory, and pricing, it becomes evident that while each stream has developed independently, its integration in the context of complementary products and game-theoretic competition has rarely been explored. Furthermore, to the best of our knowledge, no study has jointly analyzed these three factors within cooperative supply chain settings or in strategic game-theoretic frameworks aimed at identifying profit-maximizing strategies. In this direction, this article concentrates on the effect of lead time, pricing, and inventory on the competitive and cooperative behavior of firms in order to maximize profit.

In addition, today, there are products in the market that are totally complementary and interdependent, and therefore, the customers need to use them simultaneously to satisfy their needs. Thus, another innovation of this paper is to consider a model for products that are fully complementary. Despite these recent contributions (*e.g.*, [2, 14, 16, 19, 50]), no existing study has jointly integrated lead time, inventory, and pricing in the context of complementary products under both competitive and cooperative game-theoretic settings. This gap motivates the present research.

In this research, the demand function depends on the sum of the inventory of the two products which is similar to the assumption made by Wu *et al.* [49] and Wang and Gerchak [45]. On the other hand, by considering the products as complementary and assuming that the demand for one of them would lead to demand for the other, it is supposed the demand function is also dependent on the sum of both products' selling prices. To the best of our knowledge, such a demand formulation- combining inventory- and price-dependence for fully complementary products- has not been explicitly analyzed in prior works, and constitutes one of the main contributions of this paper.

As discussed earlier, an example of two mentioned complementary products is the laptop and a solid-state drive. This paper focuses on the lead time and pricing decisions when demand is influenced by the inventory. It also investigates the firms' behavior under Nash, Stackelberg, and cooperative games.

3. Problem description

In this study, the behaviors of production firms are studied under two different scenarios considering the effects of lead time, pricing and inventory on market demand. In the first scenario, a production firm intends to maximize its profit using optimal lead time, inventory and selling price. In the second scenario, the model is developed for two firms where each firm satisfies a percentage of market demand, and the demand depends on the inventory level and sale price.

Each firm produces only one product and these products are complementary. Therefore, demand for the two products are interdependent. It is also assumed that the products manufactured by these two firms, such as some electronic devices and spare parts, work together. An example of two complementary products is the laptop and a solid state drive. Many laptop users, immediately after purchasing their desirable device, in order to upgrade it, are buying an appropriate solid state drive that can meet their needs. This action causes to faster boot-up and application loading times, and more rugged data protection for the laptop. The solid state drive and laptop are two fully complementary products that their best performance happens when they work together.

As an illudtrative industry setting, consider a laptop manufacturer and an independent solid-state-drive (SSD) supplier operating in the same consumer market. The customer's effective utility is realized only when both components are available within a short time window; thus, perceived availability depends on the combined readiness of the complementary system. When both firms have comparable market power and make decisions in parallel, the interaction resembles Nash competition. If the laptop brand dominates the market and the SSD supplier reacts to its announced price and delivery promise, a leader–follower (Stackelberg) structure becomes plausible. Finally, coordination can arise through joint promotions, compatibility programs, revenue-sharing, or synchronized replenishment planning, which motivates the cooperative (centralized) benchmark used in this paper.

To study the performance and decisions of the firms in these scenarios, three game-theoretic approaches of Nash equilibrium, Stackelberg and cooperative are used. In the Nash equilibrium, both firms participate in a non-cooperative game. Under the Stackelberg game, one of the firms determines its decisions based on the other firm's decisions. Here, one of the firms, as a market leader, is informed of the decision of the other one which plays the role of follower. The follower determines the optimal value of decision variables. Then, the leader decides on own decision variables according to the best answer of the follower so that it leads to the optimal profit. In cooperative approach, in fact, both firms cooperate with each other knowing conditions of each other in order to maximize the profit.

4. Modeling

This section develops the profit function of an inventory model for two scenarios: Single firm and two competing firms. The profit of each firm is computed by the difference among sales income and production cost, backorder cost and holding cost. Table 1 presents the symbols used in this paper.

Table 1. Symbols used in the development of the inventory model.

Parameters	
w	Market potential; the base demand, $w > 0$
d	Elasticity value for measuring the degree of demand depending on inventory, $-\frac{1}{2} < d < 0$
g	Price elasticity value
R	Demand rate in units per unit time
a	Average lead time demand in units per unit time
ψ	Variance of demand during lead time units per unit time
n	Non-negative coefficient within interval $(0, 1]$ for assuring demand is always positive
S	Total inventory and expected sales in units
K	Backorder level in units
e	Production cost per unit
h	Holding cost per unit per unit time
b	Backordering cost per unit per unit time
Ω	Profit function of the firm under first scenario \$ per unit time
Ω_i	Profit function of the firm i under the second scenario \$ per unit time
Decision variables:	
I	Average on hand inventory of firm in units
V	Sale price \$ per unit
M	Lead time in unit time

It is important to remark that the parameters d and g indicate the impact of inventory and price in demand, respectively. Notice that a larger value for d indicates that inventory level has a more significant effect on demand.

It is assumed that the value of d is in the following interval: $-\frac{1}{2} < d < 0$. Firstly, note that for any value of g and $d = -\frac{1}{2}$ means that demand is independent of inventory and it is dependent on selling price only. Secondly, for any value of d in $-\frac{1}{2} < d < 0$ and $g = 0$ then demand is independent of price and it depends only on inventory. When $d = -\frac{1}{2}$ and $g = 0$, then the demand is independent of inventory and price and simply the base demand (w) occurs. These definitions are provided here for clarity, and will be used in the subsequent derivations.

4.1. First scenario: Single firm

The single firm uses an inventory model which permits shortages. The profit function of each firm is given by:

$$\max \Omega = (V - e)R - bK - hI. \tag{1}$$

Next, the market demand is defined as a function of both inventory and selling price at the same time. The demand function expressed in equation (2) is considered as multiplication of market potential (base demand), inventory and product price. From an economic perspective, the dependence of demand on inventory reflects consumers' sensitivity to product availability and service level. Higher inventory levels reduce perceived stock-out risk, increase product visibility, and enhance the probability of immediate fulfillment, thereby stimulating demand. This mechanism is consistent with the inventory billboard effect documented in the operations management literature.

$$R = wI^{2d+1}V^{-g}. \tag{2}$$

The demand exponent $(2d + 1)$ is adopted to capture the nonlinear sensitivity of demand to inventory availability. In particular, when $d < 0$ is specification reflects diminishing marginal influence of inventory:

additional stock increases the likelihood of sales, but at a decreasing rate. This interpretation motivates the term "inventory elasticity", which is analogous to conventional price elasticity. Although positive values of d might appear intuitive, we restrict $d < 0$ to ensure concavity of the profit function and the stability of the optimization problem, as discussed in the Appendices A–F.

Moreover, in this model, the selling price is endogenously chosen to maximize expected profit under the assumed stochastic demand distribution (see Eqs. (3)–(5)), consistent with stochastic pricing–inventory models (*e.g.*, [5, 45, 47, 51]).

In this setting, the average on hand inventory I is calculated as follows:

$$I = \int_{a-\psi}^{S} (S-x)\pi(x)\,\mathrm{d}x = \frac{1}{2\psi}\left(Sx - \frac{1}{2}x^2\right)\Big|_{x=a-\psi}^{x=S} = \frac{1}{4\psi}(S-a+\psi)^2 \tag{3}$$

where x represents a random demand that follows a probability density function $\pi(x)$. The firm's demand during lead time, follows a uniform distribution $U = (a-\psi, a+\psi)$, where a is the average of demand during lead time and ψ is half of the range that measures the variability of the lead time demand which are determined as follows:

$$a = MR = MwI^{2d+1}V^{-g} \tag{4}$$

$$\psi = nMR = nMwI^{2d+1}V^{-g}. \tag{5}$$

Here, n represents a non-negative coefficient in interval $(0, 1]$ in order to ensure that demand cannot be negative.

Due to the fact that each unit of demand is either a sale or a backorder then $a = \text{Expected sales} + K$. Furthermore, each unit in inventory is either sold or left in storage, thus $S = \text{Expected sales} + I$. Consequently, $\text{Expected sales} = S - I$ providing the following result $K = a + I - S$. Therefore, the total inventory and expected sales (S) can be expressed as a function of lead time, inventory and price by substituting equations (4) and (5) into equation (3), hence,

$$S = a - \psi + 2\sqrt{\psi I} = MwI^{2d+1}V^{-g}(1-n) + 2\sqrt{nMwI^{2d+2}V^{-g}}. \tag{6}$$

Similarly, the backorders (K) is expressed as a function of lead time, inventory and price as follows:

$$K = a + I - S = MwI^{2d+1}V^{-g} + I - MwI^{2d+1}V^{-g}(1-n) - 2\sqrt{nMwI^{2d+2}V^{-g}}. \tag{7}$$

Substituting equations (2) and (7) into equation (1), the profit function of the firm is obtained as follows,

$$\begin{aligned}\max\Omega = {} & wI^{2d+1}V^{1-g} - ewI^{2d+1}V^{-g} - bMwI^{2d+1}V^{-g} - bI + bMwI^{2d+1}V^{-g}(1-n) \\ & + 2bI^{d+1}\sqrt{nMwV^{-g}} - hI.\end{aligned} \tag{8}$$

It is straightforward to verify that the profit function is concave in the decision variables (see the Appendices A–F for the detailed proof). Thus, the first-order conditions guarantee a global optimum.

4.2. Second scenario: Two competing firms

This section generalizes the previous inventory model to two competing firms. Before presenting the demand function for the two-firm case, it is important to clarify how complementarity is modeled. It is worth noting that while demand for perfectly complementary products could in principle be constrained by the bottleneck component (*i.e.*, $\min(I1, I2)$), we adopt an additive specification $(I1 + I2)$ combined with prices. This additive form, which has been widely employed in the operations management literature, preserves analytical tractability while still capturing essential aspects of complementarity: greater availability of one component can enhance the bundle's attractiveness, sustain demand through backlog or marketing effects, and capture the joint role of inventory and pricing. Bottleneck-type demand constraints represent a promising direction for future research.

In the case of fully complementary products, consumer utility is derived from the joint consumption of both components. Consequently, the aggregate inventory level across firms serves as a proxy for the effective availability of the complementary system. From a behavioral and economic perspective, higher joint inventory levels signal greater system reliability and reduce consumers' perceived risk of encountering stockouts in one component, thereby increasing confidence in completing the consumption process. This formulation captures consumers' expectations regarding the simultaneous availability of complementary components and the feasibility of completing consumption without delay.

It is important to clarify the practical scope of the full-complementarity assumption adopted in this study. Full complementarity is most appropriate in markets where consumer utility is realized only when all components are jointly available and functionally interdependent, such as system-based products, core products with essential add-ons, or components required for immediate and complete consumption. In such settings, the absence of one component effectively constrains the usability of the other. However, this assumption may not hold in markets characterized by partial complementarity, substitution flexibility, or delayed consumption. Accordingly, the present model is best interpreted as a benchmark case, and extending the framework to account for partial or asymmetric complementarity represents a promising direction for future research.

The customers' demand function is shown as equation (9). Note that customers' demand rate depends on inventory levels and prices of both firms:

$$R = w(I_1 + I_2)^{2d+1}(V_1 + V_2)^{-g}. \tag{9}$$

Each firm competes to satisfy the customers' demand. Therefore, each firm satisfies a portion of market demand based on the inventory level such that aggregate demand is $R = \sum_{i=1}^{2} R_i$ for $i = 1, 2$. Where:

$$R_i = wI_i(I_i + I_{3-i})^{2d}(V_i + V_{3-i})^{-g}. \tag{10}$$

Then, considering the demand function given in equation (10) then the average demand during lead time (a_i) and variance of demand during lead time (ψ_i) are defined as follows:

$$a_i = M_iR_i = M_iwI_i(I_i + I_{3-i})^{2d}(V_i + V_{3-i})^{-g} \tag{11}$$

$$\psi_i = nM_iR_i = nM_iwI_i(I_i + I_{3-i})^{2d}(V_i + V_{3-i})^{-g}. \tag{12}$$

Hence, the sum of the inventory level and expected sales (S_i) and backorders (K_i) of firm i using equations (11) and (12) are equal to:

$$S_i = a_i - \psi_i + 2\sqrt{\psi_i I_i}$$

$$S_i = (1-n)M_iwI_i(I_i + I_{3-i})^{2d}(V_i + V_{3-i})^{-g} + 2\sqrt{nM_iw(V_i + V_{3-i})^{-g}}I_i(I_i + I_{3-i})^{d} \tag{13}$$

$$K_i = a_i + I_i - S_i$$

$$K_i = I_i + nM_iwI_i(I_i + I_{3-i})^{2d}(V_i + V_{3-i})^{-g} - 2\sqrt{nM_iw(V_i + V_{3-i})^{-g}}I_i(I_i + I_{3-i})^{d}. \tag{14}$$

Substituting equations (10) and (14) into equation (1), the profit function firm i for $i = 1, 2$ is determined as follows:

$$\begin{aligned}\max \Omega_i = {} & (V_i - e_i - nbM_i)wI_i(I_i + I_{3-i})^{2d}(V_i + V_{3-i})^{-g} \\ & + 2bI_i(I_i + I_{3-i})^{d}\sqrt{nwM_i(V_i + V_{3-i})^{-g}} - (h+b)I_i.\end{aligned} \tag{15}$$

It is straightforward to verify that the profit function is concave in the decision variables (see the Appendices A–F for the detailed proof). Thus, the first-order conditions guarantee a global optimum.

5. Solution method

This section analyzes the optimization problem under two scenarios separately. In the first scenario, the behavior of single firm is studied. In the second scenario, the behavior of two production firms is studied. In both scenarios, the main objective is to determine the optimal value for variables that maximize the total profit. It worth mentioning that in the second scenario, the decisions of two production firms are fully examined using game theory under three different ways such as simultaneous game, Stackelberg game and cooperative decision making.

5.1. First scenario: Single firm

For the single firm, the profit function has three decision variables which are selling price, inventory level and lead time. Here, the well-known calculus method is applied to optimize the profit function. Therefore, the profit function given in equation (8), is differentiated with respect to the three decision variables V, I and M. The results are:

$$\frac{\partial\Omega}{\partial M} = -bwI^{2d+1}V^{-g} + bwI^{2d+1}V^{-g}(1-n) + bI^{d+1}M^{-\frac{1}{2}}\sqrt{nwV^{-g}} = 0 \tag{16}$$

$$\begin{aligned}\frac{\partial\Omega}{\partial I} &= (2d+1)wI^{2d}V^{1-g} - (2d+1)ewI^{2d}V^{-g} - (2d+1)bMwI^{2d}V^{-g} - b \\ &\quad + (2d+1)bMwI^{2d}V^{-g}(1-n) + 2(d+1)bI^{d}\sqrt{nMwV^{-g}} - h = 0\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{\partial\Omega}{\partial V} &= (1-g)wI^{2d+1}V^{-g} + gewI^{2d+1}V^{-g-1} + gbMwI^{2d+1}V^{-g-1} \\ &\quad - gbMwI^{2d+1}V^{-g-1}(1-n) - gbI^{d+1}\sqrt{nwM}V^{-\frac{g}{2}-1} = 0.\end{aligned} \tag{18}$$

The optimal value for the decision variables is calculated by solving equations (16) to (18) simultaneously. By solving equation (16) the optimal value for the lead time is obtained which is given by

$$M^* = \frac{V^{*g}}{nwI^{*2d}}. \tag{19}$$

Notice that equation (19) shows that there exists an impact of selling price and inventory level on the lead time. Now, by substituting equation (19) into equations (17) and (18), the optimal values of selling price and inventory level are obtained and these are expressed below,

$$V^* = \frac{ge}{g-1} \tag{20}$$

$$I^* = \left[\frac{h}{(2d+1)w\left[\left(\frac{ge}{g-1}\right)^{1-g} - e\left(\frac{ge}{g-1}\right)^{-g}\right]}\right]^{\frac{1}{2d}}. \tag{21}$$

Replacing equations (20) and (21) into equation (19), the closed form for optimal value of the lead time of the firm is derived. As it can be observed from the expressions for the decision variables, only the holding cost, production cost, market potential and the elasticity of price are needed to determine the optimal values of the decision variables.

5.2. Second scenario: Two competing firms

This section extends the single firm problem to the situation with two firms competing to satisfy the customer demand. In the case of two competing firms, game theory is applied for studying the behavior of these firms. Here, the Nash equilibrium, Stackelberg and cooperative games are used for modelling the behaviors of firms.

5.2.1. Nash equilibrium

Under the Nash equilibrium, each firm makes an effort to maximize its profit function and the firms participate in a non-cooperative game without being informed of terms and conditions of each other. According to equation (15), Ω_i indicates the profit function of firm i for $i = 1, 2$. To optimize, the total profit function is derived with respect to V_i, M_i and I_i, thus

$$\frac{\partial\Omega_i}{\partial M_i} = -bnwI_i(I_i + I_{3-i})^{2d}(V_i + V_{3-i})^{-g} + bM_i^{-\frac{1}{2}}\sqrt{nw(V_i + V_{3-i})^{-g}}I_i(I_i + I_{3-i})^{d} \tag{22}$$

$$\frac{\partial\Omega_i}{\partial V_i} = wI_i(I_i + I_{3-i})^{2d}\left[\left((V_i + V_{3-i})^{-g} + (-g)(V_i + V_{3-i})^{-g-1}V_i\right) + \left((-e_i - bnM_i)(-g)(V_i + V_{3-i})^{-g-1}\right)\right] - gb\sqrt{M_inw}I_i(I_i + I_{3-i})^{d}(V_i + V_{3-i})^{-\frac{g}{2}-1} \tag{23}$$

$$\frac{\partial\Omega_i}{\partial I_i} = (V_i - e_i - bnM_i)w(V_i + V_{3-i})^{-g}\left[(I_i + I_{3-i})^{2d} + 2d(I_i + I_{3-i})^{2d-1}I_i\right] + 2b\sqrt{M_inw(V_i + V_{3-i})^{-g}}\left[(I_i + I_{3-i})^{d} + d(I_i + I_{3-i})^{d-1}I_i\right] - (b + h). \tag{24}$$

Note that each firm intends to supply its inventory in order to meet market demand. Therefore, supplying the whole market demand (I) is the common objective. Where, obviously, I is equal to $I_1 + I_2$. Each of firm has a share in supplying the inventory (I). The ε_1 and ε_2 are coefficients. It is considered that $\varepsilon_1 + \varepsilon_2 = 1$. Considering $I_1 = I\varepsilon_1$ and $I_2 = I\varepsilon_2$ then equation (15) is rewritten as.

$$\max \Omega_i = (V_i - e_i - nbM_i)w\varepsilon_i I^{2d+1}(V_i + V_{3-i})^{-g} + 2b\varepsilon_i I^{d+1}\sqrt{nwM_i(V_i + V_{3-i})^{-g}} - (h + b)I\varepsilon_i. \tag{25}$$

Now, solving equations (22) to (24), the optimal values are obtained for each firm as follows. The first derivative of the profit function (Ω_i) with respect to M_i is equal to zero and then solving it, the lead time (M_i) is:

$$M_i^{N^*} = \frac{\left(V_i^{N^*} + V_{3-i}^{N^*}\right)^g}{nw\left(I^{N^*}\right)^{2d}}, \qquad i = 1, 2. \tag{26}$$

Inserting equation (26) into equations (23) and (24) and solving these equations, the optimal values for price and inventory level are given as follows:

$$V_i^{N^*} = \frac{g^2e_i - ge_i + ge_{3-i}}{(g-1)^2 - 1}, \quad i = 1, 2 \tag{27}$$

$$I^{N^*} = \left[\frac{h}{\left(\frac{g^2e_i - ge_i + ge_{3-i}}{(g-1)^2-1} - e_i\right)w\left(\frac{g^2(e_i + e_{3-i})}{(g-1)^2-1}\right)^{-g}(1 + 2d)}\right]^{\frac{1}{2d}}. \tag{28}$$

By substituting equations (27) and (28) into equation (26), the closed form for optimal value of lead time (M_i) for firm i is determined.

5.2.2. Stackelberg game

Under Stackerlberg approach, the behavior of production firms is examined when their market powers are not identical. In the Stackelberg game one firm is assumed to be the market leader and makes its choices first (*i.e.*, firm 2) and the other is considered as the follower (*i.e.*, firm 1). Under the Stackelberg game, firm 1, as the follower, determines the optimal values of own decision variables and then, firm 2, as the leader, decides according to best responses of firm 1. In other words, the optimal values of lead time and sales price of firm 1 (*i.e.*, $M_1^{S^*}, V_1^{S^*}$) are substituted into the profit function of firm 2 in order to obtain the optimal values for firm 2.

The profit function related to the follower, firm 1, is defined as follows:

$$\max \Omega_1 = (V_1 - e_1 - nbM_1)wI_1(I_2 + I_1)^{2d}(V_2 + V_1)^{-g} + 2bI_1(I_2 + I_1)^d\sqrt{nwM_1(V_2 + V_1)^{-g}} - (h + b)I_1. \tag{29}$$

This time, total inventory of the firms is considered as I where the inventory of firm 1 and firm 2 are equal to $I\varepsilon_1$ and $I\varepsilon_2$, respectively. Therefore, equation (29) is rewritten as follows:

$$\max \Omega_1 = (V_1 - e_1 - nbM_1)wI^{2d+1}\varepsilon_1(V_2 + V_1)^{-g} + 2bI^{d+1}\varepsilon_1\sqrt{nwM_1(V_2 + V_1)^{-g}} - (h + b)\varepsilon_1 I. \tag{30}$$

Then, to determine the optimal values of decision variables of the follower, profit function of firm 1 is derived partially with respect to the lead time (M_1) and price (V_1):

$$\frac{\partial \Omega_1}{\partial M_1} = -bnwI^{2d+1}\varepsilon_1(V_1 + V_2)^{-g} + bM_1^{-\frac{1}{2}}\sqrt{nw(V_1 + V_2)^{-g}}I^{d+1}\varepsilon_1 \tag{31}$$

$$\frac{\partial \Omega_1}{\partial V_1} = wI^{2d+1}\varepsilon_1\left[\left((V_1 + V_2)^{-g} + (-g)(V_1 + V_2)^{-g-1}V_1\right) + \left((-e_1 - bnM_1)(-g)(V_1 + V_2)^{-g-1}\right)\right] - gb\sqrt{M_1 nw}I^{d+1}\varepsilon_1(V_1 + V_2)^{-\frac{g}{2}-1}. \tag{32}$$

Setting equation (31) equal to zero, the optimal lead time for the follower is obtained and it is given by:

$$M_1^{S^*} = \frac{\left(V_1^{S^*} + V_2^{S^*}\right)^g}{nw(I^{S^*})^{2d}}\cdot \tag{33}$$

Thus, substituting equation (33) into equation (32) and doing mathematical operations then the optimal price of firm 1 is obtained and it is given by:

$$V_1^{S^*} = \frac{ge_1 + V_2}{g - 1}\cdot \tag{34}$$

The profit function of firm 2 is expressed below:

$$\max \Omega_2 = (V_2 - e_2 - nbM_2)wI^{2d+1}\varepsilon_2(V_2 + V_1^{S^*})^{-g} - (h + b)I\varepsilon_2 + 2bI^{d+1}\varepsilon_2\sqrt{nwM_2(V_2 + V_1^{S^*})^{-g}}. \tag{35}$$

Now, the optimal values of the follower are substituted into the profit function of the leader (*i.e.*, firm 2). In turn, notice that the profit function of firm 2 is not dependent on firm 1's decision variables. Of course, their impacts are remained by replacing of the optimal values in the objective function. Substituting the price ($V_1^{S^*}$) given by equation (34) into $V_1^{S^*} + V_2$ then $\frac{ge_1+V_2}{g-1} + V_2 = \frac{ge_1+gV_2}{g-1}$. Therefore, the profit function of firm 2 is given by

$$\max \Omega_2 = (V_2 - e_2 - nbM_2)wI^{2d+1}\varepsilon_2\left[\frac{ge_1 + gV_2}{g - 1}\right]^{-g} + 2bI^{d+1}\varepsilon_2\sqrt{nwM_2\left[\frac{ge_1 + gV_2}{g - 1}\right]^{-g}} - (h + b)I\varepsilon_2. \tag{36}$$

Then, taking the first derivatives of the profit function of the leader (Eq. (36)) with respect to lead time (M_2) and price (V_2). Hence, the optimal values of firm 2 are determined as follows:

$$\frac{\partial \Omega_2}{\partial M_2} = -bnwI^{2d+1}\varepsilon_2\left(\frac{ge_1 + gV_2}{g - 1}\right)^{-g} + bM_2^{-\frac{1}{2}}\sqrt{nw\left(\frac{ge_1 + gV_2}{g - 1}\right)^{-g}}I^{d+1}\varepsilon_2 = 0 \tag{37}$$

$$M_2^{*S} = \frac{\left(\frac{ge_1+gV_2}{g-1}\right)^g}{nwI^{2d}} \tag{38}$$

$$\frac{\partial\Omega_2}{\partial V_2} = wI^{2d+1}\varepsilon_2\left[\begin{array}{l}\left(\left(\frac{ge_1+gV_2}{g-1}\right)^{-g} + (-g)\left(\frac{g}{g-1}\right)\left(\frac{ge_1+gV_2}{g-1}\right)^{-g-1}V_2\right)\\ +\left((-e_2-bnM_2)(-g)\left(\frac{g}{g-1}\right)\left(\frac{ge_1+gV_2}{g-1}\right)^{-g-1}\right)\end{array}\right]$$
$$- gb\sqrt{M_2nw}I^{d+1}\varepsilon_2\left(\frac{g}{g-1}\right)\left(\frac{ge_1+gV_2}{g-1}\right)^{-\frac{g}{2}-1}. \tag{39}$$

By substituting equation (38) into equation (39), then:

$$wI^{2d+1}\varepsilon_2\left[\begin{array}{l}\left(\left(\frac{ge_1+gV_2}{g-1}\right)^{-g} + (-g)\left(\frac{g}{g-1}\right)\left(\frac{ge_1+gV_2}{g-1}\right)^{-g-1}V_2\right)\\ +\left(\left(-e_2-bn\frac{\left(\frac{ge_1+gV_2}{g-1}\right)^g}{nwI^{2d}}\right)(-g)\left(\frac{g}{g-1}\right)\left(\frac{ge_1+gV_2}{g-1}\right)^{-g-1}\right)\end{array}\right]$$
$$- gb\sqrt{\frac{\left(\frac{ge_1+gV_2}{g-1}\right)^g}{nwI^{2d}}nw}I^{d+1}\varepsilon_2\left(\frac{g}{g-1}\right)\left(\frac{ge_1+gV_2}{g-1}\right)^{-\frac{g}{2}-1} = 0. \tag{40}$$

And solving equation (40), the optimal value of price (V_2) is determined as follows:

$$\rightarrow\ V_2^{*S} = \frac{ge_2+e_1}{g-1}\cdot \tag{41}$$

The optimal value of the inventory level, I, is obtained by taking first derivative of the profit function of firm 2 with regard to the inventory level (I) as follows:

$$\begin{aligned}\frac{\partial\Omega_2}{\partial I_2} &= (V_2-e_2-bnM_2^*)w\left(\frac{ge_1+gV_2}{g-1}\right)^{-g}(2d+1)I^{2d}\varepsilon_2\\ &\quad + 2b\sqrt{M_2^*nw\left(\frac{ge_1+gV_2}{g-1}\right)^{-g}}(d+1)I^d\varepsilon_2 - (b+h)\varepsilon_2\\ &= \left(V_2-e_2-bn\frac{\left(\frac{ge_1+gV_2}{g-1}\right)^g}{nwI^{2d}}\right)w\left(\frac{ge_1+gV_2}{g-1}\right)^{-g}(2d+1)I^{2d}\varepsilon_2\\ &\quad + 2b\sqrt{\frac{\left(\frac{ge_1+gV_2}{g-1}\right)^g}{nwI^{2d}}nw\left(\frac{ge_1+gV_2}{g-1}\right)^{-g}}(d+1)I^d\varepsilon_2 - (b+h)\varepsilon_2\\ &= (V_2-e_2)\varepsilon_2w\left(\frac{ge_1+gV_2}{g-1}\right)^{-g}(2d+1)I^{2d} - h\varepsilon_2 = 0.\end{aligned} \tag{42}$$

And solving equation (42),

$$I^{S^*} = \left[\frac{h}{\left(\frac{e_2+e_1}{g-1}\right)w\left(\frac{g^2(e_2+e_1)}{(g-1)^2}\right)^{-g}(1+2d)}\right]^{\frac{1}{2d}}. \tag{43}$$

5.2.3. Cooperative approach

In the cooperative approach the firms work together cooperatively under a non-zero-sum game. In this game, the profit making of one firm is not led to the loss of other firm necessarily. Both firms are aware of the conditions of each other. Then, the profit functions of the firms are mixed/merged in only one. Therefore, the firms optimize their profits using only one objective function. In this study, the cooperative game is interpreted as a centralized benchmark in which firms jointly maximize total profit by coordinating their pricing, lead time, and inventory decisions. This interpretation is consistent with recent studies (*e.g.*, [32,35]), and it serves as a reference point for evaluating decentralized equilibria such as the Nash and Stackelberg games. It should be emphasized that a formal cooperative game in the strict game-theoretic sense would additionally require explicit mechanisms for profit allocation or incentive compatibility, which lie beyond the scope of this paper but remain a valuable avenue for future research. Mathematically speaking, the total profit is:

$$\begin{aligned}\sum_{i=1}^{2}\Omega_i &= \Omega_1+\Omega_2\\ &= w(I_1+I_2)^{2d}(V_1+V_2)^{-g}[(V_1-e_1-bnM_1)I_1+(V_2-e_2-bnM_2)I_2]\\ &\quad + 2b(I_1+I_2)^d\sqrt{nw(V_1+V_2)^{-g}}\Big(M_1^{\frac{1}{2}}I_1+M_2^{\frac{1}{2}}I_2\Big)-(b+h)(I_1+I_2).\end{aligned}\tag{44}$$

Taking the first derivative with respect to decision variables yield:

$$\frac{\partial\sum\Omega_i}{\partial M_i} = -bnwI_i(I_i+I_{3-i})^{2d}(V_i+V_{3-i})^{-g}+bM_i^{-\frac{1}{2}}\sqrt{nw(V_i+V_{3-i})^{-g}}I_i(I_i+I_{3-i})^d \tag{45}$$

$$\begin{aligned}\frac{\partial\sum\Omega_i}{\partial V_i} &= wI_i(I_i+I_{3-i})^{2d}\left[\begin{array}{l}\Big((V_i+V_{3-i})^{-g}+(-g)(V_i+V_{3-i})^{-g-1}V_i\Big)\\ +\Big((-e_i-bnM_i)(-g)(V_i+V_{3-i})^{-g-1}\Big)\end{array}\right]\\ &\quad+(V_{3-i}-e_{3-i}-bnM_{3-i})wI_{3-i}(I_i+I_{3-i})^{2d}(-g)(V_i+V_{3-i})^{-g-1}\\ &\quad-gb\sqrt{M_inw}I_i(I_i+I_{3-i})^d(V_i+V_{3-i})^{-\frac{g}{2}-1}-gb\sqrt{M_{3-i}nw}I_{3-i}(I_i+I_{3-i})^d(V_i+V_{3-i})^{-\frac{g}{2}-1}\end{aligned}\tag{46}$$

$$\begin{aligned}\frac{\partial\sum\Omega_i}{\partial I_i} &= (V_i-e_i-bnM_i)w(V_i+V_{3-i})^{-g}\Big[(I_i+I_{3-i})^{2d}+2d(I_i+I_{3-i})^{2d-1}I_i\Big]\\ &\quad+(V_{3-i}-e_{3-i}-bnM_{3-i})w(V_i+V_{3-i})^{-g}\Big[2d(I_i+I_{3-i})^{2d-1}I_{3-i}\Big]\\ &\quad+2b\sqrt{M_inw(V_i+V_{3-i})^{-g}}\Big[(I_i+I_{3-i})^d+d(I_i+I_{3-i})^{d-1}I_i\Big]\\ &\quad+2b\sqrt{M_{3-i}nw(V_i+V_{3-i})^{-g}}\Big[d(I_i+I_{3-i})^{d-1}I_{3-i}\Big]-(b+h).\end{aligned}\tag{47}$$

Setting equations (45)–(47) equal to zero and solving them, for $i=1,2$, resulting:

$$M_i^{\mathrm{co}^*} = \frac{\big(V_i^{\mathrm{co}}+V_{3-i}^{\mathrm{co}}\big)^g}{nw\big(I_i^{\mathrm{co}}+I_{3-i}^{\mathrm{co}}\big)^{2d}} \tag{48}$$

$$V_i^{\mathrm{co}^*} = V_{3-i}^{\mathrm{co}}-(e_{3-i}-e_i). \tag{49}$$

The proof of equation (49) is provided in Appendix E.

$$I_i^{\mathrm{co}^*} = \frac{-g\big(e_{3-i}-V_{3-i}^{\mathrm{co}}\big)I_{3-i}^{\mathrm{co}}}{V_i^{\mathrm{co}}+V_{3-i}^{\mathrm{co}}+g(e_i-V_i^{\mathrm{co}})}. \tag{50}$$

In this approach, similar to the previous approaches, the total inventory is shown with I and each firm supplies a fraction of total inventory as ε_1 and ε_2. For tractability, we further assume a symmetric allocation of inventory, *i.e.*, $\varepsilon_1 = \varepsilon_2$. This benchmark provides a clear baseline for comparison with Nash and Stackelberg equilibria. Thus, the profit function of the firms can be written as follows:

$$\sum_{i=1}^{2} \Omega_i = \Omega_1 + \Omega_2$$
$$= wI^{2d}(V_1 + V_2)^{-g}[(V_1 - e_1 - bnM_1)I\varepsilon_1 + (V_2 - e_2 - bnM_2)I\varepsilon_2]$$
$$+ 2bI^d\sqrt{nw(V_1 + V_2)^{-g}}\left(M_1^{\frac{1}{2}}I\varepsilon_1 + M_2^{\frac{1}{2}}I\varepsilon_2\right) - (b+h)I. \tag{51}$$

Then, equations (48) to (50) change to equations (52) to (54) as follows:

$$M_i^{\mathrm{co}^*} = \frac{\left(V_i^{\mathrm{co}} + V_{3-i}^{\mathrm{co}}\right)^g}{nw\left(I^{\mathrm{co}}\right)^{2d}} \tag{52}$$

$$\frac{\varepsilon_i}{\varepsilon_{3-i}} = \frac{-g(e_{3-i} - V_{3-i}^{\mathrm{co}})}{\left(V_i^{\mathrm{co}} + V_{3-i}^{\mathrm{co}} + g(e_i - V_i^{\mathrm{co}})\right)}. \tag{53}$$

The proof of equation (53) is provided in Appendix F.

$$I^* = \left[\frac{h}{((V_1 - e_1)\varepsilon_1 + (V_2 - e_2)\varepsilon_2)w(V_1 + V_2)^{-g}(1 + 2d)}\right]^{\frac{1}{2d}}. \tag{54}$$

Solving equations (45) to (47) simultaneously by MATLAB software, optimal values for selling price, lead time and inventory are obtained. Note that, the values of ε_1 and ε_2 are equal to 0.5 because it is assumed that the firms should have equal inventory in order to obtain maximum profit for the firms under cooperative approach.

6. Numerical results and sensitivity analysis

This section solves several numerical examples in order to justify the applicability and efficiency of the models. The parameter values in the numerical examples are selected to remain consistent with ranges frequently used in the literature (*e.g.*, [45], Wu *et al.*, 2011). This approach preserves analytical tractability while offering a representative setting to compare Nash, Stackelberg, and cooperative strategies. The results of the numerical examples for different scenarios are presented in Tables 2–7.

The numerical examples presented in this section are intended to illustrate the analytical results and to provide managerial insights under controlled settings. Consistent with standard practice in analytical operations management research, parameter values are selected from ranges commonly adopted in the literature rather than calibrated to a specific firm or industry. To ensure the reliability of the conclusions, extensive sensitivity analyses are conducted over wide parameter ranges, allowing us to assess the robustness of the results and the stability of the strategic comparisons across different market conditions.

6.1. First scenario: Single firm

In this subsection, a numerical example is presented to illustrate the model of a single firm.

Example 1. The values of parameters of this example and results obtained are shown in Table 2.

According the results, by comparing the values of the first row of Table 2 with the second one, it can be seen that by reducing the amount of d, (−%25), from −0.3 to −0.375, the profit significantly decreases. Also, by the comparison between the first row and the third one, concluded that a decrease in g, (−%25), from 9 to 6.75, reduces the profit. Therefore, the parameter d affects profit more than parameter g. (The bold example is used for the sensitivity analysis).

TABLE 2. Parameters and results for single firm.

w	d	e	b	h	g	n	V^* Eq. (20)	I^* Eq. (21)	M^* Eq. (19)	Profit Eq. (8)
20 000	**−0.3**	**0.6**	**0**	**0.7**	**9**	**1**	**0.6750**	**28 110 296.4**	**0.9944**	**157 417 659.8**
20 000	−0.375	0.6	0	0.7	9	1	0.6750	486 331.45	0.6215	4 561 789.08
20 000	−0.3	0.6	0	0.7	6.75	1	0.7043	6 915 970.98	0.4473	27 836 783.22

6.2. Second scenario: Two competing firms

This subsection presents numerical examples to determine the inventory level, price and lead time under three different approaches: Nash, Stackelberg and cooperative. Also, it is assumed in all approaches that $\varepsilon_1 = \varepsilon_2 = 0.5$. Examples 2 and 3 are solved under Nash equilibrium. Examples 4 and 5 are solved under Stackelberg game. Example 6 is solved under cooperative approach.

Example 2. The parameters' values are considered as $w = 20\,000$, $d = -0.3$, $e_1 = 0.6$, $e_2 = 0.5$, $b = 0$, $h = 0.7$, $g = 9$ and $n = 1$. The results for the two firms under Nash equilibrium are shown in Table 3.

TABLE 3. Results under Nash equilibrium for $\varepsilon_1 = \varepsilon_2 = 0.5(I_i^{N^*} = \varepsilon_i I^{N^*})$.

ε_1	$M_i^{N^*}$ Eq. (26)	$V_1^{N^*}$ Eq. (27)	$V_2^{N^*}$ Eq. (27)	$I_1^{N^*} = \varepsilon_1 I^{N^*}$	$I_2^{N^*} = \varepsilon_2 I^{N^*}$	$\Omega_1 + \Omega_2$ Eq. (25)
0.5	0.0897	0.7571	0.6571	732.35	732.35	1537.95

Example 3. The parameters' values are $w = 20\,000$, $d = -0.25$, $e_1 = 0.6$, $e_2 = 0.5$, $b = 0$, $h = 0.7$, $g = 14$ and $n = 1$. The results for the two firms under Nash equilibrium are presented in Table 4.

TABLE 4. Results under Nash equilibrium for $\varepsilon_1 = \varepsilon_2 = 0.5(I_i^{N^*} = \varepsilon_i I^{N^*})$.

ε_1	$M_i^{N^*}$ Eq. (26)	$V_1^{N^*}$ Eq. (27)	$V_2^{N^*}$ Eq. (27)	$I_1^{N^*} = \varepsilon_1 I^{N^*}$	$I_2^{N^*} = \varepsilon_2 I^{N^*}$	$\Omega_1 + \Omega_2$ Eq. (25)
0.5	0.06547	0.6916	0.5916	793.76	793.76	1111.26

Example 4. The parameters' values are considered as $w = 20\,000$, $d = -0.3$, $e_1 = 0.6$, $e_2 = 0.5$, $b = 0$, $h = 0.7$, $g = 9$ and $n = 1$, the results for the two firms under Stackelberg game are tabulated in Table 5.

TABLE 5. Results under Stackelberg game for $\varepsilon_1 = \varepsilon_2 = 0.5(I_i^{S^*} = \varepsilon_i I^{S^*})$.

ε_1	$M_i^{S^*}$ Eq. (33) or Eq. (38)	$V_1^{S^*}$ Eq. (34)	$V_2^{S^*}$ Eq. (41)	$I_1^{S^*} = \varepsilon_1 I^{S^*}$	$I_2^{S^*} = \varepsilon_2 I^{S^*}$	$\Omega_1 + \Omega_2$ Eq. (30) + Eq. (36)
0.5	0.0785	0.7546	0.6375	742.44	742.44	1721.53

Example 5. The parameters' values are considered as $w = 20\,000$, $d = -0.25$, $e_1 = 0.6$, $e_2 = 0.5$, $b = 0$, $h = 0.7$, $g = 13$ and $n = 1$, the results for the two firms under Stackelberg game are presented in Table 6.

TABLE 6. Results under Stackelberg game for $\varepsilon_1 = \varepsilon_2 = 0.5 (I_i^{S^*} = \varepsilon_i I^{S^*})$.

ε_1	$M_i^{S^*}$ Eq. (33) or Eq. (38)	$V_1^{S^*}$ Eq. (34)	$V_2^{S^*}$ Eq. (41)	$I_1^{S^*} = \varepsilon_1 I^{S^*}$	$I_2^{S^*} = \varepsilon_2 I^{S^*}$	$\Omega_1 + \Omega_2$ Eq. (30) + Eq. (36)
0.5	0.0654	0.6993	0.5916	1120.35	1120.35	1699.21

TABLE 7. Results under cooperative approach for $\varepsilon_1 = \varepsilon_2 = 0.5$.

w	d	e_1	e_2	b	h	g	n	$M_i^{\text{co}^*}$	$V_1^{\text{co}^*}$	$V_2^{\text{co}^*}$	$I_i^{\text{co}^*}$	$\sum \Omega_i$
20 000	**−0.3**	**0.6**	**0.5**	**0**	**0.7**	**9**	**1**	**0.0393**	**0.6687**	**0.5687**	**1368.5**	**2873.8**
15 000	−0.3	0.6	0.5	0	0.7	9	1	0.0393	0.6687	0.5687	847.23	1779.2

The results reveal that firm 2, as the leader in Stackelberg game, always has the less price than that of firm 1.

Example 6. For the cooperative approach, it is considered that $\varepsilon_1 = \varepsilon_2 = 0.5$. This means that two firms share market with each other in order to earn maximum profit.

The results of Table 7 are obtained of MATLAB software, using equations (44) to (47). Based on the results, the best values of the decision variables are specified significantly and the related parameters used for sensitivity analysis.

The results of Table 7 are obtained with MATLAB software, using equations (44) to (47). By comparing the values of the first row of Table 7 whit the second one, it can be seen that by decreasing the amount of w, (−%25), from 20 000 to 15 000, both the profit and on-hand inventory are reduced, while lead time and prices remains constant. This reduction in profit and on-hand inventory show the impact of market potential. The reason of why prices remain constant is that the price's equation is independent of w. The reason of why lead time remain unchanged can be concluded from the equation (52) and (54), by substituting the equation (54) into equation (52) then the w is simplified from the equation. (The bold example is used for the sensitivity analysis). To improve readability, the numerical results and sensitivity analysis are presented in a unified sequence. Sections 6.1 and 6.2 reported the baseline results (Tabs. 2–7), and in what follows we examine parameter-wise sensitivities and managerial interpretation.

6.3. Sensitivity analysis

This section studies the impacts of some parameters of model on the optimal solution and the total profit under two scenarios: single firm and two competing firms. For both scenarios sensitivity analysis is done by increasing and decreasing the parameters, by 25%, 50% and 75%. The results are provided in Table 8 and diagrams related to the changes of each parameter in different models are shown in Figures 1 to 3. The diagrams related to the scenario of single firm are shown in Figure 1. In the same direction, diagrams related to the Nash, Stackelberg and cooperative approaches are shown in Figures 2 and 3 for firm 1 and 2, respectively. Figure 4 shows the diagrams of the parameters changes on the total profits of the firms.

According to the results obtained from numerical examples and sensitivity analysis, it was found that:

– Effect of parameters changes is shown on the optimal values of lead time, inventory and price for the first scenario in Figure 1. The effect of changes of the parameter w on the decision variables is examined. Once the parameter w increases, the inventory level also increases. This parameter has no effect on lead time and price significantly. Considering the impact of parameter g, on the one hand, it can be said that if parameter

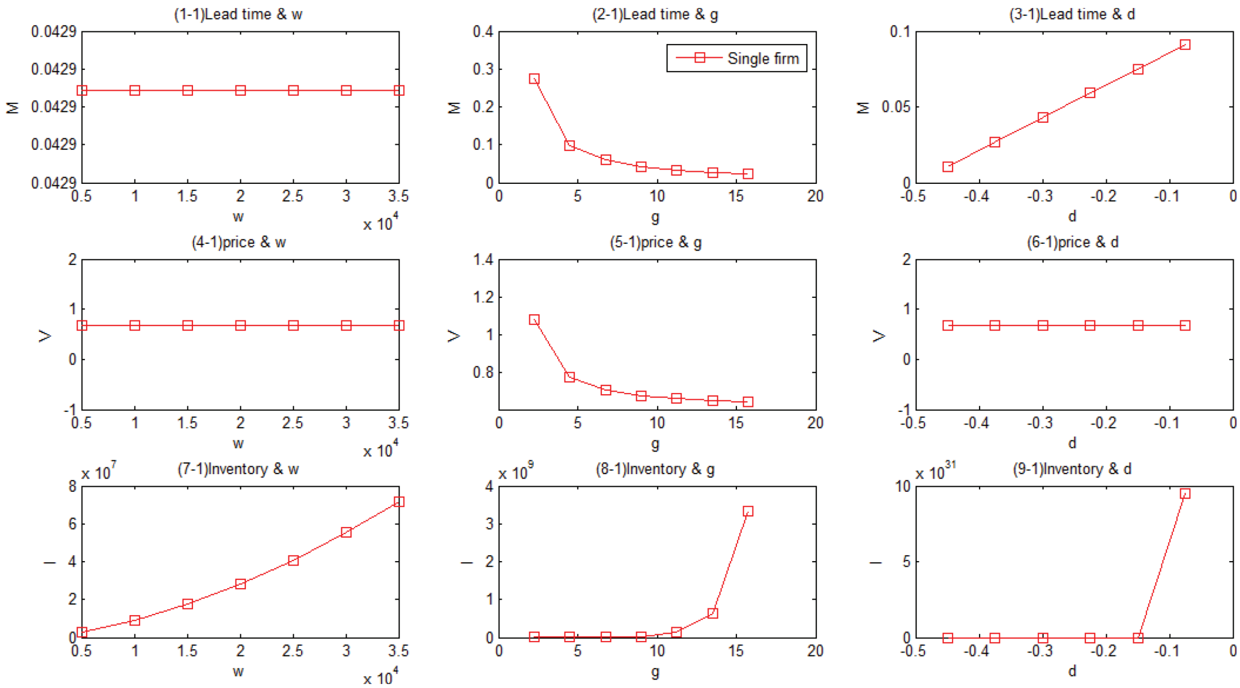

FIGURE 1. Chart of changes of parameters g, d and w on the decision variables for single firm.

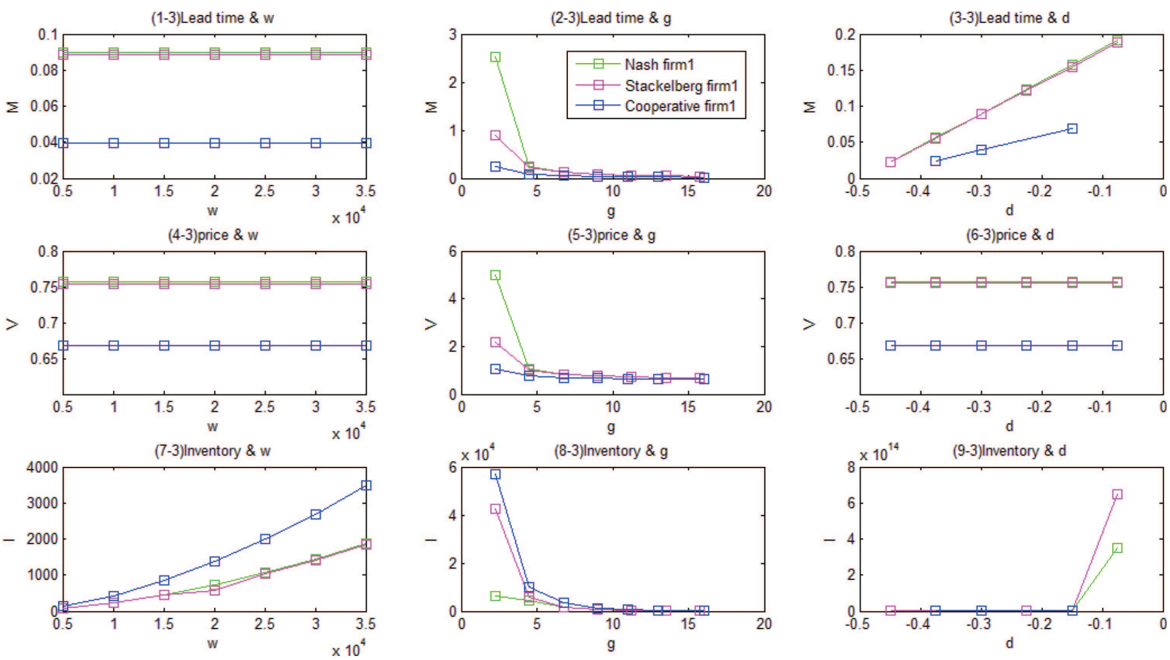

FIGURE 2. Chart of changes of parameters of the decision variables of firm 1 under Nash, Stackelberg and cooperative approaches.

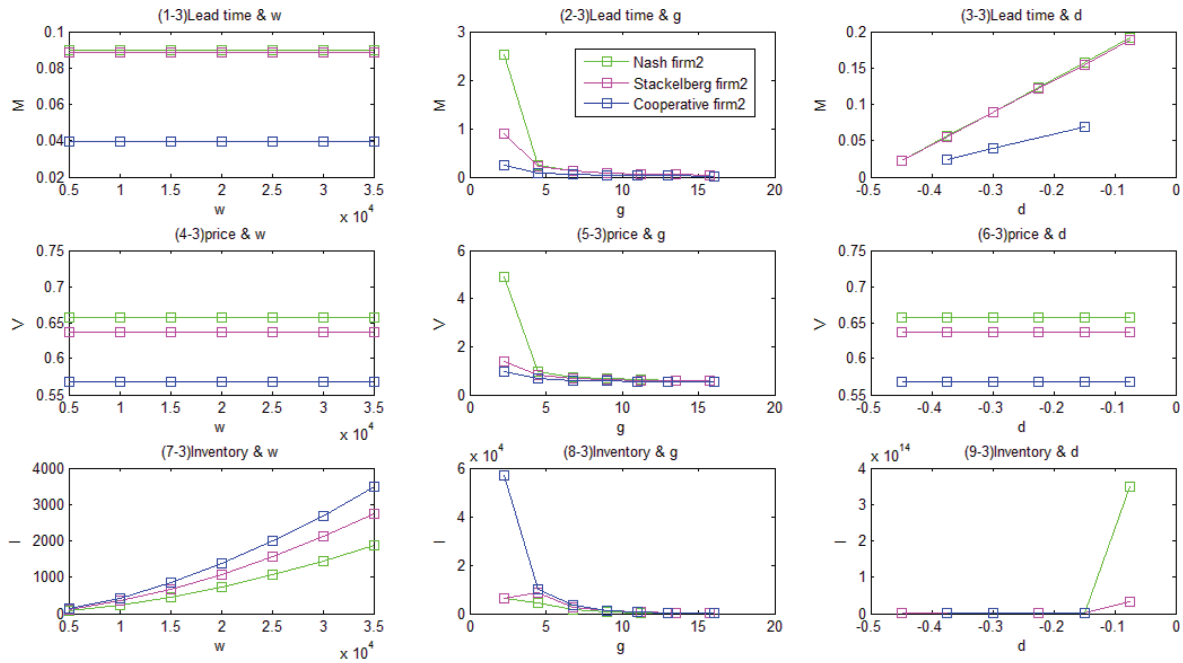

FIGURE 3. Chart of changes of parameters g, d and w on the decision variables of firm 2 under Nash, Stackelberg and cooperative approaches.

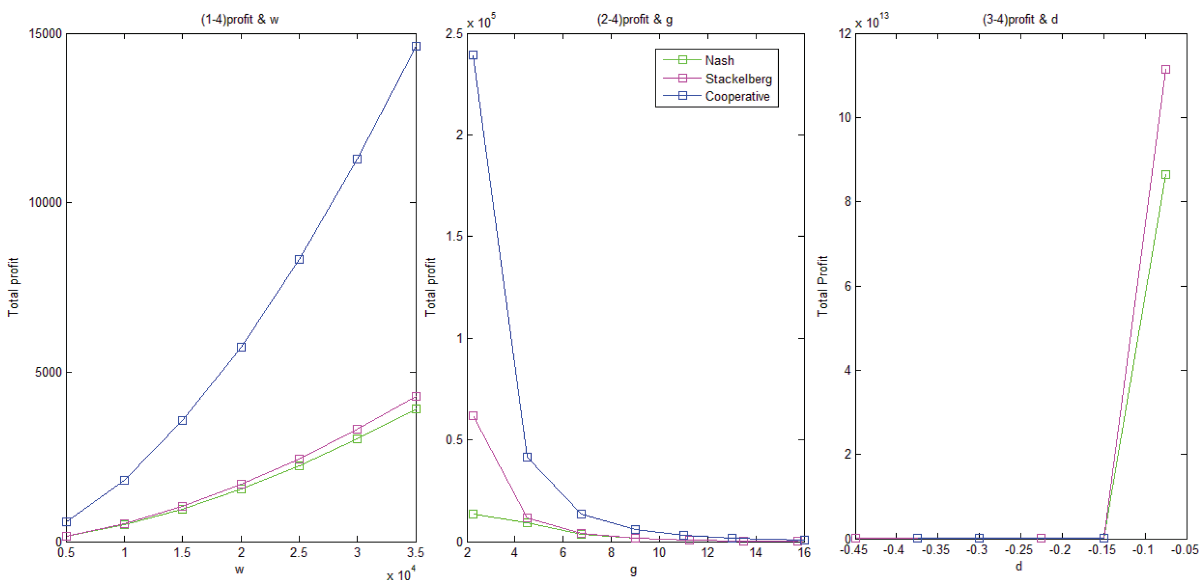

FIGURE 4. Chart of changes of parameters g, d and w on total profit of firms under Nash, Stackelberg and cooperative approaches.

g increases then both lead time and price decrease. On the other hand, if parameter g increases the inventory level increases. Also, an increasing on parameter d leads to an increasing the inventory level and lead time while it has no effect on the price.

- It is well known that lead time is one of the most effective components that affects directly to the inventory available rate in production firms. The relationship among inventory, lead time and price is in such a way that if delivery lead time is reduced then the inventory level is also decreased. When inventory level is diminished, consequently, warehousing cost is also reduced and finally, it is led to an increased profit.
- In Figures 2 and 3, the impacts of changes of parameters g, w and d on the decision variables of firms under three approaches of Nash, Stackelberg and cooperative are shown with different colors to easily analyze and compare them with each other. As it is shown in Figures 2 and 3, the cooperative approach is able to meet the demand in the shortest time and consequently it is the best approach.
- The cooperative approach leads to the best performance than other approaches due to fact that it obtains the highest profit, as it is shown in Figure 4. Notice also in Figure 4 that the profit sensitivity with respect to d, it can be said that when the elasticity of inventory, d, increases up to zero, the profit of the firms suddenly increases.
- Furthermore, it was found that the price and inventory elasticity, g and d, are more effective and influential on the optimal values of the decision variables and the profit of firms in both scenarios. The smallest change in their values leads to significant changes in the values of the decision variables. In addition, the changes on market potential (w) also are very impressive on the level of inventory of the firms so that if market potential increases (*i.e.*, increasing market share), firms keep more inventory in their warehouse in order to satisfy customers' demand.
- Cooperative approach is the best and most profitable game approach for the production firms. The reason is that, once the firms work together, can manage to send orders in the shortest time with the lower, fair and reasonable price. In this approach, the firms incur in lower holding costs because they keep a small amount of inventory.

6.4. Strategic implications of competition *vs.* cooperation

The comparative results obtained under Nash, Stackelberg, and cooperative settings provide direct insights for strategic decision-making. When firms act non-cooperatively, competitive equilibria may be suitable in markets where strategic autonomy or asymmetry in market power is dominant. However, the results consistently indicate that cooperative behavior yields higher total profits and shorter lead times in the presence of fully complementary products.

These findings suggest that firms should consider cooperation when demand interdependence is strong and when joint availability significantly affects consumer satisfaction. In contrast, competitive strategies may remain appropriate when coordination costs are high or when firms seek to preserve strategic independence. Overall, the proposed framework offers a structured basis for deciding when to compete and when to cooperate in markets characterized by complementary demand and inventory interactions.

7. Conclusion

This study examines how firms' optimal decisions emerge when demand is jointly sensitive to price and inventory levels. The analysis reveals how pricing, inventory, and lead-time decisions interact differently across alternative market structures. In the first scenario, the optimal pricing, lead time and inventory policies for a single firm are analyzed. In the second scenario; the behavior of two firms is studied using three approaches: Nash, Stackelberg and cooperative. The results highlight three key insights. First, joint optimization leads to materially different outcomes compared to sequential or isolated decision-making. Second, demand sensitivity to both price and inventory amplifies strategic interactions between firms producing complementary products. Third, the comparison across Nash, Stackelberg, and cooperative regimes clarifies how coordination and decision authority shape equilibrium outcomes and profit distribution.

TABLE 8. The impacts of parameters changes, on the decision variables for both scenarios.

%parameters changes		Single firm				Two firms														
						Nash equilibrium ($\varepsilon_1^* = 0.5$)					Stackelberg game ($\varepsilon_1^* = 0.5$)					Cooperative approach ($\varepsilon_1^* = 0.5$)				
		V^*	I^*	M^*	Ω	$V_1^{N^*}$	$V_2^{N^*}$	$I_i^{N^*}$	$M_i^{N^*}$	$\sum \Omega_i^N$	$V_1^{S^*}$	$V_2^{S^*}$	$I_i^{S^*}$	$M_i^{S^*}$	$\sum \Omega_i^S$	$V_1^{co^*}$	$V_2^{co^*}$	$I_i^{co^*}$	$M_i^{co^*}$	$\sum \Omega_i^{co}$
w	75	0.675	71 437 926.29	0.9944	400 052 387.2	0.7571	0.6571	1861.17	0.0897	3908.46	0.7546	0.6375	1886.79	0.0785	4375.01	0.6687	0.5687	3477.7	0.039	7303.2
	50	0.675	55 252 363.02	0.9944	309 413 232.9	0.7571	0.6571	1439.49	0.0897	3022.93	0.7546	0.6375	459.65	0.0785	3383.77	0.6687	0.5687	2689.8	0.039	5648.6
	25	0.675	40 773 886.83	0.9944	228 333 766.2	0.7571	0.6571	1062.28	0.0897	2230.79	0.7546	0.6375	1076.9	0.0785	2497.07	0.6687	0.5687	1985	0.039	4168.4
	−25	0.675	17 403 403.81	0.9944	97 459 061.32	0.7571	0.6571	453.41	0.0897	952.16	0.7546	0.6375	459.65	0.0785	1065.82	0.6687	0.5687	847.2	0.039	1779.2
	−50	0.675	8 854 188.536	0.9944	49 583 455.8	0.7571	0.6571	230.67	0.0897	484.42	0.7546	0.6375	233.85	0.0785	542.24	0.6687	0.5687	431	0.039	905.18
	−75	0.675	2 788 894.629	0.9944	15 617 809.92	0.7571	0.6571	72.65	0.0897	152.58	0.7546	0.6375	73.65	0.0785	170.79	0.6687	0.5687	135.7	0.039	285.11
d	75	0.675	9.50E+31	2.1132	2.15212E+32	0.7571	0.6571	3.50E+14	0.1908	8.65E+13	0.7546	0.6375	3.6993E+14	0.1669	1.2947E+14	0.6687	0.5687	Infeasible	Infeasible	Infeasible
	50	0.675	5.10E+15	1.7403	1.47998E+16	0.7571	0.6571	6 927 940.82	0.1571	4 156 764.49	0.7546	0.6375	7 120 006.91	0.1375	5 162 005.01	0.6687	0.5687	24 189 000	0.068	14 514 000
	25	0.675	17 344 424 558	1.3674	67 327 902 603	0.7571	0.6571	16 877.73	0.1234	19 332.68	0.7546	0.6375	17 188.25	0.108	22 422.85	0.6687	0.5687	Infeasible	Infeasible	Infeasible
	−25	0.675	486 331.45	0.6215	4 561 789.08	0.7571	0.6571	91.077	0.0561	382.52	0.7546	0.6375	92.07	0.0491	418.96	0.6687	0.5687	150.18	0.024	630.77
	−50	0.675	198 13.2	0.2486	485 423.52	0.7571	0.6571	13.82	0.0224	174.13	0.7546	0.6375	13.94	0.0196	187.93	0.6687	0.5687	Infeasible	Infeasible	Infeasible
	−75	0.675	Infeasible	Infeasible	Infeasible	0.7571	0.6571	Infeasible	Infeasible	Infeasible	0.7546	0.6375	Infeasible	Infeasible	Infeasible	0.6687	0.5687	Infeasible	Infeasible	Infeasible
g	75	0.6406	3 321 168 078	16.5342	34 291 060 404	0.68	0.58	99.85	0.0457	209.7	0.6796	0.5745	100.25	0.0426	222.41	0.6367	0.5367	165.26	0.021	347.05
	50	0.648	638 191 489.6	6.216	5 584 175 534	0.6956	0.5956	184.2	0.0546	386.83	0.695	0.588	185.22	0.0502	414.89	0.644	0.544	329.62	0.025	692.19
	25	0.6585	129 020 569.8	2.4207	925 722 588.1	0.7189	0.6189	355.61	0.0679	746.78	0.7177	0.6073	358.55	0.0613	814.17	0.655	0.555	700.1	0.031	1470.2
	−25	0.7043	6 915 970.98	0.4473	27 836 783.22	0.8315	0.7315	1663.11	0.1323	3492.54	0.8245	0.6913	1708.78	0.1093	4108.52	0.6957	0.5957	3268.7	0.054	6864.3
	−50	0.7714	2 148 196.59	0.2429	5 263 081.64	1.04	0.94	4395.97	0.2514	9231.55	1.004	0.8142	4752.09	0.1795	12 355.45	0.7571	0.6571	9857.2	0.089	20 700
	−75	1.08	1 278 545.60	0.2491	1 118 727.40	5	4.9	6325.26	2.5142	13 283.06	2.184	1.38	19 952.37	0.5028	69 833.32	1.04	0.94	56 957	0.251	119 610

In previous studies, lead time adjustment, pricing and inventory control for firms are normally discussed separately. While the simultaneous analysis of these decision variables and the analysis of how each of these variables affect profit functions and other decision variables has not been raised so far, for that reason this paper contemplates the joint optimization of lead time adjustment, pricing and inventory control for complementary products. This study actually considers a two-level supply chain for complementary and dependent products and examined the behavior of firms in different situations using different game theory approaches. Basically, the optimal values and closed forms for each of the decision variables are determined. Also, it is analyzed the behavior of firms where the customers are price and inventory sensitive, and the demand function is dependent on the sum of the selling prices of two completely complementary products. It is important to remark that in previous studies such a kind of demand function is not investigated by game theory approaches.

It was found that setting the decision variables (*i.e.*, lead time, available inventory and price) is essential in order to obtain the maximum profit. According to the results obtained, it was found that the cooperative approach is the best approach in terms of performance, which leads to a higher profit. Finally, it was specified that with the change in the parameters of price and inventory, which are more effective than other parameters, maximum profit is obtained. From a managerial perspective, the findings suggest that firms facing complementary demand can use the model to decide between competitive and cooperative strategies. Cooperation can reduce lead times and holding costs while leading to higher overall profits. In addition, the sensitivity of parameters provides direct guidance for pricing and inventory strategies, underlining the importance of jointly considering pricing, inventory, and lead time decisions in practice. These managerial implications ensure that the theoretical contributions also translate into clear practical takeaways for decision-maker.

It should be noted that this study can be extended in different ways as follows:

- Considering other influence factors such as quality, service level, and advertising.
- Considering stochastic demand function under competing conditions.
- Considering another trends for the demand functions.

nce 0822B01006.

References

[1] T. Avinadav, A. Herbon and U. Spiegel, Optimal ordering and pricing policy for demand functions those are separable into price and inventory age. *Int. J. Prod. Econ.* **155** (2014) 406–417.

[2] M. Benioudakis, A. Burnetas and G. Ioannou, Lead-time quotations in unobservable make-to-order systems with strategic customers: risk aversion, load control and profit maximization. *Eur. J. Oper. Res.* **289** (2021) 165–176.

[3] T. Boyaci and S. Ray, Product differentiation and capacity cost interaction in time and price sensitive markets. *Manuf. Serv. Oper. Manage.* **5** (2003) 18–36.

[4] S.K. Chaharsooghi and J. Heydari, A study on the impact of lead time statistical specifications on supply chain performance under uncertainty. *Int. J. Mech. Manuf. Syst.* **4** (2011) 95–112.

[5] X. Chen and P. Hu, Joint pricing and inventory management with deterministic demand and costly price adjustment. *Oper. Res. Lett.* **40** (2012) 385–389.

[6] J.M. Chen, I.C. Lin and H.L. Cheng, Channel coordination under consignment and vendor managed inventory in a distribution system. *Transp. Res. Part E: Logistics Transp. Rev.* **46** (2010) 831–843.

[7] J.M. Chen, H.L. Cheng and M.C. Chien, On channel coordination through revenue sharing contracts with price and shelf space dependent demand. *Appl. Math. Model.* **35** (2011) 4886–4901.
periodic review inventory systems with inventory level dependent demand. *Nav. Res. Logistics* **59** (2012) 430–440.

[8] B.R. Cobb, A.W. Johnson, R. Rumí and A. Salmerón, Accurate lead time demand modeling and optimal inventory policies in continuous review systems. *Int. J. Prod. Econ.* **163** (2015) 124–136.

[9] L. Devangan, R.K. Amit, P. Mehta, S. Swami and K. Shanker, Individually rational buyback contracts with inventory level dependent demand. *Int. J. Prod. Econ.* **143** (2013) 381–387.

[10] M.A. Edalatpour and S.M.J. Mirzapour Al-e-Hashem, Simultaneous pricing and inventory decisions for substitute and complementary items with nonlinear holding cost. *Prod. Eng.* **13** (2019) 305–315.

[11] M.A. Edalatpour, S.M.J. Mirzapour Al-e-Hashem and A.M. Fathollahi-Fard, Combination of pricing and inventory policies for deteriorating products with sustainability considerations. *Environ. Dev. Sustain.* **26** (2024) 6809–6849.

[12] R. Guan and X. Zhao, Pricing and inventory management in a system with multiple competing retailers under (r, Q) policies. *Comput. Oper. Res.* **38** (2011) 1294–1304.

[13] J. Heydari and Y. Norouzinasab, Coordination of pricing, ordering, and lead time decisions in a manufacturing supply chain. *J. Ind. Syst. Eng.* **9** (2016) 1–16.

[14] W.A. Jauhari, S.C.N. Ramadhany, C.N. Rosyidi, U. Mishra and H. Hishamuddin, Pricing and green inventory decisions for a supply chain system with green investment and carbon tax regulation. *J. Clean. Prod.* **425** (2023) 138897.

[15] S. Karray and G. Martín-Herrán, The effects of decision timing for pricing and marketing efforts in a supply chain with competing manufacturers. *Int. Trans. Oper. Res.* **31** (2024) 568–615.

[16] A. Kausar, C.K. Jaggi and S. Maheshwari, Optimizing inventory and pricing decisions in a Closed-Loop Supply Chain: a sustainable approach towards manufacturing and remanufacturing. *Clean. Logistics Supply Chain* **16** (2025) 100223.

[17] S. Kim, Y. Feng and J. Xu, Complementarity analysis of a multi-item inventory model with leading product pricing. *Prod. Oper. Manage.* **32** (2023) 4190–4211.

[18] B.K. Kumar, D. Nagaraju and S. Narayanan, Optimality of cycle time and inventory decisions in a coordinated and non-coordinated three echelon supply chain under exponential price dependent demand. *Int. J. Logistics Syst. Manage.* **25** (2016) 199–226.

[19] L. Kumar, K. Sharma and U.K. Khedlekar, Dynamic pricing strategies for efficient inventory management with auto-correlative stochastic demand forecasting using exponential smoothing method. *Results Control Opt.* **15** (2024) 100432.

[20] J.Y. Lee, An incentive contract for lead time reduction in an $(s-1, s)$ inventory system. *Comput. Oper. Res.* **36** (2009) 1633–1638.

[21] M. Leng and M. Parlar, Lead time reduction in a two level supply chain: non cooperative equilibria vs. coordination with a profit sharing contract. *Int. J. Prod. Econ.* **118** (2009) 521–544.

[22] T. Li, X. Zhao and J. Xie, Inventory management for dual sales channels with inventory level dependent demand. *J. Oper. Res. Soc.* **66** (2015) 488–499.

[23] J. Min and Y.W. Zhou, A perishable inventory model under stock dependent selling rate and shortage dependent partial backlogging with capacity constraint. *Int. J. Syst. Sci.* **40** (2009) 33–44.

[24] C. Mondal and B.C. Giri, Pricing and bundling strategies for complementary products in a closed-loop green supply chain under manufacturers' different behaviors. *Expert Syst. Appl.* **238** (2024) 121960.

[25] S.K. Mukhopadhyay, X. Yue and X. Zhu, A Stackelberg model of pricing of complementary goods under information asymmetry. *Int. J. Prod. Econ.* **134** (2011) 424–433.

[26] M. Noori, A.A. Taleizadeh and F. Jolai, Analyzing pricing, promised delivery lead time, supplier-selection, and ordering decisions of a multi-national supply chain under uncertain environment. *Int. J. Prod. Econ.* **209** (2019) 236–248

[27] V. Pando, J. García-Laguna and L.A. San-José, Optimal policy for profit maximizing in an EOQ model under non-linear holding cost and stock-dependent demand rate. *Int. J. Syst. Sci.* **43** (2011) 2160–2171.

[28] P. Pekgün, P.M. Griffin and P. Keskinocak, Coordination of marketing and production for price and lead time decisions. *IIE Trans.* **40** (2008) 12–30.

[29] S. Ray, Y. Gerchak and E.M. Jewkes, The effectiveness of investment in lead time reduction for a make to stock product. *IIE Trans.* **36** (2010) 333–344.

[30] T. Roy and K.S. Chaudhuri, A production inventory model under stock-dependent demand, Weibull distribution deterioration and shortage. *Int. Trans. Oper. Res.* **16** (2009) 325–346.

[31] S. Saha, I.E. Nielsen and I. Moon, Strategic inventory and pricing decision for substitutable products. *Comput. Ind. Eng.* **160** (2021) 107570.

[32] P. Shi and X. Chen, Supply chain decision-making and coordination for joint investment in cost and carbon emission reduction. *Int. J. Low-Carbon Technol.* **18** (2023) 306–321.

[33] C.K. Sivashankari, P. Vijayakumar and V. Choudri, Complementary products for imperfect production, reworking, scarp with advertisement-sales return–Bertrand's price dependent demand. *J. Control Decis.* (2025) 1–17.

[34] W. Soon, Investigation of Nash equilibrium existence involving complementarity constrained pricing models. *Appl. Math. Comput.* **218** (2011) 1553–1567.

[35] Y. Sun, X. Zhang, X. Huang and W. Cao, Research on logistics service supply chain coordination in the context of Green Innovation. *Sustainability* **17** (2025) 646.

[36] A.A. Taleizadeh, H. Moghadasi, S.T.A. Niaki and A.K. Eftekhari, An EOQ-joint replenishment policy to supply expensive imported raw materials with payment in advance. *J. Appl. Sci.* **8** (2009) 4263–4273.

[37] A.A. Taleizadeh, S.T.A. Niaki and V. Hosseini, Optimizing multi product multi constraints bi-objective newsboy problem with discount by hybrid method of goal programming and genetic algorithm. *Eng. Opt.* **41** (2009) 437–457.

[38] A.A. Taleizadeh, S.T.A. Niaki and M.B. Aryanezhad, Replenish-up-to multi chance-constraint inventory control system with stochastic period lengths and total discount under fuzzy purchasing price and holding costs. *Int. J. Syst. Sci.* **41** (2010) 1187–1200.

[39] A.A. Taleizadeh, S.T.A. Niaki, N. Shafii, R. Ghavamizadeh-Meibodi and A. Jabbarzadeh, A particle swarm optimization approach for constraint joint single buyer single vendor inventory problem with changeable lead-time and (r, Q) policy in supply chain. *Int. J. Adv. Manuf. Technol.* **51** (2010) 1209–1223.

[40] A.A. Taleizadeh, S.T.A. Niaki and F. Barzinpour, Multi-buyer multi-vendor multi-product multi-constraint supply chain problem with stochastic demand and variable lead time. *Appl. Math. Comput.* **217** (2011) 9234–9253.

[41] A.A. Taleizadeh, S.T.A. Niaki and A. Makui, Multi-product multi-chance constraint multi-buyer single-vendor supply chain problem with stochastic demand and variable lead time. *Expert Syst. App.* **39** (2012) 5338–5348.

[42] A.A. Taleizadeh, S.T. Niaki and H.M. Wee, Joint single vendor–single buyer supply chain problem with stochastic demand and fuzzy lead-time. *Knowl. Based Syst.* **48** (2013) 1–9.

[43] S. Transchel, S. Minner and D.F. Pyke, Capacity investment and pricing decisions in a single period two product problem, in Operation Research Proceedings, Selected Papers of the Annual International Conference of the German Operations Research Society (GOR). Springer (2007) 389–394.

[44] S.D. Treville, N. Schurhoff, L. Trigeorgis and B. Avanzi, Optimal sourcing and lead time reduction under evolutionary demand risk. *Prod. Oper. Manage.* **23** (2014) 2103–2117.

[45] Y. Wang and Y. Gerchak, Supply chain coordination when demand is shelf-space dependent. *Manuf. Serv. Oper. Manage.* **3** (2001) 82–87.

[46] J. Wei, J. Zhao and Y. Li, Pricing decisions for complementary products with firms' different market powers, *Eur. J. Oper. Res.* **224** (2013) 507–519.

[47] Z. Wu, B. Kazaz, S. Webster and K.K. Yang, Ordering, pricing, and lead-time quotation under lead-time and demand uncertainty. *Prod. Oper. Manage.* **21** (2012) 576–589.

[48] Z. Wu, Z. Liu and X. Zhai, Impacts of inventory billboard effect on RFID implementation strategy, in Enterprise Systems Conference (ES). IEEE (2014) 7–12.

[49] Z. Wu, X. Zhai and Z. Liu, The inventory billboard effect on the lead time decision. *Int. J. Prod. Econ.* **170** (2015) 45–53.

[50] Z. Xiang, J. Yang, M.H. Naseem and W. Ge, Research on dynamic cooperative replenishment optimization of shipbuilding enterprise inventory control under uncertainty. *Sustainability* **14** (2022) 2113.

[51] T. Xiao and X. Qi, A two-stage supply chain with demand sensitive to price, delivery time, and reliability of delivery. *Ann. Oper. Res.* **241** (2016) 475–496.

[52] T. Xiao, T.M. Choi and T.C.E. Cheng, Delivery lead time and channel structure decisions for make to order duopoly under different game scenarios. *Transp. Rese. Part E: Logistics Transp. Rev.* **87** (2016) 113–129.

[53] X. Yue, S.K. Mukhopadhyay and X. Zhu, A Bertrand model of pricing of complementary goods under information asymmetry. *J. Bus. Res.* **59** (2006) 1182–1192.

[54] Y. Zhai and T.C.E. Cheng, Lead-time quotation and hedging coordination in make-to-order supply chain. *Eur. J. Oper. Res.* **300** (2022) 449–460.

[55] Y. Zhai, R.Y. Zhong, Z. Li and G. Huang, Production lead time hedging and coordination in prefabricated construction supply chain management. *Int. J. Prod. Res.* **54** (2016) 1–19.

[56] S.X. Zhu, Integration of capacity, pricing, and lead time decisions in a decentralized supply chain. *Int. J. Prod. Econ.* **164** (2015) 14–23.

Appendix A. Proof of concavity of single firm's profit function

To prove the concavity of single firm's profit function, firstly Hessian matrix is formed; then the minors are examined. For concave functions, first minor is negative, other minors are positive and negative respectively. The Hessian matrix of profit function of the single firm is as follows:

$$H = \begin{bmatrix} \frac{\partial^2\Omega}{\partial V^2} & \frac{\partial^2\Omega}{\partial V\partial M} & \frac{\partial^2\Omega}{\partial V\partial I} \\ \frac{\partial^2\Omega}{\partial M\partial V} & \frac{\partial^2\Omega}{\partial M^2} & \frac{\partial^2\Omega}{\partial M\partial I} \\ \frac{\partial^2\Omega}{\partial I\partial V} & \frac{\partial^2\Omega}{\partial I\partial M} & \frac{\partial^2\Omega}{\partial I^2} \end{bmatrix} < 0 \tag{A.1}$$

where each of the element is obtained as follows:

$$\frac{\partial^2\Omega}{\partial V\partial M} = \frac{\partial^2\Omega}{\partial M\partial V} = ngbwI^{2d+1}V^{-g-1} - \frac{1}{2}M^{\frac{-1}{2}}gbI^{d+1}\sqrt{nw}V^{-\frac{g}{2}-1} \tag{A.2}$$

$$\begin{aligned}\frac{\partial^2\Omega}{\partial V^2} &= -g(1-g)wI^{2d+1}V^{-g-1} + g(-g-1)ewI^{2d+1}V^{-g-2} \\ &\quad + ng(-g-1)bMwI^{2d+1}V^{-g-2} - g(-\frac{g}{2}-1)bI^{d+1}\sqrt{nMw}V^{(-\frac{g}{2}-2)}\end{aligned} \tag{A.3}$$

$$\frac{\partial^2\Omega}{\partial V\partial I} = \frac{\partial^2\Omega}{\partial I\partial V} = (2d+1)wI^{2d}\left[(1-g)V^{-g} + geV^{-g-1} + gbMV^{-g-1}n\right] - g(d+1)bI^{d}\sqrt{nMw}V^{-\frac{g}{2}-1} \tag{A.4}$$

$$\frac{\partial^2\Omega}{\partial M^2} = -\frac{1}{2}bI^{d+1}M^{-\frac{3}{2}}\sqrt{nwV^{-g}} \tag{A.5}$$

$$\frac{\partial^2\Omega}{\partial M\partial I} = \frac{\partial^2\Omega}{\partial I\partial M} = -nbw(2d+1)I^{2d}V^{-g} + b(d+1)I^{d}M^{-\frac{1}{2}}\sqrt{nwV^{-g}} \tag{A.6}$$

$$\frac{\partial^2\Omega}{\partial I^2} = 2d(2d+1)wI^{2d-1}\left[V^{1-g} - eV^{-g} - nbMV^{-g}\right] + 2d(d+1)bI^{d-1}\sqrt{nMwV^{-g}}. \tag{A.7}$$

Now, the sign of minors is studied. Formula of first minor, which is shown with sign $|H|_1$. If equation (A.8) is found negative, next minors of function are determined. If second minor is found positive, which is shown in equation (A.9). And if the third minor is assumed negative, then the profit function is strictly concave.

$$\begin{aligned}|H|_1 = \frac{\partial^2\Omega}{\partial V^2} &= -g(1-g)wI^{2d+1}V^{-g-1} - g(g+1)ewI^{2d+1}V^{-g-2} \\ &\quad - ng(g+1)bMwI^{2d+1}V^{-g-2} + g\left(\frac{g}{2}+1\right)bI^{d+1}\sqrt{nMw}V^{(-\frac{g}{2}-2)} < 0\end{aligned} \tag{A.8}$$

$$\begin{aligned}|H|_2 &= \frac{\partial^2\Omega}{\partial V^2} \times \frac{\partial^2\Omega}{\partial M^2} - \frac{\partial^2\Omega}{\partial V\partial M} \times \frac{\partial^2\Omega}{\partial M\partial V} \geq 0 \\ &= -\begin{bmatrix} g(1-g)wI^{2d+1}V^{-g-1} + g(g+1)ewI^{2d+1}V^{-g-2} \\ +ng(g+1)bMwI^{2d+1}V^{-g-2} - g\left(\frac{g}{2}+1\right)bI^{d+1}\sqrt{nMw}V^{(-\frac{g}{2}-2)} \end{bmatrix}\end{aligned}$$

$$\times\left[-\frac{1}{2}bI^{d+1}M^{-\frac{3}{2}}\sqrt{nwV^{-g}}\right]-\left[ngbwI^{2d+1}V^{-g-1}-\frac{1}{2}M^{\frac{-1}{2}}gbI^{d+1}\sqrt{nw}V^{-\frac{g}{2}-1}\right]^2>0 \tag{A.9}$$

$$|H|_3=\frac{\partial^2\Omega}{\partial V^2}\frac{\partial^2\Omega}{\partial M^2}\frac{\partial^2\Omega}{\partial I^2}+2\frac{\partial^2\Omega}{\partial M\partial I}\frac{\partial^2\Omega}{\partial V\partial I}\frac{\partial^2\Omega}{\partial V\partial M}$$
$$-\left[\frac{\partial^2\Omega}{\partial V^2}\left(\frac{\partial^2\Omega}{\partial M\partial I}\right)^2+\left(\frac{\partial^2\Omega}{\partial V\partial M}\right)^2\frac{\partial^2\Omega}{\partial I^2}+\left(\frac{\partial^2\Omega}{\partial V\partial I}\right)^2\frac{\partial^2\Omega}{\partial M^2}\right]<0. \tag{A.10}$$

Appendix B. Proof of concavity of firm i's profit function under Nash equilibrium

To prove the concavity of single firm's profit function under Nash equilibrium, firstly Hessian matrix is built; then the minors are examined. As it is mentioned earlier, $I_1 = I\varepsilon_1$ and $I_2 = I\varepsilon_2$, if first minor is negative and other minors turn into positive and negative respectively; then the function is concave. Hessian matrix for $i = 1, 2$ is shown as follows.

$$H=\begin{bmatrix}\frac{\partial^2\Omega_i}{\partial V_i^2} & \frac{\partial^2\Omega_i}{\partial V_i\partial M_i} & \frac{\partial^2\Omega_i}{\partial V_i\partial I_i}\\ \frac{\partial^2\Omega_i}{\partial M_i\partial V_i} & \frac{\partial^2\Omega_i}{\partial M_i^2} & \frac{\partial^2\Omega_i}{\partial M_i\partial I_i}\\ \frac{\partial^2\Omega_i}{\partial I_i\partial V_i} & \frac{\partial^2\Omega_i}{\partial I_i\partial M_i} & \frac{\partial^2\Omega_i}{\partial I_i^2}\end{bmatrix}<0 \tag{B.1}$$

where,

$$\frac{\partial^2\Omega_i}{\partial V_i\partial M_i}=\frac{\partial^2\Omega_i}{\partial M_i\partial V_i}=gbnwI_i(I_i+I_{3-i})^{2d}(V_i+V_{3-i})^{-g-1}$$
$$-\frac{g}{2}bM_i^{-\frac{1}{2}}\sqrt{nw}(V_i+V_{3-i})^{-\frac{g}{2}-1}I_i(I_i+I_{3-i})^d \tag{B.2}$$

$$\frac{\partial^2\Omega_i}{\partial V_i^2}=wI_i(I_i+I_{3-i})^{2d}\begin{bmatrix}-g(V_i+V_{3-i})^{-g-1}+g(g+1)(V_i+V_{3-i})^{-g-2}V_i\\ -g(V_i+V_{3-i})^{-g-1}+(-e_i-bnM_i)g(g+1)(V_i+V_{3-i})^{-g-2}\end{bmatrix}$$
$$+gb\sqrt{M_inw}I_i(I_i+I_{3-i})^d(\frac{g}{2}+1)(V_i+V_{3-i})^{-\frac{g}{2}-2} \tag{B.3}$$

$$\frac{\partial^2\Omega_i}{\partial V_i\partial I_i}=\frac{\partial^2\Omega_i}{\partial I_i\partial V_i}$$
$$=\left[\left((V_i+V_{3-i})^{-g}-g(V_i+V_{3-i})^{-g-1}V_i\right)+\left((-e_i-bnM_i)(-g)(V_i+V_{3-i})^{-g-1}\right)\right]w$$
$$\times\left[(I_i+I_{3-i})^{2d}+2d(I_i+I_{3-i})^{2d-1}I_i\right]$$
$$-gb\sqrt{M_inw}(V_i+V_{3-i})^{-\frac{g}{2}-1}\left[(I_i+I_{3-i})^d+d(I_i+I_{3-i})^{d-1}I_i\right] \tag{B.4}$$

$$\frac{\partial^2\Omega_i}{\partial M_i^2}=-\frac{1}{2}bM_i^{-\frac{3}{2}}\sqrt{nw(V_i+V_{3-i})^{-g}}I_i(I_i+I_{3-i})^d \tag{B.5}$$

$$\frac{\partial^2\Omega_i}{\partial M_i\partial I_i}=\frac{\partial^2\Omega_i}{\partial I_i\partial M_i}=-bnw(V_i+V_{3-i})^{-g}\left[(I_i+I_{3-i})^{2d}+2d(I_i+I_{3-i})^{2d-1}I_i\right]$$
$$+bM_i^{-\frac{1}{2}}\sqrt{nw}(V_i+V_{3-i})^{-\frac{g}{2}}\left[(I_i+I_{3-i})^d+d(I_i+I_{3-i})^{d-1}I_i\right] \tag{B.6}$$

$$\frac{\partial^2\Omega_i}{\partial I_i^2}=(V_i-e_i-bnM_i)w(V_i+V_{3-i})^{-g}\left[4d(I_i+I_{3-i})^{2d-1}+2d(2d-1)(I_i+I_{3-i})^{2d-2}I_i\right]$$
$$+2b\sqrt{M_inw(V_i+V_{3-i})^{-g}}\left[2d(I_i+I_{3-i})^{d-1}+d(d-1)(I_i+I_{3-i})^{d-2}I_i\right]. \tag{B.7}$$

According to the concavity conditions, first minor is studied. In fact, the first minor is first element of the Hessian matrix and it is shown in equation (B.8). Second minor is calculated according to equation (B.9) and it is considered positive. Third minor is negative and it is shown in equation (B.10).

$$|H|_1=\frac{\partial^2\Omega_i}{\partial V_i^2}=wI_i(I_i+I_{3-i})^{2d}\begin{bmatrix}-g(V_i+V_{3-i})^{-g-1}+g(g+1)(V_i+V_{3-i})^{-g-2}V_i\\ -g(V_i+V_{3-i})^{-g-1}\\ +(-e_i-bnM_i)g(g+1)(V_i+V_{3-i})^{-g-2}\end{bmatrix}$$

$$+ gb\sqrt{M_i nw} I_i (I_i + I_{3-i})^d \left(\frac{g}{2}+1\right)(V_i + V_{3-i})^{-\frac{g}{2}-2} < 0 \tag{B.8}$$

$$|H|_2 = \frac{\partial^2 \Omega_i}{\partial V_i^2} \times \frac{\partial^2 \Omega_i}{\partial M_i^2} - \frac{\partial^2 \Omega_i}{\partial V_i \partial M_i} \times \frac{\partial^2 \Omega_i}{\partial M_i \partial V_i} > 0$$

$$|H|_2 = \left[\begin{matrix} wI_i(I_i+I_{3-i})^{2d} \left[\begin{matrix} -g(V_i+V_{3-i})^{-g-1} + g(g+1)(V_i+V_{3-i})^{-g-2} V_i \\ -g(V_i+V_{3-i})^{-g-1} + (-e_i - bnM_i) g(g+1)(V_i+V_{3-i})^{-g-2} \end{matrix} \right] \\ + gb\sqrt{M_i nw} I_i (I_i+I_{3-i})^d (\frac{g}{2}+1)(V_i+V_{3-i})^{-\frac{g}{2}-2} \end{matrix} \right]$$
$$\times \left[-\frac{1}{2} bM_i^{-\frac{3}{2}} \sqrt{nw(V_i+V_{3-i})^{-g}} I_i (I_i+I_{3-i})^d \right] - \left[\begin{matrix} gbnwI_i(I_i+I_{3-i})^{2d}(V_i+V_{3-i})^{-g-1} \\ -\frac{g}{2} bM_i^{-\frac{1}{2}} \sqrt{nw}(V_i+V_{3-i})^{-\frac{g}{2}-1} I_i (I_i+I_{3-i})^d \end{matrix} \right]^2 > 0 \tag{B.9}$$

$$|H|_3 = \frac{\partial^2 \Omega_i}{\partial V_i^2} \frac{\partial^2 \Omega_i}{\partial M_i^2} \frac{\partial^2 \Omega_i}{\partial I_i^2} + 2 \frac{\partial^2 \Omega_i}{\partial M_i \partial I_i} \frac{\partial^2 \Omega_i}{\partial V_i \partial I_i} \frac{\partial^2 \Omega_i}{\partial V_i \partial M_i}$$
$$- \left[\frac{\partial^2 \Omega_i}{\partial V_i^2} \left(\frac{\partial^2 \Omega_i}{\partial M_i \partial I_i} \right)^2 + \left(\frac{\partial^2 \Omega_i}{\partial V_i \partial M_i} \right)^2 \frac{\partial^2 \Omega_i}{\partial I_i^2} + \left(\frac{\partial^2 \Omega_i}{\partial V_i \partial I_i} \right)^2 \frac{\partial^2 \Omega_i}{\partial M_i^2} \right] < 0. \tag{B.10}$$

Therefore, the function is strictly concave.

Appendix C. Proving concavity of firm i's profit function under Stackelberg game

Under Stackelberg game, profit function must be concave as well. As it was mentioned earlier, $I_1 = I\varepsilon_1$ and $I_2 = I\varepsilon_2$. Similarly, first minor must be negative and next minors must turn into positive and negative respectively. Equation (C.1) shows Hessian matrix of the profit function for firm 1.

$$H = \begin{bmatrix} \frac{\partial^2 \Omega_1}{\partial V_1^2} & \frac{\partial^2 \Omega_1}{\partial V_1 \partial M_1} & \frac{\partial^2 \Omega_1}{\partial V_1 \partial I_1} \\ \frac{\partial^2 \Omega_1}{\partial M_1 \partial V_1} & \frac{\partial^2 \Omega_1}{\partial M_1^2} & \frac{\partial^2 \Omega_1}{\partial M_1 \partial I_1} \\ \frac{\partial^2 \Omega_1}{\partial I_1 \partial V_1} & \frac{\partial^2 \Omega_1}{\partial I_1 \partial M_1} & \frac{\partial^2 \Omega_1}{\partial I_1^2} \end{bmatrix} < 0 \tag{C.1}$$

where,

$$\frac{\partial^2 \Omega_1}{\partial V_1 \partial M_1} = \frac{\partial^2 \Omega_1}{\partial M_1 \partial V_1} = gbnwI_1(I_1+I_2)^{2d}(V_1+V_2)^{-g-1}$$
$$- \frac{g}{2} bM_1^{-\frac{1}{2}} \sqrt{nw}(V_1+V_2)^{-\frac{g}{2}-1} I_1 (I_1+I_2)^d \tag{C.2}$$

$$\frac{\partial^2 \Omega_1}{\partial V_1^2} = wI_1(I_1+I_2)^{2d} \left[\begin{matrix} -g(V_1+V_2)^{-g-1} + g(g+1)(V_1+V_2)^{-g-2} V_1 \\ -g(V_1+V_2)^{-g-1} + (-e_1 - bnM_1) g(g+1)(V_1+V_2)^{-g-2} \end{matrix} \right]$$
$$+ gb\sqrt{M_1 nw} I_1 (I_1+I_2)^d \left(\frac{g}{2}+1\right)(V_1+V_2)^{-\frac{g}{2}-2} \tag{C.3}$$

$$\frac{\partial^2 \Omega_1}{\partial V_1 \partial I_1} = \frac{\partial^2 \Omega_1}{\partial I_1 \partial V_1} = \left[\left((V_1+V_2)^{-g} - g(V_1+V_2)^{-g-1} V_1 \right) + \left((-e_1 - bnM_1)(-g)(V_1+V_2)^{-g-1} \right) \right]$$
$$\times w \left[(I_1+I_2)^{2d} + 2d(I_1+I_2)^{2d-1} I_1 \right]$$
$$- gb\sqrt{M_1 nw}(V_1+V_2)^{-\frac{g}{2}-1} \left[(I_1+I_2)^d + d(I_1+I_2)^{d-1} I_1 \right] \tag{C.4}$$

$$\frac{\partial^2 \Omega_1}{\partial M_2^2} = -\frac{1}{2} bM_1^{-\frac{3}{2}} \sqrt{nw(V_1+V_2)^{-g}} I_1 (I_1+I_2)^d \tag{C.5}$$

$$\frac{\partial^2 \Omega_1}{\partial M_1 \partial I_1} = \frac{\partial^2 \Omega_1}{\partial I_1 \partial M_1} = -bnw(V_1+V_2)^{-g} \left[(I_1+I_2)^{2d} + 2d(I_1+I_2)^{2d-1} I_1 \right]$$
$$+ bM_1^{-\frac{1}{2}} \sqrt{nw}(V_1+V_2)^{-\frac{g}{2}} \left[(I_1+I_2)^d + d(I_1+I_2)^{d-1} I_1 \right] \tag{C.6}$$

$$\frac{\partial^2 \Omega_1}{\partial I_1^2} = (V_1 - e_1 - bnM_1) w(V_1+V_2)^{-g} \left[4d(I_1+I_2)^{2d-1} + 2d(2d-1)(I_1+I_2)^{2d-2} I_1 \right]$$

$$+ 2b\sqrt{M_1 nw(V_1+V_2)^{-g}}\Big[2d(I_1+I_2)^{d-1} + d(d-1)(I_1+I_2)^{d-2}I_1\Big]. \tag{C.7}$$

According to concavity conditions, first minor is studied. The first minor is shown in equation (C.8). Second minor (Eq. (C.9)) and third minor (Eq. (C.10)) are determined. These must be positive and negative, respectively.

$$|H|_1 = \frac{\partial^2 \Omega_1}{\partial V_1^2} = wI_1(I_1+I_2)^{2d}\begin{bmatrix} -g(V_1+V_2)^{-g-1} + g(g+1)(V_1+V_2)^{-g-2}V_1 \\ -g(V_1+V_2)^{-g-1} + (-e_1 - bnM_1)g(g+1)(V_1+V_2)^{-g-2}\end{bmatrix}$$
$$+ gb\sqrt{M_1 nw}I_1(I_1+I_2)^d\Big(\frac{g}{2}+1\Big)(V_1+V_2)^{-\frac{g}{2}-2} < 0 \tag{C.8}$$

$$|H|_2 = \frac{\partial^2 \Omega_1}{\partial V_1^2}\times\frac{\partial^2 \Omega_1}{\partial M_1^2} - \frac{\partial^2 \Omega_1}{\partial V_1 \partial M_1}\times\frac{\partial^2 \Omega_1}{\partial M_1 \partial V_1} > 0$$
$$= \begin{bmatrix} wI_1(I_1+I_2)^{2d}\begin{bmatrix} -g(V_1+V_2)^{-g-1} + g(g+1)(V_1+V_2)^{-g-2}V_1 \\ -g(V_1+V_2)^{-g-1} + (-e_1 - bnM_1)g(g+1)(V_1+V_2)^{-g-2}\end{bmatrix} \\ +gb\sqrt{M_1 nw}I_1(I_1+I_2)^d(\frac{g}{2}+1)(V_1+V_2)^{-\frac{g}{2}-2}\end{bmatrix}$$
$$\times\Big[-\frac{1}{2}bM_1^{-\frac{3}{2}}\sqrt{nw(V_1+V_2)^{-g}}I_1(I_1+I_2)^d\Big]$$
$$-\Big[gbnwI_1(I_1+I_2)^{2d}(V_1+V_2)^{-g-1} - \frac{g}{2}bM_1^{-\frac{1}{2}}\sqrt{nw}(V_1+V_2)^{-\frac{g}{2}-1}I_1(I_1+I_2)^d\Big]^2 > 0 \tag{C.9}$$

$$|H|_3 = \frac{\partial^2 \Omega_1}{\partial V_1^2}\frac{\partial^2 \Omega_1}{\partial M_1^2}\frac{\partial^2 \Omega_1}{\partial I_1^2} + 2\frac{\partial^2 \Omega_1}{\partial M_1 \partial I_1}\frac{\partial^2 \Omega_1}{\partial V_1 \partial I_1}\frac{\partial^2 \Omega_1}{\partial V_1 \partial M_1}$$
$$-\left[\frac{\partial \Omega_1}{\partial V_1^2}\left(\frac{\partial \Omega_1}{\partial M_1 \partial I_1}\right)^2 + \left(\frac{\partial \Omega_1}{\partial V_1 \partial M_1}\right)^2\frac{\partial \Omega_1}{\partial I_1^2} + \left(\frac{\partial \Omega_1}{\partial V_1 \partial I_1}\right)^2\frac{\partial \Omega_1}{\partial M_1^2}\right] < 0. \tag{C.10}$$

Hence, the function is strictly concave.

Now, the concavity of profit function is studied for firm 2. The first minor must be negative and next minors must turn into positive and negative, respectively. Equation (C.11) shows Hessian matrix of the function as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \Omega_2}{\partial V_2^2} & \frac{\partial^2 \Omega_2}{\partial V_2 \partial M_2} & \frac{\partial^2 \Omega_2}{\partial V_2 \partial I_2} \\ \frac{\partial^2 \Omega_2}{\partial M_2 \partial V_2} & \frac{\partial^2 \Omega_2}{\partial M_2^2} & \frac{\partial^2 \Omega_2}{\partial M_2 \partial I_2} \\ \frac{\partial^2 \Omega_2}{\partial I_2 \partial V_2} & \frac{\partial^2 \Omega_2}{\partial I_2 \partial M_2} & \frac{\partial^2 \Omega_2}{\partial I_2^2}\end{bmatrix} < 0 \tag{C.11}$$

where,

$$\frac{\partial^2 \Omega_2}{\partial V_2 \partial M_2} = \frac{\partial^2 \Omega_2}{\partial M_2 \partial V_2} = gbnwI_2(I_1^*+I_2)^{2d}(V_1^*+V_2)^{-g-1}$$
$$-\frac{g}{2}bM_2^{-\frac{1}{2}}\sqrt{nw}(V_1^*+V_2)^{-\frac{g}{2}-1}I_2(I_1^*+I_2)^d \tag{C.12}$$

$$\frac{\partial^2 \Omega_2}{\partial V_2^2} = wI_2(I_1^*+I_2)^{2d}\begin{bmatrix} -g(V_1^*+V_2)^{-g-1} + g(g+1)(V_1^*+V_2)^{-g-2}V_2 \\ -g(V_1^*+V_2)^{-g-1} \\ +(-e_2 - bnM_2)g(g+1)(V_1^*+V_2)^{-g-2}\end{bmatrix}$$
$$+ gb\sqrt{M_2 nw}I_2(I_1^*+I_2)^d\Big(\frac{g}{2}+1\Big)(V_1^*+V_2)^{-\frac{g}{2}-2} \tag{C.13}$$

$$\frac{\partial^2 \Omega_2}{\partial V_2 \partial I_2} = \frac{\partial^2 \Omega_2}{\partial I_2 \partial V_2}$$
$$= \Big[\Big((V_1^*+V_2)^{-g} - g(V_1^*+V_2)^{-g-1}V_2\Big) + \Big((-e_2 - bnM_2)(-g)(V_1^*+V_2)^{-g-1}\Big)\Big]$$
$$\times w\Big[(I_1^*+I_2)^{2d} + 2d(I_1^*+I_2)^{2d-1}I_2\Big]$$
$$- gb\sqrt{M_2 nw}(V_1^*+V_2)^{-\frac{g}{2}-1}\Big[(I_1^*+I_2)^d + d(I_1^*+I_2)^{d-1}I_2\Big] \tag{C.14}$$

$$\frac{\partial^2\Omega_2}{\partial M_2^2} = -\frac{1}{2}bM_2^{-\frac{3}{2}}\sqrt{nw(V_1^*+V_2)^{-g}}I_2(I_1^*+I_2)^d \tag{C.15}$$

$$\frac{\partial^2\Omega_2}{\partial M_2\partial I_2} = \frac{\partial^2\Omega_2}{\partial I_2\partial M_2} = -bnw(V_1^*+V_2)^{-g}\left[(I_1^*+I_2)^{2d} + 2d(I_1^*+I_2)^{2d-1}I_2\right] + bM_2^{-\frac{1}{2}}\sqrt{nw}(V_1^*+V_2)^{-\frac{g}{2}}\left[(I_1^*+I_2)^d + d(I_1^*+I_2)^{d-1}I_2\right] \tag{C.16}$$

$$\frac{\partial^2\Omega_2}{\partial I_2^2} = (V_2-e_2-bnM_2)w(V_1^*+V_2)^{-g}\left[4d(I_1^*+I_2)^{2d-1} + 2d(2d-1)(I_1^*+I_2)^{2d-2}I_2\right] + 2b\sqrt{M_2nw(V_1^*+V_2)^{-g}}\left[2d(I_1^*+I_2)^{d-1} + d(d-1)(I_1^*+I_2)^{d-2}I_2\right]. \tag{C.17}$$

According to concavity conditions, first minor is studied. The first minor is shown in equation (C.18). The second minor and third minor given by equations (C.19) and (C.20), respectively are computed. These must be positive and negative, respectively.

$$\begin{aligned}|H|_1 &= \frac{\partial^2\Omega_2}{\partial V_2^2} < 0\\ &= wI_2(I_1^*+I_2)^{2b}\begin{bmatrix}-g(V_1^*+V_2)^{-g-1} + g(g+1)(V_1^*+V_2)^{-g-2}V_2\\ -g(V_1^*+V_2)^{-g-1} + (-e_2-bnM_2)g(g+1)(V_1^*+V_2)^{-g-2}\end{bmatrix}\\ &\quad + gb\sqrt{M_2nw}I_2(I_1^*+I_2)^d\left(\frac{g}{2}+1\right)(V_1^*+V_2)^{-\frac{g}{2}-2} < 0\end{aligned} \tag{C.18}$$

$$\begin{aligned}|H|_2 &= \frac{\partial^2\Omega_2}{\partial V_2^2}\times\frac{\partial^2\Omega_2}{\partial M_2^2} - \frac{\partial^2\Omega_2}{\partial V_2\partial M_2}\times\frac{\partial^2\Omega_2}{\partial M_2\partial V_2} > 0\\ &= \begin{bmatrix} wI_2(I_1^*+I_2)^{2d}\begin{bmatrix}-g(V_1^*+V_2)^{-g-1} + g(g+1)(V_1^*+V_2)^{-g-2}V_2\\ -g(V_1^*+V_2)^{-g-1} + (-e_2-bnM_2)g(g+1)(V_1^*+V_2)^{-g-2}\end{bmatrix}\\ +gb\sqrt{M_2nw}I_2(I_1^*+I_2)^d(\frac{g}{2}+1)(V_1^*+V_2)^{-\frac{g}{2}-2}\end{bmatrix}\\ &\quad\times\left[-\frac{1}{2}bM_2^{-\frac{3}{2}}\sqrt{nw(V_1^*+V_2)^{-g}}I_2(I_1^*+I_2)^d\right]\\ &\quad-\begin{bmatrix}gbnwI_2(I_1^*+I_2)^{2d}(V_1^*+V_2)^{-g-1}-\\ \frac{g}{2}bM_2^{-\frac{1}{2}}\sqrt{nw}(V_1^*+V_2)^{-\frac{g}{2}-1}I_2(I_1^*+I_2)^d\end{bmatrix}^2 > 0\end{aligned} \tag{C.19}$$

$$\begin{aligned}|H|_3 &= \frac{\partial^2\Omega_2}{\partial V_2^2}\frac{\partial^2\Omega_2}{\partial M_2^2}\frac{\partial^2\Omega_2}{\partial I_2^2} + 2\frac{\partial^2\Omega_2}{\partial M_2\partial I_2}\frac{\partial^2\Omega_2}{\partial V_2\partial I_2}\frac{\partial^2\Omega_2}{\partial V_2\partial M_2}\\ &\quad-\left[\frac{\partial\Omega_2}{\partial V_2^2}\left(\frac{\partial\Omega_2}{\partial M_2\partial I_2}\right)^2 + \left(\frac{\partial\Omega_2}{\partial V_2\partial M_2}\right)^2\frac{\partial\Omega_2}{\partial I_2^2} + \left(\frac{\partial\Omega_2}{\partial V_2\partial I_2}\right)^2\frac{\partial\Omega_2}{\partial M_2^2}\right] < 0.\end{aligned} \tag{C.20}$$

Therefore, the function is strictly concave.

Appendix D. Proof of firm i's profit function under cooperative approach

To prove the concavity of profit function under cooperative approach, as it is mentioned earlier, $I_1 = I\varepsilon_1$ and $I_2 = I\varepsilon_2$. Firstly, the Hessian matrix is constructed as follows:

$$H = \begin{bmatrix}
\frac{\partial^2\sum\Omega_i}{\partial V_1^2} & \frac{\partial^2\sum\Omega_i}{\partial V_1\partial V_2} & \frac{\partial^2\sum\Omega_i}{\partial V_1\partial M_1} & \frac{\partial^2\sum\Omega_i}{\partial V_1\partial M_2} & \frac{\partial^2\sum\Omega_i}{\partial V_1\partial I_1} & \frac{\partial^2\sum\Omega_i}{\partial V_1\partial I_2}\\
\frac{\partial^2\sum\Omega_i}{\partial V_2\partial V_1} & \frac{\partial^2\sum\Omega_i}{\partial V_2^2} & \frac{\partial^2\sum\Omega_i}{\partial V_2\partial M_1} & \frac{\partial^2\sum\Omega_i}{\partial V_2\partial M_2} & \frac{\partial^2\sum\Omega_i}{\partial V_2\partial I_1} & \frac{\partial^2\sum\Omega_i}{\partial V_2\partial I_2}\\
\frac{\partial^2\sum\Omega_i}{\partial M_1\partial V_1} & \frac{\partial^2\sum\Omega_i}{\partial M_1\partial V_2} & \frac{\partial^2\sum\Omega_i}{\partial M_1^2} & \frac{\partial^2\sum\Omega_i}{\partial M_1\partial M_2} & \frac{\partial^2\sum\Omega_i}{\partial M_1\partial I_1} & \frac{\partial^2\sum\Omega_i}{\partial M_1\partial I_2}\\
\frac{\partial^2\sum\Omega_i}{\partial M_2\partial V_1} & \frac{\partial^2\sum\Omega_i}{\partial M_2\partial V_2} & \frac{\partial^2\sum\Omega_i}{\partial M_2\partial M_1} & \frac{\partial^2\sum\Omega_i}{\partial M_2^2} & \frac{\partial^2\sum\Omega_i}{\partial M_2\partial I_1} & \frac{\partial^2\sum\Omega_i}{\partial M_2\partial I_2}\\
\frac{\partial^2\sum\Omega_i}{\partial I_1\partial V_1} & \frac{\partial^2\sum\Omega_i}{\partial I_1\partial V_2} & \frac{\partial^2\sum\Omega_i}{\partial I_1\partial M_1} & \frac{\partial^2\sum\Omega_i}{\partial I_1\partial M_2} & \frac{\partial^2\sum\Omega_i}{\partial I_1^2} & \frac{\partial^2\sum\Omega_i}{\partial I_1\partial I_2}\\
\frac{\partial^2\sum\Omega_i}{\partial I_2\partial V_1} & \frac{\partial^2\sum\Omega_i}{\partial I_2\partial V_2} & \frac{\partial^2\sum\Omega_i}{\partial I_2\partial M_1} & \frac{\partial^2\sum\Omega_i}{\partial I_2\partial M_2} & \frac{\partial^2\sum\Omega_i}{\partial I_2\partial I_1} & \frac{\partial^2\sum\Omega_i}{\partial I_2^2}
\end{bmatrix} \tag{D.1}$$

where the elements are shown in equations (D.2) to (D.19):

$$\begin{aligned}\frac{\partial^2 \sum \Omega_i}{\partial V_1^2} &= \left[-2g(V_1+V_2)^{-g-1} + g(g+1)(V_1+V_2)^{-g-2}V_1\right] wI_1(I_1+I_2)^{2d} \\ &+ (-e_1 - bnM_1)wI_1(I_1+I_2)^{2d}(-g)(V_1+V_2)^{-g-2}(-g-1) \\ &+ (V_2 - e_2 - bnM_2)wI_1(I_1+I_2)^{2d}(-g)(V_1+V_2)^{-g-2}(-g-1) \end{aligned} \tag{D.2}$$

$$\begin{aligned}\frac{\partial^2 \sum \Omega_i}{\partial V_2^2} &= \left[-2g(V_1+V_2)^{-g-1} + g(g+1)(V_1+V_2)^{-g-2}V_1\right] wI_2(I_1+I_2)^{2d} \\ &+ (-e_2 - bnM_2)wI_2(I_1+I_2)^{2d}(-g)(V_1+V_2)^{-g-2}(-g-1) \\ &+ (V_1 - e_1 - bnM_1)wI_2(I_1+I_2)^{2d}(-g)(V_1+V_2)^{-g-2}(-g-1) \end{aligned} \tag{D.3}$$

$$\frac{\partial^2 \sum \Omega_i}{\partial M_1^2} = -\frac{1}{2}M_1^{-\frac{3}{2}} bI_1(I_1+I_2)^d \sqrt{nw(V_1+V_2)^{-g}} \tag{D.4}$$

$$\frac{\partial^2 \sum \Omega_i}{\partial M_2^2} = -\frac{1}{2}M_2^{-\frac{3}{2}} bI_2(I_1+I_2)^d \sqrt{nw(V_1+V_2)^{-g}} \tag{D.5}$$

$$\begin{aligned}\frac{\partial^2 \sum \Omega_i}{\partial I_1^2} &= (V_1 - e_1 - bnM_1)w(V_1+V_2)^{-g}\left[4d(I_1+I_2)^{2d-1} + 2d(2d-1)(I_1+I_2)^{2d-2}I_1\right] \\ &+ (V_2 - e_2 - bnM_2)w(V_1+V_2)^{-g}\left[2dI_2(I_1+I_2)^{2d-2}(2d-1)\right] \\ &+ 2b\sqrt{M_2nw(V_1+V_2)^{-g}}\left[2d(I_1+I_2)^{d-1} + d(d-1)(I_1+I_2)^{d-2}I_1\right] \\ &+ 2b\sqrt{M_2nw(V_1+V_2)^{-g}}\left[d(d-1)(I_1+I_2)^{d-2}I_2\right] \end{aligned} \tag{D.6}$$

$$\begin{aligned}\frac{\partial^2 \sum \Omega_i}{\partial I_2^2} &= (V_2 - e_2 - bnM_2)w(V_1+V_2)^{-g}\left[4d(I_1+I_2)^{2d-1} + 2d(2d-1)(I_1+I_2)^{2d-2}I_2\right] \\ &+ (V_1 - e_1 - bnM_1)w(V_1+V_2)^{-g}\left[2dI_1(I_1+I_2)^{2d-2}(2d-1)\right] \\ &+ 2b\sqrt{M_1nw(V_1+V_2)^{-g}}\left[2d(I_1+I_2)^{d-1} + d(d-1)(I_1+I_2)^{d-2}I_2\right] \\ &+ 2b\sqrt{M_1nw(V_1+V_2)^{-g}}\left[d(d-1)(I_1+I_2)^{d-2}I_1\right] \end{aligned} \tag{D.7}$$

$$\begin{aligned}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial V_1} &= \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial V_2} = \left[-g(V_1+V_2)^{-g-1} + (-g)(-g-1)(V_1+V_2)^{-g-2}V_1\right] wI_1(I_1+I_2)^{2d} \\ &+ (-e_1 - bnM_1)wI_1(I_1+I_2)^{2d}(g)(g+1)(V_1+V_2)^{-g-2} \\ &+ wI_2(I_1+I_2)^{2d}(-g)(V_1+V_2)^{-g-1} + wI_2(I_1+I_2)^{2d}(-g)(-g-1)(V_1+V_2)^{-g-2}V_2 \\ &+ (-e_2 - bnM_2)wI_2(I_1+I_2)^{2d}(-g)(-g-1)(V_1+V_2)^{-g-2} \\ &- gb\sqrt{M_1nw}I_1(I_1+I_2)^d\left(-\frac{g}{2}-1\right)(V_1+V_2)^{-\frac{g}{2}-2} \\ &- gb\sqrt{M_2nw}I_2(I_1+I_2)^d\left(-\frac{g}{2}-1\right)(V_1+V_2)^{-\frac{g}{2}-2} \end{aligned} \tag{D.8}$$

$$\begin{aligned}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_1} &= \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_1} = \frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_2} = \frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_1} = -wI_1(I_1+I_2)^{2d}bn(-g)(V_1+V_2)^{-g-1} \\ &+ M_1^{-\frac{1}{2}}bI_1(I_1+I_2)^d\sqrt{nw}\left(-\frac{g}{2}\right)(V_1+V_2)^{-\frac{g}{2}-1} \end{aligned} \tag{D.9}$$

$$\begin{aligned}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_2} &= \frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_2} = \frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_1} = \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_2} = -wI_2(I_1+I_2)^{2d}bn(-g)(V_1+V_2)^{-g-1} \\ &+ M_2^{-\frac{1}{2}}bI_2(I_1+I_2)^d\sqrt{nw}\left(-\frac{w}{2}\right)(V_1+V_2)^{-\frac{g}{2}-1} \end{aligned} \tag{D.10}$$

$$\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial M_1} = \frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial M_2} = 0 \tag{D.11}$$

$$\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial V_1} = \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial I_1} = \left[(V_1+V_2)^{-g} - g(V_1+V_2)^{-g-1}V_1\right] w\left[(I_1+I_2)^{2d} + 2d(I_1+I_2)^{2d-1}I_1\right]$$
$$- g(V_1+V_2)^{-g-1}(-e_1 - bnM_1)w\left[(I_1+I_2)^{2d} + 2d(I_1+I_2)^{2d-1}I_1\right]$$
$$+ (V_2 - e_2 - bnM_2)w(-g)(V_1+V_2)^{-g-1}\left(2d(I_1+I_2)^{2d-1}I_2\right)$$
$$+ 2b\sqrt{M_1 nw}\left(-\frac{g}{2}\right)(V_1+V_2)^{-\frac{g}{2}-1}\left[(I_1+I_2)^d + d(I_1+I_2)^{d-1}I_1\right]$$
$$+ 2b\sqrt{M_2 nw}\left(-\frac{g}{2}\right)(V_1+V_2)^{-\frac{g}{2}-1}\left[d(I_1+I_2)^{d-1}I_2\right] \tag{D.12}$$

$$\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial V_2} = \frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial I_2} = \left[(V_1+V_2)^{-g} - g(V_1+V_2)^{-g-1}V_2\right] w\left[(I_1+I_2)^{2d} + 2d(I_1+I_2)^{2d-1}I_2\right]$$
$$- g(V_1+V_2)^{-g-1}(-e_2 - bnM_2)w\left[(I_1+I_2)^{2d} + 2d(I_1+I_2)^{2d-1}I_2\right]$$
$$+ (V_1 - e_1 - bnM_1)w(-g)(V_1+V_2)^{-g-1}\left(2d(I_1+I_2)^{2d-1}I_1\right)$$
$$+ 2b\sqrt{M_2 nw}\left(-\frac{g}{2}\right)(V_1+V_2)^{-\frac{g}{2}-1}\left[(I_1+I_2)^d + d(I_1+I_2)^{d-1}I_2\right]$$
$$+ 2b\sqrt{M_1 nw}\left(-\frac{g}{2}\right)(V_1+V_2)^{-\frac{g}{2}-1}\left[d(I_1+I_2)^{d-1}I_1\right] \tag{D.13}$$

$$\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial V_2} = \frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial I_1} = (V_1 + e_1 - bnM_1)w(V_1+V_2)^{-g-1}(-g)\left[(I_1+I_2)^{2d} + 2d(I_1+I_2)^{2d-1}I_1\right]$$
$$+ \left[(V_1+V_2)^{-g} - gV_2(V_1+V_2)^{-g-1}\right] w\left[2d(I_1+I_2)^{2d-1}I_2\right]$$
$$+ (-e_2 - bnM_2)w(-g)(V_1+V_2)^{-g-1}\left[2d(I_1+I_2)^{2d-1}I_2\right]$$
$$+ 2b\sqrt{M_1 nw}\left(-\frac{g}{2}\right)(V_1+V_2)^{-\frac{g}{2}-1}\left[(I_1+I_2)^d + d(I_1+I_2)^{d-1}I_1\right]$$
$$+ 2b\sqrt{M_2 nw}\left(-\frac{g}{2}\right)(V_1+V_2)^{-\frac{g}{2}-1}\left[d(I_1+I_2)^{d-1}I_2\right] \tag{D.14}$$

$$\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial V_1} = \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial I_2} = (V_2 + e_2 - bnM_2)w(V_1+V_2)^{-g-1}(-g)\left[(I_1+I_2)^{2d} + 2d(I_1+I_2)^{2d-1}I_2\right]$$
$$+ \left[(V_1+V_2)^{-g} - gV_1(V_1+V_2)^{-g-1}\right] w\left[2d(I_1+I_2)^{2d-1}I_1\right]$$
$$+ (-e_1 - bnM_1)w(-g)(V_1+V_2)^{-g-1}\left[2d(I_1+I_2)^{2d-1}I_1\right]$$
$$+ 2b\sqrt{M_2 nw}\left(-\frac{g}{2}\right)(V_1+V_2)^{-\frac{g}{2}-1}\left[(I_1+I_2)^d + d(I_1+I_2)^{d-1}I_2\right]$$
$$+ 2b\sqrt{M_1 nw}\left(-\frac{g}{2}\right)(V_1+V_2)^{-\frac{g}{2}-1}\left[d(I_1+I_2)^{d-1}I_1\right] \tag{D.15}$$

$$\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial M_1} = \frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial I_1} = -w\left[(I_1+I_2)^{2d} + 2d(I_1+I_2)^{2d-1}I_1\right](V_1+V_2)^{-g}bn$$
$$+ M_1^{-\frac{1}{2}}b\left[(I_1+I_2)^d + d(I_1+I_2)^{d-1}I_1\right]\sqrt{nw(V_1+V_2)^{-g}} \tag{D.16}$$

$$\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial M_2} = \frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial I_2} = -w\left[(I_1+I_2)^{2d} + 2d(I_1+I_2)^{2d-1}I_2\right](V_1+V_2)^{-g}bn$$
$$+ M_2^{-\frac{1}{2}}b\left[(I_1+I_2)^d + d(I_1+I_2)^{d-1}I_2\right]\sqrt{nw(V_1+V_2)^{-g}} \tag{D.17}$$

$$\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial M_1} = \frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial I_2} = -wI_1(I_1+I_2)^{2d-1}(2d)(V_1+V_2)^{-g}bn + M_2^{-\frac{1}{2}}bI_1(I_1+I_2)^{d-1}d\sqrt{nw(V_1+V_2)^{-g}} \tag{D.18}$$

$$\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial M_2} = \frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial I_1} = -wI_2(I_1+I_2)^{2d-1}(2d)(V_1+V_2)^{-g}bn + M_1^{-\frac{1}{2}}bI_2(I_1+I_2)^{d-1}d\sqrt{nw(V_1+V_2)^{-g}}. \tag{D.19}$$

Now, the minors of matrix are studied. To prove the concavity of this function, it is necessary that the minors must turn into negative and positive alternatively. The minors are shown in equations (D.20) to (D.25). It means that first, third and fifth minors must be negative while second, fourth and sixth minors must be positive. Establishment of these conditions for providing concavity of the profit function of cooperative approach are explained as follows:

$$|H|_1 = \frac{\partial^2 \sum \Omega_i}{\partial V_1^2} < 0 \tag{D.20}$$

$$|H|_2 = \frac{\partial^2 \sum \Omega_i}{\partial V_1^2}\frac{\partial^2 \sum \Omega_i}{\partial V_2^2} - \frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial V_2} > 0 \tag{D.21}$$

$$\begin{aligned}|H|_3 = {} & \frac{\partial^2 \sum \Omega_i}{\partial V_1^2}\frac{\partial^2 \sum \Omega_i}{\partial V_2^2}\frac{\partial^2 \sum \Omega_i}{\partial M_1^2} + 2\frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_1} \\ & - \left[\frac{\partial^2 \sum \Omega_i}{\partial V_1^2}\left(\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_1}\right)^2 + \left(\frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial V_2}\right)^2\frac{\partial^2 \sum \Omega_i}{\partial M_1^2} + \left(\frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_1}\right)^2\frac{\partial^2 \sum \Omega_i}{\partial V_2^2}\right] < 0\end{aligned} \tag{D.22}$$

$$\begin{aligned}|H|_4 = {} & \frac{\partial^2 \sum \Omega_i}{\partial V_1^2}\frac{\partial^2 \sum \Omega_i}{\partial V_2^2}\frac{\partial^2 \sum \Omega_i}{\partial M_1^2}\frac{\partial^2 \sum \Omega_i}{\partial M_2^2} + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_1} \\ & + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_2} + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial M_1} \\ & - \frac{\partial^2 \sum \Omega_i}{\partial V_1^2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial M_1^2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_2} - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial M_1} \\ & - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial V_2^2}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_2^2} - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_1} > 0\end{aligned} \tag{D.23}$$

$$\begin{aligned}|H|_5 = {} & \frac{\partial^2 \sum \Omega_i}{\partial V_1^2}\frac{\partial^2 \sum \Omega_i}{\partial V_2^2}\frac{\partial^2 \sum \Omega_i}{\partial M_1^2}\frac{\partial^2 \sum \Omega_i}{\partial M_2^2}\frac{\partial^2 \sum \Omega_i}{\partial I_1^2} + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial V_1} \\ & + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial V_2} + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial M_1} \\ & + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial M_2} - \frac{\partial^2 \sum \Omega_i}{\partial V_1^2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial V_2} \\ & - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial M_2^2}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial M_1} - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial V_2^2}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial M_2} \\ & - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial I_1^2} - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial M_1^2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial V_1} < 0\end{aligned} \tag{D.24}$$

$$\begin{aligned}|H|_6 = {} & \frac{\partial^2 \sum \Omega_i}{\partial V_1^2}\frac{\partial^2 \sum \Omega_i}{\partial V_2^2}\frac{\partial^2 \sum \Omega_i}{\partial M_1^2}\frac{\partial^2 \sum \Omega_i}{\partial M_2^2}\frac{\partial^2 \sum \Omega_i}{\partial I_1^2}\frac{\partial^2 \sum \Omega_i}{\partial I_2^2} + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial I_2}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial V_1} \\ & + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial I_2}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial V_2} + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial I_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial M_1} \\ & + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial I_2}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial M_2} + \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial I_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial I_1} \\ & - \frac{\partial^2 \sum \Omega_i}{\partial V_1^2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial I_2}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial M_2^2}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial V_2} - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial I_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial M_1} \\ & - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial V_2^2}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial I_2}\frac{\partial^2 \sum \Omega_i}{\partial I_1^2}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial M_2} - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial I_2}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial I_1} \\ & - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1^2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial V_1}\frac{\partial^2 \sum \Omega_i}{\partial I_2^2} - \frac{\partial^2 \sum \Omega_i}{\partial V_1 \partial I_2}\frac{\partial^2 \sum \Omega_i}{\partial V_2 \partial I_1}\frac{\partial^2 \sum \Omega_i}{\partial M_1 \partial M_2}\frac{\partial^2 \sum \Omega_i}{\partial M_2 \partial M_1}\frac{\partial^2 \sum \Omega_i}{\partial I_1 \partial V_2}\frac{\partial^2 \sum \Omega_i}{\partial I_2 \partial V_1} \\ & > 0.\end{aligned} \tag{D.25}$$

Appendix E. Derivation of equation (49) of cooperative approach

As shown in the Section 5.2.3, equation (47) is the first derivative with respect to inventory (I_i), for $i = 1, 2$. By substituting the optimal value of the first and second firm's lead time (Eq. (48)) then,

$$\begin{aligned}\frac{\partial \sum \Omega_i}{\partial I_i} &= \left(V_i - e_i - bn\frac{(V_i + V_{3-i})^g}{nw\left(I_i + I_{3-i}\right)^{2d}}\right)w(V_i + V_{3-i})^{-g}\left[(I_i + I_{3-i})^{2d} + 2d(I_i + I_{3-i})^{2d-1}I_i\right]\\ &+ \left(V_{3-i} - e_{3-i} - bn\frac{(V_i + V_{3-i})^g}{nw\left(I_i + I_{3-i}\right)^{2d}}\right)w(V_i + V_{3-i})^{-g}\left[2d(I_i + I_{3-i})^{2d-1}I_{3-i}\right]\\ &+ 2b\sqrt{\frac{\left(V_i + V_{3-i}\right)^g}{nw\left(I_i + I_{3-i}\right)^{2d}}nw(V_i + V_{3-i})^{-g}}\left[(I_i + I_{3-i})^d + d(I_i + I_{3-i})^{d-1}I_i\right]\\ &+ 2b\sqrt{\frac{\left(V_i + V_{3-i}\right)^g}{nw\left(I_i + I_{3-i}\right)^{2d}}nw(V_i + V_{3-i})^{-g}}\left[d(I_i + I_{3-i})^{d-1}I_{3-i}\right] - (b+h) = 0.\end{aligned} \tag{E.1}$$

After some simplification, the following expressions are obtained:

$$\begin{aligned}\frac{\partial \sum \Omega_i}{\partial I_i} &= (V_i - e_i)w(V_i + V_{3-i})^{-g}\left[(I_i + I_{3-i})^{2d} + 2d(I_i + I_{3-i})^{2d-1}I_i\right] - b\left[1 + 2d\frac{(I_i + I_{3-i})^{2d-1}I_i}{(I_i + I_{3-i})^{2d}}\right]\\ &+ (V_{3-i} - e_{3-i})w(V_i + V_{3-i})^{-g}\left[2d(I_i + I_{3-i})^{2d-1}I_{3-i}\right] - 2bd(I_i + I_{3-i})^{-1}I_{3-i}\\ &+ 2b\left[1 + \frac{d(I_i + I_{3-i})^{d-1}I_i}{(I_i + I_{3-i})^d}\right] + 2b\frac{d(I_i + I_{3-i})^{d-1}I_{3-i}}{(I_i + I_{3-i})^d} - (b+h) = 0\end{aligned} \tag{E.2}$$

$$\begin{aligned}\frac{\partial \sum \Omega_i}{\partial I_i} &= (V_i - e_i)w(V_i + V_{3-i})^{-g}\left[(I_i + I_{3-i})^{2d} + 2d(I_i + I_{3-i})^{2d-1}I_i\right]\\ &+ (V_{3-i} - e_{3-i})w(V_i + V_{3-i})^{-g}\left[2d(I_i + I_{3-i})^{2d-1}I_{3-i}\right] - h = 0.\end{aligned} \tag{E.3}$$

Equation (E.3) for i's firm when $i = 1, 2$ is expressed as follows:

$$\begin{aligned}h &= (V_1 - e_1)w(V_1 + V_2)^{-g}\left[(I_1 + I_2)^{2d} + 2d(I_1 + I_2)^{2d-1}I_1\right]\\ &\quad + (V_2 - e_2)w(V_1 + V_2)^{-g}\left[2d(I_1 + I_2)^{2d-1}I_2\right] \qquad \text{for } i = 1\end{aligned} \tag{E.4}$$

$$\begin{aligned}h &= (V_2 - e_2)w(V_2 + V_1)^{-g}\left[(I_1 + I_2)^{2d} + 2d(I_1 + I_2)^{2d-1}I_2\right]\\ &\quad + (V_1 - e_1)w(V_1 + V_2)^{-g}\left[2d(I_1 + I_2)^{2d-1}I_1\right] \qquad \text{for } i = 2.\end{aligned} \tag{E.5}$$

Both equations (E.4) and (E.5), are equal to h parameter. Thus, by subtraction both equation, (E.6) is obtained. By simplifying the mentioned expression as follows, the equation (49) is obtained.

$$\begin{aligned}&(V_1 - e_1)w(V_1 + V_2)^{-g}\left[(I_1 + I_2)^{2d} + 2d(I_1 + I_2)^{2d-1}I_1\right]\\ &\quad + (V_2 - e_2)w(V_1 + V_2)^{-g}\left[2d(I_1 + I_2)^{2d-1}I_2\right]\\ &\quad - (V_2 - e_2)w(V_2 + V_1)^{-g}\left[(I_1 + I_2)^{2d} + 2d(I_1 + I_2)^{2d-1}I_2\right]\\ &\quad - (V_1 - e_1)w(V_1 + V_2)^{-g}\left[2d(I_1 + I_2)^{2d-1}I_1\right] = 0.\end{aligned} \tag{E.6}$$

Simplifying,

$$(V_1 - e_1)w(V_1 + V_2)^{-g}(I_1 + I_2)^{2d} - (V_2 - e_2)w(V_1 + V_2)^{-g}(I_1 + I_2)^{2d} = 0. \tag{E.7}$$

Thus,

$$(V_1 - e_1) - (V_2 - e_2) = 0 \tag{E.8}$$
$$V_1 = V_2 - (e_2 - e_1). \tag{E.9}$$

In a general form,

$$V_i = V_{3-i} - (e_{3-i} - e_i) \ \text{ for } i = 1, 2 \tag{E.10}$$

Appendix F. Derivation of equation (53) of cooperative approach

As shown in the Section 5.2.3, equation (46) is the first derivative with respect to price (V_i) for $i = 1, 2$. By substituting the optimal value of the first and second firm's lead time (Eq. (48)), thus

$$\begin{aligned}
\frac{\partial \sum \Omega_i}{\partial V_i} &= wI_i(I_i + I_{3-i})^{2d} \left[\begin{array}{l} \left((V_i + V_{3-i})^{-g} + (-g)(V_i + V_{3-i})^{-g-1} V_i\right) \\ + \left(\left(-e_i - bn \frac{(V_i+V_{3-i})^g}{nw\left(I_i+I_{3-i}\right)^{2d}} \right) (-g)(V_i + V_{3-i})^{-g-1} \right) \end{array} \right] \\
&+ \left(V_{3-i} - e_{3-i} - bn \frac{(V_i + V_{3-i})^g}{nw\left(I_i + I_{3-i}\right)^{2d}} \right) wI_{3-i}(I_i + I_{3-i})^{2d}(-g)(V_i + V_{3-i})^{-g-1} \\
&- gb\sqrt{\frac{(V_i + V_{3-i})^g}{nw\left(I_i + I_{3-i}\right)^{2d}}}\, nwI_i(I_i + I_{3-i})^d (V_i + V_{3-i})^{-\frac{g}{2}-1} \\
&- gb\sqrt{\frac{(V_i + V_{3-i})^g}{nw\left(I_i + I_{3-i}\right)^{2d}}}\, nwI_{3-i}(I_i + I_{3-i})^d (V_i + V_{3-i})^{-\frac{g}{2}-1} = 0.
\end{aligned} \tag{F.1}$$

After some simplification, the following equations are obtained.

$$\begin{aligned}
&\left((V_i + V_{3-i})^{-g} + (-g)(V_i + V_{3-i})^{-g-1}V_i\right)wI_i(I_i + I_{3-i})^{2d} - e_i wI_i(I_i + I_{3-i})^{2d}(-g)(V_i + V_{3-i})^{-g-1} \\
&\quad + bg(V_i + V_{3-i})^{-1}I_i - wI_{3-i}(I_i + I_{3-i})^{2d} gV_{3-i}(V_i + V_{3-i})^{-g-1} - e_{3-i}wI_{3-i}(I_i + I_{3-i})^{2d}(-g)(V_i + V_{3-i})^{-g-1} \\
&\quad + bg(V_i + V_{3-i})^{-1}I_{3-i} - bg(V_i + V_{3-i})^{-1}I_i - bg(V_i + V_{3-i})^{-1}I_{3-i} = 0
\end{aligned} \tag{F.2}$$

$$\begin{aligned}
&(V_i + V_{3-i})^{-g}I_i - gV_i(V_i + V_{3-i})^{-g-1}I_i + e_i gI_i(V_i + V_{3-i})^{-g-1} \\
&\quad - I_{3-i}gV_{3-i}(V_i + V_{3-i})^{-g-1} + e_{3-i}gI_{3-i}(V_i + V_{3-i})^{-g-1} = 0
\end{aligned} \tag{F.3}$$

$$-(V_i + V_{3-i})^{-g}I_i = g(V_i + V_{3-i})^{-g-1}[-V_iI_i + e_iI_i - V_{3-i}I_{3-i} + e_{3-i}I_{3-i}] \tag{F.4}$$

$$I_i(V_i + V_{3-i} + g(e_i - V_i)) = -gI_{3-i}(e_{3-i} - V_{3-i}). \tag{F.5}$$

As previously mentioned, $I_i = I\varepsilon_i$ and $I_{3-i} = I\varepsilon_{3-1}$ so equation (F.5) is changed as (F.6) and equation (53) obtained as follows:

$$I\varepsilon_i(V_i + V_{3-i} + g(e_i - V_i)) = -gI\varepsilon_{3-i}(e_{3-i} - V_{3-i}) \tag{F.6}$$

$$\frac{\varepsilon_i}{\varepsilon_{3-i}} = \frac{-g(e_{3-i} - V_{3-i})}{(V_i + V_{3-i} + g(e_i - V_i))}\cdot \tag{F.7}$$