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CARBON ALLOWANCE ALLOCATION RULES AND SALES CHANNEL STRATEGIES IN AN E-COMMERCE PLATFORM SUPPLY CHAIN

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Abstract. The carbon cap-and-trade mechanism is increasingly influencing the online selling channel selection and pricing strategy in an e-commerce platform supply chain with two different sales channels (agency selling and reselling channels). This paper examines how two carbon allowance allocation rules (grandfathering and benchmarking) influence channel selection, pricing and profitability in a platform supply chain consisting of one supplier and one e-commerce platform. Using game-theoretic models under four scenarios arising from two carbon allowance allocation rules and two sales channels, we find that the benchmarking rule consistently yields lower retail prices, higher demand, and greater consumer surplus compared to grandfathering. However, the preferred carbon allowance allocation rule differs across stakeholders. In agency channel, the platform's profit is lower (higher) under benchmarking with the higher (lower) supplier's carbon allowance compared to grandfathering. In reselling channel, the platform consistently benefits more from benchmarking. Under agency channel with a high (low) platform fee, the government prefers the grandfathering (benchmarking) when the grandfathering allowance is high (low). Under reselling channel, the government prefers the benchmarking (grandfathering) when the benchmarking allowance is high (low).

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1. Introduction

With the rapid development of the economy, the global ecological environment is increasingly receiving attention from the people. A growing number of nations have proposed a series of policies and measures to control carbon emissions, the most popular of which are carbon tax [17,37] and carbon cap-and-trade regulation [14]. The latter has been widely adopted in practice and proved to be effective in encouraging enterprises to reduce carbon emissions [30]. For example, The European Union's carbon trading system was established in January 2005 and is the first carbon market in the world. In China, carbon trading market has also become one of the largest carbon markets in the world since its official launch in July 2021. At the initial stage of China's carbon market, it mainly covered the power industry, which emitted about 40% of the country's carbon dioxide. In the future, the carbon trade market is expected to gradually expand its coverage to industries such as steel, cement, chemicals, electrolytic aluminum, papermaking, and civil aviation. Under the carbon cap and trade regulation, all companies' carbon emissions are restricted, and the government allocates a carbon emission

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allowance to the company. If one company's carbon emissions exceed the allocated carbon allowance, it has to purchase extra carbon quotas from the carbon trading market, and this will result in additional expenditure. If one company reduces emissions below the cap, it can sell the unused allowances on the trading market, which will incur its revenue. Usually, there exist two typical carbon allowance allocation rules under cap-and-trade mechanisms, which are the Grandfathering mechanism (GM) and Benchmarking mechanism (BM). Under GM, a company's total carbon emission quota is established for a specific time period based on the company's prior production levels [31]. Under BM, a unit carbon quota for a company base on the unit carbon emissions is established [46]. Companies can trade carbon emissions in the carbon trading market under either BM or GM, which has already been adopted in practice [18].

With the development of e-commerce, many traditional suppliers tend to sell their product on the e-commerce platform. For example, Chinese Home appliance giants such as Haier, Midea, and Gree have official flagship stores on JD.com, selling a variety of products from refrigerators and washing machines to air conditioners, small appliances, and more. Random House sells its products through the e-commerce platform Amazon [38]. When the supplier sells through the e-commerce platform, the supplier may choose to either agency selling or reselling [43]. For instance, PepsiCo Foods, established a flagship store on JD.com to sell snacks directly to consumers by agency selling channel [4]. Chanel and Apple sell their products on JD.com by reselling channel [27]. In agent selling, a supplier determines the retail price and sells a product through the e-commerce platform by paying a fixed fraction of the sales revenue (*i.e.*, commission) to the e-commerce platform. In reselling channel, the e-commerce platform purchases products from the supplier and then sells the products to the consumers, the supplier can only affect the retail price of the product indirectly by the wholesale price.

The carbon emission regulation may have an important impact on the supplier and the e-commerce platform. For example, if the regulation tends to be stricter, the carbon allowance will decrease, which leads to a higher carbon emission cost for the supplier. The increased cost would change the sales and profits of the supplier and the e-commerce platform. Therefore, the choice of the carbon allowance rules will be worth investigating in an e-commerce platform supply chain. Furthermore, the effect of the different carbon allowance allocation regulations on the pricing strategies is also worth exploring. Hence, we also investigate the supplier's selling channels choice and the members' pricing strategies in a platform supply chain with the two different carbon cap regulations (Grandfathering and Benchmarking rules).

Based on the above discussion, we try to address the following problems. Firstly, which selling channel is beneficial to the supply chain members under the Grandfathering rule or Benchmarking rule? Secondly, under the agent selling channel or reselling channel, which carbon allowance rule is beneficial to the supply chain members and consumer? Thirdly, which carbon allowance regulation will incur more social welfare under the agent selling channel or reselling channel? And in which condition, the Grandfathering rule is better than the Benchmarking rule?

To solve these research questions, we establish a game theoretical model to investigate the carbon allowance rule selection, the sales channel selection and pricing strategy in an e-commerce platform supply chain consisting of one supplier and one e-commerce platform. We consider the combination of channel selection strategy, pricing strategy, carbon cap regulations in investigating the following four scenarios: (1) selling on the e-commerce platform by agent channel under grandfathering rule, (2) selling on the e-commerce platform by reselling channel under grandfathering rule, (3) selling on the e-commerce platform by agent selling under benchmarking rule, (4) selling on the e-commerce platform by reselling channel under benchmarking rule.

Our contributions mainly lie in three aspects. Firstly, we examine the impacts of carbon allowance rules on the pricing strategies and selling channel selection of the platform supply chain members, whereas the previous studies didn't incorporate carbon regulation in considered the pricing and selling channel choice in the platform supply chain. Secondly, we introduce two different carbon allowance regulations and compare the pricing strategies and selling channel selections of the platform supply chain members, we additionally examine how pricing decisions vary under different carbon allowance allocation mechanisms, whereas the previous studies didn't consider them in comparing the pricing and selling channel selections, and only investigate how pricing strategies vary in different selling channels. Thirdly, we compare the difference of the two different carbon allowance regulations under the agent or reselling channel in a platform supply chain, investigate the impacts of the two carbon allowance rules on the consumer surplus and social welfare, and present the conditions under which the grandfathering rule is superior to the benchmarking rule, which are seldom studied in previous research.

The rest of the paper is organized as follows. In Section 2, we review the related literature. In Section 3, we present problem description and model framework. In Section 4, we investigate the channel selection and pricing strategy with the supplier's unit carbon quota based on benchmarking rule. In Section 5, we investigate the channel selection and pricing strategy with the supplier's total carbon quota based on grandfathering rule. In Section 6, we perform the comparisons between two carbon allowance allocation rules from the perspectives of supply chain members, consumer and government, respectively. In Section 7, we conclude the paper and propose some future research directions. The proofs of all propositions are presented in the Appendix A.

2. Literature review

Our research is related to three streams of research in the literature: channel selection, pricing strategy in a platform supply chain and pricing decision in a supply chain with cap-and-trade regulation.

2.1. Channel selection

In the research related to supply chain management, the suppliers' channel selection strategy is one of the most popular topics. For the platform supply chain, previous literature mainly investigates whether the supplier directly sells his product through an agency selling channel or sells his product based on a reselling channel. Lu and Liu [26] investigated how the e-commerce channel affects the profitability and behavior of suppliers and physical retailers in a distribution system. Abhishek *et al.* [1] explored the specific circumstances for the e-commerce platform to adopt the agency or the reselling channel from the aspects of competition intensity between e-commerce platforms as well as cross channel spillover. Dong *et al.* [12] investigated the dynamic selling strategy for a firm under asymmetric market and product information over two periods. Wang *et al.* [38] investigated a supplier's e-channel decision issue in which the manufacturer selects a direct-sales channel or a third-party consignment channel, and found that the suppliers' incentive to sell through online channels is depend on the proportion of unit operational cost and income distribution of each channel. Hsiao and Chen [20] compared the suppliers' channel selection between consumers who prefer offline store shopping and those who prefer online shopping. Li *et al.* [22] empirically explored the effects of agency selling and reselling on hybrid retail platforms, noting that agency selling can positively influence reselling. Shen *et al.* [32] studied a manufacturer's channel choice between a platform retailer and a traditional reseller, and they found that whether the slotting fee is beneficial or harmful to the manufacturer or platform retailer depends on the demand substitution effect of the two retail channels. Zennyo [50] examined strategic contracting between a monopoly platform and two competing suppliers who sell their goods through the platform, in which each supplier can choose wholesale or agency contracts. Ha *et al.* [15] developed a game-theoretic model to investigate the equilibrium channel choice, wholesale price, and sale quantity with considering of three channel structures (agency channel, reselling channel and dual channel). Zhang and Hou [51] examined the manufacturers' selections for the agency selling and reselling channels, and demonstrated that manufacturers should choose agency selling (or reselling) when the percentage fee is low (or high). Chen *et al.* [6] investigated the e-tailer's logistics service strategy following the introduction of a marketplace channel, and revealed that logistics service costs were a critical determinant of the optimal logistics strategy. Chen *et al.* [7] investigated a manufacturer's online selling channel selection between wholesale selling and agency selling considering the spillovers from online to offline channel, and found that the manufacturer's optimal online selling channels vary with consumers' online channel acceptance and the spillover effect. Lin *et al.* [23] investigated the impact of the green manufacturer's distribution channel selection on the e-retailer's manufacturer selection and the green dilemma. Zhou *et al.* [53] conducted a systematic literature review of channel strategies from the perspectives of the digital platforms and the suppliers when digital platforms emerge from 2010 to 2023. Tan *et al.* [34] performed a systematic review of the agency model, and specifically identified three critical issues regarding the agency model: the impact of the agency model on channel distribution, how to effectively manage the operations and information management strategy. Yang *et al.* [49] considered the integrated dual-channel supply chains with online and offline channels, and investigated the impact of consumer channel preferences on pricing strategies and overall supply chain profitability. Chen *et al.* [8] considered an e-tailer's one direct sales channel and multiple asymmetric agency platforms, investigated the pricing strategy and channel selection strategy by using the multinomial logit (MNL) choice model. Chen *et al.* [9] conducted a comprehensive overview of logistics engineering management in platform supply chain from the perspective of logistics service strategy selection.

In summary, the aforementioned studies primarily explored the influence resulted from channel and consumer preferences, and analyzed impacts of sale channel selection on supply chain performance. The differences between our paper and existing studies lie in the following aspects: Firstly, the existing papers do not consider the two different carbon allowance rules. Similar to the above literature, we also analyze the supplier's selling channel choice, however, we address the issue under the two different carbon allowance rules and perform the comparisons from the perspectives of the supply chain members, consume and government, which is rarely covered in previous literature. Secondly, the comparison of different cap regulations in agent selling channel or reselling channel is performed, and the impacts of these carbon allowance rules on the channel selections, supply chain members' profits and governmental social welfare are investigated. Existing literature rarely addresses research in this area. Thirdly, most of existing papers compare the reselling channel and agent selling channel from aspects of retail prices, demand and members' profits. In this paper, we not only compare the retail prices, demand and members' profits between two different sale channels, but also compare the consumer surplus between two different sale channels.

2.2. Pricing strategy in a platform supply chain

Current literature on pricing strategies in a platform supply chain is mainly focused on the reselling pricing and agent selling pricing [50]. Abhishek *et al.* [1] studied the influence of two pricing strategy on dual-channel supply chain, and presented the conditions under which e-retailers will choose agency pricing or wholesale pricing. Chen *et al.* [5] investigated the impact of the wholesale pricing or agency pricing on the online retailer's profit, consumer surplus, and social welfare with considering of the customer loyalty. Hao and Tan [16] investigated a retailer's and a supplier's incentive of information disclosure under the agency pricing and the wholesale pricing strategies. Liu and Ke [24] discussed the impacts of power structure and pricing timing on firms' pricing strategies under reselling and agent selling channels, and found that the manufacturer prefers to agent selling channel when utility discount factor is sufficiently high, and to two online channels when utility discount factor is moderate. Xie *et al.* [41] explored the pricing strategies for the platform supply chain consisting of third-party seller, e-commerce platform and strategic consumers considering e-commerce consumer credit, and examined who ultimately benefits from the credit. Zhen and Xu [52] investigated whether the manufacturer or the retailer should introduce a platform's agent selling channel under unified and differential pricing strategies, and analyzed the impact of the pricing strategy on the selection of the channel structure and the joint decision of the channel selection and pricing strategy. Tian and Hu [35] investigated the impact of blockchain adoption on supply chain pricing and companies' investment decisions with information asymmetry. Xi and Zhang [40] investigated the agent selling channel addition and pricing strategy in an e-commerce supply chain consisting of one e-commerce platform, one manufacturer, and one reseller. Wan and Fan [36] considered an e-commerce supply chain consisting of a manufacturer providing products and an online platform providing service, and explored the pricing decisions under two different platform selling modes (reselling and agent) and three different channel power structures (manufacturer Stackelberg, vertical Nash and platform Stackelberg). He *et al.* [19] examined the critical determinants that influence traditional offline manufacturers' adoption of blockchain-enabled price differentiation platforms, and analyzed various pricing mechanisms: manufacturer-led uniform pricing, centralized pricing, resale-mode single-stage differentiation, resale-mode two-stage differentiation, and marketplace-mode differentiation.

The aforementioned research examined pricing strategies under reselling and agent selling modes in a platform supply chain. Our paper differs from prior studies in two key aspects: Firstly, we analyze pricing strategies across four distinct scenarios that arise from the combination of two carbon allowance allocation rules and two sales channels (reselling and agency), whereas the previous studies considered only the choice between reselling and agency channels without incorporating carbon regulation. Secondly, while earlier works compared pricing strategies solely across the two sales channels, our paper additionally examines how pricing decisions vary under different carbon allowance allocation mechanisms, thereby offering a more comprehensive analysis that integrates both channel structure and environmental policy considerations.

2.3. Pricing decision in a supply chain with cap-and-trade regulation

With the growing awareness of the low-carbon supply chain, many scholars focus on the impacts of cap-and-trade regulation on the pricing decisions. Xu *et al.* [42] studied the production and pricing problems in make-to-order supply chain consisting of a manufacturer and a retailer, in which the manufacturer is regulated by cap-and-trade regulation. Yang *et al.* [47] investigated the manufacturer's wholesale price and the retailer's retail price decisions in a remanufacturing supply chain under cap-and-trade regulation. Yang *et al.* [48] explored the optimal green technology investment and product pricing, and analyzed the impact of the cap-and-trade regulation on the pricing decisions and total emission. Cai and Jiang [2] explored the optimal pricing and carbon emission reduction decisions in a supply chain consisting of one supplier and one manufacturer by considering the impacts of cap-and-trade regulation. Ji *et al.* [21] formulated a price and low-carbon competition model for two vertically differentiated manufacturers under the cap-and-trade regulation, and analyzed the impacts of the cap-and-trade regulation on the pricing and carbon reduction of manufacturers. Shi *et al.* [33] investigated pricing and replenishment strategy for a perishable product in a sustainable three-echelon supply chain with considering of various payment schemes and cap-and-trade regulations. Liu *et al.* [25] developed a robust fuzzy bi-level optimization model to investigate the pricing decision in a low-carbon dual-channel supply chain, where the upper-level manufacturer is regulated by the cap-and-trade (CAT) mechanisms. Yan and Ni [45] studied the pricing and low-carbon decision problems in a supply chain consisting of a manufacturer and a downstream retailer, in which the manufacturer produces a single product under the cap-and-trade scheme. Pal [29] investigated the pricing strategies in a sustainable closed-loop supply chain considering carbon taxes and cap-and-trade schemes.

Existing literature on pricing decisions under cap-and-trade regulation primarily focuses on a single type of carbon allowance allocation rule, with the majority of studies adopting the grandfathering (GF) approach. Although Liu *et al.* [25] examined both the grandfathering and benchmarking (BM) mechanisms in the context of manufacturers' compliance with cap-and-trade regulations, their work did not extend to platform-based supply chains. In this paper, we address this gap by investigating pricing decisions in an e-commerce platform supply chain under two distinct carbon allowance allocation rules (grandfathering and benchmarking) within a cap-and-trade framework, a setting that has received little attention in prior research.

Table 1 summarizes the most relevant papers from this literature review. In this paper, we focus on the governmental carbon allowance selection, a supplier's sale channel selection and the members' pricing decisions in a platform supply chain under two different cap and trade regulations (grandfathering and benchmarking rules), which is seldom studied in previous research. Our main contributions lie in the following aspects. Firstly, most of current studies explored the pricing strategy and channel selection in a platform supply chain, however, they didn't consider the cap-and-trade regulations and investigate the impacts of the cap-and-trade regulations on the pricing decision and channel selection. Although Liu and Ke [24] explored the impacts of the cap-and-trade regulation on the sale channel selection by considering a supply chain composed of an e-tailer and a manufacturer, they only consider one kinds of cap-and-trade regulation whereas our paper consider two different cap and trade regulations. Secondly, we compare the difference of the pricing strategy, demand, members' profits, consumer surplus between agent selling and reselling channels under grandfathering or benchmarking rules, and the difference of the pricing strategy, demand, members' profits, consumer surplus and social welfare between grandfathering and benchmarking rules under agent selling or reselling channels, which is seldom studied in

Table 1. Difference between this paper and the existing literature.

Relevant literature	Sale channel selection			Pricing decision in a supply chain		
	Carbon allowance	Reselling and agent selling	Consumer surplus and social welfare	Grandfathering rule	Benchmarking rule	Platform supply chain
Lu and Liu [26], Abhishek *et al.* [1], Wang *et al.* [38], Chen *et al.* [6], Chen *et al.* [7], Yang *et al.* [49], Chen *et al.* [8], Chen *et al.* [9], Tan *et al.* [34], Zhou *et al.* [53]		✓				✓
Xu *et al.* [42], Yang *et al.* [47], Cai and Jiang [2], Ji *et al.* [21], Shi *et al.* [33]				✓		✓
Liu *et al.* [25]				✓	✓	
Yan and Ni [45], Pal [29]				✓		
Abhishek *et al.* [1], Chen *et al.* [5], Hao and Tan [16], Liu and Ke [24], Xie *et al.* [41], Zhen and Xu [52], Tian and Hu [35], Xi and Zhang [40], Wan and Fan [36], He *et al.* [19]		✓				✓
This paper	✓	✓	✓	✓	✓	✓

previous research. Thirdly, we compare the difference of the two different cap and trade regulation under the agent or reselling channel in a platform supply chain, and present the conditions under which the grandfathering or benchmarking rule is better.

3. Problem description and modeling

We consider a platform supply chain which is composed of a supplier and an e-commerce platform, where the supplier sells his product by two channels on the e-commerce platform: agent selling and reselling channel.

We assume the carbon emissions are incurred from the supplier who is responsible for manufacturing and distribution, and the carbon emissions generated by e-commerce platforms are negligible. This assumption is consistent with Xu *et al.* [44] and Cao *et al.* [3]. In the life cycle of most goods or services, manufacturing, raw material extraction and physical logistics are the primary sources of carbon emissions. By contrast, the core functions of sales platforms (such as information matching, order processing, and payment interfaces) are light-asset, low-energy activities. Although platforms do generate indirect emissions (*e.g.*, from data center electricity consumption, employee commuting, and office energy use), their carbon emissions per transaction are substantially lower than those associated with physical supply chain activities. Based on the above assumption, only the supplier is constrained by cap-and-trade regulations.

TABLE 2. Notations.

Notation	Definition
Parameters	
d^R	Demand from reselling channel
d^A	Demand from agent selling channel
c	Unit production cost of supplier
e	The carbon trading price
β	Carbon emission per unit product
φ	Environment damage per unit carbon emission
μ	The commission rate of e-commerce platform
m	Supplier's unit carbon quota
Q	Supplier's total carbon quota
CS	Consumer surplus
SW	Social welfare
ED	Environment damage
Decision variables	
w	Supplier's wholesale price in reselling channel
p_s	Supplier's retail price in agent selling channel
p_r	E-commerce platform's retail price in reselling channel

Under the cap-and-trade regulations, the free carbon emission quota can be allocated by two rules: Grandfathering and Benchmarking. Under the rule of grandfathering, the supplier is initially allocated a fixed free emission quota. Whereas, using the Benchmarking method, the carbon emission quota is granted based on the unit production emission.

We use superscripts V and T to denote the scenarios of supplier's unit carbon quota under Benchmarking rule and fixed carbon quota under Grandfathering rule, respectively. We use the superscripts R and A to represent the scenarios of selling on the e-commerce platform by reselling channel, and selling on the e-commerce platform by agency selling, respectively.

The notations used in the model are summarized in Table 2.

In order to ensure the equilibrium solutions and the members' profits are non-negative, we assume $a(1-\mu) > b(c+\beta e)$.

4. Channel selection and pricing strategy with the benchmarking rule

In this section, we assume the cap-and-trade regulation is based on benchmarking rule, in which the government establishes a unit carbon quota for a supplier to limit unit carbon emissions. The supplier can trade carbon emissions in the carbon trading market, the government allocates the cap for the unit carbon emissions.

4.1. Agent selling with the supplier's unit carbon quota (VA)

Here, we consider the scenario that the supplier sells products on the e-commerce platform by agency selling. Given the fee rate of e-commerce platform μ, the supplier decides the retail price directly. The profit functions of the supplier and e-commerce platform are as follows:

$$\begin{aligned}
&\text{Max } \pi_s^{\text{VA}} = (1-\mu)p_a\left(a-bp_a\right) + e(m-\beta)(a-bp_a) - c(a-bp_a)\\
&\text{Max } \pi_p^{\text{VA}} = \mu p_a\left(a-bp_a\right).
\end{aligned}$$

The equilibrium results are presented in Proposition 1.

Proposition 1. *When the e-commerce chooses agent selling mode with the carbon cap for per unit product, the supplier's optimal decision and corresponding demand are as*

$$p^{\mathrm{VA}} = \frac{a(1-\mu)+b(c-me+\beta e)}{2b(1-\mu)}, d^{\mathrm{VA}} = \frac{a(1-\mu)-b(c-me+\beta e)}{2(1-\mu)}.$$

Based on the assumption, the demand and the members' profits non-negative are obviously ensured. From Proposition 1, we can obtain the supplier's profit is

$$\pi_s^{\mathrm{VA}} = \frac{[a(1-\mu)-b(c-me+\beta e)]^2}{4b(1-\mu)}. \tag{1}$$

The platform's profit is

$$\pi_p^{\mathrm{VA}} = \frac{\mu[a(1-\mu)-b(c-me+\beta e)][a(1-\mu)+b(c-me+\beta e)]}{4b(1-\mu)^2}. \tag{2}$$

According to the concept of economic surplus presented by Currie *et al.* [11], consumer surplus refers to the difference between the maximum price consumers are willing to pay and the actual price they pay for a certain number of goods. Following Mankiw [28] and Wang *et al.* [39], consumer surplus is expressed as the upper triangle area of the inverse demand curve. Therefore, we can get the consumer surplus as:

$$\mathrm{CS}^{\mathrm{VA}} = \frac{[a(1-\mu)-b(c-me+\beta e)]^2}{8b(1-\mu)^2}. \tag{3}$$

4.2. Reselling with supplier's unit carbon quota (VR)

In this case, the supplier sells its product on the e-commerce platform by wholesale price, and then the e-commerce sells the products to the market. The supplier firstly determines the wholesale price, and the e-commerce platform then decides the retail price. The decision model can be represented as follows:

$$\begin{aligned}
&\text{Max } \pi_s^{\mathrm{VR}} = (w-c)(a-bp_r) + e(m-\beta)(a-bp_r)\\
&\text{Max } \pi_p^{\mathrm{VR}} = (p_r - w)(a-bp_r).
\end{aligned}$$

By using the backward induction method, the equilibrium results are presented in Proposition 2.

Proposition 2. *When the supplier chooses reselling channel with the carbon cap for per unit product, the two members' optimal decisions and corresponding demand are as follows:*

$$w^{\mathrm{VR}} = \frac{a+b(c-me+\beta e)}{2b}, p^{\mathrm{VR}} = \frac{3a+b(c-me+\beta e)}{4b}, d^{\mathrm{VR}} = \frac{a-b(c-me+\beta e)}{4}.$$

Based on the assumption, the demand and the members' profits non-negative are obviously ensured. According to the above optimal solutions, the supplier's and the e-commerce's optimal profits can be obtained as:

$$\pi_s^{\mathrm{VR}} = \frac{[a-b(c-me+\beta e)]^2}{8b}, \tag{4}$$

$$\pi_p^{\mathrm{VR}} = \frac{[a-b(c-me+\beta e)]^2}{16b}. \tag{5}$$

Similarly, the consumer surplus can be obtained as

$$\mathrm{CS}^{\mathrm{VR}} = \frac{[a-b(c-me+\beta e)]^2}{32b}. \tag{6}$$

4.3. Comparison of different sale channels with benchmarking rule

Based on the optimal decisions obtained in the previous subsections, we compare the retail price and the members' profits between reselling and agent selling situations. Furthermore, we reveal the supplier's choice of retailing channel and the e-commerce platform's corresponding response

Proposition 3. *By comparing the reselling and agent selling, we can get the following results:*

(1) *If* $\mu_1 < \mu < 1$ *and* $m > m_1$*, then* $p^{\mathrm{VR}} > p^{\mathrm{VA}}, d^{\mathrm{VR}} < d^{\mathrm{VA}}$*; If* $\mu_1 < \mu < 1$ *and* $0 < m < m_1$*, then* $p^{\mathrm{VR}} < p^{\mathrm{VA}}, d^{\mathrm{VR}} > d^{\mathrm{VA}}$*. If* $0 < \mu < \mu_1$*, then* $p^{\mathrm{VR}} > p^{\mathrm{VA}}$*,* $d^{\mathrm{VR}} < d^{\mathrm{VA}}$*. Here,* $\mu_1 = \frac{a-b(c+\beta e)}{a+b(c+\beta e)}$*,* $m_1 = \beta + \frac{c}{e} - \frac{a(1-\mu)}{be(1+\mu)}$*.*

(2) *If* $0 < \mu < \mu_2$*, or* $\mu_3 < \mu < 1$*, then* $\pi_s^{\mathrm{VR}} < \pi_s^{\mathrm{VA}}$*;*
If $\mu_2 < \mu < \mu_3$*, when* $0 < m < m_2$ *then* $\pi_s^{\mathrm{VR}} > \pi_s^{\mathrm{VA}}$*; when* $m > m_2$*, then* $\pi_s^{\mathrm{VR}} < \pi_s^{\mathrm{VA}}$*. Here,*

$$\mu_2 = \frac{3a^2 - 2ab\,(c+\beta e) - b^2(c+\beta e)^2 - [a - b\,(c+\beta e)]\sqrt{a^2 + 6ab\,(c+\beta e) + b^2(c+\beta e)^2}}{4a^2},$$

$$\mu_3 = \frac{3a^2 - 2ab\,(c+\beta e) - b^2(c+\beta e)^2 + [a - b\,(c+\beta e)]\sqrt{a^2 + 6ab\,(c+\beta e) + b^2(c+\beta e)^2}}{4a^2}$$

$$m_2 = \beta + \frac{c}{e} - \frac{a(1-\mu)}{be(1+\mu)} + \frac{a\mu\sqrt{2(1-\mu)}}{be(1+\mu)}.$$

(3) *If* $0 < \mu < \min\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\}$*, then* $\pi_p^{\mathrm{VR}} > \pi_p^{\mathrm{VA}}$*.*
If $\max\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\} < \mu < 1$ *and* $m > m_3$*, then* $\pi_p^{\mathrm{VR}} > \pi_p^{\mathrm{VA}}$*. Here,* $m_3 = \beta + \frac{c}{e} - \frac{a(1-\mu)^2}{be(1+\mu)^2} + \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}$*.*
If $\max\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\} < \mu < 1$*,* $0 < m < m_4$*, and* $c > -\beta e - \frac{a(1-\mu)^2}{b(1+\mu)^2} + \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}$*, then* $\pi_p^{\mathrm{VR}} > \pi_p^{\mathrm{VA}}$*.*
Here, $m_4 = \beta + \frac{c}{e} - \frac{a(1-\mu)^2}{be(1+\mu)^2} - \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}$*.*
If $\max\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\} < \mu < 1$*,* $m_4 < m < m_3$*, then* $\pi_p^{\mathrm{VR}} < \pi_p^{\mathrm{VA}}$*.*
If $\max\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\} < \mu < 1$*,* $0 < m < m_4$ *and* $c < -\beta e - \frac{a(1-\mu)^2}{b(1+\mu)^2} + \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}$*, then* $\pi_p^{\mathrm{VR}} < \pi_p^{\mathrm{VA}}$*.*

As stated in Proposition 3, the e-commerce platform's fee rate and the supplier's unit carbon quota are key factors influencing supply chain pricing strategies under different selling modes.

Specifically, Proposition 3(1) indicates that when the e-commerce platform charges a low fee rate, the retail price under the reselling model consistently exceeds that under the agency selling model, irrespective of whether the supplier's unit carbon quota is high or low. This pricing disparity leads to lower market demand under reselling compared to agency selling. This occurs because, at a low fee rate, the supplier shares more profit with the platform under the agency mode. As a result, the platform can afford to set a lower retail price in the agency mode to stimulate the consumer demand. This not only boosts sales volume but also enhances the overall profitability of the supply chain through higher turnover.

In contrast, when the e-commerce platform imposes a higher fee rate, the supplier's profit margin becomes substantially compressed, and the interplay between the fee structure and environmental constraints particularly the unit carbon quota begins to dominate pricing decisions.

When the supplier's unit carbon quota is low (*i.e.*, the supplier faces tighter emissions constraints), the retail price under the reselling mode tends to be lower than that under the agency selling mode. Conversely, when the unit carbon quota is high (indicating greater emissions flexibility), the retail price under reselling is higher than that under agency selling. This suggests that under a low unit carbon quota scenario, the supplier seeks to raise retail price of agent channel in order to increase marginal profit and simultaneously curb demand. By doing so, the supplier reduces total production volume, thereby lowering carbon emissions and avoiding excessive carbon compliance costs. On the other hand, when the unit carbon quota is high, the supplier leverages its abundant carbon allowance to pursue a volume-driven strategy in the agency mode. By setting a lower retail price, the supplier stimulates demand, increases sales, and generates additional revenue not only from product sales but also from potential carbon credit trading.

Proposition 3(2) reveals that the supplier can achieve higher profits through the agency selling channel compared to the reselling channel, regardless of whether the e-commerce platform's fee rate is low or high. This advantage stems from the supplier's direct control over the retail price in the agency mode, which avoids the double marginalization inherent in the reselling arrangement. Consequently, the agency channel is generally more favorable to the supplier.

However, when the e-commerce platform's fee rate falls within a moderate range, the relative profitability of the agency selling channel becomes highly contingent on environmental regulatory factors, particularly the unit carbon quota allocated to the supplier. Under such conditions, the unit carbon quota acts as a pivotal determinant of the supplier's net earnings. If the unit quota is set at a low level, the supplier faces stringent emissions constraints and must either reduce production or purchase additional allowances in the carbon market at prevailing prices. Both options entail significant costs. These added expenditures disproportionately affect the agency mode, where the supplier bears full responsibility for production and compliance decisions, thereby eroding its profit margin even though it retains pricing autonomy. Conversely, when the unit carbon quota is high, the supplier not only avoids costly carbon purchases but may also generate surplus allowances through energy-efficient operations or lower-than-allocated emissions. These excess quotas can be sold in the carbon trading market, creating a new stream of ancillary revenue. In the agency mode, this supplementary income compounds with the sales revenue, resulting in a substantially enhanced overall profitability.

Proposition 3(3) indicates the platform's channel preference under different combinations of the e-commerce fee rate and the supplier's unit carbon emission quota. Specifically, the platform prefers the reselling channel in the following scenarios: (1) when the e-commerce fee rate is low; (2) when the fee rate is high and the unit carbon emission quota is also high; or (3) when the fee rate is high, the unit carbon emission quota is low, and the total unit cost (including production and carbon emission costs) is high. In contrast, the agency selling channel is more beneficial to the platform when: (1) the fee rate is high and the unit carbon emission quota is moderate, or (2) the fee rate is high, the unit carbon emission quota is low, and the unit cost is low.

The underlying rationale is as follows. Under a low fee rate, the platform earns limited profit from the agency channel, making reselling more attractive. When the fee rate and the unit carbon emission quota are both high, the supplier has an incentive to stimulate demand by lowering the retail price. In this case, the platform benefits more from the reselling channel by capturing a larger share of the sales revenue. On the other hand, if the fee rate is high, the unit carbon emission quota is low, and the total unit cost is high, any price increase needed to cover costs leads to a significant drop in demand, and the platform still secures higher profits under the reselling mode due to its control over pricing and margins. In all other situations, the agency selling channel becomes the platform's preferred choice.

Proposition 4. *By comparing the consumer surplus of reselling and agent selling, we can get the following results:*

If $\mu_1 < \mu < 1$ *and* $m > m_1$, *then* $\mathrm{CS}^{\mathrm{VR}} < \mathrm{CS}^{\mathrm{VA}}$*; If* $\mu_1 < \mu < 1$ *and* $0 < m < m_1$, *then* $\mathrm{CS}^{\mathrm{VR}} > \mathrm{CS}^{\mathrm{VA}}$. *If* $0 < \mu < \mu_1$, *then* $\mathrm{CS}^{\mathrm{VR}} < \mathrm{CS}^{\mathrm{VA}}$.

From Proposition 4, it is shown that when the e-commerce's fee rate is low or when the e-commerce's fee rate is high and the unit carbon quota is high, the consumer surplus generated from agent selling channel is more than that from reselling channel, the consumer prefers the agent selling channel. However, when the e-commerce's fee rate is high and the unit carbon quota is low, the consumer surplus under the agent selling channel is less than that under the reselling channel, in this case, the consumer prefers the reselling channel. This can be explained from Proposition 3(1), which demonstrates that, when the e-commerce's fee rate is low or when it is high alongside a high unit carbon quota, the retail price in the agent selling channel is lower than that in the reselling channel, and the corresponding demand from agent selling channel is higher than that from reselling channel. Conversely, when the e-commerce fee rate is high and the unit carbon quota is low, the retail price in the agent selling channel is higher than that in the reselling channel, resulting in lower demand.

5. Channel selection and pricing strategy with the grandfathering rule

In this section, we consider the governmental carbon allowance allocation based on grandfathering rule, in which the supplier obtains a free total carbon allowance Q based on its historical carbon emissions quantity. The supplier can trade carbon emissions in the carbon trading market, the government allocates the cap according to the supplier's total carbon emissions.

5.1. Agent selling with supplier's total carbon quota (TA)

Here, we consider the scenario that the supplier sells products on the e-commerce platform by agency selling. Given the fee rate of e-commerce platform μ, the supplier decides the retail price directly. The profit functions of the supplier and e-commerce platform are as follows:

$$\begin{aligned}&\text{Max } \pi_s^{\mathrm{VA}} = (1-\mu)(p-c)\,(a-bp) + e[Q-\beta(a-bp)]\\&\text{Max } \pi_p^{\mathrm{VA}} = \mu(p-c)\,(a-bp)\,.\end{aligned}$$

The equilibrium results are presented in Proposition 5.

Proposition 5. *When the supplier chooses agent selling channel with governmental carbon cap for total carbon emission, the supplier's optimal decision and corresponding demand are as*

$$p^{\mathrm{TA}} = \frac{a(1-\mu)+b(c+\beta e)}{2b(1-\mu)},\quad d^{\mathrm{TA}} = \frac{a(1-\mu)-b(c+\beta e)}{2(1-\mu)}.$$

To ensure the equilibrium solutions and the members' profits are non-negative, we assume $a(1-\mu) > b(c+\beta e)$, *i.e.*, $0 < \mu < \frac{a-b(c+\beta e)}{a}$. We defined $\mu_0 = \frac{a-b(c+\beta e)}{a}$.

From Proposition 5, we can get the profits of the supplier as

$$\pi_s^{\mathrm{TA}} = Qe + \frac{[a(1-\mu)-b(c+\beta e)]^2}{4b(1-\mu)}. \tag{7}$$

The platform's profit is

$$\pi_p^{\mathrm{TA}} = \frac{[a(1-\mu)-b(c+\beta e)][a(1-\mu)+b(c+\beta e)]\mu}{4b(1-\mu)^2}. \tag{8}$$

The consumer surplus is

$$\mathrm{CS}^{\mathrm{TA}} = \frac{[a(1-\mu)-b(c+\beta e)]^2}{8b(1-\mu)^2}. \tag{9}$$

5.2. Reselling channel with supplier's total carbon quota (TR)

In this case, the decision model can be represented as follows:

$$\begin{aligned}&\text{Max } \pi_s^{\mathrm{TR}} = (w-c)\,(a-bp) + e[Q-\beta(a-bp)]\\&\text{Max } \pi_p^{\mathrm{TR}} = (p-w)\,(a-bp)\,.\end{aligned}$$

By using the backward induction method, the equilibrium results are presented in Proposition 6.

Proposition 6. *When the supplier chooses reselling channel with governmental carbon cap for total carbon emission, the two members' optimal decisions and corresponding demand are as follows:*

$$w^{\mathrm{TR}} = \frac{a + b(c + \beta e)}{2b}, p^{\mathrm{TR}} = \frac{3a + b(c + \beta e)}{4b}, d^{\mathrm{TR}} = \frac{a - b(c + \beta e)}{4}\cdot$$

Based on the assumptions in Proposition 5, the demand and the members' profits are obviously non-negative. Therefore, the supplier's profit can be obtained as

$$\pi_s^{\mathrm{TR}} = Qe + \frac{[a - b(c + \beta e)]^2}{8b}, \tag{10}$$

the platform's profit can be obtained as

$$\pi_p^{\mathrm{TR}} = \frac{[a - b(c + \beta e)]^2}{16b}\cdot \tag{11}$$

The consumer surplus can be obtained as

$$\mathrm{CS}^{\mathrm{TR}} = \frac{[a - b(c + \beta e)]^2}{32b}\cdot \tag{12}$$

5.3. Comparison of two different sale channels with total carbon quota

Based on the optimal decisions obtained in the previous subsections, we compare the retail price and the members' profits in reselling and agent selling channels with total carbon quota. Furthermore, we show the supplier's choice of retailing channel and the e-commerce platform's corresponding response.

Proposition 7. *By comparing the pricing strategies, demand and profits from reselling and agent selling channel with governmental carbon cap of total carbon emission, we can get the following results:*

(1) *If* $0 < \mu < \mu_1$, $p^{\mathrm{TR}} > p^{\mathrm{TA}}$, $d^{\mathrm{TR}} < d^{\mathrm{TA}}$, *and If* $\mu_1 < \mu < \mu_0$, $p^{\mathrm{TR}} < p^{\mathrm{TA}}$, $d^{\mathrm{TR}} > d^{\mathrm{TA}}$.

(2) *If* $0 < \mu < \mu_4$, $\pi_s^{\mathrm{TR}} < \pi_s^{\mathrm{TA}}$, *if* $\mu_4 < \mu < \mu_0$, $\pi_s^{\mathrm{TR}} > \pi_s^{\mathrm{TA}}$, *in which* $\mu_4 = \frac{[a-b(c+\beta e)][3a+b(c+\beta e)-\sqrt{B}]}{4a^2}$, $B = a^2 + 6ab\,(c + \beta e) + b^2(c + \beta e)^2$.

(3) *If* $\frac{1}{4} < \mu < 1$ *and* $0 < c + \beta e < c_1$, *then* $\pi_p^{\mathrm{TR}} < \pi_p^{\mathrm{TA}}$; *if* $0 < \mu < \frac{1}{4}$ *and* $c_1 < c + \beta e < c_2$, *then* $\pi_p^{\mathrm{TR}} < \pi_p^{\mathrm{TA}}$; *if* $0 < \mu < \frac{1}{4}$ *and* $0 < c + \beta e < c_1$, *then* $\pi_p^{\mathrm{TR}} > \pi_p^{\mathrm{TA}}$; *if* $c + \beta e > c_2$, *then* $\pi_p^{\mathrm{TR}} > \pi_p^{\mathrm{TA}}$. *Here,* $c_1 = \frac{a(1-\mu)^2 - 2a\mu(1-\mu)\sqrt{\mu+2}}{b(1+\mu)^2}$, $c_2 = \frac{a(1-\mu)^2 + 2a\mu(1-\mu)\sqrt{\mu+2}}{b(1+\mu)^2}$.

From Proposition 7, it is known that the carbon cap of total carbon emission doesn't affect the sales channel selection of the supplier. The fee rate of e-commerce platform plays an important role in the sales channel selection. Specifically, Proposition 7(1) shows that under the agency selling channel, a high platform fee rate leads the supplier to set a higher retail price, which in turn reduces market demand. Conversely, when the fee rate is low, the supplier lowers the retail price, thereby stimulating demand. This behavior stems from the supplier's need to maintain its marginal profit. In response to a higher platform fee, the supplier raises the retail price to offset the increased cost burden.

Proposition 7(2) further indicates that the supplier prefers the reselling channel when the platform's fee rate is high, but favors the agency channel when the fee rate is low. Although the agency mode mitigates the double marginalization problem inherent in reselling, a high fee rate forces the supplier to share a larger portion of revenue with the platform, thereby diminishing its net profit, thus making reselling relatively more attractive despite the inefficiency of double marginalization.

Proposition 7(3) indicates that the e-commerce platform prefers the agent selling channel when the fee rate of e-commerce platform is high and the unit total cost (including production and carbon emission cost) is low, or when the fee rate of e-commerce platform is low and the unit total cost is moderate. When the fee rate of e-commerce platform is low and the unit total cost is low, or when the unit cost is high, the e-commerce platform prefers the reselling channel.

The reason is that, when the fee rate is high, the platform captures a larger share of revenue under the agency mode, especially if the supplier's unit cost is low which allowing for competitive pricing and robust demand. When the fee rate is low and unit cost is moderate, the agency channel helps alleviate double marginalization, enhancing overall channel efficiency and platform profitability. However, if both the fee rate and unit cost are low, the supplier can afford to set a low retail price under reselling channel, boosting demand enable the platform to earn higher absolute profits. Similarly, when the unit total cost is high, the supplier has to set a high retail price to cover the unit production and carbon emission cost, the increase in the retail price result in the decrease in demand, but under reselling channel, the platform retains full control over pricing and margin, resulting in greater profit than under the agency mode where its earnings are limited to a fixed percentage of a shrinking revenue base. The insights from Proposition 7(3) are nearly consistent with the Proposition 3 in Dong *et al.* [13].

Proposition 8. *By comparing the consumer surplus of reselling and agent selling with carbon cap of total carbon emission, we can get the following results: If* $0 < \mu < \mu_1$, $\mathrm{CS}^{\mathrm{TR}} < \mathrm{CS}^{\mathrm{TA}}$*; If* $\mu_1 < \mu < \mu_0$, $\mathrm{CS}^{\mathrm{TR}} > \mathrm{CS}^{\mathrm{TA}}$.

Proposition 8 indicates that consumers prefer the agent selling channel when the e-commerce platform's fee rate is low, whereas they tend to favor the reselling channel when the revenue sharing rate is relatively high. This can be explained by Proposition 7(1), which shows that, under a low fee rate, the agent selling channel enables suppliers to retain greater control over pricing, resulting in lower retail prices and consequently higher consumer demand, which clearly enhances consumer surplus. In contrast, when the e-commerce platform's fee rate is relatively high, high fee rate erodes supplier margins in the agency selling channel, and prompt supplier to raise retail prices to preserve profitability, thereby dampening demand. By contrast, under the reselling channel, the platform assumes inventory and pricing responsibilities, and with a higher revenue-sharing rate, it has stronger incentives to optimize the retail price for volume maximization. As Proposition 7(1) demonstrates, this often leads to more competitive pricing and higher sales volumes in the reselling model, making it the more attractive option from the consumer's standpoint.

6. Comparison of different cap regulations

In this section, we perform the comparison of the retail price, demand and the members' profits under different cap regulations in agent selling channel or reselling channel. We initially compare the outcomes of Scenarios VA and TA in which the supplier sells the products by agency channel. Then, we compare the outcomes of Scenarios VR and TR in which the supplier sells the products by reselling channel.

6.1. Comparison of different cap regulations in agent selling channel

In this subsection, by comparing the pricing strategies, demand and the members' profits under two different cap regulations in agent selling channel, we can drive the following results.

Proposition 9. *In agent selling channel, the cap regulation with unit carbon quota will:*

(1) *Decrease the retail price (i.e.,* $p^{\mathrm{VA}} < p^{\mathrm{TA}}$*) and increase the demand (i.e.,* $d^{\mathrm{VA}} > d^{\mathrm{TA}}$*);*
(2) *Decrease the profit of the supplier when the unit carbon quota is low. (i.e.,* $\pi_s^{\mathrm{TA}} > \pi_s^{\mathrm{VA}}$ *if* $0 < m < m_5$*), and increase the profit of the supplier when the unit carbon quota is high. (i.e.,* $\pi_s^{\mathrm{TA}} < \pi_s^{\mathrm{VA}}$ *if* $m > m_5$*).*
In which, $m_5 = \beta + \frac{c}{e} - \frac{a(1-\mu)}{be} + \frac{\sqrt{A}}{be}$,

$$A = \sqrt{\left[a\left(1-\mu\right) - b\left(c+\beta e\right)\right]^2 + 4bQe(1-\mu)}.$$

(3) *Increase the profit of the e-commerce platform when the unit carbon quota is low. (i.e.,* $\pi_p^{\mathrm{TA}} < \pi_p^{\mathrm{VA}}$ *if* $0 < m < 2\beta + \frac{2c}{e}$*), and decrease the profit of the e-commerce platform when the unit carbon quota is high. (i.e.,* $\pi_p^{\mathrm{TA}} > \pi_p^{\mathrm{VA}}$ *if* $m > 2\beta + \frac{2c}{e}$*).*

It can be obviously shown from Proposition 9 that, when a unit carbon quota (per-unit allowance) is implemented, the free allocation of carbon allowances is strictly proportional to the supplier's output value. In other words, for the benchmarking rule, the allocated allowance is not a fixed value, but is in proportion to the supplier's output value or sales quantity. In this case, the supplier is willing to increase the demand in order to get more free allocated allowance.

In the case of higher unit carbon quota, the supplier can get additional free allocated allowance through output expansion. By lowering sale prices to stimulate demand, they simultaneously increase their carbon trading revenue from selling excess allowances and total sales volume. Although the lower price reduces per-unit sales revenue, the net revenue effect (carbon trading gains + sale volume expansion) dominates, leading to higher total supplier profit. Meanwhile, the sales revenue is possibly reduced due to the lower sales price, and the sales revenue shared by the e-commerce platform is accordingly decreased.

In the case of lower unit carbon quota, the supplier cannot leverage output expansion to gain significant free allowances as the quota ceiling restricts allowance allocation. Facing higher marginal carbon costs from mandatory allowance purchases), the supplier prioritizes maximizing sales revenue to offset emissions expenses. Therefore, the sales revenue shared by the e-commerce platform is accordingly increased.

6.2. Comparison of different cap regulations in reselling channel

In this subsection, by comparing the pricing strategies, demand and the members' profits under two different cap regulations in reselling channel, we can drive the following results.

Proposition 10. *In reselling channel, the cap regulation with unit carbon quota will:*

(1) *Decrease the retail price (i.e.,* $p^{\mathrm{VR}} < p^{\mathrm{TR}}$*) and increase the demand (i.e.,* $d^{\mathrm{VR}} > d^{\mathrm{TR}}$*);*

(2) *Decrease the profit of the supplier when the supplier's unit carbon quota is low. (i.e.,* $\pi_s^{\mathrm{TR}} > \pi_s^{\mathrm{VR}}$ *if* $0 < m < m_6$*), and increase the profit of the supplier when the unit carbon quota is high. (i.e.,* $\pi_s^{\mathrm{TR}} < \pi_s^{\mathrm{VR}}$ *if* $m > m_6$*).*

In which, $m_6 = \beta + \frac{c}{e} - \frac{a}{be} + \frac{\sqrt{[a-b(c+\beta e)]^2 + 8bQe}}{be}$.

(3) *Increase the profit of the e-commerce platform. (i.e.,* $\pi_p^{\mathrm{TR}} < \pi_p^{\mathrm{VR}}$*).*

The insights from Proposition 10(1) and (2) are nearly the same as the Proposition 9(1) and (2). However, different from Proposition 9(3), Proposition 10(3) demonstrate that, in the reselling channel, the e-commerce platform's profit when the supplier is allocated by the unit carbon quota is always higher than that when the supplier is allocated by the fixed total carbon quota. This outcome stems from a strategic alignment of incentives between the supplier and platform. When the cap regulation with the unit carbon quota is used for the supplier, the supplier would like to increase the demand in order to get more free allocated allowance since the allowance is proportional to production volume. To stimulate higher demand, suppliers aggressively reduce wholesale prices, which simultaneously increases platform sales volume and revenue.

Proposition 10 indicates that, in the reselling channel, the e-commerce platform prefers for the unit carbon allowance based on the benchmarking rule. The supplier's preference for the carbon allowance rules is affected by the unit carbon quota. When the unit carbon quota is low, the supplier prefers the total carbon allowance rule based on the grandfathering, and when the unit carbon quota is high, the supplier prefers the unit carbon allowance rule based on the benchmarking.

6.3. Comparison of consumer surplus under different cap regulations

By comparing the consumer surplus under total carbon quota and unit carbon quota, the following results can be obtained.

Proposition 11. *Whether the agent selling channel or the reselling channel is used, the consumer surplus resulted from the supplier's unit carbon quota is greater than that from the supplier's total carbon quota (i.e.,* $\mathrm{CS}^{\mathrm{TA}} < \mathrm{CS}^{\mathrm{VA}}$*,* $\mathrm{CS}^{\mathrm{TR}} < \mathrm{CS}^{\mathrm{VR}}$*).*

Proposition 11 can be interpreted by Propositions 9(1) and 10(1), which demonstrates that, both in reselling channel and in the agent selling channel, the cap regulation with unit carbon quota will decrease the retail price and increase the demand. Therefore, the decrease in sales price usually leads to an overall increase in consumer surplus. Because the lower price enables more consumers to purchase goods at prices lower than their personal valuation, while also encouraging existing consumers to increase their demand. This indicates that whether the agent selling or reselling channel in a platform supply chain, the consumer prefers the unit carbon quota based on benchmarking rule.

This finding underscores a critical policy implication: consumers universally prefer benchmarking rule over alternative grandfathering rule, as it yield superior welfare outcomes across all sale channel in a platform supply chain. The consistency of this result across channel types further validates the robustness of the benchmarking rule in enhancing consumer welfare.

6.4. The carbon allowance allocation rule selection

In the previous section, we compare two different cap regulations from the perspective of supplier, e-commerce platform and consumer, respectively. In this section, we further compare the social welfare of two carbon allowance allocation rules, and investigate the carbon allowance allocation rule selection from the viewpoints of the government.

Under the carbon cap and trade mechanism, grandfathering and benchmarking are the two currently popular carbon allowance allocation rules adopted by the government, which have their pros and cons. The government usually chooses the suitable carbon allowance allocation rule based on the social welfare. It is necessary to compare the two different allocation rules from the perspective of social welfare in order for the government to identify which rule is better in a platform supply chain.

Following Cui and Notteboom [10], the social welfare function of the supply chain is defined as the sum of consumer surplus, the supplier profit, and the e-commerce platform profit minus environmental pollution damage in the supply chain. Therefore, the social welfare SW in a platform supply chain is specified as:

$\mathrm{SW} = \pi_p + \pi_s + \mathrm{CS} - \mathrm{ED}$, where ED represents the environment damage incurred from carbon emission, $\mathrm{ED} = d\beta\varphi$.

Base on the results in Sections 4.1, 4.2, 5.1 and 5.2, the social welfare functions in a platform supply chain with two different sales channels under two different cap regulations are obtained respectively as

$$\mathrm{SW}^{\mathrm{VA}} = \frac{[b(c - me + \beta e) - a(1-\mu)][b(c - me + \beta e)(3 - 4\mu) - 3a(1-\mu)] + A}{8b(1-\mu)^2}, \tag{13}$$

where $A = 4b\beta[b(1 + c - \mu - me + \beta e) - 2a(1-\mu)](1-\mu)\varphi$.

$$\mathrm{SW}^{\mathrm{TA}} = Qe + \frac{[a(1-\mu) - b(c + \beta e)][(4b\beta\varphi - 3a)(1-\mu) - (3 - 4\mu)b(c + \beta e)]}{8b(1-\mu)^2} \tag{14}$$

$$\mathrm{SW}^{\mathrm{VR}} = \frac{7[a - b(c - me + \beta e)]^2}{32b} - \frac{[a - b(c - me + \beta e)]\beta\varphi}{4} \tag{15}$$

$$\mathrm{SW}^{\mathrm{TR}} = Qe + \frac{7[a - b(c + \beta e)]^2}{32b} - \frac{[a - b(c + \beta e)]\beta\varphi}{4}. \tag{16}$$

By comparing the social welfare under total carbon quota and unit carbon quota, the following results can be obtained.

Proposition 12. *In the agent selling channel,*

If $\mu > \frac{3}{4}$ *and* $Q > \frac{(b-a)\beta\varphi}{2e}$, *then* $\mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$;

If $\mu > \frac{3}{4}$, $Q < \frac{(b-a)\beta\varphi}{2e}$, *and* $m > m_7$, *then* $\mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$;

If $\mu > \frac{3}{4}$, $Q < \frac{(b-a)\beta\varphi}{2e}$, *and* $0 < m < m_7$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$;

If $0 < \mu < \frac{3}{4}$, *and* $Q < \frac{A^2}{8be(4\mu-3)(1-\mu)^2} - \frac{(a-b)\beta\varphi}{2e}$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$;

If $0 < \mu < \frac{3}{4}, Q > \frac{(b-a)\beta\varphi}{2e}$, *and* $0 < m < m_7'$, $\mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$;

If $0 < \mu < \frac{3}{4}, Q > \frac{(b-a)\beta\varphi}{2e}$, *and* $m > m_7'$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$;

In which, $m_7' = \beta + \frac{c}{e} - \frac{(1-\mu)(2a\mu+2b\beta\varphi-3a)+\sqrt{A^2-4b(4\mu-3)(1-\mu)^2(2Qe+a\beta\varphi-b\beta\varphi)}}{(4\mu-3)be}$,

$m_7 = \beta + \frac{c}{e} - \frac{(1-\mu)(2a\mu+2b\beta\varphi-3a)-\sqrt{A^2-4b(4\mu-3)(1-\mu)^2(2Qe+a\beta\varphi-b\beta\varphi)}}{(4\mu-3)be}$,

$A = (1-\mu)(2a\mu + 2b\beta\varphi - 3a) - b(c + \beta e)(4\mu - 3)$.

From Proposition 12, it can be shown that, the social welfare difference between the grandfathering and benchmarking carbon allowance regulations in agent selling channel is depend on the total carbon quota, the unit carbon quota and the fee rate of e-commerce platform.

When the fee rate of e-commerce platform and the supplier's total carbon quota are high, or when the fee rate of e-commerce platform is high, the total carbon quota is low and the unit carbon quota is high, the social welfare in a platform supply chain resulted from the grandfathering rule is greater than that from the benchmarking rule. This occurs because grandfathering rule provides greater cost predictability and operational flexibility under these conditions, thereby mitigating inefficiencies that might otherwise arise from stringent or misaligned regulatory incentives.

When the fee rate of e-commerce platform is high, the total carbon quota and the unit carbon quota are low, the social welfare in a platform supply chain resulted from the grandfathering rule is less than that from the benchmarking rule. This outcome arises because, under tight carbon constraints (*i.e.*, limited total and per-unit allowances), the grandfathering approach fails to incentivize efficiency improvements or emission reductions among suppliers. In contrast, the benchmarking rule ties allowance allocations to performance standards, rewarding more efficient operations and encouraging innovation even in resource-constrained environments. Consequently, when regulatory stringency is high and platform fees amplify cost pressures, the dynamic efficiency gains induced by benchmarking lead to higher overall social welfare, including greater consumer surplus and more effective environmental outcomes.

When the fee rate of e-commerce platform and the supplier's total carbon quota are low, or when the fee rate of e-commerce platform is low, the total carbon quota is high, and the supplier's unit carbon quota is high, the social welfare in a platform supply chain resulted from the grandfathering rule is less than that from the benchmarking rule. In these scenarios, the regulatory environment is relatively permissive (due to abundant allowances), and platform transaction costs are modest. Under such conditions, benchmarking effectively rewards best practices, fosters competition among suppliers, and promotes continuous productivity improvement. In contrast, grandfathering, which allocates permits based on past emissions, offers weaker incentives for innovation and may perpetuate inefficiencies, thereby resulting in lower overall welfare. When the fee rate of e-commerce platform is low, the total carbon quota is high, and the unit carbon quota is low, the social welfare in a platform supply chain resulted from the grandfathering rule is greater than that from the benchmarking rule. In this setting, the low platform fee reduces cost pressure on suppliers, while the high total quota provides sufficient room for production scale. However, the low unit quota imposes a binding constraint at the operational level. Under grandfathering, suppliers that historically operated efficiently benefit from larger initial allocations, avoiding abrupt adjustments and maintaining stable output. Benchmarking, by contrast, may penalize even moderately efficient suppliers if they fall short of an ambitious sectoral standard, leading to over-compliance costs or reduced supply, ultimately diminishing consumer surplus and total welfare.

Proposition 12 indicates that, in an e-commerce platform supply chain with agent selling channel, in which the e-commerce has a higher fee rate, once the carbon allowance allocated by grandfathering rule is high, or the carbon allowance allocated by grandfathering rule is low and the carbon allowance allocated by benchmarking rule is high, it is beneficial for the government to adopt the grandfathering rule. In the other cases, the benchmarking rule is beneficial for the government.

In an e-commerce platform supply chain with agent selling channel, in which the e-commerce has a lower fee rate, once the carbon allowance allocated by grandfathering rule is low, or the carbon allowance allocated by grandfathering rule is high and the carbon allowance allocated by benchmarking rule is high, it is beneficial for the government to adopt the benchmarking rule. When the carbon allowance allocated by grandfathering rule is high and the carbon allowance allocated by benchmarking rule is low, it is beneficial for the government to adopt the grandfathering rule.

Proposition 13. *In the reselling channel, if* $0 < m < m_8$*, then* $\mathrm{SW}^{\mathrm{TR}} > \mathrm{SW}^{\mathrm{VR}}$*; if* $m > m_8$, $\mathrm{SW}^{\mathrm{TR}} < \mathrm{SW}^{\mathrm{VR}}$*, where,* $m_8 = \frac{-7a+7bc+7b\beta e+4b\beta\varphi}{7be} + \frac{\sqrt{224bQe+(-7a+7bc+7b\beta e+4b\beta\varphi)^2}}{7be}$.

Proposition 13 reveals the comparative performance of grandfathering and benchmarking rules within the context of a reselling channel in an e-commerce platform supply chain. It demonstrates that, in the reselling channel, the social welfare difference between the grandfathering and benchmarking cap regulations is depend on the unit carbon quota. When the unit carbon quota is relatively low, the total carbon allowance granted to the supplier under the grandfathering rule yields a higher level of social welfare compared to the benchmarking rule. In this case, the more generous aggregate allocation under grandfathering enables greater production flexibility and market responsiveness, thereby enhancing overall social welfare.

Conversely, when the unit carbon quota is sufficiently high, the benchmarking regulation outperforms grandfathering in terms of social welfare. Under such conditions, the benchmarking rule which links emissions allowances directly to production efficiency provides stronger incentives for the supplier to adopt cleaner technologies and operate more sustainably, without unduly constraining output. This leads to a more efficient allocation of resources and a better balance between environmental and economic objectives.

In summary, Proposition 13 underscores that the superiority of one carbon allowance rule over the other is not absolute but contingent on the stringency of the unit carbon quota. Policymakers must therefore carefully calibrate the level of the unit quota when designing emissions regulations, as this parameter fundamentally shapes the welfare implications of their chosen instrument in a platform supply chains.

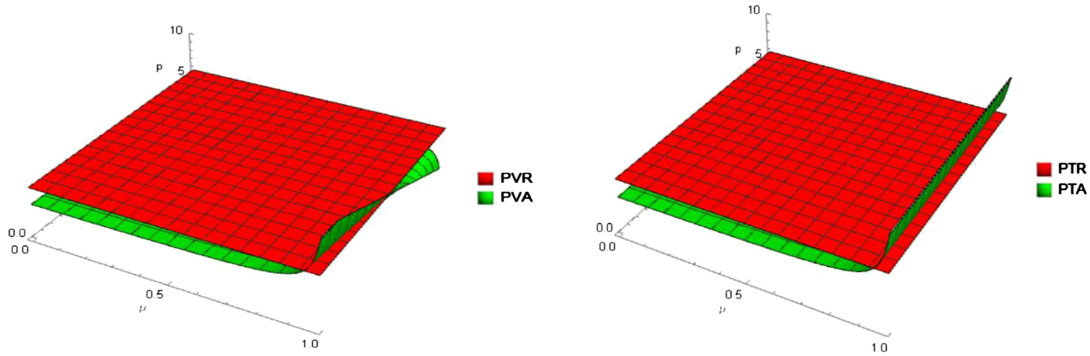
7. Numerical analysis

In this section, some numerical analyses are conducted to demonstrate the obtained analytical expressions in Sections 4–6. Following the related papers [2,45], the parameters are set as: $a = 4$, $b = 0.5$, $c = 0.2$, $\beta = 0.4$, $e = 0.2$, $Q = 8$.

7.1. Comparison of retail price

The retail prices of different sale channels with benchmarking rule and grandfathering rule are shown in Figures 1a and 1b respectively. As shown in Figure 1a, under benchmarking rule, when $0 < \mu < \mu_1 = 0.9323$, the retail price of agency channel is less than that of reselling channel. When $\mu_1 = 0.9323 < \mu < 1$, as m varies from 0 to a certain critical value (m_1), the retail price of agency channel is greater than that of reselling channel; as m exceeds the critical value (m_1), the retail price of agency channel is less than that of reselling channel. This is consistent with Proposition 3(1). As shown in Figure 1b, under grandfathering rule, when $0 < \mu < \mu_1 = 0.9323$, the retail price of agency channel is less than that of reselling channel. When $\mu_1 = 0.9323 < \mu < 1$, the retail price of agency channel is greater than that of reselling channel. This is consistent with Proposition 7(1).

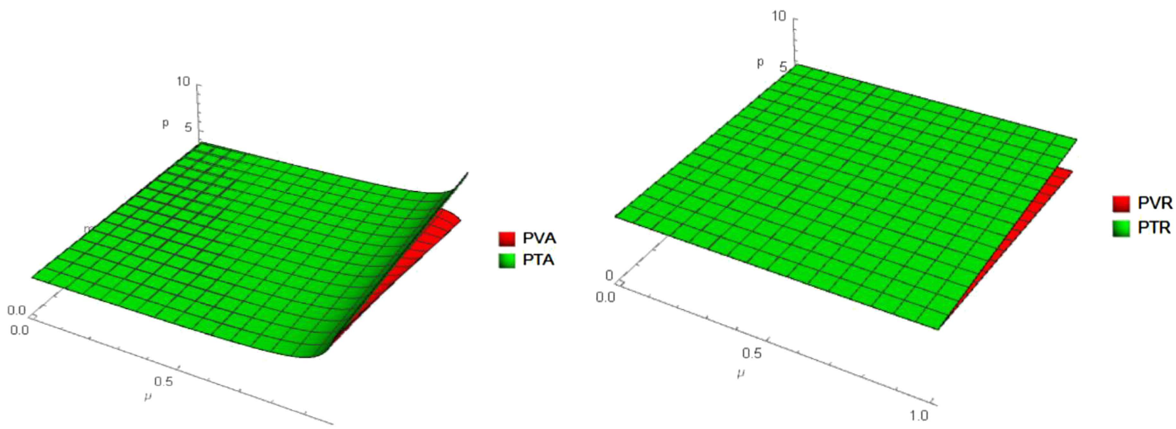
The retail prices under different carbon allowance rules in agency channel and reselling channels are shown in Figures 2a and 2b respectively. As shown in Figures 2a and 2b, whether in agency channel or reselling channel, the retail price under grandfathering rule is greater than that under benchmarking. This verifies Propositions 9(1) and 10(1).



(a) The retail prices of different sale channels with benchmarking rule

(b) The retail prices of different sale channels with grandfathering rule

FIGURE 1. The retail prices of different sale channels.



(a) The retail prices under different carbon allowance rules in agency channel

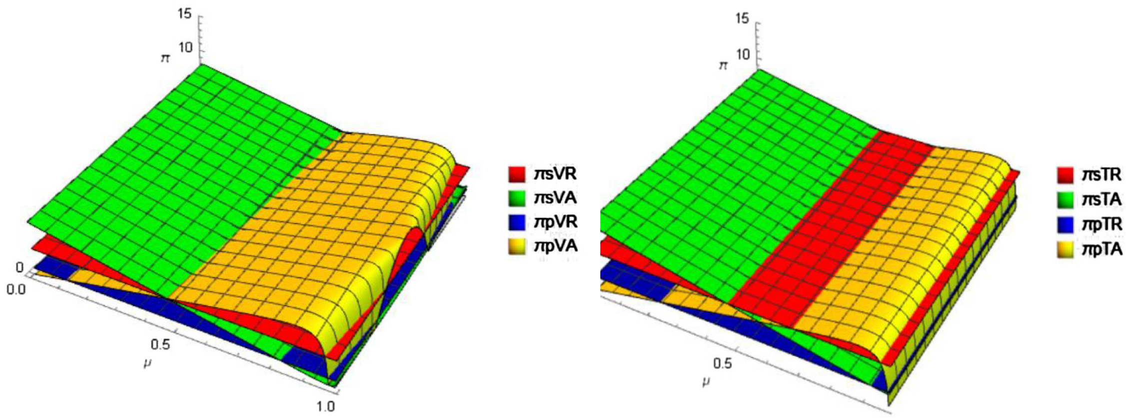
(b) The retail prices under different carbon allowance rules in reselling channel

FIGURE 2. The retail prices under different carbon allowance rules.

7.2. Comparison of the supplier and platform' profits

7.2.1. The supplier and platform's profit comparison in different sale channels

The supplier and platform's profit in different sale channels with benchmarking rule and grandfathering rule are shown in Figures 3a and 3b respectively. As shown in Figure 3a, under benchmarking rule, when $0 < \mu < \mu_2 = 0.4667$ or $\mu_3 = 0.9977 < \mu < 1$, the supplier's profit in agency channel is greater than that of reselling channel. When $\mu_2 = 0.4667 < \mu < 0.9977$, as m varies from 0 to a certain critical value (m_2), the supplier's profit in agency channel is less than that in reselling channel; as m exceeds the critical value (m_2), the supplier's profit in agency channel is greater than that in reselling channel. This is consistent with Proposition 3(2). When $0 < \mu < 0.25$, the platform's profit in agency channel is less than that in reselling channel. When $0.6848 < \mu < 1$, as m is moderate, the platform's profit in agency channel is greater than that



(a) The members' profits in different channels under benchmarking

(b) The members' profits in different channels under grandfathering

FIGURE 3. The members' profits in different channels.

in reselling channel. As m exceeds a certain critical value (m_3), the platform's profit in agency channel is less than that in reselling channel. This is consistent with Propositions 3(2) and 3(3).

As shown in Figure 3b, under grandfathering rule, if $0 < \mu < \mu_4 = 0.4667$, the supplier's profit in agency channel is greater than that in reselling channel. If $\mu_4 = 0.4667 < \mu < 1$, the supplier's profit in agency channel is less than that in reselling channel. This is consistent with Proposition 7(1).

7.2.2. The supplier and platform's profit comparison under different carbon allowance

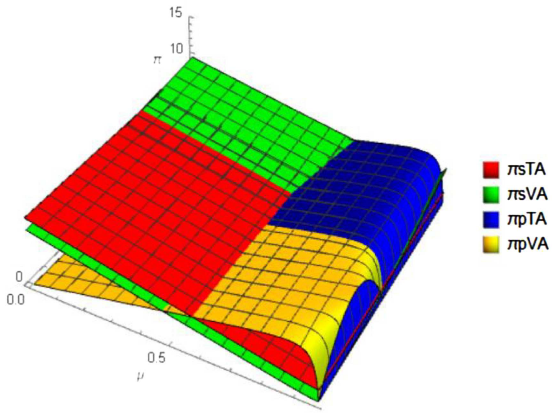
The supplier and platform's profit under different carbon allowance in agency channel and reselling channel are shown in Figures 4a and 4b respectively. As shown in Figure 4a, in agency channel, if the unit carbon quota (m) is low, the supplier's profit under grandfathering rule is greater than that under benchmarking, the platform's profit under grandfathering rule is less than that under benchmarking. If the unit carbon quota (m) is high, the supplier's profit under grandfathering rule is less than that under benchmarking rule, the platform's profit under grandfathering rule is greater than that under benchmarking. This is consistent with Propositions 9(2) and 9(3).

As shown in Figure 4b, in reselling channel, if the unit carbon quota is low, the supplier's profit under grandfathering rule is greater than that under benchmarking rule, if the unit carbon quota is high, the supplier's profit under grandfathering rule is less than that under benchmarking rule. However, the platform's profit under grandfathering rule is always less than that under benchmarking. This is consistent with Propositions 10(2) and 10(3).

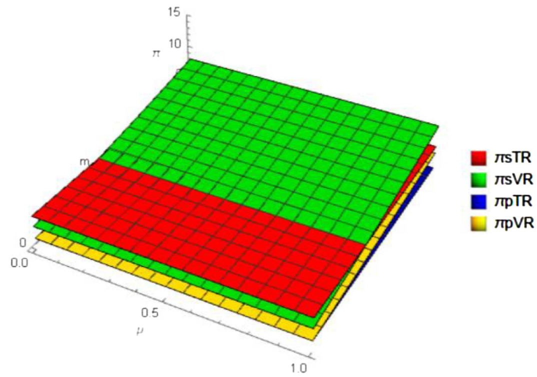
7.3. Comparison of the consumer surplus

7.3.1. The consumer surplus comparison in different sale channels

The consumer surplus in different sale channels with benchmarking rule and grandfathering rule are shown in Figures 5a and 5b respectively. As shown in Figure 5a, under benchmarking rule, when $0 < \mu < \mu_1 = 0.9323$, the consumer surplus in agency channel is greater than that in reselling channel. When $\mu_1 = 0.9323 < \mu < 1$, as m varies from 0 to a certain critical value (m_1), the consumer surplus in agency channel is less than that in reselling channel; as m exceeds the critical value (m_1), the consumer surplus in agency channel is greater that in reselling channel. This is consistent with Proposition 4. As shown in Figure 5b, under grandfathering rule, when $0 < \mu < \mu_1 = 0.9323$, the consumer surplus in agency channel is greater than that in reselling channel.

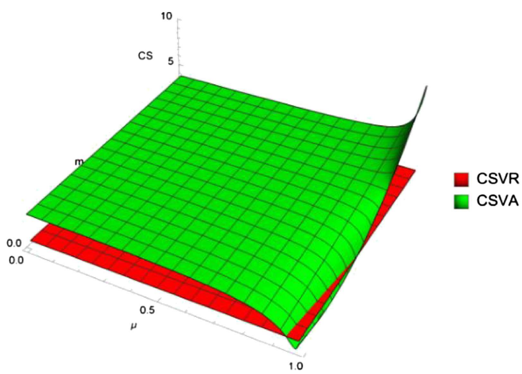


(a) The members' profits under different carbon allowance in agency channel

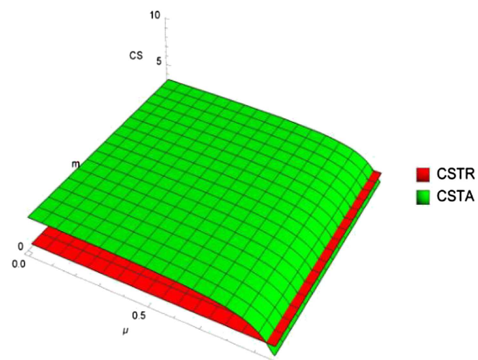


(b) The members' profits under different carbon allowance in reselling channel

FIGURE 4. The members' profits under different carbon allowance.



(a) The consumer surplus in different sale channel under benchmarking



(b) The consumer surplus in different sale channel under grandfathering

FIGURE 5. The consumer surplus in different channels.

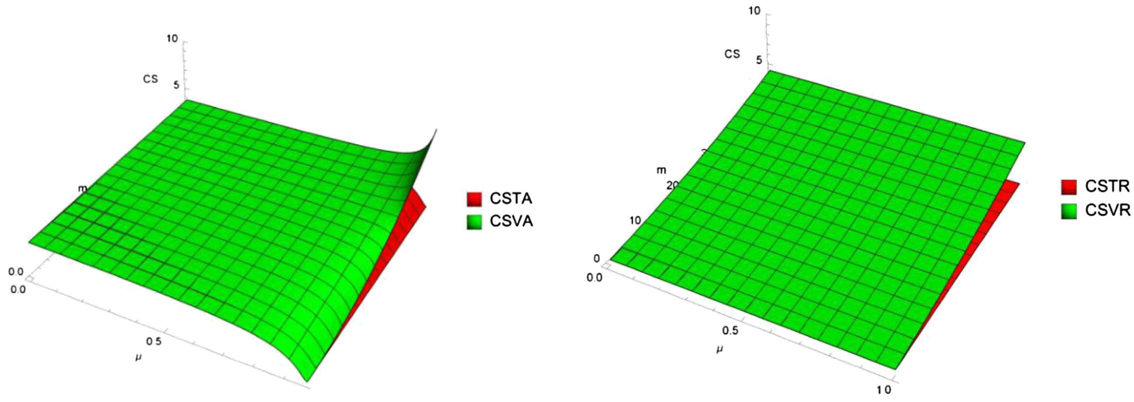
When $\mu_1 = 0.9323 < \mu < 1$, the consumer surplus in agency channel is less than that in reselling channel. This is consistent with Proposition 8.

7.3.2. The consumer surplus comparison under different carbon allowance

The consumer surplus under different carbon allowance rules in agency channel and reselling channels are shown in Figures 6a and 6b respectively. As shown in Figures 6a and 6b, whether in agency channel or reselling channel, the consumer surplus under grandfathering rule is less than that under benchmarking. This verifies Proposition 11.

7.4. Comparison of the social welfare

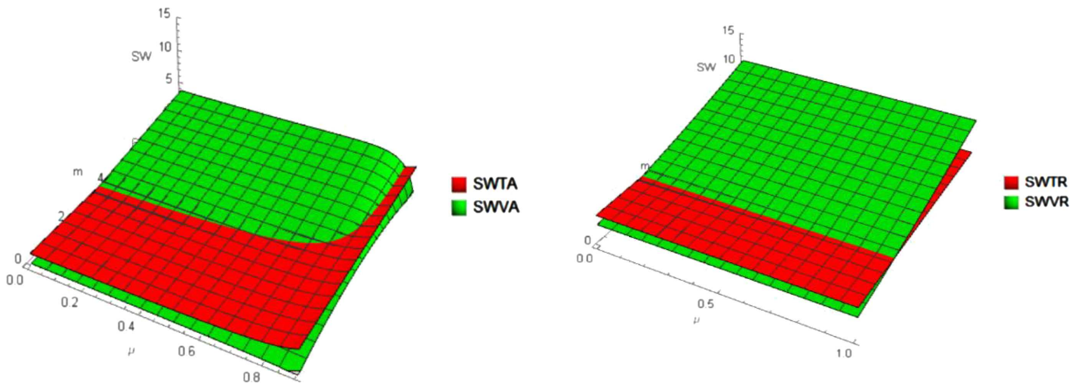
The consumer surplus under different carbon allowance rules in agency channel and reselling channels are shown in Figures 7a and 7b respectively.



(a) The consumer surplus under different carbon allowance in agency channel

(b)The consumer surplus under different carbon allowance in reselling channel

FIGURE 6. The consumer surplus under different carbon allowance.



(a) The social welfare under different carbon allowance in agency channel

(b) The social welfare under different carbon allowance in reselling channel

FIGURE 7. The social welfare under different carbon allowance.

As shown in Figure 7a, in agency channel, when the commission rate of e-commerce platform (μ) is greater than 0.75, the social welfare under grandfathering rule is always greater than that under benchmarking. When μ is less than 0.75, if the unit carbon quota (m) is low, the social welfare under grandfathering rule is greater than that under benchmarking, else if the unit carbon quota (m) is high, the social welfare under grandfathering rule is less than that under benchmarking. This is consistent with Proposition 12.

As shown in Figure 7b, in reselling channel, if the unit carbon quota (m) is low, the social welfare under grandfathering rule is greater than that under benchmarking. If the unit carbon quota (m) is high, the social welfare under grandfathering rule is less than that under benchmarking. This is consistent with Proposition 13.

8. Conclusions and future research directions

8.1. Conclusions

This paper investigated how the different carbon allowance allocation rules affect the supply chain members' preferences for online sales channels and pricing strategies. In this paper, we consider a supply chain consisting of one supplier and one e-commerce platform, in which the platform offers two sales channel options: agent selling and reselling. The supplier may be allocated free carbon allowance under two different allocation rules: benchmarking or grandfathering. Based on our analysis, some meaningful findings and managerial insights are summarized as follows.

Firstly, when the e-commerce's fee rate is low, the reselling channel results in a higher retail price (and lower demand) than the agent selling channel, regardless of whether carbon allowances are allocated *via* benchmarking or grandfathering. When the fee rate is high, under grandfathering rule, the agency channel has a higher retail price (lower demand) than reselling channel. Under benchmarking rule, the outcome depends on the unit carbon quota. If the unit carbon quota is low, the reselling channel yields a lower retail price (higher demand) than agency channel. If the unit carbon quota is high, the reselling channel yields a higher retail price (lower demand) than agency channel.

Secondly, under the benchmarking rule, the supplier's channel choice depends on both the platform's fee rate and the unit carbon allowance. The supplier prefers agency selling when the e-commerce fee rate is either relatively low or high. For example, Chanel and Apple can sell their products on JD.com by agency channel instead of reselling channel. When the fee rate is moderate, a higher (lower) unit carbon allowance leads to a preference for agency (reselling) channel. For example, PepsiCo Foods can sell its products on JD.com by reselling channel instead of agency channel if the fee rate is moderate and the unit carbon allowance is lower. Under the grandfathering rule, the supplier's channel preference is influenced solely by the platform's fee rate. The supplier favors the reselling (agent) channel when the fee rate is high (low). In this case, PepsiCo Foods can choose reselling channel on JD.com if the fee rate is high. Chanel and Apple can choose agency channel on JD.com if the fee rate is low.

Thirdly, under benchmarking rule, the platform prefers the reselling channel when the e-commerce's fee rate is low. When the e-commerce's fee rate is high, it favors reselling channel only if the supplier's unit carbon quota or unit production and carbon emission cost is high. Otherwise, it prefers agency channel. Under the grandfathering rule, when the fee rate of platform is low and the supplier's unit total cost is low, or the supplier's unit total cost is high, it prefers the reselling channel. When the fee rate of platform is high and the supplier's unit total cost is low, or when the fee rate of platform is low and the supplier's unit total cost is moderate, it prefers the agent selling channel.

Fourthly, under benchmarking rule, when the e-commerce's fee rate is low or when the e-commerce's fee rate is high and the unit carbon quota is high, the consumer prefers the agent selling channel. Otherwise, the consumer prefers the reselling channel. Under the grandfathering rule, when the fee rate of e-commerce platform is low, the consumer prefers the agent selling channel. Otherwise, the consumer prefers the reselling channel.

Fifthly, in an e-commerce platform supply chain, whether in agent selling or reselling channel, the retail price (demand) under the Benchmarking rule is lower (higher) than that under the grandfathering rule. Accordingly, the consumer surplus resulted from the Benchmarking rule is higher than that from the grandfathering rule.

Sixthly, for the supplier, it prefers the carbon allowance with the higher unit carbon quota to the fixed total carbon quota, conversely, the supplier prefers the fixed total carbon quota to the carbon allowance with the lower unit carbon quota. For the e-commerce platform,in agent selling channel, it prefers the supplier's fixed total carbon quota to the higher unit carbon quota, and *vice versa*. In reselling channel, it prefers the supplier's unit carbon quota to the fixed total carbon quota.

Lastly, in the agent selling channel in which the e-commerce has a higher fee rate, the government prefers the higher carbon allowance allocated by grandfathering rule or prefers the benchmarking rule when the carbon allowances allocated by grandfathering and benchmarking rules are lower. In agent selling channel in which the e-commerce has a lower fee rate, the government prefers the benchmarking rule when the carbon allowances allocated by grandfathering rule is low or both the carbon allowances allocated by grandfathering and benchmarking rules are higher. the government prefers the grandfathering rule when the carbon allowance allocated by grandfathering rule is high and the carbon allowance allocated by benchmarking rule is low.

In the reselling channel, the government prefers to adopt the benchmarking rule when the carbon allowance allocated by benchmarking rule is higher, and *vice versa*.

8.2. Limitations and future research directions

There are still some limitations which motivates us some potential research directions in our paper. Firstly, our research focuses on a simple supply chain consisting of a supplier and an e-commerce platform. However, there may be multiple suppliers or multiple platforms in reality. Hence, we will investigate carbon allowance allocation rules, selling channel selections and pricing decisions in a more complex e-commerce platform supply chain.

Secondly, we assume that carbon emissions are generated solely by the supplier and that the platform's emissions are negligible. However, this assumption has some limitations. Firstly, many contemporary platforms actively participate in physical operations – such as logistics, fulfillment centers, packaging systems, or last-mile delivery (*e.g.*, Amazon, JD.com), in such cases, the platform's direct and indirect emissions can be substantial and should not be overlooked. Secondly, even "light-asset" digital platforms contribute to emissions through energy-intensive data centers, cloud computing, employee operations, and user-facing digital services. Although emissions per transaction may be low, they can become non-negligible at scale – especially as platform-mediated commerce grows exponentially. Future research could relax this assumption by incorporating e-commerce platform emissions. It is more realistic to consider carbon allowances for both suppliers and e-commerce platforms simultaneously in future research.

Thirdly, in addition to the cap-and-trade regulation, a carbon tax is also widely adopted by some countries. Therefore, we can address the same issue under carbon tax policy and compare it with the results of this paper. Thirdly, in reality, the e-commerce platform may also bear the carbon emissions from the retail process. It is more realistic to consider carbon allowances for both suppliers and e-commerce platforms simultaneously in future research.

Lastly, we assume that the demand is a deterministic linear function of the retail price. However, in a platform supply chain, demand depends not only on retail price but may also be influenced by platform services. Moreover, demand is often uncertain. Therefore, future research should further consider demand as a function of both retail price and platform service level, or model demand as a stochastic variable, to investigate carbon allowance allocation rules and channel strategy in a platform supply chain.

Acknowledgments

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Appendix A.

Proof of Proposition 1. It is easily proofed by taking the derivative of the supplier's profit function with respect to the retail price. Therefore, it is omitted. □

Proof of Proposition 2. It is easily proofed by the backward induction method. Therefore, it is omitted. □

Proof of Proposition 3(1).

$$p^{\mathrm{VR}} - p^{\mathrm{VA}} = \frac{a\,(1-\mu) - b\,(1+\mu)\,(c - me + \beta e)}{4b(1-\mu)}.$$

It is shown that the denominator $4b(1-\mu)$ is positive. Next, we only consider whether the numerator $a\,(1-\mu) - b\,(1+\mu)\,(c-me+\beta e)$ is positive or negative.

To this end, we consider $a\,(1-\mu) - b\,(1+\mu)\,(c-me+\beta e)$ as the function of m, by solving the inequality $a\,(1-\mu) - b\,(1+\mu)\,(c-me+\beta e) > 0$ or $a\,(1-\mu) - b\,(1+\mu)\,(c-me+\beta e) < 0$, we can obtain that: If $\mu_1 < \mu < 1$ and $m > m_1$, in which $\mu_1 = \frac{a-b(c+\beta e)}{a+b(c+\beta e)} m_1 = \beta + \frac{c}{e} - \frac{a(1-\mu)}{be(1+\mu)}$, then $p^{\mathrm{VR}} > p^{\mathrm{VA}}$. If $\mu_1 < \mu < 1$ and $0 < m < m_1$, then $p^{\mathrm{VR}} < p^{\mathrm{VA}}$. If $0 < \mu < \mu_1$, then $p^{\mathrm{VR}} > p^{\mathrm{VA}}$.

Similarly, it can be proofed for the difference of $d^{\mathrm{VR}} - d^{\mathrm{VA}}$. □

Proof of Proposition 3(2). The difference between π_s^{VR} and π_s^{VA} can be written as the quadratic function of m:

$$\pi_s^{\mathrm{VR}} - \pi_s^{\mathrm{VA}} = -\frac{b^2e^2(1+\mu)}{8b(1-\mu)}m^2 + \frac{be(c+\beta e)(1+\mu) - ae(1-\mu)}{4b(1-\mu)}m + \frac{a(c+\beta e)}{4} + \frac{a^2(2\mu-1)}{8b} - \frac{b(c+\beta e)^2(1+\mu)}{8(1-\mu)}. \tag{A.1}$$

Obviously, $\pi_s^{\mathrm{VR}} - \pi_s^{\mathrm{VA}}$ is concave in m by solving the quadratic equation $\pi_s^{\mathrm{VR}} - \pi_s^{\mathrm{VA}} = 0$, we get the following two roots:

$$m_2 = \beta + \frac{c}{e} - \frac{a(1-\mu)}{be(1+\mu)} + \frac{a\mu\sqrt{2(1-\mu)}}{be(1+\mu)}, m_2^* = \beta + \frac{c}{e} - \frac{a(1-\mu)}{be(1+\mu)} - \frac{a\mu\sqrt{2(1-\mu)}}{be(1+\mu)}.$$

We can get

$$\pi_s^{\mathrm{VR}} - \pi_s^{\mathrm{VA}} = -\frac{b^2e^2\,(1+\mu)}{8b\,(1-\mu)}\,(m-m_2)\,(m-m_2^*)\,. \tag{A.2}$$

Since $m_2^* = \beta + \frac{c}{e} - \frac{a(1-\mu)}{be(1+\mu)} - \frac{a\mu\sqrt{2(1-\mu)}}{be(1+\mu)} < \beta + \frac{c}{e} - \frac{a(1-\mu)}{be} < 0$, we need to identify whether m_2 is positive or negative. According to Vieta's formulas,

$$m_2 \times m_2^* = -\left[\frac{a(c+\beta e)}{4} + \frac{a^2(2\mu-1)}{8b} - \frac{b(c+\beta e)^2\,(1+\mu)}{8\,(1-\mu)}\right] / \frac{b^2e^2(1+\mu)}{8b(1-\mu)}. \tag{A.3}$$

Next, we can identify the sign of m_2 by determining the sign of $\frac{a(c+\beta e)}{4} + \frac{a^2(2\mu-1)}{8b} - \frac{b(c+\beta e)^2(1+\mu)}{8(1-\mu)}$.

$$\begin{aligned}&\frac{a(c+\beta e)}{4} + \frac{a^2(2\mu-1)}{8b} - \frac{b(c+\beta e)^2(1+\mu)}{8(1-\mu)}\\ &\quad= \frac{2ab\,(c+\beta e)\,(1-\mu) + a^2\,(2\mu-1)\,(1-\mu) - b^2(c+\beta e)^2(1+\mu)}{8b(1-\mu)}.\end{aligned}$$

Let $f\,(\mu) = 2ab\,(c+\beta e)\,(1-\mu) + a^2\,(2\mu-1)\,(1-\mu) - b^2(c+\beta e)^2(1+\mu)$, we can get

when $0 < \mu < \mu_2$, or $\mu_3 < \mu < 1$, $f\,(\mu) < 0$, it can be obtained from (A.3) that $m_2 < 0$. Here, $\mu_2 = \frac{3a^2-2ab(c+\beta e)-b^2(c+\beta e)^2-[a-b(c+\beta e)]\sqrt{a^2+6ab(c+\beta e)+b^2(c+\beta e)^2}}{4a^2}$, $\mu_3 = \frac{3a^2-2ab(c+\beta e)-b^2(c+\beta e)^2+[a-b(c+\beta e)]\sqrt{a^2+6ab(c+\beta e)+b^2(c+\beta e)^2}}{4a^2}$.

When $\mu_2 < \mu < \mu_3$, $f\,(\mu) > 0$, therefore $m_2 > 0$.

Then, we can get the following results:

(i) When $0 < \mu < \mu_2$ or $\mu_3 < \mu < 1$, $m_2^* < m_2 < 0$. We can obtain from (A.2) that $\pi_s^{\mathrm{VR}} - \pi_s^{\mathrm{VA}} < 0$ for $m > 0$. Thus $\pi_s^{\mathrm{VR}} < \pi_s^{\mathrm{VA}}$.

(ii) When $\mu_2 < \mu < \mu_3$, $m_2^* < 0$, $m_2 > 0$. In this case, we can further obtain from (A.2) that, if $0 < m < m_2$ then $\pi_s^{\mathrm{VR}} > \pi_s^{\mathrm{VA}}$; if $m > m_2$, then $\pi_s^{\mathrm{VR}} < \pi_s^{\mathrm{VA}}$.

(3) The difference between π_p^{VR} and π_p^{VA} can be written as the quadratic function of m:

$$\begin{aligned}\pi_p^{\mathrm{VR}} - \pi_p^{\mathrm{VA}} &= \frac{be^2(1+\mu)^2}{16(1-\mu)^2}m^2 + \left[\frac{ae}{8} - \frac{be\,(c+\beta e)\,(1+\mu)^2}{8(1-\mu)^2}\right]m\\ &\quad+ \frac{a(c+\beta e)}{8} + \frac{a^2(1-4\mu)}{16b} + \frac{b^2(1+\mu)^2(c+\beta e)^2}{16b(1-\mu)^2}.\end{aligned} \tag{A.4}$$

Obviously, $\pi_p^{\mathrm{VR}} - \pi_p^{\mathrm{VA}}$ is convex in m. It can be written as $\pi_p^{\mathrm{VR}} - \pi_p^{\mathrm{VA}} = \frac{be^2(1+\mu)^2}{16(1-\mu)^2}(m-m_3)(m-m_4)$.

Here, $m_3 = \beta + \frac{c}{e} - \frac{a(1-\mu)^2}{be(1+\mu)^2} + \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}$,

$$m_4 = \beta + \frac{c}{e} - \frac{a(1-\mu)^2}{be(1+\mu)^2} - \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}.$$

In order to identify the sign of $\pi_p^{\mathrm{VR}} - \pi_p^{\mathrm{VA}}$, we need to identify whether m_3 and m_4 are negative or positive.

Firstly, we analyze the constant term $\frac{a(c+\beta e)}{8} + \frac{a^2(1-4\mu)}{16b} + \frac{b^2(1+\mu)^2(c+\beta e)^2}{16b(1-\mu)^2}$ in (A.4). We let $B = \frac{a(c+\beta e)}{8} + \frac{a^2(1-4\mu)}{16b} + \frac{b^2(1+\mu)^2(c+\beta e)^2}{16b(1-\mu)^2}$.

It is easily obtained that:

If $0<\mu<\frac{1}{4}$, $B>0$; If $\frac{1}{4}<\mu<1$ and $0<c<-\beta e - \frac{a(1-\mu)^2}{b(1+\mu)^2} + \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}$, then $B<0$; If $\frac{1}{4}<\mu<1$ and $c>-\beta e - \frac{a(1-\mu)^2}{b(1+\mu)^2} + \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}$, then $B>0$.

Secondly, we analyze the coefficient of m in (A.4). We let

$$C = \frac{ae}{8} - \frac{be\,(c+\beta e)\,(1+\mu)^2}{8(1-\mu)^2}.$$

It can be obtained that:

If $0<\mu<\frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}$, then $C>0$; If $\frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}<\mu<1$, then $C<0$.

Thirdly, we can get the following results:

(i) If $0<\mu<\min\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\}$, then $B>0$ and $C>0$. According to Vieta's formulas, it can be obtained from (A.4) that $m_3<0$, $m_4<0$. For $m>0, \pi_p^{\mathrm{VR}} - \pi_p^{\mathrm{VA}}>0$.

(ii) If $\max\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\}<\mu<1$ and $0<c<-\beta e - \frac{a(1-\mu)^2}{b(1+\mu)^2} + \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}$, then $B<0$ and $C<0$. According to Vieta's formulas, it can be obtained from (A.4) that $m_3>0$, $m_4<0$, thus, we can get:
If $0<m<m_3$, $\pi_p^{\mathrm{VR}} - \pi_p^{\mathrm{VA}}<0$; If $m>m_3$, $\pi_p^{\mathrm{VR}} - \pi_p^{\mathrm{VA}}>0$.

(iii) If $\max\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\}<\mu<1$ and $c>-\beta e - \frac{a(1-\mu)^2}{b(1+\mu)^2} + \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}$, then $B>0$ and $C<0$. According to Vieta's formulas, it can be obtained from (A.4) that $m_3>0$, $m_4>0$, thus, we can get:
If $0<m<m_4$ or $m>m_3$, $\pi_p^{\mathrm{VR}} - \pi_p^{\mathrm{VA}}>0$; If $m_4<m<m_3$, $\pi_p^{\mathrm{VR}} - \pi_p^{\mathrm{VA}}<0$.
In summary, we can get,
If $0<\mu<\min\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\}$, then $\pi_p^{\mathrm{VR}} > \pi_p^{\mathrm{VA}}$.
If $\max\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\}<\mu<1$ and $m>m_3$, then $\pi_p^{\mathrm{VR}} > \pi_p^{\mathrm{VA}}$.
If $\max\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\}<\mu<1$, $0<m<m_4$, and $c>-\beta e - \frac{a(1-\mu)^2}{b(1+\mu)^2} + \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}$, then $\pi_p^{\mathrm{VR}} > \pi_p^{\mathrm{VA}}$.
If $\max\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\}<\mu<1$, $m_4<m<m_3$, then $\pi_p^{\mathrm{VR}} < \pi_p^{\mathrm{VA}}$.
If $\max\{\frac{1}{4}, \frac{\sqrt{a}-\sqrt{b(c+\beta e)}}{\sqrt{a}+\sqrt{b(c+\beta e)}}\}<\mu<1$, $0<m<m_4$ and $c<-\beta e - \frac{a(1-\mu)^2}{b(1+\mu)^2} + \frac{2a\mu(1-\mu)\sqrt{\mu+2}}{be(1+\mu)^2}$, then $\pi_p^{\mathrm{VR}} < \pi_p^{\mathrm{VA}}$.

□

Proof of Proposition 4. The difference between $\mathrm{CS}^{\mathrm{VR}}$ and $\mathrm{CS}^{\mathrm{VA}}$ can be written as the quadratic function of m:

$$\begin{aligned}\mathrm{CS}^{\mathrm{VR}} - \mathrm{CS}^{\mathrm{VA}} &= \frac{[(1-\mu)^2-4]b^2e^2}{32b(1-\mu)^2}m^2 + \frac{2be[b(c+\beta e)(3-\mu)(1+\mu)-a(1-\mu)(\mu+3)]}{32b(1-\mu)^2}m \\ &\quad + \frac{2ab(c+\beta e)(\mu+3)}{32b(1-\mu)} - \frac{3a^2}{32b} - \frac{b^2(c+\beta e)^2(3-\mu)(1+\mu)}{32b(1-\mu)^2} \end{aligned} \tag{A.5}$$

Obviously, $\mathrm{CS}^{\mathrm{VR}}-\mathrm{CS}^{\mathrm{VA}}$ is concave in m. Then, we can get $\mathrm{CS}^{\mathrm{VR}}-\mathrm{CS}^{\mathrm{VA}}=\frac{[(1-\mu)^2-4]b^2e^2}{32b(1-\mu)^2}(m-m_1)(m-m_1')$, in which $m_1'=\beta+\frac{c}{e}-\frac{3a(1-\mu)}{be(3-\mu)}$. Since $m_1'<m_0<m_1$, in which $m_0=\beta+\frac{c}{e}-\frac{a(1-\mu)}{be}<0$, we need to identify whether m_1 is positive or negative. It is obviously obtained that, if $\mu_1<\mu<1$, then $m_1>0$, $0<\mu<\mu_1$, $m_1<0$.

Therefore, we can get, if $\mu_1<\mu<1$ and $0<m<m_1$, $\mathrm{CS}^{\mathrm{VR}}>\mathrm{CS}^{\mathrm{VA}}$. If $\mu_1<\mu<1$ and $m>m_1$, $\mathrm{CS}^{\mathrm{VR}}<\mathrm{CS}^{\mathrm{VA}}$. If $0<\mu<\mu_1$, $\mathrm{CS}^{\mathrm{VR}}<\mathrm{CS}^{\mathrm{VA}}$. □

Proof of Proposition 5. It is easily proofed by the backward induction method. Therefore, it is omitted. □

Proof of Proposition 6. It is easily proofed by the backward induction method. Therefore, it is omitted. □

Proof of Proposition 7. (1) It is verified by $p^{\mathrm{TR}}-p^{\mathrm{TA}}$ and $d^{\mathrm{TR}}-d^{\mathrm{TA}}$.

(2) $\pi_s^{\mathrm{TR}}-\pi_s^{\mathrm{TA}}=\frac{a^2(1-\mu)(2\mu-1)+2ab(c+\beta e)(1-\mu)-b^2(c+\beta e)^2(1+\mu)}{8b(1-\mu)}$. Since the denominator $8b\,(1-\mu)>0$, we only consider the numerator $a^2\,(1-\mu)\,(2\mu-1)+2ab\,(c+\beta e)\,(1-\mu)-b^2(c+\beta e)^2(1+\mu)$.

Let $g\,(\mu)=a^2\,(1-\mu)\,(2\mu-1)+2ab\,(c+\beta e)\,(1-\mu)-b^2(c+\beta e)^2(1+\mu)$, it is shown that $g\,(\mu)$ is concave in μ. We can obtain that $g\,(\mu)=-2a^2\,(\mu-\mu_4)\,(\mu-\mu_4')$, in which $\mu_4=\frac{[a-b(c+\beta e)][3a+b(c+\beta e)-\sqrt{B}]}{4a^2}$, $\mu_4'=\frac{[a-b(c+\beta e)][3a+b(c+\beta e)+\sqrt{B}]}{4a^2}$, $B=a^2+6ab\,(c+\beta e)+b^2(c+\beta e)^2$.

It is shown from Proposition 5 that the condition $0<\mu<\mu_0=\frac{a-b(c+\beta e)}{a}$ should be satisfied to ensure the equilibrium solutions and the members' profits are non-negative. It is easily obtained that $\mu_4<\mu_0$ and $\mu_4'>\mu_0$. Therefore, we can get $g\,(\mu)<0$ when $0<\mu<\mu_4$, $g\,(\mu)>0$ when $\mu_4<\mu<\mu_0$. It means $\pi_s^{\mathrm{TR}}<\pi_s^{\mathrm{TA}}$ when $0<\mu<\mu_4$ and $\pi_s^{\mathrm{TR}}>\pi_s^{\mathrm{TA}}$ when $\mu_4<\mu<\mu_0$.

(3) $\pi_p^{\mathrm{TR}}-\pi_p^{\mathrm{TA}}=\frac{b^2(c+\beta e)^2(1+\mu)^2+a^2(1-4\mu)(1-\mu)^2-2ab(c+\beta e)(1-\mu)^2}{16b(1-\mu)^2}$.

Since $16b(1-\mu)^2>0$, we only consider the numerator.

For the convenience of analysis, let $c'=c+\beta e$, which means the supplier's unit cost including the unit production cost and unit carbon emission cost. The numerator can be considered as the function of c', *i.e.*, $h\,(c')=b^2c'^2(1+\mu)^2+a^2\,(1-4\mu)\,(1-\mu)^2-2abc'(1-\mu)^2$ which is a convex function of c'.

Obviously, $h\,(c')=b^2(1+\mu)^2\,(c'-c_1)\,(c'-c_2)$, in which $c_1=\frac{a(1-\mu)^2-2a\mu(1-\mu)\sqrt{\mu+2}}{b(1+\mu)^2}$, $c_2=\frac{a(1-\mu)^2+2a\mu(1-\mu)\sqrt{\mu+2}}{b(1+\mu)^2}$.

It can be shown from $c_1=\frac{a(1-\mu)^2-2a\mu(1-\mu)\sqrt{\mu+2}}{b(1+\mu)^2}$ that, if $\frac{1}{4}<\mu<1$, $c_1<0$, and if $0<\mu<\frac{1}{4}$, $c_1>0$.

Therefore, we can obtain the following results:

(i) if $\frac{1}{4}<\mu<1$ and $0<c'<c_2$, then $h\,(c')<0,\pi_p^{\mathrm{TR}}<\pi_p^{\mathrm{TA}}$;
(ii) if $0<\mu<\frac{1}{4}$ and $c_1<c'<c_2$, then $h\,(c')<0,\pi_p^{\mathrm{TR}}<\pi_p^{\mathrm{TA}}$;
(iii) if $0<\mu<\frac{1}{4}$ and $0<c'<c_2$, then $h\,(c')>0,\pi_p^{\mathrm{TR}}>\pi_p^{\mathrm{TA}}$;
(iv) if $c'>c_2$, then $h\,(c')>0$, $\pi_p^{\mathrm{TR}}>\pi_p^{\mathrm{TA}}$.

□

Proof of Proposition 8.

$$\mathrm{CS}^{\mathrm{TR}}-\mathrm{CS}^{\mathrm{TA}}=\frac{[a-b\,(c+\beta e)]^2(1-\mu)^2-4[a\,(1-\mu)-b\,(c+\beta e)]^2}{32b(1-\mu)^2}.$$

Let $f_2\,(\mu)=[a-b\,(c+\beta e)]^2(1-\mu)^2-4[a\,(1-\mu)-b\,(c+\beta e)]^2$, we can obtain that

$$\begin{aligned}f_2\,(\mu)&=-\left[3a-b\,(c+\beta e)\right]\left[a+b\,(c+\beta e)\right]\mu^2+2\left[a-b\,(c+\beta e)\right]\left[3a+b\,(c+\beta e)\right]\mu\\&\quad-3[a-b\,(c+\beta e)]^2\\f_2\,(\mu)&=\left(\mu-\frac{3a-3b\,(c+\beta e)}{3a-b\,(c+\beta e)}\right)\left(\mu-\frac{a-b\,(c+\beta e)}{a+b\,(c+\beta e)}\right).\end{aligned}$$

From Proposition 5, it is satisfied that $0<\;<\frac{a-b(c+\beta e)}{a}$.

Therefore, we can obtain that $f_2\,(\mu)<0$ when $0<\mu<\frac{a-b(c+\beta e)}{a+b(c+\beta e)}$, and $f_2\,(\mu)<0$ when $\frac{a-b(c+\beta e)}{a+b(c+\beta e)}<\mu<\frac{a-b(c+\beta e)}{a}$. □

Proof of Proposition 9. (1) It is easily obtained by calculating $p^{\mathrm{TA}}-p^{\mathrm{VA}}$ and $d^{\mathrm{TA}}-d^{\mathrm{VA}}$.

(2) We compare π_s^{TA} and π_s^{VA}. We can obtain from (3) and (13) that

$$\begin{aligned}\pi_s^{\mathrm{TA}}-\pi_s^{\mathrm{VA}} &= Qe+\frac{[a(1-\mu)-b(c+\beta e)]^2}{4b(1-\mu)}-\frac{[a(1-\mu)-b(c-me+\beta e)]^2}{4b(1-\mu)}\\ &= \frac{-be^2}{4(1-\mu)}m^2+\frac{(bc+b\beta e-a)\,e}{2(1-\mu)}m+Qe.\end{aligned}$$

Let $\pi_s^{\mathrm{TA}}-\pi_s^{\mathrm{VA}}=f(m)$, it is obviously that $f(m)$ is concave in m.
We can obtain that

$$f\left(m\right)=\frac{-be^2}{4(1-\mu)}\left(m-m_5\right)\left(m-m_5'\right), \tag{A.6}$$

in which $m_5=\beta+\frac{c}{e}-\frac{a(1-\mu)}{be}+\frac{\sqrt{A}}{be}$, $m_5'=\beta+\frac{c}{e}-\frac{a(1-\mu)}{be}-\frac{\sqrt{A}}{be}$ and $A=\sqrt{[a\left(1-\mu\right)-b\left(c+\beta e\right)]^2+4bQe(1-\mu)}$. It can be shown that $m_5'<m_0=\beta+\frac{c}{e}-\frac{a(1-\mu)}{be}<0$. We only consider $f(m)$ when $m>0$. Then we can easily get from (A6) that$f\left(m\right)>0$ when $0<m<m_5$, $f\left(m\right)<0$ when $m>m_5$.

(3) We compare π_p^{TA} and π_p^{VA}. It can be obtained that

$$\begin{aligned}\pi_p^{\mathrm{TA}}-\pi_p^{\mathrm{VA}} &= \frac{\mu[a(1-\mu)-b(c+\beta e)][a(1-\mu)+b(c+\beta e)]}{4b(1-\mu)^2}\\ &\quad-\frac{\mu[a(1-\mu)-b(c-me+\beta e)][a(1-\mu)+b(c-me+\beta e)]}{4b(1-\mu)^2}\\ &= \frac{be\mu m(me-2c-2\beta e)}{4(1-\mu)^2}.\end{aligned}$$

We can get, if $0<m<2\beta+\frac{2c}{e}$, $\pi_p^{\mathrm{TA}}<\pi_p^{\mathrm{VA}}$, if $m>2\beta+\frac{2c}{e}$, $\pi_p^{\mathrm{TA}}>\pi_p^{\mathrm{VA}}$.

□

Proof of Proposition 10. (1) It is easily proofed by calculating $p^{\mathrm{TR}}-p^{\mathrm{VR}}$ and $d^{\mathrm{TR}}-d^{\mathrm{VR}}$.
(2) In order to compare π_s^{TR} and π_s^{VR}, we calculate the $\pi_s^{\mathrm{TR}}-\pi_s^{\mathrm{VR}}$.

$$\begin{aligned}\pi_s^{\mathrm{TR}}-\pi_s^{\mathrm{VR}} &= Qe+\frac{[a-b(c+\beta e)]^2}{8b}-\frac{[a-b(c-me+\beta e)]^2}{8b}\\ &= -\frac{be^2}{8}m^2-\frac{e(a-bc-b\beta e)}{4}m+Qe.\end{aligned}$$

Let $f'\left(m\right)=-\frac{be^2}{8}m^2-\frac{e(a-bc-b\beta e)}{4}m+Qe$. Obviously,f$'\left(m\right)$ is concave in m. We can get

$$f'\left(m\right)=-\frac{be^2}{8}\left(m-m_6\right)\left(m-m_6'\right),\ \text{here}$$

$$m_6=\beta+\frac{c}{e}-\frac{a}{be}+\frac{\sqrt{[a-b\left(c+\beta e\right)]^2+8bQe}}{be},$$

$$m_6'=\beta+\frac{c}{e}-\frac{a}{be}-\frac{\sqrt{[a-b\left(c+\beta e\right)]^2+8bQe}}{be}.$$

According to Proposition 2, $\beta+\frac{c}{e}-\frac{a}{be}<0$, thus $m_6'<0$. We can get
$f'\left(m\right)>0$ when $0<m<m_6=\beta+\frac{c}{e}-\frac{a}{be}+\frac{\sqrt{[a-b(c+\beta e)]^2+8bQe}}{be}$, and
$f'\left(m\right)<0$ when $m>m_6=\beta+\frac{c}{e}-\frac{a}{be}+\frac{\sqrt{[a-b(c+\beta e)]^2+8bQe}}{be}$.

(3)

$$\pi_p^{\mathrm{TR}}-\pi_p^{\mathrm{VR}}=\frac{[a-b(c+\beta e)]^2}{16b}-\frac{[a-b(c-me+\beta e)]^2}{16b}=\frac{-bem(bem+2a-2bc-2b\beta e)}{16b}<0.$$

□

Proof of Proposition 11. It can be obtained that

$$\mathrm{CS}^{\mathrm{TA}} - CS^{\mathrm{VA}} = \frac{[a(1-\mu)-b(c+\beta e)]^2}{8b(1-\mu)^2} - \frac{[a(1-\mu)-b(c-me+\beta e)]^2}{8b(1-\mu)^2}$$
$$= \frac{-be^2m^2 - 2e[a(1-\mu)-b(c+\beta e)]m}{8(1-\mu)^2} < 0$$
$$\mathrm{CS}^{\mathrm{TR}} - \mathrm{CS}^{\mathrm{VR}} = \frac{[a-b(c+\beta e)]^2}{32b} - \frac{[a-b(c-me+\beta e)]^2}{32b} = \frac{-bem(bem+2a-2bc-2b\beta e)}{32b} < 0.$$

□

Proof of Proposition 12. We can compare the social welfare of two carbon allowance allocation rules in agency channel by calculating the $\mathrm{SW}^{\mathrm{TA}} - \mathrm{SW}^{\mathrm{VA}}$. From (**??**) and (**??**), we can obtain that

$$\mathrm{SW}^{\mathrm{TA}} - \mathrm{SW}^{\mathrm{VA}} = \frac{1}{8(1-\mu)^2}\{(4\mu-3)\,be^2m^2 + [2ae\,(1-\mu)\,(2\mu-3) - 2be\,(c+\beta e)\,(4\mu-3)$$
$$+ 4b\beta e\,(1-\mu)\,\varphi]m + 4(1-\mu)^2(2Qe + a\beta\varphi - b\beta\varphi)\}.$$

$\mathrm{SW}^{\mathrm{TA}} - \mathrm{SW}^{\mathrm{VA}}$ can be considered as a function of m, thus, we let

$$f\,(m) = (4\mu-3)\,be^2m^2 + [2ae\,(1-\mu)\,(2\mu-3) - 2be\,(c+\beta e)\,(4\mu-3) + 4b\beta e\,(1-\mu)\,\varphi]\,m$$
$$+ 4(1-\mu)^2(2Qe + a\beta\varphi - b\beta\varphi).$$

We can compare the $\mathrm{SW}^{\mathrm{TA}}$ and SW^{VA} by identifying the sign of $f(m)$.
The discriminant for roots of $f\,(m) = 0$ is

$$\Delta = [2ae\,(1-\mu)\,(2\mu-3) - 2be\,(c+\beta e)\,(4\mu-3) + 4b\beta e\,(1-\mu)\,\varphi]^2$$
$$- 16\,(4\mu-3)\,(1-\mu)^2(2Qe + a\beta\varphi - b\beta\varphi)be^2.$$

We consider the following three cases based on the sign of the coefficient of m^2 in $f(m)$.

(1) If $\mu > \frac{3}{4}, \Delta \geq 0 \Leftrightarrow Q \leq \frac{A^2}{8be(4\mu-3)(1-\mu)^2} - \frac{(a-b)\beta\varphi}{2e}$, here

$$A = (1-\mu)\,(2a\mu + 2b\beta\varphi - 3a) - b\,(c+\beta e)\,(4\mu-3)\,.$$

Therefore, we can obtain that

$$f\,(m) = (4\mu-3)\,be^2\,(m-m_7)\,(m-m_7'),\ \text{here}$$
$$m_7' = \frac{-A - \sqrt{A^2 - 4b(4\mu-3)(1-\mu)^2(2Qe+a\beta\varphi-b\beta\varphi)}}{(4\mu-3)\,be}$$
$$= \beta + \frac{c}{e} - \frac{(1-\mu)\,(2a\mu+2b\beta\varphi-3a) + \sqrt{A^2 - 4b(4\mu-3)(1-\mu)^2(2Qe+a\beta\varphi-b\beta\varphi)}}{(4\mu-3)\,be} \tag{A.7}$$
$$m_7 = \frac{-A + \sqrt{A^2 - 4b(4\mu-3)(1-\mu)^2(2Qe+a\beta\varphi-b\beta\varphi)}}{(4\mu-3)\,be}$$
$$= \beta + \frac{c}{e} - \frac{(1-\mu)\,(2a\mu+2b\beta\varphi-3a) - \sqrt{A^2 - 4b(4\mu-3)(1-\mu)^2(2Qe+a\beta\varphi-b\beta\varphi)}}{(4\mu-3)\,be}. \tag{A.8}$$

In order to identify the sign of $f(m)$, we consider the following three cases:

(i) When $-\frac{(a-b)\beta\varphi}{2e} < Q \leq \frac{A^2}{8be(4\mu-3)(1-\mu)^2} - \frac{(a-b)\beta\varphi}{2e}$, $2Qe + a\beta\varphi - b\beta\varphi > 0$, in this case, the constant term in $f(m)$ is positive. According to Vieta's formulas, $m_7m_7' > 0$. According to (A.7) and (A.8), it is obvious that $m_7 < 0$, and $m_7' < m_7$. In this case, $m_7' < 0$, $m_7 < 0$. In reality, $m > 0$, thus $f\,(m) > 0$, *i.e.*, $\mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$.

(ii) When $Q < -\frac{(a-b)\beta\varphi}{2e}$, $2Qe + a\beta\varphi - b\beta\varphi < 0$, the constant term in $f(m)$ is negative. According to Vieta's formulas, $m_7 m_7' < 0$. In this case, $m_7 > 0$, $m_7' < 0$. Therefore, if $0 < m < m_7$, then $f(m) < 0$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$, if $m > m_7$, then $f(m) > 0$, $\mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$.

(iii) When $Q > \frac{A^2}{8be(4\mu-3)(1-\mu)^2} - \frac{(a-b)\beta\varphi}{2e}$, $\Delta < 0$. $f(m) > 0$, $\mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$.

(2) If $\mu = \frac{3}{4}$, $f(m) = \frac{(4be\beta\varphi - 3ae)m + 2Qe + a\beta\varphi - b\beta\varphi}{4}$.

In order to identify the sign of $f(m)$, we consider the following three cases:

(i) When $4b\beta\varphi > 3a$, $m > \frac{-2Qe - a\beta\varphi + b\beta\varphi}{4be\beta\varphi - 3ae}$, or when $4b\beta\varphi < 3a$, $m < \frac{-2Qe - a\beta\varphi + b\beta\varphi}{4be\beta\varphi - 3ae}$, $f(m) > 0$, $\mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$;

(ii) When $4b\beta\varphi > 3a$, $m < \frac{-2Qe - a\beta\varphi + b\beta\varphi}{4be\beta\varphi - 3ae}$, or when $4b\beta\varphi < 3a$, $m > \frac{-2Qe - a\beta\varphi + b\beta\varphi}{4be\beta\varphi - 3ae}$, $f(m) < 0$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$.

(3) If $0 < \mu < \frac{3}{4}$, $\Delta \geq 0 \Leftrightarrow Q \geq \frac{A^2}{8be(4\mu-3)(1-\mu)^2} - \frac{(a-b)\beta\varphi}{2e}$.

$$A = (1-\mu)(2a\mu + 2b\beta\varphi - 3a) - b(c + \beta e)(4\mu - 3).$$

In order to identify the sign of $f(m)$, we consider the following three cases:

(i) When $\frac{A^2}{8be(4\mu-3)(1-\mu)^2} - \frac{(a-b)\beta\varphi}{2e} < Q < -\frac{(a-b)\beta\varphi}{2e}$, according to Vieta's formulas, $m_7 m_7' > 0$. Next, we have to compare m_7 and m_7' in order to compare $\mathrm{SW}^{\mathrm{TA}}$ and $\mathrm{SW}^{\mathrm{VA}}$.

If $A > 0$, it can be obtained from (A.7) and (A.8) that $m_7' > m_7 > 0$, in this case, if $m_7 < m < m_7'$, $f(m) > 0$, $\mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$; if $m > m_7'$, $f(m) < 0$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$.

If $A < 0$, then $m_7 < m_7' < 0$. When $m > 0$, $f(m) < 0$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$.

(ii) When $Q > -\frac{(a-b)\beta\varphi}{2e}$, $m_7 m_7' < 0$. $m_7' > 0$, $m_7 < 0$. Therefore, $0 < m < m_7'$, $f(m) > 0$, $\mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$; if $m > m_7'$, $f(m) < 0$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$.

(iii) When $Q < \frac{A^2}{8be(4\mu-3)(1-\mu)^2} - \frac{(a-b)\beta\varphi}{2e}$, $\Delta < 0$. In this case, $f(m) < 0$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$.

In summary, we can get the following results:

If $\mu > \frac{3}{4}$ and $Q > -\frac{(a-b)\beta\varphi}{2e}$, then $\mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$; If $\mu > \frac{3}{4}$, $Q < -\frac{(a-b)\beta\varphi}{2e}$, and $m > m_7$, then $\mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$; If $\mu > \frac{3}{4}$, $Q < -\frac{(a-b)\beta\varphi}{2e}$, and $0 < m < m_7$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$.

If $0 < \mu < \frac{3}{4}$, and $Q < \frac{A^2}{8be(4\mu-3)(1-\mu)^2} - \frac{(a-b)\beta\varphi}{2e}$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$; If $0 < \mu < \frac{3}{4}, Q > -\frac{(a-b)\beta\varphi}{2e}$, and $0 < m < m_7', \mathrm{SW}^{\mathrm{TA}} > \mathrm{SW}^{\mathrm{VA}}$; If $0 < \mu < \frac{3}{4}, Q > -\frac{(a-b)\beta\varphi}{2e}$, and $m > m_7'$, $\mathrm{SW}^{\mathrm{TA}} < \mathrm{SW}^{\mathrm{VA}}$;

Where

$$m_7' = \beta + \frac{c}{e} - \frac{(1-\mu)(2a\mu + 2b\beta\varphi - 3a) + \sqrt{A^2 - 4b(4\mu-3)(1-\mu)^2(2Qe + a\beta\varphi - b\beta\varphi)}}{(4\mu-3)\,be},$$

$$m_7 = \beta + \frac{c}{e} - \frac{(1-\mu)(2a\mu + 2b\beta\varphi - 3a) - \sqrt{A^2 - 4b(4\mu-3)(1-\mu)^2(2Qe + a\beta\varphi - b\beta\varphi)}}{(4\mu-3)\,be},$$

$$A = (1-\mu)(2a\mu + 2b\beta\varphi - 3a) - b(c + \beta e)(4\mu - 3).$$

□

Proof of Proposition 13.

$$\mathrm{SW}^{\mathrm{TR}} - \mathrm{SW}^{\mathrm{VR}} = \frac{-7be^2m^2}{32} - \frac{(7a - 7bc - 7b\beta e - 4b\beta\varphi)m}{16} + Qe.$$

Let $f(m) = \frac{-7be^2m^2}{32} - \frac{(7a - 7bc - 7b\beta e - 4b\beta\varphi)m}{16} + Qe$, by solving $f(m) = 0$, we can get

$$f(m) = -\frac{7be^2}{32}(m - m_8')(m - m_8), \text{ here}$$

$$m_8' = \frac{-7a + 7bc + 7b\beta e + 4b\beta\varphi}{7be} - \frac{\sqrt{224bQe + (-7a + 7bc + 7b\beta e + 4b\beta\varphi)^2}}{7be} \tag{A.9}$$

$$m_8 = \frac{-7a + 7bc + 7b\beta e + 4b\beta\varphi}{7be} + \frac{\sqrt{224bQe + (-7a + 7bc + 7b\beta e + 4b\beta\varphi)^2}}{7be}. \tag{A.10}$$

From (A.9), it can be easily obtained that $m_8' < 0$.

Thus, if $0 < m < m_8 f(m) > 0$, *i.e.*, $\mathrm{SW}^{\mathrm{TR}} > \mathrm{SW}^{\mathrm{VR}}$, and if $m > m_8$, $f(m) < 0$, *i.e.*, $\mathrm{SW}^{\mathrm{TR}} > \mathrm{SW}^{\mathrm{VR}}$. □