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DUAL-CHANNEL PERISHABLE SUPPLY CHAIN: OPTIMAL PRICING, QUALITY, AND SCREENING RATE OF DAMAGE WITH RETURNS UNDER STOCHASTIC DEMAND

Santanu Kumar Ghosh[1,*], Chandan Pathak[1], Sudhansu Khanra[2], Saswati Mukherjee[1] and Bivas Dinda[1]

Abstract. This paper investigates a dual-channel, two-echelon supply chain comprising a manufacturer and a retailer for a single perishable product. In today's global marketplace, a well-designed return policy is a crucial element that can lead to higher revenues, reduced expenses, improved profitability, and enhanced customer loyalty. Offering the option to return perishable goods can significantly increase consumer willingness to purchase them. The model incorporates a return policy, recognizing its importance in enhancing consumer acceptance and demand in both traditional retail and the manufacturer's e-tail channels. Analytical solutions are derived for centralized and decentralized systems to optimize decision variables that maximize total profit. Numerical examples illustrate the model's findings, revealing that a return policy for perishable goods significantly benefits the supply chain members by increasing profit, particularly with higher demand.

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1. Introduction

The contemporary marketing landscape is undergoing significant transformation. Manufacturers are increasingly leveraging online channels to cultivate specialized markets, attract diverse customer segments, build brand trust, and secure better profit margins by reducing reliance on traditional retailers. Simultaneously, conventional retail channels continue to serve customers who lack internet access or prefer the tangible experience of inspecting products in physical stores before purchase. Current market analyses indicate a growing consumer preference for purchasing through both online and physical channels. To maintain a competitive edge and enhance both demand and profitability, many established manufacturers, such as Apple, HP, Reliance, Sony, Samsung, etc., have adopted a dual-channel strategy, integrating direct online sales with their existing retail networks. However, this rise in consumerism, particularly facilitated by e-commerce, has also contributed to increased product wastage. The convenience of online shopping can lead to impulsive purchases and subsequent disposal after limited use. Furthermore, e-commerce returns pose a substantial challenge, significantly eroding profit margins and

Keywords. Dual channel supply chain, quality of product, damage rate, return policy, stochastic demand.

[1] Department of Mathematics, Kazi Nazrul University, Asansol 713340, West Bengal, India.

[2] Department of Mathematics, Tamralipta Mahavidyalaya, Tamluk 721636, West Bengal, India.

*Corresponding author: santanukumar.ghosh@knu.ac.in

negatively impacting conversion rates. In 2023 alone, the total value of returned goods reached an astounding $743 billion, representing approximately 14.5% of total retail sales. The National Retail Federation estimates that these returns resulted in a $400 billion loss in sales for U.S. retailers, exacerbating environmental concerns due to disposal and logistical processes.

To address the growing challenge of product returns, companies are expanding their workforce, warehouse capacity, and establishing specialized reverse logistics operations. Returns have become an integral part of the customer journey, but they don't necessarily have to be a negative aspect. However, managing the costs associated with returns presents a significant challenge for supply chain managers. For instance, reports indicate that a substantial portion of online sales, such as 30% on platforms like Flipkart, result in returns, often due to product damage. This significantly elevates transportation expenses and, consequently, the overall cost of goods. This paper examines the rate at which damaged products are returned within the market. Effectively managing these returns offers an opportunity to recover substantial waste materials, preventing their disposal into the environment. It is hypothesized that implementing customer-centric policies, such as a clear return policy, can further encourage this process. A return occurs when a customer sends back a previously purchased online item for various reasons, including product dissatisfaction, receiving the wrong item, damage, etc. The typical process involves the customer initiating a return request, shipping the item back, and receiving a refund or exchange. A well-defined return policy outlines the conditions under which a retailer accepts returns, which, paradoxically, can also help in minimizing excessive returns.

Deterministic models yield outcomes entirely dictated by their initial states and parameter settings. Conversely, stochastic models inherently possess a degree of variability. Customer demand is a critical factor, fundamentally influencing decisions such as facility location. In practice, demand varies across customers and can be best represented as random variables following specific probability distributions. This demand uncertainty often arises from external factors, leading to unforeseen fluctuations. Events like public health emergencies or shifts in consumer preferences exemplify such situations. Given the unpredictability of consumer behavior, incorporating the stochastic nature of demand provides a more realistic representation of market dynamics for manufacturers and retailers.

This research introduces a dual-channel supply chain model under stochastic demand, where demand is influenced by selling price, product quality, return rate, and damage rate. In today's market, offering return policies has become a powerful strategy for boosting sales by fostering customer trust and brand loyalty. Consumers are more inclined to purchase from retailers they trust and are willing to recommend. Return options reduce the perceived risk associated with purchases, particularly online, thereby encouraging experimentation and improving conversion rates. Notably, studies indicate that a significant majority of consumers consult reviews before making purchasing decisions (*e.g.*, Spiegel, Bright Local, and Testimonial Engine). This consumer behaviour motivates the development of our model, which incorporates a return policy for damaged goods alongside quality considerations within a dual-channel supply chain framework. A novel aspect of this model is the formulation of the return rate (R) as a function of both product quality (q) and the damage rate (λ). This unique approach is adopted to:

– Establish a more distinctive and dependable parameter compared to fixed return rates.
– Potentially yield maximum profit by allowing manufacturers to strategically manage product quality and damage rates, which directly influence the return rate.

While much research has focused on supply chain coordination and integration, this study addresses dual-channel coordination by resolving channel conflicts. Notably, it incorporates the return rate of damaged products and their quality, aspects often overlooked in previous literature. Recognizing the inherent uncertainties in real-world scenarios, recent studies have increasingly adopted probabilistic demand functions to enhance model realism. This research builds upon this trend by stochastic optimization of demand.

Prior research on dual-channel supply chains with stochastic demand has often overlooked the influence of the return rate (R), which is inherently linked to product quality and damage occurrences. This paper addresses this gap by integrating the quality factor, return rate, and damage rate into a stochastic model featuring a

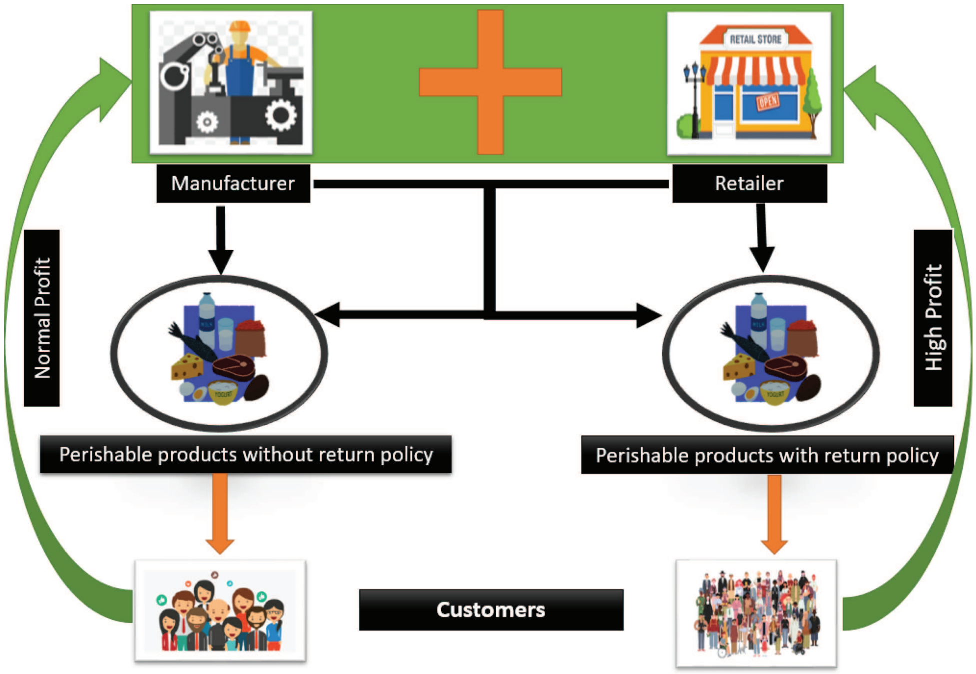


FIGURE 1. Pictorial representation of the present model.

dual-channel supply chain with a single manufacturer and retailer. The manufacturer utilizes both a traditional retail channel and an online (e-tail) channel. Within this stochastic demand framework, we analyze the impact of retail price, e-tail price, product quality, return rate, damage rate, and channel compatibility. Under the assumption that the manufacturer holds the position of channel leader, the model has been developed considering both the centralized and the decentralized situations.

The subsequent sections of this paper are structured as follows: Section 2 provides a review of relevant literature pertaining to the proposed model. Section 3 outlines the notations and fundamental assumptions employed in the model's formulation. The mathematical framework and analysis of the model are detailed in Section 4. Section 5 focuses on the analytical solution for a centralized system, including the presentation of a key proposition. Section 6 describes the solution approach for the decentralized system. Section 7 illustrates the optimal solutions for both centralized and decentralized scenarios through numerical examples. Following this, Section 8 presents a sensitivity analysis of the optimal solutions, and finally, Section 9 concludes the paper with a summary of the findings and potential future research directions. The schematic diagram illustrates the model in Figure 1.

2. Literature review

While designing a model of a dual channel supply chain, managers has to keep in mind the possibility of returned products by the customers. One of the major challenges of online channel is to reduce too many returns and minimizing the number of damaged products. For that the manufacturers have to first decide the permissible return rate and damage rate for a particular product. The existing literature deals with such problems individually. However we have tried to formulate a relation between them incorporating quality factors as well to create an integrated channel dynamics.

2.1. Dual-channel supply chain

Dual-channel supply chain takes an influential impact in Supply Chain Management (SCM) in the recent drifts of the worldwide marketing framework. Along with retail channel, an online channel operated by manufacturer is likely to lessen the overpowering of the retailers, thus accounting for the segments of individual customers, with a view to gain a better profit margin. For example, as more and more customers are drifting towards the internet usage, leading manufacturers like Infosys, IBM, Nike, Titan, Caratlane etc., have started selling in direct channels, together with the conventional retail channels. Several electronics based companies that includes Havells, Philips, Samsung, Xiomi, etc. have established their franchise stores in high-end localities. The leading English-language publishers, Random House has declared on-screen that, it may sell directly books to the readers that have put them in direct contest with Amazon.com [1].

Now a days many customary companies are extracting their trade through direct access via Internet at retail shops. For instance, Dell has set up kiosks in shopping complexes and now sells its' products through Castco [2]. Recent studies reveals that the customers prefers to enjoy multiple channels rather than single channel to meet their shopping needs [1]. There is always a cross-effect in between the customers who prefer retail channel shopping and the customers who prefer online shopping. As a result, the companies revamp their usual channel structures by emphasizing more in direct sales to access different customers in locations that can not be reached by the retailers, thus leading to emergence of dual-channel. Many problems have been addressed by several researchers in the field of dual-channel supply chains. Chen *et al.* [3] analyzed the pricing strategies in a dual-channel by introducing co-ordination contracts. Yue and Liu [4] examined the usage of sharing demand forecast information in a dual-channel supply chain by taking Gaussian primary demand as a linear function of price. Chiang *et al.* [5] considered the effects of consumers' compatibility to the online channels. Dan *et al.* [6] conceptualized the maximum service level and pricing factors in a supply chain model considering dual-channel scenario. Hua *et al.* [7] determined the impact of products delivery lead time on the prices in a dual-channel supply chain. Cao *et al.* [8] has modeled the wholesale contract design problem under asymmetry of information. Chen *et al.* [3] formulated a dual-chain supply chain model that proposed such pricing strategy which shall maximize the profit in decentralized dual-channel scenarios. Panda *et al.* [9] scrutinized the recycling and pricing policies in case of high-end product in a supply chain model considering dual-channel scenario. Batarfi *et al.* [10] claimed that dual-channel supply chain has the ability to maximize the channel profit in centralized scenario. However Sharma and Mehrotra [11] argued that, dual-channel supply chain has the tendency to increase the channel conflicts in spite of having the ability to increase the subsequent customers' demand. Dumrongsiri *et al.* [12] suggested that manufacturer has potential to be better in case of dual-channel than that in a single channel, provided that the retailer's cost is high in comparison to the wholesale price with demand certainty. Panda *et al.* [9] and Batarfi *et al.* [13] studied the optimal pricing and recycling factors in a dual-channel supply chain by considering unit cost decrease in continuous manner. Modak and Kelle [14] coordinated a dual channel supply chain model while considering price and delivery-time dependent uncertain demand. Their studies established that under suitable policy for channel coordination, it yields better results. Recent studies by [15] investigated a supply chain model considering dual-channel scenario with price competitiveness and green sensitive demand. They coordinated the channel to optimize their results in more green oriented and feasible manner. All the above models studied the conflicts among the retail and e-tail channels under certainty in demand but none of them have neither accounted for the uncertainty in demand nor they have taken quality dependent return rate under consideration.

2.2. Stochastic demand

Given the unpredictable nature of market demand, numerous researchers have developed supply chain models that incorporate stochastic rather than deterministic demand. Currently, most supply chain researchers acknowledge that accounting for the randomness of demand leads to more realistic models. Consequently, over the past few decades, many supply chain models have been formulated by researchers who have considered stochastic demand. For instance, [16] analyzed supply chain coordination under channel concessions and sales efforts, where the manufacturer incentivizes the retailer to increase efforts and order quantities by enhancing their marginal revenue through a linear rebate structure. Zhang *et al.* [17] examined a Stackelberg approach in a single vendor-single manufacturer inventory model, analyzing individual profit structures under different scenarios. Gautam *et al.* [18] developed a vendor-buyer inventory model that considered carbon emissions. Sana *et al.* [19] investigated a manufacturer-distributor system, focusing on controlling cash flow in a supply chain model using feedback loops. He *et al.* [20] studied supply chain coordination with stochastic demand that was sensitive to both sales effort and retail price. Xie and Wei [21] proposed pricing and coordinated advertising strategies within a manufacturer-retailer supply chain. Gurnani and Erkoc [22] analyzed a decentralized two-level supply chain model where demand was influenced by the product's selling price and the retailer's selling efforts. Sana [23] conducted a comparative analysis of non-green and green supply chains, considering the corporate social responsibility of the companies. Chen and Bell [24] designed a price-dependent supply chain model with stochastic demand and a buyback policy. In their model, they achieved optimal coordination between the manufacturer and retailer in a decentralized supply chain where the retailer determined the order quantity and retail price. Taleizadeh *et al.* [25] explored a single vendor, multi-product, multi-buyer supply chain model with lead time dependent on lot size under various constraints and uncertain demand. Hisham and Magy [26] developed a three-layer lot-sizing problem involving a single supplier, single manufacturer, and multiple retailers facing stochastic demand. Govindan *et al.* [27] formulated a multi-objective supply chain model with sustainable order distribution and stochastic demand, utilizing a reliable supply chain network. Rezaee *et al.* [28] investigated a green supply chain model considering the uncertainty of product demand and carbon price in a carbon trading environment. Giri and Dey [29] analyzed a closed-loop dual-channel supply chain model using a game-theoretic approach, where the manufacturer used both fresh and recycled materials for production. Malik and Sarkar [30] examined cost-sharing and optimal coordination among supply chain partners, considering random demand and a flexible, complex production system. However, despite this extensive body of literature, the impact of manufacturers' return policies on damage rates and quality factors in a dual-channel supply chain with stochastic demand remains unexplored. This is particularly relevant when the manufacturer operates a traditional retail channel alongside an e-tail channel as an equally important avenue to boost product sales.

2.3. Return policy

When an online shopper sends back a previously bought item, this is known as a return. This can happen for various reasons, such as the customer being unhappy with the item, receiving the wrong order, or the product being damaged. Typically, this process involves the customer initiating a return request, sending the product back to the seller, and receiving either a replacement or their money back. A return policy is a documented set of guidelines outlining the conditions under which a store will accept returned items. He *et al.* [31] examined a model for determining the best return strategies for products sold within a limited timeframe, considering uncertain market demand and potential risks. Jian *et al.* [32] found that customer returns influence a retailer's ordering decisions, the manufacturer's earnings, and the retailer's earnings. Their research highlighted that when the refund amount differs significantly, a buyback agreement can help align the supply chain. Guo *et al.* [33] investigated a two-channel supply chain and concluded that a full-refund policy offered solely through the indirect channel yields greater strategic advantages and retailer profits when the return rate is low. Xu *et al.* [34] explored the potential negative impacts of return policies on retailers by analyzing the profitable effects of consumer returns across customers, retailers, and the supply chain. Tyagi and Dhingra [35] presented a study based on the return policies of several prominent online retailers. They emphasized the importance of returns in online retail and proposed a theoretical framework based on how e-retailers' return policies affect consumer purchasing behaviour, as well as the mediating influence of consumers' knowledge, understanding, awareness, and perception of online shopping habits on the relationship between their buying behaviour and the impact of e-retailers' return policies. Anthony and Julia [36] reviewed that exploring methods to improve return policies offers potential benefits for customer satisfaction and environmental sustainability. They also identified areas for future research, suggesting continued investigation into predictive analytics and customer behaviour to refine returns management practices further. However, the existing literature contains very few mathematical approaches to this subject. Our study offers a different and original perspective in this regard. Specifically, there is limited research that considers the return rate as dependent on both product quality and the rate at which products are damaged.

3. Notations and assumptions

The following notations and assumptions have been undertaken for this study:

3.1. Notations

θ : Product's compatibility with the retail channel ($0 < \theta < 1$).
a : Market potential ($a > 0$).
α_1 : Price sensitivity in the retail channel.
α_2 : Price sensitivity in the online channel.
α : Cross price effect (degree of price competition).
w_m : Manufacturer's wholesale price to the retailer in retail channel (\$).
q : Quality of the product (\$).
R : Return rate of the product (*units*).
λ : Damage rate of the product.
β : Sensitivity parameter for quality of the product.
γ : Sensitivity parameter for damage rate of the product.
p_r : Retailer's selling price (\$).
p_d : Manufacturer's selling price of in online channel (\$).
Q_r : Retailer's order quantity for retail channel (*units*).
Q_d : Manufacturer's inventory for online channel (*units*).
Q : Total inventory $Q = Q_r + Q_d$.
x : A part of demand quantity which is a random variable, following a probability distribution. $f(x)$: Probability density function of x.
$F(x)$: Probability distribution function of x.
r : Unit salvage value.
t_r : Retailer's shortage cost.
t_d : Manufacturer's shortage cost in the online channel.
x^+ : Max$[x, 0]$ the positive part of x.
c_r : Manufacturer's purchasing cost in the retail channel.
c_d : Manufacturer's procurement cost in the retail online channel.
π_r : Retailer's profit.
π_m : Manufacturer's profit.
π_c : Profit of the centralized system.

3.2. Assumptions

(1) Consider a supply chain with two levels and two sales avenues, comprising one retailer and one manufacturer. The manufacturer produces goods and supplies them to the retailer, who then offers these goods to consumers. The manufacturer's price to the retailer is invariably less than the price at which the retailer sells to the end customer. Additionally, the manufacturer possesses the option to directly market and sell their products to consumers via an online channel. This framework has been the subject of investigation in numerous studies, including those by [37–40], and others.
(2) The presence of dual distribution channels for a single product creates a cross-price effect. This situation is posited to foster a competitive environment for the same product across different channels. Furthermore, it is hypothesized that the cross-price elasticity within each channel positively influences its own demand.

(3) The demand function incorporates a stochastic element, representing the market's potential demand x, which follows a probability distribution $f(x)$. Additionally, demand is influenced by the product return rate and quality level. Current models often primarily focus on the selling price as the main determinant of demand.
(4) In both online and physical retail, the potential market demand is segmented based on the consistency or compatibility of the product. This aligns with research conducted by [41] and numerous other scholars. Consequently, we have defined θ as the product compatibility factor, which introduces dynamism to the demand variable.
(5) A novel assumption is introduced: the product return rate (R) is influenced by both the quality factor (q) and the rate at which sold products are damaged (λ). This premise stems from the understanding that online customers readily return damaged goods, underscoring the importance of maintaining product quality. The manufacturer bears the expense of handling these returns, which, in turn, can incentivize improvements in product quality and potentially boost product demand.
(6) The persistence of consumer demand during periods of stockouts is a well-recognized phenomenon. This study operates under the premise that such shortages can manifest in both digital and physical retail environments. Informed by previous scholarly investigations, notably the work of [42], and the broader body of literature where numerous researchers have integrated the concept of shortages into their models, this analysis proceeds with this understanding.
(7) A fundamental assumption of this study is the flexibility of lead time in both online and traditional retail avenues. The topic of lead time has been further developed by numerous researchers, including [25, 30, 43].

4. Mathematical formulation and analysis of the model

A supply chain model featuring a single manufacturer and a single retailer, operating through both offline (traditional retail) and online (e-tail) channels is investigated. The manufacturer's products are distributed across these two channels, with pricing strategies, promotional efforts, and feedback effects influenced by demand, which is uncertain and subject to variability. Building on prior research by [4, 44–46], the demand function is considered to be linearly dependent on the individual pricing of each channel, as well as the competitive pricing interactions between them.

The demand functions of the retail and online channels are

$$D_r = \theta x - \alpha_1 p_r + \alpha(p_d - p_r) + \beta q - \gamma\lambda + \delta R \tag{1}$$

$$\text{and} \quad D_d = (1-\theta)x - \alpha_2 p_d + \alpha(p_r - p_d) + \beta q - \gamma\lambda + \delta R. \tag{2}$$

Therefore, the expected demand in the retail channel is

$$E(D_r) = \theta\mu - \alpha_1 p_r + \alpha(p_d - p_r) + \beta q - \gamma\lambda + \delta R$$

and in the e-tail channel the expected demand is

$$E(D_d) = (1-\theta)\mu - \alpha_2 p_d + \alpha(p_r - p_d) + \beta q - \gamma\lambda + \delta R.$$

The schematic diagram in Figure 2 illustrates the model.

In this model, market demand exhibits variability, represented by $x > 0$, which signifies the base potential market size. This follows the probability density function $f(x)$, ensuring $\int_{-\infty}^{\infty} f(x)\mathrm{d}x = 1$. For the retail channel, demand decreases as the retailer's selling price rises, while in the e-tail channel, demand increases with the manufacturer's price. The parameter θ ($0 \le \theta \le 1$) defines the product's compatibility within the retail sector, indicating how well the product aligns with consumer needs, beliefs, and customs. Higher compatibility means greater market acceptance. The demand allocation is determined by θ, with θ representing the proportion directed to retail and $(1-\theta)$ corresponding to manufacturer-led demand. A higher θ implies greater compatibility. Generally, books, clothing, electronics, and gadgets favor direct sales channels, whereas items like groceries, fuel, and automobiles are more suited to retail distribution. Price sensitivity in both channels is represented

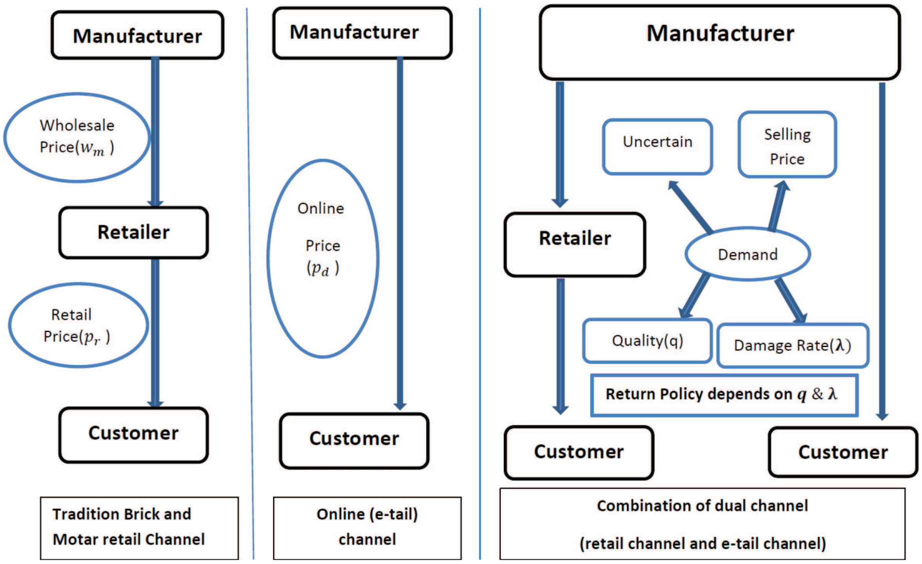


Figure 2. Pictorial representation of the present model.

by α_1 (>0) and α_2 (>0), while α (>0) denotes the competitive price effect between the two channels. To reflect real-world conditions, it is assumed that the e-tail selling price exceeds the manufacturer's wholesale price ($p_d > w_m$), preventing retailers from directly purchasing from the e-tail channel. Additionally, for retailer profitability, its selling price must be higher than the manufacturer's wholesale price ($p_r > w_m$). Customer feedback plays a crucial role in assessing satisfaction levels, allowing manufacturers to refine their products to better meet consumer expectations. This ultimately enhances the company's goodwill and market reputation.

5. Centralized model

In a centralized system, decisions are consistently optimized for all participants within the supply chain. It is assumed that a sole decision-maker is responsible for maximizing overall profitability. The total profit of the system is determined by the combined earnings from retail sales (π_r) and the profits generated through the e-tail channel (π_m).

The profit of retailer in retail channel,

$$\pi_r = p_r E(D_r) - w_m Q_r + r\int_{-\infty}^{A}(A-x)f(x)\mathrm{d}x - t_r\int_{A}^{\infty}(x-A)f(x)\mathrm{d}x - (1-m)\gamma\lambda^2 - n\delta R. \tag{3}$$

The manufacturer's profit is given by π_m = (Profit in retail channel) + (profit in e-tail channel) − (salvage cost) − (shortage cost in e-tail channel) − (Damage cost)-(Partial return cost). *i.e.*,

$$\begin{aligned}\pi_m = (w_m - c_r)Q_r + p_d E(D_r) - c_d Q_d - r\int_{-\infty}^{A}(A-x)f(x)\mathrm{d}x \\ -t_d\int_{B}^{\infty}(x-B)f(x)\mathrm{d}x - \beta q - m\gamma\lambda^2 - (1-n)\delta R.\end{aligned} \tag{4}$$

Where, $R = \frac{k_1}{q} + k_2\lambda$

$$A = Q_r + \alpha_1 p_r - \alpha(p_d - p_r) - \beta q + \gamma\lambda - \delta R$$

$$\text{and} \quad B = Q_d + \alpha_2 p_d - \alpha(p_r - p_d) - \beta q + \gamma\lambda - \delta R.$$

Therefore, the total profit of the system is,

$$\begin{aligned}
\pi_c &= \pi_r + \pi_m \\
&= p_r[\theta\mu - \alpha_1 p_r + \alpha(p_d - p_r) + \beta q - \gamma\lambda + \delta R] && (5) \\
&\quad -t_r \int_A^\infty [x - Q_r - \alpha_1 p_r + \alpha(p_d - p_r) + \beta q - \gamma\lambda + \delta R] f(x) \mathrm{d}x \\
&\quad -c_r Q_r + p_d[(1-\theta)\mu - \alpha_2 p_d + \alpha(p_r - p_d) + \beta q - \gamma\lambda + \delta R] - c_d Q_d - \beta q - \gamma\lambda^2 - \delta R \\
&\quad -t_d \int_B^\infty [x - Q_d - \alpha_2 p_d + \alpha(p_r - p_d) + \beta q - \gamma\lambda + \delta R] f(x) \mathrm{d}x. && (6)
\end{aligned}$$

The necessary condition for π_c to be maximum are; $\frac{\partial \pi_c}{\partial p_r} = 0$, $\frac{\partial \pi_c}{\partial p_d} = 0$, $\frac{\partial \pi_c}{\partial Q_r} = 0$, $\frac{\partial \pi_c}{\partial Q_d} = 0$ $\frac{\partial \pi_c}{\partial q} = 0$, $\frac{\partial \pi_c}{\partial \lambda} = 0$, where,

$$\begin{aligned}
\frac{\partial \pi_c}{\partial p_r} &= \mu\theta - 2(\alpha + \alpha_1)p_r + 2\alpha p_d + \beta q + \frac{k_1\delta}{q} + (k_2\delta - \gamma)\lambda \\
&\quad + (\alpha + \alpha_1)t_r[1 - F(A)] - \alpha t_d[1 - F(B)] && (7) \\
\frac{\partial \pi_c}{\partial p_d} &= \mu(1-\theta) - 2(\alpha + \alpha_2)p_d + 2\alpha p_r + \beta q + \frac{k_2\delta}{q} + (k_2\delta - \gamma)\lambda \\
&\quad + (\alpha + \alpha_2)t_d[1 - F(B)] - \alpha t_r[1 - F(A)] && (8) \\
\frac{\partial \pi_c}{\partial Q_r} &= t_r[1 - F(A)] - c_r && (9) \\
\frac{\partial \pi_c}{\partial Q_d} &= t_d[1 - F(B)] - c_d && (10) \\
\frac{\partial \pi_c}{\partial q} &= \left(\beta - \frac{k_1\delta}{q^2}\right)(p_r + p_d) - \left(\beta - \frac{k_1\delta}{q^2}\right)t_r[1 - F(A)] \\
&\quad - t_d\left(\beta - \frac{k_1\delta}{q^2}\right)[1 - F(B)] - \left(\beta - \frac{k_1\delta}{q^2}\right) && (11) \\
\frac{\partial \pi_c}{\partial \lambda} &= (k_2\delta - \gamma)(p_d + p_r) - (k_2\delta - \gamma)t_r[1 - F(A)] \\
&\quad - (k_2\delta - \gamma)t_d[1 - F(B)] - 2\gamma\lambda - k_2\gamma. && (12)
\end{aligned}$$

From the equations $\frac{\partial \pi_c}{\partial Q_r} = 0$ and $\frac{\partial \pi_c}{\partial Q_d} = 0$, we get

$$Q_r = F^{-1}\left[\frac{t_r - c_r}{t_r}\right] + \alpha p_d - (\alpha + \alpha_1)p_r + \beta q + \frac{\delta k_1}{q} + (\delta k_2 - \gamma)\lambda \tag{13}$$

$$Q_d = F^{-1}\left[\frac{t_d - c_d}{t_d}\right] + \alpha p_r - (\alpha + \alpha_2)p_d + \beta q + \frac{\delta k_1}{q} + (\delta k_2 - \gamma)\lambda. \tag{14}$$

Again from the equations $\frac{\partial \pi_c}{\partial p_r} = 0$; $\frac{\partial \pi_c}{\partial p_d} = 0$;$\frac{\partial \pi_c}{\partial q} = 0$ and $\frac{\partial \pi_c}{\partial \lambda} = 0$ and solving we get the optimal values of retail, e-tail price, quality of the product and damage rate are,

$$p^*_{rc} = \frac{[(2\theta - 1)\mu + 4\alpha + 2\alpha_2 + (6\alpha + \alpha_1 + 2\alpha_2)c_r + (2\alpha + \alpha_2)c_d]}{2(4\alpha + \alpha_1 + \alpha_2)} \tag{15}$$

$$p^*_{dc} = \frac{4\alpha + 2\alpha_1 + (1 - 2\theta)\mu - (2\alpha + \alpha_1)c_r + (6\alpha + \alpha_1 + \alpha_2)c_d}{2(4\alpha + \alpha_1 + \alpha_2)} \tag{16}$$

$$q_c^* = \frac{1}{2L}(-M + \sqrt{M^2 - 4LN}) \tag{17}$$

$$\lambda_c^* = \frac{1}{2\gamma}(2k_2\delta - \gamma). \tag{18}$$

Where, $L = 2\beta\gamma(4\alpha + \alpha_1 + \alpha_2), N = 2k_1\gamma\delta(4\alpha + \alpha_1 + \alpha_2)$ and $M = 2\gamma q[(\alpha_2 - \alpha_1)\theta\mu + (2\alpha + \alpha_1)\mu - (\alpha\alpha_1 + \alpha_1\alpha_2 + \alpha_2\alpha_1)(c_r + c_d - 2) + (4\alpha + \alpha_1 + \alpha_2)(2k_2^2\delta^2 - 3k_2\gamma\delta + \gamma^2)]$.

Putting the values of equations (15)–(18) in equations (13) and (14), we get the optimal lot sizes as

$$Q_{rc}^* = F^{-1}\left[\frac{t_r - c_r}{t_r}\right] + \alpha p_{dc}^* - (\alpha + \alpha_1)p_{rc}^* + \beta q^* + \frac{\delta k_1}{q^*} + (\delta k_2 - \gamma)\lambda^* \tag{19}$$

$$Q_{dc}^* = F^{-1}\left[\frac{t_d - c_d}{t_d}\right] + \alpha p_{rc}^* - (\alpha + \alpha_2)p_{dc}^* + \beta q^* + \frac{\delta k_1}{q^*} + (\delta k_2 - \gamma)\lambda^*. \tag{20}$$

Proposition 5.1. *The total profit of the centralized system* (π_c) *will be maximum if* $24(\alpha + \alpha_1)f^2 > (\gamma + 6\beta f^2)$ *and* $96f^2(\alpha\alpha_1 + \alpha_1\alpha_2 + \alpha\alpha_2) > 4(6f^2\beta + \gamma)\{(\alpha\alpha_1 + \alpha_1\alpha_2 + \alpha\alpha_2)t_d f(B) + (\alpha_1 + \alpha_2 + 4\alpha)\}$.

Proof. See Appendix A. □

Proposition 5.1 establishes two sufficient conditions for maximizing π_c. Consequently, π_c reaches its optimal value when its decision variables satisfy six necessary conditions – specifically, (4), (5), (6), (7), (8), (9) along with the two sufficient conditions outlined in the proposition. Therefore, the decision variables that meet conditions (4), (5), (6), (7), (8) and (9) are considered optimal, provided the two required conditions from the proposition hold true.

This proposed model highlights that the centralized system achieves maximum profitability only when the specified conditions in Proposition 1 are fulfilled.

6. Decentralized model

In a decentralized supply chain, individual units make decisions based on localized information. This structure enables participants to collaborate effectively from the outset, streamlining operations across the entire supply network. By fostering cooperation rather than competition, a decentralized system enhances overall coordination among entities within the supply chain. The profit of the manufacturer is

$$\pi_m = (w_m - c_r)Q_r + p_d E(D_r) - c_d Q_d - r\int_{-\infty}^{A}(A - x)f(x)\mathrm{d}x \tag{21}$$
$$-t_d\int_{B}^{\infty}(x - B)f(x)\mathrm{d}x - \beta q - m\gamma\lambda^2 - (1 - n)\delta R$$

where,

$$E[B - x]^+ = \int_{-\infty}^{B}(B - x)f(x)\mathrm{d}x$$
$$E[x - B]^+ = \int_{B}^{\infty}(x - B)f(x)\mathrm{d}x$$
$$\int_{-\infty}^{\infty}f(x)\mathrm{d}x = 1$$
$$F(B) = \int_{-\infty}^{B}f(x)\mathrm{d}x.$$

The necessary conditions for π_m to be maximum are given by $\frac{\partial \pi_m}{\partial p_m} = 0$, $\frac{\partial \pi_m}{\partial Q_d} = 0$, $\frac{\partial \pi_m}{\partial q} = 0$, where,

$$\begin{aligned}\frac{\partial \pi_m}{\partial p_d} &= (1-\theta)\mu - \alpha_2 p_d - \alpha(p_r - p_d) + \beta q - \gamma\lambda + \delta\left(\frac{k_1}{q} + k_2\lambda\right) \\ &\quad - p_d(\gamma + \alpha_2) + \alpha r \alpha F(A) + t_d(\alpha + \alpha_2)[1 - F(B)] \\ \frac{\partial \pi_m}{\partial Q_d} &= -c_d + t_d\{1 - F(B)\} \\ \frac{\partial \pi_m}{\partial q} &= p_d\left(\beta - \frac{\delta k_1}{q^2}\right) + r\left(\beta - \frac{\delta k_1}{q^2}\right)F(A) - t_d\left(\beta - \frac{\delta k_1}{q^2}\right)[1 - F(B)] - \beta + (1-n)\frac{\delta k_1}{q^2}.\end{aligned} \tag{22}$$

Now solving the equations $\frac{\partial \pi_m}{\partial p_d} = 0$, $\frac{\partial \pi_m}{\partial q} = 0$, $\frac{\partial \pi_m}{\partial Q_d} = 0$, we get,

$$(1-\theta)\mu + \alpha p_r - (\alpha + \alpha_2)(2p_d - c_d) + \beta q + \frac{\delta k_1}{q} - (\gamma - k_2\delta)\lambda + \alpha r \frac{w_m - t_r}{r - t_r} = 0 \tag{23}$$

$$p_d\left(\beta - \frac{\delta k_1}{q^2}\right) + r\left(\beta - \frac{\delta k_1}{q^2}\right)F(A) - t_d\left(\beta - \frac{\delta k_1}{q^2}\right)[1 - F(B)] - \beta + (1-n)\frac{\delta k_1}{q^2} \tag{24}$$

$$Q_d = F^{-1}\left[\frac{t_d - c_d}{t_d}\right] - \alpha_2 p_d + \alpha(p_r - p_d) - \beta q - \gamma\lambda + \delta R. \tag{25}$$

Proposition 6.1. *The manufacturer's total profit* (π_m) *is concave if* $2(\alpha + \alpha_2) + \alpha^2 r f(A) > 0$ *and* $\frac{f(A)}{q^3}[2k_1\delta r\alpha^2 t_d + 2k_1\delta r^2\alpha^2 f(A) + 2k_1\delta(n-1)r\alpha^2 - 2k_1\delta r\alpha^2 t_d\{1 - f(A)\}] < (\beta - \frac{k_1\delta}{q^2})^2[2(\alpha + \alpha_2)rf(A) + 2\alpha r f(A) - 1] + \frac{\alpha+\alpha_2}{q^3}[4(1-n)k_1\delta - 4k_1\delta p_d - 4k_1\delta r f(A) + 4k_1\delta t_d\{1 - f(A)\}]$.

Proof. See Appendix B. □

Proposition 6.1 establishes two sufficient conditions for maximizing π_m. Consequently, π_m reaches its optimal value when its decision variables satisfy four necessary conditions – specifically, (19), (20), (21), (22) along with the two sufficient conditions outlined in the proposition. Therefore, the decision variables that meet conditions (19), (20), (21) and (22) are considered optimal, provided the two required conditions from the proposition hold true.

This proposed model highlights that the decentralized system achieves maximum profitability for manufacturer only when the specified conditions in Proposition 2 are fulfilled. The manufacturer can initiate most profit in decentralized scenario.

In the decentralized model, the profit of the retailer is given by

$$\begin{aligned}\pi_r &= p_r E(D_r) - w_m Q_r + r\int_{-\infty}^{A}(A - x)f(x)\mathrm{d}x \\ &\quad - t_r\int_{A}^{\infty}(x - A)f(x)\mathrm{d}x - (1-m)\gamma\lambda^2 - n\delta R.\end{aligned} \tag{26}$$

The necessary condition for π_r to be maximum are ; $\frac{\partial \pi_r}{\partial p_r} = 0$, $\frac{\partial \pi_r}{\partial Q_r} = 0$, $\frac{\partial \pi_r}{\partial \lambda} = 0$ where,

$$\begin{aligned}\frac{\partial \pi_r}{\partial p_r} &= \mu\theta - 2(\alpha + \alpha_1)p_r + \alpha p_d + \beta q - \gamma\lambda \\ &\quad + \delta\left(\frac{k_1}{q} + k_2\lambda\right) + (r - t_r)(\alpha + \alpha_1)F(A) - (\alpha + \alpha_1)t_r \\ \frac{\partial \pi_r}{\partial Q_r} &= -w_m + rF(A) + t_r(1 - F(A))\end{aligned}$$

$$\frac{\partial \pi_r}{\partial \lambda} = (-\gamma + k_2\delta)p_r + r(\gamma - k_2\delta)F(A) \\ -t_r(-\gamma + k_2\delta)[1 - F(A)] - 2(1-m)\gamma\lambda - n\delta k_2.$$

The equations $\frac{\partial \pi_r}{\partial p_r} = 0$, $\frac{\partial \pi_r}{\partial \lambda} = 0$ and $\frac{\partial \pi_r}{\partial Q_r} = 0$ gives,

$$\mu\theta - 2(\alpha+\alpha_1)p_r + \alpha p_d + \beta q - \gamma\lambda + \delta\left(\frac{k_1}{q} + k_2\lambda\right) + (r - t_r)(\alpha+\alpha_1)F(A) - (\alpha+\alpha_1)t_r = 0 \tag{27}$$

$$(-\gamma + k_2\delta)p_r + r(\gamma - k_2\delta)F(A) - t_r(-\gamma + k_2\delta)[1 - F(A)] - 2(1-m)\gamma\lambda - n\delta k_2 = 0 \tag{28}$$

$$\text{and,}\quad Q_r = F^{-1}\left[\frac{w_m - t_r}{r - t_r}\right] - \alpha_1 p_r + \alpha(p_d - p_r) + \beta q - \gamma\lambda + \delta R. \tag{29}$$

Proposition 6.2. *The retailer's total profit (π_r) is concave if $t_r > r$, $(\alpha+\alpha_1)(t_r - r)f(A) > 0$ and $(t_r - r)f(A)[(\gamma - k_2\delta)^2 - 4(1-m)(\alpha+\alpha_1)\delta] > 0$.*

Proof. Follow Appendix C. □

Proposition 6.2 establishes two sufficient conditions for maximizing π_r. Consequently, π_r reaches its optimal value when its decision variables satisfy three necessary conditions – specifically, (24), (25), (26) along with the two sufficient conditions outlined in the proposition. Therefore, the decision variables that meet conditions (24), (25), and (26) are considered optimal, provided the two required conditions from the proposition hold true.

This proposed model highlights that the decentralized system achieves maximum profitability for a retailer only when the specified conditions in Proposition 3 are fulfilled.

Two conditions which are expressed in Proposition 3 are sufficient conditions for π_r to be maximum. Thus, optimal value of π_r takes place if the decision variables of π_r persuade three necessary conditions (24), (25), (26) and two sufficient conditions which is given in this proposition. Consequently, the set of all decision variables satisfying (24), (25) and (26) are optimal if two stated conditions of this proposition are satisfied.

The above proposed Proposition divulges that only under certain conditions (as stated in Prop. 3) the retailer can bring back maximum profit in decentralized scenario.

Now, solving the equations, (12), (23), (25), (27), (28) and (29) we get, a bi-quadratic equation of the decision variable quality q as

$$\beta^2 q^4 + B_1\beta q^3 - (A_1 + k_1\delta B_1)q - k_1^2\delta^2 = 0. \tag{30}$$

Where, $A_1 = [\frac{2(\alpha+\alpha_1)}{B'} - \alpha - \frac{CD}{B'E} + \frac{k_2 CD}{B'}]k_1 n\delta$
$B_1 = \theta\mu - 2(\alpha+\alpha_1)\{\frac{1}{B'}[C(A' - c_d - 1) + F + \alpha A' + G]\} + \frac{1}{E}[Dw_m - \frac{D}{B'}\{C(A' - c_d - 1) + F + \alpha A' + G\}]$
$A' = r(\frac{w_m - t_r}{r - t_r})$; $B' = 3\alpha + 2\alpha_1$; $C = 3\alpha + 3\alpha_2$; $D = \gamma - k_2\delta$; $E = 2(1-m)$; $F = x - \theta(\mu + x)$ *and* $G = c_d(\alpha + \alpha_2)$.

Applying Descartes' Rule of signs on equation (30), we observe that the equation has only one change of sign. Therefore the equation (30) has only one positive real root. Let us consider it will be q_c^*.

Therefore the optimal values of the decision variables (p_r), (p_d), (λ), (Q_r) and (Q_d) as

$$p_{dd}^* = \frac{n\delta k_1}{\beta q^2 - \delta k_1} - c_d + A' \tag{31}$$

$$p_{rd}^* = \frac{1}{B'}\left[C\left(\frac{n\delta k_1}{\beta q^2 - \delta k_1} - c_d - 1 + A'\right) + F + \alpha A' + G\right] \tag{32}$$

$$\lambda_d^* = \frac{1}{\gamma E}\left[D\left\{w_m - \frac{1}{B'}\left(C\left(\frac{n\delta k_1}{\beta q^2 - \delta k_1} - c_d - 1 + A'\right) + F + \alpha A' + G\right)\right\} - n\delta k_2\right].$$

For decentralized model, the optimal lot sizes is deduced by assigning the values of p_{rd}^*, p_{dd}^*, q_d^* and λ_d^* in equations (29) and (25)

$$Q_{rd}^* = F^{-1}\left[\frac{w_m - t_r}{r - t_r}\right] - \alpha_1 p_r^* + \alpha(p_d^* - p_r^*) + \beta u_d^* + \gamma f_d^* \tag{33}$$

TABLE 1. Optimal solutions for centralized and decentralized case with return policy ($\delta = 0.2$).

Scenarios	p_r (units)	p_d (units)	Q_r (units)	Q_d (units)	q (Rs.)	λ(Rs.)	π_m (Rs.)	π_r (Rs.)	π_c(Rs.)
Centralized	65.57	76.20	588.32	387.55	65.50	18.99	–	–	11 266.20
Decentralized	58.72	68.53	323.67	390.72	52.61	17.54	7489.5	2917.4	–

TABLE 2. Optimal solutions for centralized and decentralized case without return policy ($\delta = 0$).

Scenarios	p_r (units)	p_d (units)	Q_r (units)	Q_d (units)	q (Rs.)	π_m (Rs.)	π_r (Rs.)	π_c(Rs.)
Centralized	64.53	75.29	585.67	384.90	64.46	–	–	10 630.6
Decentralized	55.32	61.2	312.91	401.17	48.09	6454.3	2704.07	–

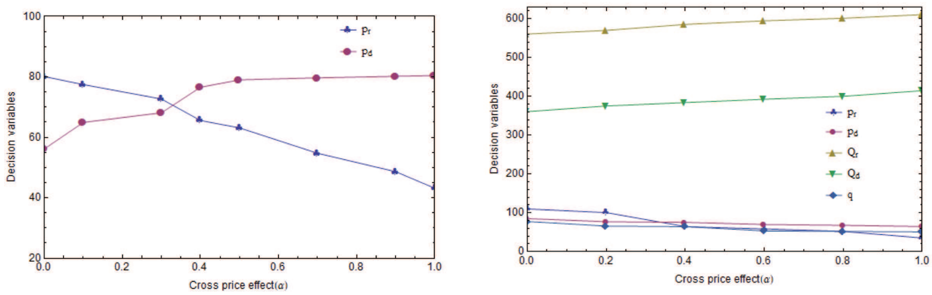
$$Q_{dd}^* = F^{-1}\left[\frac{t_d - c_d}{t_d}\right] + \alpha(p_r^* - p_d^*) - \alpha_2 p_d^* + \beta u_d^* + \gamma f_d^*. \tag{34}$$

7. Numerical example

We evaluate the values of key parameters in appropriate units as follows. Demand is assumed to follow an exponential distribution, characterized by the probability density function $f(x) = a\, e^{-ax}$, $-\infty \leq x \leq \infty$, where $a = 0.001428$ units and the mean $\mu = 700$ units. The exponential probability distribution is employed here to determine optimal decision variables, as this type of distribution has been widely used in various research models ([41] and others). While other distributions, such as uniform or normal distributions, could also be viable, the exponential model is utilized due to the lack of precise industry data. Consequently, estimated parameters are applied to numerically derive optimal decision variables, following the approach adopted by previous researchers ([41] and others).

Additional parameters are examined as follows: $m = 1.2$, $n = 1.3$, $t_r = 14$ units, $t_d = 14$ units, $\alpha = 0.4$ units, $\alpha_1 = 2.5$ units, $\alpha_2 = 3$ units, $\theta = 0.4$ units, $c_d = 6$ units, $c_r = 5$ units, $\mu = 700$ units, and $r = 1$ unit. Subsequently, optimal solutions for both decentralized and centralized cases, incorporating feedback with factor $\gamma = 0.6$, are presented in Table 1. Based on Table 1, the centralized approach is identified as the most effective strategy for supply chain participants. The profit distribution across direct and online channels, along with the total profit, is detailed in the table. Previous sections have already demonstrated that the profit functions for both the manufacturer and the retailer exhibit declines under certain conditions, depending on decision variables. This model remains applicable in cases where shortages and excess inventory are not allowed. In the decentralized framework, the retailer determines the optimal price p_{rd} after receiving the manufacturer's optimal wholesale price w_m, whereas in the direct channel, the manufacturer sets the optimal price p_{dd} based on the retailer's price p_{rd}.

Tables 1 and 2 from the numerical examples demonstrate that adopting a centralized supply chain consistently yields higher profits compared to a decentralized approach. Additionally, the tables reveal that when a return policy is considered, the profit margins of individual channel members surpass their net profits. This outcome is intuitive, as effective coordination among supply chain participants enhances overall performance, leading to increased profitability both for the centralized system and individual members. Moreover, the implementation of a return policy contributes to profit growth by driving market demand. It also provides manufacturers with valuable customer feedback, allowing them to refine product quality in alignment with consumer expectations and preferences.



(A) centralized scenario (B) Decentralized scenario

FIGURE 3. Graphical representation of p_r, p_d, Q_r, Q_d, q and λ *vs.* α.

8. Sensitivity analysis

8.1. Impact of the cross-price effect

In a centralized supply chain, the cross-price effect – represented by the competitive sensitivity parameter α – has a favorable influence on the e-tail channel. As α increases, heightened competition leads consumers to prefer the retail channel due to the reduction in retail price p_r. However, the pricing for the online channel p_d rises due to the cross-price relationship. Figure 3a illustrates that optimal prices p_r and p_d are inversely related to the cross-price effect α.

On the other hand, the decentralized scenario, depicted in Figure 3b, shows an increase in order quantity for the retail channel and a higher inventory level for the manufacturer. This trend is driven by a decrease in the retailer's selling price for the retail channel, influenced by the cross-price effect, whereas the opposite occurs for the manufacturer's online channel. Additionally, the competitive sensitivity parameter α does not have a significant impact on product quality. This result aligns with practical business expectations.

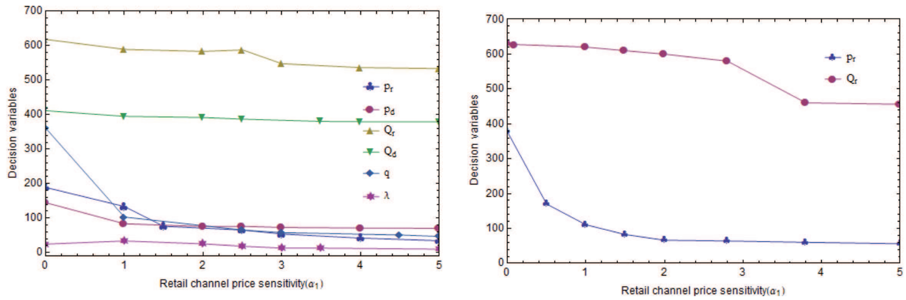
8.2. Impact of the price sensitivity parameters

In Figure 4a, within the centralized framework, an increase in the price sensitivity parameter of the retail channel (α_1) leads to an initial sharp decline in optimal retail pricing (p_r) and online channel pricing (p_d), followed by a more gradual decrease. As the unit price of the product drops, consumer demand rises, causing order quantities in both channels to diminish. Additionally, fluctuations in the quality parameter (q) and the screening rate for damaged goods (λ) are noteworthy. In contrast, the decentralized scenario illustrated in Figure 4b shows a significant decline in the retail channel's optimal price (p_r), which, in turn, stimulates demand for the product. As a result, the retailer's order quantity gradually decreases as the price sensitivity parameter (α_1) of the retail channel increases.

In a centralized scenario, an increase in the price sensitivity parameter (α_2) for the online channel leads to a decline in the retail price (p_r), direct channel price (p_d), and product quality (q). As a result, the retailer's order quantity (Q_r) decreases, and the manufacturer's inventory level (Q_d) also reduces, as illustrated in Figure 5a. However, in the decentralized case shown in Figure 5b, the inventory level of the retail channel (Q_d) experiences an initial moderate decline before stabilizing. Despite this trend, both the online channel price (p_d) and product quality (q) exhibit a significant decrease as the price sensitivity parameter (α_2) for the online channel expands.

8.3. Impact of the quality parameter

In a centralized system, an increase in the quality sensitivity parameter (β) leads to higher order quantities and unit prices across both channels. Consequently, this results in a significant rise in profitability for both



(A) centralized scenario

(B) Decentralized scenario

FIGURE 4. Graphical representation of p_r, p_d, Q_r, Q_d, q and λ *vs.* α_1.

channels, as depicted in Figure 6a. This outcome is expected, as improved quality enhances market value. Conversely, in the decentralized model shown in Figure 6b, an increase in the quality sensitivity parameter (β) causes order quantities and unit prices in both channels to decline. This is due to a decrease in product returns, resulting from improved product quality. Despite the reduction in orders, the overall profitability of both channels sees an upward trend.

8.4. Impact of the screening rate for damage

In a centralized framework, an increase in the screening rate for damaged products (γ) leads to an improvement in quality (q), as depicted in Figure 7a. This effect is expected since a higher screening rate helps eliminate defective items, thereby enhancing overall product quality. In a decentralized scenario, the rise in γ similarly contributes to better quality (q), though at a more gradual pace, as shown in Figure 7b. This outcome aligns with practical observations, reflecting the inherent differences in quality control efficiency between centralized and decentralized models.

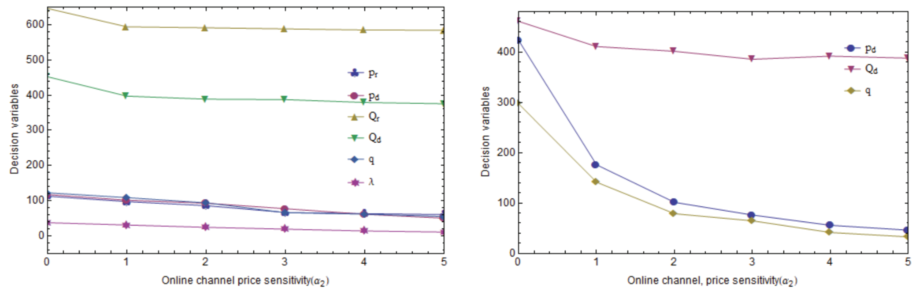
8.5. Impact of the compatibility parameter

As illustrated in Figures 8a and 8b, both centralized and decentralized models exhibit a similar pricing trend – an increase in the optimal retail price (p_r) leads to a decline in the retailer's order quantity (Q_r). Conversely, a decrease in the online channel price (p_d) results in a rise in the manufacturer's inventory level (Q_d). This pattern is a logical outcome of pricing dynamics across different channels. Furthermore, the data highlights that the quality parameter affects the two scenarios in opposite ways. In a centralized framework, it contributes positively to overall performance, whereas in a decentralized setting, it has a detrimental impact when the product compatibility parameter (θ) increases.

8.6. Impact of all parameters on profit

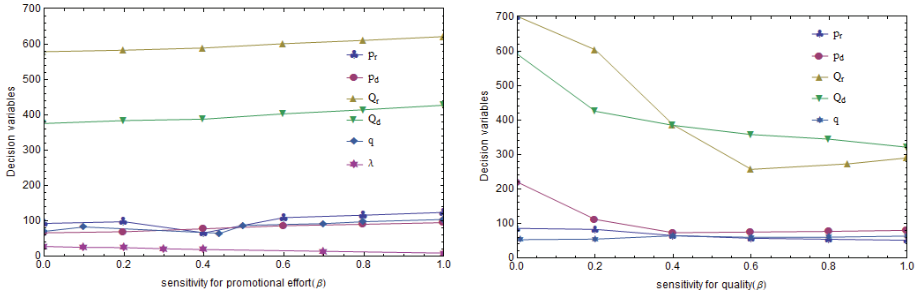
(1) The price sensitivity parameter (α_1) of the retail channel negatively impacts its expected demand while simultaneously increasing demand for the e-tail channel. As a result, online channel profitability steadily rises, whereas retail channel profits decline sharply when (α_1) increases. Consequently, the profit of the centralized model experiences an initial sharp drop, followed by a more gradual decline, as depicted in Figure 9a. Figure 9b further illustrates that as the price sensitivity parameter (α_2) for the online channel increases, the reduction in online channel profits surpasses the gain in retail channel profitability. This leads to a rapid initial decline in the centralized model's profit function, followed by a slower decrease over time.

(2) An increase in the quality sensitivity parameter (β) enhances profitability in both the online and retail channels. Figure 10a shows a notable rise in profits within the centralized model. Additionally, as the



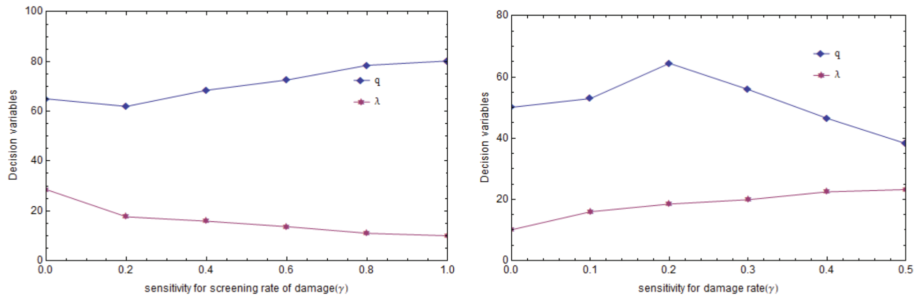
(A) centralized scenario (B) Decentralized scenario

FIGURE 5. Graphical representation of p_r, p_d, Q_r, Q_d and q *vs.* α_2.



(A) centralized scenario (B) Decentralized scenario

FIGURE 6. Graphical representation of p_r, p_d, Q_r, Q_d and q *vs.* β.

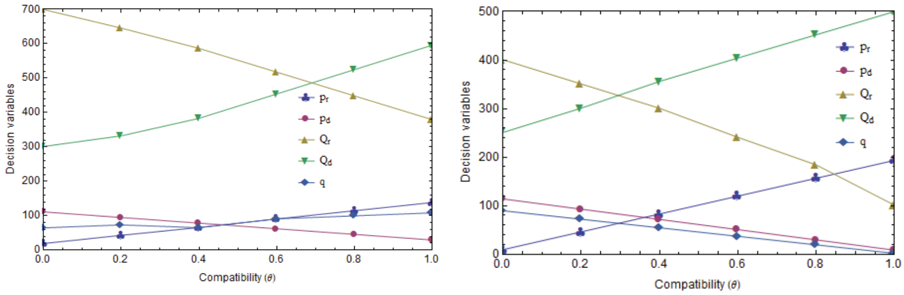


(A) centralized scenario (B) Decentralized scenario

FIGURE 7. Graphical representation of q and λ *vs.* γ.

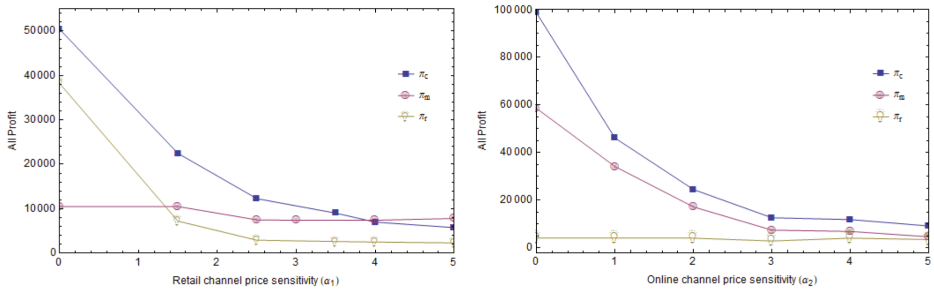
feedback sensitivity parameter (γ) grows, profits for both channels consistently increase. Similarly, a higher damage screening sensitivity parameter (γ) positively affects demand in both channels, as demonstrated in Figure 10b.

(3) Figure 11a indicates that retail channel profits decline, while the online channel sees an increase as the sensitivity parameter (α) for the cross-price effect grows, leading to a slight reduction in overall centralized profits. Furthermore, as the product compatibility parameter (θ) rises, profitability in the retail channel



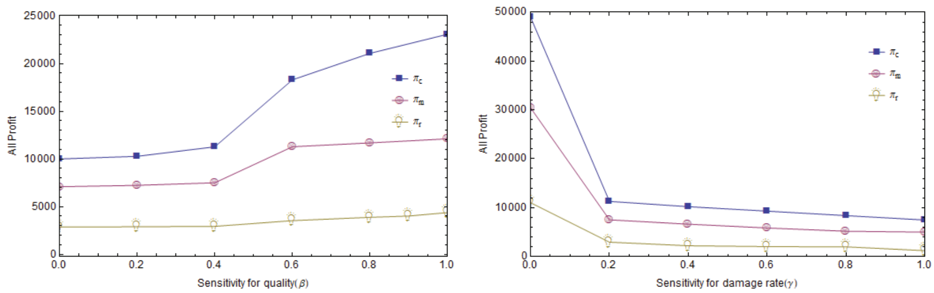
(A) centralized scenario (B) Decentralized scenario

FIGURE 8. Graphical representation of p_r, p_d, Q_r, Q_d and q *vs.* θ.



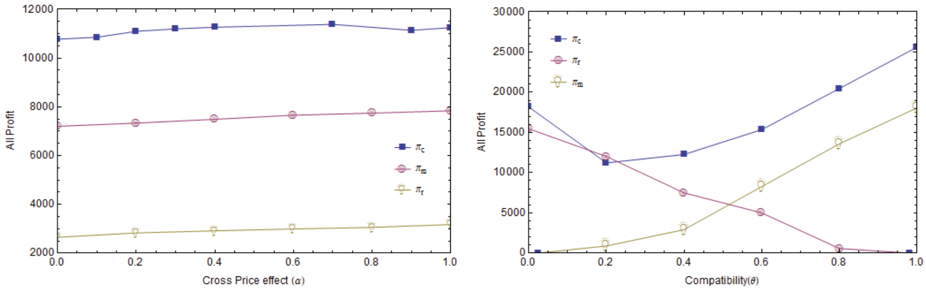
(A) Price sensitivity in retail channel (B) Price sensitivity in online channel

FIGURE 9. Graphical representation of π_r, π_m and π_r *vs.* Profit sensitivity profit.



(A) Sensitivity parameter for promotion (B) Sensitivity parameter for feedback

FIGURE 10. Graphical representation of π_r, π_m and π_r *vs.* β and γ respectively.



(A) Degree of price competetion (B) Compatibility of the product

FIGURE 11. Graphical representation of π_r, π_m and π_r *vs.* α and θ respectively.

improves, while profits in the e-tail channel diminish. This trend is expected, as retail consumers typically demonstrate greater acceptance of new products compared to e-tail shoppers. Figure 11b illustrates that although centralized system profitability initially drops, it later experiences a rapid increase.

9. Significant new findings

(1) This paper signifies and quantifies the effect of consumers' acceptance (return rate, R) on product demands in retail as well as e tail channels.
(2) The model reveals that implementing a return policy for perishable goods significantly benefits supply chain members by increasing profit, especially under conditions of higher demand for both centralized and decentralized scenario.

10. Conclusion

The rise of online retail has amplified product returns, often due to damage, creating new marketing dynamics around viral consumer feedback. These increased returns can inflate manufacturer costs, potentially raising prices and dampening demand. Conversely, prioritizing product quality and implementing effective return policies can bolster a manufacturer's reliability and trustworthiness, yielding positive outcomes. Prior research supports the notion that better product compatibility, often a result of quality improvements, enhances profitability. Building upon this understanding, this research developed a two-echelon supply chain model that incorporates the impact of retail and e-tail pricing, quality investments, and damage-dependent return rates on stochastic demand. The model analyzed both centralized and decentralized decision-making scenarios. Our numerical findings consistently demonstrate the superior profitability of a centralized supply chain structure compared to a decentralized one. Furthermore, the analysis reveals that enhancing product quality leads to reduced damage and consequently lower return rates in both channels, significantly boosting the profit margins for all supply chain members. The study also investigated the interplay between return rates and product compatibility under conditions of uncertain demand. Future research could fruitfully expand upon this work by considering supply chains dealing with multiple products rather than a single item. Another promising direction for future exploration involves analyzing the influence of service facilities and product warranties on consumer purchasing behaviour.

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Appendix A.

Proof of Proposition 1. The corresponding Hessian matrix is now expressed as

$$H_c = \begin{bmatrix} \frac{\partial^2 \pi_c}{\partial {Q_r}^2} & \frac{\partial^2 \pi_c}{\partial Q_r \partial Q_d} & \frac{\partial^2 \pi_c}{\partial Q_r \partial q} & \frac{\partial^2 \pi_c}{\partial Q_r \partial \lambda} & \frac{\partial^2 \pi_c}{\partial Q_r \partial p_r} & \frac{\partial^2 \pi_c}{\partial Q_r \partial p_d} \\ \frac{\partial^2 \pi_c}{\partial Q_d \partial Q_r} & \frac{\partial^2 \pi_c}{\partial {Q_d}^2} & \frac{\partial^2 \pi_c}{\partial Q_d \partial u} & \frac{\partial^2 \pi_c}{\partial Q_d \partial \lambda} & \frac{\partial^2 \pi_c}{\partial Q_d \partial p_r} & \frac{\partial^2 \pi_c}{\partial Q_d \partial p_d} \\ \frac{\partial^2 \pi_c}{\partial q \partial Q_r} & \frac{\partial^2 \pi_c}{\partial q \partial Q_d} & \frac{\partial^2 \pi_c}{\partial q^2} & \frac{\partial^2 \pi_c}{\partial q \partial f} & \frac{\partial^2 \pi_c}{\partial q \partial p_r} & \frac{\partial^2 \pi_c}{\partial q \partial p_d} \\ \frac{\partial^2 \pi_c}{\partial \lambda \partial Q_r} & \frac{\partial^2 \pi_c}{\partial \lambda \partial Q_d} & \frac{\partial^2 \pi_c}{\partial \lambda \partial q} & \frac{\partial^2 \pi_c}{\partial \lambda^2} & \frac{\partial^2 \pi_c}{\partial \lambda \partial p_r} & \frac{\partial^2 \pi_c}{\partial \lambda \partial p_d} \\ \frac{\partial^2 \pi_c}{\partial p_r \partial Q_r} & \frac{\partial^2 \pi_c}{\partial p_r \partial Q_d} & \frac{\partial^2 \pi_c}{\partial p_r \partial q} & \frac{\partial^2 \pi_c}{\partial p_r \partial \lambda} & \frac{\partial^2 \pi_c}{\partial {p_r}^2} & \frac{\partial^2 \pi_c}{\partial p_r \partial p_d} \\ \frac{\partial^2 \pi_c}{\partial p_d \partial Q_r} & \frac{\partial^2 \pi_c}{\partial p_d \partial Q_d} & \frac{\partial^2 \pi_c}{\partial p_d \partial q} & \frac{\partial^2 \pi_c}{\partial p_d \partial \lambda} & \frac{\partial^2 \pi_c}{\partial p_d \partial p_r} & \frac{\partial^2 \pi_c}{\partial {p_d}^2} \end{bmatrix}$$

where, $\frac{\partial^2 \pi_c}{\partial {Q_r}^2} = -t_r f(A)$

$\frac{\partial^2 \pi_c}{\partial {Q_d}^2} = -t_d f(B)$

$\frac{\partial^2 \pi_c}{\partial q^2} = \frac{2\delta k_1}{q^3}(p_r + p_d) - \left(\frac{\beta - \delta k_1}{q^2}\right)^2 t_r f(A) - t_r \frac{2\delta k_1}{q^3}[1 - F(A)]$
$- \left(\frac{\beta - \delta k_1}{q^2}\right)^2 t_d f(B) - t_d \frac{2\delta k_1}{q^3}[1 - F(B)] - \frac{2\delta k_1}{q^3} = -x_1(say),$

$\frac{\partial^2 \pi_c}{\partial \lambda^2} = -(k_2\delta - \gamma)^2[t_r f(A) + t_d f(B)] - 2\gamma = -x_2(say),$

$\frac{\partial^2 \pi_c}{\partial {p_r}^2} = -\left[2(\alpha + \alpha_1) + \alpha^2 t_d f(B) + (\alpha + \alpha_1)^2 t_r f(A)\right] = -x_3(say),$

$\frac{\partial^2 \pi_c}{\partial {p_d}^2} = -\left[2(\alpha + \alpha_2) + \alpha^2 t_r f(A) + (\alpha + \alpha_2)^2 t_d f(B)\right] = -x_4(say),$

$\frac{\partial^2 \pi_c}{\partial Q_r \partial Q_d} = 0 = \frac{\partial^2 \pi_c}{\partial Q_d \partial Q_r}$

$\frac{\partial^2 \pi_c}{\partial Q_r \partial q} = \frac{\partial^2 \pi_c}{\partial q \partial Q_r} = \left(\beta - \frac{\delta k_1}{q^2}\right) t_r f(A)$

$\frac{\partial^2 \pi_c}{\partial Q_r \partial \lambda} = \frac{\partial^2 \pi_c}{\partial \lambda \partial Q_r} = (\delta k_2 - \gamma) t_r f(A)$

$\frac{\partial^2 \pi_c}{\partial Q_r \partial p_r} = \frac{\partial^2 \pi_c}{\partial p_r \partial Q_r} = -(\alpha + \alpha_1) t_r f(A)$

$\frac{\partial^2 \pi_c}{\partial Q_r \partial p_d} = \frac{\partial^2 \pi_c}{\partial p_d \partial Q_r} = \alpha t_r f(A)$

$\frac{\partial^2 \pi_c}{\partial Q_d \partial q} = \frac{\partial^2 \pi_c}{\partial q \partial Q_d} = \left(\beta - \frac{\delta k_1}{q^2}\right) t_d f(B)$

$\frac{\partial^2 \pi_c}{\partial Q_d \partial \lambda} = \frac{\partial^2 \pi_c}{\partial \lambda \partial Q_d} = (\delta k_2 - \gamma) t_d f(B)$

$\frac{\partial^2 \pi_c}{\partial Q_d \partial p_r} = \frac{\partial^2 \pi_c}{\partial p_r \partial Q_d} = \alpha t_d f(B)$

$\frac{\partial^2 \pi_c}{\partial Q_d \partial p_d} = \frac{\partial^2 \pi_c}{\partial p_d \partial Q_d} = -(\alpha + \alpha_2) t_d f(B)$

$\frac{\partial^2 \pi_c}{\partial q \partial \lambda} = \frac{\partial^2 \pi_c}{\partial \lambda \partial q} = \left(\beta - \frac{\delta k_1}{q^2}\right)(\gamma - \delta k_2)[t_d f(B) + t_r f(A)] = x_5(say),$

$\frac{\partial^2 \pi_c}{\partial q \partial p_r} = \frac{\partial^2 \pi_c}{\partial p_r \partial q} = \left(\beta - \frac{\delta k_1}{q^2}\right)[1 + (\alpha + \alpha_2) t_d f(B) - \alpha t_r f(A)] = x_6(say),$

$\frac{\partial^2 \pi_c}{\partial q \partial p_d} = \frac{\partial^2 \pi_c}{\partial p_d \partial q} = \left(\beta - \frac{\delta k_1}{q^2}\right)[1 - \alpha t_r f(A) + (\alpha + \alpha_2) t_d f(B)] = x_7(say),$

$\frac{\partial^2 \pi_c}{\partial \lambda \partial p_r} = \frac{\partial^2 \pi_c}{\partial p_r \partial \lambda} = (\delta k_2 - \gamma)[1 - \alpha t_d f(B) + (\alpha + \alpha_1) t_r f(A)] = x_8(say),$

$\frac{\partial^2 \pi_c}{\partial \lambda \partial p_d} = \frac{\partial^2 \pi_c}{\partial p_d \partial \lambda} = (\delta k_2 - \gamma)[1 - \alpha t_r f(A) + (\alpha + \alpha_2) t_d f(B)] = x_9(say),$

$\frac{\partial^2 \pi_c}{\partial p_r \partial p_d} = \frac{\partial^2 \pi_c}{\partial p_d \partial p_r} = [2\alpha + (\alpha + \alpha_2)\alpha t_d f(B) + (\alpha + \alpha_1)\alpha t_r f(A)] = x_{10}(say),$

$$= \begin{bmatrix} -t_rf(A) & 0 & \left(\beta-\frac{\delta k_1}{q^2}\right)t_rf(A) & (\delta k_2-\gamma)t_rf(A) & -(\alpha+\alpha_1)t_rf(A) & \alpha t_rf(A) \\ 0 & -t_df(B) & \left(\beta-\frac{\delta k_1}{q^2}\right)t_df(B) & (\delta k_2-\gamma)t_df(B) & \alpha t_df(B) & -(\alpha+\alpha_2)t_df(B) \\ \left(\beta-\frac{\delta k_1}{q^2}\right)t_rf(A) & \left(\beta-\frac{\delta k_1}{q^2}\right)t_df(B) & -x_1 & x_5 & x_6 & x_7 \\ (\delta k_2-\gamma)t_rf(A) & (\delta k_2-\gamma)t_df(B) & x_5 & -x_2 & x_8 & x_9 \\ -(\alpha+\alpha_1)t_rf(A) & \alpha t_df(B) & x_6 & x_8 & -x_3 & x_{10} \\ \alpha t_rf(A) & -(\alpha+\alpha_2)t_df(B) & x_7 & x_9 & x_{10} & -x_4 \end{bmatrix}.$$

The principal minors of the Hessian matrix are,

$$\Delta_{c1} = -t_rf(A) < 0$$

$$\Delta_{c2} = \begin{vmatrix} -t_rf(A) & 0 \\ 0 & -t_df(B) \end{vmatrix} = t_rt_df(A)f(B) > 0$$

$$\Delta_{c3} = \begin{vmatrix} -t_rf(A) & 0 & \left(\beta-\frac{\delta k_1}{q^2}\right)t_rf(A) \\ 0 & -t_df(B) & \alpha t_df(B) \\ \left(\beta-\frac{\delta k_1}{q^2}\right)t_rf(A) & \left(\beta-\frac{\delta k_1}{q^2}\right)t_df(B) & -x_1 \end{vmatrix} < 0 \text{ if}$$

$$-2\left(\beta-\frac{\delta k_1}{q^2}\right)t_rt_df(A)f(B) < 0$$

and

$$\Delta_{c4} = \begin{vmatrix} -t_rf(A) & 0 & \left(\beta-\frac{\delta k_1}{q^2}\right)t_rf(A) & (\delta k_2-\gamma)t_rf(A) \\ 0 & -t_df(B) & \left(\beta-\frac{\delta k_1}{q^2}\right)t_df(B) & (\delta k_2-\gamma)t_df(B) \\ \left(\beta-\frac{\delta k_1}{q^2}\right)t_rf(A) & \left(\beta-\frac{\delta k_1}{q^2}\right)t_df(B) & -x_1 & x_5 \\ (\delta k_2-\gamma)t_rf(A) & (\delta k_2-\gamma)t_df(B) & x_5 & -x_2 \end{vmatrix} > 0 \text{ if}$$

$$24\left(\beta-\frac{\delta k_1}{q^2}\right)\gamma t_rt_df^2f(A)f(B) > 0 \text{ and}$$

$$\Delta_{c5} = \begin{vmatrix} -t_rf(A) & 0 & \left(\beta-\frac{\delta k_1}{q^2}\right)t_rf(A) & (\delta k_2-\gamma)t_rf(A) & -(\alpha+\alpha_1)t_rf(A) \\ 0 & -t_df(B) & \left(\beta-\frac{\delta k_1}{q^2}\right)t_df(B) & (\delta k_2-\gamma)t_df(B) & \alpha t_df(B) \\ \left(\beta-\frac{\delta k_1}{q^2}\right)t_rf(A) & \left(\beta-\frac{\delta k_1}{q^2}\right)t_df(B) & -x_1 & x_5 & x_6 \\ (\delta k_2-\gamma)t_rf(A) & (\delta k_2-\gamma)t_df(B) & x_5 & -x_2 & x_8 \\ -(\alpha+\alpha_1)t_rf(A) & \alpha t_df(B) & x_6 & x_8 & -x_3 \end{vmatrix} < 0 \text{ if}$$

$$-2\left(\beta-\frac{\delta k_1}{q^2}\right)(\delta k_2-(\delta k_2-\gamma))t_rt_df(A)f(B)$$

$$\left[24(\alpha+\alpha_1)\lambda^2-\left((\delta k_2-\gamma)+6\left(\beta-\frac{\delta k_1}{q^2}\right)\lambda^2\right)\right] < 0 \text{ and}$$

$$\Delta_{c6} = \begin{vmatrix} -t_rf(A) & 0 & \left(\beta-\frac{\delta k_1}{q^2}\right)t_rf(A) & (\delta k_2-\gamma)t_rf(A) & -(\alpha+\alpha_1)t_rf(A) & \alpha t_rf(A) \\ 0 & -t_df(B) & \left(\beta-\frac{\delta k_1}{q^2}\right)t_df(B) & (\delta k_2-\gamma)t_df(B) & \alpha t_df(B) & -(\alpha+\alpha_2)t_df(B) \\ \left(\beta-\frac{\delta k_1}{q^2}\right)t_rf(A) & \left(\beta-\frac{\delta k_1}{q^2}\right)t_df(B) & -x_1 & x_5 & x_6 & x_7 \\ (\delta k_2-\gamma)t_rf(A) & (\delta k_2-\gamma)t_df(B) & x_5 & -x_2 & x_8 & x_9 \\ -(\alpha+\alpha_1)t_rf(A) & \alpha t_df(B) & x_6 & x_8 & -x_3 & x_{10} \\ \alpha t_rf(A) & -(\alpha+\alpha_2)t_df(B) & x_7 & x_9 & x_{10} & -x_4 \end{vmatrix} > 0 \text{ if}$$

$$\left(\beta-\frac{\delta k_1}{q^2}\right)(\delta k_2-\gamma)t_r t_d f(A)f(B)\left[96\lambda^2(\alpha\alpha_1+\alpha_1\alpha_2+\alpha\alpha_2)-4\left(6\lambda^2\left(\beta-\frac{\delta k_1}{q^2}\right)+(\delta k_2-\gamma)\right)\right.$$
$$\left.\{(\alpha\alpha_1+\alpha_1\alpha_2+\alpha\alpha_2)t_d f(B)+(\alpha_1+\alpha_2+4\alpha)\}\right]>0.$$

This completes the proof. □

Appendix B.

Proof of Proposition 2. The corresponding Hessian matrix for the manufacturer is presented as follows

$$H_{dm}=\begin{bmatrix}\frac{\partial^2\pi_m}{\partial Q_d^2} & \frac{\partial^2\pi_m}{\partial Q_d\partial p_d} & \frac{\partial^2\pi_m}{\partial Q_d\partial q}\\ \frac{\partial^2\pi_m}{\partial p_d\partial Q_d} & \frac{\partial^2\pi_m}{\partial p_d^2} & \frac{\partial^2\pi_m}{\partial p_d\partial q}\\ \frac{\partial^2\pi_m}{\partial q\partial Q_d} & \frac{\partial^2\pi_m}{\partial q\partial p_d} & \frac{\partial^2\pi_m}{\partial q^2}\end{bmatrix}$$

where,

$\frac{\partial^2\pi_m}{\partial Q_d^2}=-t_d f(B)$

$\frac{\partial^2\pi_m}{\partial p_d^2}=-2(\alpha+\alpha_2)-r\alpha^2 f(A)-t_d(\alpha+\alpha_2)^2 f(B)$

$\frac{\partial^2\pi_m}{\partial q^2}=\frac{2k_1\delta p_d}{q^3}+\frac{2k_1\delta r}{q^3}F(A)-r\left(\beta-\frac{k_1\delta}{q^2}\right)^2 f(A)-2\frac{k_1\delta t_d}{q^3}\{1-F(B)\}$

$\frac{\partial^2\pi_m}{\partial Q_d\partial p_d}=\frac{\partial^2\pi_m}{\partial p_d\partial Q_d}=-(\alpha+\alpha_2)t_d f(B)$

$\frac{\partial^2\pi_m}{\partial Q_d\partial q}=\frac{\partial^2\pi_m}{\partial q\partial Q_d}=t_d f(B)\left(\beta-\frac{k_1\delta}{q^2}\right)$

$\frac{\partial^2\pi_m}{\partial p_d\partial q}=\frac{\partial^2\pi_m}{\partial q\partial p_d}=\left(\beta-\frac{k_1\delta}{q^2}\right)-r\alpha\left(\beta-\frac{k_1\delta}{q^2}\right)f(A)+t_d(\alpha+\alpha_2)\left(\beta-\frac{k_1\delta}{q^2}\right)f(B)$

$$=\begin{bmatrix}-t_d f(B) & -(\alpha+\alpha_2)t_d f(B) & t_d f(B)\left(\beta-\frac{k_1\delta}{q^2}\right)\\ -(\alpha+\alpha_2)t_d f(B) & \begin{array}{c}-2(\alpha+\alpha_2)-r\alpha^2 f(A)\\ -t_d(\alpha+\alpha_2)^2 f(B)\end{array} & \begin{array}{c}\left(\beta-\frac{k_1\delta}{q^2}\right)-r\alpha\left(\beta-\frac{k_1\delta}{q^2}\right)f(A)\\ +t_d(\alpha+\alpha_2)\left(\beta-\frac{k_1\delta}{q^2}\right)f(B)\end{array}\\ t_d f(B)\left(\beta-\frac{k_1\delta}{q^2}\right) & \begin{array}{c}\left(\beta-\frac{k_1\delta}{q^2}\right)-r\alpha\left(\beta-\frac{k_1\delta}{q^2}\right)f(A)\\ +t_d(\alpha+\alpha_2)\left(\beta-\frac{k_1\delta}{q^2}\right)f(B)\end{array} & \begin{array}{c}\frac{2k_1\delta p_d}{q^3}+\frac{2k_1\delta r}{q^3}F(A)-r\left(\beta-\frac{k_1\delta}{q^2}\right)^2 f(A)\\ -2\frac{k_1\delta t_d}{q^3}\{1-F(B)\}\end{array}\end{bmatrix}.$$

The principal minors of the Hessian matrix H_{dm} are,

$$\Delta_{dm1}=-t_d f(B)<0$$

$$\Delta_{dm2}=\begin{vmatrix}-t_d f(B) & -(\alpha+\alpha_2)t_d f(B)\\ -(\alpha+\alpha_2)t_d f(B) & -2(\alpha+\alpha_2)-r\alpha^2 f(A)-t_d(\alpha+\alpha_2)^2 f(B)\end{vmatrix}>0,\quad\text{if}$$

$$t_d f(B)[2(\alpha+\alpha_2)+r\alpha^2 f(A)]>0$$

$$\Delta_{dm3}=\begin{vmatrix}-t_d f(B) & -(\alpha+\alpha_2)t_d f(B) & t_d f(B)\left(\beta-\frac{k_1\delta}{q^2}\right)\\ -(\alpha+\alpha_2)t_d f(B) & -2(\alpha+\alpha_2)-r\alpha^2 f(A)-t_d(\alpha+\alpha_2)^2 f(B) & x_{11}\\ t_d f(B)\left(\beta-\frac{k_1\delta}{q^2}\right) & \left(\beta-\frac{k_1\delta}{q^2}\right)-r\alpha\left(\beta-\frac{k_1\delta}{q^2}\right)f(A)+t_d(\alpha+\alpha_2)\left(\beta-\frac{k_1\delta}{q^2}\right)f(B) & x_{12}\end{vmatrix}<0,$$

$$\text{if}\quad\frac{f(A)}{q^3}[2k_1\delta r\alpha^2 t_d+2k_1\delta r^2\alpha^2 f(A)+2k_1\delta(n-1)r\alpha^2-2k_1\delta r\alpha^2 t_d\{1-f(A)\}]$$
$$<\left(\beta-\frac{k_1\delta}{q^2}\right)^2 x_{13}.$$

if $\beta\gamma t_d f(B)[4k\gamma r f(A)(2\alpha+\alpha_2)+12f^2\{2\beta r(2\alpha+\alpha_2)f(A)+4(\alpha+\alpha_2)k$
$+2kr\alpha^2 f(A)\}-2k\gamma]>0$

$x_{11}=(\beta-\frac{k_1\delta}{q^2})-r\alpha(\beta-\frac{k_1\delta}{q^2})f(A)+t_d(\alpha+\alpha_2)(\beta-\frac{k_1\delta}{q^2})f(B)$

$x_{12}=\frac{2k_1\delta p_d}{q^3}+\frac{2k_1\delta r}{q^3}F(A)-r(\beta-\frac{k_1\delta}{q^2})^2 f(A)-2\frac{k_1\delta t_d}{q^3}\{1-F(B)\}$

$x_{13}=[2(\alpha+\alpha_2)rf(A)+2\alpha rf(A)-1]+\frac{\alpha+\alpha_2}{q^3}[4(1-n)k_1\delta-4k_1\delta p_d$

$-4k_1\delta rf(A)+4k_1\delta t_d\{1-f(A)\}]$.

This completes the proof. □

Appendix C.

Proof of Proposition 3. The corresponding Hessian matrix for the retailer is presented as follows

$$H_{dr}=\begin{bmatrix}\frac{\partial^2\pi_r}{\partial {Q_r}^2} & \frac{\partial^2\pi_r}{\partial Q_r\partial p_r} & \frac{\partial^2\pi_r}{\partial Q_r\partial\lambda}\\ \frac{\partial^2\pi_r}{\partial p_r\partial Q_r} & \frac{\partial^2\pi_r}{\partial {p_r}^2} & \frac{\partial^2\pi_r}{\partial p_r\partial\lambda}\\ \frac{\partial^2\pi_r}{\partial\lambda\partial Q_r} & \frac{\partial^2\pi_r}{\partial\lambda\partial Q_r} & \frac{\partial^2\pi_r}{\partial\lambda^2}\end{bmatrix}$$

where,

$\frac{\partial^2\pi_r}{\partial {Q_r}^2}=-(t_r-r)f(A)$

$\frac{\partial^2\pi_r}{\partial {p_r}^2}=-2(\alpha+\alpha_1)-(\alpha+\alpha_1)^2(t_r-r)f(A)$

$\frac{\partial^2\pi_r}{\partial\lambda^2}=-(t_r-r)(\gamma-k_2\delta)f(A)-2(1-m)\gamma$

$\frac{\partial^2\pi_r}{\partial p_r\partial Q_r}=\frac{\partial^2\pi_r}{\partial Q_r\partial p_r}=(\alpha+\alpha_1)(r-t_r)f(A)$

$\frac{\partial^2\pi_r}{\partial\lambda\partial Q_r}=\frac{\partial^2\pi_r}{\partial Q_r\partial\lambda}=(r-t_r)(\gamma-k_2\delta)f(A)$

$\frac{\partial^2\pi_r}{\partial p_r\partial\lambda}=\frac{\partial^2\pi_r}{\partial\lambda\partial p_r}=-(\gamma-k_2\delta)+(\gamma-k_2\delta)(\alpha+\alpha_1)(r-t_r)f(A)$

$$=\begin{bmatrix}-(t_r-r)f(A) & (\alpha+\alpha_1)(r-t_r)f(A) & (r-t_r)(\gamma-k_2\delta)f(A)\\ (\alpha+\alpha_1)(r-t_r)f(A) & -2(\alpha+\alpha_1)-(\alpha+\alpha_1)^2(t_r-r)f(A) & -(\gamma-k_2\delta)+(\gamma-k_2\delta)(\alpha+\alpha_1)(r-t_r)f(A)\\ (r-t_r)(\gamma-k_2\delta)f(A) & -(\gamma-k_2\delta)+(\gamma-k_2\delta)(\alpha+\alpha_1)(r-t_r)f(A) & -(t_r-r)(\gamma-k_2\delta)f(A)-2(1-m)\gamma\end{bmatrix}.$$

The principal minors of the Hessian matrix H_{dr} are,

$$\Delta_{dr1}=-(t_r-r)f(A)<0$$

$$\Delta_{dr2}=\begin{vmatrix}-(t_r-r)f(A) & (\alpha+\alpha_1)(r-t_r)f(A)\\ (\alpha+\alpha_1)(r-t_r)f(A) & -2(\alpha+\alpha_1)-(\alpha+\alpha_1)^2(t_r-r)f(A)\end{vmatrix}>0 \text{ if}$$

$$2(\alpha+\alpha_1)(t_r-r)f(A)(t_r-r)>0$$

$$\Delta_{dr3}=\begin{vmatrix}-(t_r-r)f(A) & (\alpha+\alpha_1)(r-t_r)f(A) & (r-t_r)(\gamma-k_2\delta)f(A)\\ (\alpha+\alpha_1)(r-t_r)f(A) & \begin{matrix}-2(\alpha+\alpha_1)\\-(\alpha+\alpha_1)^2(t_r-r)f(A)\end{matrix} & \begin{matrix}-(\gamma-k_2\delta)+(\gamma-k_2\delta)\\(\alpha+\alpha_1)(r-t_r)f(A)\end{matrix}\\ (r-t_r)(\gamma-k_2\delta)f(A) & \begin{matrix}-(\gamma-k_2\delta)+(\gamma-k_2\delta)\\(\alpha+\alpha_1)(r-t_r)f(A)\end{matrix} & -(t_r-r)(\gamma-k_2\delta)f(A)-2(1-m)\gamma\end{vmatrix}<0 \text{ if}$$

$$\frac{f(A)}{q^3}[2k_1\delta r\alpha^2 t_d+2k_1\delta r^2\alpha^2 f(A)+2k_1\delta(n-1)r\alpha^2-2k_1\delta r\alpha^2 t_d\{1-f(A)\}]$$
$$<\left(\beta-\frac{k_1\delta}{q^2}\right)^2 \qquad (35)$$

$$[2(\alpha+\alpha_2)rf(A)+2\alpha rf(A)-1]+\frac{\alpha+\alpha_2}{q^3}[4(1-n)k_1\delta-4k_1\delta p_d-4k_1\delta rf(A)$$
$$+4k_1\delta t_d\{1-f(A)\}].$$

This completes the proof. $\square$