

WHICH GROUP SHOULD GOVERNMENTAL POLICIES TARGET?

EFFECTS OF INCENTIVE POLICY FOR REMANUFACTURING

INDUSTRY

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Abstract: Although previous research ignores other subsidy groups, we conduct a comprehensive study. In our closed-loop supply chain model, the manufacturer can produce either new products or remanufactured products of end-of-life products recycled by independent recyclers. Our core objective is to explore the production and recycling strategies of the two products under different government subsidy plans and which groups the government subsidy plans should target. Considering the consumers' discount perception and uncertainty of recycling quantity, we construct four game scenarios: (1) no subsidy (Scenario N); (2) subsidy for the manufacturer (Scenario M); (3) subsidy for the recycler (Scenario R); (4) subsidy for consumers (Scenario C). The results show that the corporate profits and social welfare under scenario C are higher than those under other scenarios. If the government adopts scenario C, enterprise profits and social welfare will always be higher, but the downside is that government expenditure will also be higher. If considering the limited government funds, the government can set a medium subsidy level. Scenario M will bring sub-optimal social welfare to the government. For scenario R, government subsidies stimulate the recycling of end-of-life products and help recyclers increase their profits, but from the manufacturer's perspective, manufacturers may resist this scenario.

Keywords: Remanufacturing strategy; Government subsidy; Consumer preference; Stackelberg game

1. INTRODUCTION

In recent years, with the acceleration of the upgrading of electronic products, the amount of e-waste (e.g. TV, refrigerator, printer, etc.) has increased dramatically, and the ecological environment has continued to deteriorate [62]. As reported, it is estimated that in 2021, more than 50 million tons of e-waste will be generated, but only 15 percent will be recycled [10]. It's worth noticing the growth of second-hand electronic products also aggravates resource consumption and environmental pollution, in which the percentage of recycled material in discarded electronic products is relatively low. Still, the recycling potential of materials is great. If the resource such as gold, silver, platinum and other resources contained in them can be reasonably used, the economic and social benefits will be obtained. In addition, the governments have also taken legal measures, and required manufacturers to recycle end-of-life (EOL) products and further utilize scrapped products.

According to Mcconocha and Spoh [33], remanufacturing has created many benefits, such as saving labor, material and energy consumption, shortening the lead time of enterprises, expanding new markets and improving the social image of enterprises. In fact, much of Apple's supply chain is driven by renewable energy, announced in 2017 that all final products would be green and environmental friendly. Therefore, remanufacturing, as an effective method to decrease cost and improve efficiency, has become the focus of the industry and academia. Consumer perceptions of remanufactured products will affect their

buying behavior and decision making [52]. However, in the market economic environment, consumer's attitude has changed greatly, and remanufacturing products are difficult to sell. From the consumer's perspective, the incentive to buy remanufactured products may be lower. From the perspective of collection enterprises, recycling is uncertain since arrival time, quality, and quantity of recycled products directly affect the enterprise's profit [21, 55]. Hence, it's advisable to investigate the uncertainty of consumer preference and recycling in closed-loop supply chain (CLSC).

However, it is unrealistic for enterprises to develop remanufacturing industries by themselves, mainly because the required investment is at great expense. Government subsidy as a powerful tool to stimulate enterprises to promote the circular economy plays a vital role in CLSC. For instance, auto replacement plan, replacement of household electrical appliance program [63]. In November 2015, other policies were formulated for recycling WEEE, which subsidized recycler to encourage recyclers to recycle EOL electronic products. Although the above policies have achieved good results, such as promoting the development of remanufactured products and reducing environmental pollution, they have also brought enormous financial pressure on the government. Therefore, ensuring the effective implementation of government subsidy policies is inseparable from the joint participation of governments, enterprises, and consumers.

Different subsidy modes have different effects on the equilibrium decision and social welfare of each member in the CLSC. In the existing literature on government subsidies, most scholars only consider one kind of government subsidy mode, such as consumer subsidies (*e.g.*, [32]). Some scholars compared the differences between government subsidies for manufacturers and consumers (*e.g.*, [53, 64]). Yet, we consider all participants in the CLSC, which is more comprehensive. We attempt to answer the following question: Which group should governmental subsidy policies target? Specifically, we address the following issues:

- For members, what is the optimal level of subsidy for different modes of government subsidies? And what impact will it have on the recycling quantity and production quantity of the two products?
- For consumers, enterprises, or society, which subsidy mode is most beneficial? And what factors will be critical to the results?
- Do remanufacturing cost savings and consumer preferences affect CLSC decision strategies? What measures should the government take to improve social welfare?

Aiming at these problems above, we use the scenario of no government subsidy as a benchmark model and then expand on three scenarios in which the government subsidizes manufacturers, recyclers, or consumers. In the benchmark model, both the manufacturer (Leader) and the recycler (Follower) aim to maximize their own profits. The game order is as follows: the manufacturer first determine the sales quantity of the two products, and then the recycler determine the quantity of recycling. In other government subsidy models, the government makes the first decision, and its goal is to maximize social welfare. The game order in these three models is as follows: the government moves first than the manufacturer and the recycler, that is, the government first determine the level of subsidy, then the manufacturer move, and finally the recyclers move after observing the manufacturer's decision.

The rest of this paper is described below: Section 2 combed the relevant literature from three perspectives. We describe and assume the model in Section 3. The optimal decision

solutions are obtained for the benchmark model (excluding government subsidies) and the other models with three types of subsidy (subsidizing manufacturer, recycler, and consumer) in Section 4. Section 5 presents the equilibrium results and compares the difference of optimal results among the three models. In Section 6, we analyze and compare the effects of related parameters. In Section 7, a summary of this paper and prospects. The proofs are in the Appendix.

2. LITERATURE REVIEW

The literature in this article is divided into three categories: Section 2.1 focus on consumer preference in a supply chain. And then, we discuss CLSC decision in Section 2.2. In Section 2.3, we discuss the government subsidy in a CLSC. Section 2.4 introduces the research gaps and contributions of this paper.

2.1. Consumer preference in a supply chain

Different consumers have different types of preferences for products, which is an essential factor affecting product sale [1, 44]. Therefore, some consumers are more environmentally conscious and prefer environmentally friendly products when choosing products, such as low-carbon product [8, 41, 62] and remanufactured product [7, 62, 64]. Besides, consumer preference will also make a difference to enterprise profits and the supply chain operation efficiency [14]. Atasu *et al.* [2] studied the impact of different willingness to pay for consumers (ordinary consumers and green consumers) on purchasing behavior. In a two-period CLSC network, Xiong *et al.* [54] investigated the impact of consumer preference on equilibrium output of different products. Du *et al.* [9] found that consumer preference was positively correlated with channel profits. Some scholars also analyzed the influence of consumer preference from the perspective of dual-channel and multi-channel [28, 29]. Furthermore, some scholars have made analyses from the empirical perspective. Shuai *et al.* [47] studied the low-carbon preferences of consumers in six Chinese cities and found that consumers' willingness to pay was significantly different. Motoshita *et al.* [36] investigated consumers' daily shopping choices and analyzed the changes in consumers' buying intentions to products. Thus, the effect of consumer preferences on enterprise profits and operations is evident. Similarly, we assume that consumers are environmentally conscious, but we focus on recycling and production decisions.

2.2. The CLSC decision

In recent years, the influence of product recycling and remanufacturing on CLSC decisions has attracted extensive attention from scholars. Many other studies have also examined CLSC under uncertain conditions, such as uncertain demand [25, 26], uncertain recycling quality [24, 37], or uncertain recycling quantity [19, 38]. In terms of pricing and coordination, Banker [3] explored the influence of uncertain demand on production and pricing strategies. Ferrer and Swaminathan [12] analyzed the remanufacturing and pricing strategies of two products in different cycles. Based on the uncertainty of supply and demand, Hu and Feng [22] investigated the optimal price decision and coordination conditions under service demand. Also, several scholars have studied the uncertainty of relevant parameters in the design of the CLSC network [18, 40]. Moreover, in terms of CLSC recycling channels, Savaskan *et al.* [45] analyzed the choice of recycling channels and pricing decisions of manufacturers and retailers. Feng *et al.* [13] studied the supply chain

coordination strategies of three recycling channels based on consumer preferences. Meanwhile, Li *et al.* [29] extended study to the impact of random demand for remanufactured products on the reverse supply chain. These pieces of literature have made essential contributions to the uncertainty analysis of CLSC and the comparison of recovery channels from different perspectives, these literature studies focus on demand uncertainty. Different from them, the influence of the uncertain recycling quantity on an enterprise's equilibrium decision in a realistic situation, this is our following work.

In fact, enterprises not only pay attention to profit issues when making decisions, but risks may also have a greater impact on the enterprise [27, 31, 42, 48]. Therefore, it is necessary for enterprises to consider risk factors in a CLSC. Choi *et al.* [5] studied how a risk averse manufacturer and two risk averse competing retailers affect supply chain profitability and pricing decisions. Qu *et al.* [43] incorporates shipping policies and default risks into the green supply chain, and the manufacturer may deliver free shipping to the retailer, depending on the retailer's order quantity. Gupta and Ivanov [16] further studied how the competitive supplier's risk aversion and supply disruption affects firms' pricing decisions. However, the above literature focuses on the impact of enterprise risk aversion on operation decisions of CLSC, while ignoring consumer's risk aversion. Recently, Deng [8] analyzed the impact of consumer's risk aversion on enterprise pricing decisions and the environment. Different from the research of Deng [8], we explored the impact of consumer's risk aversion on operation decisions and profits under different subsidy models.

2.3 Government subsidy in a CLSC

Developing circular economy and to realize green transformation, the government playing a significant role in the CLSC characterized by EOL product recycling [17, 58]. By providing subsidies, tax exemptions, loans, and other measures, the government can not only stimulate the enthusiasm of supply chain participants in recycling, but also effectively promote remanufacturing activities, bringing social, economic, and environmental benefits [49, 56, 57]. The government subsidy policies related to this article mainly involve subsidy for the manufacturer, subsidy for the recycler, and subsidy for consumers. In terms of subsidy for the manufacturer, Mitra and Webster [34] analyzed the practical significance of government subsidies to manufacturers and remanufacturers' profits, and the results showed that part of the subsidy should be given to the manufacturer. Nielsen *et al.* [39] studied the impact of subsidy for the manufacturer (to stimulating product recycling), subsidy for the manufacturer (to improving product quality) and subsidy for consumers on pricing and social welfare under different power structures. The results show that if the government's goal is to increase green consumption, the effect of subsidy for consumers is better, but if the government's goal is to effectively use subsidy expenditures, the government should choose to subsidy for the manufacturer (to improving product quality). The difference is that we are to reduce the cost of remanufacturing products. In terms of subsidy for the recycler, Wang *et al.* [51] analyzed the impact of recycling subsidy on the remanufacturing industry based on system dynamics, and the results showed that recycling subsidy should be set at medium or high level. Liu *et al.* [30] analyzed the impact of equilibrium recycling prices and subsidy strategy on formal and informal channels, where the government provides subsidies to formal recyclers.

Also, for consumer subsidy, based on government budget constraints, Huang *et al.* [23] explored the influence of consumer subsidies on energy-efficient product

prices and product energy efficiency, and the results showed that the government's certification standard and subsidy level should take into account the capabilities of enterprises themselves, and unfavorable and exorbitant. Cohen *et al.* [6] studied the impact of consumer subsidies on supplier production and pricing strategies based on uncertain demand and minimization of government expenditures, and found that demand uncertainty is positively correlated with production quantity, but is opposite to price. However, the above research did not consider social welfare. From the social welfare point of view, Shao *et al.* [46] compared two different incentive plans and found that the government tends to use consumer subsidy plans, while environmental factors have a great impact on social welfare. Xu *et al.* [55] established a supply chain model in which the government taxed high-carbon manufacturers and provided subsidy to low-carbon consumers, and studied the impact of horizontal integration strategies on social welfare. Yu *et al.* [61] discussed the effect of different government subsidy plans and government goals on the equilibrium strategy of enterprises and social welfare. Their results showed that when the government aims to improve social welfare, subsidizing consumers is more robust. He *et al.* [20] analyzed the optimal government subsidy level and social welfare under three channel structures, and the results showed that high subsidy can benefit consumers and enterprises, but may not be conducive to the environment. Furthermore, He *et al.* [20] only considered consumer subsidy and did not consider risk factors. Recently, Bian *et al.* [4] discussed the impact of manufacturer subsidy and consumer subsidy on investment in emission reduction technologies, and which subsidy the government should implement depends on whether the government focuses on cost or social welfare. Our study differs from Bian *et al.* [4] in two aspects. First, unlike Bian *et al.* [4], we not only consider the manufacturer subsidy and consumer subsidy, but also consider the scenarios of without subsidy and recycler subsidy. Moreover, Bian *et al.* [4] consider the financial risk of the manufacturer, but they did not consider the consumers' risk aversion.

2.4. Research gaps and our contributions

According to the differences between the existing literature and this paper (shown in Table 1), this article has contributed in the following areas: First, we have studies four different subsidy objects, which are more comprehensive. Second, previous papers considered the uncertainty of demand and quality in the supply chain, we further consider the uncertainty of recycling quantity in the three-tier supply chain Stackelberg game. Third, we conduct a comprehensive analysis of different subsidy scenarios from the perspectives of consumers, enterprises and government and give reasonable policy recommendations and managerial insights. Fourth, we further explore the impact of consumers' risk aversion on the equilibrium strategy under different subsidy scenarios.

TABLE 1. The difference between our work and the related literature.

Author(s)	Consumer preferences				Subsidy object				Consider uncertainty		Social welfare	
	Price preference	Low-carbon preference	Channel preference	Green preference	Manufacturer	Remanufacturer	Recycler	Consumer	Certain	Uncertain	Uncorrelated	Correlated
Shao <i>et al.</i> [46]		✓						✓	✓			✓
Mitra and Webster [34]	✓				✓	✓			✓		✓	
Nielsen <i>et al.</i> [39]				✓	✓			✓	✓			✓
Wang <i>et al.</i> [51]					✓		✓		✓		✓	
Liu <i>et al.</i> [30]							✓		✓		✓	
Huang <i>et al.</i> [23]	✓							✓	✓		✓	
Yu <i>et al.</i> [61]					✓			✓		✓		✓
Xu <i>et al.</i> [55]		✓						✓	✓			✓
He <i>et al.</i> [20]	✓		✓					✓	✓			✓
Cohen <i>et al.</i> [6]								✓		✓	✓	
Bian <i>et al.</i> [4]				✓	✓			✓		✓		✓
Our work	✓				✓		✓	✓		✓		✓

3. MODEL DESCRIPTION AND NOTATIONS

We consider a CLSC consisting of the government, one manufacturer (M), and one recycler (R). The manufacturer can produce new products and remanufactured products, and sell them to consumers at prices p_n and p_r , respectively. At the same time, the manufacturer entrusts the recycler to recycle the EOL products, the recycling price is v , and the manufacturer's unit transfer price is w_r . In real life, the government often adopts different means of fiscal policy to achieve specific political and economic goals [35]. The government subsidy is a powerful tool to promote the development of the remanufacturing industry. Different subsidies have a significant impact on the efficiency of CLSC operations. Therefore, we designed four different subsidy models (without government subsidy, subsidy for the manufacturer, subsidy for recycler and subsidy for consumers), the purpose of which is to determine which subsidy model is more effective.

We only consider consumers' purchase of a single product, i.e., a new product or remanufactured product. All EOL products recovered by recyclers from consumers can be processed into remanufactured products. $\Delta = c_n - c_r > 0$ represents the unit cost savings, where the unit cost of a new product produced by the manufacturer is c_n , the unit cost of a remanufactured product is c_r , and the unit recycling price for the recycler is v , both of which include transportation costs (e.g. [11,59]). To ensure the economy of the remanufacturing activity and ensure the non-negative profit of the recycler, the following conditions need to be met: $w_r > c_1 > v > k$.

In the research, the remanufactured product has no difference from the new product in quality and function. Due to consumers' inconsistent sense of identity to the two products, it will affect consumers' purchasing behaviour and decision. We assume that the market size is 1, the utility of consumers buying a new product is e , and follow uniform distribution between 0 and 1. Relative to the recycling price of the new product, the lowest acceptance of the recycling price of the remanufactured product is θ and $\theta \in [0,1]$. Therefore, in the without government subsidy scenario, the utility functions of the two products are $U_n^N = e - p_n$, $U_r^N = \theta e - p_r$, respectively. So, the linear inverse demand functions are $P_r^N = \theta(1 - Q_n - Q_r)$ and $P_n^N = 1 - Q_n - \theta Q_r$, respectively.

The government and enterprises take the absolute initiative in the game relationship between product recycling and remanufacturing. By formulating subsidy policies, the government provides subsidies to manufacturers, recyclers, and consumers are m , k , and s . Maximizing social welfare is widely applied in remanufacturing literature, and according to the government objective function proposed by [14,60], which contains the following four key elements: (1) The manufacturers' profit π_m^i ; (2) The recycler' profit π_r^i ; (3) The total consumer surplus CS^i ; (4) The total government subsidy expenditure GS^i . Therefore, it can be expressed as:

$$SW^i = \pi_m^i + \pi_r^i + CS^i - GS^i = \pi_m^i + \pi_r^i + (\int_{1-Q_n^*}^1 (e - p_n^*)de + \int_{1-Q_n^*-Q_r^*}^{1-Q_n^*} (\theta e - p_r^*)de) - iQ_r^{i*}.$$

Besides, the recovery of the EOL product is distributed uniformly between 0 and 1. The probability density function is $f(\lambda)$ and the cumulative distribution function is $F(\lambda)$. π_l^i is used to represent the profit of enterprises l in model i ($i = \{N, M, R, C\}$). m^* , k^* , and s^* represents the optimal level of government unit subsidies to the manufacturer, recycler, and consumers, respectively. Among of them, the anarchic subsidy model is the benchmark model, which is used to compare the effects of different government subsidy plans on

equilibrium strategies and economic benefits. All notations are shown in Table 2.

TABLE 2. Notation and explanation

Notation	Explanation
p_j	The sales price of the new /remanufactured products, $j = n, r$.
Q_j	The quantity of products.
c_j	The unit cost of products.
Δ	Unit cost savings for remanufactured products.
c_1	Emergency procurement cost.
w_r	Unit transfer price of EOL products.
θ	Consumers' perception of discount on remanufactured products, $0 < \theta < 1$.
R	Recycling quantity of EOL products.
λ	Recovery rate.
e	The willingness of consumers to buy new products.
v	Recycling price.

4. SUBSIDY MECHANISM DESIGN

We first discuss the case of no subsidy as a benchmark model and then investigate the subsidy for the manufacturer, subsidy for the recycler, subsidy for consumers, and we sequentially analyze the equilibrium results.

4.1. Benchmark model: decisions without government subsidy (Scenario N)

In the benchmark model (without government subsidy), the manufacturer(Leader) first determines the sales quantity of new and remanufactured products. After observing the manufacturer's remanufactured product sales, the recycler (Follower) decides the quantity of recycled products. If the recycler finds that the quantity recycled is less than the observed quantity of remanufactured products, it will adopt an urgent ordering strategy. Otherwise, it will result in a loss of profits. Hence, the manufacturer's and recycler's profit functions are obtained as:

$$\pi_m^N = (p_n - c_n)Q_n + (p_r - c_r)Q_r - w_r Q_r \quad (4.1)$$

$$\pi_r^N = w_r Q_r - c_1 [Q_r - \lambda R]^+ - vR \quad (4.2)$$

In (4.1), the first term represents the profit from selling the new products. The second term represents the profit from selling remanufactured products. The last term represents the transfer payment cost that the manufacturer pays to the recycler. In (4.2), the first term is the recycler's revenue. The second term represents the emergency procurement costs. The last term represents the cost of recycling EOL products.

To solve the above problems, we can get the optimal sales quantity of the manufacturer's two products, the optimal recycled quantity of the recycler, and their own profits. The superscript * and i to represent the optimal solution for the four subsidy scenarios

Proposition 1. In scenario N, the optimal recycling quantity is $R^{N*} = \frac{w_r - \theta c_n + c_r}{2\theta(-1+\theta)} \sqrt{\frac{c_1}{2v}}$. The

sales quantity and sales price of the two products are

$$Q_r^{N*} = \frac{w_r - \theta c_n + c_r}{2(-1+\theta)}, Q_n^{N*} = \frac{1+w_r - \theta - c_n + c_r}{2-2\theta}, p_r^{N*} = \frac{w_r + \theta + c_r}{2}, p_n^{N*} = \frac{1+c_n}{2}.$$

By Proposition 1, the optimal recycling quantity R^{N*} is positively correlated with the sales quantity of the remanufactured product Q_r^{N*} , both Q_r^{N*} and Q_n^{N*} are determined by the parameters w_r , θ , c_n and c_r . p_r^{N*} is positively correlated with w_r , θ and c_r , while p_n^{N*} is only positively correlated with c_n .

4.2. Subsidy for the manufacturer (Scenario M)

As the core role of the CLSC, remanufacturing is one of the important development directions of manufacturers. The government has formulated relevant policies to provide financial subsidies to energy-conserving production enterprises. For example, On May 26, 2010, the Chinese government issued the "Energy Saving Products Benefit People Project" implementation rules [23]. Chinese automakers have received government subsidies to promote energy-efficient vehicles, with a subsidy standard of 3,000 yuan per vehicle. This section assumes that the government encourages the manufacturer to continuously invest in remanufacturing activities by subsidizing m per unit of the remanufactured product produced by the manufacturer. Financial subsidy indirectly reduces the manufacturer's remanufacturing costs, thus increasing its profits. The manufacturer's and recycler's profit functions are:

$$\pi_m^M = (p_n - c_n)Q_n + (p_r - c_r + m)Q_r - w_r Q_r \quad (4.3)$$

$$\pi_r^M = w_r Q_r - c_1 [Q_r - \lambda R]^+ - v R \quad (4.4)$$

Proposition 2. *In scenario M, the optimal recycling quantity of the recycler is $R^{M*} = \frac{m - w_r + \theta c_n - c_r}{2\theta(1-\theta)} \sqrt{\frac{c_1}{2v}}$. The sales quantity and sales price of the new and remanufactured products are, $Q_n^{M*} = \frac{1 - m + w_r - \theta - c_n + c_r}{2(1-\theta)}$, $Q_r^{M*} = \frac{m - w_r + \theta c_n - c_r}{2\theta - 2\theta^2}$, $p_n^{M*} = \frac{1 + c_n}{2}$, $p_r^{M*} = \frac{w_r + \theta - m + c_r}{2}$. The optimal subsidy value is $m^* = w_r - \sqrt{2vc_1} + \theta c_n - c_r$.*

By Proposition 2, the optimal recycling quantity R^{M*} is positively correlated with the sales quantity of the remanufactured product Q_r^{M*} . Also, the recycling quantity and the remanufactured product sales quantity have been improved due to the influence of unit subsidy (m), while the new product sales quantity is opposite. p_r^{M*} is negatively correlated with m , while p_n^{M*} is only positively correlated with c_n . The optimal unit subsidy levels of subsidy for manufacturers are related to parameters w_r , θ , c_n , c_r , c_1 and v .

4.3. Subsidy for the recycler (Scenario R)

Recycler, as a recycling enterprise of EOL product, guarantee the smooth operation of the whole product CLSC. To recycle EOL products, government agencies stimulate recyclers to put more efforts into recycling EOL products by providing unit subsidy k to the recycler. For example, according to the Waste Electrical and Electronic Products Recycling Management Regulations issued by the Chinese government, recycling enterprises can get subsidies for disassembling waste products (e.g. 85 yuan per TV set and 80 yuan per

refrigerator). Hence, the manufacturer's and recycler's profit functions are:

$$\pi_m^R = (p_n - c_n)Q_n + (p_r - c_r)Q_r - w_r Q_r \quad (4.5)$$

$$\pi_r^R = w_r Q_r - c_1[Q_r - \lambda R]^+ - (v - k)R \quad (4.6)$$

Proposition 3. *In scenario R, the optimal recycling quantity of the recycler is $R^{R*} =$*

$\frac{\theta c_n - c_r - w_r}{2\theta(1-\theta)} \sqrt{\frac{c_1}{2v-2k}}$. The sales quantity and sales price of the two products are $Q_r^{R*} = \frac{\theta c_n - c_r - w_r}{2\theta(1-\theta)}$,

$Q_n^{R*} = \frac{1+w_r-\theta-c_n+c_r}{2-2\theta}$, $p_r^{R*} = \frac{w_r+\theta+c_r}{2}$, $p_n^{R*} = \frac{1+c_n}{2}$. The optimal subsidy value is $k^* = v$.

According to Proposition 3, the result is similar to Proposition 1. The optimal unit subsidy levels of subsidy for the recycler is related to parameter v . The optimal recycling quantity R^{R*} is positively correlated with the sales quantity of the remanufactured product Q_r^{R*} . The optimal sales quantity and price of two products are not related to the subsidy.

4.4. Subsidy for consumers (Scenario C)

Government agencies often promote the upgrading of the remanufacturing industry through direct subsidies to consumers. When consumers buy remanufactured products that meet the requirements of national policies, the government directly gives a certain percentage of the sales price as a financial subsidy.

Suppose the government subsidizes per unit s for consumers who purchase a remanufactured product [32]. The utility function becomes $U_r^C = \theta e - p_r + s$ and $U_n^C = e - p_n$, respectively. Linear inverse demand function becomes $p_r = \theta(1 - Q_n - Q_r) + s$, $p_n = 1 - Q_n - \theta Q_r$, respectively. The manufacturer's and recycler's profit functions are:

$$\pi_m^C = (p_n - c_n)Q_n + (p_r - c_r)Q_r - w_r Q_r \quad (4.7)$$

$$\pi_r^C = w_r Q_r - c_1[Q_r - \lambda R]^+ - vR \quad (4.8)$$

Proposition 4. *In scenario C, the optimal recycling quantity of the recycler is $R^{C*} =$*

$\frac{s-w_r+\theta c_n-c_r}{2\theta-2\theta^2} \sqrt{\frac{c_1}{2v}}$. The sales quantity and sales price of the two products are $Q_r^{C*} = \frac{s-w_r+\theta c_n-c_r}{2(1-\theta)\theta}$,

$Q_n^{C*} = \frac{1-s+w_r-\theta-c_n+c_r}{2(1-\theta)}$, $p_r^{C*} = \frac{w_r+\theta+s(2\theta-1)+c_r}{2}$, $p_n^{C*} = \frac{1+c_n}{2}$. The optimal subsidy value is $s^* =$

$$\frac{-2\sqrt{2vc_1+w_r(2\theta-1)+(3-2\theta)\theta c_n+(2\theta-3)c_r}}{-3+4\theta}.$$

By Proposition 4, the result is similar to Proposition 2. R^{C*} and Q_r^{C*} are positively correlated with unit subsidy s . p_r^{C*} is positively affected by unit subsidy s and increases the selling price, while p_n^{C*} is only positively correlated with c_n . The optimal unit subsidy levels of subsidy for consumers are related to parameters w_r , θ , c_n , c_r , c_1 and v .

5. DECISIONS ANALYSIS IN THE FOUR DIFFERENT SUBSIDY MODELS

We compare the level of government subsidies and the equilibrium results of the manufacturer and recycler under different subsidy models.

Theorem 1

The optimal government subsidy levels under different subsidy modes are ordered as the following.

TABLE 3. Value range and size relation

Value range	Size relationship
$w_r > v + 2\sqrt{2vc_1} - \theta c_n + c_r$	$k < m < s$
$\frac{v(4\theta - 3) + 2\sqrt{2vc_1}}{2\theta - 1} + \frac{(2\theta - 3)(\theta c_n - c_r)}{2\theta - 1} < w_r < v + 2\sqrt{2vc_1} - \theta c_n + c_r$	$m < k < s$
$w_r < \frac{v(4\theta - 3) + 2\sqrt{2vc_1}}{2\theta - 1} + \frac{(2\theta - 3)(\theta c_n - c_r)}{2\theta - 1}$	$m < s < k$

Theorem 1 shows that under different government subsidy models, the amount of unit subsidy is different. Here, it can be divided into three cases. When w_r is greater than a value, the unit subsidy amount of S mode is the highest, and the unit subsidy amount of K mode is the lowest; When w_r is in a certain range, the unit subsidy amount of s mode is the highest, while m mode is the lowest; When w_r is less than a value, the unit subsidy amount of k mode is the highest, while that of m mode is the lowest. Therefore, we need to study who is most effective for government subsidies.

Theorem 2

The optimal sales prices of new product are ordered as the following.

$$p_n(N) = p_n(R) = p_n(M) = p_n(C)$$

Theorem 2 shows that under different government subsidy models, the sales price of new products is only related to the unit production cost of new products, and the unit government subsidies have no effect on the sales price of new products.

Theorem 3

The relationship between the sales price of remanufactured products is as follows.

$$p_r(M) < p_r(N) = p_r(R) < p_r(C)$$

By Theorem 3, for the sales price of remanufactured products, the price of subsidy for consumers is the highest, while the price subsidy for the manufacturer is lower. The main reason is that the subsidy for consumers will inevitably increase the demand for remanufactured products, which will cause the price of remanufactured products to rise. Subsidy to manufacturers directly leads to the reduction of remanufacturing costs, which indirectly leads to the reduction of the product sales price.

Theorem 4

When $\max\{v, T_1\} < c_1 < \min\{T_2, w_r\}$, $Q_n(M) > Q_n(C)$. The production quantities of new product face the following relationships $Q_n(C) < Q_n(M) < Q_n(R) = Q_n(N)$. Where

$$T_1 = \frac{[(-1+\theta)(3-2w_r+4(-1+w_r)\theta)+(3+\theta(-7+6\theta-4\theta^2))c_n+2\theta(-1+2\theta)c_r]^2}{8v(1-2\theta)^2}, \quad T_2 = \frac{[w_r(2\theta-1)+(3-2\theta)(\theta c_n-c_r)]^2}{8v}$$

Theorem 4 shows that the quantity of new products produced in both subsidy for the manufacturer and subsidy for consumers are reduced compared to the no subsidy scenario, among which, the result of the subsidy recycler is consistent with no government subsidy. By comparing the quantity of new products produced by scenario M and scenario C, we find that when $\max\{v, T_1\} < c_1 < \min\{T_2, w_r\}$, $Q_n(M) > Q_n(C)$, the quantity of new products produced by scenario M is greater than that produced by scenario C.

Theorem 5

When $v < c_1 < \min\left\{\frac{(w_r+3\theta c_n-3c_r)^2}{32v}, w_r\right\}$, the relationship is: $Q_r(N) = Q_r(R) < Q_r(M) < Q_r(C)$.

Theorem 6

When $v < c_1 < \min\left\{\frac{(w_r+3\theta c_n-3c_r)^2}{32v}, w_r\right\}$, $R(M) > R(C)$, and when the government subsidy k is trending to v , the relationship is: $R(N) < R(C) < R(M) < R(R)$.

By Theorem 5, the quantity of remanufactured products produced in both subsidy for the manufacturer and subsidy for consumers are increased compared to the no subsidy scenario, among which, the result of the subsidy recycler is consistent with no government subsidy. By comparing the optimal quantity of new products produced by scenario M and scenario C, we find that when $v < c_1 < \min\{(w_r + 3\theta c_n - 3c_r)^2/32v, w_r\}$, the quantity of remanufactured products scenario C is higher than other scenarios. It is mainly because consumers are the end of the market and the beginning of the reverse supply chain. By Theorem 6, under certain conditions, scenario R has the best recycling effect on EOL products, indicating that government subsidies for recyclers can effectively promote the recycling of EOL products.

6. NUMERICAL ANALYSIS

From the perspective of consumers, enterprises, and government, we use numerical experiments to illustrate the impacts of consumer discount perception and cost savings on the optimal decision of CLSC, different government subsidy levels, and the profits of manufacturers and recyclers. From this, we can compare the pricing, demand, profit and social welfare of new and remanufactured products under different models. Following assumptions for relevant literature [15], the values of the parameter in these papers can be adopted as follows: $c_n = 0.06$, $c_r = 0.01$, $c_1 = 0.035$, $\theta = 0.9$, $w_r = 0.042$, $v = 0.014$.

6.1. Consumer's Perspective

6.1.1. The influence of different parameters on the sales price of remanufactured products

We compared the sales prices of remanufactured products among different models and designed a variety of scenarios. It can be seen from Table 4 that recycling prices, cost savings and emergency procurement costs have an important role in the sales price of

remanufactured products, where $p_r(C)$ has the highest sales price and $p_r(M)$ has the lowest sales price. This shows that scenario C has the most significant influence on the sales price of remanufactured products. Besides, when the values of Δ and c_1 remain unchanged, as the recycling price increases, the value of $p_r(M)$ increases, while the value of $p_r(C)$ decreases. But it has no effect on the values of $p_r(N)$ and $p_r(R)$; When the values of Δ and v remain unchanged, the impact of emergency procurement costs on the sales price of remanufactured products is consistent with the above; When the values of v and c_1 are fixed, as the cost saving increases, the value of $p_r(C)$ shows an upward trend and the value of $p_r(N)$, $p_r(M)$ and $p_r(R)$ shows a downward trend.

TABLE 4. The comparison of sales prices of remanufactured products under different modes ($\theta = 0.9$)

Parameters			Sales prices			
v	Δ	c_1	$p_r(N)$	$p_r(M)$	$p_r(R)$	$p_r(C)$
0.014	0.05	0.015	0.476	0.453	0.476	0.506
		0.03	0.476	0.462	0.476	0.495
		0.035	0.476	0.464	0.476	0.492
	0.055	0.015	0.474	0.449	0.474	0.508
		0.03	0.474	0.457	0.474	0.496
		0.035	0.474	0.459	0.474	0.493
0.02	0.05	0.03	0.476	0.468	0.476	0.487
		0.035	0.476	0.470	0.476	0.484
	0.055	0.03	0.474	0.463	0.474	0.489
		0.035	0.474	0.465	0.474	0.485

6.1.2. The effect of different parameters on sales quantity of two products

The sales quantity of new/remanufactured products of the four subsidy modes under different parameters is shown in Table 5. It can be shown that $Q_n(C) < Q_n(M) < Q_n(R) = Q_n(N)$, $Q_r(N) = Q_r(R) < Q_r(M) < Q_r(C)$. This indicates that government subsidies to manufacturers or consumers have a significant influence on the sales of products, and the lowest sales quantity of new products is $Q_n(C)$. In contrast, for remanufactured products, the sales quantity of $Q_r(C)$ is the highest. This verifies the Theorem 4 and Theorem 5. In addition, when the values of Δ and c_1 remain unchanged, as the recycling price increases, the value of $Q_n(M)$ and $Q_n(C)$ increases, but it has no effect on the values of $Q_n(N)$ and $Q_n(R)$. Similarly, for remanufactured products, the values of $Q_n(M)$ and $Q_n(C)$ tend to decrease. When the values of Δ and v remain unchanged, as the emergency procurement cost increases, the value of $Q_n(M)$ and $Q_n(C)$ increases, while the value of $Q_r(M)$ and $Q_r(C)$ decreases. When the values of v and c_1 are fixed, as the cost saving increases, the sales quantity of new products shows a downward trend, while the sales quantity of remanufactured products shows an upward trend.

TABLE 5. The comparison of sales quantity of new/remanufactured products under different modes ($\theta = 0.9$)

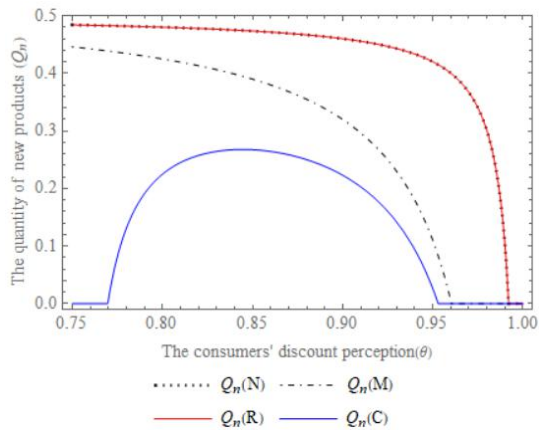
Parameters	Sales quantity (New product)	Sales quantity (Remanufactured product)
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v	Δ	c_1	$Q_n(N)$	$Q_n(M)$	$Q_n(R)$	$Q_n(C)$	$Q_r(N)$	$Q_r(M)$	$Q_r(R)$	$Q_r(C)$
0.014	0.05	0.015	0.46	0.235	0.46	0.082	0.011	0.261	0.011	0.432
		0.03	0.46	0.32	0.46	0.22	0.011	0.167	0.011	0.274
		0.035	0.46	0.343	0.46	0.262	0.011	0.141	0.011	0.231
	0.055	0.015	0.435	0.185	0.435	0.007	0.039	0.317	0.039	0.515
		0.03	0.435	0.27	0.435	0.148	0.039	0.222	0.039	0.358
		0.035	0.435	0.293	0.435	0.187	0.039	0.197	0.039	0.315
0.02	0.05	0.03	0.46	0.376	0.46	0.317	0.011	0.104	0.011	0.169
		0.035	0.46	0.404	0.46	0.364	0.011	0.073	0.011	0.118
	0.055	0.03	0.435	0.326	0.435	0.242	0.039	0.16	0.039	0.253
		0.035	0.435	0.354	0.435	0.289	0.039	0.129	0.039	0.202

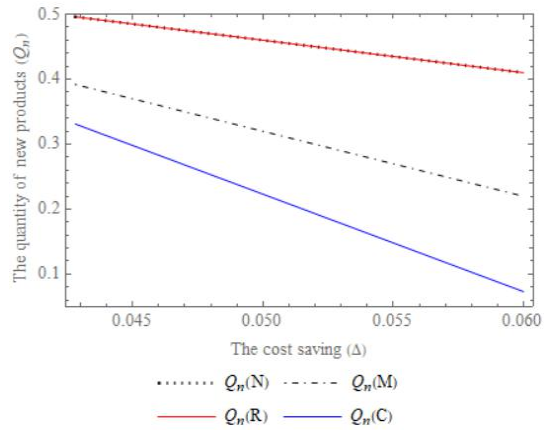
6.1.3. The impact of different parameters on quantity of two products

Figure 1(a) shows, the number of new products of scenario M, N, and R is negatively correlated with consumers' perception and tends to 0 with the increase of consumers' perception. However, only when consumers' discount perception is within a certain range, the quantity of new products of scenario C is meaningful, which first increases and then decreases. We can see that the quantity of new products in scenario M, R, and C is less than or equal to scenario N. In contrast, Figure 1(c) indicates that the quantity of remanufactured products is greater than or equal to the quantity of non-government subsidies. This further indicates that government subsidies are conducive to promoting the recycling of remanufactured products, improving the market demand for remanufactured products, stimulating consumers' consumption of remanufactured products, and thereby improving the environmental benefits of social production.

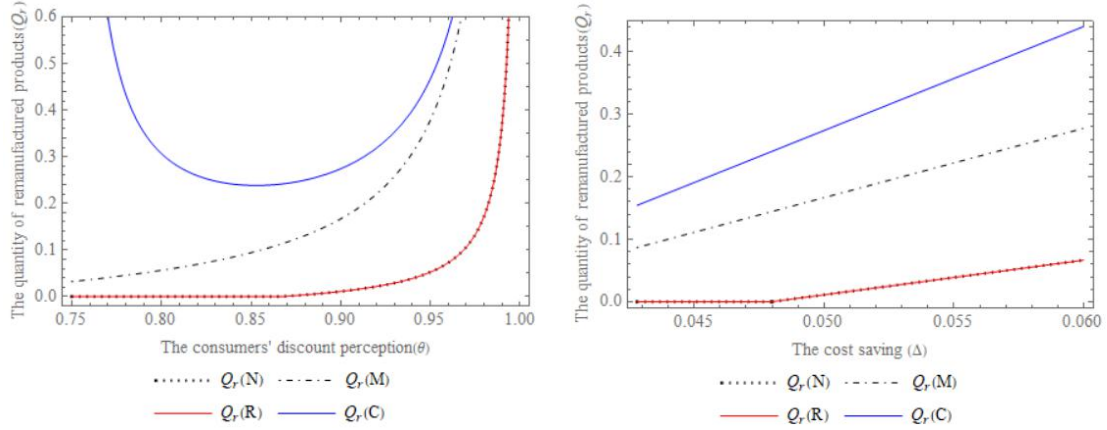
Figure 1(b) and Figure 1(d) show that with the increase of cost saving, the quantity of new products will gradually decrease and the quantity of remanufactured products will gradually increase within a certain range regardless of the operation mode. Interestingly, when consumers were subsidized, the quantity of new products drops the most and the quantity of remanufactured products increases the most. In other words, the government subsidy plan promotes the upgrading of the remanufacturing industry and raises the level of remanufacturing cost saving.



(a) The impact of θ on quantity of new product



(b) The impact of Δ on quantity of new product



(c) The impact of θ on quantity of remanufactured product

(d) The impact of Δ on quantity of remanufactured product

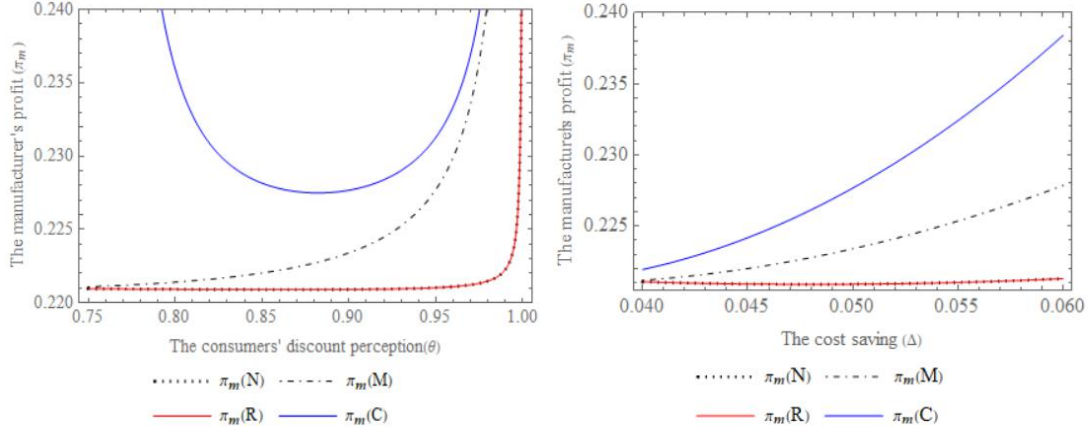
FIGURE 1. The impact of different parameters on quantity of two products

6.2. Enterprise's Perspective

6.2.1. The impact of different parameters on manufacturer's profit

According to Figure 2(a), the manufacturer's profits of scenario N, R and M is positively correlated with consumers' discount perception. However, in scenario C, when Consumers' discount perception was relatively small, the manufacturer's profit decreased with the increase of consumers' discount perception. Instead, when consumers' discount perception was relatively large, the manufacturer's profit gradually increased, presenting a process of first decreasing and then increasing. This may be because relatively hours in remanufacturing consumer preferences, consumer subsidies policy of the government, won the consumers a great deal of attention, but as the consumer, to the promotion of remanufacturing product awareness attention drops, also reflected in demand as you can see new products increased first and then manufactured goods to reduce, so cause manufacturers profit present from high to low down the process. When consumers have a relatively large preference for remanufactured products, they generally accept remanufactured products. We know the number of new products is gradually decreased and tends to zero, while the number of remanufactured products gradually increases, so does the manufacturers' profits rise accordingly. Meanwhile, when the government subsidizes consumers, manufacturers' profits increase the most. This is because consumers' acceptance of the remanufactured product gradually increases when consumers' discount perception increases and the remanufactured product has a stronger competitive advantage in market competition. And because consumers are closer to the market, manufacturers make the most money when the government subsidizes them.

Figure 2 (b) shows that there is a necessary connection between remanufacturing cost savings and manufacturer's profit. Among of them, the manufacturer's profit value and growth rate of scenario C are the largest, followed by scenario M, while cost saving has no significant impact on scenario R and scenario N. In conclusion, if manufacturers want to gain more profits, they should constantly improve the remanufacturing technology level and result in significant cost savings.

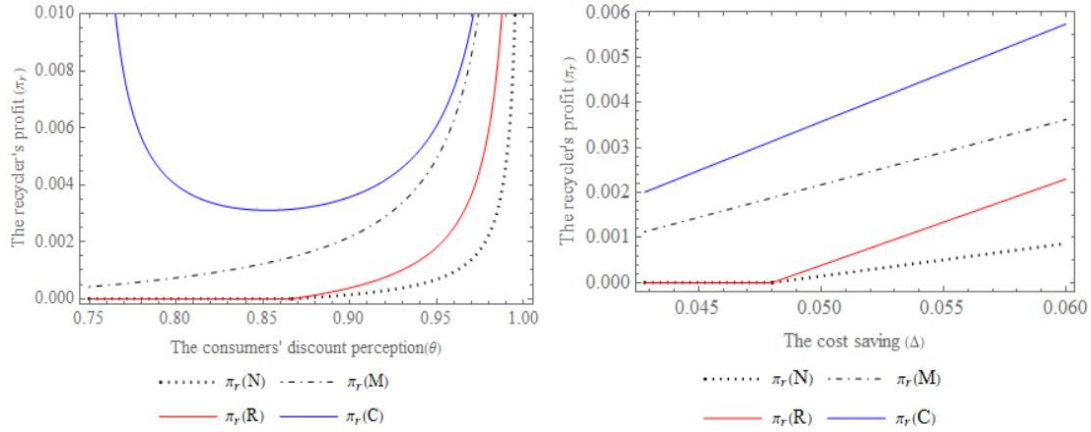


(a) The impact of θ on manufacturer's profit (b) The impact of Δ on manufacturer's profit

FIGURE 2. The impact of different parameters on manufacturer's profit

6.2.2. The impact of different parameters on recycler's profit

It can be seen from Figure 3(a) that Consumers' discount perception has a similar effect on profits of recyclers as shown in Figure 2(a). Moreover, when consumers' discount perception is larger than a certain value, profits of recyclers will be larger than that of non-government subsidies. From Figure 3(b), with the increase of cost saving, the profits of recyclers will increase regardless of the subsidy model. Moreover, when cost saving is larger than a certain value, the profits of recyclers will be larger than that in the case of no subsidy. This shows that the profits of recyclers benefit from government subsidies. The above analysis shows that to gain more profits, recyclers should strengthen advocacy to improve consumers' awareness of the remanufactured product.



(a) The impact of θ on recycler's profit (b) The impact of Δ on recycler's profit

FIGURE 3. The impact of different parameters on recycler's profit

6.3. Government's Perspective

6.3.1. The comparison of optimal government subsidy levels under different scenarios

In this section, we compared the sales prices of remanufactured products among different subsidy scenarios and designed a variety of scenarios, where $v = 0.014$ represents a relatively low recycling price situation, and $v = 0.02$ represents a relatively high recycling

price situation; $\Delta = 0.05$ represents a relatively low cost savings situation and $\Delta = 0.055$ represents a relatively high cost savings situation. For parameter c_1 , we set three different values to represent low, medium, and high levels, respectively. As shown in Table 6.

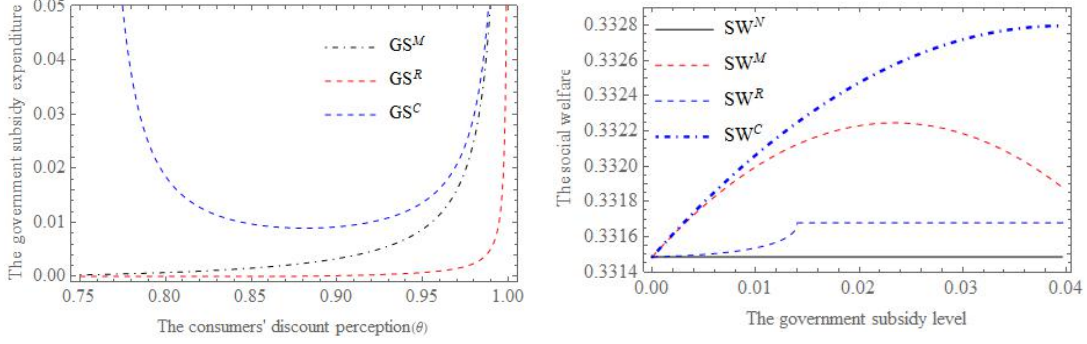
According to Table 6, the recycling prices, cost savings and emergency procurement costs plays an important role on the government's optimal subsidy level. Consistent with the results of Theorem 1, the optimal government subsidy level is divided into three situations, namely $k < m < s$, $m < k < s$, and $m < s < k$.

TABLE 6. The optimal government subsidies under different parameters

Parameters				The amount of subsidy			Size relationship
v	Δ	c_1	θ	m	k	s	
0.014	0.05	0.015	0.9	0.045	0.014	0.0757	
		0.03	0.9	0.028	0.014	0.0474	
		0.035	0.9	0.0234	0.014	0.0397	
	0.055	0.015	0.8	0.044	0.014	0.2221	
		0.03	0.8	0.027	0.014	0.1372	$k < m < s$
		0.035	0.8	0.0224	0.014	0.1139	
		0.015	0.9	0.05	0.014	0.0857	
		0.03	0.9	0.033	0.014	0.0574	
		0.035	0.9	0.0284	0.014	0.0496	
0.02	0.05	0.03	0.8	0.0107	0.02	0.0456	$m < k < s$
		0.03	0.9	0.0167	0.02	0.0285	
		0.035	0.8	0.0052	0.02	0.0178	$m < s < k$
	0.055	0.035	0.9	0.0112	0.02	0.0193	
		0.03	0.8	0.0157	0.02	0.0806	$m < k < s$
		0.03	0.9	0.0217	0.02	0.0385	$k < m < s$
		0.035	0.8	0.0101	0.02	0.0528	$m < k < s$
		0.035	0.9	0.0162	0.02	0.0293	

6.3.2. The total government expenditure and social welfare under different scenarios

As shown in Figure 4., the government subsidy have improved social welfare to a certain extent, but higher subsidies do not always bring higher social welfare. Specifically, in scenario M, the social welfare shows a trend of firstly rises and then declines. When the government's optimal subsidy reaches a certain level, it will bring huge financial pressure to the government and offset the positive benefits brought by the manufacturer's production of remanufactured products; In scenario C, the government expenditure and social welfare are always higher; However, in scenario R, although the social welfare is always the lowest compared with other subsidy scenarios (scenario M, and C), but scenario R collects more EOL products with lower government expenditures under certain conditions.



(a) The total government expenditure

(b) The social welfare

FIGURE 4. The total government expenditure and social welfare in Scenarios N, M, R, and C

6.4. Extension

In the previous analysis, we assume there is no consumer's risk aversion. However, consumers may have risk aversion, which is also consistent with existing literature such as Deng [8]. Therefore, we extend the model to include consumer's risk aversion and discuss how the degree of consumer's risk aversion affects the optimal decisions of manufacturer and recycler. Beside, we use the superscript NN, MM, RR and CC to represent the four possible subsidy plans (corresponding to Model NN, Model MM, Model RR and Model CC).

6.4.1. Consumers' purchase decisions

Based on the Section 3, we consider consumer's risk aversion in this section, we can derive the linear inverse demand functions in Model NN, Model MM, Model RR and Model CC. Specifically, we assume the consumer's risk aversion degree is r , and follow uniform distribution between 0 and 1. Here, a consumer's willingness to pay for the remanufactured product is adjusted from θe to $\theta e - r$, and the willingness to pay for the new product is adjusted from e to $e - hr$, where h ($h < 1$) is the ratio of consumer's risk aversion to the two types of products. Thus, for a consumer, the utility obtained by purchasing a new product and a remanufactured product is $U_{nn}^{NN} = e - hr - p_{nn}$, $U_{rr}^{NN} = \theta e - r - p_{rr}$ respectively. When the consumer's risk aversion is r_0 , consumers will neither purchasing a new product nor a remanufactured product, where $r_0 = [p_{nn} - p_{rr} - e(1 - \theta)] / (1 - h)$. When the consumer's risk aversion is $r_0 < r < (e - p_{nn}) / h$, the consumers tend to purchase a new product, while when $0 < r < r_0$, the consumers tend to purchase a remanufactured product. Hence, we can derive the linear inverse demand functions as follows: $P_{nn}^{NN} = e - h(Q_{nn} + Q_{rr})$, $P_{rr}^{NN} = \theta e - hQ_{nn} - Q_{rr}$. Note, the linear inverse demand functions for models NN, MM and RR are the same. Similarly, in model CC, if the government subsidizes consumers who purchase a remanufactured product (s per unit), then the linear inverse demand functions as follows: $P_{nn}^{CC} = e - h(Q_{nn} + Q_{rr})$, $P_{rr}^{CC} = \theta e - hQ_{nn} - Q_{rr} + s$.

6.4.2. The model results

Substituting P_{nn}^{NN} and P_{rr}^{NN} into (4.1, 4.2), Eq. (4.3, 4.4) and Eq. (4.5, 4.6), respectively. Similarly, substituting P_{nn}^{CC} and P_{rr}^{CC} into (4.7) and (4.8), respectively. We can get the manufacturer's and recycler's profit functions under different subsidy schemes. Solving the four possible subsidy models can get the equilibrium solution of the supply chain members,

which are summarized in Table 7 in the Appendix.

TABLE 7. The optimal solutions under different modes

	Scenario NN	Scenario MM	Scenario RR	Scenario CC
p_{nn}^{i*}	$\frac{1}{2}(e + c_n)$	$\frac{1}{2}(e + c_n)$	$\frac{1}{2}(e + c_n)$	$\frac{1}{2}(e + c_n)$
p_{rr}^{i*}	$\frac{1}{2}(w_r + e\theta + c_r)$	$\frac{1}{2}(w_r + e\theta + c_r - m)$	$\frac{1}{2}(w_r + e\theta + c_r)$	$\frac{1}{2}(w_r + e\theta + c_r + s)$
Q_{nn}^{i*}	$\frac{e(1-h\theta) + hw_r - c_n + hc_r}{2h(1-h)}$	$\frac{e(1-h\theta) + hw_r - hm - c_n + hc_r}{2h(1-h)}$	$\frac{e(1-h\theta) + hw_r - c_n + hc_r}{2h(1-h)}$	$\frac{e(1-h\theta) + hw_r - hs - c_n + hc_r}{2h(1-h)}$
Q_{rr}^{i*}	$\frac{\Delta - e(1-\theta) - w_r}{2(1-h)}$	$\frac{\Delta - e(1-\theta) - w_r + m}{2(1-h)}$	$\frac{\Delta - e(1-\theta) - w_r}{2(1-h)}$	$\frac{\Delta - e(1-\theta) - w_r + s}{2(1-h)}$
R^{i*}	$\frac{\Delta - e(1-\theta) - w_r}{2(1-h)} \sqrt{\frac{c_1}{2v}}$	$\frac{\Delta - e(1-\theta) - w_r + m}{2(1-h)} \sqrt{\frac{c_1}{2v}}$	$\frac{\Delta - e(1-\theta) - w_r}{2(1-h)} \sqrt{\frac{c_1}{2(v-k)}}$	$\frac{\Delta - e(1-\theta) - w_r + s}{2(1-h)} \sqrt{\frac{c_1}{2v}}$
π_m^{i*}	$\frac{(e - c_n)^2 + h(e\theta - w_r - c_r)[\Delta - w_r + c_n - e(2 - \theta)]}{4(1-h)h}$	$\frac{(e - c_n)^2 + h(e\theta - w_r - c_r + m)[\Delta - w_r + c_n + m - e(2 - \theta)]}{4(1-h)h}$	$\frac{(e - c_n)^2 + h(e\theta - w_r - c_r)[\Delta - w_r + c_n - e(2 - \theta)]}{4(1-h)h}$	$\frac{(e - c_n)^2 + h(e\theta - w_r - c_r + s)[\Delta - w_r + c_n + s - e(2 - \theta)]}{4(1-h)h}$
π_r^{i*}	$\frac{(w_r - \sqrt{2vc_1})[\Delta - w_r - e(1 - \theta)]}{2(1-h)}$	$\frac{(w_r - \sqrt{2vc_1})[\Delta - w_r + m - e(1 - \theta)]}{2(1-h)}$	$\frac{(w_r - \sqrt{2(v-k)c_1})[\Delta - w_r - e(1 - \theta)]}{2(1-h)}$	$\frac{(w_r - \sqrt{2vc_1})[\Delta - w_r + s - e(1 - \theta)]}{2(1-h)}$

It can be seen from Table 7 that consumers' risk aversion has no influence on the sales price of new products. For the price of remanufactured products, the relationship is as follows: $p_{rr}(MM) < p_{rr}(NN) = p_{rr}(RR) < p_{rr}(CC)$, which is consistent with our previous research (Theorem 3). The scenario MM and CC increased the sales of remanufactured products, while scenario RR has no effect on the sales of remanufactured products. Moreover, when the government subsidy k is trending to v , the scenario RR may be the government's best choice if the government is more concerns about the recycling of EOL products.

7. CONCLUSIONS

In this paper, CLSC with the guidance of government is considered by using game theory, in which the manufacturer can produce new products or products manufactured products through waste products recovered by an independent recycler. The government needs to decide whom the subsidies should be provided to maximize social welfare. We focus on four possible models, i.e., scenario N, scenario M, scenario R, and scenario C, and analyze the optimal recycling decision of recyclers and the optimal production decision of manufacturers under different government subsidy policies. First, the demand function comes from the utility function. After obtaining the optimal response from the recycler, we discuss the optimal subsidy level of the government under different scenarios, which provided help for the government and enterprises to make decisions. Moreover, we extend our main model and confirm the robustness of our findings.

The main conclusions are as the follows: First, we obtain the government's optimal subsidy decisions with different subsidy plans and find that consumer preferences and recycling prices have different effects on government subsidy levels. Second, higher government subsidy tends to push up the price of the remanufactured product, but the price of the new product remains unchanged. Moreover, in scenarios N, M, and R, consumers' discount perception always stimulates manufacturers to sell remanufactured products,

which has a negative impact on the sales of new products. In scenario C, the sales quantity of the remanufactured product decreased first and then increased, while the sales quantity of the new product is opposite. Third, our research results show that compared with other subsidy scenarios, the scenario C is the optimal choice for the government because it can always bring higher corporate profits and social welfare. However, the scenario C will increase government expenditures and bring fiscal pressure to the government. This is similar to the results of Nielsen *et al.* [39], but they focus on green technologies and do not consider risk factors. For scenario M, government subsidies reduce the cost of remanufactured products, similar to the results of Mitra and Webster [34], both increase the sales of remanufactured products, but the effect is inferior to the scenario C. For Scenario R, the government subsidies for recyclers mobilize the enthusiasm of recyclers to collect EOL products and thus improve their own profits, but the profits of manufacturers will remain unchanged. Therefore, the scenario R may be resisted by manufacturers. Different from the research results of Wang *et al.* [51], the government subsidy should not be set too high. Fourth, our research results show that, compared with other subsidy plans, the consumer subsidy plan is the optimal choice for the government, and it always bring higher enterprise profits and social welfare. Furthermore, we also gave some policy recommendations, if the government wants to increase the output of remanufactured products or care about social welfare, the government can implement scenario C, but the government expenditure will increase substantially. If government funds are limited, the scenario M can be considered, it should be noted that the subsidy scenario may cause social welfare losses due to higher subsidies, so setting a reasonable subsidy level is particularly important. If the purpose of the government is to effectively collect EOL products, then scenario R can bring excellent results, but social welfare is significantly lower than the other two subsidy scenarios (scenario M, and C).

In addition, the following managerial insights can be drawn:

(1) Faced with different groups, the government should comprehensively consider the impact of each group's behaviour on decision-making and implement different subsidy policies to improve the operation efficiency of enterprises and social welfare according to the needs of economic and social progress.

(2) In terms of profit maximization, government subsidies will always increase profits for enterprises, but government subsidies may not necessarily increase the profits of manufacturers and recyclers at the same time, such as scenario R. Therefore, the government should choose the appropriate subsidy plan.

(3) As far as the government is concerned, the government must first define its goals, which is to care about social welfare, the environment, or other things; Second, consider the fiscal pressures on governments; Finally, the government should also promote environmental protection through various means (TV, radio, etc.) to enhance consumers' awareness of environmental protection.

Our research still has some limitations and can be extended in multiple directions in future work. (1) We studied the CLSC formed by a single manufacturer and recycler, and did not consider the participation of transporters, such as the impact of government subsidies on transporters. Undoubtedly, considering transporters can make our model more interesting and reasonable, which is worthwhile of independent research in the future. (2) There may be one-to-many, many-to-one, or many-to-many situations between the manufacturer and the recycler. How to integrate multiple different recyclers into the model

will be our next research work. (3) We can study the competition between manufacturers and recyclers, and the difference in the quality of recycled EOL product.

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APPENDIX

Proofs for Proposition 1

In this case, the decision of the CLSC can be regarded as a two-stage Stackelberg game problem, in which the manufacturer and the recycler are the leader and follower respectively. According to the idea of backward induction, the expected profit of the recycler is a concave function of the recycling quantity R , and there is a unique optimal R to maximize the profit of the recycler. Where, R should satisfy:

$$\frac{d\pi_r^N}{dR} = -v - \left(\int_0^{\frac{Q_r}{R}} -\lambda f[\lambda] d\lambda \right) c_1$$

$$\frac{d^2\pi_r^N}{dR^2} = \frac{-F\left[\frac{Q_r}{R}\right]c_1 Q_r^2}{R^3} < 0$$

Therefore, according to the hypothesis and when $\frac{d\pi_r^N}{dR} = 0$, the optimal recycling amount R can be obtained to maximize the profits of the recycler

$$R^* = \frac{w_r - \theta c_n + c_r}{2\theta(-1 + \theta)} \sqrt{\frac{c_1}{2v}}$$

The Hessian matrix of the manufacturer's profit function regarding the number of sales of the new product and the remanufactured product is

$$H_m^N(Q_n^N, Q_r^N) = \begin{bmatrix} \frac{\partial^2 \pi_m^N}{\partial Q_n^2} & \frac{\partial^2 \pi_m^N}{\partial Q_n \partial Q_r} \\ \frac{\partial^2 \pi_m^N}{\partial Q_r \partial Q_n} & \frac{\partial^2 \pi_m^N}{\partial Q_r^2} \end{bmatrix} = \begin{bmatrix} -2 & -2\theta \\ -2\theta & -2\theta \end{bmatrix}$$

According to the hypothesis $0 < \theta < 1$, it is known $\partial^2 \pi_m / \partial Q_n^2 = -2 < 0$ and $|H_m^N| = 4\theta(1 - \theta) > 0$. Therefore, the manufacturer's profit function is a joint concave function about the sales volume of new products and remanufactured products, so there is an optimal solution.

Set the first order partial derivative $\partial \pi_m^N / \partial Q_n^N = 0$, $\partial \pi_m^N / \partial Q_r^N = 0$, The optimal response of

retailers can be obtained as follows $Q_r^N = \frac{w_r - \theta c_n + c_r}{2(-1 + \theta)\theta}$, $Q_n^N = \frac{1 + w_r - \theta - c_n + c_r}{2 - 2\theta}$. By substituting it into the

inverse demand function, the sales price of the new product and the remanufactured product can be obtained as $p_r^N = \frac{w_r + \theta + c_r}{2}$, $p_n^N = \frac{1 + c_n}{2}$.

The proof of Propositions 2 , 3 and 4 are similar to proposition 1.

Proof of Proposition 5

According to assumption, we can get the social welfare function. Therefore, the social welfare function of scenario M can be expressed as

$$\begin{aligned} SW^M &= \pi_m^M + \pi_r^M + CS^M - GS^M \\ &= \frac{1}{8(-1 + \theta)\theta} [(m - w_r)^2 + 3(-1 + \theta)\theta - 3\theta c_n^2 + 4c_2(m - w_r + \theta c_n - c_r) \\ &\quad + (2m - 2w_r - 3c_r)c_r + 2\theta c_n(3 - m + w_r - 3\theta + 3c_r)] \end{aligned}$$

We first check the condition of optimal solution existing. By taking the second-order derivative of SW^M with respect to resulting in $\frac{d^2SW^M}{dm^2} = \frac{1}{4(-1+\theta)\theta} < 0$, which means that the model in M Scenario exists unique optimal solution: $m^* = w - \frac{2c_1}{\sqrt{\frac{c_1}{2v}}} + \theta c_n - c_r$.

Likewise, we can get the social welfare function of the M Scenario:

$$\begin{aligned} SW^R &= \pi_m^R + \pi_r^R + CS^R - GS^R \\ &= \frac{1}{8(-1 + \theta)\theta} [w_r^2 - \theta + 2(-1 + \theta)\theta + \theta^2 - 3\theta c_n^2 - 2w_r c_r - 3c_r^2 + 4\theta c_n(1 \\ &\quad - \theta + c_r) + 2\theta c_n(1 + w_r - \theta + c_r) - 4k(w_r - \theta c_n + c_r) - \frac{4c_1(w_r - \theta c_n + c_r)}{\sqrt{\frac{c_1}{2v - 2k}}} \\ &\quad - \frac{4c_2\left(-1 + \sqrt{\frac{c_1}{2v - 2k}}\right)(w_r - \theta c_n + c_r)}{\sqrt{\frac{c_1}{2v - 2k}}}] \end{aligned}$$

Taking first-order derivatives of SW^R with respect to k yields the following results:

$$\frac{\partial SW^R}{\partial k} = \frac{(-1 + \sqrt{\frac{c_1}{2v - 2k}})(w_r - \theta c_n + c_r)}{2(-1 + \theta)\theta}$$

While $g(k) = \frac{(-1 + \sqrt{\frac{c_1}{2v - 2k}})(w_r - \theta c_n + c_r)}{2(-1 + \theta)\theta}$, then $g(0) = \frac{(-1 + \sqrt{\frac{c_1}{2v}})(w_r - \theta c_n + c_r)}{2(-1 + \theta)\theta}$. While $v \leq \frac{1}{2}c_1$, get $g(0) \geq 0$. While $g(v) = \frac{(-1 + \sqrt{\frac{c_1}{2v - 2k}})(w_r - \theta c_n + c_r)}{2(-1 + \theta)\theta}$. When v is infinitely close to k , get $g(v) > 0$. So $g(k)$ is an increasing function, and the model in R scenario exists unique optimal solution: $k = v$.

Similarly, while $g(0) < 0, g(v) < 0$, then $k = 0$, $g(k)$ is a descending function. While $g(0) < 0, g(v) > 0$, then $g(k)$ decreased and then increased.

Substitute $k = v$ and $k = 0$ into the SW^K function, and subtract them to get $\frac{1}{2}v(-1 + \sqrt{\frac{2c_1}{v}}) > 0$. So the optimal value of k is $k^* = v$.

The proofs of government subsidies in C Scenario is similar to that in M Scenario. By taking the second-order derivative of SW^C with respect to resulting in $\frac{d^2SW^C}{ds^2} = \frac{3-4\theta}{4\theta-4\theta^2}$. Therefore, we give the existence condition of the optimal solution $\frac{3}{4} < \theta < 1$. The expressions of social welfare in C Scenario can be given by.

$$\begin{aligned} SW^C &= \pi_m^C + \pi_r^C + CS^C - GS^C \\ &= \frac{1}{8(-1+\theta)\theta} [-3s^2 + 2sw_r + w_r^2 - 3\theta + 4s^2\theta - 4sw_r\theta + 3\theta^2 - 3\theta c_n^2 \\ &\quad + 4\sqrt{2c_1v}(s - w_r + \theta c_n - c_r) - 2(w_r + s(-3 + 2\theta))c_r - 3c_r^2 \\ &\quad + 2\theta c_n(3 + w_r - 3\theta + s(-3 + 2\theta) + 3c_r)] \end{aligned}$$

The government subsidies to Recycler in C Scenario exists unique optimal solution:

$$s^* = \frac{-2c_1 + \sqrt{\frac{c_1}{2v}}(w_r(-1 + 2\theta) + (3 - 2\theta)\theta c_n + (-3 + 2\theta)c_r)}{(-3 + 4\theta)\sqrt{\frac{c_1}{2v}}}$$

The expressions of social welfare in N Scenario can be given by.

$$\begin{aligned} SW^N &= \pi_m^N + \pi_r^N + CS^N - GS^N \\ &= \frac{w_r^2 - 3\theta + 3\theta^2 - 3\theta c_n^2 - 2w_r c_r - 3c_r^2 - 4\sqrt{2c_1v}(w_r - \theta c_n + c_r) + 2\theta c_n(3 + w_r - 3\theta + 3c_r)}{8(-1+\theta)\theta} \end{aligned}$$

We first take the optimal responsive solutions of each model into (4.1)-(4.8) to obtain the following equations.

$$\begin{aligned} \pi_m^N &= \frac{w_r^2 + \theta - \theta^2 + \theta c_n^2 + 2w_r c_r + c_r^2 - 2\theta c_n(1 + w_r - \theta + c_r)}{4(-1+\theta)\theta} \\ \pi_m^M &= \frac{(m - w_r)^2 + \theta - \theta^2 + \theta c_n(2(-1 + m - w_r + \theta) + c_n) - 2(m - w_r + \theta c_n)c_r + c_r^2}{4(-1+\theta)\theta} \\ \pi_m^R &= \frac{w_r^2 + \theta - \theta^2 + \theta c_n^2 + 2w_r c_r + c_r^2 - 2\theta c_n(1 + w_r - \theta + c_r)}{4(-1+\theta)\theta} \\ \pi_m^C &= \frac{(s - w_r)^2 + \theta - \theta^2 + \theta c_n(2(-1 + s - w_r + \theta) + c_n) - 2(s - w_r + \theta c_n)c_r + c_r^2}{4(-1+\theta)\theta} \end{aligned}$$

$$\begin{aligned}
\pi_r^N &= \frac{(c_1 - w_r \sqrt{\frac{c_1}{2v}} + c_2(-1 + \sqrt{\frac{c_1}{2v}}))(w_r - \theta c_n + c_r)}{2(-1 + \theta)\theta \sqrt{\frac{c_1}{2v}}} \\
\pi_r^M &= \frac{(c_1 - w_r \sqrt{\frac{c_1}{2v}})(m - w_r + \theta c_n - c_r)}{2(-1 + \theta)\theta \sqrt{\frac{c_1}{2v}}} \\
\pi_r^R &= \frac{(w_r \sqrt{\frac{c_1}{2v - 2k}} - \sqrt{2c_1})(w_r - \theta c_n + c_r)}{2(-1 + \theta)\theta \sqrt{\frac{c_1}{2v - 2k}}} \\
\pi_r^C &= \frac{(c_1 - w_r \sqrt{\frac{c_1}{2v}} + c_2(-1 + \sqrt{\frac{c_1}{2v}}))(s - w_r + \theta c_n - c_r)}{2(-1 + \theta)\theta \sqrt{\frac{c_1}{2v}}} \\
CS^N &= \frac{w_r^2 + \theta - \theta^2 + \theta c_n^2 + 2w_r c_r + c_r^2 - 2\theta c_n(1 + w_r - \theta + c_r)}{8(-1 + \theta)\theta} \\
CS^M &= \frac{(m - w_r)^2 + \theta - \theta^2 + \theta c_n(2(-1 + m - w_r + \theta) + c_n) - 2(m - w_r + \theta c_n)c_r + c_r^2}{8(-1 + \theta)\theta} \\
CS^R &= \frac{w_r^2 + \theta - \theta^2 + \theta c_n^2 + 2w_r c_r + c_r^2 - 2\theta c_n(1 + w_r - \theta + c_r)}{8(-1 + \theta)\theta} \\
CS^C &= \frac{w_r^2 + s^2(5 - 4\theta) + \theta - \theta^2 + 2sw_r(-3 + 2\theta) + \theta c_n^2 - 2\theta c_n(1 - 3s + w_r - \theta + 2s\theta + c_r)}{8(-1 + \theta)\theta} \\
GS^N &= - \\
GS^M &= \frac{m(m - w_r + \theta c_n - c_r)}{2\theta - 2\theta^2} \\
GS^R &= \frac{k(w_r - \theta c_n + c_r)}{2\theta^2 - 2\theta} \\
GS^C &= \frac{s(s - w_r + \theta c_n - c_r)}{2\theta - 2\theta^2}
\end{aligned}$$

Proof of Theorem 1

According to conditional $w_r > c_1 > v$, $c_n > w_r + c_r$, $\theta - 1 < w_r - \theta c_n + c_r < 0$. We can get $\theta c_n - c_r > \sqrt{2vc_1}$, $w_r > \sqrt{2vc_1}$. $m - s = \frac{2(-1+\theta)(w-4\sqrt{2vc_1}+3\theta c_n-3c_r)}{-3+4\theta} < 0$, m is always less than s .

Therefore, government subsidies have three forms of size. $k < m < s$, $m < k < s$ and $m < s < k$.

Then, pairwise comparison can determine the boundary of government subsidies in three cases.

Proof of Theorem 2

It can be easily proved by calculating $p_n(N) = p_n(R) = p_n(M) = p_n(C)$.

Proof of Theorem 3

It can be easily proved by calculating $p_r(M) - p_r(N) < 0$, $p_r(C) - p_r(N) > 0$. $p_r(M) < p_r(N) = p_r(R) < p_r(C)$.

Proof of Theorem 4

It can be easily proved by calculating $Q_n(M) - Q_n(N) < 0$, $Q_n(C) - Q_n(N) < 0$. $Q_n(M) - Q_n(S) = \frac{w_r - 4\sqrt{2}v\sqrt{\frac{c_1}{v} + 3\theta c_n - 3c_r}}{-3 + 4\theta}$. Therefore, when $c_1 < \frac{(w_r + 3\theta c_n - 3c_r)^2}{32v}$, $Q_n(M) > Q_n(S)$. $Q_n(S) < Q_n(M) < Q_n(R) = Q_n(N)$; When $c_1 > \frac{(w_r + 3\theta c_n - 3c_r)^2}{32v}$, $Q_n(M) < Q_n(C)$. $Q_n(M) < Q_n(C) < Q_n(R) = Q_n(N)$.

Proof of Theorem 5

It can be easily proved by calculating $Q_r(M) - Q_r(N) > 0$, $Q_r(C) - Q_r(N) > 0$. $Q_r(M) - Q_r(C) = \frac{w_r - 4\sqrt{2}v\sqrt{\frac{c_1}{v} + 3\theta c_n - 3c_r}}{3\theta - 4\theta^2}$. Therefore, when $c_1 > \frac{(w_r + 3\theta c_n - 3c_r)^2}{32v}$, $Q_r(M) > Q_r(C)$. $Q_r(N) = Q_r(R) < Q_r(C) < Q_r(M)$. When $c_1 < \frac{(w_r + 3\theta c_n - 3c_r)^2}{32v}$, $Q_r(M) < Q_r(C)$. When $\min\{\frac{(w_r + 3\theta c_n - 3c_r)^2}{32v}, w_r\}$, $Q_r(N) = Q_r(R) < Q_r(M) < Q_r(C)$.

Proof of Theorem 6

$$\text{Similarly, } R(M) - R(N) > 0, R(C) - R(N) > 0. R(M) - R(C) = \frac{\sqrt{\frac{c_1}{v}}(-w_r + 4\sqrt{2}v\sqrt{\frac{c_1}{v} - 3\theta c_n + 3c_r})}{\sqrt{2}\theta(-3 + 4\theta)}.$$

Therefore, When $c_1 < \frac{(w_r + 3\theta c_n - 3c_r)^2}{32v}$, $R(C) < R(M)$. When $\min\{\frac{(w_r + 3\theta c_n - 3c_r)^2}{32v}, w_r\}$, $R(N) < R(C) < R(M) < R(R)$

Proof of Section 6.4

The utility obtained by purchasing a new product and a remanufactured product is: $U_{nn} = e - hr - p_{nn}$; $U_{rr} = \theta e - r - p_{rr}$. When $U_{nn} = U_{rr}$, we can get $r_0 = [p_{nn} - p_{rr} - e(1 - \theta)] / (1 - h)$. $U_{nn} > 0$, we can get $r < (e - p_{nn}) / h$. Hence, when the consumer's risk aversion is $r_0 < r < (e - p_{nn}) / h$, the consumers tend to purchase a new product, while when $0 < r < r_0$, the consumers tend to purchase a remanufactured product. The demand functions for new products and remanufactured products can be expressed as:

$$\begin{cases} Q_{nn} = \int_{\frac{p_{nn} - p_{rr} - e(1-\theta)}{1-h}}^{\frac{e - p_{nn}}{h}} 1 dr = \frac{e(1-h\theta) - p_{nn} + hp_{rr}}{h(1-h)} \\ Q_{rr} = \int_0^{\frac{p_{nn} - p_{rr} - e(1-\theta)}{1-h}} 1 dr = \frac{p_{nn} - p_{rr} - e(1-\theta)}{1-h} \end{cases}$$

Hence, we can derive the linear inverse demand functions as follows:

$$\begin{cases} p_{nn} = e - h(Q_{nn} + Q_{rr}) \\ p_{rr} = e\theta - hQ_{nn} - Q_{rr} \end{cases}$$

The proof in this section is similar to that in section 4 and is omitted.