A note on the paper "Necessary and sufficient optimality conditions using convexifactors for mathematical programs with equilibrium constraints"

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Abstract. In this work, some counterexamples are given to refute some results in the paper by Kohli (RAIRO-Oper. Res. 53, 1617-1632, 2019). We correct the fault in some of his results.

Keywords Convexifactor; Constraint qualifications; Mathematical programs with equilibrium constraints; Optimality conditions.
AMS Subject Classifications: 90C30; 90C46; 49J52

1 Introduction

Mathematical programs with equilibrium constraints have been investigated by many authors. In the paper [8], the author investigated the following mathematical programs with equilibrium constraints

$$(MPEC):\begin{cases}\text{Minimize } \ f(x)\\ \text{s.t.}\begin{cases} g(x)\leq 0,\ h(x)=0,\\ G(x)\geq 0,\ H(x)\geq 0,\ G(x)^{\top}H(x)=0,\end{cases}\end{cases}$$

where $f:\mathbb{R}^n\to\mathbb{R},\ g:\mathbb{R}^n\to\mathbb{R}^m,\ h:\mathbb{R}^n\to\mathbb{R}^p,\ G:\mathbb{R}^n\to\mathbb{R}^l$ and $H:\mathbb{R}^n\to\mathbb{R}^l$. Under a nonsmooth constraint qualification (∂^*-GCQ) given in terms of convexifactors, the author established first order necessary optimality condition for $(MPEC)$. The main theorem, where the author gave necessary optimality conditions, is Theorem 4.4 [8].

In this article, we show that necessary optimality conditions given by Kohli in [8] are not correct. In support of our remarks, some counterexamples are given (see Example 2 and Remark 4) and some reasoning mistakes in the proof of the main result ([8, Theorem 4.4]) are highlighted (see Remark 3, Remark 4 and Remark 7). Finally, we present corrected versions of his results. Theorem 10 is actually a corrected version of Theorem 4.4 in [8].

The rest of the paper is organized in this way: Section 2 contains basic definitions and preliminary material. Counterexamples and comments are given in Section 3. Section 4 addresses our main results (corrected optimality conditions). A conclusion is given in Section 5.

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2 Preliminaries

Throughout this section, let $\mathbb{R}^n$ be the usual n-dimensional Euclidean space. Given a nonempty subset S of $\mathbb{R}^n$, the closure, convex hull, and convex cone (including the origin) generated by S are denoted respectively by $cl\ S$, $conv\ S$ and $pos\ S$. The negative polar cone of S is defined by

$$S^{-} := \{v \in \mathbb{R}^n \mid \langle x, v\rangle \leq 0,\ \forall x \in S\}.$$

The contingent cone $T(S,x)$ to S at $x \in cl\ S$ is defined by

$$T(S,x) = \{v \in \mathbb{R}^n : \exists t_n \downarrow 0 \text{ and } \exists v_n \rightarrow v \text{ such that } x + t_n v_n \in S,\ \forall n \in \mathbb{N}\}.$$

Let $f : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{+\infty\}$ be a given function and let $x \in \mathbb{R}^n$ where $f(x)$ is finite. The expressions

$$f_d^{-}(x,v) = \liminf_{t\searrow 0}\ [f(x+tv) - f(x)]/t \text{ and } f_d^{+}(x,v) = \limsup_{t\searrow 0}\ [f(x+tv) - f(x)]/t$$

signify, respectively, the lower and upper Dini directional derivatives of f at x in the direction v.

Definition 1 *[2]The function $f : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{+\infty\}$ is said to have an upper convexifactor $\partial^u f(x)$ at x if $\partial^u f(x) \subseteq \mathbb{R}^n$ is closed and, for each $v \in \mathbb{R}^n$,*

$$f_d^{-}(x,v) \leq \sup_{x^* \in \partial^u f(x)} \langle x^*, v\rangle.$$

The function $f : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{+\infty\}$ is said to have an upper semiregular convexifactor $\partial^{us} f(x)$ at x if $\partial^{us} f(x)$ is an upper convexifactor at x and, for each $v \in \mathbb{R}^n$,

$$f_d^{+}(x,v) \leq \sup_{x^* \in \partial^{us} f(x)} \langle x^*, v\rangle.$$

3 Counterexamples and comments

The following example shows that [8, Theorem 4.4] is not correct.

Example 2 *Consider the optimization problem $(MPEC)$ where*

$$f(x_1,x_2,x_3) := x_1 + x_2 - 2x_3,\ g(x_1,x_2,x_3) := x_3,$$

$$h(x_1,x_2,x_3) := 0,\ G_1(x_1,x_2,x_3) := x_1,\ G_2(x_1,x_2,x_3) := x_2,\ H_1(x_1,x_2,x_3) := x_2 \text{ and } H_2(x_1,x_2,x_3) := x_1.$$

On the one hand, the origin is the unique minimizer of $(MPEC)$. On the other hand, it can be seen that $\partial^{us} f(\overline{x}) := \{(1,1,-2)\}$ is a bounded upper semiregular convexifactor of f at $\overline{x} := (0,0,0)$. Moreover,

$$\partial^u g(\overline{x}) := \{(0,0,1)\},\ \partial^u h(\overline{x}) := \{(0,0,0)\},$$

$$\partial^u G_1(\overline{x}) := \{(1,0,0)\},\ \partial^u(-G_1)(\overline{x}) := \{(-1,0,0)\},\ \partial^u G_2(\overline{x}) := \{(0,1,0)\},\ \partial^u(-G_2)(\overline{x}) := \{(0,-1,0)\},$$

$$\partial^u H_1(\overline{x}) := \{(0,1,0)\},\ \partial^u(-H_1)(\overline{x}) := \{(0,-1,0)\},\ \partial^u H_2(\overline{x}) := \{(1,0,0)\} \text{ and } \partial^u(-H_2)(\overline{x}) := \{(-1,0,0)\}$$

are upper convexifactors of $g,\ h,\ G_1,\ -G_1,\ G_2,\ -G_2,\ H_1,\ -H_1,\ H_2$ and $-H_2$ at $\overline{x}$ respectively. Remark that $B = \{1,2\}$.

- *The feasible set* K *of* $(MPEC)$ *is* $K = (\{0\} \times \mathbb{R}^+ \times \mathbb{R}^-) \cup (\mathbb{R}^+ \times \{0\} \times \mathbb{R}^-)$. *Consequently,*

$$T(K, \overline{x}) = \left(\{0\} \times \mathbb{R}^+ \times \mathbb{R}^-\right) \cup \left(\mathbb{R}^+ \times \{0\} \times \mathbb{R}^-\right) \text{ and } clconv\ T(K, \overline{x}) = \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^-.$$

- $\partial^* - GCQ(B_1, B_2)$ *holds for all* $(B_1, B_2) \in P(B)$ *at* $\overline{x}$.
  - *If* $B_1 = \{1\}$ *and* $B_2 = \{2\}$, *then* $\partial^* - GCQ(B_1, B_2)$ *holds at* $\overline{x}$. *Indeed,*

$$\begin{pmatrix} (conv\ \partial^u g(\overline{x})) \bigcup (conv\ \partial^u h(\overline{x})) \bigcup (conv\ \partial^u G_2(\overline{x}) \cup conv\ \partial^u(-G_2)(\overline{x})) \\ \bigcup (conv\ \partial^u H_1(\overline{x}) \cup conv\ \partial^u(-H_1)(\overline{x})) \bigcup conv\ \partial^u(-G_1)(\overline{x}) \bigcup (conv\ \partial^u(-H_2)(\overline{x})) \end{pmatrix}^- = \mathbb{R}^+ \times \{0\} \times \mathbb{R}^-.$$

  - *If* $B_1 = \{2\}$ *and* $B_2 = \{1\}$, *then* $\partial^* - GCQ(B_1, B_2)$ *holds at* $\overline{x}$. *Indeed,*

$$\begin{pmatrix} (conv\ \partial^u g(\overline{x})) \bigcup (conv\ \partial^u h(\overline{x})) \bigcup (conv\ \partial^u G_1(\overline{x}) \cup conv\ \partial^u(-G_1)(\overline{x})) \\ \bigcup (conv\ \partial^u H_2(\overline{x}) \cup conv\ \partial^u(-H_2)(\overline{x})) \bigcup conv\ \partial^u(-G_2)(\overline{x}) \bigcup (conv\ \partial^u(-H_1)(\overline{x})) \end{pmatrix}^- = \{0\} \times \mathbb{R}^+ \times \mathbb{R}^-.$$

  - *If* $B_1 = \emptyset$ *and* $B_2 = \{1, 2\}$, *then* $\partial^* - GCQ(B_1, B_2)$ *holds at* $\overline{x}$. *Indeed,*

$$\begin{pmatrix} (conv\ \partial^u g(\overline{x})) \bigcup (conv\ \partial^u h(\overline{x})) \bigcup (conv\ \partial^u G_1(\overline{x}) \cup conv\ \partial^u(-G_1)(\overline{x})) \\ \bigcup (conv\ \partial^u G_2(\overline{x}) \cup conv\ \partial^u(-G_2)(\overline{x})) \bigcup conv\ \partial^u(-H_1)(\overline{x}) \bigcup (conv\ \partial^u(-H_2)(\overline{x})) \end{pmatrix}^- = \{0\} \times \{0\} \times \mathbb{R}^-.$$

  - *If* $B_1 = \{1, 2\}$ *and* $B_2 = \emptyset$, *then* $\partial^* - GCQ(B_1, B_2)$ *holds at* $\overline{x}$. *Indeed,*

$$\begin{pmatrix} (conv\ \partial^u g(\overline{x})) \bigcup (conv\ \partial^u h(\overline{x})) \bigcup (conv\ \partial^u H_1(\overline{x}) \cup conv\ \partial^u(-H_1)(\overline{x})) \\ \bigcup (conv\ \partial^u H_2(\overline{x}) \cup conv\ \partial^u(-H_2)(\overline{x})) \bigcup conv\ \partial^u(-G_1)(\overline{x}) \bigcup (conv\ \partial^u(-G_2)(\overline{x})) \end{pmatrix}^- = \{0\} \times \{0\} \times \mathbb{R}^-.$$

- $\partial^* - GCQ$ *holds at* $\overline{x}$. *Indeed,*

$$\begin{pmatrix} (conv\ \partial^u g(\overline{x})) \bigcup (conv\ \partial^u h(\overline{x})) \\ \bigcup (conv\ \partial^u(-G_1)(\overline{x}) \cup conv\ \partial^u(-G_2)(\overline{x})) \\ \bigcup (conv\ \partial^u(-H_1)(\overline{x}) \cup conv\ \partial^u(-H_2)(\overline{x})) \end{pmatrix}^- = \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^- \subseteq clconv\ T(K, \overline{x}).$$

- *Observe that all hypotheses of Theorem 4.4 in [8] are satisfied, but* $\overline{x}$ *is not a* ∂^**-strong stationary point as defined by Kohli [8, Definition 4.1]. Indeed, if there exists a vector* $0 \neq \left(\lambda^g, \lambda^h, \lambda^G, \lambda^H, \mu^G, \mu^H\right) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$ *such that*

$$\lambda^g,\ \lambda^h,\ \lambda_1^G,\ \lambda_2^G,\ \lambda_1^H,\ \lambda_2^H,\ \mu_1^G,\ \mu_2^G,\ \mu_1^H,\ \mu_2^H \geq 0, \tag{1}$$

$$\lambda^g + \lambda^h + \lambda_1^G + \lambda_2^G + \lambda_1^H + \lambda_2^H + \mu_1^G + \mu_2^G + \mu_1^H + \mu_2^H = 1 \tag{2}$$

and

$$0 \in cl \begin{bmatrix} conv\ \partial^{us} f(\overline{x}) + \lambda^g\ conv\ \partial^u g(\overline{x}) + \lambda^h\ conv\ \partial^u h(\overline{x}) + \lambda_1^G\ conv\ \partial^u(-G_1)(\overline{x}) \\ +\lambda_2^G\ conv\ \partial^u(-G_2)(\overline{x}) + \lambda_1^H\ conv\ \partial^u(-H_1)(\overline{x}) + \lambda_2^H\ conv\ \partial^u(-H_2)(\overline{x}) \\ +\mu_1^G\ conv\ \partial^u G_1(\overline{x}) + \mu_1^H\ conv\ \partial^u H_1(\overline{x}) + \mu_2^G\ conv\ \partial^u G_2(\overline{x}) + \mu_2^H\ conv\ \partial^u H_2(\overline{x}) \end{bmatrix}$$

we get

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in cl \left\{ \begin{pmatrix} 1 - \lambda_1^G - \lambda_2^H + \mu_1^G + \mu_2^H \\ 1 - \lambda_2^G - \lambda_1^H + \mu_1^H + \mu_2^G \\ -2 + \lambda^g \end{pmatrix} \right\}.$$

Then,

$$\begin{cases} 0 = 1 - \lambda_1^G - \lambda_2^H + \mu_1^G + \mu_2^H, \\ 0 = 1 - \lambda_2^G - \lambda_1^H + \mu_1^H + \mu_2^G, \\ \quad 0 = -2 + \lambda^g. \end{cases}$$

We have $\lambda^g = 2$ *while* $\lambda^g \leq 1$ *due to* (1) *and* (2). *A contradiction.*

Remark 3 *Contrary to what is stated on page 1625 (line -1), it is impossible to deduce*

$$\lim_{k\to\infty} \left[\sum_{i\in I(\overline{x})} \lambda_{ik}^g + \sum_{i=1}^{p} \lambda_{ik}^h + \sum_{i\in A\cup B_1\cup B_2} \lambda_{ik}^G + \sum_{i\in D\cup B_1\cup B_2} \lambda_{ik}^H + \sum_{i\in A\cup B_2} \mu_{ik}^G + \sum_{i\in D\cup B_1} \mu_{ik}^H \right] := 1. \tag{3}$$

The author did not pay attention to the cone that precedes the convex hull in the previous formula (see line -6 on page 1625). This error has seriously impacted the remaining of the proof of [8, Theorem 4.4]. Since (3) *is an essential part of the definition of the* ∂^**- strong stationarity property, [8, Theorem 4.4] is also not correct. Notice that the boundedness of the sequence of the multipliers is neither acquired nor insured.*

Remark 4 *The main result [8, Theorem 4.4], is based on Lemma 2.3 [8]. However, this latter ([8, Lemma 2.3]) is clearly incorrect, as setting*

$$A := \left\{(x, y) \in \mathbb{R}^2 : x < 0,\ y < 0\right\} \cup \{(0, 0)\} \ \text{ and } B := \{(1, 0)\}$$

yields a simple counterexample. Unfortunately, this error impacted [8, Theorem 4.4] and forced the author to add useless and cumbersome closures and convex hulls.

The following result is a corrected version of Lemma 2.3 [8]. Being standard, the proof has been omitted.

Lemma 5 *Let* $\mathcal{B}$ *a nonempty, convex and compact set and* $\mathcal{A}$ *be a convex cone. If*

$$\sup_{v\in\mathcal{B}} \langle v, d\rangle \geq 0, \quad \text{for all } d \in \mathcal{A}^-$$

then $0 \in \mathcal{B} + cl\mathcal{A}$.

4 Optimality conditions

In the following definition, we recall the generalized alternatively stationarity concept given by Ardali et al. [1, Definition 4.3].

Definition 6 *[1] A feasible point* $\overline{x}$ *of MPEC is said to be a generalized alternatively stationary point if there exists a vector* $\left(\lambda^g, \lambda^h, \mu^h, \lambda^G, \lambda^H, \mu^G, \mu^H\right) \in \mathbb{R}^m \times \mathbb{R}^{2p} \times \mathbb{R}^{2l} \times \mathbb{R}^{2l}$ *such that*

$$0 \in \begin{bmatrix} conv\ \partial^{us} f(\overline{x}) + \sum_{i=1}^{m} \lambda_i^g\ conv\ \partial^u g_i(\overline{x}) + \sum_{i\in I'} \mu_i^h\ conv\ \partial^u h_i(\overline{x}) + \sum_{i\in I'} \lambda_i^h\ conv\ \partial^u(-h_i)(\overline{x}) \\ + \sum_{i=1}^{l} \lambda_i^G\ conv\ \partial^u(-G_i)(\overline{x}) + \sum_{i=1}^{l} \lambda_i^H\ conv\ \partial^u(-H_i)(\overline{x}) + \sum_{i=1}^{l} \mu_i^G\ conv\ \partial^u G_i(\overline{x}) + \sum_{i=1}^{l} \mu_i^H\ conv\ \partial^u H_i(\overline{x}) \end{bmatrix} \tag{4}$$

with

$$\lambda_i^g\ g_i(\overline{x}) = 0,\ \forall i \in I \tag{5}$$

and

$$\begin{cases} \mu_i^G = 0 \ \text{or}\ \mu_i^H = 0, \ \forall i \in B, \\ \lambda_i^G = 0, \ \mu_i^G = 0, \ \forall i \in D, \\ \lambda_i^H = 0, \ \mu_i^H = 0, \ \forall i \in A, \\ \lambda_i^G, \ \lambda_i^H, \ \mu_i^G, \ \mu_i^H \geq 0, \ \forall i \in \{1, \ ..., \ l\}, \\ \lambda_i^g \geq 0, \ \forall i \in I = \{1, \ ..., \ m\}, \ \text{and}\ \lambda_i^h \geq 0, \ \mu_i^h \geq 0, \ \forall i \in I' = \{1, \ ..., \ p\}. \end{cases} \tag{6}$$

Here,

$$A := \{i \in \{1, \ ..., \ l\} : G_i(\overline{x}) = 0, \ H_i(\overline{x}) > 0\},$$
$$B := \{i \in \{1, \ ..., \ l\} : G_i(\overline{x}) = 0, \ H_i(\overline{x}) = 0\},$$
$$D := \{i \in \{1, \ ..., \ l\} : G_i(\overline{x}) > 0, \ H_i(\overline{x}) = 0\}.$$

Remark 7 *Contrary to Definition 10 [8],*

$$\sum_{i=1}^{m} \lambda_i^g + \sum_{i=1}^{p} \lambda_i^h + \sum_{i=1}^{l} \lambda_i^G + \sum_{i=1}^{l} \lambda_i^H + \sum_{i=1}^{l} \mu_i^G + \sum_{i=1}^{l} \mu_i^H = 1$$

is not an integral part of Definition 6. It is this equality that distorted Kohli's result. Notice that [8, Remark 4.2] is not correct since condition for the sum of multipliers does not exist in [4, 6, 10].

Remark 8 *Notice that if all the functions are differentiable and the upper convexifactor is replaced by the upper regular convexifactor in the above stationary notion, then this notion reduces to the A-stationary condition given by Flegel and Kanzow in [6] and by Flegel in [3].*

We shall need the following nonsmooth constraint qualification.

Definition 9 *Let $\overline{x} \in K$ and (B_1, B_2) be a partition of $B \neq \emptyset$. Suppose that $g_i, \ i \in I, \ h_i, \ -h_i, \ i \in J, \ -G_i, \ G_i, \ i \in A \cup B, \ -H_i, \ H_i, \ i \in D \cup B$, admit upper convexifactors $\partial^u g_i(\overline{x}), \ i \in I, \ \partial^u h_i(\overline{x}), \ \partial^u(-h_i)(\overline{x}), \ i \in J, \ \partial^u(-G_i)(\overline{x}), \ \partial^u G_i(\overline{x}), \ i \in A \cup B, \ \partial^u(-H_i)(\overline{x}), \ \partial^u H_i(\overline{x}), \ i \in D \cup B$, respectively at $\overline{x}$. We say that $\partial^* - ACQ(B_1, B_2)$ holds at $\overline{x}$ if*

$$\mathcal{A}^- \subseteq \ \text{clconv}\,(T(K, \overline{x})),$$

where K is the feasible set of $(MPEC)$ and

$$\begin{aligned} \mathcal{A} := & \left(\bigcup_{i \in I(\overline{x})} \text{conv}\ \partial^u g_i(\overline{x})\right) \cup \left(\bigcup_{i \in I'} \text{conv}\ \partial^u h_i(\overline{x})\right) \cup \left(\bigcup_{i \in I'} \text{conv}\ \partial^u(-h_i)(\overline{x})\right) \\ & \cup \left(\bigcup_{i \in A \cup B_2} (\text{conv}\ \partial^u G_i(\overline{x}) \cup \text{conv}\ \partial^u(-G_i)(\overline{x}))\right) \cup \left(\bigcup_{i \in D \cup B_1} (\text{conv}\ \partial^u H_i(\overline{x}) \cup \text{conv}\ \partial^u(-H_i)(\overline{x}))\right) \\ & \cup \left(\bigcup_{i \in B_1} \text{conv}\ \partial^u(-G_i)(\overline{x})\right) \cup \left(\bigcup_{i \in B_2} \text{conv}\ \partial^u(-H_i)(\overline{x})\right). \end{aligned}$$

The following result is the corrected version of [8, Theorem 4.4].

Theorem 10 *Let $\overline{x}$ be a local optimal solution of $MPEC$. Assume that f is locally Lipschitz and admits a bounded upper semiregular convexifactor $\partial^{us} f(\overline{x})$ at $\overline{x}$. Let $g_i, \ i \in I, \ -h_i, \ h_i, \ i \in I', \ -G_i, \ G_i, \ i \in A \cup B, \ -H_i, \ H_i, \ i \in D \cup B$, admit upper convexifactors $\partial^u g_i(\overline{x}), \ i \in I, \ \partial^u(-h_i)(\overline{x}), \ \partial^u h_i(\overline{x}), \ i \in$*

I', $\partial^u(-G_i)(\overline{x})$, $\partial^u G_i(\overline{x})$, $i \in A \cup B$, $\partial^u(-H_i)(\overline{x})$, $\partial^u H_i(\overline{x})$, $i \in D \cup B$, respectively at $\overline{x}$. Suppose that pos $\mathcal{A}$ is closed and that there exists a partition (B_1, B_2) of B such that $\partial^* - ACQ(B_1, B_2)$ holds at $\overline{x}$. Then, $\overline{x}$ is a generalized alternatively stationary point.

Proof. The beginning of the proof of [8, Theorem 4.4] remains correct. However, from line 6 on page 1624 until the end of the proof, the argument should be corrected as the following.

$$\sup_{\eta \in conv \partial^{us} f(\overline{x})} \langle \eta, v \rangle \geq 0, \quad \text{for all } v \in \mathcal{A}^-.$$

- Since $\mathcal{A} \subseteq pos\ \mathcal{A}$, we get

$$\sup_{\eta \in conv \partial^{us} f(\overline{x})} \langle \eta, v \rangle \geq 0, \quad \text{for all } v \in (pos\ \mathcal{A})^-.$$

- Since $\partial^{us} f(\overline{x})$ is also a closed set, $conv\ \partial^{us} f(\overline{x})$ is a compact set (see [7, Theorem 1.4.3]). By Lemma 5, we get

$$0 \in conv\ \partial^{us} f(\overline{x}) + cl\,(pos\ \mathcal{A}).$$

  - Since $pos\ \mathcal{A}$ is closed, we obtain

$$0 \in conv\ \partial^{us} f(\overline{x}) + pos\ \mathcal{A}.$$

  Then, there exist scalars $\lambda_i^g \geq 0$, $i \in I(\overline{x})$, $\mu_i^h \geq 0$, $\lambda_i^h \geq 0$, $i \in I'$, $\mu_i^G \geq 0$, $i \in A \cup B_2$, $\lambda_i^G \geq 0$, $i \in A \cup B$, $\mu_i^H \geq 0$, $i \in D \cup B_1$, and $\lambda_i^H \geq 0$, $i \in D \cup B$, such that

$$0 \in \left[\begin{array}{c} conv\ \partial^{us} f(\overline{x}) + \sum_{i \in I(\overline{x})} \lambda_i^g\ conv\ \partial^u g_i(\overline{x}) + \sum_{i \in I'} \mu_i^h\ conv\ \partial^u h_i(\overline{x}) + \sum_{i \in I'} \lambda_i^h\ conv\ \partial^u(-h_i)(\overline{x}) \\ + \sum_{i \in A \cup B_2} \mu_i^G\ conv\ \partial^u G_i(\overline{x}) + \sum_{i \in A \cup B} \lambda_i^G\ conv\ \partial^u(-G_i)(\overline{x}) \\ + \sum_{i \in D \cup B_1} \mu_i^H\ conv\ \partial^u H_i(\overline{x}) + \sum_{i \in D \cup B} \lambda_i^H\ conv\ \partial^u(-H_i)(\overline{x}) \end{array} \right].$$

  - Setting

$$\begin{cases} \mu_i^G = 0, \ \forall i \in D \cup B_1 \\ \mu_i^H = 0, \ \forall i \in A \cup B_2 \\ \lambda_i^G = 0, \ \forall i \in D \\ \lambda_i^H = 0, \ \forall i \in A \end{cases}$$

  we obtain (4), (5) and (6). The proof is then finished.

■

5 Conclusions

In the paper [8], the author investigated a mathematical programs with equilibrium constraints. The main result, Theorem 4.4 [8], and the lemma (Lemma 2.3 [8]) on which the author is based are false. In this work, counterexamples are given to refute [8, Theorem 4.4] and [8, Lemma 2.3]. Furthermore, we correct the flaws.

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