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STRATEGIC BI-DIRECTION SIGNAL SHARING WITH SUPPLIER COMPETITION IN A HYBRID-CHANNEL PLATFORM SUPPLY CHAIN

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Abstract. Due to data source and forecast competency differences, an increasing number of enterprises in hybrid platform supply chains implement forecast signal sharing to eliminate information asymmetry. Existing studies mostly examine vertical-sharing incentive of the platform retailer with suppliers, while neglecting horizontal sharing among suppliers. This paper explores the optimal bi-direction signal-sharing strategy including vertical sharing of platform retailer and horizontal sharing between two suppliers adopting reselling channel and agency channel respectively to improve supply chain performance. By analyzing the established Bayesian Stackelberg game models, the study finds that the two types of suppliers are always willing to implement horizontal sharing, while the vertical sharing strategy of platform retailers depends on channel competition and signal correlation. Specially, when channel competition intensity and signal correlations are sufficiently high, the platform retailer would like to share signal only with the supplier adopting reselling channel, which is different from previous studies. Moreover, it is found that horizontal sharing weakens the vertical-sharing incentive of the platform retailer and reduces the benefits of vertical sharing for two suppliers. This paper enriches the existing literature by combining the research on horizontal and vertical sharing within the context of hybrid-channel platform supply chains, and systematically clarifies the impacts of channel competition and signal correlation on the optimal strategy. Some managerial insights and practical schemes derived from findings may guide enterprises in making decisions on effective bi-direction information cooperation to improve supply chain performance.

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1. Introduction

With the prosperous development of online retailing, many platform retailers have provided hybrid channel including agency and reselling channels to sell products [1–3]. For competitive suppliers selling substitutable products on the online platform, some suppliers choose to adopt the agency channel, whereas some suppliers choose to adopt the reselling channel [4]. For example, on Amazon, Apple adopts the agency channel to sell iPhones, whereas Samsung distributes smartphones through the reselling channel[1]; on JD, HUVE adopts the

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[1] https://www.amazon.com/s?k=samsung; https://www.amz123.com/ask/2sUIUBY1, accessed on October 3, 2025.

agency channel to sell electronic products whereas Lenovo adopts the reselling channel[2]. The operational difference between the two channels is as follows: in the agency channel, suppliers sell products directly to consumers and pay a proportion of sales revenue (*i.e.*, a commission fee) to the platform retailer; in the reselling channel, the platform retailer first wholesales products from suppliers and then resells them to consumers [3, 5]. With the market competition becoming increasingly fierce, product competition combined with hybrid-channel competition between agency channel and reselling channel is becoming a mutual recognition, but how competition affects the decision-making process on substitutable products remains unclear.

On the other hand, demand uncertainty arising from intense market competition and complex consumer behavior poses a considerable challenge for supply chain members in effectively making strategic and operational decisions [6, 7]. With the rapid development of information technology, many platform retailers have conducted accurate analysis of potential demand by installing data collection infrastructures on customer browsing history and sales data, and some suppliers have also made substantial investments in data analysis tools and forecast algorithms to enhance their ability on forecasting potential demand [8, 9]. For example, JD has set up the Y Division (a unit focusing on building smart supply chain) utilizing data from product sales and customer feedback to obtain demand forecast information about consumer behavior and preferences [10]; Lenovo has implemented technology to predict consumer preferences[3]. However, demand forecast asymmetry exists among supply chain members due to inherent differences in data sources and forecast algorithms. In addition, the platform retailer, compared with suppliers, has a clear advantage in collecting consumer data derived from direct user interaction with the platform's websites [5, 7]. This asymmetry, coupled with market demand uncertainty, exacerbates the "information silo" phenomenon and ultimately affects the performance of the hybrid-channel platform supply chain[4].

Demand forecast sharing may be an effective collaborative initiative to tackle information asymmetry [11,12]. In practice, some suppliers selling substitutable products have shared consumer demand information among themselves. For example, Apple and Huawei, two well-known smartphone brands, have carried out information sharing and technical cooperation to jointly promote the development of smartphone industry[5]. Unilever and Procter & Gamble (P&G), two major daily necessities giants, have leveraged big data analytics to accurately predict demand in some emerging markets[6]. Fastener industry enterprises have shared data such as demand information to analyze customer needs accurately and control collaborative order delivery times, with collaborative order production efficiency averaging a 10% increase, labor costs falling by 10%, and on-time delivery rates rising from 90% to 95%[7]. Meanwhile, the platform retailer has also conducted cooperation with some suppliers by establishing demand forecast-sharing mechanism. For instance, Amazon shared demand information with the suppliers adopting agency channel through its Brand Analytics [13]. JD has shared real-time sales forecasts of maternal and infant products with P&G to reduce the return rate and minimize JD's losses[8]. In an announcement of JD, the sales of Huggies on JD increased by 60% following the application of JD's data analytics information [14]. A leafy vegetable purchaser increased its procurement accuracy rate from 60% to 90% and reduced losses by 15% based on terminal sales data provided by the platform retailer[9].

Within the hybrid-channel supply chain with supplier competition, the mechanisms of demand signal sharing require suppliers and the platform retailer to assess the benefits and potential drawbacks of sharing. For competitive suppliers, driven by market expansion and product optimization, they are willing to share demand

[2] `https://mall.jd.com/index-11749107.html?from=pc`; `https://mall.jd.com/index-1000000157.html?from=pc`, accessed on December 1, 2024.

[3] `http://ex.chinadaily.com.cn/exchange/partners/82/rss/channel/cn/columns/6ldgif/stories/WS5e240e2ba3107bb6b579acfa.html`, accessed on December 1, 2024.

[4] `https://www.sohu.com/a/898695323_120004713`, accessed on October 6, 2025.

[5] `https://max.book118.com/html/2025/0422/7043132140010063.shtm`, accessed on October 5, 2025.

[6] `https://bk.taobao.com/k/shenghuo_241/d8aaa44dc790068f241d88cb58ffcef7.html`, accessed on October 3, 2025.

[7] `https://www.toutiao.com/article/7540850243198272054/?upstream_biz=doubao&source=m_redirect`, accessed on October 25, 2025.

[8] `https://www.renrendoc.com/paper/336564050.html`, accessed on October 5, 2025.

[9] `https://m.sohu.com/a/942228977_100041230/`, accessed on October 25, 2025.

information, but they may also be concerned about potential core-technology leakage or competitors getting a head start. For the platform retailer offering both reselling channel and agency channel, it exists co-operative relationships with suppliers, which affect the incentive of demand forecast sharing [15]. Specifically, the platform retailer may be reluctant to share information with the supplier adopting reselling channel due to the double marginalization effect, which can be exacerbated by the sharing forecast [16, 17]. In contrast, while the revenue-sharing mechanism gives the platform retailer an incentive to share information with the supplier adopting agency channel, it still needs to consider the competitive relationship with this supplier, as such collaboration involves hybrid-channel competition [4]. Previous studies have explored vertical-sharing incentive of platform retailer in some specific situations (here, vertical sharing refers to sharing among supply chain members between upstream and downstream echelons [9,18]), but few studies have also paid attention to horizontal sharing between suppliers (here, horizontal sharing refers to sharing among competitors in the same echelon of supply chains [9,18]). In the context of a hybrid-channel platform supply chain with supplier competition, cooperative and competitive relationships between enterprises complicate the integration and control of information resources. With information asymmetry among supply chain members, gaining insights into how to strategically conduct bi-direction forecast sharing including horizontal sharing and vertical sharing for practical benefits is a crucial issue.

Motivated by above observations and business practices, this study focuses on a hybrid-channel platform supply chain where two competitive suppliers sell substitutable products through agency channel and reselling channel, respectively. The platform retailer and two suppliers possess their own demand forecast signal. This study aims to address the following questions.

(i) What is the optimal bi-direction signal-sharing strategy in the hybrid-channel platform supply chain? Specifically, what is the optimal vertical-sharing strategy of the platform retailer and what is the optimal horizontal sharing between two suppliers?
(ii) What is the impact of horizontal sharing on the optimal vertical-sharing strategy of the platform retailer?
(iii) How do hybrid-channel competition intensity and signal correlations between the platform retailer and suppliers affect the values of vertical sharing and horizontal sharing?

To tackle the above questions, the Bayesian Stackelberg game model is established in a hybrid-channel platform supply chain, which integrates the leader-follower hierarchy of Stackelberg game and Bayesian method to update information using prior and observed data [19, 20]. In business practice, some powerful suppliers (*e.g.*, Microsoft and Intel) maintain a more dominant position than platform retailers in supply chains [21]. Hence, this paper assumes that two suppliers act as leaders in Stackelberg game, with the platform retailer making decisions as a follower. At the strategic decision-making level, suppliers first formulate horizontal-sharing strategy, followed by the platform retailer determining vertical-sharing strategy. At the operational decision-making level, we similarly assume that suppliers first make pricing decisions, followed by the platform retailer. In most relevant studies, Bayesian method is applied to address information asymmetry among supply chain members by updating information with prior and observed data [10, 22, 23]. Therefore, compared with other methods for solving multi-player decision-making problems, it is appropriate for this paper to establish Bayesian Stackelberg game model to explore the optimal bi-direction signal-sharing strategy between powerful suppliers and the platform retailer in a hybrid-channel platform supply chain.

Through analyzing equilibrium solutions, we obtain the following results. First, the optimal bi-direction signal-sharing strategy is that two suppliers always engage in horizontal sharing whereas the platform retailer's vertical-sharing strategy depends on channel competition intensity and correlation coefficients. With horizontal sharing, the platform retailer may share signal with the two suppliers separately or not share signal with either of them. Without horizontal sharing, the platform retailer always shares signal with the supplier adopting agency channel. Second, through comparison with the optimal vertical-sharing strategy without and with horizontal sharing, it is found that suppliers' horizontal sharing weakens the vertical-sharing incentive of the platform retailer and reduces the benefits from vertical sharing for two suppliers and the whole supply chain. Third, numerical analysis reveals that as hybrid-channel competition intensifies and signal correlation coefficients decrease, the values of the optimal vertical sharing for the whole supply chain and its members increase. The more intense the hybrid-channel competition, the higher the values of horizontal sharing for two suppliers.

The main contributions of this paper are threefold. First, previous literature on bi-direction demand signal sharing has been conducted in reselling channel supply chains, *e.g.*, Luo *et al.* [24] and Qiu *et al.* [25]. Given the prevalence of hybrid channel in platform supply chains, hybrid-channel competition makes the decision-making process related to signal sharing more complicated. We find that the hybrid-channel competition intensity affects the optimal bi-direction signal-sharing strategy, and the values of signal sharing sharply change when hybrid-channel competition intensity is higher. Second, studies on signal sharing in platform supply chains have primarily focused on the vertical-sharing incentive of the platform retailer with supplier(s), *e.g.*, Yang *et al.* [26] and Liang and Ye [10]. To the best of our knowledge, this paper is the first to study both vertical sharing and horizontal sharing in a hybrid-channel platform supply chain, considering that all supply chain members possess demand forecast signals. This research explores the impacts of signal correlation and hybrid-channel competition on the optimal bi-direction signal-sharing strategy, and reveals the influence of horizontal sharing on vertical-sharing incentive of the platform retailer, which enriches demand signal-sharing theory. Finally, this study provides managerial implications for the platform retailer and suppliers on how to strategically implement demand signal-sharing strategy and decide equilibrium prices under different scenarios. In summary, this paper not only fills the research gap on demand signal-sharing in platform supply chains, but also provides practical guidance for enterprises to make strategic and operational decisions in the face of uncertain market environments.

The remainder of this paper is organized as follows. Section reviews the relevant literature. Section 3 presents model description and assumption. In Section 4, we derive the equilibrium solutions and explore the effects of signal sharing on equilibrium prices. Section 5 analyses the optimal bi-direction signal-sharing strategy. In Section 6, this study explores the impacts of hybrid-channel competition intensity and correlation coefficients on the values of signal sharing. Finally, conclusions are summarized in Section 7. Parameters in tables and propositions are presented in Appendix A, and all proofs of the lemma and propositions are presented in Appendix B.

2. Literature review

Our paper is related to the following two research streams: supplier competition and demand signal sharing.

2.1. Supplier competition

In recent decades, scholars have examined strategic and operational decisions with supplier competition under different sales channels. The reselling channel was popular in offline retailing, and early scholars examined operational decisions in a reselling channel supply chain consisting of multiple suppliers and one retailer, such as optimal pricing strategy [21, 27, 28], product and service quality decisions [29], cooperation effort degrees [30]. The agency channel has become widespread with the development of online retail platforms, and some scholars examined strategic decisions in an agency channel supply chain with multiple suppliers and a platform retailer, such as product bundling strategies [31], direct sales establishment of manufacturers [32], cooperative promotion strategies [33] and blockchain adoption [34]. In addition, some scholars investigated channel selection with supplier competition between reselling channel and agency channel. For example, Hagiu and Wright [35] investigated the impact of product types and channel cost on the platform retailer's channel selection, and Kwark *et al.* [36] investigated the impact of reviews information on platform retailer' channel preference. The above studies assumed all suppliers adopt the same sales channel (reselling or agency), and did not consider channel competition between reselling channel and agency channel.

In recent years, some studies have further extended investigations to the setting in which reselling and agency channels co-exist in a platform supply chain and competitive suppliers may adopt hybrid channel to sell products. For example, Tian *et al.* [37] studied the platform retailer's channel selection considering two competing suppliers adopting a pure reselling channel, a pure agency channel and a hybrid channel. They showed that the hybrid channel was preferred when competition and order-fulfillment costs were moderate.

Zennyo [38] expanded Tian *et al.* [37] to two asymmetric suppliers with different potential demands and studied suppliers' channel selection. The results showed that when the degree of product substitution was intermediate, hybrid channel arose in equilibrium where the supplier with smaller demand adopted agency channel. Matsui [39] studied two competing suppliers' channel selection with direct channel and the results showed that equilibrium distribution channel was asymmetric between suppliers. Yan *et al.* [40] explored the contract selection strategies for an incumbent supplier, a potential supplier and a retailer among long-term wholesale contract, short-term wholesale contract and agency contract. The above studies were all in deterministic demand environments, and did not consider information asymmetry between supply chain members. Different from them, this study focuses on demand signal sharing in the presence of market demand uncertainty and examines the impacts of hybrid-channel competition on the bi-direction signal-sharing strategy.

2.2. Demand signal sharing

Our research pertains to the studies on the incentives of demand signal sharing, which have gained much attention in previous literature. Early scholars studied horizontal sharing in an oligopoly market, such as Vives [41], Gal-Or [42], Li [43], Shapiro [44] and Raith [45]. They demonstrated that horizontal sharing improves the precision accuracy of market conditions due to pooled information. Some scholars examined the incentives of horizontal sharing among downstream firms in a reselling channel supply chain. For example, Shamir and Shin [46] studied the incentives to exchange private forecast signals for a group of retailers sourcing the product from a single manufacturer. Wu *et al.* [47] showed that downstream manufacturers' horizontal-sharing strategy can be affected by the upstream supplier's pricing decision. Afterward, extensive literature examined the vertical-sharing incentives of downstream retailer(s) with upstream supplier(s) in reselling channel supply chains, such as Li [48], Li and Zhang [49], Ha *et al.* [50], Shang *et al.* [51], Shamir and Shin [52], Huang *et al.* [53], Zhang *et al.* [9], Yan *et al.* [54], Zhao *et al.* [55], Xu *et al.* [56] etc. They demonstrated that the retailer has no incentive to share signal due to the exacerbated double marginalization effect in the reselling channel. These studies only examined unidirectional (*i.e.*, horizontal or vertical) sharing strategy.

A few studies examined bi-direction signal sharing including vertical-sharing and horizontal-sharing strategies in a reselling channel supply chain. Luo *et al.* [24] and Qiu *et al.* [25] assumed that only retailers have access to demand signals, where Qiu *et al.* [25] considered market competition in price and in quantity scenarios. Lei *et al.* [57] considered a dual-channel supply chain consisting of a manufacturer and multiple retailers in which all firms possess demand signals. These studies examined the bi-direction signal-sharing incentives from the perspective of downstream retailers. Hao and Jiang [8] examined vertical-sharing strategy of retailers and horizontal-sharing strategy of suppliers in reselling channel supply chains. They showed that two suppliers engage in horizontal sharing voluntarily, but the retailer is reluctant to share signal with suppliers. Dissimilar to the above studies in reselling channel supply chains, this paper focus on channel competition between reselling channel and agency channel in a hybrid-channel platform supply chain, and highlight the influence of forecast signal correlations between the platform retailer and suppliers on the bi-direction signal-sharing strategy.

Recently, some scholars have investigated the vertical-sharing incentive of the platform retailer offering reselling and agency channels in different supply chain structures, involving one-to-one supply chain [2,26,58,59], downstream competition [13,60] and upstream competition [3,10,16,23,61]. For example, Liang and Ye [10] studied the platform retailer's channel selection and signal-sharing strategies, and found that the platform retailer is willing to share signal with the supplier adopting agency channel if commission rate is large and competition is weak in a hybrid-channel supply chain. In these studies, Bayesian Stackelberg game model was established, the equilibrium solution was obtained by using backward induction, and further the optimal vertical-sharing strategy of the platform retailer was derived through comparative analysis. Chen *et al.* [3] established a Stackelberg game model involving a platform retailer and two risk-averse suppliers (one adopting reselling channel and the other adopting agency channel) and studied the platform retailer's vertical-sharing incentives by applying the mean-variance theory. The above literature only considered the vertical-sharing incentive of the platform retailer owning demand signal. Different from them, we assume that all supply chain members possess demand forecast signals in a hybrid-channel supply chain with supplier competition and examine bi-direction

TABLE 1. Comparison with the most related literature.

Literature	Channel R	Channel A	Channel H	Supplier competition	Signal sharing HS	Signal sharing VS	Research problem
Wu *et al.* [47]	✓				✓		Explore horizontal-sharing strategy of two downstream manufacturers with asymmetric capacity constraint in a two-echelon supply chain.
Shamir and Shin [46]	✓				✓		Study the incentives of a group of retailers on exchanging forecast signal under strategic wholesale price set by a manufacturer.
Luo *et al.* [24]	✓				✓	✓	Investigate horizontal-sharing and vertical-sharing strategies of two retailers who order the same supplier in common sourcing but from different suppliers in independent sourcing.
Hao and Jiang [8]	✓			✓	✓	✓	Study horizontal sharing (between horizontal competitors) and vertical sharing (between vertical partners) under bilateral monopoly, retailer competition and supplier competition supply chain considering direct selling role.
Qiu *et al.* [25]	✓				✓	✓	Establish price and quantity models to explore two retailers' strategies about horizontal sharing with each other and vertical sharing with a supplier under competition or cooperative decision regime.
Wang *et al.* [7]	✓			✓		✓	Study which types of suppliers varying product quality should be shared demand information by the retail platform with risk aversion applying mean-variance theory.
Lin *et al.* [16]	✓	✓		✓		✓	Explore platform's signal-sharing strategy considering two manufacturers' eco-friendly efforts in the reselling or agency selling.
Chen *et al.* [3]			✓	✓		✓	Investigate the risk-neutral platforms signal-sharing strategy with two risk-averse suppliers adopting reselling selling and agency selling, respectively.
Liu and Wang [23]	✓	✓	✓	✓		✓	Study signal-sharing strategy for three types of platforms (a pure reselling platform, a pure commission platform, and a hybrid platform) with two competing manufacturers.
This paper			✓	✓	✓	✓	Explore two competing suppliers' horizontal-sharing and a platform's vertical-sharing strategies in a hybrid-channel supply chain considering signal correlation.

Notes. R, A and H represent the reselling channel, agency channel and hybrid channel, respectively. HS and VS represent horizontal sharing and vertical sharing, respectively.

signal-sharing strategy including vertical sharing of the platform retailer and horizontal sharing between two suppliers.

2.3. Research gap

While existing literature contributes to demand signal sharing, three aspects remain insufficiently explored.

- Regarding the direction of demand signal sharing, existing studies mainly focus on unidirectional sharing (either horizontal among competitors or vertical between upstream and downstream members), and rarely consider bi-directional sharing.
- The limited literature concentrating on bi-direction signal sharing is based on reselling-channel supply chains, and it mainly considers competition among downstream retailers rather than upstream suppliers.
- Regarding the members with demand signal, most literature assumes that only downstream retailer or platform holds demand signal, while ignoring the situation that upstream suppliers also have the capacity to forecast market demand.

In summary, existing literature has not examined bi-direction signal sharing in the hybrid-channel platform supply chain with supplier competition, and most of studies overlook the fact that suppliers also possess demand forecasting signals. To fill this gap, this paper explores bi-direction signal-sharing strategy including vertical sharing between the platform retailer and suppliers, as well as horizontal sharing between two competing suppliers in a hybrid-channel supply chain.

Table 1 presents a comparison between this paper and the most closely related literature.

TABLE 2. Summary of notations.

Notation	Description
Index	
i	Subscript, index of product i or supplier S_i, $i = 1, 2$
j	Subscript, index of firm, $j = 1, 2, p$
Y	Superscript, index of horizontal-sharing strategy, $Y \in \{N, I\}$
(v_1, v_2)	Superscript, index of vertical-sharing strategy, $v_i \in \{0, 1\}$
Parameters	
a	Potential market demand
α	Commission rate
θ	Uncertainty in market potential
f_j	Exclusive signal received by firm j
ε_j	Error in firm j's signal, $f_j = \theta + \varepsilon_j$
σ^2	Variance of market uncertainty
σ_j^2	Variance of firm j's signal error
r	Competitive intensity of hybrid channel
Θ_j	Signal set to firm j
$\hat{\theta}_{\Theta_j}$	Conditional expectation for market under Θ_j
$\hat{\theta}_{j_1 \mid \Theta_{j_2}}$	Conditional expectation for signal f_{j_1} under Θ_{j_2}, $j_1, j_2 = 1, 2, p$
$\pi_j^{Y(v_1,v_2)}$	Ex ante profit for firm j under strategy $Y(v_1, v_2)$
$V_j^{Y(v_1,v_2)}$	Value of vertical sharing for firm j under strategy $Y(v_1, v_2)$
$V^{Y(v_1,v_2)}$	Value of vertical sharing for the whole supply chain under strategy $Y(v_1, v_2)$
H_i	Value of horizontal sharing for supplier S_i
Decision variables	
$w_2^{Y(v_1,v_2)}$	Wholesale price for product 2 under strategy $Y(v_1, v_2)$
$p_i^{Y(v_1,v_2)}$	Retail price for product i under strategy $Y(v_1, v_2)$

3. Problem description and assumptions

Consider a hybrid-channel platform supply chain with two competing suppliers S_1 and S_2 and one platform retailer P (she) providing agency and reselling channels. Supplier S_1 (he) sells product 1 through agency channel and supplier S_2 (he) sells product 2 through reselling channel. Products 1 and 2 are substitutable. Under the agency channel, supplier S_1 determines retail price p_1 and shares revenue with the platform retailer at a commission rate α. Under the reselling channel, the platform retailer wholesales product 2 at a wholesale price w_2 determined by supplier S_2 and sells it to consumers at a retail price p_2. Table 2 shows summary of all notations in this paper.

Based on the massive data collected from online consumers, supplier S_1, supplier S_2 and the platform retailer can forecast market demand, and their forecast signals are denoted by f_1, f_2 and f_p, respectively. In line with industry practices, this research considers whether two suppliers exchange their forecast signals with each other, *i.e.*, implement horizontal signal-sharing strategy Y ($Y \in \{N, I\}$), hereinafter referred to as horizontal sharing, where $Y = N$ indicates no horizontal sharing and $Y = I$ indicates horizontal sharing. The platform retailer decide her vertical signal-sharing strategy (v_1, v_2) ($v_i \in \{0, 1\}$) with two suppliers, hereinafter referred to as

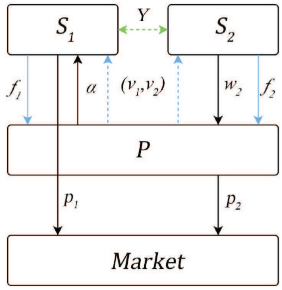


FIGURE 1. Possible signal-sharing strategies under hybrid channel.

vertical sharing, where $v_i = 0$ indicates no vertical sharing with S_i and $v_i = 1$ indicates the platform retailer shares signal with S_i. Figure 1 illustrates possible signal-sharing strategies under hybrid channel.

To make the analysis tractable, this research introduces the following assumptions regarding commission rate, cost, demand function, forecast signal and signal set.

Commission rate and cost: The commission rate α is assumed to be exogenous because it is fixed for products of the same category, such as 5% for sporting goods and 6% for luxury items charged by JD [62] and 15% for apparel and skincare products charged by Amazon [63]. Without loss of generality, production costs and other operating costs are normalized to zero [37].

Demand function: Following the marketing and operational management literature (*e.g.*, [41,64]), the demand function of product i ($i = 1, 2$) is formulated in equation (1).

$$d_i = a - p_i + rp_{3-i}, \tag{1}$$

where a denotes the potential demand, p_i and p_{3-i} denote the retail prices of product i and p_{3-i}, respectively. Parameter r $(0 < r < 1)$ is cross-price sensitivity, which represents the competitive intensity of hybrid channel. To represent the demand uncertainty, this study assumes that $a = a_0 + \theta$, where a_0 is a positive constant and θ is normally distributed with mean zero and variance σ^2.

Forecast signal: Relying on product sales data, the platform retailer can observe a forecast signal $f_p = \theta + \varepsilon_p$, where forecast error $\varepsilon_p \sim N(0, \sigma_p^2)$ and is independent of θ. Similarly, supplier S_i $(i = 1, 2)$ can also observe a forecast signal $f_i = \theta + \varepsilon_i$, where forecast error $\varepsilon_i \sim N(0, \sigma_i^2)$ and is independent of θ. Forecast variance σ_j^2 represents forecast accuracy of signal f_j $(j = 1, 2, p)$. Note that higher (lower) variance means less (more) accurate demand forecast signal. We assume that the forecast accuracy of platform retailer is higher than those of two suppliers due to her superiority in gathering and analyzing rich data of online consumers, *i.e.*, $\sigma_p^2 < \min\{\sigma_1^2, \sigma_2^2\}$ [8]. In addition, ε_1 and ε_2 are assumed to be independent, *i.e.*, $\mathrm{Cov}(\varepsilon_1, \varepsilon_2) = 0$, which is because two suppliers have different consumer data sources from different sales channels [22]. Forecast errors ε_i and ε_p may be correlated and the correlation coefficient is ρ_i $(i = 1, 2)$, which is because supplier S_i sells the product through the online platform, resulting in similar data source [6]. Following Mishra *et al.* [64] and Wei *et al.* [18], the covariance is not greater than individual variances, *i.e.*, $\rho_1\sigma_1\sigma_p < \min\{\sigma_1^2, \sigma_p^2\}$ and $\rho_2\sigma_2\sigma_p < \min\{\sigma_2^2, \sigma_p^2\}$.

Signal set: Suppose Θ_i and Θ_p are sets of demand signals received by supplier S_i and the platform retailer, respectively. This research considers that two suppliers and the platform retailer play Stackelberg game. Two suppliers as the leaders first make their pricing decisions and then the platform retailer as the follower makes her pricing decision. Following Lei *et al.* [57] and Hao and Jiang [8], no matter which signal-sharing scenario, the platform retailer can infer two suppliers' demand signals from their optimal pricing strategies, thus $\Theta_p =$

TABLE 3. The signal sets under possible signal-sharing strategies.

Y	(v_1, v_2)	Θ_1	Θ_2	Θ_p
N	$(0,0)$	$\{f_1\}$	$\{f_2\}$	$\{f_1, f_2, f_p\}$
	$(1,0)$	$\{f_1, f_p\}$	$\{f_2\}$	
	$(0,1)$	$\{f_1\}$	$\{f_2, f_p\}$	
	$(1,1)$	$\{f_1, f_p\}$	$\{f_2, f_p\}$	
I	$(0,0)$	$\{f_1, f_2\}$	$\{f_1, f_2\}$	
	$(1,0)$	$\{f_1, f_2, f_p\}$	$\{f_1, f_2\}$	
	$(0,1)$	$\{f_1, f_2\}$	$\{f_1, f_2, f_p\}$	
	$(1,1)$	$\{f_1, f_2, f_p\}$	$\{f_1, f_2, f_p\}$	

TABLE 4. Conditional expectations for market condition.

Θ	$\{f_i\}$	$\{f_p\}$	$\{f_i, f_p\}$	$\{f_1, f_2\}$	$\{f_1, f_2, f_p\}$
$\hat{\theta}_\Theta = E[\theta\|\Theta]$	$T_i f_i$	$T_p f_p$	$G_i f_i + \tilde{G}_i f_p$	$L_1 f_1 + L_2 f_2$	$K_1 f_1 + K_2 f_2 + K_3 f_p$

Notes. Where $T_i = \frac{\sigma^2}{\sigma^2+\sigma_i^2}$, $T_p = \frac{\sigma^2}{\sigma^2+\sigma_p^2}$, $G_i = \frac{\sigma^2(\sigma_p^2-\rho_i\sigma_i\sigma_p)}{\sigma_i^2\sigma_p^2(1-\rho_i^2)+\sigma^2(\sigma_i^2+\sigma_p^2-2\rho_i\sigma_i\sigma_p)}$, $\tilde{G}_i = \frac{\sigma^2(\sigma_i^2-\rho_i\sigma_i\sigma_p)}{\sigma_i^2\sigma_p^2(1-\rho_i^2)+\sigma^2(\sigma_i^2+\sigma_p^2-2\rho_i\sigma_i\sigma_p)}$, $L_i = \frac{\sigma^2\sigma_{3-i}^2}{\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2}$, $K_1 = \frac{\sigma^2\sigma_2\sigma_p(\sigma_2\sigma_p(1-\rho_2^2)+\rho_1\sigma_1(\rho_2\sigma_p-\sigma_2))}{k}$, $K_2 = \frac{\sigma^2\sigma_1\sigma_p(\sigma_1\sigma_p(1-\rho_1^2)+\rho_2\sigma_2(\rho_1\sigma_p-\sigma_1))}{k}$, $K_3 = \frac{\sigma^2\sigma_1\sigma_2(\sigma_1\sigma_2-\sigma_p(\rho_1\sigma_2+\rho_2\sigma_1))}{k}$, $k = \sigma_1^2\sigma_2^2\sigma_p^2[(1-\sum_{i=1}^2\rho_i^2)+\sigma^2(\frac{1}{\sigma_p^2}-(\frac{\rho_1}{\sigma_2}-\frac{\rho_2}{\sigma_1})^2+\sum_{i=1}^2\frac{1}{\sigma_i^2}(1-\frac{2\rho_i\sigma_i}{\sigma_p}))]$.

TABLE 5. Conditional expectations for the shared demand signal.

Θ	$\{f_i\}$		$\{f_p\}$	$\{f_i, f_p\}$	$\{f_1, f_2\}$
f	f_{3-i}	f_p	f_i	f_{3-i}	f_p
$\hat{\theta}_{f\|\Theta} = E[f\|\Theta]$	$T_i f_i$	$U_i f_i$	$\tilde{U}_i f_p$	$J_i f_i + \tilde{J}_i f_p$	$\tilde{L}_1 f_1 + \tilde{L}_2 f_2$

Notes. Where $T_i = \frac{\sigma^2}{\sigma^2+\sigma_i^2}$, $U_i = \frac{\sigma^2+\rho_i\sigma_i\sigma_p}{\sigma^2+\sigma_i^2}$, $\tilde{U}_i = \frac{\sigma^2+\rho_i\sigma_i\sigma_p}{\sigma^2+\sigma_p^2}$, $J_i = \frac{\sigma^2(\sigma_p^2-\sigma_p(\rho_1\sigma_1+\rho_2\sigma_2))-\rho_1\rho_2\sigma_1\sigma_2\sigma_p^2}{\sigma_i^2\sigma_p^2(1-\rho_i^2)+\sigma^2(\sigma_i^2+\sigma_p^2-2\rho_i\sigma_i\sigma_p)}$, $\tilde{J}_i = \frac{\sigma^2(\sigma_i^2+\sigma_p(\rho_{3-i}\sigma_{3-i}-\rho_i\sigma_i))+\rho_{3-i}\sigma_i^2\sigma_{3-i}\sigma_p}{\sigma_i^2\sigma_p^2(1-\rho_i^2)+\sigma^2(\sigma_i^2+\sigma_p^2-2\rho_i\sigma_i\sigma_p)}$, $\tilde{L}_i = \frac{\sigma^2(\sigma_{3-i}^2+\sigma_p(\rho_i\sigma_i-\rho_{3-i}\sigma_{3-i}))+\rho_i\sigma_i\sigma_{3-i}^2\sigma_p}{\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2}$.

$\{f_1, f_2, f_p\}$. The signal sets of two suppliers and platform retailer under different signal-sharing strategies are shown in Table 3.

The firm relies on available demand signal(s) to update the forecast for market condition and the understanding of demand signals shared by other firms [8]. The corresponding outcomes are given in Lemma 1.

Lemma 1. (a) *The conditional expectations for market condition given the demand signal set* Θ *are in Table 4:* (b) *The conditional expectations for the shared demand signal given the demand signal set* Θ *are in Table 5:*

The proof of Lemma 1 is detailed in Appendix B.

Sequence of events: According to Hao and Jiang [8] and Qiu *et al.* [25], the selection of signal-sharing strategy is a long-term decision and horizontal sharing between suppliers precedes vertical sharing of platform retailer. The outcomes in the sequence where vertical sharing precedes horizontal sharing are similar, and the detailed proof about this sequencing issue refers to Appendix B. In addition, we assume that the supplier is the leader in Stackelberg game and makes the pricing decision first. In practice, powerful suppliers, such as Microsoft and Intel, play a more dominant role than downstream members in some chains and thus make pricing decisions first [21]. The sequence of events is as follows.

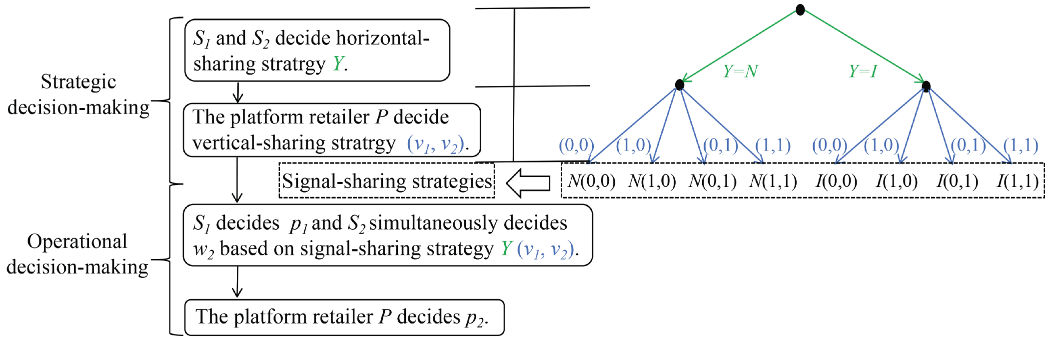


FIGURE 2. Sequence of events.

- The suppliers decide whether or not to share demand signal with each other, *i.e.*, horizontal-sharing strategy Y, where $Y \in \{N, I\}$.
- The platform retailer makes vertical-sharing strategy (v_1, v_2) to decide whether or not to share her demand signal with two suppliers, where $v_i \in \{0, 1\}$.
- Based on signal-sharing strategy $Y(v_1, v_2)$, supplier S_1 determines the retail price p_1 to maximize his conditional expected profit as formulated in equation (2), and simultaneously supplier S_2 sets the wholesale price w_2 to maximize his conditional expected profit as formulated in equation (3).

$$E[\pi_1|\Theta_1] = E[(1-\alpha)p_1 d_1|\Theta_1], \tag{2}$$
$$E[\pi_2|\Theta_2] = E[w_2 d_2|\Theta_2], \tag{3}$$

  where the signal set Θ_i is in Table 3 and the demand d_i is given in equation (1) $(i = 1, 2)$.
- The platform retailer decides the retail price p_2 based on the signal set $\Theta_p = \{f_1, f_2, f_p\}$ to maximize her conditional expected profit as formulated in equation (4).

$$E[\pi_p|\Theta_p] = E[\alpha p_1 d_1 + (p_2 - w_2)d_2|\Theta_p]. \tag{4}$$

A visual figure about sequence of events is given below (Fig. 2).

4. Equilibrium analysis

In this section, the study derives the equilibrium prices and ex ante profits under various signal-sharing strategies, and further explores the impacts of vertical and horizontal sharing on the equilibrium prices.

4.1. Equilibrium solutions

Using backward induction method, we solve the Bayesian Stackelberg game models under signal-sharing strategy $Y(v_1, v_2)$ where $Y \in \{N, I\}$ and $v_i \in \{0, 1\}$. Given p_1 and w_2 determined respectively by two suppliers, the best response function of the platform retailer can be obtained by maximizing the expected profit $E[\pi_p|\Theta_p]$, which is presented in equation (5).

$$p_2^{Y(v_1,v_2)}(p_1, w_2) = \frac{(1+\alpha)r}{2}p_1 + \frac{1}{2}w_2 + \frac{a_0 + E[\theta|\Theta_p]}{2}. \tag{5}$$

Substituting equation (5) into equations (2) and (3), two suppliers respectively determine the retail price p_1 and wholesale price w_2 to maximize their own expected profits $E[\pi_1|\Theta_1]$ and $E[\pi_2|\Theta_2]$. The signal sets Θ_1 and

TABLE 6. Equilibrium prices under different signal-sharing strategies.

(v_1, v_2)	No horizontal sharing (N)	Horizontal sharing (I)
$(0,0)$	$p_1^{N(0,0)} = B_1(a_0 + \hat{\theta}_1)$ $w_2^{N(0,0)} = B_2(a_0 + \hat{\theta}_2)$ $p_2^{N(0,0)} = B_3 a_0 + \frac{1}{2}(\hat{\theta}_{12p} + (1+\alpha)rB_1\hat{\theta}_1 + B_2\hat{\theta}_2)$	$p_1^{I(0,0)} = B_1(a_0 + \hat{\theta}_{12})$ $w_2^{I(0,0)} = B_2(a_0 + \hat{\theta}_{12})$ $p_2^{I(0,0)} = B_3 a_0 + \frac{1}{2}(\hat{\theta}_{12p} + ((1+\alpha)rB_1 + B_2)\hat{\theta}_{12})$
$(1,0)$	$p_1^{N(1,0)} = B_1(a_0 + \hat{\theta}_{1p})$ $w_2^{N(1,0)} = B_2(a_0 + \hat{\theta}_2)$ $p_2^{N(1,0)} = B_3 a_0 + \frac{1}{2}(\hat{\theta}_{12p} + (1+\alpha)rB_1\hat{\theta}_{1p} + B_2\hat{\theta}_2)$	$p_1^{I(1,0)} = B_1(a_0 + \hat{\theta}_{12p})$ $w_2^{I(1,0)} = B_2(a_0 + \hat{\theta}_{12})$ $p_2^{I(1,0)} = B_3 a_0 + \frac{1}{2}((1 + (1+\alpha)rB_1)\hat{\theta}_{12p} + B_2\hat{\theta}_{12})$
$(0,1)$	$p_1^{N(0,1)} = B_1(a_0 + \hat{\theta}_1)$ $w_2^{N(0,1)} = B_2(a_0 + \hat{\theta}_{2p})$ $p_2^{N(0,1)} = B_3 a_0 + \frac{1}{2}(\hat{\theta}_{12p} + (1+\alpha)rB_1\hat{\theta}_1 + B_2\hat{\theta}_{2p})$	$p_1^{I(0,1)} = B_1(a_0 + \hat{\theta}_{12})$ $w_2^{I(0,1)} = B_2(a_0 + \hat{\theta}_{12p})$ $p_2^{I(0,1)} = B_3 a_0 + \frac{1}{2}((1 + B_2)\hat{\theta}_{12p} + (1+\alpha)rB_1\hat{\theta}_{12})$
$(1,1)$	$p_1^{N(1,1)} = B_1(a_0 + \hat{\theta}_{1p})$ $w_2^{N(1,1)} = B_2(a_0 + \hat{\theta}_{2p})$ $p_2^{N(1,1)} = B_3 a_0 + \frac{1}{2}(\hat{\theta}_{12p} + (1+\alpha)rB_1\hat{\theta}_{1p} + B_2\hat{\theta}_{2p})$	$p_1^{I(1,1)} = B_1(a_0 + \hat{\theta}_{12p})$ $w_2^{I(1,1)} = B_2(a_0 + \hat{\theta}_{12p})$ $p_2^{I(1,1)} = B_3 a_0 + \frac{1}{2}(1 + (1+\alpha)rB_1 + B_2)\hat{\theta}_{12p}$

Notes. Where $\hat{\theta}_i = E[\theta|f_i]$, $\hat{\theta}_{12} = E[\theta|f_1, f_2]$, $\hat{\theta}_{ip} = E[\theta|f_i, f_p]$, $\hat{\theta}_{12p} = E[\theta|f_1, f_2, f_p]$ $(i = 1, 2)$.

Θ_2 under signal-sharing strategy $Y(v_1, v_2)$ are given in Table 3. The equilibrium retail price of product 1 and equilibrium wholesale price of product 2 are present in equations (6) and (7), respectively.

$$p_1^{Y(v_1,v_2)} = B_1(a_0 + E[\theta|\Theta_1]), \tag{6}$$

$$w_2^{Y(v_1,v_2)} = B_2(a_0 + E[\theta|\Theta_2]), \tag{7}$$

where $B_1 = \frac{4+3r}{8-(5+3\alpha)r^2}$ and $B_2 = \frac{4+2(1-\alpha)r-(1+3\alpha)r^2}{8-(5+3\alpha)r^2}$. Then, by substituting equations (6) and (7) into equation (5), the equilibrium retail price of product 2 can be derived, as shown in equation (8).

$$p_2^{Y(v_1,v_2)} = B_3 a_0 + \frac{1}{2}((1+\alpha)rB_1 E[\theta|\Theta_1] + B_2 E[\theta|\Theta_2] + E[\theta|\Theta_p]), \tag{8}$$

where $B_3 = \frac{12+2(3+\alpha)r-3(1+\alpha)r^2}{16-2(5+3\alpha)r^2}$. The equilibrium prices under different signal-sharing strategies are summarized in Table 6.

Substituting the equilibrium prices in Table 6 into equations (2)–(4) and taking expectation, we obtain the ex ante profits of supply chain members under different signal-sharing strategies, which are summarized in Table 7.

4.2. Effects of signal sharing on equilibrium prices

By comparing the equilibrium prices under different signal-sharing strategies in Table 6, this study analyzes the effects of signal sharing on equilibrium prices. It first examines the effects of vertical sharing on equilibrium prices and then explores those of horizontal sharing.

4.2.1. Effects of vertical sharing

In this subsection, this paper examines the effects of vertical sharing on the equilibrium prices. Through comparing the equilibrium prices under signal-sharing strategies $N(v_1, v_2)$ where $v_i \in \{0, 1\}$ in Table 6, Proposition 1 is obtained as follows.

Proposition 1. (a) *If* $\hat{\theta}_{1p} > \hat{\theta}_1$, $p_1^{N(1,1)} = p_1^{N(1,0)} > p_1^{N(0,1)} = p_1^{N(0,0)}$*; if* $\hat{\theta}_{2p} > \hat{\theta}_2$, $w_2^{N(1,1)} = w_2^{N(0,1)} > w_2^{N(1,0)} = w_2^{N(0,0)}$*, and vice versa.*

TABLE 7. Ex ante profits under different signal-sharing strategies.

(v_1, v_2)	No horizontal sharing (N)	Horizontal sharing (I)
$(0,0)$	$\pi_1^{N(0,0)} = \tilde{\pi}_1 + \frac{(1-\alpha)B_1}{2}E[B_4\hat{\theta}_1^2 + rB_2\hat{\theta}_1\hat{\theta}_{2\|1} + r\hat{\theta}_1\hat{\theta}_{12p\|1}]$	$\pi_1^{I(0,0)} = \tilde{\pi}_1 + \frac{(1-\alpha)B_1}{2}E[(B_4 + rB_2)\hat{\theta}_{12}^2 + r\hat{\theta}_{12}\hat{\theta}_{12p\|12}]$
	$\pi_2^{N(0,0)} = \tilde{\pi}_2 + \frac{B_2}{2}E[(2-B_2)\hat{\theta}_2^2 + (1-\alpha)rB_1\hat{\theta}_2\hat{\theta}_{1\|2} - \hat{\theta}_2\hat{\theta}_{12p\|2}]$	$\pi_2^{I(0,0)} = \tilde{\pi}_2 + \frac{B_2}{2}E[(2 + (1-\alpha)rB_1 - B_2)\hat{\theta}_{12}^2 - \hat{\theta}_{12}\hat{\theta}_{12p\|12}]$
	$\pi_p^{N(0,0)} = \tilde{\pi}_p + \frac{1}{4}E[\hat{\theta}_{12p}^2 + B_5\hat{\theta}_1^2 + B_2^2\hat{\theta}_2^2 + 2\alpha rB_1(\hat{\theta}_1\hat{\theta}_{12p\|1} + B_2\hat{\theta}_1\hat{\theta}_{2\|1}) + 2rB_1(\hat{\theta}_1\hat{\theta}_{12p} - B_2\hat{\theta}_1\hat{\theta}_2) - 2B_2\hat{\theta}_2\hat{\theta}_{12p}]$	$\pi_p^{I(0,0)} = \tilde{\pi}_p + \frac{1}{4}E[\hat{\theta}_{12p}^2 + (B_5 + B_2^2 - 2rB_1B_2 + 2\alpha rB_1B_2)\hat{\theta}_{12}^2 + 2(rB_1 - B_2)\hat{\theta}_{12}\hat{\theta}_{12p} + 2\alpha rB_1\hat{\theta}_{12}\hat{\theta}_{12p\|12}]$
$(1,0)$	$\pi_1^{N(1,0)} = \tilde{\pi}_1 + \frac{(1-\alpha)B_1}{2}E[B_4\hat{\theta}_{1p}^2 + rB_2\hat{\theta}_{1p}\hat{\theta}_{2\|1p} + r\hat{\theta}_{1p}\hat{\theta}_{12p\|1p}]$	$\pi_1^{I(1,0)} = \tilde{\pi}_1 + \frac{(1-\alpha)B_1}{2}E[(B_4+r)\hat{\theta}_{12p}^2 + rB_2\hat{\theta}_{12p}\hat{\theta}_{12}]$
	$\pi_2^{N(1,0)} = \tilde{\pi}_2 + \frac{B_2}{2}E[(2 - B_2)\hat{\theta}_2^2 + (1-\alpha)rB_1\hat{\theta}_2\hat{\theta}_{1p\|2} - \hat{\theta}_2\hat{\theta}_{12p\|2}]$	$\pi_2^{I(1,0)} = \tilde{\pi}_2 + \frac{B_2}{2}E[(2 - B_2)\hat{\theta}_{12}^2 + ((1-\alpha)rB_1 - 1)\hat{\theta}_{12}\hat{\theta}_{12p\|12}]$
	$\pi_p^{N(1,0)} = \tilde{\pi}_p + \frac{1}{4}E[\hat{\theta}_{12p}^2 + B_5\hat{\theta}_{1p}^2 + B_2^2\hat{\theta}_2^2 + 2\alpha rB_1(\hat{\theta}_{1p}\hat{\theta}_{12p\|1p} + B_2\hat{\theta}_{1p}\hat{\theta}_{2\|1p}) + 2rB_1(\hat{\theta}_{1p}\hat{\theta}_{12p} - B_2\hat{\theta}_{1p}\hat{\theta}_2) - 2B_2\hat{\theta}_2\hat{\theta}_{12p}]$	$\pi_p^{I(1,0)} = \tilde{\pi}_p + \frac{1}{4}E[B_2^2\hat{\theta}_{12}^2 + (1 + B_5 + 2rB_1(\alpha + 1))\hat{\theta}_{12p}^2 - 2B_2(1 + rB_1 - \alpha rB_1)\hat{\theta}_{12}\hat{\theta}_{12p}]$
$(0,1)$	$\pi_1^{N(0,1)} = \tilde{\pi}_1 + \frac{(1-\alpha)B_1}{2}E[B_4\hat{\theta}_1^2 + rB_2\hat{\theta}_1\hat{\theta}_{2p\|1} + r\hat{\theta}_1\hat{\theta}_{12p\|1}]$	$\pi_1^{I(0,1)} = \tilde{\pi}_1 + \frac{(1-\alpha)B_1}{2}E[B_4\hat{\theta}_{12}^2 + r(B_2 + 1)\hat{\theta}_{12}\hat{\theta}_{12p\|12}]$
	$\pi_2^{N(0,1)} = \tilde{\pi}_2 + \frac{B_2}{2}E[(2 - B_2)\hat{\theta}_{2p}^2 + (1-\alpha)rB_1\hat{\theta}_{2p}\hat{\theta}_{1\|2p} - \hat{\theta}_{2p}\hat{\theta}_{12p\|2p}]$	$\pi_2^{I(0,1)} = \tilde{\pi}_2 + \frac{B_2}{2}E[(1 - B_2)\hat{\theta}_{12p}^2 + (1-\alpha)rB_1\hat{\theta}_{12p}\hat{\theta}_{12}]$
	$\pi_p^{N(0,1)} = \tilde{\pi}_p + \frac{1}{4}E[\hat{\theta}_{12p}^2 + B_5\hat{\theta}_1^2 + B_2^2\hat{\theta}_{2p}^2 + 2\alpha rB_1(\hat{\theta}_1\hat{\theta}_{12p\|1} + B_2\hat{\theta}_1\hat{\theta}_{2p\|1}) + 2rB_1(\hat{\theta}_1\hat{\theta}_{12p} - B_2\hat{\theta}_1\hat{\theta}_{2p}) - 2B_2\hat{\theta}_{2p}\hat{\theta}_{12p}]$	$\pi_p^{I(0,1)} = \tilde{\pi}_p + \frac{1}{4}E[B_5\hat{\theta}_{12}^2 + (1 + B_2(B_2 - 2))\hat{\theta}_{12p}^2 + 2rB_1(1 - B_2)\hat{\theta}_{12}\hat{\theta}_{12p} + 2\alpha rB_1(1 + B_2)\hat{\theta}_{12}\hat{\theta}_{12p\|12}]$
$(1,1)$	$\pi_1^{N(1,1)} = \tilde{\pi}_1 + \frac{(1-\alpha)B_1}{2}E[B_4\hat{\theta}_{1p}^2 + rB_2\hat{\theta}_{1p}\hat{\theta}_{2p\|1p} + r\hat{\theta}_{1p}\hat{\theta}_{12p\|1p}]$	$\pi_1^{I(1,1)} = \tilde{\pi}_1 + \frac{(1-\alpha)B_1}{2}(B_4 + rB_2 + r)E[\hat{\theta}_{12p}^2]$
	$\pi_2^{N(1,1)} = \tilde{\pi}_2 + \frac{B_2}{2}E[(2 - B_2)\hat{\theta}_{2p}^2 + (1-\alpha)rB_1\hat{\theta}_{2p}\hat{\theta}_{1p\|2p} - \hat{\theta}_{2p}\hat{\theta}_{12p\|2p}]$	$\pi_2^{I(1,1)} = \tilde{\pi}_2 + \frac{B_2}{2}(1 + (1-\alpha)rB_1 - B_2)E[\hat{\theta}_{12p}^2]$
	$\pi_p^{N(1,1)} = \tilde{\pi}_p + \frac{1}{4}E[\hat{\theta}_{12p}^2 + B_5\hat{\theta}_{1p}^2 + B_2^2\hat{\theta}_{2p}^2 + 2\alpha rB_1(\hat{\theta}_{1p}\hat{\theta}_{12p\|1p} + B_2\hat{\theta}_{1p}\hat{\theta}_{2p\|1p}) + 2rB_1(\hat{\theta}_{1p}\hat{\theta}_{12p} - B_2\hat{\theta}_{1p}\hat{\theta}_{2p}) - 2B_2\hat{\theta}_{2p}\hat{\theta}_{12p}]$	$\pi_p^{I(1,1)} = \tilde{\pi}_p + \frac{1}{4}(1 + B_5 + B_2(B_2 - 2) + 2rB_1(1 + \alpha + \alpha B_2 - B_2))E[\hat{\theta}_{12p}^2]$

Notes. Where $\tilde{\pi}_1 = (1-\alpha)B_1(1 - B_1 + rB_3)a_0^2$, $\tilde{\pi}_2 = B_2(1 - B_3 + rB_1)a_0^2$, $\tilde{\pi}_p = [\alpha B_1(1 - B_1 + rB_3) + (B_3 - B_2)(1 - B_3 + rB_1)]a_0^2$, $B_4 = 2 - 2B_1 + (1+\alpha)r^2B_1$, $B_5 = B_1(4\alpha - 4\alpha B_1 + (1+\alpha)^2r^2B_1)$. $\hat{\theta}_{3-i|i} = E[f_{3-i}|f_i]$, $\hat{\theta}_{12p|i} = E[f_1, f_2, f_p|f_i]$, $\hat{\theta}_{3-i|ip} = E[f_{3-i}|f_i, f_p]$, $\hat{\theta}_{12p|ip} = E[f_1, f_2, f_p|f_i, f_p]$, $\hat{\theta}_{ip|3-i} = E[f_i, f_p|f_{3-i}]$, $\hat{\theta}_{3-i,p|ip} = E[f_{3-i}, f_p|f_i, f_p]$ $(i = 1, 2)$, $\hat{\theta}_{12p|12} = E[f_1, f_2, f_p|f_1, f_2]$. The detailed expansions of conditional expectations are provided in Appendix A.

(b) *If* $\hat{\theta}_{1p} > \hat{\theta}_1$, $p_2^{N(1,v_2)} > p_2^{N(0,v_2)}$, $v_2 \in \{0,1\}$; *if* $\hat{\theta}_{2p} > \hat{\theta}_2$, $p_2^{N(v_1,1)} > p_2^{N(v_1,0)}$, $v_1 \in \{0,1\}$; *if* $\hat{\theta}_{1p} > \hat{\theta}_1$ *and* $\hat{\theta}_{2p} > \hat{\theta}_2$, $p_2^{N(1,1)} > p_2^{N(0,0)}$, *and vice versa.*

Proposition 1(a) shows that when $\hat{\theta}_{1p} > \hat{\theta}_1$, *i.e.*, the conditional expectation of market condition given the signal f_p shared by the platform retailer is greater than that without it, which reflects that the shared signal is *optimistic* with supplier S_1, the retail price of product 1 determined by supplier S_1 is higher than that without the shared signal. Similarly, when the signal shared by the platform retailer is optimistic with supplier S_2, (*i.e.*, $\hat{\theta}_{2p} > \hat{\theta}_2$), the wholesale price of product 2 determined by supplier S_2 is higher than that without the shared signal. Otherwise, when the signal shared by the platform retailer is *pessimistic* (*i.e.*, $\hat{\theta}_{ip} < \hat{\theta}_i$), the supplier $S_i (i = 1, 2)$ will make the lower price than that without the shared signal. In addition, there is no change in the equilibrium price determined by the supplier, no matter whether the platform retailer shares signal with the other supplier or not.

Proposition 1(b) shows that regardless of which sharing strategy, the platform retailer will make the higher retail price of product 2 when the shared signal is optimistic; otherwise, when the shared signal is pessimistic, the equilibrium retail price of product 2 will be lower. Specifically, as long as the signal shared with one supplier is optimistic, the equilibrium retail price of product 2 will be higher no matter whether the platform retailer shares signal with the other supplier or not. When both suppliers receive the optimistic signal (*i.e.*, $\hat{\theta}_{1p} > \hat{\theta}_1$ and $\hat{\theta}_{2p} > \hat{\theta}_2$), the equilibrium retail price of product 2 is higher than that without sharing signal, and *vice versa*. This is because the retail price of product 2 is determined by the platform retailer as the follower of Stackelberg game, which is influenced by two suppliers' pricing strategies. This result suggests that whether the platform retailer shares signal and with which supplier will affect her pricing decision.

Then we compare the equilibrium prices under signal-sharing strategies $I(v_1, v_2)$ where $v_i \in \{0, 1\}$ in Table 6 and obtain the effects of vertical sharing on the equilibrium prices with horizontal sharing, which are presented in Proposition 2.

Proposition 2. *If* $\hat{\theta}_{12p} > \hat{\theta}_{12}$, $p_1^{I(1,1)} = p_1^{I(1,0)} > p_1^{I(0,1)} = p_1^{I(0,0)}$, $w_2^{I(1,1)} = w_2^{I(0,1)} > w_2^{I(1,0)} = w_2^{I(0,0)}$, $p_2^{I(1,1)} > p_2^{I(1,0)}(p_2^{I(0,1)}) > p_2^{I(0,0)}$, *and vice versa.*

Proposition 2 shows that when the shared signal is optimistic with horizontal sharing, *i.e.*, $\hat{\theta}_{12p} > \hat{\theta}_{12}$, which represents that the conditional expectation of market condition given signal f_p shared by the platform retailer is greater than that without it, the retail price of product 1 determined by supplier S_1 and the wholesale price of product 2 determined by supplier S_2 are higher than those without the shared signal. In addition, no matter whether or not the platform retailer shares signal with supplier S_i, given the vertical-sharing strategy v_{3-i}, there is no change in the equilibrium price determined by supplier $S_{3-i} (i = 1, 2)$. For the platform retailer, when the shared signal is optimistic with horizontal sharing, the more suppliers with receiving signal, the higher equilibrium price of product 2 is determined. Otherwise, when the signal shared by the platform retailer is pessimistic (*i.e.*, $\hat{\theta}_{12p} < \hat{\theta}_{12}$), supply chain members will make the lower equilibrium prices than those without the shared signal.

4.2.2. Effects of horizontal sharing

Through comparing the equilibrium prices between two horizontal sharing strategies, the effects of horizontal sharing are obtained, as presented in Proposition 3.

Proposition 3. (a) *If* $\hat{\theta}_{12} > \hat{\theta}_1$, $p_1^{I(0,v_2)} > p_1^{N(0,v_2)}$*; if* $\hat{\theta}_{12p} > \hat{\theta}_{1p}$, $p_1^{I(1,v_2)} > p_1^{N(1,v_2)}$, $v_2 \in \{0, 1\}$*;*
(b) *if* $\hat{\theta}_{12} > \hat{\theta}_2$, $w_2^{I(v_1,0)} > w_2^{N(v_1,0)}$*; if* $\hat{\theta}_{12p} > \hat{\theta}_{2p}$, $w_2^{I(v_1,1)} > w_2^{N(v_1,1)}$, $v_1 \in \{0, 1\}$, *and vice versa.*
(c) *When* $\hat{\theta}_{12} > \hat{\theta}_1$, *if* $\hat{\theta}_{12} > \hat{\theta}_2$, $p_2^{I(0,0)} > p_2^{N(0,0)}$*; if* $\hat{\theta}_{12p} > \hat{\theta}_{2p}$, $p_2^{I(0,1)} > p_2^{N(0,1)}$, *and vice versa.*

When $\hat{\theta}_{12p} > \hat{\theta}_{1p}$, if $\hat{\theta}_{12} > \hat{\theta}_2$, $p_2^{I(1,0)} > p_2^{N(1,0)}$; if $\hat{\theta}_{12p} > \hat{\theta}_{2p}$, $p_2^{I(1,1)} > p_2^{N(1,1)}$, and *vice versa*.

Proposition 3(a) shows that for supplier S_1, no matter whether or not the platform retailer shares signal, the higher retail price of product 1 is determined only when the signal shared by supplier S_2 is optimistic with supplier S_1 (*i.e.*, $\hat{\theta}_{12} > \hat{\theta}_1$ or $\hat{\theta}_{12p} > \hat{\theta}_{1p}$, which represents that the conditional expectation of market condition given the signal f_2 shared is greater than that without it). Otherwise, when the signal shared by supplier S_2 is pessimistic (*i.e.*, $\hat{\theta}_{12} < \hat{\theta}_1$ or $\hat{\theta}_{12p} < \hat{\theta}_{1p}$), the supplier S_1 will make the lower price than that without the shared signal f_2. Similarly, proposition 3(b) shows that for supplier S_2, no matter whether or not the platform retailer shares signal, the higher wholesale price of product 2 is determined only when the signal shared by supplier S_1 is optimistic with supplier S_2 (*i.e.*, $\hat{\theta}_{12} > \hat{\theta}_2$ or $\hat{\theta}_{12p} > \hat{\theta}_{2p}$). Otherwise, when the signal shared by supplier S_1 is pessimistic (*i.e.*, $\hat{\theta}_{12} < \hat{\theta}_2$ or $\hat{\theta}_{12p} < \hat{\theta}_{2p}$), supplier S_2 will make the lower price than that without the shared signal f_1.

Proposition 3(c) shows that for the platform retailer, no matter whether or not she shares signal and no matter she shares signal with which supplier, the equilibrium retail price of product 2 becomes higher when the horizontal shared signal is optimistic with both suppliers; otherwise, it becomes lower. For example, when the platform retailer shares her signal with two suppliers, she will make the higher retail price of product 2 if two suppliers acquire the optimistic horizontal-sharing signal (*i.e.*, $\hat{\theta}_{12p} > \hat{\theta}_{1p}$ and $\hat{\theta}_{12p} > \hat{\theta}_{2p}$); otherwise, the retail price of product 2 is lower. This reflects the effect of horizontal sharing on the pricing strategy of the platform retailer as a follower of Stackelberg game.

Propositions 1–3 indicate that supply chain member with the optimistic signal under a signal-sharing strategy has the incentive to set the higher equilibrium wholesale/retail price, otherwise, the lower equilibrium wholesale/retail price will be set, which reflects the role of signal-sharing in adjusting the equilibrium prices.

5. The optimal bi-direction signal-sharing strategy

In this section, this study applies the backward induction method to analyze the optimal bi-directional signal-sharing strategy. According to the sequence of events in Section 3, it first studies the platform retailer's vertical-sharing strategy without and with horizontal sharing, respectively. Then, the suppliers' horizontal-sharing strategy is examined by comparing the suppliers' ex ante profits with the optimal vertical-sharing strategy.

5.1. Vertical-sharing strategy

In this subsection, we first analyze the optimal vertical-sharing strategy of the platform retailer without and with horizontal sharing, and then explore the impact of horizontal sharing on vertical sharing.

Following Wei *et al.* [18] and Zha *et al.* [14], the values of vertical sharing (v_1, v_2) under horizontal-sharing strategy $Y(Y \in \{N, I\})$ for supply chain members are defined as $V_j^{Y(v_1,v_2)} = \pi_j^{Y(v_1,v_2)} - \pi_j^{Y(0,0)}$ where $(v_1, v_2) \in \{(1,0),(0,1),(1,1)\}$ and $j = 1, 2, p$, which represent the profit differences between the vertical-sharing strategy (v_1, v_2) and no vertical-sharing strategy $(0,0)$ under horizontal-sharing strategy Y. The value of vertical sharing for the whole supply chain is defined as $V^{Y(v_1,v_2)} = V_1^{Y(v_1,v_2)} + V_2^{Y(v_1,v_2)} + V_p^{Y(v_1,v_2)}$.

5.1.1. *Vertical-sharing strategy without horizontal sharing*

The values of vertical sharing for the whole supply chain and its members without horizontal sharing are summarized in Table 8.

Through analyzing the values of vertical sharing in Table 8, the following results are obtained.

Proposition 4. (a) $V_1^{N(1,0)} > 0$; $V_2^{N(1,0)} > 0$; $V_p^{N(1,0)} > 0$; $V^{N(1,0)} > 0$;
(b) $V_1^{N(0,1)} > 0$; $V_2^{N(0,1)} > 0$; $V_p^{N(0,1)} < 0$; $V^{N(0,1)} > 0$ *iff* $\Phi^{N(0,1)} > 0$, *and vice versa;*
(c) $V_1^{N(1,1)} > 0$; $V_2^{N(1,1)} > 0$; $V^{N(1,1)} > 0$; $V_p^{N(1,1)} > 0$ *iff* $\Phi_p^{N(1,1)} > 0$, *and vice versa.*
Where $\Phi^{N(0,1)} = (2(1-\alpha)(3r+4)r\rho_1\sigma_1\sigma_p - ((7-3\alpha)r^2 + 6(1-\alpha)r + 4)(\sigma_2^2 - \rho_2\sigma_2\sigma_p))\sigma^2 + (2(1-\alpha)(3r+4)r\rho_1\sigma_2\sigma_p + ((3\alpha+1)r^2 - 2(1-\alpha)r - 4)(\sigma_1\sigma_2 - \rho_2\sigma_1\sigma_p))\sigma_1\sigma_2$, $\Phi_p^{N(1,1)} = (B_5 + 2rB_1(1+\alpha))E[\hat{\theta}_{1p}^2 - \hat{\theta}_1^2] - B_2(2-B_2)E[\hat{\theta}_{2p}^2 - \hat{\theta}_2^2] - 2rB_1B_2(1-\alpha)E[\hat{\theta}_{1p}\hat{\theta}_{2p} - \hat{\theta}_1\hat{\theta}_2]$.

The proof is detailed in Appendix B.

From Proposition 4(a), we can see that when the platform retailer shares signal only with the supplier adopting agency channel, all values of vertical sharing are positive, which shows that vertical-sharing strategy $(1,0)$ without horizontal sharing is beneficial to each supply chain member and the whole supply chain (Pareto Improvement). This is because combining with the platform retailer's signal, supplier S_1 can strategically adjust retail price p_1 to respond to demand uncertainty more effectively, which affects the equilibrium retail price p_2 and increases the ex ante profits of supplier S_1 and the platform retailer. Furthermore, due to the spillover effect of signal sharing, it is beneficial to supplier S_2. Therefore, the platform retailer has incentive to share signal with the supplier adopting agency channel.

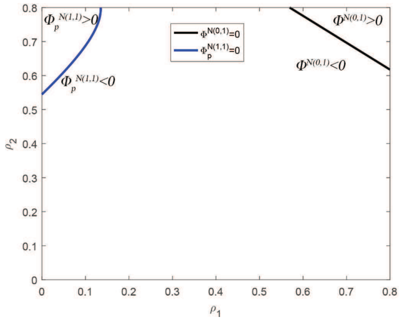
TABLE 8. The values of vertical sharing without horizontal sharing.

(v_1, v_2)	No horizontal sharing (N)
$(1,0)$	$V_1^{N(1,0)} = \frac{(1-\alpha)B_1}{2}((B_4+r)E[\hat\theta_{1p}^2-\hat\theta_1^2]+rB_2E[\hat\theta_{1p}\hat\theta_2-\hat\theta_1\hat\theta_2])$
	$V_2^{N(1,0)} = \frac{(1-\alpha)rB_1B_2}{2}E[\hat\theta_{1p}\hat\theta_2-\hat\theta_1\hat\theta_2]$
	$V_p^{N(1,0)} = \frac{1}{4}((B_5+2\alpha rB_1+2rB_1)E[\hat\theta_{1p}^2-\hat\theta_1^2]-2rB_1B_2(1-\alpha)E[\hat\theta_{1p}\hat\theta_2-\hat\theta_1\hat\theta_2])$
	$V^{N(1,0)} = \frac{B_5+4rB_1+2(1-\alpha)B_1B_4}{4}E[\hat\theta_{1p}^2-\hat\theta_1^2]+\frac{(1-\alpha)rB_1B_2}{2}E[\hat\theta_{1p}\hat\theta_2-\hat\theta_1\hat\theta_2]$
$(0,1)$	$V_1^{N(0,1)} = \frac{(1-\alpha)rB_1B_2}{2}E[\hat\theta_1\hat\theta_{2p}-\hat\theta_1\hat\theta_2]$
	$V_2^{N(0,1)} = \frac{B_2}{2}((1-B_2)E[\hat\theta_{2p}^2-\hat\theta_2^2]+(1-\alpha)rB_1E[\hat\theta_1\hat\theta_{2p}-\hat\theta_1\hat\theta_2])$
	$V_p^{N(0,1)} = -\frac{1}{4}(B_2(2-B_2)E[\hat\theta_{2p}^2-\hat\theta_2^2]+2rB_1B_2(1-\alpha)E[\hat\theta_1\hat\theta_{2p}-\hat\theta_1\hat\theta_2])$
	$V^{N(0,1)} = \frac{(1-\alpha)rB_1B_2}{2}E[\hat\theta_1\hat\theta_{2p}-\hat\theta_1\hat\theta_2]-\frac{B_2^2}{4}E[\hat\theta_{2p}^2-\hat\theta_2^2]$
$(1,1)$	$V_1^{N(1,1)} = \frac{(1-\alpha)B_1}{2}((B_4+r)E[\hat\theta_{1p}^2-\hat\theta_1^2]+rB_2E[\hat\theta_{1p}\hat\theta_{2p}-\hat\theta_1\hat\theta_2])$
	$V_2^{N(1,1)} = \frac{B_2}{2}((1-B_2)E[\hat\theta_{2p}^2-\hat\theta_2^2]+(1-\alpha)rB_1E[\hat\theta_{1p}\hat\theta_{2p}-\hat\theta_1\hat\theta_2])$
	$V_p^{N(1,1)} = \frac{1}{4}((B_5+2rB_1(1+\alpha))E[\hat\theta_{1p}^2-\hat\theta_1^2]-B_2(2-B_2)E[\hat\theta_{2p}^2-\hat\theta_2^2]-2rB_1B_2(1-\alpha)E[\hat\theta_{1p}\hat\theta_{2p}-\hat\theta_1\hat\theta_2])$
	$V^{N(1,1)} = \frac{B_5+4rB_1+2(1-\alpha)B_1B_4}{4}E[\hat\theta_{1p}^2-\hat\theta_1^2]-\frac{B_2^2}{4}E[\hat\theta_{2p}^2-\hat\theta_2^2]+\frac{(1-\alpha)rB_1B_2}{2}E[\hat\theta_{1p}\hat\theta_{2p}-\hat\theta_1\hat\theta_2]$

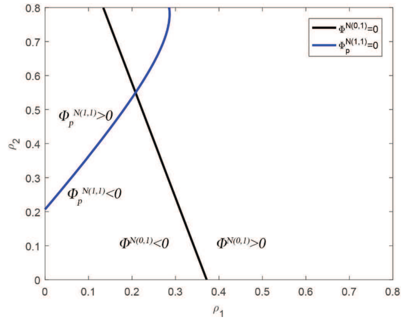
Proposition 4(b) indicates that when the platform retailer shares signal only with the supplier adopting reselling channel, it is beneficial to supplier S_2 and detrimental to the platform retailer because combining with the platform retailer's signal, supplier S_2 can strategically adjust wholesale price w_2 to respond to demand uncertainty more effectively, which affects the equilibrium retail price p_2 and aggravates the double marginalization effect. Furthermore, due to the spillover effect of signal sharing, it is beneficial to supplier S_1. Whether or not the whole supply chain benefits depends on parameter condition: $\Phi^{N(0,1)} > 0$, where the region becomes bigger with channel competition intensity (illustrated in Fig. 3). It's worth noting that $V_2^{N(0,1)} < |V_p^{N(0,1)}|$, thus it is impossible to share signal for the platform retailer by implementing a compensation contract with supplier S_2. Therefore, the platform retailer would not share signal only with supplier adopting reselling channel without horizontal sharing.

Proposition 4(c) shows that vertical sharing with both two suppliers is always beneficial for two suppliers and the whole supply chain; whether or not it is beneficial for the platform retailer depends on parameter condition: $\Phi_p^{N(1,1)} > 0$, where correlation coefficient ρ_1 is lower whereas ρ_2 is higher (illustrated in Fig. 3). This is because the lower correlation coefficient ρ_1, the more benefit the platform retailer is gained from sharing signal in agency channel; whereas the higher correlation coefficient ρ_2, the less loss the platform retailer is gained from sharing signal in reselling channel. That is to say, when the platform retailer's forecast signal has a lower correlation with the forecast signal of supplier S_1 and a higher correlation with the forecast signal of supplier S_2, she is willing to share signal with both suppliers; otherwise, she is reluctant to share with two suppliers simultaneously.

In order to derive the optimal vertical-sharing strategy of the platform retailer without horizontal sharing, we further compare vertical-sharing values of the platform retailer. According to the analysis of proposition 4, the platform retailer always benefits from sharing signal only with supplier S_1, and benefits from sharing signal with both suppliers only when $\Phi_p^{N(1,1)} > 0$. We compare two scenarios and find that it is more beneficial for the platform retailer to share signal only with supplier S_1. Therefore, the vertical-sharing strategy $(1,0)$ is optimal without horizontal sharing.



(a) $\Phi^{N(0,1)}$ and $\Phi_p^{N(1,1)}(r=0.3)$



(b) $\Phi^{N(0,1)}$ and $\Phi_p^{N(1,1)}(r=0.5)$

FIGURE 3. Illustration of parameter conditions without horizontal sharing.

TABLE 9. The values of vertical sharing with horizontal sharing.

(v_1, v_2)	Horizontal sharing (I)
$(1,0)$	$V_1^{I(1,0)} = \frac{(1-\alpha)B_1(B_4+r)}{2} E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$ $V_2^{I(1,0)} = 0$ $V_p^{I(1,0)} = \frac{B_5+2(1+\alpha)rB_1}{4} E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$ $V^{I(1,0)} = \frac{B_5+4rB_1+2(1-\alpha)B_1B_4}{4} E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$
$(0,1)$	$V_1^{I(0,1)} = 0$ $V_2^{I(0,1)} = \frac{B_2(1-B_2)}{2} E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$ $V_p^{I(0,1)} = -\frac{B_2(2-B_2)}{4} E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$ $V^{I(0,1)} = -\frac{B_2^2}{4} E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$
$(1,1)$	$V_1^{I(1,1)} = \frac{(1-\alpha)B_1(B_4+r(1+B_2))}{2} E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$ $V_2^{I(1,1)} = \frac{B_2(1-B_2+(1-\alpha)B_1)}{2} E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$ $V_p^{I(1,1)} = \frac{B_5-B_2(2-B_2)+2\alpha rB_1(1+B_2)+2rB_1(1-B_2)}{4} E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$ $V^{I(1,1)} = \frac{B_5+4rB_1-B_2^2+2(1-\alpha)B_1(B_2+B_4)}{4} E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$

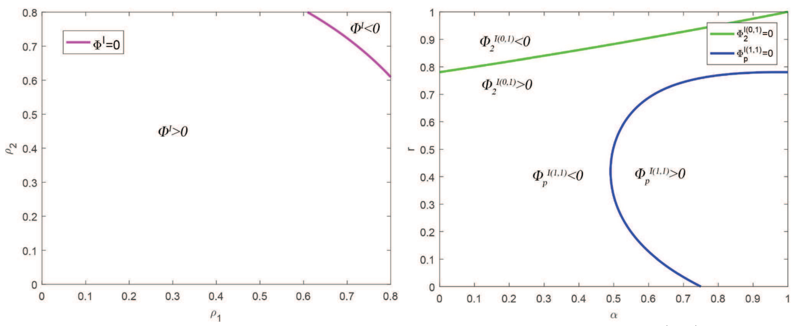
5.1.2. Vertical-sharing strategy with horizontal sharing

In this subsection, this study analyzes the optimal vertical-sharing strategy of the platform retailer with horizontal sharing. The values of vertical sharing for the whole supply chain and its members are summarized in Table 9.

Through analyzing the values of vertical sharing with horizontal sharing in Table 9, the following results are obtained.

Proposition 5. *If* $\Phi^I > 0$, *then*

(a) $V_1^{I(1,0)} > 0$, $V_2^{I(1,0)} = 0$, $V_p^{I(1,0)} > 0$, $V^{I(1,0)} > 0$;
(b) $V_1^{I(0,1)} = 0$, $V_p^{I(0,1)} < 0$, $V^{I(0,1)} < 0$; $V_2^{I(0,1)} > 0$ *iff* $\Phi_2^{I(0,1)} > 0$;
(c) $V_1^{I(1,1)} > 0$, $V_2^{I(1,1)} > 0$, $V^{I(1,1)} > 0$; $V_p^{I(1,1)} > 0$ *iff* $\Phi_p^{I(1,1)} > 0$, *and vice versa.*



(a) Φ^I (b) $\Phi_2^{I(0,1)}$ and $\Phi_p^{I(1,1)}$

FIGURE 4. Illustration of parameter conditions with horizontal sharing.

Where $\Phi^I = [((1-\rho_1^2)\sigma_1^2+(1-\rho_2^2)\sigma_2^2+2\sigma_1\sigma_2\rho_1\rho_2)\sigma_p^2+\sigma_1^2\sigma_2^2-2\rho_1\sigma_1\sigma_p\sigma_2^2-2\rho_2\sigma_2\sigma_p\sigma_1^2]\sigma^2+\sigma_1^2\sigma_2^2\sigma_p^2(1-\rho_1^2-\rho_2^2)$, $\Phi_2^{I(0,1)} = 8-2(\alpha^2+\alpha+6)r^2-3(\alpha-1)^2r^3+(6\alpha+2)r^4$, $\Phi_p^{I(1,1)} = -12+(4+28\alpha)r-(19-15\alpha+11\alpha^2)r^2+(1-13\alpha+18\alpha^2)r^3+(6-12\alpha-9\alpha^2)r^4$.

The proof is detailed in Appendix B.

Proposition 5 indicates that all values of vertical sharing depend on the sign of parameter Φ^I except for the value of the supplier without the signal shared by platform retailer, which is different from the results without horizontal sharing in proposition 4. Specifically, Proposition 5(a) shows that the platform retailer would share signal only with supplier S_1 only when $\Phi^I > 0$ where two correlation coefficients are not large (illustrated in Fig. 4a), and it is beneficial to supplier S_1, the platform retailer and whole supply chain, whereas it has no effect on supplier S_2. Otherwise, when $\Phi^I \le 0$ where two correlation coefficients are larger, the platform retailer is reluctant to share signal with supplier S_1.

From Proposition 5(b), we can see that when $\Phi^I > 0$, the platform retailer is not willing to share signal only with supplier S_2 because it is detrimental to the platform retailer and the whole supply chain. When $\Phi^I \le 0$, sharing signal only with supplier S_2 is beneficial to the platform retailer and the whole supply chain, and supplier S_2 also benefits when $\Phi_2^{I(0,1)} \le 0$. Therefore, when $\Phi^I \le 0$ and $\Phi_2^{I(0,1)} \le 0$ where both signal correlations are high and channel competition is also intense (illustrated in Figs. 4a and 4b), the platform retailer voluntarily shares signal with the supplier adopting reselling channel. This result is very different from previous studies in a reselling channel where the platform retailer would not share signal with suppliers. The main reason is that two suppliers are assumed to have forecast signals in this paper and there exists correlations between the platform retailer's signal and suppliers' signals. When the signal correlations are higher, sharing signal with the supplier adopting reselling channel may mitigate the negative impact of the double marginalization, which is beneficial to the platform retailer and the whole supply chain.

It follows from Proposition 5(c) that when $\Phi^I > 0$, sharing signal with both suppliers is beneficial to two suppliers and the whole supply chain. For the platform retailer, her value of vertical sharing also depends on parameter $\Phi_p^{I(1,1)}$. If $\Phi_p^{I(1,1)} > 0$ where the channel competition intensity r is not too high and the commission rate α is higher (illustrated in Fig. 4b), the platform retailer benefits from sharing signal with two suppliers and she would share with both suppliers. Otherwise, $(1,1)$ strategy cannot be implemented.

In summary, when $\Phi^I > 0$, the platform retailer voluntarily shares signal with the supplier adopting agency channel and is reluctant to share signal with the supplier adopting reselling channel. The platform retailer is willing to share signal with both suppliers only when $\Phi^I > 0$ and $\Phi_p^{I(1,1)} > 0$. Through computation, we have that $V_p^{I(1,0)} > V_p^{I(1,1)}$ when $\Phi^I > 0$. When $\Phi^I \le 0$, the platform retailer voluntarily shares signal with the supplier adopting reselling channel who benefits if $\Phi_2^{I(0,1)} \le 0$, otherwise she shares no signal. Therefore, with horizontal sharing, the optimal vertical-sharing strategy is

$$\begin{cases}(1,0), & \Phi^I > 0;\\ (0,1), & \Phi^I \le 0 \text{ and } \Phi_2^{I(0,1)} \le 0;\\ (0,0), & \Phi^I \le 0 \text{ and } \Phi_2^{I(0,1)} > 0.\end{cases}$$

5.1.3. The impacts of horizontal sharing on vertical sharing

Through comparing the optimal vertical-sharing strategy for the platform retailer without and with horizontal sharing, it can be found that the platform retailer is willing to share signal only with the supplier adopting agency channel in most scenarios, regardless of whether there is horizontal sharing between two suppliers or not. On the other hand, it also found that horizontal sharing between two suppliers weakens the platform retailer's incentive of vertical sharing. The internal reason is supplier S_1 adopting agency channel relies less on the platform retailer's shared signal with horizontal sharing than without horizontal sharing, which less the platform retailer's benefit from signal sharing. Moreover, when two correlation coefficients of signals between suppliers and platform retailer are sufficiently large, *i.e.*, $\Phi^I \le 0$, supplier S_1 may be less profitable from signal sharing, which is not beneficial to the platform retailer. Therefore, when $\Phi^I \le 0$, the platform retailer is no longer willing to share signal with supplier S_1. Interestingly, when $\Phi^I \le 0$ and $\Phi_2^{I(0,1)} \le 0$, horizontal sharing between two suppliers promotes the platform retailer's sharing with the supplier adopting reselling channel. This may be because horizontal sharing reduces the utilization efficiency of platform retailer's signal, which weakens the platform retailer's loss due to double marginalization effect from sharing with the supplier adopting reselling channel.

To further explore the impacts of horizontal sharing on vertical sharing, we compare the values of vertical sharing without and with horizontal sharing and obtain Proposition 6.

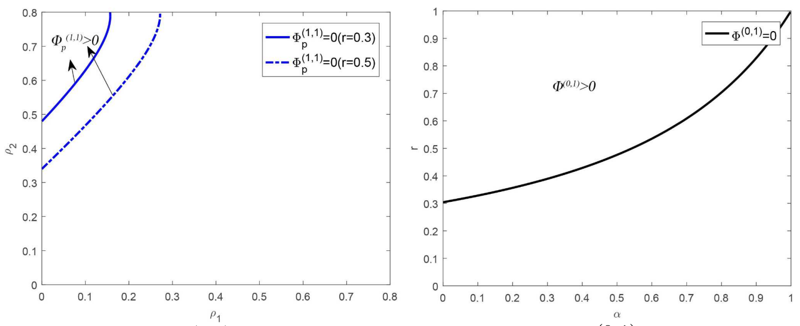
Proposition 6. (a) $V_i^{I(v_1,v_2)} < V_i^{N(v_1,v_2)},\ v_i \in \{0,1\},\ i = 1,2$;
(b) $V_p^{I(1,0)} < V_p^{N(1,0)}$, $V_p^{I(0,1)} > V_p^{N(0,1)}$; $V_p^{I(1,1)} < V_p^{N(1,1)}$ *if* $\Phi_p^{(1,1)} > 0$, *and vice versa;*
(c) $V^{I(1,0)} < V^{N(1,0)}$, $V^{I(1,1)} < V^{N(1,1)}$; $V^{I(0,1)} < V^{N(0,1)}$ *if* $\Phi^{(0,1)} > 0$, *and vice versa.*

Where $\Phi_p^{(1,1)} = (B_5 + 2r(1+\alpha)B_1)E[(\hat\theta_{1p}^2 - \hat\theta_1^2) - (\hat\theta_{12p}^2 - \hat\theta_{12}^2)] - B_2(2-B_2)E[(\hat\theta_{2p}^2 - \hat\theta_2^2) - (\hat\theta_{12p}^2 - \hat\theta_{12}^2)] - 2r(1-\alpha)B_1B_2E[(\hat\theta_{1p}\hat\theta_{2p} - \hat\theta_1\hat\theta_2) - (\hat\theta_{12p}^2 - \hat\theta_{12}^2)]$, $\Phi^{(0,1)} = 2(1-\alpha)rB_1B_2E[\hat\theta_1\hat\theta_{2p} - \hat\theta_1\hat\theta_2] - B_2^2E[(\hat\theta_{2p}^2 - \hat\theta_2^2) - (\hat\theta_{12p}^2 - \hat\theta_{12}^2)]$.

The proof is detailed in Appendix B.

Proposition 6(a) shows that the values of vertical sharing for both suppliers decrease with horizontal sharing, regardless of the vertical-sharing strategy. This is because when suppliers engage in horizontal sharing, they can obtain more precise forecast signals, which weakens the utilization of the platform retailer's signal f_p in forecasting market demand. It follows from Proposition 6(b) that for the platform retailer, horizontal sharing decreases her value under strategy $(1,0)$ and increases her value under strategy $(0,1)$, which shows that horizontal sharing has an opposite impact on the vertical-sharing values of platform retailer in agency and reselling channels. Thus, the change of vertical-sharing value under strategy $(1,1)$ is uncertain, which depends on two correlation coefficients. Specifically, horizontal sharing decreases the value of vertical sharing for the platform retailer when $\Phi_p^{(1,1)} > 0$ where correlation coefficient ρ_1 is lower and ρ_2 is higher and the region becomes bigger with channel competition intensity (illustrated in Fig. 5a); otherwise, it increases her value under strategy $(1,1)$ when $\Phi_p^{(1,1)} < 0$.

From Proposition 6(c), it can be obtained that for the whole supply chain, horizontal sharing decreases the values of vertical sharing under strategies $(1,0)$ and $(0,1)$. This is because all values of vertical sharing for supply chain members decrease with horizontal sharing under strategy $(1,0)$. Even if when $\Phi_p^{(1,1)} < 0$, the decrease values of two suppliers still exceed the increase value of the platform retailer under strategy $(1,1)$. Under strategy $(0,1)$, since horizontal sharing has an opposite impact on the values of vertical sharing for suppliers and platform retailer, the change of vertical-sharing value for the whole supply chain is uncertain, which depends on parameter condition. Specifically, when $\Phi^{(0,1)} > 0$ where competition intensity is higher



(a) $\Phi_p^{(1,1)}$ (b) $\Phi^{(0,1)}$

FIGURE 5. Illustration of parameter conditions on the impacts of horizontal sharing.

(illustrated in Fig. 5b), horizontal sharing decreases the value of vertical sharing for the whole supply chain; otherwise, it increases the value under strategy $(0, 1)$.

5.2. Horizontal-sharing strategy

The ex ante profits of suppliers under the optimal vertical-sharing strategy without and with horizontal sharing $(i = 1, 2)$ are summarized as follows

$$\pi_i^N = \pi_i^{N(1,0)};\ \pi_i^I = \begin{cases} \pi_i^{I(1,0)}, & \Phi^I > 0; \\ \pi_i^{I(0,1)}, & \Phi^I \le 0 \text{ and } \Phi_2^{I(0,1)} \le 0; \\ \pi_i^{I(0,0)}, & \Phi^I \le 0 \text{ and } \Phi_2^{I(0,1)} > 0; \end{cases} \qquad i = 1, 2.$$

To examine the optimal horizontal-sharing strategy, we define the value of horizontal sharing for supplier S_i as

$$H_i = \pi_i^I - \pi_i^N = \begin{cases} \pi_i^{I(1,0)} - \pi_i^{N(1,0)}, & \Phi^I > 0; \\ \pi_i^{I(0,1)} - \pi_i^{N(1,0)}, & \Phi^I \le 0 \text{ and } \Phi_2^{I(0,1)} \le 0; \\ \pi_i^{I(0,0)} - \pi_i^{N(1,0)}, & \Phi^I \le 0 \text{ and } \Phi_2^{I(0,1)} > 0. \end{cases}$$

The values of horizontal sharing for supplier S_1 and supplier S_2 under the optimal vertical-sharing strategy are computed, with the respective results shown in equations (9) and (10), respectively.

$$H_1 = \begin{cases} \frac{(1-\alpha)B_1}{2}((B_4 + r)E[\hat{\theta}_{12p}^2 - \hat{\theta}_{1p}^2] + rB_2E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}\hat{\theta}_2]), & \Phi^I > 0; \\ \frac{(1-\alpha)B_1}{2}((B_4 + r)E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}^2] + rB_2E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}\hat{\theta}_2]), & \Phi^I \le 0. \end{cases} \tag{9}$$

$$H_2 = \begin{cases} \frac{B_2}{2}((1 - B_2)E[\hat{\theta}_{12p}^2 - \hat{\theta}_2^2] + (1-\alpha)rB_1E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}\hat{\theta}_2]), & \Phi^I \le 0 \text{ and } \Phi_2^{I(0,1)} \le 0; \\ \frac{B_2}{2}((1 - B_2)E[\hat{\theta}_{12}^2 - \hat{\theta}_2^2] + (1-\alpha)rB_1E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}\hat{\theta}_2]), & \text{else}. \end{cases} \tag{10}$$

Through analyzing the values of horizontal sharing in equations (9) and (10), the following results can be obtained.

Proposition 7. $H_1 > 0, H_2 > 0$. *The proof is detailed in Appendix B.*

Proposition 7 shows that under the optimal vertical-sharing strategy, no matter which strategy the platform retailer implements, both suppliers benefit from horizontal sharing. For supplier S_1, when $\Phi^I > 0$ where two signal correlations are not too high, he can receive supplier S_2's signal from horizontal sharing, which improves forecast accuracy and obtains more ex ante profits than without horizontal sharing. When $\Phi^I \leq 0$ where two signal correlations are sufficiently high, supplier S_1 can obtain more ex ante profits from horizontal sharing even if he doesn't have the platform retailer's signal than without horizontal sharing, which is because the sufficiently high signal correlation decreases his benefit from the platform retailer's sharing without horizontal sharing. Thus supplier S_1 always prefers to have access to signal f_2 through horizontal sharing. Similarly, for supplier S_2, no matter when $\Phi^I \leq 0$ and $\Phi_2^{I(0,1)} \leq 0$ where signal correlations and channel competition are high or in other scenarios, horizontal sharing enables him to improve forecast accuracy and obtain more ex ante profits than without horizontal sharing. Therefore, supplier S_2 is willing to share signal with supplier S_1. It is worth noting that combining Proposition 8(a), it can be found that even though horizontal sharing reduces two suppliers' benefits from vertical sharing, they prefer to engage in horizontal sharing, which is because the benefit from horizontal sharing exceeds the loss on the value of vertical sharing.

In summary, two suppliers voluntarily share signals with each other under the optimal vertical-sharing strategy of the platform retailer. The results imply that even in the context of competitive hybrid-channel market, suppliers may engage in collaboration on forecasting demand information. Through exchanging signal with each other, two suppliers can obtain a more comprehensive understanding of market demand, enabling them to adjust their pricing strategies appropriately and increase their ex ante profits.

5.3. The optimal bi-direction signal-sharing strategy

In this subsection, this study explores the optimal bi-directional signal-sharing strategy. By combining the analysis in Sections 5.1 and 5.2, Proposition 8 is obtained.

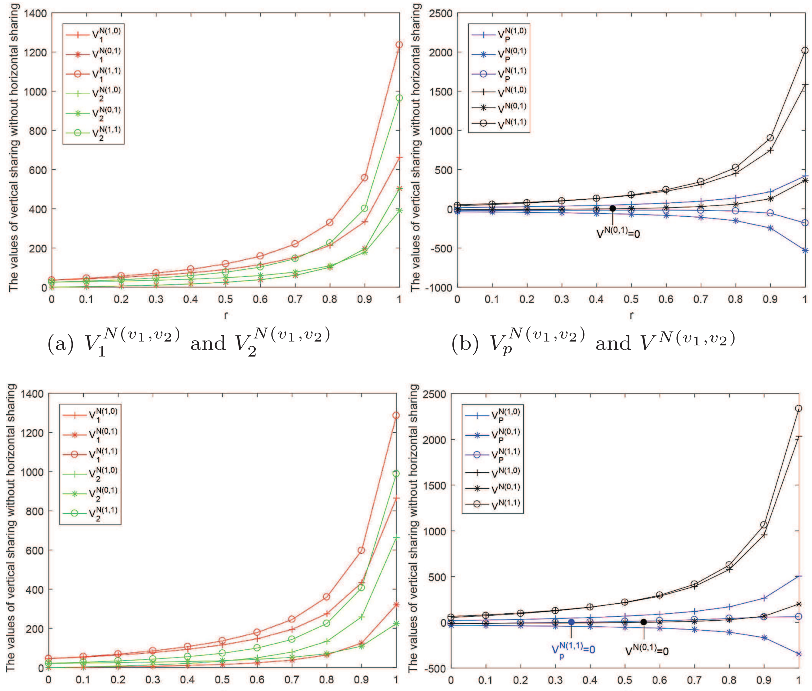
Proposition 8. *The optimal bi-direction signal-sharing strategy in the hybrid-channel platform chain is*

$$\begin{cases} I(1,0), & \Phi^I > 0; \\ I(0,1), & \Phi^I \leq 0 \text{ and } \Phi_2^{I(0,1)} \leq 0; \\ I(0,0), & \Phi^I \leq 0 \text{ and } \Phi_2^{I(0,1)} > 0. \end{cases}$$

The expansions of Φ^I and $\Phi_2^{I(0,1)}$ are provided in Appendix A, and the proof of Proposition 8 is detailed in Appendix B.

From Proposition 8, we can obtain that when $\Phi^I > 0$ where the correlation coefficients ρ_1 and ρ_2 are not large, the optimal bi-direction signal-sharing strategy is that the platform retailer shares signal only with the supplier adopting agency channel and two suppliers share signal with each other. This is because, under this scenario, the signal shared by the platform retailer can bring more benefits in agency channel. Moreover, horizontal sharing between two suppliers can make them more effectively respond to uncertain demand by applying the shared signal and thus improve the performance of supply chain and its members. If $\Phi^I \leq 0$ and $\Phi_2^{I(0,1)} \leq 0$ where correlation coefficients and channel competition intensity are high, the optimal bi-direction signal-sharing strategy is that the platform retailer shares signal only with the supplier adopting reselling channel and two suppliers share signal with each other. This is because the platform retailer's vertical sharing with S_2 under the higher signal correlations and channel competition can benefit both parties. In addition, horizontal sharing can bring more ex ante profits to both suppliers. When $\Phi^I \leq 0$ and $\Phi_2^{I(0,1)} > 0$ where correlation coefficients are high and channel competition intensity is not high, the optimal bi-direction signal-sharing strategy is that the platform retailer shares no signal with two suppliers and two suppliers share signal with each other. This is because horizontal sharing can bring more benefits to two suppliers, whereas the higher correlation coefficients and lower channel competition intensity make the values of vertical sharing for supply chain and its members become negative.

Note that the optimal bi-direction signal-sharing strategy will always involve at least one direction, regardless of the parameter condition. Furthermore, the co-existence of horizontal and vertical sharing occurs in the majority of possible scenarios. This implies that in order to enhance the performance of the hybrid-channel platform supply chain, it is essential for all members to forecast demand and obtain more signals from multiple directions.



(a) $V_1^{N(v_1,v_2)}$ and $V_2^{N(v_1,v_2)}$ (b) $V_p^{N(v_1,v_2)}$ and $V^{N(v_1,v_2)}$

(c) $V_1^{N(v_1,v_2)}$ and $V_2^{N(v_1,v_2)}$ (d) $V_p^{N(v_1,v_2)}$ and $V^{N(v_1,v_2)}$

FIGURE 6. Impacts of r on values of vertical sharing without horizontal sharing.

6. Numerical analysis

To gain greater insights into managerial implications, this study conducts numerical analysis to explore the impacts of key parameters including hybrid-channel competition intensity and correlation coefficients on the values of vertical and horizontal sharing. With reference to Mishra *et al.* [64] and Liang and Ye [10], the default parameter values are set as: $\sigma = 50, \sigma_1 = \sigma_2 = 25, \sigma_p = 20, \alpha = 0.3, r = 0.5, \rho_1 = \rho_2 = 0.4$.

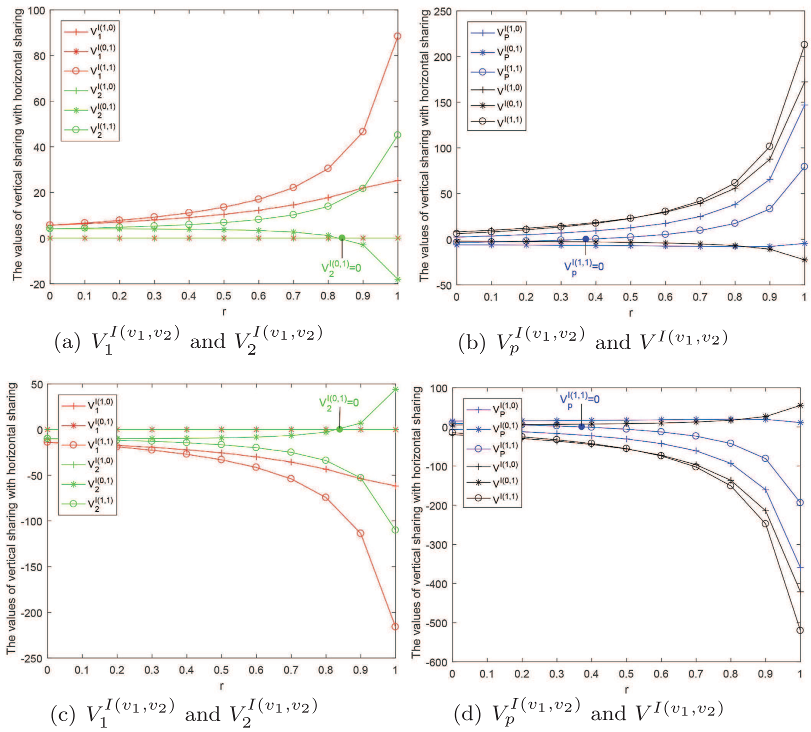
6.1. Impacts of key parameters on values of vertical sharing

In this subsection, this study examines the impacts of key parameters on the values of vertical sharing for the whole supply chain and its members.

6.1.1. Impacts of hybrid-channel competition intensity

We first examine the impacts of hybrid-channel competitive intensity r on the values of vertical sharing without horizontal sharing. According to Proposition 4 and Figure 3, two parameter scenarios are selected to illustrate different vertical sharing strategies without horizontal sharing. Specifically, $\rho_1 = \rho_2 = 0.4$ in Figures 6a and 6b, and $\rho_1 = 0.1$ and $\rho_2 = 0.6$ in Figures 6c and 6d. The changes of $V_1^{N(v_1,v_2)}$ and $V_2^{N(v_1,v_2)}$ with r are shown in Figures 6a and 6c, and the changes of $V_p^{N(v_1,v_2)}$ and $V^{N(v_1,v_2)}$ are shown in Figures 6b and 6d.

From Figures 6a and 6c, it can be observed that no matter which vertical-sharing strategy the platform retailer implements without horizontal sharing, two suppliers' values of vertical sharing increase with channel competition intensity. The more intense the channel competition, the more sharply two suppliers' values of vertical sharing increase. It can be visualized that $V_1^{N(v_1,v_2)}$ and $V_2^{N(v_1,v_2)}$ are positive, suggesting that vertical sharing is always beneficial to both suppliers without horizontal sharing, which is consistent with Proposition 4.



(a) $V_1^{I(v_1,v_2)}$ and $V_2^{I(v_1,v_2)}$ (b) $V_p^{I(v_1,v_2)}$ and $V^{I(v_1,v_2)}$

(c) $V_1^{I(v_1,v_2)}$ and $V_2^{I(v_1,v_2)}$ (d) $V_p^{I(v_1,v_2)}$ and $V^{I(v_1,v_2)}$

FIGURE 7. Impacts of r on values of vertical sharing with horizontal sharing.

In addition, it can be seen that $V_1^{N(1,1)} \geq V_1^{N(1,0)} > V_1^{N(0,1)}$ and $V_2^{N(1,1)} \geq V_2^{N(1,0)}(V_2^{N(0,1)})$, which show that vertical sharing with both suppliers is the most beneficial for two suppliers.

It follows from Figures 6b and 6d that as the channel competition intensifies, the values of vertical sharing $V_p^{N(v_1,v_2)}$ and $V^{N(v_1,v_2)}$ change sharply. All values of vertical sharing for the whole supply chain increase with r no matter which vertical-sharing strategy the platform retailer implements without horizontal sharing. It can be visualized that when $r > 0.6$, $V^{N(1,1)} > V^{N(1,0)} > V^{N(0,1)} > 0$, which shows that when channel competition intensity is higher, for the whole supply chain, vertical sharing with both suppliers is superior to sharing only with one supplier. For the platform retailer, as the channel competition intensifies, $V_p^{N(1,0)}(> 0)$ increases and $V_p^{N(0,1)}(< 0)$ decreases, whereas the change of $V_p^{N(1,1)}$ is uncertain. Specifically, in Figure 6b where the correlation coefficients are median ($\rho_1 = \rho_2 = 0.4$), $V_p^{N(1,1)}(< 0)$ decreases slowly with r; in Figure 6d where ρ_1 is lower and ρ_2 is high ($\rho_1 = 0.1, \rho_2 = 0.6$), $V_p^{N(1,1)}$ increases from negative to positive with r. Furthermore, it follows from $V_p^{N(1,0)} > V_p^{N(1,1)} > V_p^{N(0,1)}$ that the optimal vertical-sharing strategy for platform retailer is only to share with supplier S_1 without horizontal sharing.

Then, we examine the impacts of r on the values of vertical sharing with horizontal sharing. According to Proposition 5 and Figure 4a, two parameter scenarios are selected to illustrate different vertical sharing strategies with horizontal sharing. Specifically, $\rho_1 = \rho_2 = 0.4$ satisfying $\Phi^I > 0$ in Figures 7a and 7b, and $\rho_1 = \rho_2 = 0.75$ satisfying $\Phi^I < 0$ in Figures 7c and 7d. The changes of $V_1^{I(v_1,v_2)}$ and $V_2^{I(v_1,v_2)}$ with r are shown in Figures 7a and 7c, and the changes of $V_p^{I(v_1,v_2)}$ and $V^{I(v_1,v_2)}$ are shown in Figures 7b and 7d.

From Figures 7a and 7b where two correlation coefficients are not high, we can see that different from Figures 6a and 6b without horizontal sharing, (1) $V_1^{I(0,1)} = V_2^{I(1,0)} = 0$, which shows that two suppliers' values of vertical sharing without the platform retailer's signal f_p are unchanged with r; (2) as the channel competition r increases, $V_2^{I(0,1)}$ decreases from positive to negative, while $V_p^{I(1,1)}$ increases from negative to positive; (3) $V^{I(0,1)}(<0)$ decreases with r. Similarly, when channel competition r is high (about greater than 0.6), vertical sharing with both suppliers is the most beneficial for two suppliers and the whole supply chain with horizontal sharing, and the optimal vertical-sharing strategy for platform retailer is only sharing with supplier S_1. Moreover, the more intense the channel competition, the more benefits for the whole supply chain and its members.

It follows from Figures 7c and 7d where correlation coefficients are sufficiently high, the changes of values of vertical sharing are opposite with channel competition r compared with Figures 7a and 7b. In addition, it can be obtained that $V_1^{I(1,1)} \leq V_1^{I(1,0)} < V_1^{I(0,1)} = 0$; $V_2^{I(1,1)} < V_2^{I(0,1)} < V_2^{I(1,0)} = 0$ if $r < 0.84$ and $V_2^{I(1,1)} < V_2^{I(1,0)} = 0 < V_2^{I(0,1)}$ if $r > 0.84$ in Figure 7c, which show that the platform retailer's vertical sharing strategies with horizontal sharing are detrimental for both suppliers, except that sharing only with supplier S_2 is beneficial for him when r is sufficiently high. It follows from Figure 7d that $V_p^{I(1,0)} < V_p^{I(1,1)} < V_p^{I(0,1)}(>0)$ and $V^{I(1,1)}(V^{I(1,0)}) < 0 < V^{I(0,1)}$, which show that vertical sharing only with supplier S_2 is the optimal for the platform retailer and the whole supply chain.

Through comparing Figures 6a and 6b with Figures 7a and 7b, we analyze the difference in the values of vertical sharing without and with horizontal sharing. Specifically, (1) $V_1^{I(v_1,v_2)} < V_1^{N(v_1,v_2)}$ and $V_2^{I(v_1,v_2)} < V_2^{N(v_1,v_2)}$, which show that two suppliers' values of vertical sharing become less valuable with horizontal sharing than without horizontal sharing. (2) $0 < V_p^{I(1,0)} < V_p^{N(1,0)}$, $V_p^{N(0,1)} < V_p^{I(0,1)} < 0$ and $V_p^{N(1,1)} < V_p^{I(1,1)}$, which show that horizontal sharing undermines the platform retailer's benefits of vertical sharing with supplier S_1 and mitigates her disadvantages of vertical sharing with supplier S_2. (3) $V^{I(1,0)} < V^{N(1,0)}$ and $V^{I(1,1)} < V^{N(1,1)}$, which show that horizontal sharing decreases the value of vertical sharing in strategies $(1,0)$ and $(1,1)$. In summary, when channel competition is greater than 0.5, horizontal sharing reduces the benefits of vertical sharing for the whole supply chain and its members, except for the benefit for platform retailer under strategy $(1,1)$ which increases with horizontal sharing.

6.1.2. Impacts of correlation coefficients

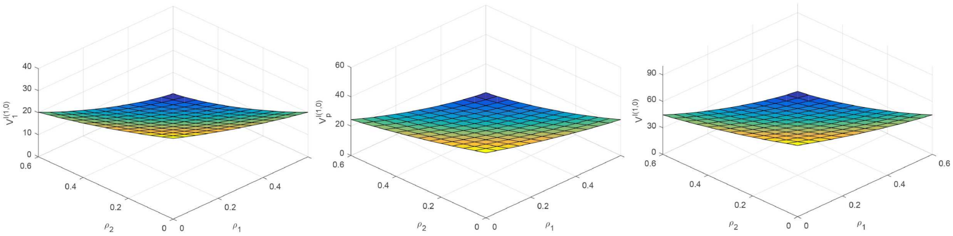
This paper examines the impacts of correlation coefficients (*i.e.*, ρ_1 and ρ_2) on the values of vertical sharing. The optimal vertical-sharing strategy with horizontal sharing is $(1,0)$ or $(0,1)$[10], which depends on certain parameter conditions. Therefore, we take $\rho_1, \rho_2 \in (0, 0.6)$ satisfying $\Phi^I > 0$ in Figures 8a–8c where respectively illustrate the corresponding changes of $V_1^{I(1,0)}, V_p^{I(1,0)}$ and $V^{I(1,0)}$. We take $\rho_1, \rho_2 \in (0.75, 0.8)$ and $r = 0.9$ satisfying $\Phi^I < 0$ and $\Phi_2^{I(0,1)} < 0$ in Figures 8d–8f where respectively illustrate the corresponding changes of values $V_2^{I(0,1)}, V_p^{I(0,1)}$ and $V^{I(0,1)}$. Furthermore, according to proposition 5, $V_2^{I(1,0)} = 0$ and $V_1^{I(0,1)} = 0$, and these values are therefore omitted.

From Figures 8a–8c where correlation coefficients are not too high, it can be intuitively seen that $V_1^{I(1,0)}$, $V_p^{I(1,0)}$ and $V^{I(1,0)}$ have a similar decrease trend with correlation coefficients ρ_1 and ρ_2. In Figures 8d–8f where correlation coefficients and channel competition are high, $V_2^{I(0,1)}$, $V_p^{I(0,1)}$ and $V^{I(0,1)}$ all decrease with ρ_1 and ρ_2. Therefore, no matter under which sharing strategy, the lower two signal correlations, the higher the values of vertical sharing for the whole supply chain and its members. This is because the higher signal correlations suggest that there is a higher degree of overlap between the signals of suppliers and platform retailer, and therefore less value is obtained from vertical sharing.

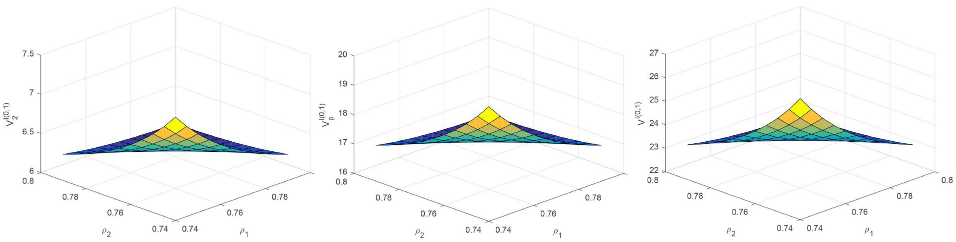
6.2. Impacts of key parameters on values of horizontal sharing

In this subsection, this study examines the impacts of key parameters on the values of horizontal sharing for two suppliers (*i.e.*, H_1 and H_2).

[10] Given that the values of vertical sharing under the optimal vertical-sharing strategy $(0,0)$ are always zero, this scenario is not taken into account in numerical analysis.

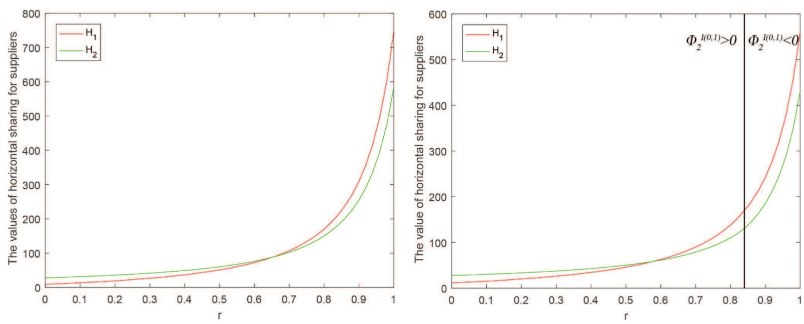


(a) $V_1^{I(1,0)}$ (b) $V_p^{I(1,0)}$ (c) $V^{I(1,0)}$



(d) $V_2^{I(0,1)}$ (e) $V_p^{I(0,1)}$ (f) $V^{I(0,1)}$

FIGURE 8. Impacts of ρ_1 and ρ_2.



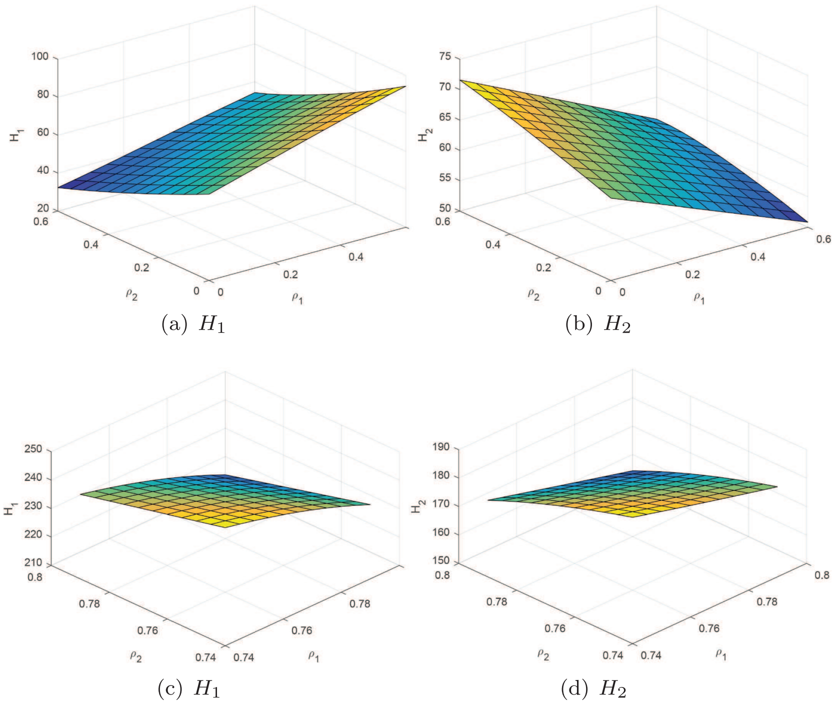
(a) $\rho_1 = \rho_2 = 0.4$ (b) $\rho_1 = \rho_2 = 0.75$

FIGURE 9. Impacts of r on the values of horizontal sharing.

Firstly, we examine the impacts of hybrid-channel competition intensity r on the values of horizontal sharing. Combining with equations (9) and (10), we take that $\rho_1 = \rho_2 = 0.4$ satisfying $\Phi^I > 0$ in Figure 9a, and $\rho_1 = \rho_2 = 0.75$ satisfying $\Phi^I < 0$ in Figure 9b where $\Phi_2^{I(0,1)}$ changes from positive to negative as r increases.

From Figures 9a and 9b, it can be seen that no matter what the values of correlation coefficients, both H_1 and H_2 slowly increase at first and then sharply increase with channel competition intensity r. This result suggests that the more intense the channel competition, the more both suppliers can benefit from horizontal sharing regardless of signal correlation coefficients. In addition, horizontal sharing is more beneficial for supplier S_2 when channel competition intensity r is low, whereas it is more beneficial for supplier S_1 when r is high.

Then, this study examines the impacts of correlation coefficients on H_1 and H_2. According to equations (9) and (10), we take that $\rho_1, \rho_2 \in (0, 0.6)$ satisfying $\Phi^I > 0$ in Figures 10a and 10b, where respectively illustrate the changes of H_1 and H_2. Similarly, we take $\rho_1, \rho_2 \in (0.75, 0.8)$ and $r = 0.9$ satisfying $\Phi^I < 0$ and $\Phi_2^{I(0,1)} < 0$ in Figures 10c and 10d, where respectively illustrate the changes of H_1 and H_2.



(a) H_1 (b) H_2

(c) H_1 (d) H_2

FIGURE 10. Impacts of ρ_1 and ρ_2 on the values of horizontal sharing.

From Figures 10a and 10b where correlation coefficients are not high, it can be seen that H_i increases with ρ_i while decreases with ρ_{3-i} $(i = 1, 2)$, which shows that the value of horizontal sharing is higher when the supplier's own signal has a high correlation with the platform retailer's signal f_p and the other supplier's signal has a low correlation with f_p. This also suggests that the supplier can compensate for his informational deficiency caused by high overlap and enrich his forecast information through horizontal sharing with another supplier. Moreover, through comparing Figure 10a with Figure 10b, it can be seen that correlation coefficients ρ_1 and ρ_2 have a greater impact on H_1 than H_2. This is because when correlation coefficients are not high ($i.e.$, $\Phi^I > 0$), only supplier S_1 can obtain the platform retailer's signal f_p, thus H_1 is more sensitive to the changes in correlation coefficients.

From Figures 10c and 10d where correlation coefficients and channel competition are high, both H_1 and H_2 decrease with ρ_1 and ρ_2, which indicates that when correlation coefficients fluctuate within a range of high values, horizontal sharing between two suppliers is more valuable with a lower correlation. This is because when the correlation coefficient is sufficiently large, the signals of two suppliers may have some degree of overlap, and thus horizontal sharing cannot provide more additional information about market demand.

7. DISCUSSION OF IMPLICATIONS

This section discusses the theoretical, managerial, and practical implications.

In the theoretical aspect, this paper enriches the existing literature on demand signal sharing in supply chain management by studying bi-direction signal-sharing strategy in a hybrid-channel platform supply chain. Specifically, it explores vertical-sharing strategy of platform retailer with suppliers and horizontal-sharing strategy between two competing suppliers who adopt reselling channel and agency channel, respectively. In addition, this paper considers that all supply chain members possess demand forecast signals, and systematically analyzes how hybrid-channel competition intensity and signal correlation influence the incentives for bi-direction signal sharing, providing a more in-depth understanding and offering additional insights for related theory.

In the managerial aspect, this paper provides some managerial insights for enterprises on demand signal sharing in a hybrid-channel platform supply chain. From the suppliers' standpoint, we suggest that the suppliers (adopting reselling channel and agency channel respectively) share their signals with each other regardless of the platform retailer's vertical-sharing strategy. A typical real-world example is Alibaba's digital supply chain (*i.e.*, Alibaba DChain): more than 50 000 suppliers have accessed its dedicated supply chain system to share sales forecast data, jointly improve the accuracy of demand prediction algorithms, and ultimately achieve efficient collaboration in sales forecasting[11]. From platform retailer's perspective, if two suppliers implement horizontal sharing, we suggest her sharing signal only with the supplier adopting agency channel when signal correlations between the platform retailer and two suppliers are not too high, and sharing signal only with the supplier adopting reselling channel when both signal correlations and hybrid-channel competition intensity are high. Otherwise, it is advisable for the platform retailer not to share signals with any supplier. If horizontal sharing is not implemented, the platform retailer should always share signal only with the supplier adopting agency channel. A real research on platform signal-sharing states that when competition among suppliers is intense, platform retailer can adjust their signal-sharing partner (reselling supplier or agency supplier) based on actual scenarios such as high demand uncertainty or precise demand signal[12].

In the practical aspect, the model established in this paper can be applied to hybrid-channel platform supply chains that feature supplier competition and involve all members possessing demand forecasting capabilities, such as hybrid e-commerce platforms (Amazon, Walmart, JD) and suppliers (Apple, Huawei). A decision matrix tailored to signal correlation and competition intensity can be developed to guide bi-direction signal-sharing. Taking the retail platform JD as an example, the decision matrix can be designed with clear decision rules: if signal correlation between JD and suppliers is low and hybrid-channel competition is moderate, JD should share signals only with supplier adopting agency channel while encouraging horizontal sharing among suppliers; if correlation is high and competition is intense, JD shifts vertical sharing to reselling supplier. This decision matrix can be embedded in JD's supplier management system, allowing category managers to input real-time data, such as monthly signal correlation calculated *via* sales data, to get instant optimal signal-sharing recommendations.

8. Conclusion

Our research investigates the optimal bi-direction signal-sharing strategy in a hybrid-channel platform supply chain where two suppliers sell substitutable products through agency channel and reselling channel, respectively. Through theoretical analysis, this research gives the optimal bi-direction signal-sharing strategy. It is found that two suppliers are always willing to conduct horizontal sharing, while the platform retailer's optimal vertical-sharing strategy depends on hybrid-channel competition intensity and signal correlations between platform retailer and suppliers. Different from the results of previous studies that the platform retailer never voluntarily shares signal in the reselling channel, this study shows that when signal correlations and channel competition are high, the platform retailer is willing to share signal with the supplier adopting reselling channel. The reason is that this study considers suppliers' forecast signals, which exist correlation with that of the platform retailer. Under this framework, high signal correlation weakens double marginalization caused by signal sharing, while intense competition enables signal sharing to bring benefits to the supplier adopting reselling channel. We also find that horizontal sharing weakens the vertical-sharing incentive of platform retailer and reduces the benefits from vertical sharing for two suppliers and the whole supply chain. Numerical analysis further reveals the impacts of hybrid-channel competition intensity and signal correlation on signal-sharing strategy and the values of signal sharing.

[11] https://ascp.alibaba.com/#/, accessed on October 8, 2025.

[12] https://utl.sztu.edu.cn/info/1087/3099.htm, accessed on October 8, 2025.

Given that our study has certain limitations, there are some directions for future research. First, this study assumes that suppliers and the platform retailer play Stackelberg game, with suppliers acting as leaders and the platform retailer as the follower. With the development of e-commerce platform, some platform retailers have equivalent bargaining power with suppliers, which motivates us to explore bi-direction signal-sharing under Nash game. Second, this paper assumes that each firm possesses a private demand signal but does not consider any costs associated with acquiring or processing these signals. Further research may explore the trade-off between the costs of investing in novel technology and the benefits of improved signal accuracy. Finally, considering the trend of numerous suppliers selling products in omni-channel supply chains, the inclusion of offline channel becomes crucial in established models, which may result in information leakage and influence the signal-sharing strategy.

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Data availability statement

The research data associated with this article are included in the article.

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Appendix A.

The expansions of conditional expectations in Table 7

$$
\begin{aligned}
E[\hat{\theta}_i^2] &= T_i^2(\sigma^2+\sigma_i^2)\\
E[\hat{\theta}_{12}^2] &= L_1^2(\sigma^2+\sigma_1^2)+2L_1L_2\sigma^2+L_2^2(\sigma^2+\sigma_2^2)\\
E[\hat{\theta}_{ip}^2] &= G_i^2(\sigma^2+\sigma_i^2)+2G_i\tilde{G}_i(\sigma^2+\rho_i\sigma_i\sigma_p)+\tilde{G}_i^2(\sigma^2+\sigma_p^2)\\
E[\hat{\theta}_{12p}^2] &= (K_1+K_2+K_3)^2\sigma^2+K_1^2\sigma_1^2+K_2^2\sigma_2^2+K_3^2\sigma_p^2+2K_1K_3\rho_1\sigma_1\sigma_p+2K_2K_3\rho_2\sigma_2\sigma_p\\
E[\hat{\theta}_1\hat{\theta}_2] &= T_1T_2\sigma^2\\
E[\hat{\theta}_{ip}\hat{\theta}_{3-i}] &= G_iT_{3-i}\sigma^2+T_{3-i}\tilde{G}_i(\sigma^2+\rho_{3-i}\sigma_{3-i}\sigma_p)\\
E[\hat{\theta}_{1p}\hat{\theta}_{2p}] &= \tilde{G}_1\tilde{G}_2(\sigma^2+\sigma_p^2)+G_1G_2\sigma^2+G_1\tilde{G}_2(\sigma^2+\rho_1\sigma_1\sigma_p)+G_2\tilde{G}_1(\sigma^2+\rho_2\sigma_2\sigma_p)\\
E[\hat{\theta}_{12p}\hat{\theta}_{12}] &= (K_1+K_2+K_3)(L_1+L_2)\sigma^2+K_1L_1\sigma_1^2+K_2L_2\sigma_2^2+L_1K_3\rho_1\sigma_1\sigma_p+L_2K_3\rho_2\sigma_2\sigma_p\\
E[\hat{\theta}_{12p}\hat{\theta}_i] &= (K_1+K_2+K_3)T_i\sigma^2+K_iT_i\sigma_i^2+T_iK_3\rho_i\sigma_i\sigma_p\\
E[\hat{\theta}_{12p}\hat{\theta}_{ip}] &= K_iG_i(\sigma^2+\sigma_i^2)+K_3\tilde{G}_i(\sigma^2+\sigma_p^2)+K_{3-i}G_i\sigma^2+(K_i\tilde{G}_i+K_3G_i)(\sigma^2+\rho_i\sigma_i\sigma_p)\\
&\quad+K_{3-i}\tilde{G}_i(\sigma^2+\rho_{3-i}\sigma_{3-i}\sigma_p)\\
E[\hat{\theta}_i\hat{\theta}_{3-i|i}] &= T_{3-i}T_i^2(\sigma^2+\sigma_i^2)\\
E[\hat{\theta}_{3-i}\hat{\theta}_{ip|3-i}] &= T_{3-i}(G_iT_{3-i}+\tilde{G}_iU_{3-i})(\sigma^2+\sigma_{3-i}^2)\\
E[\hat{\theta}_i\hat{\theta}_{12p|i}] &= T_i(K_i+K_{3-i}T_i+K_3U_i)(\sigma^2+\sigma_i^2)\\
E[\hat{\theta}_{12}\hat{\theta}_{12p|12}] &= (L_1+L_2)(K_1+K_3\tilde{L}_1+K_2+K_3\tilde{L}_2)\sigma^2+L_1(K_1+K_3\tilde{L}_1)\sigma_1^2+L_2(K_2+K_3\tilde{L}_2)\sigma_2^2\\
E[\hat{\theta}_{ip}\hat{\theta}_{3-i|ip}] &= T_{3-i}(G_iJ_i(\sigma^2+\sigma_i^2)+(G_i\tilde{J}_i+\tilde{G}_iJ_i)(\sigma^2+\rho_i\sigma_i\sigma_p)+\tilde{G}_i\tilde{J}_i(\sigma^2+\sigma_p^2))\\
E[\hat{\theta_{ip}}\hat{\theta}_{(3-i)p|ip}] &= G_1G_2J_i(\sigma^2+\sigma_i^2)+(G_{3-i}(G_i\tilde{J}_i+\tilde{G}_iJ_i)+G_i\tilde{G}_{3-i})(\sigma^2+\rho_i\sigma_i\sigma_p)\\
&\quad+\tilde{G}_i(G_{3-i}\tilde{J}_i+\tilde{G}_{3-i})(\sigma^2+\sigma_p^2)\\
E[\hat{\theta}_{ip}\hat{\theta}_{12p|ip}] &= (G_i+\tilde{G}_i)(K_i+K_{3-i}J_i+K_3+K_{3-i}\tilde{J}_i)\sigma^2+G_i(K_i+K_{3-i}J_i)\sigma_i^2+\tilde{G}_i(K_3+K_{3-i}\tilde{J}_i)\sigma_p^2\\
&\quad+(\tilde{G}_i(K_i+K_{3-i}J_i)+G_i(K_3+K_{3-i}\tilde{J}_i))\rho_i\sigma_i\sigma_p.
\end{aligned}
$$

The conditional expectations below are proven to be equivalent after simplification.

$$
\begin{aligned}
&E[\hat{\theta}_i^2]=E[\hat{\theta}_{12p}\hat{\theta}_i]=E[\hat{\theta}_i\hat{\theta}_{12p|i}],\quad E[\hat{\theta}_{12}^2]=E[\hat{\theta}_{12}\hat{\theta}_{12p}]=E[\hat{\theta}_{12}\hat{\theta}_{12p|12}]\\
&E[\hat{\theta}_{ip}^2]=E[\hat{\theta}_{ip}\hat{\theta}_{12p}]=E[\hat{\theta}_{ip}\hat{\theta}_{12p|ip}],\quad E[\hat{\theta}_{ip}\hat{\theta}_{3-i}]=E[\hat{\theta}_{3-i}\hat{\theta}_{ip|3-i}]=E[\hat{\theta}_{ip}\hat{\theta}_{3-i|ip}]\\
&E[\hat{\theta}_1\hat{\theta}_2]=E[\hat{\theta}_i\hat{\theta}_{3-i|i}],\quad E[\hat{\theta}_{1p}\hat{\theta}_{2p}]=E[\hat{\theta}_{ip}\hat{\theta}_{(3-i)p|ip}].
\end{aligned}
$$

Parameters in Propositions 4–8

$$
\begin{aligned}
\Phi^{N(0,1)} &= (2(1-\alpha)r(3r+4)\rho_1\sigma_1\sigma_p-((7-3\alpha)r^2+6(1-\alpha)r+4)(\sigma_2^2-\rho_2\sigma_2\sigma_p))\sigma^2\\
&\quad+(2(1-\alpha)r(3r+4)\rho_1\sigma_2\sigma_p+((3\alpha+1)r^2-2(1-\alpha)r-4)(\sigma_1\sigma_2-\rho_2\sigma_1\sigma_p))\sigma_1\sigma_2\\
\Phi_p^{N(1,1)} &= (B_5+2rB_1(1+\alpha))E[\hat{\theta}_{1p}^2-\hat{\theta}_1^2]+B_2(B_2-2)E[\hat{\theta}_{2p}^2-\hat{\theta}_2^2]+2rB_1B_2(\alpha-1)E[\hat{\theta}_{1p}\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2]\\
\Phi^I &= \sigma^2(\sigma_1^2(\sigma_2^2+\sigma_p^2-\rho_1^2\sigma_p^2-2\rho_2\sigma_2\sigma_p)-2\rho_1\sigma_1\sigma_2\sigma_p(\sigma_2-\rho_2\sigma_p)+(1-\rho_2^2)\sigma_2^2\sigma_p^2)+\sigma_1^2\sigma_2^2\sigma_p^2(1-\rho_1^2-\rho_2^2)\\
\Phi_2^{I(0,1)} &= 8-2(\alpha^2+\alpha+6)r^2-3(\alpha-1)^2r^3+(6\alpha+2)r^4\\
\Phi_p^{I(1,1)} &= \alpha(16+28r+15r^2-13r^3-12r^4)-\alpha^2(11+18r+9r^2)r^2-(12-4r+19r^2-r^3-6r^4)\\
\Phi_p^{(1,1)} &= (B_5+2r(1+\alpha)B_1)E[(\hat{\theta}_{1p}^2-\hat{\theta}_1^2)-(\hat{\theta}_{12p}^2-\hat{\theta}_{12}^2)]-B_2(2-B_2)E[(\hat{\theta}_{2p}^2-\hat{\theta}_2^2)-(\hat{\theta}_{12p}^2-\hat{\theta}_{12}^2)]\\
&\quad-2r(1-\alpha)B_1B_2E[(\hat{\theta}_{1p}\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2)-(\hat{\theta}_{12p}^2-\hat{\theta}_{12}^2)]\\
\Phi^{(0,1)} &= 2(1-\alpha)rB_1B_2E[\hat{\theta}_1\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2]-B_2^2E[(\hat{\theta}_{2p}^2-\hat{\theta}_2^2)-(\hat{\theta}_{12p}^2-\hat{\theta}_{12}^2)].
\end{aligned}
$$

Appendix B.

Proof of Lemma 1. (a) Based on the assumption of signal structure, $(\theta,\ f_1,\ f_2,\ f_p)$ is a multivariate normal vector with the following covariance matrix:

$$\Sigma_{(\theta,f_1,f_2,f_p)} = \left[\begin{array}{c|ccc} \sigma^2 & \sigma^2 & \sigma^2 & \sigma^2 \\ \hline \sigma^2 & \sigma^2+\sigma_1^2 & \sigma^2 & \sigma^2+\rho_1\sigma_1\sigma_p \\ \sigma^2 & \sigma^2 & \sigma^2+\sigma_2^2 & \sigma^2+\rho_2\sigma_2\sigma_p \\ \sigma^2 & \sigma^2+\rho_1\sigma_1\sigma_p & \sigma^2+\rho_2\sigma_2\sigma_p & \sigma^2+\sigma_p^2 \end{array}\right] = \left[\begin{array}{c|c} \Sigma_{11} & \Sigma_{12} \\ \hline \Sigma_{21} & \Sigma_{22} \end{array}\right].$$

If the signal set $\Theta = \{f_1, f_2, f_p\}$, the conditional expectation for market condition is $E[\theta|\Theta] = E[\theta|f_1, f_2, f_p] = \Sigma_{12}\Sigma_{22}^{-1}(f_1, f_2, f_p)^T = K_1 f_1 + K_2 f_2 + K_3 f_p \triangleq E[\hat{\theta}_{12p}]$, where

$$K_1 = \frac{\sigma^2\sigma_2\sigma_p(\sigma_2\sigma_p(1-\rho_2^2)+\rho_1\sigma_1(\rho_2\sigma_p-\sigma_2))}{\sigma_1^2\sigma_2^2\sigma_p^2\left[\left(1-\sum_{i=1}^2\rho_i^2\right)+\sigma^2\left(\frac{1}{\sigma_p^2}-\left(\frac{\rho_1}{\sigma_2}-\frac{\rho_2}{\sigma_1}\right)^2+\sum_{i=1}^2\frac{1}{\sigma_i^2}\left(1-\frac{2\rho_i\sigma_i}{\sigma_p}\right)\right)\right]},$$

$$K_2 = \frac{\sigma^2\sigma_1\sigma_p\left(\sigma_1\sigma_p\left(1-\rho_1^2\right)+\rho_2\sigma_2(\rho_1\sigma_p-\sigma_1)\right)}{\sigma_1^2\sigma_2^2\sigma_p^2\left[\left(1-\sum_{i=1}^2\rho_i^2\right)+\sigma^2\left(\frac{1}{\sigma_p^2}-\left(\frac{\rho_1}{\sigma_2}-\frac{\rho_2}{\sigma_1}\right)^2+\sum_{i=1}^2\frac{1}{\sigma_i^2}\left(1-\frac{2\rho_i\sigma_i}{\sigma_p}\right)\right)\right]},$$

$$K_3 = \frac{\sigma^2\sigma_1\sigma_2(\sigma_1\sigma_2-\sigma_p(\rho_1\sigma_2+\rho_2\sigma_1))}{\sigma_1^2\sigma_2^2\sigma_p^2\left[\left(1-\sum_{i=1}^2\rho_i^2\right)+\sigma^2\left(\frac{1}{\sigma_p^2}-\left(\frac{\rho_1}{\sigma_2}-\frac{\rho_2}{\sigma_1}\right)^2+\sum_{i=1}^2\frac{1}{\sigma_i^2}\left(1-\frac{2\rho_i\sigma_i}{\sigma_p}\right)\right)\right]}.$$

Similarly, the other conditional expectations for market conditions, as shown in Table 4, can be calculated.

(b) $(f_1,\ f_2,\ f_p)$ is a multivariate normal vector with the following covariance matrix:

$$\Sigma_{(f_1,f_2,f_p)} = \left[\begin{array}{c|cc} \sigma^2+\sigma_1^2 & \sigma^2 & \sigma^2+\rho_1\sigma_1\sigma_p \\ \hline \sigma^2 & \sigma^2+\sigma_2^2 & \sigma^2+\rho_2\sigma_2\sigma_p \\ \sigma^2+\rho_1\sigma_1\sigma_p & \sigma^2+\rho_2\sigma_2\sigma_p & \sigma^2+\sigma_p^2 \end{array}\right] = \left[\begin{array}{c|c} \Sigma_{13} & \Sigma_{14} \\ \hline \Sigma_{23} & \Sigma_{24} \end{array}\right].$$

If demand signal $f = f_p$ and signal set $\Theta = \{f_1, f_2\}$, the conditional expectation for the demand signal is $E[f|\Theta] = E[f_p|f_1, f_2] = \Sigma_{14}\Sigma_{24}^{-1}(f_1, f_2)^T = \tilde{L}_1 f_1 + \tilde{L}_2 f_2 \triangleq E[\hat{\theta}_{p|12}]$, where $\tilde{L}_1 = \frac{\sigma^2(\sigma_2^2+\sigma_p(\rho_1\sigma_1-\rho_2\sigma_2))+\rho_1\sigma_1\sigma_2^2\sigma_p}{\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2}$, $\tilde{L}_2 = \frac{\sigma^2(\sigma_1^2+\sigma_p(\rho_2\sigma_2-\rho_1\sigma_1))+\rho_2\sigma_2\sigma_1^2\sigma_p}{\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2}$.

Through similar calculation described above, the results of the other conditional expectations for the shared demand signal, as shown in Table 5, can be obtained. □

Proof of Propositions 1–3. From Table 6, the results can be obtained straightforwardly, thus the detailed calculations are omitted. □

Proof of Proposition 4. (a) Through computation, the following result can be obtained:

$$\begin{aligned} V_1^{N(1,0)} &= F_1(1-\alpha)(\sigma^2((3r+4)\rho_1\sigma_1\sigma_p((\alpha+1)r^2-2)+r\rho_2\sigma_2\sigma_p(4-(3\alpha+1)r^2-2(\alpha-1)r) \\ &\quad +(3r+4)\sigma_1^2(2-(\alpha+1)r^2))+\sigma_1^2\sigma_2(r\rho_2\sigma_p(4-(3\alpha+1)r^2 \\ &\quad +2(1-\alpha)r)+(2\sigma_2-2\rho_1\sigma_2\sigma_p)(4-(\alpha+3)r^2-r^3+r))); \\ V_2^{N(1,0)} &= F_1(1-\alpha)r(4-(3\alpha+1)r^2+2(1-\alpha)r)(\sigma^2(\sigma_1^2-\rho_1\sigma_1\sigma_p+\rho_2\sigma_2\sigma_p)+\rho_2\sigma_1^2\sigma_2\sigma_p); \\ V_p^{N(1,0)} &= F_1\alpha(r(\sigma^2((8-5r^2)(\sigma_1^2-\rho_1\sigma_1\sigma_p)+2(r^2-2r-4)\rho_2\sigma_2\sigma_p)+\sigma_1\sigma_2((16-7r^2+4r)\rho_1\sigma_2\sigma_p \\ &\quad +2(4-r^2+2r)\rho_2\sigma_1\sigma_p+(7r^2-4r-16)\sigma_1\sigma_2))+2\alpha(\sigma^2(2r(r^2+2r+2)\rho_2\sigma_2\sigma_p \\ &\quad +(8-3r^3-2r^2+6r)(\sigma_1^2-\rho_1\sigma_1\sigma_p))+\sigma_1\sigma_2(2r(r^2+2r+2)\rho_2\sigma_1\sigma_p+(8-5r^3-6r^2+2r)(\sigma_1\sigma_2 \\ &\quad -\rho_1\sigma_2\sigma_p)))-\alpha^2r^2(\sigma^2(2(3r+2)\rho_2\sigma_2\sigma_p+3(3r+4)(\sigma_1^2-\rho_1\sigma_1\sigma_p))+\sigma_1\sigma_2(2(3r+2)\rho_2\sigma_1\sigma_p \\ &\quad +(3r+8)\sigma_2(\sigma_1-\rho_1\sigma_p)))), \end{aligned}$$

where

$$F_1 = \frac{(3r+4)\sigma^4\sigma_1(\sigma_1-\rho_1\sigma_p)}{2((3\alpha+5)r^2-8)^2(\sigma^2+\sigma_1^2)(\sigma^2+\sigma_2^2)(\sigma^2(\sigma_1^2+\sigma_p^2-2\rho_1\sigma_1\sigma_p)+(1-\rho_1^2)\sigma_1^2\sigma_p^2)} > 0.$$

According to parameter assumption $\rho_i\sigma_i\sigma_p < \min\{\sigma_i^2,\sigma_p^2\}$, it is easy to deduce $V_j^{N(1,0)} > 0$ where $j=1,2,p$. Hence, $V^{N(1,0)} = V_1^{N(1,0)} + V_2^{N(1,0)} + V_p^{N(1,0)} > 0$.

(b) Through computation, the following result can be obtained:

$$\begin{aligned}
V_1^{N(0,1)} &= F_2(1-\alpha)r(3r+4)(\sigma^2(\rho_1\sigma_1\sigma_p-\rho_2\sigma_2\sigma_p+\sigma_2^2)+\rho_1\sigma_1\sigma_2^2\sigma_p);\\
V_2^{N(0,1)} &= F_2(\sigma^2((4-(3\alpha+1)r^2+2(1-\alpha)r)(\sigma_2^2-\rho_2\sigma_2\sigma_p)+(1-\alpha)r(3r+4)\rho_1\sigma_1\sigma_p)\\
&\quad+\sigma_1\sigma_2(2\sigma_1(2+\alpha r-2r^2-r)(\sigma_2-\rho_2\sigma_p)+(1-\alpha)r(3r+4)\rho_1\sigma_2\sigma_p));\\
V_p^{N(0,1)} &= -\frac{1}{2}F_2(\sigma^2(3(4-(3\alpha+1)r^2+2(1-\alpha)r)(\sigma_2^2-\rho_2\sigma_2\sigma_p)+2(1-\alpha)r(3r+4)\rho_1\sigma_1\sigma_p)\\
&\quad+\sigma_1\sigma_2(\sigma_1(12-3(\alpha+3)r^2-2(1-\alpha)r)(\sigma_2-\rho_2\sigma_p)+2(1-\alpha)r(3r+4)\rho_1\sigma_2\sigma_p),
\end{aligned}$$

where

$$F_2 = \frac{\sigma^4\sigma_2(4-(3a+1)r^2+2(1-\alpha)r)(\sigma_2-\rho_2\sigma_p)}{2((3a+5)r^2-8)^2(\sigma^2+\sigma_1^2)(\sigma^2+\sigma_2^2)(\sigma^2(\sigma_2^2+\sigma_p^2-2\rho_2\sigma_2\sigma_p)+(1-\rho_2^2)\sigma_2^2\sigma_p^2)} > 0.$$

According to parameter assumption $\rho_i\sigma_i\sigma_p < \min\{\sigma_i^2,\sigma_p^2\}$, it is easy to deduce $V_i^{N(1,0)} > 0 (i=1,2)$, $V_p^{N(1,0)} < 0$. Through computation, $V^{N(0,1)} = \frac{1}{2}F_2\Phi^{N(0,1)} = \frac{1}{2}F_2(\sigma^2(2(1-\alpha)r(3r+4)\rho_1\sigma_1\sigma_p-((7-3\alpha)r^2+6(1-\alpha)r+4)(\sigma_2^2-\rho_2\sigma_2\sigma_p))+\sigma_1\sigma_2(2(1-\alpha)r(3r+4)\rho_1\sigma_2\sigma_p-\sigma_1(4-(3\alpha+1)r^2+2(1-\alpha)r)(\sigma_2-\rho_2\sigma_p)))$, thus the value of $V^{N(0,1)}$ depends on the parameter $\Phi^{N(0,1)}$, which is illustrated in Figure 3.

(c) Through computation, the following result can be obtained:

$$\begin{aligned}
V_1^{N(1,1)} &= \frac{(1-\alpha)B1}{2}((B_4+r)E[\hat{\theta}_{1p}^2-\hat{\theta}_1^2]+rB_2E[\hat{\theta}_{1p}\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2])\\
&= \frac{(3r+4)(1-\alpha)}{2(8-(3\alpha+5)r^2)}\left(\frac{2(4+r-(\alpha+3)r^2-r^3)}{(8-(3\alpha+5)r^2)}E[\hat{\theta}_{1p}^2-\hat{\theta}_1^2]\right.\\
&\quad\left.+\frac{r(4+2(1-\alpha)r-(1+3\alpha)r^2)}{(8-(3\alpha+5)r^2)}E[\hat{\theta}_{1p}\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2]\right);\\
V_2^{N(1,1)} &= \frac{B_3}{2}((1-B_2)E[\hat{\theta}_{2p}^2-\hat{\theta}_2^2]+(1-\alpha)rB_1E[\hat{\theta}_{1p}\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2])\\
&= \frac{4+2(1-\alpha)r-(3\alpha+1)r^2}{2(8-(3\alpha+5)r^2)}\left(\frac{4-2(1-\alpha)r-4r^2}{8-(3\alpha+5)r^2}E[\hat{\theta}_{2p}^2-\hat{\theta}_2^2]\right.\\
&\quad\left.+\frac{(1-\alpha)r(3r+4)}{8-(3\alpha+5)r^2}E[\hat{\theta}_{1p}\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2]\right);\\
V_p^{N(1,1)} &= \frac{1}{4}\Phi_p^{N(1,1)} = \frac{1}{4}((B_5+2rB_1(1+\alpha))E[\hat{\theta}_{1p}^2-\hat{\theta}_1^2]-B_2(2-B_2)E[\hat{\theta}_{2p}^2-\hat{\theta}_2^2]\\
&\quad-2rB_1B_2(1-\alpha)E[\hat{\theta}_{1p}\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2])\\
&= \frac{1}{4}\left(\frac{(3r+4)(a^2(3r+8)r^2+2a(5r^3+6r^2-2r-8)+(7r^2-4r-16)r)}{2(8-(3\alpha+5)r^2)}E[\hat{\theta}_{1p}^2-\hat{\theta}_1^2]\right.\\
&\quad-\frac{(12-3(\alpha+3)r^2-2(1-\alpha)r)(4-(3\alpha+1)r^2+2(1-\alpha)r)}{(8-(3\alpha+5)r^2)^2}E[\hat{\theta}_{2p}^2-\hat{\theta}_2^2]\\
&\quad\left.-\frac{2(1-\alpha)r(3r+4)(4-(3\alpha+1)r^2+2(1-\alpha)r)}{(8-(3\alpha+5)r^2)^2}E[\hat{\theta}_{1p}\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2]\right);\\
V^{N(1,1)} &= \frac{2B_1B_4(1-\alpha)+4rB_1+B_5}{4}E[\hat{\theta}_{1p}^2-\hat{\theta}_1^2]-\frac{B_2^2}{4}E[\hat{\theta}_{2p}^2-\hat{\theta}_2^2]+\frac{(1-\alpha)rB_1B_2}{2}E[\hat{\theta}_{1p}\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2]\\
&= \frac{(3r+4)(16-(3\alpha^2+6\alpha+11)r^3-4(\alpha^2+\alpha+2)r^2+20r)}{4(8-(3\alpha+5)r^2)^2}E[\hat{\theta}_{1p}^2-\hat{\theta}_1^2]
\end{aligned}$$

$$- \frac{(4+2(1-\alpha)r-(3\alpha+1)r^2)^2}{4(8-(3\alpha+5)r^2)^2} E[\hat{\theta}_{2p}^2 - \hat{\theta}_2^2] + \frac{r(1-\alpha)(3r+4)(4+2(1-\alpha)r-(3\alpha+1)r^2)}{2(8-(3\alpha+5)r^2)^2} E[\hat{\theta}_{1p}\hat{\theta}_{2p} - \hat{\theta}_1\hat{\theta}_2],$$

where $E[\hat{\theta}_{ip}^2 - \hat{\theta}_i^2] = \frac{(\sigma^4\sigma_i^2(\sigma_i-\rho_i\sigma_p)^2)}{(\sigma^2+\sigma_i^2)(\sigma^2(\sigma_i^2+\sigma_p^2-2\rho_i\sigma_i\sigma_p)+\sigma_i^2\sigma_p^2(1-\rho_i^2))} > 0$, $E[\hat{\theta}_{1p}\hat{\theta}_{2p} - \hat{\theta}_1\hat{\theta}_2] = (\sigma^8(\sigma_1^2(\sigma_2^4 + \sigma_2^2\sigma_p^2 - \rho_1^2\rho_2\sigma_2\sigma_p^3 + \rho_1^2\sigma_p^4 - 2\rho_2\sigma_2^3\sigma_p) - \rho_1\sigma_1\sigma_2\sigma_p(\rho_2^2\sigma_2\sigma_p^2 - 4\rho_2\sigma_2^2\sigma_p + \rho_2\sigma_p^3 + 2\sigma_2^3) - 2\rho_1\sigma_1^3\sigma_p(\sigma_2^2+\sigma_p^2-2\rho_2\sigma_2\sigma_p) + \sigma_1^4(\sigma_2^2+\sigma_p^2-2\rho_2\sigma_2\sigma_p) + \sigma_2^2\sigma_p^2(\sigma_2-\rho_2\sigma_p)^2) + \sigma^6\sigma_1\sigma_2(\sigma_1^3(\sigma_2\sigma_p^2(2-\rho_1^2-\rho_2^2) + \rho_1^2\rho_2\sigma_p^3 - 2\rho_2\sigma_2^2\sigma_p + \sigma_2^3) + \sigma_1\sigma_2\sigma_p^2((\sigma_2^2(2-\rho_1^2-\rho_2^2)) + \sigma_p^2(\rho_1^2+\rho_2^2-\rho_1^2\rho_2^2) + (\rho_1^2-2)\rho_2\sigma_2\sigma_p) + \rho_1\sigma_1^2\sigma_p((\rho_2^2-2)\sigma_2\sigma_p^2 + 4\rho_2\sigma_2^2\sigma_p - \rho_2\sigma_p^3 - 2\sigma_2^3) + \rho_1\rho_2\sigma_2^2\sigma_p^3(\rho_2\sigma_2-\sigma_p)) + \sigma^4\sigma_1^3\sigma_2^3\sigma_p^2(\sigma_1\sigma_2(1-\rho_1^2-\rho_2^2) + \rho_1^2\rho_2\sigma_1\sigma_p + \rho_1\rho_2\sigma_p(\rho_2\sigma_2-\sigma_p))) \prod_{i=1}^2 \frac{1}{(\sigma^2+\sigma_i^2)(\sigma^2(\sigma_i^2+\sigma_p^2-2\rho_i\sigma_i\sigma_p)+(1-\rho_i^2)\sigma_i^2\sigma_p^2)} > 0$. According to parameter assumption $\rho_i\sigma_i\sigma_p < \min\{\sigma_i^2, \sigma_p^2\}$, it can deduce $V_i^{N(1,1)} > 0$ $(i = 1, 2)$, $V^{N(1,1)} > 0$. However, $V_p^{N(1,1)}$ depends on $\Phi_p^{N(1,1)}$, which is illustrated in Figure 3.

□

Proof of the optimal vertical-sharing strategy without horizontal sharing

Proof. According to the analysis of Proposition 4, $V_p^{N(1,0)} > 0$ and $V_p^{N(0,1)} < 0$, whereas $V_p^{N(1,1)}$ depends on parameter $\Phi_p^{N(1,1)}$. Therefore, it is only necessary to compare the strategies $(1,0)$ and $(1,1)$.

We make the following discussions.

(i) When $\Phi_p^{N(1,1)} \leq 0$, it can be obtained that $V_p^{N(1,1)} < 0$, thus platform retailer will not voluntarily share signal with two suppliers. Because $V_p^{N(1,0)} > 0$, the optimal vertical-sharing strategy is $(1,0)$ in this scenario.

(ii) When $\Phi_p^{N(1,1)} > 0$, a comparison is made between the values of $V_p^{N(1,0)}$ and $V_p^{N(1,1)}$. Through computation,

$$V_p^{N(1,0)} - V_p^{N(1,1)} = \frac{1}{4}(B_2(2-B_2)E[\hat{\theta}_{2p}^2 - \hat{\theta}_2^2] - 2rB_1B_2(1-\alpha)E[\hat{\theta}_{1p}\hat{\theta}_{2p} - \hat{\theta}_{1p}\hat{\theta}_2]).$$

According to parameter assumption $\rho_i\sigma_i\sigma_p < \min\{\sigma_i^2, \sigma_p^2\}$, it can be concluded that $V_p^{N(1,0)} > V_p^{N(1,1)}$, which indicates that the platform retailer can obtain more benefits from strategy $(1,0)$.

In summary, the optimal vertical-sharing strategy without horizontal sharing is $(1,0)$. □

Proof of Proposition 5. (a) Through computation, the following result can be obtained:

$$V_1^{I(1,0)} = \frac{(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)((1-\alpha)(3r+4)\sigma^4\sigma_1^2\sigma_2^2(4-r^3-(\alpha+3)r^2+r)(\rho_1\sigma_2\sigma_p+\rho_2\sigma_1\sigma_p-\sigma_1\sigma_2)^2)}{((3\alpha+5)r^2-8)^2\Phi^I};$$

$$V_2^{I(1,0)} = 0;$$

$$V_p^{I(1,0)} = \frac{(3r+4)\sigma^4\sigma_1^2\sigma_2^2(2\alpha(8-5r^3-6r^2+2r)+(16-7r^2+4r)r-\alpha^2(3r+8)r^2)(\rho_1\sigma_2\sigma_p+\rho_2\sigma_1\sigma_p-\sigma_1\sigma_2)^2}{4((3\alpha+5)r^2-8)^2(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)\Phi^I};$$

$$V^{I(1,0)} = \frac{(3r+4)\sigma^4\sigma_1^2\sigma_2^2(16-(3\alpha^2+6\alpha+11)r^3-4(\alpha^2+\alpha+2)r^2+20r)(\rho_1\sigma_2\sigma_p+\rho_2\sigma_1\sigma_p-\sigma_1\sigma_2)^2}{4((3\alpha+5)r^2-8)^2(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)\Phi^I}.$$

Excluding $\Phi^I = \sigma^2(\sigma_1^2(\sigma_2^2+\sigma_p^2-\rho_1^2\sigma_p^2-2\rho_2\sigma_2\sigma_p) - 2\rho_1\sigma_1\sigma_2\sigma_p(\sigma_2-\rho_2\sigma_p) + (1-\rho_2^2)\sigma_2^2\sigma_p^2) + \sigma_1^2\sigma_2^2\sigma_p^2(1-\rho_1^2-\rho_2^2)$, it is obvious that the remainders of $V_1^{I(1,0)}, V_p^{I(1,0)}$ and $V^{I(1,0)}$ are positive according to parameter assumption $\rho_i\sigma_i\sigma_p < \min\{\sigma_i^2, \sigma_p^2\}$, thus the signs of $V_1^{I(1,0)}, V_p^{I(1,0)}$ and $V^{I(1,0)}$ depend on Φ^I, which is affected by correlation coefficients ρ_1 and ρ_2 and illustrated in Figure 4a.

(b) Through computation, the following result can be obtained:

$$V_1^{I(0,1)} = 0;$$

$$V_2^{I(0,1)} = \frac{\sigma^4\sigma_1^2\sigma_2^2(\rho_1\sigma_2\sigma_p+\rho_2\sigma_1\sigma_p-\sigma_1\sigma_2)^2\Phi_2^{I(0,1)}}{((3\alpha+5)r^2-8)^2(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)\Phi^I};$$

$$V_p^{I(0,1)} = -\frac{\begin{array}{c}\sigma^4\sigma_1^2\sigma_2^2((9\alpha^2+30\alpha+9)r^4-4(\alpha^2+10\alpha+13)r^2+16(\alpha-1)r^3\\ -16(\alpha-1)r+48)(\rho_1\sigma_2\sigma_p+\rho_2\sigma_1\sigma_p-\sigma_1\sigma_2)^2\end{array}}{4((3\alpha+5)r^2-8)^2(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)\Phi^I};$$

$$V^{I(0,1)} = -\frac{\sigma\alpha^4\sigma_1^2\sigma_2^2((3\alpha+1)r^2+2(\alpha-1)r-4)^2(\rho_1\sigma_2\sigma_p+\rho_2\sigma_1\sigma_p-\sigma_1\sigma_2)^2}{4((3\alpha+5)r^2-8)^2(s^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)\Phi^I}.$$

Similarly, it is easy to see that the signs of $V_p^{I(0,1)}$ and $V^{I(0,1)}$ depend on Φ^I. And the sign of $V_2^{I(0,1)}$ depends on Φ^I and $\Phi_2^{I(0,1)}$, where $\Phi_2^{I(0,1)} = 8-2(\alpha^2+\alpha+6)r^2-3(\alpha-1)^2r^3+(6\alpha+2)r^4$, which is illustrated in Figures 4b.

(c) Through computation, the following result can be obtained:

$$V_1^{I(1,1)} = \frac{(1-\alpha)(3r+4)^2\sigma^4\sigma_1^2\sigma_2^2((2-\alpha+1)r^2)(\rho_1\sigma_2\sigma_p+\rho_2\sigma_1\sigma_p-\sigma_1\sigma_2)^2}{2((3\alpha+5)r^2-8)^2(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)\Phi^I};$$
$$V_2^{I(1,1)} = \frac{\sigma^4\sigma_1^2\sigma_2^2((3\alpha+1)r^2+2(\alpha-1)r-4)^2(\rho_1\sigma_2\sigma_p+\rho_2\sigma_1\sigma_p-\sigma_1\sigma_2)^2}{2((3\alpha+5)r^2-8)^2(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)\Phi^I};$$
$$V_p^{I(1,1)} = \frac{\sigma^4\sigma_1^2\sigma_2^2(\rho_1\sigma_2\sigma_p+\rho_2\sigma_1\sigma_p-\sigma_1\sigma_2)^2\Phi_p^{I(1,1)}}{(3\alpha+5)r^2-8)^2(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)\Phi^I};$$
$$V^{I(1,1)} = \frac{\sigma^4\sigma_1^2\sigma_2^2(12+(18-\alpha^2-10\alpha)r^2-(9\alpha+10)r^4-(17\alpha+15)r^3+4(9-\alpha))r(\rho_1\sigma_2\sigma_p+\rho_2\sigma_1\sigma_p-\sigma_1\sigma_2)^2}{(3\alpha+5)r^2-8)^2(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)\Phi^I}.$$

Similarly, it can be seen that the remainders of $V_i^{I(1,1)}$ and $V^{I(1,1)}$ are positive excluding Φ^I, thus the signs of them depend on Φ^I. Whereas $V_p^{I(1,1)}$ is affected by Φ^I and $\Phi_p^{I(1,1)}$, where $\Phi_p^{I(1,1)} = \alpha(16+28r+15r^2-13r^3-12r^4)-\alpha^2(11+18r+9r^2)r^2-(12-4r+19r^2-r^3-6r^4)$, which is illustrated in Figure 4b.

□

Proof of the optimal vertical-sharing strategy with horizontal sharing

Proof. According to the analysis of Proposition 5, the values of vertical sharing with horizontal sharing are all affected by parameter Φ^I , thus the following two scenarios are discussed.

(i) When $\Phi^I > 0$, it has been proved that $V_p^{I(1,0)} > 0$, $V_p^{I(0,1)} < 0$ and $V_p^{I(1,1)}$ depends on $\Phi_p^{I(1,1)}$, which indicates that strategy $(0,1)$ will not occur, thus we compare the values of $V_p^{I(1,0)}$ and $V_p^{I(1,1)}$. Through computation, $V_p^{I(1,0)} - V_p^{I(1,1)} = (1+\frac{(5-r-3r^2)(4+2(1-\alpha)r-(3\alpha+1)r^2)}{2(8-(3\alpha+1)r^2)^2})\frac{\sigma^4\sigma_1^2\sigma_2^2(\rho_1\sigma_2\sigma_p+\rho_2\sigma_1\sigma_p-\sigma_1\sigma_2)^2}{(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)\Phi^I} > 0$. Thus, the optimal vertical-sharing strategy in this scenario is $(1,0)$.

(ii) When $\Phi^I \leq 0$, it has been proved that $V_p^{I(1,0)} < 0$, $V_p^{I(0,1)} > 0$ and $V_p^{I(1,1)}$ depends on $\Phi_p^{I(1,1)}$. It indicates that strategy $(1,0)$ will not occur. Because $V_1^{I(1,1)} < 0$ and $V_2^{I(1,1)} < 0$ when $\Phi^I \leq 0$, vertical-sharing strategy $(1,1)$ cannot be implemented between platform retailer and two suppliers, thus strategy $(1,1)$ will also not occur. In addition, it follows from proposition 5 that $V_2^{I(0,1)} > 0$ if $\Phi_2^{I(0,1)} \leq 0$, thus the optimal vertical-sharing strategy is $(0,1)$, otherwise, the optimal strategy is $(0,0)$.

□

Proof of Proposition 6. (a) Through computation, the following result can be obtained:

$$V_1^{I(1,0)} - V_1^{N(1,0)} = -\frac{(1-\alpha)B_1(B_4+r)}{2}E[(\hat{\theta}_{1p}^2-\hat{\theta}_1^2)-(\hat{\theta}_{12p}^2-\hat{\theta}_{12}^2)] - \frac{(1-\alpha)rB_1B_2}{2}E[\hat{\theta}_{1p}\hat{\theta}_2-\hat{\theta}_1\hat{\theta}_2];$$
$$V_1^{I(0,1)} - V_1^{N(0,1)} = -\frac{(1-\alpha)rB_1B_2}{2}E[\hat{\theta}_1\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2];$$
$$V_1^{I(1,1)} - V_1^{N(1,1)} = -\frac{(1-\alpha)B_1(B_4+r)}{2}E[(\hat{\theta}_{1p}^2-\hat{\theta}_1^2)-(\hat{\theta}_{12p}^2-\hat{\theta}_{12}^2)] - \frac{(1-\alpha)rB_1B_2}{2}E[(\hat{\theta}_{1p}\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2)-(\hat{\theta}_{12p}^2-\hat{\theta}_{12}^2)];$$
$$V_2^{I(1,0)} - V_2^{N(1,0)} = -\frac{(1-\alpha)rB_1B_2}{2}E[\hat{\theta}_{1p}\hat{\theta}_2-\hat{\theta}_1\hat{\theta}_2];$$
$$V_2^{I(0,1)} - V_2^{N(0,1)} = -\frac{B_2(1-B_2)}{2}E[(\hat{\theta}_{2p}^2-\hat{\theta}_2^2)-(\hat{\theta}_{12p}^2-\hat{\theta}_{12}^2)] - \frac{(1-\alpha)rB_1B_2}{2}E[\hat{\theta}_1\hat{\theta}_{2p}-\hat{\theta}_1\hat{\theta}_2];$$
$$V_2^{I(1,1)} - V_2^{N(1,1)} = -\frac{B_2(1-B_2)}{2}E[(\hat{\theta}_{2p}^2-\hat{\theta}_2^2)-(\hat{\theta}_{12p}^2-\hat{\theta}_{12}^2)]$$

$$- \frac{(1-\alpha)rB_1B_2}{2}E[(\hat{\theta}_{1p}\hat{\theta}_{2p} - \hat{\theta}_1\hat{\theta}_2) - (\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2)].$$

According to parameter assumptions $\rho_i\sigma_i\sigma_p < \min\{\sigma_i^2, \sigma_p^2\}$ and the higher forecast accuracy of platform retailer, we can deduce that $E[\hat{\theta}_{ip}^2 - \hat{\theta}_i^2] > E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$, $E[\hat{\theta}_{1p}\hat{\theta}_{2p} - \hat{\theta}_1\hat{\theta}_2] > E[\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2]$ and $E[\hat{\theta}_{ip}\hat{\theta}_{3-i} - \hat{\theta}_1\hat{\theta}_2] > 0,\ (i = 1, 2)$. Thus, it is proven that $V_i^{I(v_1,v_2)} < V_i^{N(v_1,v_2)},\ v_i \in \{0, 1\},\ i = 1, 2$.

(b) Through computation, the following result can be obtained:

$$\begin{aligned}
V_p^{I(1,0)} - V_p^{N(1,0)} &= -\frac{B_5 + 2r(1+\alpha)B_1}{4}E[(\hat{\theta}_{1p}^2 - \hat{\theta}_1^2) - (\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2)] + \frac{(1-\alpha)rB_1B_2}{2}E[\hat{\theta}_{1p}\hat{\theta}_2 - \hat{\theta}_1\hat{\theta}_2];\\
V_p^{I(0,1)} - V_p^{N(0,1)} &= \frac{B_2(2-B_2)}{4}E[(\hat{\theta}_{2p}^2 - \hat{\theta}_2^2) - (\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2)] + \frac{(1-\alpha)rB_1B_2}{2}E[\hat{\theta}_1\hat{\theta}_{2p} - \hat{\theta}_1\hat{\theta}_2];\\
V_p^{I(1,1)} - V_p^{N(1,1)} &= -\frac{1}{4}\Phi_p^{(1,1)} = -\frac{1}{4}((B_5 + 2r(1+\alpha)B_1)E[(\hat{\theta}_{1p}^2 - \hat{\theta}_1^2) - (\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2)] - B_2(2-B_2)E[(\hat{\theta}_{2p}^2\\
&\quad - \hat{\theta}_2^2) - (\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2)] - 2r(1-\alpha)B_1B_2E[(\hat{\theta}_{1p}\hat{\theta}_{2p} - \hat{\theta}_1\hat{\theta}_2) - (\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2)]).
\end{aligned}$$

Though computation, $\frac{B_5+2r(1+\alpha)B_1}{4} = \frac{(3r+4)(16\alpha+4(4+\alpha)r-4(2\alpha^2+3\alpha-1)r^2-(3\alpha^2+10\alpha+7)r^3)}{4(8-(3\alpha+5)r^2)^2}$ and $\frac{(1-\alpha)rB_1B_2}{2} = \frac{r(1-\alpha)(3r+4)(12+3(1+\alpha)r-3(1+\alpha)r^2)}{2(8-(3\alpha+5)r^2)^2}$. Since $\frac{B_5+2r(1+\alpha)B_1}{4} > \frac{(1-\alpha)rB_1B_2}{2}$, we can obtain that $V_p^{I(1,0)} < V_p^{N(1,0)}$. It is easy to deduce that $V_p^{I(0,1)} > V_p^{N(0,1)}$. The value of $V_p^{I(1,1)} - V_p^{N(1,1)}$ depends on parameter $\Phi_p^{(1,1)}$, which is affected by correlation coefficients and competition intensity, as illustrated in Figure 5a.

(c) Through computation, the following result can be obtained:

$$\begin{aligned}
V^{I(1,0)} - V^{N(1,0)} &= -\frac{B_5 + 4rB_1 + 2(1-\alpha)B_1B_4}{4}E[(\hat{\theta}_{1p}^2 - \hat{\theta}_1^2) - (\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2)]\\
&\quad - \frac{(1-\alpha)rB_1B_2}{2}E[\hat{\theta}_{1p}\hat{\theta}_2 - \hat{\theta}_1\hat{\theta}_2];\\
V^{I(0,1)} - V^{N(0,1)} &= -\frac{1}{4}\Phi^{(0,1)} = -\frac{1}{4}(2(1-\alpha)rB_1B_2E[\hat{\theta}_1\hat{\theta}_{2p} - \hat{\theta}_1\hat{\theta}_2] - B_2^2E[(\hat{\theta}_{2p}^2 - \hat{\theta}_2^2) - (\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2)]);\\
V^{I(1,1)} - V^{N(1,1)} &= -\frac{B_5 + 4rB_1 + 2(1-\alpha)B_1B_4}{4}E[(\hat{\theta}_{1p}^2 - \hat{\theta}_1^2) - (\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2)] + \frac{B_2^2}{4}E[(\hat{\theta}_{2p}^2 - \hat{\theta}_2^2)\\
&\quad - (\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2)] - \frac{(1-\alpha)rB_1B_2}{2}E[(\hat{\theta}_{1p}\hat{\theta}_{2p} - \hat{\theta}_1\hat{\theta}_2) - (\hat{\theta}_{12p}^2 - \hat{\theta}_{12}^2)].
\end{aligned}$$

It is easy to deduce that $V^{I(1,0)} < V^{N(1,0)}$. The values of $V^{I(0,1)} - V^{N(0,1)}$ depends on parameter $\Phi^{(0,1)}$, which is affected by competition intensity, as illustrated in Figure 5b. Through computation, $\frac{B_5+4rB_1+2(1-\alpha)B_1B_4}{4} = \frac{(3r+4)(16+20r-4(2+2\alpha+\alpha^2)r^2-(11+6\alpha+3\alpha^2)r^3)}{2(8-(3\alpha+5)r^2)^2}$, $\frac{B_2^2}{4} = \frac{(4+2(1-\alpha)r-(1+3\alpha)r^2)^2}{4(8-(3\alpha+5)r^2)^2}$ and $\frac{(1-\alpha)rB_1B_2}{2} = \frac{r(3r+4)(1-\alpha)(4+2(1-\alpha)r-(1+3\alpha)r^2)}{2(8-(3\alpha+5)r^2)^2}$. Since $\frac{B_5+4rB_1+2(1-\alpha)B_1B_4}{4} > \frac{B_2^2}{4}$ and $\frac{(1-\alpha)rB_1B_2}{2} > 0$, it can be deduced that $V^{I(1,1)} < V^{N(1,1)}$.

□

Proof of Proposition 7. For supplier S_1, (i) when $\Phi^I > 0$, $H_1 = \pi_1^{I(1,0)} - \pi_1^{N(1,0)} = \frac{(1-\alpha)B_1}{2}((B_4 + r)E[\hat{\theta}_{12p}^2 - \hat{\theta}_{1p}^2] + rB_2E[\hat{\theta}_{12p}\hat{\theta}_{12} - \hat{\theta}_{1p}\hat{\theta}_2])$; (ii) when $\Phi^I \leq 0$, $H_1 = \pi_1^{I(0,1)}(\pi_1^{I(0,0)}) - \pi_1^{N(1,0)} = \frac{(1-\alpha)B_1}{2}((B_4+r)E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}^2] + rB_2E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}\hat{\theta}_2])$. Through computation, we can obtain that $\frac{(1-\alpha)B_1}{2} = \frac{(1-\alpha)(3r+4)}{2(8-(3\alpha+5)r^2)} > 0$, $B_4 + r = \frac{2(4+r-(3+\alpha)r^2-r^3)}{8-(3\alpha+5)r^2} > 0$, $rB_2 = \frac{r(4+2(1-\alpha)r-(1+3\alpha)r^2)}{8-(3\alpha+5)r^2} > 0$. In addition, (1) if $\Phi^I > 0$, $E[\hat{\theta}_{12p}^2 - \hat{\theta}_{1p}^2] = \frac{\sigma^4\sigma_1^2\sigma_p^2((\rho_1^2-1)\sigma_1\sigma_p+\rho_2\sigma_2(\sigma_1-\rho_1\sigma_p))^2}{(\sigma^2(\sigma_1^2+\sigma_p^2-2\rho_1\sigma_1\sigma_p)+(1-\rho_1^2)\sigma_1^2\sigma_p^2)\Phi^I} > 0$; (2) when ρ_1 is high, $E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}^2] = \frac{\sigma^4\sigma_1^2(\sigma_1^2(\sigma_p^2-\sigma_2^2)+2\rho_1\sigma_1\sigma_2^2\sigma_p-\rho_1^2\sigma_p^2(\sigma_1^2+\sigma_2^2))}{(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)(\sigma^2(\sigma_1^2+\sigma_p^2-2\rho_1\sigma_1\sigma_p)+(1-\rho_1^2)\sigma_1^2\sigma_p^2)} > 0$; (3) $E[\hat{\theta}_{12p}\hat{\theta}_{12} - \hat{\theta}_{1p}\hat{\theta}_2] = E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}\hat{\theta}_2] = \frac{1}{(\sigma^2+\sigma_2^2)(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)(\sigma^2(\sigma_1^2+\sigma_p^2-2\rho_1\sigma_1\sigma_p)+(1-\rho_1^2)\sigma_1^2\sigma_p^2)}(\sigma^6(\sigma_1^4\sigma_p((1-\rho_1^2)\sigma_p - \rho_2\sigma_2) + \sigma_1^2\sigma_2^2(\sigma_2^2 + \sigma_p^2 - \rho_1^2\sigma_p^2 - \rho_2\sigma_2\sigma_p) + \rho_1\rho_2\sigma_1^3\sigma_2\sigma_p^2 + \rho_1\sigma_1\sigma_2^3\sigma_p(\rho_2\sigma_p - 2\sigma_2) + \sigma_2^4\sigma_p^2) + (1-\rho_1^2)\sigma_p(\sigma_1^2 + \sigma_2^2) + \sigma^4\sigma_1^2\sigma_2^2\sigma_p(\rho_2\sigma_1\sigma_2(\rho_1\sigma_p - \sigma_1)))) > 0$. Thus, it can be deduced that $H_1 > 0$.

For supplier S_2, (i) when $\Phi^I \leq 0$ and $\Phi_2^{I(0,1)} \leq 0$, $H_2 = \pi_2^{I(0,1)} - \pi_2^{N(1,0)} = \frac{B_2}{2}((1 - B_2)E[\hat{\theta}_{12p}^2 - \hat{\theta}_2^2] + (1-\alpha)rB_1E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}\hat{\theta}_2])$; (ii) under other scenarios, $H_2 = \pi_2^{I(1,0)}(\pi_2^{I(0,0)}) - \pi_2^{N(1,0)} = \frac{B_2}{2}((1 - B_2)E[\hat{\theta}_{12}^2 - \hat{\theta}_2^2] + (1-\alpha)rB_1E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}\hat{\theta}_2])$. Through computation, $\frac{B_2}{2} = \frac{4+2(1-\alpha)r-(1+3\alpha)r^2}{2(8-(3\alpha+5)r^2)} > 0$, $1 - B_2 =$

$\frac{4-2(1-\alpha)r-4r^2}{8-(5+3\alpha)r^2} < 0$ when r is high, $\frac{(1-\alpha)rB_1}{2} = \frac{(1-\alpha)r(3r+4)}{8-(3\alpha+5)r^2} > 0$. In addition, (1) when $\Phi^I \le 0$, $E[\hat{\theta}_{12p}^2 - \hat{\theta}_2^2] = \frac{\sigma^4\sigma_2^2((1-\rho_2^2)\sigma_2^2\sigma_p^2+\sigma_1^2(\sigma_2-\rho_2\sigma_p)^2-2\rho_1\sigma_1\sigma_2\sigma_p(\sigma_2-\rho_2\sigma_p))}{(\sigma^2+\sigma_2^2)\Phi^I} < 0$; (2) $E[\hat{\theta}_{12}^2 - \hat{\theta}_2^2] = \frac{\sigma^4\sigma_2^4}{(\sigma^2+\sigma_2^2)(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)} > 0$; (3) $E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}\hat{\theta}_2] = \frac{1}{(\sigma^2+\sigma_2^2)(\sigma^2(\sigma_1^2+\sigma_2^2)+\sigma_1^2\sigma_2^2)(\sigma^2(\sigma_1^2+\sigma_p^2-2\rho_1\sigma_1\sigma_p)+(1-\rho_1^2)\sigma_1^2\sigma_p^2)}(\sigma^6(\sigma_1^4\sigma_p((1-\rho_1^2)\sigma_p - \rho_2\sigma_2) + \sigma_1^2\sigma_2^2(\sigma_2^2 + \sigma_p^2 - \rho_1^2\sigma_p^2 - \rho_2\sigma_2\sigma_p) + \rho_1\rho_2\sigma_1^3\sigma_2\sigma_p^2 + \rho_1\sigma_1\sigma_2^3\sigma_p(\rho_2\sigma_p - 2\sigma_2) + \sigma_2^4\sigma_p^2) + (1-\rho_1^2)\sigma_p(\sigma_1^2+\sigma_2^2) + \sigma^4\sigma_1^2\sigma_2^2\sigma_p(\rho_2\sigma_1\sigma_2(\rho_1\sigma_p - \sigma_1))) > 0$. Thus, it can be deduced that $H_2 > 0$. □

Proof of Proposition 8. From Propositions 6 and 7, the result can be obtained straightly, and thus the details are omitted. □

Proof of vertical sharing precedes horizontal sharing scenario

Proof. Through computation, the following result can be obtained:

$$\pi_1^{I(0,0)} - \pi_1^{N(0,0)} = \frac{(1-\alpha)B_1}{2}((B_4+r)E[\hat{\theta}_{12}^2 - \hat{\theta}_1^2] + rB_2E[\hat{\theta}_{12}^2 - \hat{\theta}_1\hat{\theta}_2]);$$
$$\pi_1^{I(1,0)} - \pi_1^{N(1,0)} = \frac{(1-\alpha)B_1}{2}((B_4+r)E[\hat{\theta}_{12p}^2 - \hat{\theta}_{1p}^2] + rB_2E[\hat{\theta}_{12p}\hat{\theta}_{12} - \hat{\theta}_{1p}\hat{\theta}_2]);$$
$$\pi_1^{I(0,1)} - \pi_1^{N(0,1)} = \frac{(1-\alpha)B_1}{2}((B_4+r)E[\hat{\theta}_{12}^2 - \hat{\theta}_1^2] + rB_2E[\hat{\theta}_{12}^2 - \hat{\theta}_1\hat{\theta}_{2p}]);$$
$$\pi_1^{I(1,1)} - \pi_1^{N(1,1)} = \frac{(1-\alpha)B_1}{2}((B_4+r)E[\hat{\theta}_{12p}^2 - \hat{\theta}_{1p}^2] + rB_2E[\hat{\theta}_{12p}^2 - \hat{\theta}_{1p}\hat{\theta}_{2p}]).$$

According to the previous proof, we know that $\frac{(1-\alpha)B_1}{2} > 0$, $B_4 + r > 0$, $rB_2 > 0$, $E[\hat{\theta}_{12}^2 - \hat{\theta}_i^2] > 0$, $E[\hat{\theta}_{12}^2 - \hat{\theta}_1\hat{\theta}_2] > 0$, $E[\hat{\theta}_{12p}^2 - \hat{\theta}_{ip}^2] > 0$ when $\Phi^I > 0$, $E[\hat{\theta}_{12p}^2 - \hat{\theta}_{ip}^2] > 0$ when $\Phi^I > 0$ (the forecasting capabilities of the two suppliers are similar), $E[\hat{\theta}_{12p}\hat{\theta}_{12} - \hat{\theta}_{ip}\hat{\theta}_{3-i}] > 0$, $E[\hat{\theta}_{12}^2 - \hat{\theta}_i\hat{\theta}_{(3-i)p}] > 0$. Then, it can be deduced that $\pi_1^{I(0,0)} > \pi_1^{N(0,0)}$, $\pi_1^{I(0,1)} > \pi_1^{N(0,1)}$, $\pi_1^{I(1,0)} > \pi_1^{N(1,0)}$ and the sign of $\pi_1^{I(1,1)} > \pi_1^{N(1,1)}$ depends on Φ^I.

$$\pi_2^{I(0,0)} - \pi_2^{N(0,0)} = \frac{B_2}{2}((1-B_2)E[\hat{\theta}_{12}^2 - \hat{\theta}_2^2] + (1-\alpha)rB_1E[\hat{\theta}_{12}^2 - \hat{\theta}_1\hat{\theta}_2]);$$
$$\pi_2^{I(1,0)} - \pi_2^{N(1,0)} = \frac{B_2}{2}((1-B_2)E[\hat{\theta}_{12}^2 - \hat{\theta}_2^2] + (1-\alpha)rB_1E[\hat{\theta}_{12}^2 - \hat{\theta}_{1p}\hat{\theta}_2]);$$
$$\pi_2^{I(0,1)} - \pi_2^{N(0,1)} = \frac{B_2}{2}((1-B_2)E[\hat{\theta}_{12p}^2 - \hat{\theta}_{2p}^2] + (1-\alpha)rB_1E[\hat{\theta}_{12p}\hat{\theta}_{12} - \hat{\theta}_1\hat{\theta}_{2p}]);$$
$$\pi_2^{I(1,1)} - \pi_2^{N(1,1)} = \frac{B_2}{2}((1-B_2)E[\hat{\theta}_{12p}^2 - \hat{\theta}_{2p}^2] + (1-\alpha)rB_1E[\hat{\theta}_{12p}^2 - \hat{\theta}_{1p}\hat{\theta}_{2p}]).$$

Similarly, we know that $\frac{B_2}{2} > 0$, $1 - B_2 = \frac{4-2(1-\alpha)r-4r^2}{8-(5+3\alpha)r^2} > 0$ when r is not too high, $(1-\alpha)rB_1 > 0$. Then, it can be deduced that $\pi_2^{I(0,0)} > \pi_2^{N(0,0)}$, $\pi_2^{I(0,1)} > \pi_2^{N(0,1)}$, $\pi_2^{I(1,0)} > \pi_2^{N(1,0)}$ and the sign of $\pi_2^{I(1,1)} > \pi_2^{N(1,1)}$ depends on Φ^I. Therefore, except in the situation where the platform retailer shares signal with two suppliers simultaneously and $\Phi^I < 0$, horizontal sharing is beneficial for two suppliers.

Given the suppliers' horizontal-sharing strategy, the platform retailer decides on the vertical-sharing strategy. According to the analysis of Proposition 5, the optimal vertical-sharing strategy is

$$\begin{cases} (1,0), & \Phi^I > 0; \\ (0,1), & \Phi^I \le 0 \text{ and } \Phi_2^{I(0,1)} \le 0; \\ (0,0), & \Phi^I \le 0 \text{ and } \Phi_2^{I(0,1)} > 0. \end{cases}$$

Therefore, the optimal bi-direction sharing strategy is

$$\begin{cases} I(1,0), & \Phi^I > 0; \\ I(0,1), & \Phi^I \le 0 \text{ and } \Phi_2^{I(0,1)} \le 0; \\ I(0,0), & \Phi^I \le 0 \text{ and } \Phi_2^{I(0,1)} > 0. \end{cases}$$

□