RAIRO-Oper. Res. 60 (2026) 2297–2309
https://doi.org/10.1051/ro/2026071

RAIRO Operations Research
www.rairo-ro.org

AN INNOVATIVE TACTIC TO TRANSPORTATION PROBLEM IN FERMATEAN FUZZY SETTING

K. Hemalatha[1] and B. Venkateswarlu[2,*]

Abstract. In operations research, a transportation problem (TP) is a sort of optimization problem in which the goal is to determine the best cost-effective way to convey items from many different places of origin and destination. The basic goal of the transportation challenge is to reduce the overall expense of transportation when moving a product from a source to a destination. It is extremely challenging to characterize the transportation problem's data, like cost, demand, and supply, because of the unpredictable economic and environmental elements. In this paper, the fermatean fuzzy transportation problem (FFTP) is utilized to calculate the lowest cost of transporting products from origin to destination. It is believed that the most recent tools for effectively managing ambiguity and fuzziness are fermatean fuzzy sets. In this work, a new ranking function that converts Fermatean uncertain numbers into crisp form is developed. An initial basic feasible solution (IBFS) for three different kinds of transportation problems in fermatean settings is subsequently provided using a novel method known as the inverse coefficient of a range. Furthermore, we used the modified distribution (MODI) method to obtain the ideal outcome. To illustrate the suggested methodology, we tackled a series of numerical problems, and outcomes are offered together with a comparison of results to the findings of previous research. The significance of the study and the direction of future research are then emphasised.

Mathematics Subject Classification. 90C70, 03E72.

Received July 10, 2023. Accepted June 1, 2026.

1. Introduction

A unique form of optimization technique is the transportation problem. It is a crucial tool utilized by decision-makers. To achieve the optimum outcome, such as profitability or the lowest price, one can use linear programming. In a transportation problem, the objective is to figure out the minimal cost of moving a homogeneous unit from a particular number of starting points to a specified assortment of locations [1]. Many logistical as well as supply chain uses for transportation models will save expenses. There are primarily three components that must be carried into account while solving a transportation challenge. Goods accessibility at origins, the single unit cost of items from ith starting to jth endpoint, and demand for commodities at a specific point. The quantity of units delivered determines how much it costs to ship from an initial location to a destination [2]. Hitchcock [3]

Keywords. Fermatean fuzzy transportation problem, initial basic feasible solution, inverse coefficient of range, MODI method, transportation problem.

[1] Department of Mathematics, Easwari Engineering College, Chennai, Tamil Nadu, India.

[2] Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu, India.

*Corresponding author: venkatesh.reddy@vit.ac.in

made the initial suggestion for the fundamental transportation dilemma. In authentic circumstances, the means of transport problems' costs, availability, and requirements are not exact. The significance of transportation problems gained enormous popularity with the advent of fuzzy set approaches. This is because it's possible to deal with unclear values that emerge while optimization. For the improvement of transportation problems, numerous writers developed several techniques [4]. Many fuzzy set extensions, among them fermatean fuzzy set (FFS), have been used by academics to better reflect such imprecision [5,6].

The most important area of study that facilitates selecting the finest option among all viable options is decision-making. In any sensible choice process, uncertainty is a noteworthy factor. The idea of a fuzzy set was first articulated by Zadeh [7]. It merely has a single membership degree [8,9]. The traditional fuzzy set has taken on multiple forms in recent times. The goal of established kinds of fuzzy sets is to clearly identify the ambiguity and provide improved maximizing results. In consideration of this, Atanassov [10] expanded the fuzzy idea to include intuitionistic fuzzy set (IFS). Both the membership (α) as well as non-membership (β) grades have the intent of accomplishing the requirement that $0 \leq \alpha + \beta \leq 1$. Since it was first suggested, the intuitionistic fuzzy set has drawn significant interest from a variety of domains. Nonetheless, there may be circumstances where decision-makers separately assess the range of membership and non-membership and the aggregate turns out to be higher than 1. Subsequently, Yager [11] devised a pythagorean fuzzy set (PFS) to fix this problem when the squared sum of the membership and non-membership quantities is less than or equal to one. In this regard, an expert might provide a value of 0.5 for inclusion and 0.7 for non-belongingness. In this case where IFS fails that is $0.5+0.7 \nleq 1$, however it is possible when we apply PFS in the scenario which implies $(0.5)^2+(0.7)^2 \leq 1$. When we apply the parameters 0.6 for membership and 0.9 for non-membership, the PFS criterion is not filled in this occasion. We can definitely get the value more than 1 furthermore, $(0.9)^2 + (0.6)^2 \nleq 1$. Thus, Senapati and Yager [12] presented fermatean fuzzy set to clarify this phenomenon that is $(0.9)^3+(0.6)^3 \leq 1$. By widening the territorial span of membership and non-membership, the FFS improves its capacity to characterize ambiguous data when compared to IFS and PFS [13]. One of the newest types of conventional fuzzy sets that effectively describe uncertainty is the fermatean fuzzy set. In the process of making decisions, a FFS, which has a framework that is more flexible compared to an IFS as well as a PFS, is a useful tool to manage uncertain data using membership levels and absence from membership level [14–17]. The graphic illustration of FFS is shown in Figure 1.

A notable advancement in multi-criteria group decision making was made by Akram and Bibi [18] which integrated the preference ranking organization approach for enhanced evaluation using 2-tuple linguistic fermatean fuzzy sets. Employing fermatean fuzzy soft notations, Zeb *et al.* [19] offered a situational analysis of COVID-19 patient records. A new method for determining decisions using fermatean fuzzy sets was created by Sindhu *et al.* [20]. Akram *et al.* [21] solved a completely fermatean multi-objective issue with transportation using a mathematical approach known as data envelope analysis. Jeevaraj [22] proposed the concept of interval-valued fermatean fuzzy sets (IVFFN) and developed some mathematical operations on the class of IVFFNs. The multi-level substantial transportation problem was resolved by Ali and Javaid [23] utilizing fermatean uncertain parameters with a new score function. A unique compromise technique centered on the sequence of triangular fermatean fuzzy numbers (FFN) was used by Akram *et al.* [24] to resolve the multi-objective mobility issue.

A combined fermatean fuzzy model has been recommended by Simic *et al.* [25] for urban mobility planning during the COVID-19 outbreak. Akram *et al.* [26] employed a decision-making model with sine-trigonometric fermatean fuzzy data to assist online financial soft power and sportswear companies.

Initially, a unique ranking algorithm was used by Kumar *et al.* [27] to answer the three different forms of Pythagorean fuzzy transportation problems (PyFTP). Later, Hemalatha and Venkateswarlu [28] created a new ranking function and methodology for PyFTP. Afterwards, Sharma *et al.* [29] proposed a new ranking function for fermatean fuzzy sets and used vogle's approximation method (VAM) for solving FFTP. Sahoo [30] used Excel Solver to solve FFTP and came up with the best solution. Simic *et al.* created a novel method for group decision-making concerning the taxing of expenses in public transportation by employing fermatean fuzzy. Following that, Sahoo [31] described three score formulas for the sorting of fermatean fuzzy sets along

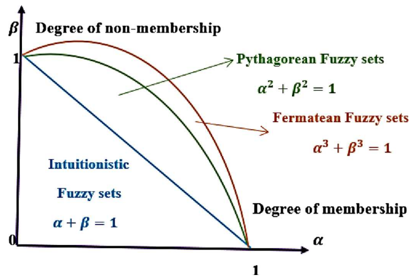

FIGURE 1. Pictorial representation of FFS.

with employing the TOPOSIS method to resolve the multi-criteria decision-making challenge. A fractional transportation dilemma with interval-valued fermatean ambiguous sets was reported by Akram *et al.* [32].

Only a small number of studies have been conducted on determining the IBFS of FFTP, according to the literature mentioned above. Furthermore, none of the writers have created a new ranking for FFS while also developing an innovative algorithm for solving FFTP. This research gap prompted the authors to develop a new ranking for FFSs that converts fermatean uncertainty to crisp, as well as a novel technique for determining IBFS of FFTP. The main goal of this study is to lower overall transportation costs when supply, demand, and transportation costs are all in FFNs. The aforementioned are this paper's significant contributions:

(i) For FFSs, created a unique ranking function, which transforms the fermatean fuzzy into crisp form.
(ii) A fresh approach known as the inverse coefficient of range is established for arriving an IBFS of FFTP.
(iii) Validating the proposed methodologies using existing and random numerical instances of three kinds of transportation dilemma in the fermatean fuzzy setting.

The remaining content of the article is as described in the following: Preliminaries are in Section 2, after which in Section 3, we developed a score function. Section 4 defines the mathematical model of the FFTP. A recommended technique is given in Section 5, and some illustrations are presented in Section 6. Outcomes are explored in Section 7 and a conclusion as well as the direction of further investigation are stated in Section 8.

2. PRELIMINARIES

This section presents the study's terminology and ranking function. Throughout this study, let $\check{X}$ be a universe of discourse.

2.1. Intuitionistic fuzzy set [9, 33]

A representation of an IFS is $\hat{I} = \{\langle \varsigma,\, \zeta_I(\varsigma),\, \xi_I(\varsigma)\rangle \mid \varsigma \in \check{X}\}$ where, $\zeta_I, \xi_I : \check{X} \to [0,1]$ are the functions of affiliation and non-membership correspondingly in such a way that $0 \leq \zeta_I(\varsigma) + \xi_I(\varsigma) \leq 1$ and hesitancy segment $\pi_I(\varsigma) = 1 - \ \zeta_I(\varsigma) - \xi_I(\varsigma)$ in which $0 \leq \pi_I(\varsigma) \leq 1$ for every $\varsigma \in \check{X}$ and $\zeta_I(\varsigma)$ signifies the affiliation level, $\xi_I(\varsigma)$ signifies the non-membership level and $\pi_I(\varsigma)$ so called measure of hesitancy of $\varsigma \in \check{X}$.

2.2. Pythagorean fuzzy set [11]

A pythagorean fuzzy set $\mathcal{P}$ in $\check{X}$ is described by $\mathcal{P} = \{\langle \varsigma,\, (\zeta_P(\varsigma), \xi_P(\varsigma))\rangle \mid \varsigma \in \check{X}\}$, where, $\zeta_P(\varsigma),\ \xi_P(\varsigma) :\ \check{X} \to [0,1]$ are the degrees of membership and non-membership of the element $\varsigma \in \check{X}$ in $\mathcal{P}$, respectively. Moreover, for every $\varsigma \in \check{X}$, $0 \leq (\zeta_P(\varsigma))^2 + (\xi_P(\varsigma))^2 \leq 1$.

2.3. Fermatean fuzzy set [12]

A fermatean fuzzy set $\mathcal{F}$ in $\breve{X}$ is described by $\mathcal{F} = \{\langle \varsigma, (\zeta_F(\varsigma), \xi_F(\varsigma))\rangle \mid \varsigma \in \breve{X}\}$, where, $\zeta_F : \breve{X} \to [0,1]$ and $\xi_F : \breve{X} \to [0,1]$, subject to the condition $0 \leq (\zeta_F(\varsigma))^3 + (\xi_F(\varsigma))^3 \leq 1, \quad \forall \varsigma \in \breve{X}$. The values $\zeta_F(\varsigma)$ and $\xi_F(\varsigma)$ represent the degrees of membership and non-membership, respectively, of the element ς in the set $\mathcal{F}$.

For every Fermatean fuzzy set (FFS) $\mathcal{F}$ and $\varsigma \in \breve{X}$, $\pi_{\mathcal{F}}(\varsigma) = \sqrt[3]{1 - \left(\zeta_F(\varsigma)\right)^3 - \left(\xi_F(\varsigma)\right)^3}$, is called the degree of indeterminacy of ς with respect to $\mathcal{F}$.

2.4. Elementary operations on FFS [34]

Consider three Fermatean fuzzy sets (FFSs) on the universal set $\breve{X}$, namely $\mathcal{F} = \langle \zeta_F, \xi_F\rangle$, $\mathcal{F}_1 = \langle \zeta_{F_1}, \xi_{F_1}\rangle$, $\mathcal{F}_2 = \langle \zeta_{F_2}, \xi_{F_2}\rangle$, and let $\lambda > 0$. The fundamental operations on FFSs are defined as follows.

(1) Addition: $\mathcal{F}_1 \oplus \mathcal{F}_2 = \left\{\sqrt[3]{(\zeta_{F_1})^3 + (\zeta_{F_2})^3 - (\zeta_{F_1})^3 \cdot (\zeta_{F_2})^3},\ \xi_{F_1}.\xi_{F_2}\right\}$.
(2) Multiplication: $\mathcal{F}_1 \otimes \mathcal{F}_2 = \left\{\zeta_{F_1}.\zeta_{F_2},\ \sqrt[3]{(\xi_{F_1})^3 + (\xi_{F_2})^3 - (\xi_{F_1})^3 \cdot (\xi_{F_2})^3}\right\}$.
(3) Scalar Multiplication: $\lambda \odot \mathcal{F} = \left\{\sqrt[3]{1 - (1 - \zeta_F^3)^\lambda},\ (\xi_F)^\lambda\right\}$.
(4) Exponentiation: $\mathcal{F}^\lambda = \left\{(\zeta_F)^\lambda,\ \sqrt[3]{1 - (1 - \xi_F^3)^\lambda}\right\}$.
(5) Union: $\mathcal{F}_1 \cup \mathcal{F}_2 = \{\max(\zeta_{F_1}, \zeta_{F_2}),\ \min(\xi_{F_1}, \xi_{F_2})\}$.
(6) Intersection: $\mathcal{F}_1 \cap \mathcal{F}_2 = \{\min(\zeta_{F_1}, \zeta_{F_2}),\ \max(\xi_{F_1}, \xi_{F_2})\}$.
(7) Complement: $\mathcal{F}^c = \{\xi_F, \zeta_F\}$.

2.5. Definition [35]

If $\mathcal{F} = \{\zeta_F, \xi_F\}$ is a Fermatean fuzzy set (FFS), then its score function, denoted by $S_F(\mathcal{F})$, is defined as $S_F(\mathcal{F}) = \left(\zeta_F^3 - \xi_F^3\right)$. The value of the score function satisfies $S_F(\mathcal{F}) \in [-1, 1]$.

2.6. Definition [35]

If $\mathcal{F} = \{\zeta_F, \xi_F\}$ is a Fermatean fuzzy set (FFS), then the accuracy function of $\mathcal{F}$, denoted by $H_F(\mathcal{F})$, is defined as $H_F(\mathcal{F}) = \left(\zeta_F^3 + \xi_F^3\right)$.

2.7. Definition [25, 26]

Let $\mathcal{F}_1 = \langle \zeta_{F_1}, \xi_{F_1}\rangle$ and $\mathcal{F}_2 = \langle \zeta_{F_2}, \xi_{F_2}\rangle$ be two FFSs. The following describes the ranking relations between $\mathcal{F}_1$ and $\mathcal{F}_2$:

(1) If $\mathcal{F}_1 > \mathcal{F}_2$, then $S_F(\mathcal{F}_1) > S_F(\mathcal{F}_2)$ and $H_F(\mathcal{F}_1) > H_F(\mathcal{F}_2)$.
(2) If $\mathcal{F}_1 < \mathcal{F}_2$, then $S_F(\mathcal{F}_1) < S_F(\mathcal{F}_2)$ and $H_F(\mathcal{F}_1) < H_F(\mathcal{F}_2)$.
(3) If $\mathcal{F}_1 \simeq \mathcal{F}_2$, then $S_F(\mathcal{F}_1) = S_F(\mathcal{F}_2)$ and $H_F(\mathcal{F}_1) = H_F(\mathcal{F}_2)$.

3. Proposed ranking function

Let $\mathcal{F} = (\zeta_F(\varsigma), \xi_F(\varsigma))$ be a Fermatean Fuzzy Number (FFN). The new score function $S_F(\mathcal{F})$, where $S_F(\mathcal{F}) \in [0, 1]$, is defined as

$$S_F(\mathcal{F}) = \frac{1}{2}\left\{\left|(\zeta_F - \xi_F)^2 + \min(\zeta_F, \xi_F)^2 - \max(\zeta_F, \xi_F)^2\right|\right\} \cdot \min(\zeta_F, \xi_F).$$

Two fermatean uncertain numbers can be analyzed using the suggested ranking mechanism. Consider two FFNs $\mathcal{F}_1 = \langle \zeta_{F_1}, \xi_{F_1}\rangle$ and $\mathcal{F}_2 = \langle \zeta_{F_2}, \xi_{F_2}\rangle$ their relationship is shown below.

Table 1. Fermatean fuzzy transportation problem.

	Destinations				
Sources	$\mathcal{D}_1$	$\mathcal{D}_2$	$\dots$	$\mathcal{D}_n$	Supply
$\mathcal{S}_1$	$c_{11}^{\mathcal{F}}$	$c_{12}^{\mathcal{F}}$	$\dots$	$c_{1n}^{\mathcal{F}}$	$a_1^{\mathcal{F}}$
$\mathcal{S}_2$	$c_{21}^{\mathcal{F}}$	$c_{22}^{\mathcal{F}}$	$\dots$	$c_{2n}^{\mathcal{F}}$	$a_2^{\mathcal{F}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathcal{S}_m$	$c_{m1}^{\mathcal{F}}$	$c_{m2}^{\mathcal{F}}$	$\dots$	$c_{mn}^{\mathcal{F}}$	$a_m^{\mathcal{F}}$
Demand	$b_1^{\mathcal{F}}$	$b_2^{\mathcal{F}}$	$\dots$	$b_n^{\mathcal{F}}$	

(1) $\mathcal{F}_1 > \mathcal{F}_2$ iff $S_F(\mathcal{F}_1) > S_F(\mathcal{F}_2)$.
(2) $\mathcal{F}_1 < \mathcal{F}_2$ iff $S_F(\mathcal{F}_1) < S_F(\mathcal{F}_2)$.
(3) $\mathcal{F}_1 = \mathcal{F}_2$ iff $S_F(\mathcal{F}_1) = S_F(\mathcal{F}_2)$.

4. Mathematical formulations of FFTP

Suppose there are m resources and n recipients that need to be reached shown in Table 1. The challenge is to choose a transportation tactic that will minimize complete transportation expenses while yet meeting availability and demand requirements.

$$\text{Minimize } (\theta_{z_0}, \delta_{z_0}) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\theta_{c_{ij}^F}, \delta_{c_{ij}^F}\right)\widetilde{\S}_{ij}$$

subject to

$$\begin{aligned}
&\sum_{j=1}^{n}\widetilde{\S}_{ij} = \left(\theta_{a_i^F}, \delta_{a_i^F}\right), && i = 1, 2, \dots, m,\\
&\sum_{i=1}^{m}\widetilde{\S}_{ij} = \left(\theta_{b_j^F}, \delta_{b_j^F}\right), && j = 1, 2, \dots, n,\\
&\widetilde{\S}_{ij} \geq 0, && \forall\, i = 1, 2, \dots, m,\ j = 1, 2, \dots, n
\end{aligned}$$

where,

a_i^F – The supply item is given in a fermatean fuzzy number.
b_j^F – The market demand item in a fermatean fuzzy number.
$\widetilde{\S}_{ij}$ – Quantity of products moved from source to destination in units.
c_{ij}^F – The shifting price for a single product unit in a fermatean fuzzy number.

5. Proposed algorithm for solving fermatean fuzzy transportation model

This section provides steps for the proposed algorithm and the steps are shown in Figure 2.

Step 1: Determine the fermatean fuzzy transportation issue.
Step 2: Convert expense, need, and availability Fermatean fuzzy values into crisp values according to the provided ranking function in Section 3.
Step 3: Verify whether the provided fermatean fuzzy transportation issue is balanced or not.

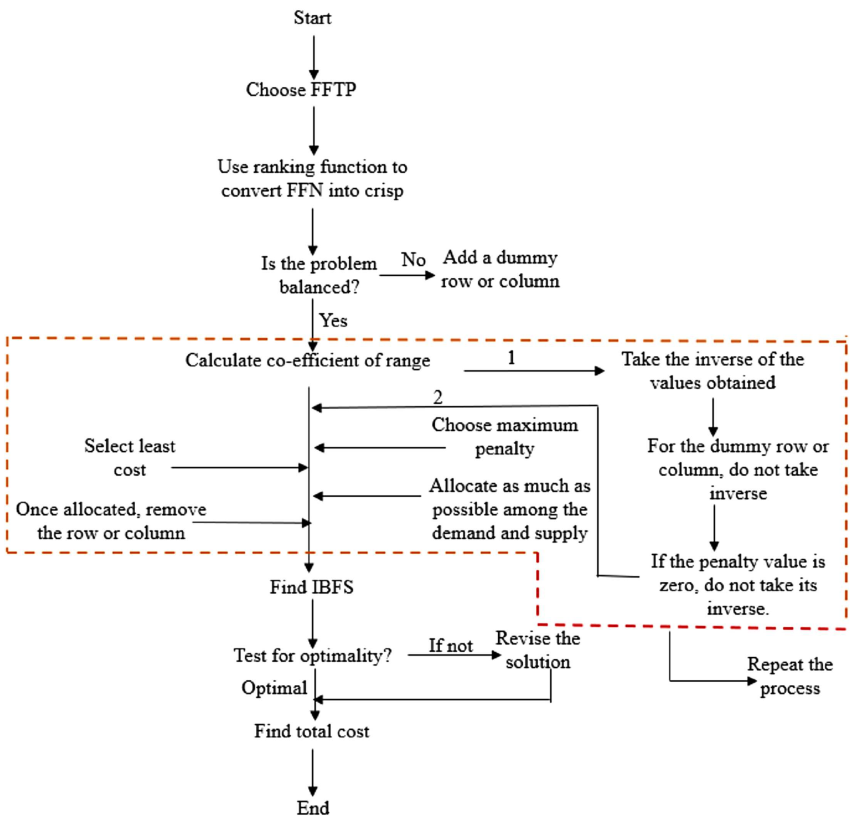

FIGURE 2. Flowchart for the proposed algorithm.

(a) If it is balanced, move on to step 5, where total demand and total supply must equal one another after being converted to crisp values.
(b) Proceed to step 4 if there is an imbalance between the total supply and total demand.

Step 4: With zero cost, add a dummy row or column to balance the overall supply and demand.
Step 5: Calculate coefficient of range $\frac{L-S}{L+S}$ where, L denotes largest expense and S denotes smallest expense for ever row and column.
Step 6: After finding coefficient of range, take the inverse of them for every row and column.
Step 7: If the entire row or column is zero, do not apply the inverse; instead, retain the penalty at zero for that specific row or column.
Step 8: Determine the biggest value of the inverse between the row and the column. Choose any row or column if both have the same value.
Step 9: Choose the relevant row or column with the lowest cost.
Step 10: Choose the minimum value in demand and supply and assign them to the least cost of the corresponding row or column.
Step 11: Once allocation is complete, remove the entire row or column.
Step 12: Continue with steps 5 to 11 until all of the supply and demand have been fulfilled.
Step 13: To achieve the optimal solution, the MODI approach should be applied after determining IBFS.

TABLE 2. Input data for Type I [15].

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	$\mathcal{D}_4$	Supply
$\mathcal{S}_1$	$(0.4, 0.7)$	$(0.5, 0.4)$	$(0.8, 0.3)$	$(0.6, 0.3)$	26
$\mathcal{S}_2$	$(0.4, 0.2)$	$(0.7, 0.3)$	$(0.4, 0.8)$	$(0.7, 0.3)$	24
$\mathcal{S}_3$	$(0.7, 0.1)$	$(0.8, 0.1)$	$(0.6, 0.4)$	$(0.9, 0.1)$	30
Demand	17	23	28	12	

TABLE 3. Defuzzified values.

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	$\mathcal{D}_4$	Supply
$\mathcal{S}_1$	0.048	0.016	0.045	0.027	26
$\mathcal{S}_2$	0.008	0.036	0.064	0.036	24
$\mathcal{S}_3$	0.006	0.007	0.032	0.008	30
Demand	17	23	28	12	

TABLE 4. First allocation.

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	$\mathcal{D}_4$	Supply	Row penalty
$\mathcal{S}_1$	0.048	0.016	0.045	0.027	26	2
$\mathcal{S}_2$	0.008	0.036	0.064	0.036	24	1.287
$\mathcal{S}_3$	0.006	0.007	0.032^{28}	0.008	30	1.461
Demand	17	23	28	12		
Column penalty	1.287	1.48	3.00	1.572		

6. NUMERICAL ILLUSTRATIONS

To demonstrate the usefulness of the suggested approach, the problems listed below are taken randomly for each of the three forms of the fermatean fuzzy transportation issue (FFTP).

Existing data

6.1. Assume a TP having type-1 PyFTP contributions from Kumar *et al.* [15], for which expenses are treated as fermatean fuzzy values while demand and supply remain crisp as shown in Table 2.

Step 1: Select the transportation dilemma in the Fermatean context that is fuzzy.
Step 2: Using the suggested ranking mechanism, fermatean uncertain figures should be transformed into crisp values.
Step 3: Table 3 does not require an extra dummy row or column because the issue chosen is balanced.
Step 5: Determine each row's as well as column's coefficient of range and the inverse of those values, follow the steps of 6 and 7 for finding penalties.
Step 8: Determine the biggest value of the inverse among the row's and the column's. Choose any row or column if both have the same value.
Step 9: Choose the relevant row or column with the lowest cost.
Step 10: Choose the minimum value in demand and supply and assign them to the least cost of the corresponding row or column. Table 4 shows this.
Step 11: Once allocation is complete, remove the entire row or column. Table 5 displays the modified transportation cost matrix.

Table 5. Second allocation.

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_4$	Supply	Row penalty
$\mathcal{S}_1$	0.048	0.016	0.027	26	2
$\mathcal{S}_2$	0.008	0.036	0.036	24	1.57
$\mathcal{S}_3$	0.006^2	0.007	0.008	2	7.042
Demand	17	23	12		
Column penalty	1.287	1.48	1.572		

Table 6. Third allocation.

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_4$	Supply	Row penalty
$\mathcal{S}_1$	0.048	0.016	0.027^{12}	26	2
$\mathcal{S}_2$	0.008	0.036	0.036	24	1.57
Demand	15	23	12		
Column penalty	1.40	2.604	7.04		

Table 7. Final allocations.

	$\mathcal{D}_1$	$\mathcal{D}_2$	Supply	Row penalty
$\mathcal{S}_1$	0.048	0.016^{14}	26	2
$\mathcal{S}_2$	0.008^{15}	0.036^9	24	1.57
Demand	15	23		
Column penalty	1.40	2.604		

Table 8. Input data for Type II [15].

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	$\mathcal{D}_4$	Supply
$\mathcal{S}_1$	0.0335	0.0545	0.0775	0.0635	$(0.7, 0.1)$
$\mathcal{S}_2$	0.056	0.07	0.026	0.07	$(0.8, 0.1)$
$\mathcal{S}_3$	0.074	0.0815	0.06	0.09	$(0.9, 0.1)$
Demand	$(0.4, 0.7)$	$(0.7, 0.3)$	$(0.8, 0.1)$	$(0.60832, 0.4)$	

Step 12: Continue in the same manner until all allotments have been fulfilled. The repeated procedures are shown in Tables 6 and 7.

Tables 3–7 show the allocations obtained through the recommended method. Additionally, Tables 7–10 display the allocations as superscripts. After using the modified distribution approach, the optimum value is 1.556.

6.2. Assume a TP that incorporates information of type-2 PyFTP form Kumar *et al.* [15], as shown in Table 8, where expenses are crisp and availability as well as demand remains fermatean fuzzy parameters.

Verify that the supply and demand defuzzified values are equal; if not, include a fake row or column with no cost. The suggested algorithm calculates the IBFS of the overall costs associated with transportation in Table 8 above; therefore, the ideal transportation expense is 0.000975.

6.3. Assume a TP having type-3 PyFTP contributions from Kumar *et al.* [15], in which the entire expenses, availability, and requirement parameters are fermatean fuzzy values as indicated in Table 9.

TABLE 9. Input data for Type III [15].

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	$\mathcal{D}_4$	Supply
$\mathcal{S}_1$	$(0.1, 0.9)$	$(0.2, 0.8)$	$(0.1, 0.8)$	$(0.1, 0.9)$	$(0.7, 0.1)$
$\mathcal{S}_2$	$(0.01, 0.99)$	$(0.3, 0.9)$	$(0.3, 0.9)$	$(0.1, 0.7)$	$(0.8, 0.1)$
$\mathcal{S}_3$	$(0.1, 0.8)$	$(0.4, 0.8)$	$(0.4, 0.9)$	$(0.2, 0.9)$	$(0.9, 0.1)$
Demand	$(0.4, 0.7)$	$(0.7, 0.3)$	$(0.8, 0.1)$	$(0.60832, 0.4)$	

TABLE 10. Fermatean fuzzy TP of Type I.

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	$\mathcal{D}_4$	Supply
$\mathcal{S}_1$	$(0.6, 0.5)$	$(0.7, 0.3)$	$(0.7, 0.8)$	$(0.4, 0.8)$	22
$\mathcal{S}_2$	$(0.9, 0.5)$	$(0.4, 0.7)$	$(0.8, 0.6)$	$(0.7, 0.5)$	24
$\mathcal{S}_3$	$(0.7, 0.3)$	$(0.9, 0.6)$	$(0.8, 0.7)$	$(0.5, 0.8)$	34
Demand	20	18	31	11	

TABLE 11. Fermatean fuzzy TP of Type II.

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	$\mathcal{D}_4$	Supply
$\mathcal{S}_1$	25	36	49	64	$(0.6, 0.4)$
$\mathcal{S}_2$	10	48	72	50	$(0.3, 0.8)$
$\mathcal{S}_3$	36	108	49	75	$(0.7, 0.2)$
Demand	$(0.5, 0.8)$	$(0.7, 0.4)$	$(0.4, 0.8)$	$(0.9, 0.2)$	

Using the provided method, the best answer for the previously described table emerged, and the total cost of transportation arrived at 0.0000986.

Random data

6.4. Type I: Three branches of a warehouse, each of which is responsible for delivering food items to one, two, three, and four ($\mathcal{D}1, \mathcal{D}2, \mathcal{D}3$, and $\mathcal{D}4$) distinct places. Consider branches S1, S2, and S3 that require supplies of groundnut, sesame seeds, and niger seeds, respectively. Here, supply and demand were expressed in sharp values, whereas the cost is expressed in FFNs. Determine the transportation expense for the following Table 10.

Using the provided ranking strategy and algorithm, IBFS for the above table emerged, and the optimum solution is 3.406.

6.5. Type II: A warehouse contains three branches, each of which is responsible for delivering food items to separate places ($\mathcal{D}1, \mathcal{D}2, \mathcal{D}3$, and $\mathcal{D}4$). Consider branches S1, S2, and S3 that require supplies of groundnut, sesame seeds, and niger seeds, respectively. Here, supply and demand were expressed as FFNs, whereas the cost that we are using is expressed as crisp values. Calculate the transportation cost for the following Table 11.

Verify that the supply and demand defuzzified values are equal; if not, include a fake row or column with no cost. The suggested algorithm calculates the IBFS of the overall costs associated with transportation in Table 8 above; therefore, the ideal transportation expense is 3.662.

6.6. Type III: Three branches of a warehouse, each of which is responsible for delivering food items four distinct places ($\mathcal{D}1, \mathcal{D}2, \mathcal{D}3$, and $\mathcal{D}4$). Consider the demands of branches S1, S2, and S3 to supply groundnut, sesame, and niger seeds respectively. Fermatean fuzzy numbers have been utilized to represent the expense, availability, and requirement in this case. Calculate the transportation cost for the following Table 12.

Table 12. Fermatean fuzzy TP of Type III.

	$\mathcal{D}_1$	$\mathcal{D}_2$	$\mathcal{D}_3$	$\mathcal{D}_4$	Supply
$\mathcal{S}_1$	$(0.6, 0.5)$	$(0.7, 0.3)$	$(0.7, 0.8)$	$(0.4, 0.8)$	$(0.6, 0.4)$
$\mathcal{S}_2$	$(0.9, 0.5)$	$(0.4, 0.7)$	$(0.8, 0.6)$	$(0.7, 0.5)$	$(0.3, 0.8)$
$\mathcal{S}_3$	$(0.7, 0.3)$	$(0.9, 0.6)$	$(0.8, 0.7)$	$(0.5, 0.8)$	$(0.7, 0.2)$
Demand	$(0.5, 0.8)$	$(0.7, 0.4)$	$(0.4, 0.8)$	$(0.9, 0.2)$	

Table 13. Comparative study.

Existing data			
Examples	Types	Existing method [29]	Proposed method
6.1	I	9.161	1.556
6.2	II	0.011483	0.000975
6.3	III	0.002818	0.0000986
Random data			
Examples	Types	Existing method [29]	Proposed method
6.4	I	18.346	3.406
6.5	II	15.315	3.662
6.6	III	0.071133	0.004845

Using the provided ranking strategy and algorithm, IBFS for the above table emerged, and the optimum solution is 0.004845.

7. Results and discussion

Numerous studies that address TP in various uncertain circumstances have drawn attention from researchers in a variety of domains. There are always difficult and complex situations in the real world that require processing ambiguous information. Fuzzy sets and its extensions have made it possible to successfully deal with uncertainty in a variety of scenarios. Table 13 demonstrates that our proposed method addresses FFTP better than other procedures. Using fermatean uncertain for transportation problems will make it simpler for researchers to account for all possible costs at each node. In comparison to the transport costs determined by Sharma *et al.* for such three forms of FFTP, Table 13 demonstrates that by using the suggested technique, we have been successful in determining the least transportation costs with all three kinds of FFTP. Furthermore, this study illustrated the utility of the proposed score function and technique using random numerical problems in addition to an existing issue. Our suggested method results in low transit costs for each of them. Consequently, the output of the proposed technique satisfied the goal of the study. Hence, we can say that our suggested approach is a novel strategy to deal with ambiguity in the Fermatean fuzzy setting. The suggested solution outlined here is highly straightforward, easy to implement, and applicable for resolving practical decision-making issues with fermatean fuzzy parameters. The efficiency of the suggested approach is demonstrated in Figures 3 and 4.

8. Conclusion

Firms are facing mounting pressure to find more efficient ways to deliver goods to customers in the fiercely competitive market of today. To deal with this issue, the transportation model provides a solid basis. As a result, different organizations work to deliver items to customers in the most economical or time-efficient way

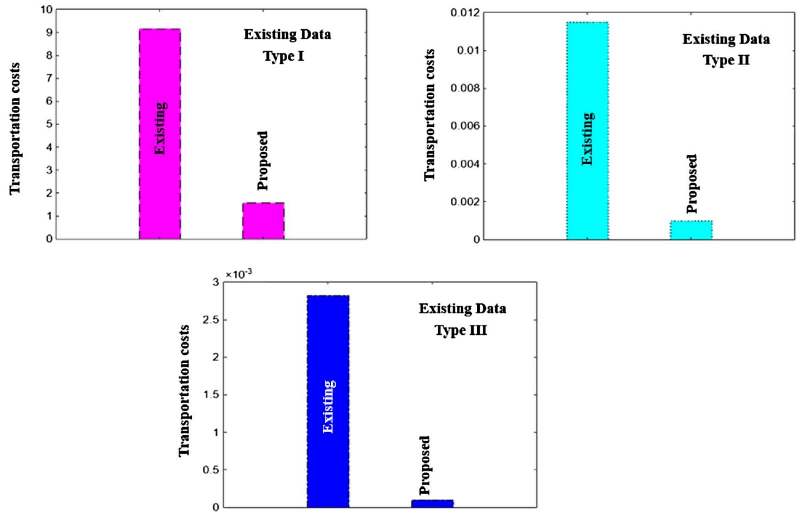

FIGURE 3. Comparison of proposed and existing methods for existing data.

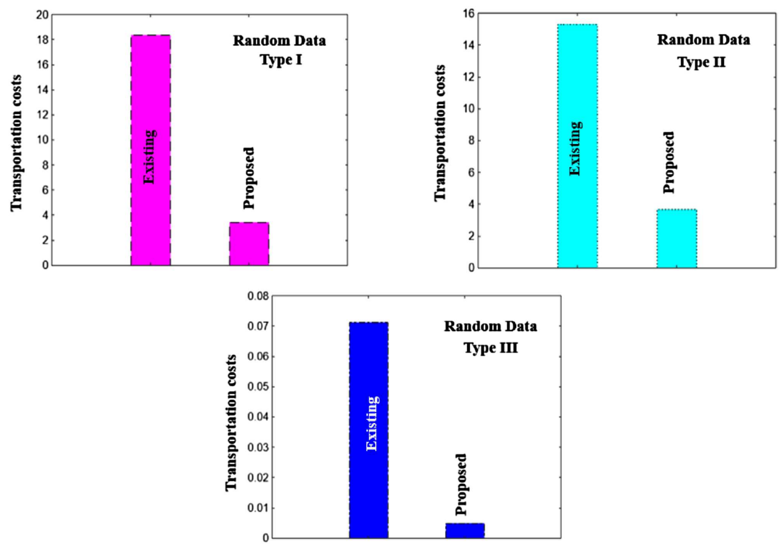

FIGURE 4. Comparison of proposed and existing methods for random data.

possible. Numerous researchers have created many transportation problems over the years, as well as diverse methods for tackling them. This paper quantifies imprecise, incomplete, and ambiguous information using fermatean fuzzy sets. In this investigation, we originally suggested a rank function for fermatean fuzzy numbers that is more trustworthy than the current score formula. The second claim argues that an innovative strategy known as the inverse coefficient of range is suggested for identifying an initial fundamental sensible solution to a balanced transportation dilemma. The recommended method might provide a compelling IBFS that resolves the fermatean transportation challenge by ensuring the lowest feasible transit costs. It is proven that the inverse coefficient of range Method achieves a considerably superior outcome. Using illustrative simulations, the suggested methodology has been described, and the results have been examined with the existing research. Additionally, we can determine that our proposed algorithm outperforms previous techniques based on the comparative figures. This research will assist in achieving the goal of increasing revenues by lowering transportation costs. The given technique can be used to overcome ambiguity in real-world transportation issues. Multi-objective fermatean fuzzy transportation frameworks can be developed in the future using this work to manage competing objectives like time and cost minimization.

Conflicts of interest

The authors of this paper declare that there are no competing interests.

Ethical approval

The creation of this work did not include any people or animals.

References

[1] S.K. Sharma and K. Goel, Analysis of IBFS for transportation problem by using various methods. *Math. Stat.* **10** (2022) 741–746.

[2] M.K. Sharma, S.P. Yadav and R. Yadav, Fermatean fuzzy programming with new score function: a new methodology to multi-objective transportation problems. *Electronics* **12** (2023) 277.

[3] F.L. Hitchcock, The distribution of a product from several sources to numerous localities. *J. Math. Phys.* **20** (1941) 224–230.

[4] S. Kalyani and S. Nagarani, A fully fuzzy transportation problem with hexagonal fuzzy number. *AIP Conf. Proc.* **2261** (2020) 030078.

[5] G. Gupta and K. Anupum, An efficient method for solving intuitionistic fuzzy transportation problem of type-2. *Int. J. Appl. Comput. Math.* **3** (2017) 3795–3804.

[6] J. Ahmed and S. Bashir, Fully bipolar single-valued neutrosophic transportation problems. *Math. Probl. Eng.* **2022** (2022) 1839028.

[7] L.A. Zadeh, Fuzzy sets. *Inf. Control* **8** (1965) 338–353.

[8] R.M. Umamageswari and G. Uthra, A Pythagorean fuzzy approach to solve transportation problem. *Adalya J.* **9** (2020) 1301–1308.

[9] H. Karunakaran and V. Bhumireddy, Utilising neutrosophic logic in the design of a smart air-conditioning system. *Appl. Sci.* **12** (2022) 9776.

[10] K.T. Atanassov, New operations defined over the intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **20** (1986) 87–96.

[11] R.R. Yager, Pythagorean membership grades in multicriteria decision making. *IEEE Trans. Fuzzy Syst.* **22** (2013) 958–965.

[12] T. Senapati and R.R. Yager, Fermatean fuzzy sets. *J. Ambient Intell. Humaniz. Comput.* **11** (2020) 663–674.

[13] M. Akram, U. Amjad, J.C.R. Alcantud and G. Santos-García, Complex Fermatean fuzzy N-soft sets: a new hybrid model with applications. *J. Ambient Intell. Humaniz. Comput.* **14** (2023) 8765–8798.

[14] Z. Deng and J. Wang, New distance measure for Fermatean fuzzy sets and its application. *Int. J. Intell. Syst.* **37** (2022) 1903–1930.

[15] M. Kirisci, Correlation coefficients of Fermatean fuzzy sets with a medical application. *J. Math. Sci. Model.* **5** (2022) 16–23.

[16] M. Kirişci, New cosine similarity and distance measures for Fermatean fuzzy sets and TOPSIS approach. *Knowl. Inf. Syst.* **65** (2023) 855–868.

[17] M. Keshavarz-Ghorabaee, M. Amiri, M. Hashemi-Tabatabaei, E.K. Zavadskas and A. Kaklauskas, A new decision-making approach based on Fermatean fuzzy sets and WASPAS for green construction supplier evaluation. *Mathematics* **8** (2020) 2202.

[18] M. Akram and R. Bibi, Multi-criteria group decision-making based on an integrated PROMETHEE approach with 2-tuple linguistic Fermatean fuzzy sets. *Granul. Comput.* **8** (2023) 917–941.

[19] A. Zeb, A. Khan, M. Juniad and M. Izhar, Fermatean fuzzy soft aggregation operators and their application in symptomatic treatment of COVID-19. *J. Ambient Intell. Humaniz. Comput.* **14** (2023) 11607–11624.

[20] M.S. Sindhu, I. Siddique, M. Ahsan, F. Jarad and T. Altunok, An approach of decision-making under the framework of Fermatean fuzzy sets. *Math. Probl. Eng.* **2022** (2022) 8442123.

[21] M. Akram, S. Shahzadi, S.M.U. Shah and T. Allahviranloo, A fully Fermatean fuzzy multi-objective transportation model using an extended DEA technique. *Granul. Comput.* **8** (2023) 1173–1204.
[22] S. Jeevaraj, Ordering of interval-valued Fermatean fuzzy sets and its applications. *Expert Syst. Appl.* **185** (2021) 115613.
[23] W. Ali and S. Javaid, Enhancing Fermatean fuzzy transportation problems: an innovative score function-based optimization strategy. *Int. Res. J. Econ. Manag. Stud.* **2** (2023) 546–558.
[24] M. Akram, S. Shahzadi, S.M.U. Shah and T. Allahviranloo, An extended multi-objective transportation model based on Fermatean fuzzy sets. *Soft Comput.* (2023) 1–23.
[25] V. Simić, I. Ivanović, V. Đorić and A.E. Torkayesh, Adapting urban transport planning to the COVID-19 pandemic: an integrated Fermatean fuzzy model. *Sustain. Cities Soc.* **79** (2022) 103669.
[26] M. Akram, G. Shahzadi and B. Davvaz, Decision-making model for internet finance soft power and sportswear brands based on sine-trigonometric Fermatean fuzzy information. *Soft Comput.* **27** (2023) 1971–1983.
[27] R. Kumar, S.A. Edalatpanah, S. Jha and R. Singh, A Pythagorean fuzzy approach to the transportation problem. *Complex Intell. Syst.* **5** (2019) 255–263.
[28] K. Hemalatha and B. Venkateswarlu, Pythagorean fuzzy transportation problem: new way of ranking for Pythagorean fuzzy sets and mean square approach. *Heliyon* **9** (2023) e20775.
[29] M.K. Sharma, A.K. Bhargava, S. Kumar, L. Rathour, L.N. Mishra and S. Pandey, A Fermatean fuzzy ranking function in optimization of intuitionistic fuzzy transportation problems. *Adv. Math. Models Appl.* **7** (2022) 191–204.
[30] L. Sahoo, Transportation problem in Fermatean fuzzy environment. *RAIRO-Oper. Res.* **57** (2023) 145–156.
[31] L. Sahoo, A new score function based Fermatean fuzzy transportation problem. *Results Control Optim.* **4** (2021) 100040.
[32] M. Akram, S.M.U. Shah, M.M.A. Al-Shamiri and S.A. Edalatpanah, Fractional transportation problem under interval-valued Fermatean fuzzy sets. *AIMS Math.* **7** (2022) 17327–17348.
[33] Kamini and M.K. Sharma, Zero-point maximum allocation method for solving intuitionistic fuzzy transportation problem. *Int. J. Appl. Comput. Math* **6** (2020) 115.
[34] T. Senapati and R.R. Yager, Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision-making methods. *Eng. Appl. Artif. Intell.* **85** (2019) 112–121.
[35] L. Sahoo, Some score functions on Fermatean fuzzy sets and its application to bride selection based on TOPSIS method. *Int. J. Fuzzy Syst. Appl.* **10** (2021) 8–29.