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ERRATUM TO "OPTIMALITY CONDITIONS FOR NONSMOOTH INTERVAL-VALUED AND MULTIOBJECTIVE SEMI-INFINITE PROGRAMMING" *

Mohsine Jennane[1], El Mostafa Kalmoun[2] and Lahoussine Lafhim[3]

Abstract. This note corrects an error in our paper *RAIRO-Operations Research /doi.org/10.1051/ro/2020066* as we should drop the expression "*with at least one strict inequality*" in the definition of interval order in Section 2. Instead of proposing this short amendment, the authors of *RAIRO-Operations Research doi.org/10.1051/ro/2020107* gave a proposition that requires an additional condition on the constraint functions. However, we claim that all the results of our paper are correct once the modification above is done.

Keywords: Multiobjective semi-infinite programming; Interval-valued functions; Optimality conditions

Mathematics Subject Classification. 49J52, 90C46, 58E35

In defining the interval order on page 3 (line 19), the condition "*with at least one strict inequality*" is incorrect and should be dropped. Indeed, if we denote the class of all closed intervals in $\mathbb{R}$ by $\mathcal{I}$, then a partial order on $\mathcal{I}$ would be defined for two elements $A = [a^L, a^U]$ and $B = [b^L, b^U]$ in $\mathcal{I}$ by $A \leq_{LU} B$ if $a^L \leq b^L$ and $a^U \leq b^U$. We write $A <_{LU} B$ if $a^L < b^L$ and $a^U < b^U$. On the other hand, $A = (A_1, \ldots, A_p)$ is called an interval-valued vector if $A_k = [a_k^L, a_k^U] \in \mathcal{I}$ for each $k = 1, \ldots, p$. For two interval-valued vectors $A = (A_1, \ldots, A_p)$ and $B = (B_1, \ldots, B_p)$, we write $A \leq_{LU} B$ if $A_k \leq_{LU} B_k$ for each $k = 1, \ldots, p$, and $A <_{LU} B$ if $A_k <_{LU} B_k$ for each $k = 1, \ldots, p$.

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* *Corresponding author: ekalmoun@qu.edu.qa*

[1] FSDM, Department of mathematics Sidi Mohamed Ben Abdellah University, Fez, Morocco

[2] Department of Mathematics, Statistics and Physics, Qatar University, Doha, Qatar

[3] FSDM, Department of mathematics Sidi Mohamed Ben Abdellah University, Fez, Morocco

When the interval order is defined as above, we claim all the results in our paper are correct. In particular, Example 5 and Example 6 in [1] will not stand anymore as counter-examples of Lemma 3.3 in our paper, because we will have $\Omega = \{x \in \mathbb{R} : g_t(x) \leq 0,\ \forall t \in T\} = \{0\}$ for the first example, and $\Omega = \{(x_1, x_2) \in \mathbb{R}^2 : g_t(x_1, x_2) \leq 0,\ \forall t \in T\} = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 \geq |x_1|\}$ with $(0, 0) \in \Omega$ for the second, which contradicts what the authors of [1] have asserted. Consequently, [1, Remark 11] being based on [1, Example 6] will not remain valid since $\overline{x}$ will be a weak efficient solution of Problem (11) and Ω will be locally star-shaped at $\overline{x}$. Moreover, contrary to [1, Remark 7] the feasible set of Problem (2) will always be equal to $\{x \in \mathbb{R} : g_t(x) \leq 0,\ \forall t \in T\}$.

Instead of proposing the short amendment in the LU order that we explained above, the authors of [1] gave a proposition that requires an additional condition on the constraint functions [1, Assumption 3, page 7]. This strong assumption is not needed in our case, and hence, [1, Lemma 13] turns out to be superfluous.

The arguments presented in [1, emark 8] are misleading. The authors said that the equality given in Lemma 3.4 is false because "the structure of Problem (4) requires that $\overline{x}$ be already a weak efficient solution of Problem (2)" without explaining how this supports their claim and how is against our statement in Lemma 3.4. Their proposed version [1, Lemma 14]) need Assumption 3 in [1, page 27] to be hold. In order to clarify our lemma 3.4, we give a new reformulation as follows.

Lemma 3.4. *Let $\overline{x} \in \Omega$ and consider the maps f_1 and f_2 given by* (3.4)*. Then, $\overline{x}$ is a weak efficient solution of* (3.1) *if and only if it is a weak minima of* (3.3)*.*

In [1, Remark 9], the authors said that the following equality we used

$$N_\Omega(\overline{x}) = cl\, cone(\Gamma(\overline{x})) = cl\, cone\left(\bigcup_{t \in T(\overline{x})} co(\partial^* g_t(\overline{x}))\right),$$

due to [2], is based on a condition that we did not check. However, after going back to the indicated reference [2, Theorem 3.3(iii)], we found this condition does not appear anywhere in the statement of this result nor in its proof. Hence, this condition is not necessary in [1, Remark 12].

On the other hand, in [1, Example 10], the authors stated that "The following example shows that the set $cone\big(\cup_{t \in T(\overline{x})} \partial^{us} g_t(\overline{x})\big)$ is not necessarily closed even if the Abadie constraint qualification is satisfied". However, using the relation $\cup_{t \in T(\overline{x})} \partial^{us} g_t(\overline{x}) = T \times \{-1\}$, which is shown in this example, we get $cone\big(\cup_{t \in T(\overline{x})} \partial^{us} g_t(\overline{x})\big) = \mathbb{R} \times \mathbb{R}_-$ is closed. Therefore, we can see that this example does not support their claim. It should be pointed that with the exception of the last two lines which are incorrect (because $cone\big(\cup_{t \in T(\overline{x})} co\big(\partial^{us} g_t(\overline{x})\big)\big) = cone\big(\cup_{t \in T(\overline{x})} \partial^{us} g_t(\overline{x})\big) = \mathbb{R} \times \mathbb{R}_-$), the example is the same as Example 3 (ii) in [2] but was not cited by the authors.

Finally, we add to the statement of our theorem 4.5 a condition that we use implicitly in its proof.

Theorem 4.5. *Let Ω be locally star-shaped at $\overline{x} \in \Omega$, and let F_k^L, F_k^U, G_t^L and G_t^U ($i \in \{1, \ldots, p\}$ and $t \in T$), admit respectively USRCs, $\partial^* F_k^L(\overline{x})$, $\partial^* F_k^U(\overline{x})$, $\partial^* G_t^L(\overline{x})$ and $\partial^* G_t^U(\overline{x})$ at $\overline{x}$. Moreover, assume that ACQ holds at $\overline{x}$, the set cone $\Gamma(\overline{x})$ is closed and Assumption 4.1 is fulfilled. If $\overline{x}$ is a weak efficient solution of (3.1), then there exist an index set $T' \subseteq T(\overline{x})$ with $|T'| \leq n$, $\alpha \in \mathbb{R}_+^{|I^L(\overline{x})|}$, $\beta \in \mathbb{R}_+^{|I^U(\overline{x})|}$, $\mu \in \mathbb{R}_+^{|T'|}$, $\gamma_t^L \in \mathbb{R}_+^{|T'|}$, $\gamma_t^U \in \mathbb{R}_+^{|T'|}$ and $\lambda \in \mathbb{R}_+^2$ with*

$$\lambda_1 + \lambda_2 = \sum_{k \in I^L(\overline{x})} \alpha_k = \sum_{k \in I^U(\overline{x})} \beta_k = \sum_{t \in T'} \gamma_t^L = \sum_{t \in T'} \gamma_t^U = 1,$$

such that

$$\begin{aligned} 0 \in \quad & cl\Big[\lambda_1 \sum_{k \in I^L(\overline{x})} \alpha_k co(\partial^* F_k^L(\overline{x})) + \lambda_2 \sum_{k \in I^U(\overline{x})} \beta_k co(\partial^* F_k^U(\overline{x})) \\ & + \sum_{t \in T'} \mu_t \gamma_t^L co(\partial^* G_t^L(\overline{x})) + \sum_{t \in T'} \mu_t \gamma_t^U co(\partial^* G_t^U(\overline{x}))\Big]. \end{aligned}$$

In summary, the expression "with at least one strict inequality" which was inadvertently missed in the definition of interval order in our paper but was never used. In dropping this expression, we claim that all the results and proofs of our paper are correct. Instead of simply proposing this amendment, the authors in [1] made remarks that most of them lack accuracy. For the convenience of the reader, we have shown that [1, Example 5, Example 6, Remark 7, Remark 8, Example 10, Assumption 3, Lemma 13, Lemma 14] are all superfluous. They also claimed providing a new and short proof. However, we found that they rewrote ours but only replacing a part of it with a result in [2, Theorem 4.1], whose proof is based exactly on the same arguments we employed.

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