

Sustainable integrated and pricing decisions for two-echelon supplier-retailer supply chain of growing Items

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Abstract

In today's market, supply chain players have to cooperate mutually for extra benefits, long lasting paybacks, and to control carbon emission for a clean environment. In this study, a two-echelon sustainable supply chain model with a supplier-retailer scenario is considered to investigate the overlooked area of growing items and reducing carbon emissions. These joint effects will benefit the firms for interim financing as well as minimize carbon emission for a clean environment. The main task for the supplier is to breed new-born animals with respect to a biologic growth pattern, then slaughters them and controls the carbon emission to maintain the sustainability. The supplier then delivers the slaughtered items to the retailer where it is used as final products to satisfy customers demand and also experienced deterioration during the inventory replenishment cycle. Carbon emission is considered due to transportation of slaughtered items to the retailer. The main goal of this paper is to analyze the model under decentralized and centralized chain structures and in the centralized case profit-sharing contract is incorporated as the cooperation tool. The model has been solved with an analytic solution approach to obtain the global optimum solution. Sensitivity analysis is carried out to investigate the impact of different input parameters. The results support the claim that centralized chain structure can provide the partners with more benefits if an appropriate coordination mechanism is applied. Moreover, it is shown that the unit purchasing cost of each echelon has a significant effect on the profit in comparison to the other cost factors. Finally the results reveal that the supplier's inventory cycle is more dependent on the growth pattern rather than external cost factors.

Keywords: Inventory management; revenue management; growth; deterioration; supply chain; profit-sharing

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Highlights

- A Two-level sustainable supply chain, including a new type of inventory, is studied.
- Inventory and revenue management through supply chain is proposed by introducing the concept of growth.
- The items experience growth and then deterioration through the chain.
- The impact of quality losses is considered in the model to prevent overbreeding and disease.
- Carbon emissions is considered due to transportation
- Centralized and decentralized chain structures are analyzed.
- Theoretical results are developed to form an exact analytic solution procedure.
- Experimental results validate the proposed structure and provide fruitful managerial insights

1. Introduction

In the current business environment, relations between retailer and supplier have undergone significant changes with increasing emphasis on cooperation and information sharing. A retailer and a supplier together constitute a simple two-echelon supply chain. [Woo et al. \(2001\)](#) suggested that the cooperation between vendor and buyer for improving the performance of a supply chain has received a great deal of attention from researchers. Recently, supply chains with multiple decision makers have begun to receive considerable interest due to the fact that independent entities in the supply chain acting in their own self-interest often make decisions that are sub-optimal. It is well established that the total supply chain profit under decentralized decision scenario is less than the profit achievable by a single central decision maker with complete information. Therefore decentralized control is believed to be inefficient as compared to centralized decision making.

Supply chain management (SCM) involves a large class of activities over a wide spectrum of members from raw material suppliers to the end customers. The demand of the end customer influences all the activities through the chain. In order to meet this demand in an efficient way, which results in customer satisfaction, keeping inventory is compulsive. Holding inventory incurs different costs to the system. So inventory management can play a key role in enhancing the profit of the system by balancing these costs.

In addition to the aforementioned relation between demand and inventory, it has been observed from the purchasing behavior of the customers that price of a product is a significant factor that influences the demand pattern. According to traditional marketing and economic theory, the higher the price, the lower the demand. Hence, the price cannot be kept in isolation way from the demand. Then setting the right price which is known as pricing can considerably raise the profit of the system. After all, pricing, as a key implement of revenue management, can achieve its ultimate purpose on the condition that inventory considerations are regarded. So, joint inventory and revenue management policies are treated as influential tasks in SCM which can highly improve the chain performance.

In traditional inventory models, it is assumed that the commodities preserve their physical characteristics during the storage time. This is not always authentic. As an instance, the inventory may experience deterioration and quality loss or growth during stocking. Deterioration is generally understood as falling from a higher to a lower level in quality, it also simply implies a change, decay, damage, obsolescence, collapse, pinch, spoilage, loss of utility or loss of marginal value of goods that results in a decrease of the usefulness of the original item. Therefore, deterioration of items plays a vital role in the determination of an inventory model and has to be taken into account. On the other hand, the issue of growth suffers from lack of academic endeavor. Growing items involve in a group of inventories whose levels increases during their stocking period due to the weight increase of the initial items. Expressly, as the growing inventory enters the system, it starts to flourish resulting in weight increase. This is usual in farming, fisheries, poultry and livestock industry. Each inventory cycle of a growing item involves two sub-cycles: breeding and consumption. During the breeding period, the inventory grows owing to feeding and nourishment. The inventory level keeps enhancing until slaughtering time which is the ending point of the breeding period. Afterwards, the consumption period starts and the slaughtered items are depleted to zero due to demand fulfillment and deterioration.

The increasing amount of toxic gases like carbon dioxide (CO_2), carbon monoxide and many greenhouse gases in the environment are responsible for global warming effect which is becoming a major critical threat on earth resulting in climate changes in the world. Many governments and researchers have begun to pay close attention to green logistic concepts to reduce carbon dioxide emission caused by transportation. Though supply chain activities that include production, transportation and inventory emit CO_2 , transportation module gets the main attention due to larger CO_2 emission and is the fastest growing major contributor to global climate change, accounting for 23% of energy-related carbon dioxide (CO_2) emissions (Dekker et al. 2012).

The inventory control of growing items is highly crucial. This importance stems from the fact that the items are prone to different diseases and quality decrements. Consequently any

improper decision not only leads to high financial losses but also is directly linked to food safety issues. On the other hand, the items after being slaughtered fall into the category of deteriorating items. Optimizing inventory and pricing decisions for these items can effectively decrease the inventory wastes and undesirable costs. Motivated by the aforementioned significances and the positive impact of SCM on performance of the firms, this paper investigates the inventory management problems in the supply chain by addressing the concept of growth. The paper outlines a two-echelon SC involving one supplier and one retailer. The supplier holds the growing items in its system during the breeding period. The inventory growth is measured by a common biological weight increase function. The items are slaughtered and sent to the retailer according to its ordering system. The items start to deteriorate during the consumption period in the retailer side. The retailer seeks to optimize its order size as well as the retailing price. The supplier intends to optimize the breeding period length based on the retailer's decisions. The problem is investigated under decentralized and centralized scenarios. Profit sharing contract is implemented as the coordination tool under the centralized case.

The area of the revenue and inventory management of growing items is still in its infancy and has not been dealt with as it deserves. When it comes to supply chain, this defect is even much more highlighted. The proposed research attempts to atone for the mentioned deficiency as its most prevalent contribution. The procedure of growth is modeled by a common biologic weight increase function which can effectively depict the growth behavior of the new-born animals. As the other novel feature of this study, an age-dependent cost term comprising feeding, nourishment and inventory holding expenditures is taken into account. Furthermore, the negative impact of overbreeding, disease and quality losses is considered in order to preserve the quality standards of the items.

The rest of the paper is organized as follows. Section 2 provides a brief review of the literature. Section 3 presents the mathematical model. The solution approach is outlined in Section 4. Section 5 provides numerical results, sensitivity analysis and managerial insights. Finally the conclusions and future research directions are drawn in Section 6.

2. Literature review

The literature body of the problem is perused from two streams: deterioration and growth. Deterioration is among the most wide-spread investigated subjects in the scope of inventory management. On the other hand, growth is still an overlooked area and has not been studied in depth.

Deterioration

Although management of deteriorating items through supply chain has not been investigated as deeply as the single echelon case, they still are regarded as a trending research area. The related papers fall into two general classes including two-echelon and three-echelon SCs.

Non-delayed equal-sized shipment is a usual ordering policy through the supply chain. [Chen and Chang \(2010\)](#) incorporated this ordering policy in their proposed one-manufacturer, multi-retailer SC. The problem is to determine the optimal retailing price, order size and number of shipments. They showed that the centralized chain structure provides better results than the decentralized one in their problem. [Maihami et al. \(2017\)](#) extended this research by considering stochastic demand. [Chen and Sarkar \(2017\)](#) added the possibility of shortage to [Chen and Chang \(2010\)](#). Delayed equal-sized shipment is the other ordering policy appeared in the literature body of the problem. [Shah et al. \(2011\)](#) studied a two-echelon supply chain where the demand rate is dependent on time as well as the selling price. They also applied delayed-payment as a revenue management policy in their model. Their result suggested that price discount and delay in payments are influential tools in cooperation to reach a win-win outcome.

Supply chain contracts are potent implements of chain coordination which can enhance the benefits of distinct chain members. This is surveyed in several papers. [Giri and Bardhan \(2012\)](#) analyzed the impact of revenue-sharing contract in a pricing and inventory control model. [Yu et al. \(2013\)](#) studied a two-level SC where notable set-up costs are imposed on the supplier. Then profit-sharing contract is implemented to convince the supplier to coordinate. Similarly, [Zhang et al. \(2015\)](#) incorporated revenue-sharing contract. Besides, they exploited cooperative investment on inventory holding technology to diminish the negative result of deterioration. All of these mentioned studies suggested that the coordination mechanisms have

positive effect on the profit of the system. [Xiao and Xu \(2013\)](#), [Kawatkasu et al. \(2013\)](#) and [Bai et al. \(2017\)](#) also applied revenue-sharing, retailing discount and profit-sharing respectively in a game-theoretic structure. Despite the two-echelon chain, three-echelon cases are still poorly investigated due to the complexity of their modeling and solution approach. [Cai et al. \(2013\)](#) explored a chain involving supplier, distributor and retailer. Product freshness has a major impact on demand and pricing procedure is propounded in each of three levels. They applied two coordination mechanisms and showed that in their model, there is no necessity for price-discount contract to be accompanied by buy-back contract. [Daryanto et al. \(2019\)](#) analyzed an integrated three level SC under environmental considerations. They considered that transportation, warehousing and deteriorated items disposal accompany carbon emission. Their results showed that integration can considerably reduce the costs and carbon emission.

This brief literature review contains studies which particularly consider deterioration in the context of SC. For more studies on deterioration in inventory management, see [Bakker et al. \(2012\)](#), [Janssen et al. \(2016\)](#), [Rabbani et al. \(2015\)](#).

Growth

Inventory and revenue management of growing items is an almost inadvertent research district suffering from lack of academic endeavor. A few numbers of papers are observed in this area which is discussed as follows. [Rezaei \(2014\)](#) introduced the concept of growth in inventory management by developing an EOQ model for growing items. He modeled growth as a common weight increasing function in poultry and livestock literature. The feeding procedure is highlighted by considering feeding costs and production function in his study. While the paper has incorporated a number of simplistic assumptions, it can be treated as an appropriate basis for later extensions. To put it in a nutshell, our study has exploited this paper as the primal structure to develop its own framework. [Zhang et al. \(2016\)](#) extended [Rezaei \(2014\)](#) model considering environmental issues and carbon emission. [Nobil et al. \(2019\)](#) generalized [Rezaei \(2014\)](#) to allow for shortages. To overcome the complexity raised by admissible shortages, their solution procedure treats the weight of the items as a constant value independent of the period length which is a major shortcoming questioning the validity of their structure. In addition to [Rezaei's \(2014\)](#) findings, their results revealed that shortage cost has no impact on breeding

period. [Sebatjane \(2018\)](#) proposed an EOQ model for growing items with quality considerations. In his model, screening procedure is carried out to distinguish poor quality items which should be salvaged. Investigating different growth functions is a noble feature of this study. Their results indicated that logistic function can project the growth procedure better than split linear and linear functions. On the other hand, split linear function can reduce the complexity of the equations and is still able to reflect growth better than linear one. [Khalilpourazari and Pasandideh \(2019\)](#) provided a multi-item, multi-constrained EOQ model for growing items. They considered three operational constrains including on-hand budget, warehouse capacity and total allowable holding cost. To solve the problem in small sizes, sequential quadratic programming is incorporated. Moreover the paper uses two meta-heuristics in medium and large sizes. [Sebatjane and Adetunji \(2019a\)](#) established an EOQ model for growing items connected with imperfect quality. [Sebatjane and Adetunji \(2019b\)](#) presented an inventory control system of growing items in three echelon system. Recently, [Mahato et al. \(2021\)](#) analyzed joint pricing and inventory management for growing items in a supply chain under trade credit. [Rana et al. \(2021\)](#) worked on an Inventory control model for growing items considering various aspects simultaneously such as trade credit, partial backlogging, time dependent demand and carbon emission. [De-la-Cruz-Márquez et al. \(2021\)](#) developed an inventory model for growing items with imperfect quality with price sensitive demand under carbon emissions and shortages. [Sebatjane and Adetunji \(2021\)](#) presented an optimal lot-sizing and shipment decisions in a three-echelon supply for growing items with inventory level- and expiration date-dependent demand.

Table 1: Comparative study of existing literature of inventory models of growing items

Authors	Growing items	Deterioration	Demand pattern	Carbon emission	Number of supply chain echelons	Solution approach	Additional characteristics
Rezaei (2014)	√	×	Constant	×	One	Not exact	
Malekitabar et al. (2019)	√	×	Constant	×	Two	Not exact	Revenue sharing
Nobil et al. (2019)	√	×	Constant	×	One	Not exact	Shortages
Sebatjane and Adetunji (2019a)	√	×		×	One	Not exact	Imperfect quality
Sebatjane and	√	×	Constant	×	One	Not exact	Quality discounts

Adetunji (2019b)							
Khalilpourazari and Pasandideh (2019)	√	×	Constant	×	One	Not exact	Multiple items
De-la-Cruz-Márquez et al. (2021)	√	×	Price dependent	√	One	Not exact	Imperfect quality, shortages
Rana et al. (2021)	√	√	Time dependent	√	One	Not exact	Permissible delay in payment
Sebatjane and Adetunji (2020)	√	×	Constant	×	Four	Not exact	Imperfect quality
Sebatjane and Adetunji (2021)	√	×	Stock and expiration date dependent	×	Three	Not exact	Integrated supply chain
This paper	√	√	Price dependent	√	Two	Exact analytic solution procedure	Decentralized and centralized chain structures and in centralized case Profit-sharing contract

A comparison of the proposed inventory system and related published works in the literature is provided in Table 1, which also shows the contributions of this paper to inventory management research for growing items. In addition to highlighting the contributions made by various researchers, the comparison also shows the research gap identified in literature. The table gives the impression that no published inventory control model for growing items has considered a two-echelon supply chain system with both growing and deteriorating items is of paramount importance. The novelty of the model lies in the fact that it incorporates a novel mathematical inventory model in a two-echelon supply chain consisting of one supplier and one retailer dealing with items having growth experience and then deterioration through the chain under carbon emissions. Carbon emission is considered due to transportation of slaughtered items to the retailer. The supplier holds the growing items in its system during the breeding period. The inventory growth is measured by a common biological weight increase function. Considering age-dependent breeding and holding costs of the items during the growth is another noble feature of the current paper. Indeed, this is the first attempt to heed the quality reduction of the items due to overbreeding. The items are slaughtered and sent to the retailer

according to its ordering system. Finally, the quality loss of the inventory during the consumption period is taken into account in terms of deterioration in the retailer side. The main goal of this paper is to analyze the model under decentralized and centralized chain structures and in the centralized case profit-sharing contract is incorporated as the cooperation tool. The model has been solved with an analytic solution approach to obtain the global optimum solution.

3. Model development

The following notations are used throughout the paper to formulate the model:

3.1. Notations

Parameters

p_s	Supplier's unit selling price (€/gr)
c_p	Supplier's unit purchasing cost (€/gr)
c_b	Supplier's breeding (feeding, nourishment and holding) cost per unit item (€/unit)
h	Retailer's unit holding cost per unit time (€/gr/year)
A_R	Retailer's ordering cost per order (€/order)
A_S	Supplier's ordering cost per order (€/order)
F_S	Supplier's Transportation cost per shipment (€/shipment)
v_t	Supplier's Variable transportation cost per unit (€/gr)
c_f	Supplier's constant carbon emission cost per shipment (€/shipment)
c_v	Supplier's variable carbon emission cost per unit (€/gr)
θ	Constant deterioration rate at the retailer side
y	Number of growing items purchased at the beginning of a cycle (unit items)
ρ	Profit-sharing ratio at the supplier

Decision variables

p_r	Retailer's unit selling price (€/gr)
T_S	Inventory cycle at the supplier (breeding period) (time unit)
T_R	Inventory cycle at the retailer (consumption period) (time unit)

Functions

$D(p_r)$	Price-dependent demand at the retailer
w_t	Weight of a unit item at time t
$\lambda(\cdot)$	Fraction of discarded items during quality control
$B(t)$	Increasing breeding cost at time t
$I_S(t)$	Supplier's inventory level at time t
$I_R(t)$	Retailer's inventory level at time t
Q_0	Supplier's order quantity (units)
Q_R	Retailer's order quantity (units)
TP_S	Supplier's total profit per unit time
TP_R	Retailer's total profit per unit time
OP_S	Supplier's obtained profit per unit time under centralized structure
OP_R	Retailer's obtained profit per unit time under centralized structure
TP	Total profit of the chain per unit time

3.2. Assumptions

The basic assumptions shaping the mathematical model are as follow.

- (i) The planning horizon is infinite and shortages are not admissible.
- (ii) Replenishment at both sides is instantaneous with infinite rate and negligible lead-time.
- (iii) The growth starts to proceed as the items are effectively in stock.
- (iv) The deterioration occurs only at the retailer side (during consumption period).
- (v) Growth is modeled as a common biological weight function (Goliomytis et al., 2003). The weight of a unit item at time t is formulated as $w_t = A(1 + be^{-gt})^{-1}$. Where A is ultimate limiting value ($A > 0$) representing the maximum possible weight of the item, b is the integration constant which reflects the choice of zero time ($b > 0$), g is a constant rate which determines the spread of growth curve during the time axis ($0 < g < 1$). In this formulation, time is expressed in days. Since the time basis of our inventory model is year, $k = 365g$ is substituted to change the time basis (i.e., $w_t = A(1 + be^{-kt})^{-1}$, t in years).
- (vi) As the items grow, the ratio of useless weight (such as fat) to the whole weight increases. Moreover, they might lose quality standards due to illness and overbreeding. So, at the

end of the breeding period quality control is performed and a fraction of the inventory units are disposed. This process is assumed to be instantaneous.

- (vii) The supplier applies “delayed equal-size shipments” policy and delivers the slaughtered items to the retailer.
- (viii) The slaughtered items are transported through a single-setup-single-delivery (SSSD) policy. The model considers the transportation cost and carbon emission cost to transport to the retailer.
- (ix) The demand rate is a function of the retailer’s selling price as $D(p_r) = MB - \omega p_r$. MB is the potential demand where price is equal to zero and $\omega > 0$ is the price sensitivity factor.

4. Mathematical formulation

Consider a typical supplier-retailer scenario. The supplier buys newborn animals at the beginning of its inventory cycle and raises them during the breeding period. The growth is modeled as a common biological weight increasing function (Richards, 1959), where the weight of a unit item at time t is formulated as

$$w_t = \frac{A}{1 + be^{-gt}}$$

A is ultimate limiting value ($A > 0$) representing the maximum possible weight of the item, b is the integration constant which is biologically unimportant, since it reflects the choice of zero time ($b > 0$) and g is a constant rate which determines the spread of growth curve during the time axis ($0 < g < 1$). In this formulation, time is expressed in days. Since the time basis of our inventory model is year, $k = 365g$ is substituted to change the time basis. (i.e. $w_t = A(1 + be^{-kt})^{-1}$, t in years).

While the items flourish in the supplier’s system, their breeding costs increase. That is because the items’ feeding costs rises due to weight increase. Nourishment and holding costs raises as the items are growing. The impact of this cost increase could be regarded by a time-dependent function $B(t)$. There are a number of functions for $B(t)$ in existing literature, where polynomial and exponential functions are the most commonly applied ones (Goliomytis et al., 2003). In this paper, the exponential function ($B(t) = e^{\beta t}$, $\beta > 0$) is selected.

As the items grow, the ratio of useless weight (such as fat) to the whole weight increases (Jensen et al., 1974). Moreover, the items might lose quality standards due to illness and over breeding. So, at the end of breeding period, i.e. after slaughtering the items, quality control is performed and a fraction of the inventory units are disposed. This process is assumed to be instantaneous. The slaughtered items are then sent to the retailer based on the ordered quantity. As the inventory enters the retailer's system, it faces quality losses owing to deterioration. Accordingly, during the retailer's inventory cycle, which is called consumption period, the inventory level depletes to zero due to both fulfilling the customer demand and deterioration. The retailer seeks to optimize its profit by determining the appropriate retailing price and ordering quantity, based on which the supplier tries to distinguish its optimal breeding period and initial ordering quantity of newborn animals. Fig. 1 depicts the inventory system.

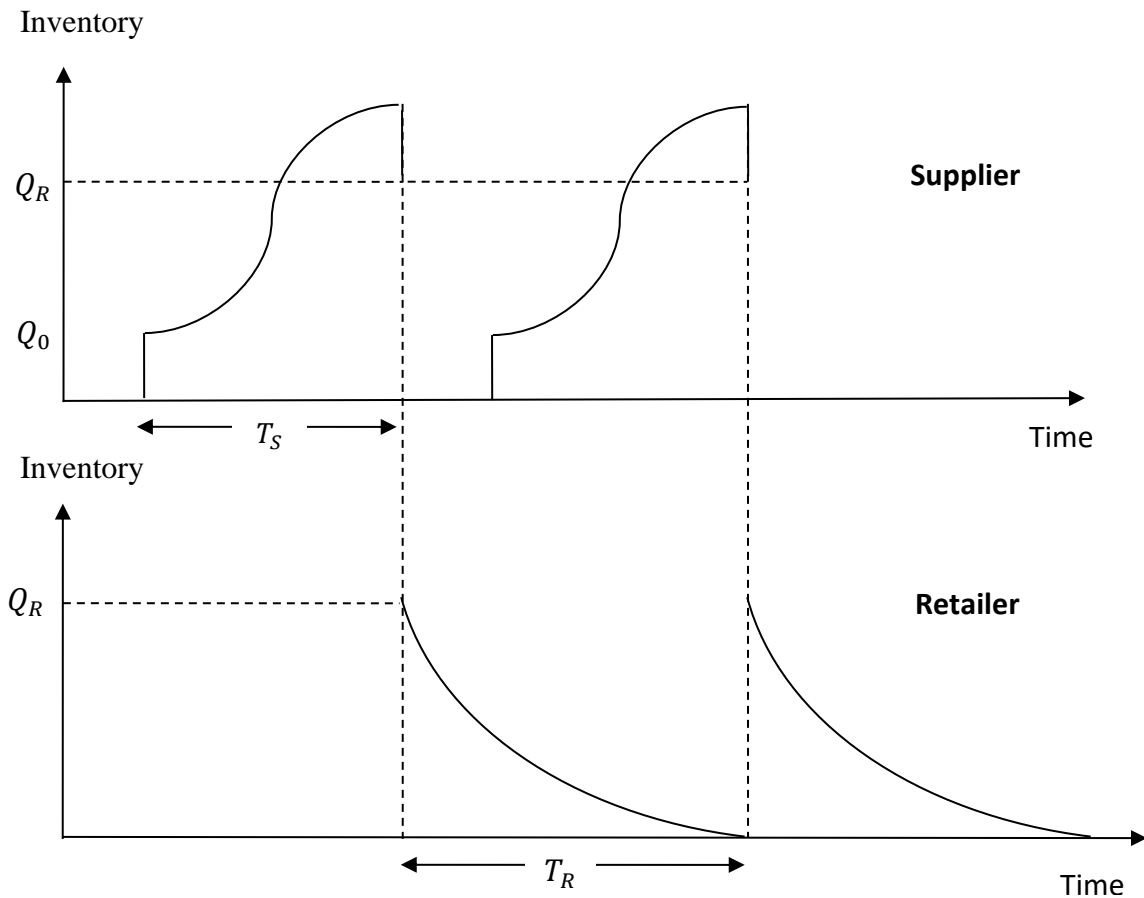


Fig. 1: The prescribed inventory system

The inventory system at the retailer side

During $[0, T_R]$ the inventory status is ruled by the following differential equation:

$$\frac{dI_R(t)}{dt} = -D(p_r) - \theta I(t), \quad 0 \leq t \leq T_R \quad (1)$$

With the boundary condition $I_R(T_R) = 0$ solving Eq. (1) yields:

$$I_R(t) = \frac{D(p_r)}{\theta} [e^{\theta(T_R-t)} - 1], \quad 0 \leq t \leq T_R \quad (2)$$

From Eq. (2), the order quantity for the retailer is given by

$$Q_R = I_R(0) = \frac{MB - \omega p_r}{\theta} (e^{\theta T_R} - 1) \quad (3)$$

The total profit of the inventory system at the retailer comprises the following components:

The sales revenue

$$SR_R = p_r \int_0^{T_R} D(p_r) dt = p_r (MB - \omega p_r) T_R \quad (4)$$

The purchasing cost

$$PC_R = p_s Q_R = p_s \frac{MB - \omega p_r}{\theta} (e^{\theta T_R} - 1). \quad (5)$$

The inventory holding cost

$$HC_R = h \int_0^{T_R} I_R(t) dt = h \frac{MB - \omega p_r}{\theta^2} (e^{\theta T_R} - \theta T_R - 1). \quad (6)$$

The ordering cost

$$OC_R = A_R. \quad (7)$$

Accordingly, the total profit per unit time of the retailer's inventory system is expressed as

$$TP_R(p_r, T_R) = \frac{SR_R - PC_R - HC_R - OC_R}{T_R}$$

The inventory system at the Supplier side

Suppose y unit items are ordered by the supplier at time zero. Since each unit item weights

$w_t = \frac{A}{1 + be^{-kt}}$ at time t , the inventory level during $t \in [0, T_s]$ is illustrated by:

$$I_s(t) = y w_t = \frac{yA}{1 + be^{-kt}}, \quad 0 \leq t \leq T_s. \quad (8)$$

Then the initial inventory level (Supplier's ordering quantity) is

$$Q_0 = I_s(0) = \frac{yA}{1+b}. \quad (9)$$

As y introduces the number of new born animals, it should take integer values; which transforms the model into an integer non-linear programming. To overcome this intricacy, the

equations could be reformulated in order to use Q_0 instead of y . Eq. (9) gives $y = \frac{Q_0(1+b)}{A}$. So Eq. (8) would be written as:

$$I_s(t) = \frac{Q_0(1+b)}{1+be^{-kt}}, \quad 0 \leq t \leq T_s \quad (10)$$

Recall that a fraction of inventory loses its quality during breeding period which is unfolded by quality control of items at point T_s . Vividly, this fraction should be an increasing function of T_s . It should hold two more features. First, in time zero this fraction is negligible (i.e., $\lambda(0) = 0$). Second, as breeding period takes very large values, this approaches one (i.e., $\lim_{T_s \rightarrow \infty} \lambda(T_s) = 1$).

The following function holds these features:

$$\lambda(T_s) = 1 - e^{-\alpha T_s}, \quad \alpha > 0. \quad (11)$$

The inventory level before inspection at time T_s is depicted by $I'_s(T_s) = \frac{Q_0(1+b)}{1+be^{-kT_s}}$, then the disposal quantity after quality control would be expressed as:

$$\lambda(T_s)I'_s(T_s) = (1 - e^{-\alpha T_s}) \frac{Q_0(1+b)}{1+be^{-kT_s}}. \quad (12)$$

Subsequently, the inventory level of inspected and useable items is outlined by:

$$I_s(T_s) = (1 - \lambda(T_s))I'_s(T_s) = (1 - \lambda(T_s)) \frac{Q_0(1+b)}{1+be^{-kT_s}} = Q_0 \frac{e^{-\alpha T_s}(1+b)}{(1+be^{-kT_s})}. \quad (13)$$

This is the inventory quantity which enters the retailer's system. This inventory level $I_s(T_s)$ should match the retailer's ordering quantity Q_R , i.e.,

$$Q_0 \frac{e^{-\alpha T_s}(1+b)}{(1+be^{-kT_s})} = \frac{MB - \omega p_r}{\theta} (e^{\theta T_R} - 1) \quad (14)$$

Using this equation, Q_0 can be expressed as a function of T_s and T_R :

$$Q_0 = \frac{MB - \omega p_r}{\theta(1+b)} (1 + be^{-kT_s}) e^{\alpha T_s} (e^{\theta T_R} - 1) \quad (15)$$

Subsequently, the total profit of the inventory system at the supplier embodies the pursuant components:

RE_s : The sales revenue

$$RE_s = p_s Q_R = \frac{p_s(MB - \omega p_r)}{\theta} (e^{\theta T_R} - 1) \quad (16)$$

PC_s : The purchasing cost

$$PC_s = c_p Q_0 = \frac{c_p(MB - \omega p_r)}{\theta(1+b)} (1 + be^{-kT_s}) e^{\alpha T_s} (e^{\theta T_R} - 1) \quad (17)$$

BC_s : The breeding cost

$$\begin{aligned}
BC_S &= c_b y \int_0^{T_S} B(t) dt = \frac{c_b Q_0 (1+b)}{A} \int_0^{T_S} e^{\beta t} dt \\
&= \frac{c_b (MB - \omega p_r) e^{\alpha T_S}}{\theta \beta A} (e^{\beta T_S} - 1) (1 + b e^{-k T_S}) (e^{\theta T_R} - 1)
\end{aligned} \tag{18}$$

OC_S : The ordering cost

$$OC_S = A_S. \tag{19}$$

TPC_S : The transportation cost

During the transporting time, the supplier transports the final products to the retailers by investing the fixed amount F_S per shipment. Depending on the demand, the variable transportation cost is $v_t Q_R$. Therefore, the total transportation cost is

$$TPC_S = F_S + v_t Q_R = F_S + \frac{v_t (MB - \omega p_r)}{\theta} (e^{\theta T_R} - 1) \tag{20}$$

EC_S : Carbon emission cost

The supplier has to bare fixed and variable transportation costs. There are fixed carbon emission cost c_f per shipment and variable cost with respect to the quantity as $c_v Q_R$. Thus, the total carbon emission cost is

$$EC_S = c_f + c_v Q_R = c_f + \frac{c_v (MB - \omega p_r)}{\theta} (e^{\theta T_R} - 1) \tag{21}$$

As Fig. 1 depicts, the supplier's inventory cycle is repeated every T_R units of time. Accordingly, the total profit per unit time of the supplier's inventory system is obtained as:

$$TP_S(T_S) = \frac{RE_S - PC_S - BC_S - OC_S - TPC_S - EC_S}{T_R} \tag{22}$$

The centralized chain model

In this case, the decisions are made simultaneously for each of the chain echelons so that the total profit of the chain would be optimized. Therefore

$$TP = TP_R + TP_S \tag{23}$$

Although the cumulative profit through the chain increases in this case, one of the echelons might experience lower profit in comparison to the decentralized structure. Then this echelon will be reluctant to enter the centralized optimization structure. Profit-sharing seems to be an influential coordination mechanism to convince it (Cachon, 2003; Yao et al., 2008 and Taleizadeh et al., 2018). In this regard, total profit of the chain is apportioned according to the profit-sharing ratio. The ratio can be specified based on the profit contribution of each level in

the decentralized structure. Therefore, the obtained profits per unit time by each of the SC echelons are equal to:

$$OP_S = \rho TP = \frac{TP_S}{TP_S + TP_R} TP \quad (24)$$

$$OP_R = (1 - \rho) TP = \frac{TP_R}{TP_S + TP_R} TP \quad (25)$$

5. Solution approach

The inventory system at the retailer's side

The necessary conditions for TP_R to be optimal are $\frac{\partial TP_R}{\partial T_R} = 0$ and $\frac{\partial TP_R}{\partial p_r} = 0$. Besides, it should be demonstrated that these equations give unique optimal solutions. Due to the complexity of the formulations, the concavity of the profit function cannot be demonstrated using Hessian matrix, which is why we use an approach developed by [Pentico and Drake \(2009\)](#).

Lemma 1. For fixed T_R , there exists a unique value p_r^* which maximizes TP_R , where $\frac{\partial TP_R}{\partial p_r} \Big|_{p_r=p_r^*} = 0$.

Proof: The first order optimality condition for the selling price gives:

$$\frac{\partial TP_R}{\partial p_r} = \left[(MB - 2\omega p_r) T_R + \frac{p_s \omega}{\theta} (e^{\theta T_R} - 1) + \frac{h\omega}{\theta^2} (e^{\theta T_R} - \theta T_R - 1) \right] = 0 \quad (26)$$

After some algebra, Eq. (26) can be rewritten as:

$$p_r = \frac{MB}{2\omega} - \frac{h}{2\theta} + \left(\frac{p_s}{2\theta T_R} + \frac{h}{2\theta^2 T_R} \right) (e^{\theta T_R} - 1) \quad (27)$$

In order to show that Eq. (27) gives the unique optimal value when T_R is treated as a fixed value, it is enough to demonstrate TP_R is concave with respect to p_r , which is:

$$\frac{\partial^2 TP_R}{\partial p_r^2} = -2\omega T_R < 0, \quad \forall T_R \in (0, \infty) \quad (28)$$

Hence Lemma 1 is proved.

Lemma 2. For fixed p_r , there exists a unique value T_R^* which maximizes TP_R where

$$\frac{\partial TP_R}{\partial T_R} \Big|_{T_R=T_R^*} = 0.$$

Proof: The first order optimality condition for the consumption period gives:

$$\begin{aligned} \frac{\partial TP_R}{\partial T_R} = \frac{(MB - 2\omega p_r)}{T_R^2} \left[\frac{p_s}{\theta} (e^{\theta T_R} - 1) - p_s T_R e^{\theta T_R} + \frac{h}{\theta^2} (e^{\theta T_R} - \theta T_R - 1) \right. \\ \left. - \frac{h}{\theta^2} (\theta e^{\theta T_R} - \theta) T_R + \frac{A_R}{MB - \omega p_r} \right] = 0 \end{aligned} \quad (29)$$

Then

$$\left[\left(p_s + \frac{h}{\theta} \right) (e^{\theta T_R} - \theta T_R e^{\theta T_R} - 1) + \frac{\theta A_R}{MB - \omega p_r} \right] = 0 \quad (30)$$

Motivated by Eq. (30) the auxiliary function $F(T_R)$ is defined as the phrases in []. Since $\frac{\partial TP_R}{\partial T_R} = 0$ and $F(T_R) = 0$ are equivalent, it is enough to demonstrate that $F(T_R) = 0$ gives a unique solution. Then

$$\frac{dF(T_R)}{dT_R} = \left(p_s + \frac{h}{\theta} \right) (-\theta^2 T_R e^{\theta T_R}) < 0, \text{ for } 0 < T_R < \infty. \quad (31)$$

Consequently, for any $0 < T_R < \infty$, $F(T_R)$ is strictly decreasing function of T_R . Besides,

$\lim_{T_R \rightarrow 0} F(T_R) = \frac{\theta A_R}{MB - \omega p_r} > 0$ and $\lim_{T_R \rightarrow \infty} F(T_R) = -\infty < 0$. Thus, for fixed selling price, there exists

a unique value of T_R for which $F(T_R) = 0$. As $\frac{\partial TP_R}{\partial T_R} = \frac{(MB - 2\omega p_r)}{T_R^2} F(T_R)$, at the point $T_R = T_R^*$:

$$\left. \frac{\partial^2 TP_R}{\partial T_R^2} \right|_{T_R=T_R^*} = \left. \frac{(MB - \omega p_r) F'(T_R) - 2F}{\theta T_R^3} \right|_{T_R=T_R^*} = \left. \frac{(MB - \omega p_r) F'}{\theta (T_R^*)^2} \right|_{T_R=T_R^*} < 0. \quad (32)$$

Hence Lemma 2 is proved.

Substituting Eq. (27) into Eq. (30) gives:

$$\left(p_s + \frac{h}{\theta} \right) (e^{\theta T_R} - \theta T_R e^{\theta T_R} - 1) + \frac{\theta A_R}{\frac{MB}{2} - \omega \left(\frac{p_s}{2\theta T_R} + \frac{h}{2\theta^2 T_R} \right) (e^{\theta T_R} - 1) - \frac{\omega h}{2\theta}} = 0. \quad (33)$$

Eq. (33) does not represent closed-form formula for T_R^* . Then a numerical root-finding method such as Newton-Raphson should be applied. To speed-up the convergence process, the initial solution can be obtained by replacing the first-order Taylor approximation of exponential function in Eq. (33), which gives:

$$T_R^0 = \sqrt{\frac{2A_R}{(MB - \omega p_s)(\theta p_s + h)}} \quad (34)$$

In sum, the solution procedure is depicted as follows:

- Calculate the optimal inventory cycle T_R^* using Eq. (33).

- Then obtain optimal selling price p_r^* applying Eq. (27) where $T_R = T_R^*$.

The inventory system at the supplier side

The necessary condition for TP_S in (22) to reach its optimal value is $\frac{dTP_S}{dT_S} = 0$. On the other hand, we need to demonstrate that this equation provides unique optimal value for T_S . The first-order optimality condition gives:

$$\frac{dTP_S}{dT_S} = -\frac{c_p(MB-\omega p_r)(e^{\theta T_R}-1)e^{\alpha T_S}}{\theta T_R(1+b)} [\alpha(1+be^{-kT_S}) - bke^{-kT_S}] - \frac{c_b(MB-\omega p_r)(e^{\theta T_R}-1)e^{\alpha T_S}}{\theta \beta T_{RA}} \times [\alpha(e^{\beta T_S}-1)(1+be^{-kT_S}) + \beta e^{\beta T_S}(1+be^{-kT_S}) - bke^{-kT_S}(e^{\beta T_S}-1)] = 0. \quad (35)$$

Then

$$\frac{c_p}{1+b} [\alpha + b(\alpha - k)e^{-kT_S}] + \frac{c_b}{\beta A} [(\alpha + \beta)e^{\beta T_S} + (bk - \alpha b)e^{-kT_S} + (\alpha b - bk + \beta b)e^{-kT_S}e^{\beta T_S} - \alpha] = 0. \quad (36)$$

To illustrate that Eq. (36) provides unique optimal solution, it is enough to show TP_S is concave:

$$\frac{d^2TP_S}{dT_S^2} = -\frac{(MB-\omega p_r)(e^{\theta T_R}-1)}{\theta T_R} \left[\frac{c_p}{1+b} e^{\alpha T_S} \{\alpha^2 + b\alpha(\alpha - k)e^{-kT_S} - bk(\alpha - k)e^{-kT_S}\} + \frac{c_b}{\beta A} e^{\alpha T_S} \{(\alpha + \beta)^2 e^{\beta T_S} + b(k - \alpha)(\alpha - k)e^{-kT_S} + b(\alpha - k + \beta)^2 e^{-kT_S} e^{\beta T_S} - \alpha^2\} \right] \quad (37)$$

Suppose

$$I = e^{\alpha T_S} [\alpha^2 + b(\alpha - k)^2 e^{-kT_S}] \quad (38)$$

$$J = e^{\alpha T_S} [(\alpha + \beta)^2 e^{\beta T_S} - b(\alpha - k)^2 e^{-kT_S} + b(\alpha - k + \beta)^2 e^{-kT_S} e^{\beta T_S} - \alpha^2] \quad (39)$$

Since $\beta \geq 0$, Eqs. (38) and (39) are always non-negative. Then the Eq. (37) is always non-positive. Accordingly, TP_S is concave and Eq. (36) provides unique optimal value for T_S .

Again, Eq. (36) does not provide closed-form formula for the optimal breeding period. Then Newton-Raphson is used to solve this equation. To speed-up the convergence process, the initial solution is distinguished by replacing the first-order Taylor approximation of exponential function in Eq. (36), which gives:

$$\left[\frac{c_b bk(k - \alpha - \beta)}{A} \right] T_S^2 + \left[\frac{c_p}{1+b} (bk(k - \alpha)) + \frac{c_b}{A} ((\alpha + \beta)(1+b) - 2bk) \right] T_S + \left[\frac{c_p}{1+b} (\alpha + \alpha b - bk) + \frac{c_b}{A} (1+b) \right] = 0 \quad (40)$$

Letting,

$$X = \left[\frac{c_b b k (k - \alpha - \beta)}{A} \right], Y = \left[\frac{c_p}{1+b} (b k (k - \alpha)) + \frac{c_b}{A} ((\alpha + \beta)(1 + b) - 2 b k) \right], \text{ and}$$

$$Z = \left[\frac{c_p}{1+b} (\alpha + \alpha b - b k) + \frac{c_b}{A} (1 + b) \right].$$

Then,

$$T_S^0 = \frac{-Y + \sqrt{Y^2 - 4XZ}}{2X} \quad (41)$$

The centralized chain

The necessary conditions for TP to be optimal, are $\frac{\partial TP}{\partial T_S} = 0$, $\frac{\partial TP}{\partial T_R} = 0$ and $\frac{\partial TP}{\partial p_r} = 0$. Again, the uniqueness of the optimal solutions is authenticated through the procedure which was provided by [Pentico and Drake \(2009\)](#). Optimal solutions of T_S and p_r are obtained from Eq. (27) and (36) and sufficient optimality conditions of these two decision variables are identical to the decentralized case. On the other hand, for T_R we have the following Lemma.

Lemma 3. For fixed p_r and T_S , there exists a unique value T_R^* which maximizes TP where

$$\left. \frac{\partial TP}{\partial T_R} \right|_{T_R=T_R^*} = 0.$$

Proof: The first-order optimality condition gives:

$$\begin{aligned} \frac{\partial TP}{\partial T_R} = & \frac{(MB - \omega p_r)}{T_R^2} \left[\frac{h}{\theta^2} [e^{\theta T_R} - \theta T_R - 1 - (\theta e^{\theta T_R} - \theta) T_R] + \left(\frac{A_r + A_s + F_s + c_f}{(MB - \omega p_r)} \right) \right. \\ & + \frac{c_p e^{\alpha T_S} (1 + b e^{-k T_S})}{\theta (1 + b)} [e^{\theta T_R} - \theta e^{\theta T_R} T_R - 1] \\ & + \frac{c_b e^{\alpha T_S} (1 + b e^{-k T_S}) (e^{\beta T_S} - 1)}{\theta \beta A} [e^{\theta T_R} - \theta e^{\theta T_R} T_R - 1] \\ & \left. + \frac{[e^{\theta T_R} - 1 - \theta T_R e^{\theta T_R}]}{\theta} (v_t + c_v) \right] = 0 \end{aligned} \quad (42)$$

Considering $\gamma = e^{\alpha T_S} (1 + b e^{-k T_S})$ and $\eta = (e^{\beta T_S} - 1)$, after some algebra we have

$$\left[\left(\frac{h}{\theta^2} + \frac{c_p \gamma}{\theta (1 + b)} + \frac{c_b \gamma \eta}{\theta \beta A} + \frac{(v_t + c_v)}{\theta} \right) (e^{\theta T_R} - \theta e^{\theta T_R} T_R - 1) + \left(\frac{A_r + A_s + F_s + c_f}{(MB - \omega p_r)} \right) \right] = 0 \quad (43)$$

Motivated by Eq. (43), the auxiliary function $F(T_R)$ is defined as the left-hand phrases. Since

$\frac{\partial TP}{\partial T_R} = 0$ and $F(T_R) = 0$ are equivalent, it is enough to demonstrate that $F(T_R) = 0$ gives a

unique solution. Then

$$\frac{dF(T_R)}{dT_R} = - \left[\frac{h}{\theta^2} + \frac{c_p \gamma}{\theta(1+b)} + \frac{c_b \gamma \eta}{\theta \beta A} + \frac{(v_t + c_v)}{\theta} \right] \theta^2 e^{\theta T_R} T_R < 0, \quad \forall 0 < T_R < \infty \quad (44)$$

Hence $F(T_R)$ is strictly decreasing function of T_R . Furthermore

$$\lim_{T_R \rightarrow 0} F(T_R) = \left(\frac{A_r + A_s + F_s + c_f}{(MB - \omega p_r)} \right) > 0 \text{ and } \lim_{T_R \rightarrow \infty} F(T_R) = -\infty < 0. \text{ Consequently, there exists a}$$

unique solution for the function $F(T_R) = 0$.

$$\text{As } \frac{\partial TP}{\partial T_R} = F(T_R) \frac{(MB - \omega p_r)}{\theta T_R^2}, \text{ at the point } T_R = T_R^*$$

$$\left. \frac{\partial^2 TP}{\partial T_R^2} \right|_{T_R = T_R^*} = \left. \frac{(MB - \omega p_r) F' T_R - 2F}{\theta T_R^3} \right|_{T_R = T_R^*} = \frac{(MB - \omega p_r) F'}{\theta (T_R^*)^2} < 0 \quad (45)$$

Hence Lemma 3 is proven.

Substituting Eq. (27) into Eq. (43) gives

$$\left[\left(\frac{h}{\theta^2} + \frac{c_p \gamma}{\theta(1+b)} + \frac{c_b \gamma \eta}{\theta \beta A} + \frac{(v_t + c_v)}{\theta} \right) (e^{\theta T_R} - \theta e^{\theta T_R} T_R - 1) + \left(\frac{A_r + A_s + F_s + c_f}{\frac{MB}{2} - \omega \left(\frac{p_s}{2\theta T_R} + \frac{h}{2\theta^2 T_R} \right)} (e^{\theta T_R} - 1) - \frac{h\omega}{2\theta} \right) \right] = 0 \quad (46)$$

Replacing the first-order Taylor approximation of the exponential function in Eq. (46), to get the initial solution in Newton-Raphson method, gives:

$$T_R^0 = \sqrt{\frac{2A_R}{(MB - \omega p_s) \left(h + \frac{c_p \gamma \theta}{(1+b)} + \frac{c_b \gamma \eta \theta}{\beta A} \right)}} \quad (47)$$

Then the procedure to derive the optimal solutions can be summarized as follows:

- First Calculate the optimal breeding period (T_S^*) using Eq. (36).
- Then obtain the optimal consumption period (T_R^*) applying Eq. (46).
- Finally derive the optimal selling price (p_r^*) using Eq. (27) where $T_R = T_R^*$.

6. Numerical Results

In order to illustrate the proposed model and applicability of its solution approach, numerical results for a specific type of newborn animals, broiler chicken, are provided. The parameters of the growth curve are estimated by real data for weight chart of the chickens in a poultry farm:

$A = 3200$, $b = 69.4$ and $g = 0.12$, $k = 0.12 * 365 = 43.8$. Then the growth function is $w_t = 3200(1 + 69.4e^{-43.8t})^{-1}$ (see Fig. 2).

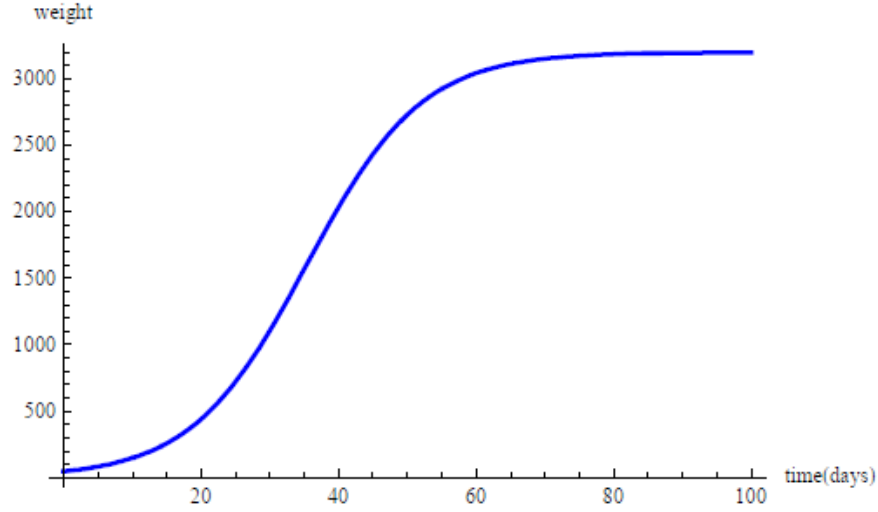


Fig. 2: Richard's Growth Curve

Moreover, the exponential breeding function $B(t)$ is ruled by: $B(t) = e^{76t}$.

Other identical parameters are taken from [Rezaei \(2014\)](#) and adapted to our model:

Supplier side: $c_p = 0.005\text{€/gr}$, $c_b = 0.02 \text{ €/unit item}$, $A_s = 5000\text{€/cycle}$, $p_s = 0.006\text{€/gr}$ and $\alpha = 1$, $F_s = 0.3\text{€/shipment}$, $v_t = 0.0002\text{€/gr}$, $c_f = 0.2\text{€/shipment}$, $c_v = 0.0005\text{€/gr}$

Retailer side: $h = 0.001\text{€/gr/year}$, $A_R = 400\text{€/cycle}$, $\theta = 0.2$, $MB = 100 \times 10^6\text{gr/year}$ and $\omega = 600 \times 10^7\text{€/gr}$.

Solving the outlined problem provides the following solutions:

Decentralized Case:

Supplier: $T_S = 0.08175 \text{ year}$, $Q_0 = 511489.35 \text{ gr}$, $TP_S = 111139.1\text{€}$

Retailer: $T_R = 0.1064359 \text{ year}$, $p_r = 0.01139229 \text{ €/gr}$, $Q_R = 3404404 \text{ gr}$, $TP_R = 163156.1\text{€}$.

This indicates that the retailer orders 3404.4 kg of slaughtered inventory at the beginning of each cycle. As the items enter the retailer's system, they are sold at the retailing price $p_r = 0.01139229$. The retailer's inventory is depleted to zero in $T_R = 39$ days due to the customer demand and deterioration. On the other hand, based on the retailer's order quantity, the supplier buys $Q_0 = 511.48$ kg of newborn chickens at the beginning of the breeding period. These items are flourished during $T_S = 30$ days. So the final weight of each growing item reaches 1.091 kg. The items are slaughtered and 7.85 % of the inventory is disposed after quality control as useless portion. As shown, the retailer's total profit per unit time is around

32% more than the supplier's one. Two major factors may have caused this: First, the supplier's inventory system runs based on the retailer's order. Second, the retailer's selling price is much more than the supplier's price. Fig. 3 projects total unit profit of the supplier's inventory system for this problem, which as shown, is a concave function of T_S . Also, Fig. 4 projects total unit profit of the retailer's inventory system for this problem, which as shown, is a concave function of p_r and T_R .

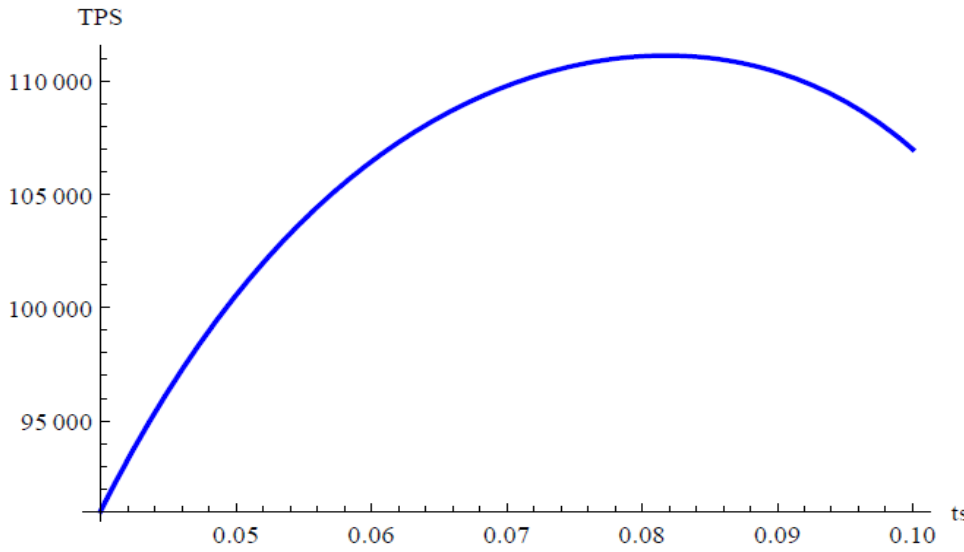


Fig. 3: Total unit profit of the supplier's inventory system

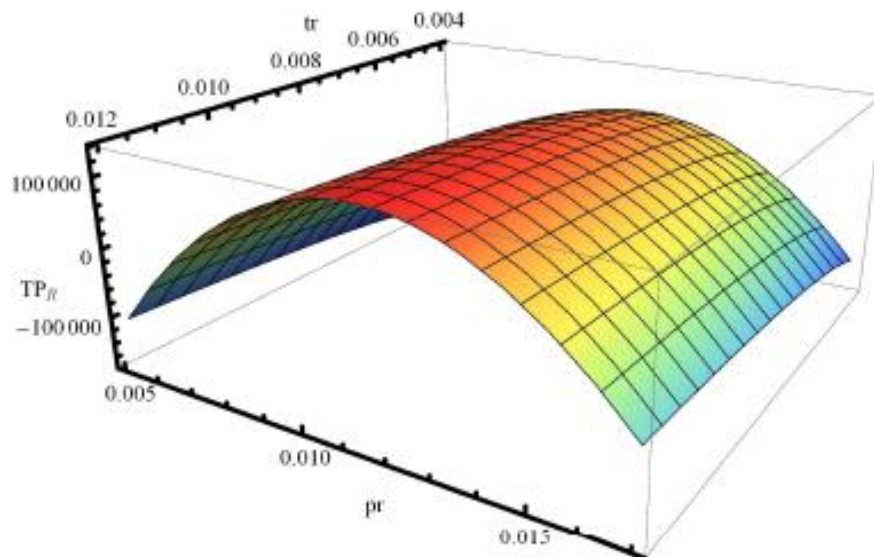


Fig. 4: Total unit profit of the retailer's inventory system

Centralized Case:

$T_S = 0.08175$ year, $T_R = 0.1118854$ year, $p_r = 0.011395\text{€}/\text{gr}$, $Q_0 = 511489.35$ gr

$Q_R = 3580411.68$ gr, $TP = 274627.09$ €, $OP_S = 111281.358$ € and $OP_R = 163343.729$ €.

The total unit profit of the chain rises in the centralized case, such that 331.89 € profit increase is observed. By the profit-sharing contract the obtained profit of each echelon enhances as well. Comparing the decentralized and centralized chain structures reveals that the chain participants benefit from the centralized case more than the other one. While the breeding period remains unchanged, an increase in selling price is observed. This causes demand reduction which results in a drastic increase in consumption period.

Table 2: Sensitivity Analysis on cost parameters- Decentralized Structure

parameter	% Changes	T_S	T_R	p_r	Q_0	Q_R	TP_S	TP_R
c_p	-50%	0.07471 (-8.6%)	0.1064359 (0%)	0.01139229 (0%)	597815.64 (22.92%)	3404404 (0%)	114828.91 (3.32%)	163156.1 (0%)
	-25%	0.07886 (-3.53%)	0.1064359 (0%)	0.01139229 (0%)	528025 (+8.57%)	3404404 (0%)	112895.09 (1.58%)	163156.1 (0%)
	+25%	0.08412 (+2.9%)	0.1064359 (0%)	0.01139229 (0%)	455851.44 (-6.27%)	3404404 (0%)	109383.10 (-1.58%)	163156.1 (0%)
	+50%	0.08575 (+4.91%)	0.1064359 (0%)	0.01139229 (0%)	436543.54 (-10.24%)	3404404 (0%)	107960.52 (-2.86%)	163156.1 (0%)
c_b	-50%	0.0885 (+8.3%)	0.1064359 (0%)	0.01139229 (0%)	406633.3 (-16.39%)	3404404 (0%)	113406.33 (+2.04%)	163156.1 (0%)
	-25%	0.0846 (+3.5%)	0.1064359 (0%)	0.01139229 (0%)	450063.94 (-7.46%)	3404404 (0%)	112183.80 (+0.94%)	163156.1 (0%)
	+25%	0.0795 (-2.7%)	0.1064359 (0%)	0.01139229 (0%)	518152.28 (6.54%)	3404404 (0%)	110216.64 (-0.83%)	163156.1 (0%)
	+50%	0.0776 (-4.81%)	0.1064359 (0%)	0.01139229 (0%)	546895.28 (12.45%)	3404404 (0%)	109383.10 (-1.58%)	163156.1 (0%)
A_S	-50%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	139712.96 (25.71%)	163156.1 (0%)
	-25%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	125431.58 (12.86%)	163156.1 (0%)
	+25%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	95712.99 (-13.88%)	163156.1 (0%)
	+50%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	82565.23 (-25.71%)	163156.1 (0%)
p_s	-50%	0.08175 (0%)	0.131980 (24%)	0.0098805 (-13.27%)	779806 (+60.34%)	5458621 (60.34%)	55825.2 (-49.77%)	274722 (68.38%)
	-25%	0.08175 (0%)	0.1171859 (10.1%)	0.01063698 (-6.63%)	613379 (26.12%)	4293634 (26.12%)	89633.7 (-19.35%)	215415 (+32.03%)
	+25%	0.08175 (0%)	0.09824 (-7.7%)	0.01214759 (+6.63%)	383775 (-21.09%)	2686415 (-21.09%)	120252.5 (+8.2%)	117913 (-27.73%)
	+50%	0.08175 (0%)	0.0916413 (-13.9%)	0.012901 (+13.25%)	297789 (-38.77%)	2084517 (-38.77%)	122508.6 (+10.23%)	79669.1 (-51.17%)

h	-50%	0.08175 (0%)	0.1045200 (-1.8%)	0.01138101 (-0.099%)	478369 (-1.64%)	3348572 (-1.64%)	110394.5 (-0.67%)	163792.4 (0.39%)
	-25%	0.08175 (0%)	0.1061165 (-1.3%)	0.01138625 (-0.053%)	480412 (-1.22%)	3362870 (-1.22%)	110539 (-0.54%)	163466 (0.19%)
	+25%	0.08175 (0%)	0.1087748 (+2.2%)	0.01139901 (+0.059%)	496607 (+2.11%)	3476237 (+2.11%)	112206 (+0.96%)	162829.8 (-0.2%)
	+50%	0.08175 (0%)	0.1122685 (+5.48%)	0.01140699 (+0.129%)	511976 (+5.27%)	3583816 (+5.27%)	113784.2 (+2.38%)	162471 (-0.42%)
A_R	-50%	0.08175 (0%)	0.0754524 (-29.11%)	0.01141268 (0.179%)	344478.4 (-29.17%)	2411339 (-29.17%)	87522 (-21.25%)	165946 (+1.71%)
	-25%	0.08175 (0%)	0.092279 (-13.3%)	0.01138295 (-0.082%)	421515.5 (-13.33%)	2950597 (-13.33%)	102303.5 (-7.95%)	164428.7 (0.78%)
	+25%	0.08175 (0%)	0.118878 (+11.69%)	0.0114005 (0.072%)	543394 (+11.73%)	3803741 (+11.73%)	117174 (+5.43%)	162030.3 (-0.69%)
	+50%	0.08175 (0%)	0.1301178 (+22.25%)	0.011408 (0.138%)	594898 (+22.32%)	4164267 (+22.32%)	121641.7 (+9.45%)	161018.8 (-1.31%)
F_S	-50%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	111141.8 (0.00242%)	163156.1 (0%)
	-25%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	111141.7 (0.00233%)	163156.1 (0%)
	+25%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	111141.6 (0.00224%)	163156.1 (0%)
	+50%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	114337.5 (2.8778%)	163156.1 (0%)
v_t	-50%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	114337.7 (2.87%)	163156.1 (0%)
	-25%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	112738.4 (1.439%)	163156.1 (0%)
	+25%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	109539.9 (-1.438%)	163156.1 (0%)
	+50%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	107940.6 (-2.87%)	163156.1 (0%)
c_f	-50%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	111140.1 (0.00089%)	163156.1 (0%)
	-25%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	111139.6 (0.0004%)	163156.1 (0%)
	+25%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	111138.7 (-0.0003%)	163156.1 (0%)
	+50%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	111138.2 (-0.0008%)	163156.1 (0%)
c_v	-50%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	119135.5 (7.194%)	163156.1 (0%)
	-25%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	115137.3 (3.597%)	163156.1 (0%)
	+25%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	107140.9 (-3.597%)	163156.1 (0%)
	+50%	0.08175 (0%)	0.1064359 (0%)	0.01139229 (0%)	486345.3 (0%)	3404404 (0%)	103142.8 (-7.194%)	163156.1 (0%)

Table 3: Sensitivity analysis on cost parameters- Centralized Structure

parameter	% Changes	T_S	T_R	p_r	Q_0	Q_R	TP	OP_S	OP_R
	-50%	0.07472	0.11433	0.011405	642431	3658465	277813	113952	163686.8(

c_p		(-8.59%)	(+2.19%)	(+0.089%)	(25.6%)	(+2.18%)	(1.16%)	(2.40%)	0.21%
	-25%	0.07887 (-3.52%)	0.113032 (+1.03%)	0.011340 (0.042%)	561002 (9.68%)	3616932 (1.02%)	276137.5 (+0.55%)	112206 (0.831%)	163897.5 (0.339%)
	+25%	0.08412 (+2.89%)	0.11078 (-0.98%)	0.011390 (-0.041%)	474662 (-7.2%)	3544966 (-0.99%)	273116.6 (-0.55%)	108966.7 (-2.08%)	164307.5 (0.59%)
	+50%	0.08575 (+4.89%)	0.109922 (-1.75%)	0.011386 (-0.072%)	450980 (-11.83%)	3517038 (-1.77%)	271853.4 (-1.01%)	107675.8 (-3.24%)	164438 0.67%
c_b	-50%	0.08854 (8.31%)	0.11337 (+1.33%)	0.011401 (+0.054%)	433283 (-15.29%)	3627673 (1.32%)	276577 (0.71%)	112672.4 (1.25%)	163843.6 0.306%
	-25%	0.0846 (3.49%)	0.11255 (0.6%)	0.011397 (+0.025%)	476145 (-6.91%)	3601894 (+0.6%)	275533.4 (0.33%)	111540.6 (0.233%)	163990.6 0.396%
	+25%	0.0795 (-2.73%)	0.111292 (-0.53%)	0.011392 (-0.021%)	542128 (+5.99%)	3561794 (-0.52%)	273830.7 (-0.29%)	109745.7 (-1.38%)	164209.5 0.53%
	+50%	0.0776 (-4.99%)	0.11081 (-0.96%)	0.01139 (-0.04%)	569492 (11.34%)	3544966 (-0.99%)	273116.6 (-0.55%)	108966.7 (-2.08%)	164307.5 0.59%
A_S	-50%	0.08175 (0%)	0.0827 (-26.03%)	0.011275 (-1.05%)	377530 (-26.19%)	2642702 (-26.19%)	283360 (3.18%)	129197.7 (16.1%)	152628.4 -6.56%
	-25%	0.08175 (0%)	0.09844 (-12.01%)	0.01133 (-0.488%)	449855 (-12.05%)	3148972 (-12.05%)	278691.6 (1.48%)	120061.5 (7.89%)	157904.4 -3.33%
	+25%	0.08175 (0%)	0.12470 (11.46%)	0.01144 (+0.473%)	570208 (11.48%)	3991443 (11.48%)	270700 (-1.43%)	99786 (-10.33%)	171903 5.24%
	+50%	0.08175 (0%)	0.13462 (20.32%)	0.01149 (0.0844%)	615577 (20.35%)	4309025 (20.35%)	267596.6 (-2.56%)	89915.3 (-19.2%)	179580 9.94%
p_s	-50%	0.08175 (0%)	0.11125 (-0.56%)	0.00978 (-14.09%)	681355 (33.21%)	4769466 (33.21%)	314475.5 (14.51%)	53971.5 (-51.5%)	267965.4 64.05%
	-25%	0.08175 (0%)	0.11159 (-0.26%)	0.010591 (-7.05%)	596703 (16.66%)	4176908 (16.66%)	298162.6 (8.57%)	88023.6 (-20.9%)	213539.3 30.73%
	+25%	0.08175 (0%)	0.11222 (0.3%)	0.012198 (+7.05%)	425713 (-16.77%)	2979977 (-16.77%)	243869 (-11.2%)	121341.2 (9.04%)	120384.3 -26.3%
	+50%	0.08175 (0%)	0.11255 (0.6%)	0.013002 (+14.11%)	339373 (-33.65%)	2375603 (-33.65%)	205833 (-25.05%)	121864.2 (9.51%)	80201.8 -50.9%
h	-50%	0.08175 (0%)	0.133188 (19.04%)	0.011368 (-0.231%)	619158 (21.05%)	4334088 (21.05%)	278197 (1.3%)	111332.5 (0.046%)	167007.5 2.243%
	-25%	0.08175 (0%)	0.11941 (+6.73%)	0.011384 (-0.095%)	550056 (7.54%)	3850375 (7.54%)	276604.4 (0.72%)	110903 (-0.34%)	165794 1.50%
	+25%	0.08175 (0%)	0.108014 (-3.46%)	0.01140 (0.129%)	490467 (-4.11%)	3433257 (-4.11%)	272457.5 (-0.79%)	110447.9 (-0.749%)	161988 -0.83%
	+50%	0.08175 (0%)	0.106850 (-4.5%)	0.011435 (0.353%)	482028 (-5.76%)	3374180 (-5.76%)	269986 (-1.69%)	110446.7 (-0.75%)	159439.8 -2.39%
A_R	-50%	0.08175 (0%)	0.10986 (-1.81%)	0.011386 (-0.074%)	502180 (-1.82%)	3515248 (-1.82%)	275231 (+0.22%)	94956.4 (-14.67%)	181948.6 11.39%
	-25%	0.08175 (0%)	0.110878 (-0.9%)	0.011390 (-0.037%)	506835 (-0.91%)	3547830 (-0.91%)	274929 (0.11%)	105010.7 (-5.635%)	170565 4.421%
	+25%	0.08175 (0%)	0.112903 (+0.91%)	0.01139 (0.036%)	516042 (+0.89%)	3612277 (0.89%)	274325 (-0.11%)	114263.7 (2.68%)	159737 -2.208%
	+50%	0.08175 (0%)	0.11388 (+1.79%)	0.011403 (0.073%)	520594 (+1.78%)	3644143 (1.78%)	273995.4 (-0.23%)	116934.5 (5.08%)	156483.3 -4.20%

To deep dive into the problem, it is efficient to investigate the result of changes in input parameters. In order to study the components of profit function of the echelons, analyzing the changes in the values of c_p , c_b , A_S and as well as p_s , h and A_R seems advantageous. The sensitivity analysis is carried out by changing each parameter by -50%, -25%, +25% and +50%,

taking one parameter at a time and keeping the others constant. The related results for the decentralized and centralized chain structures are provided in Table 2 and 3. Figures (5) to (15) depict the results graphically.

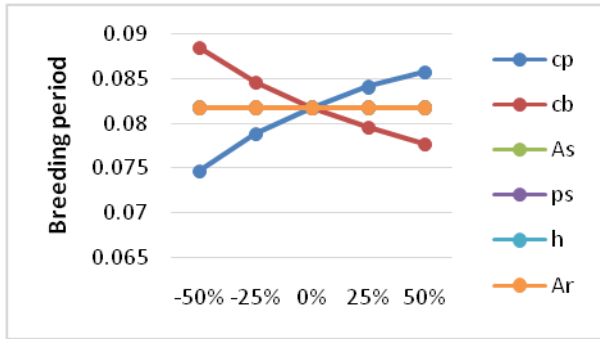


Fig 5: Changes in the optimal T_S with variations in cost parameters – Decentralized case

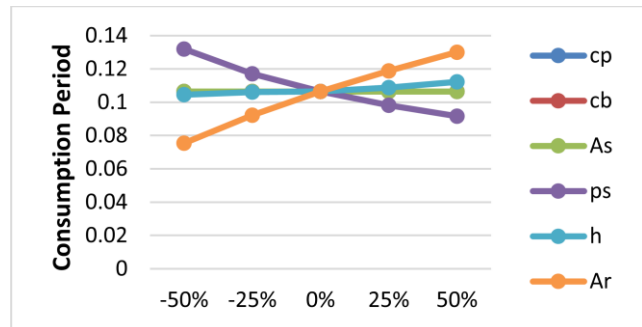


Fig 6: Changes in the optimal T_R with variations in cost parameters – Decentralized case

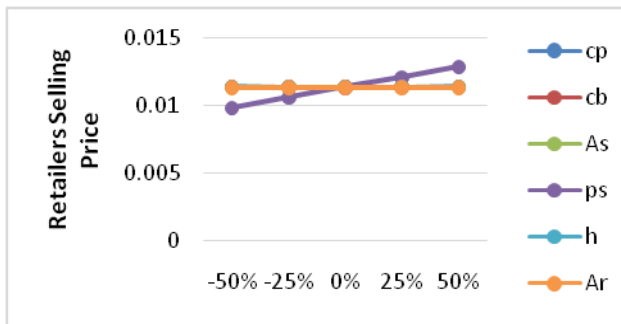


Fig 7: Changes in the optimal P_R with variations in cost parameters – Decentralized case

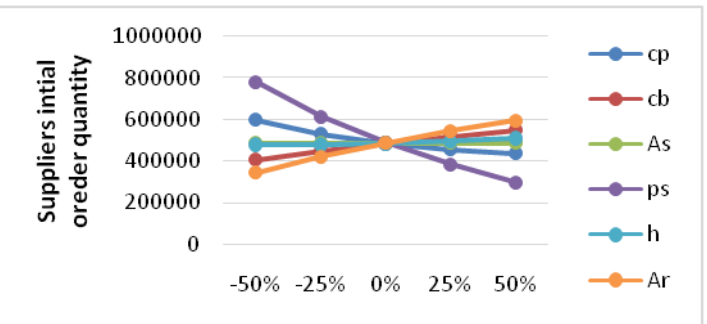


Fig 8: Changes in the optimal Q_0 with variations in cost parameters – Decentralized case

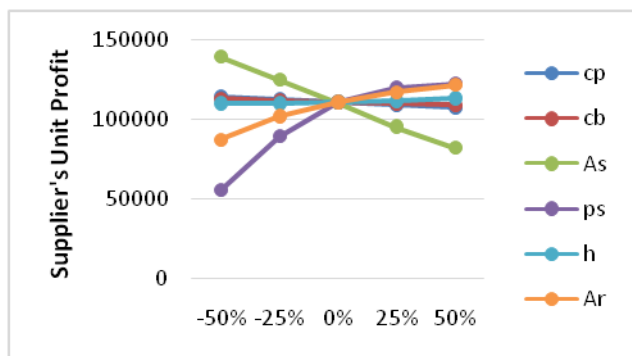


Fig 9: Changes in the optimal TP_S with variations in cost parameters – Decentralized case

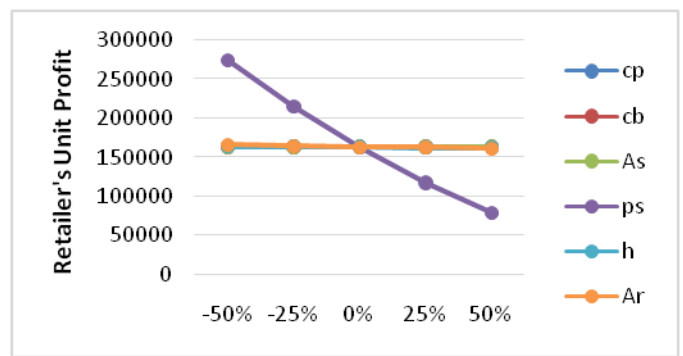


Fig 10: Changes in the optimal TP_R with variations in cost parameters – Decentralized case

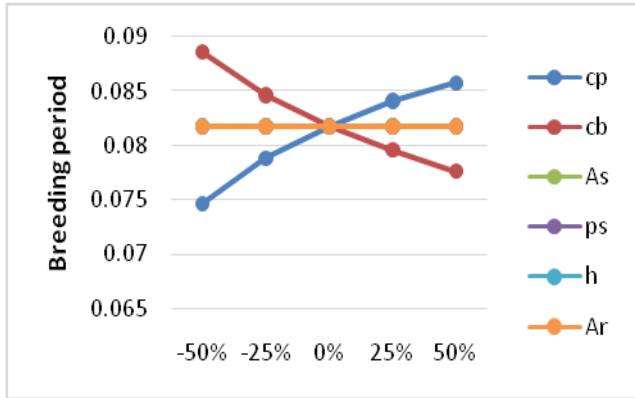


Fig 11: Changes in the optimal T_S with variations in cost parameters – Centralized case

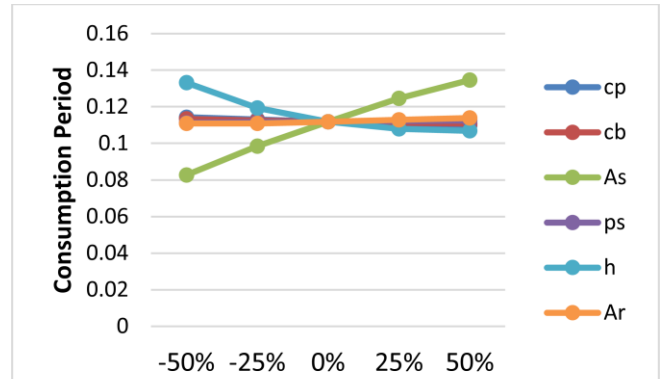


Fig 12: Changes in the optimal T_R with variations in cost parameters – Centralized case

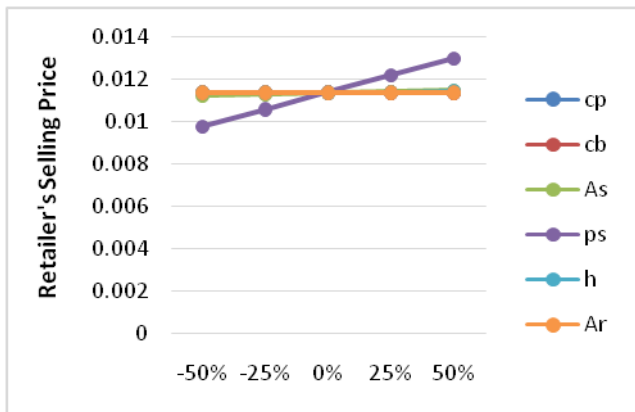


Fig 13: Changes in the optimal p_r with variations in cost parameters – Centralized case

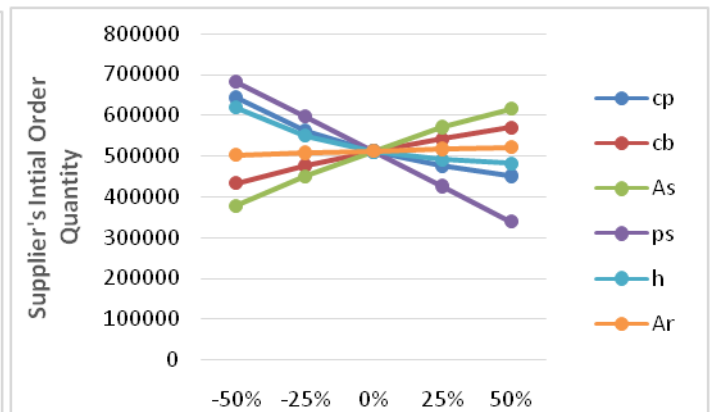


Fig 14: Changes in the optimal Q_0 with variations in cost parameters – Centralized case

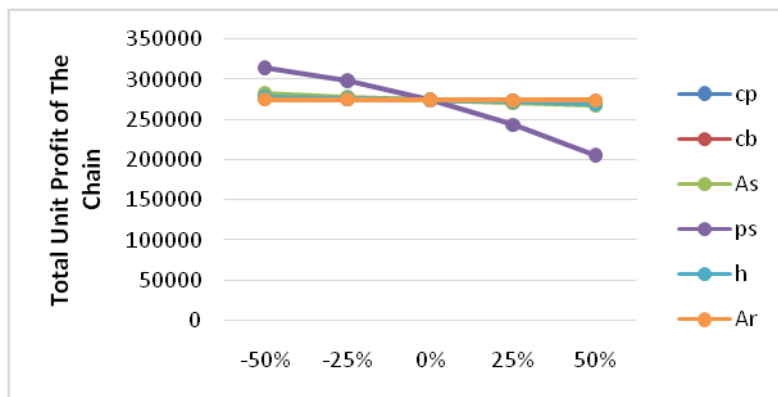


Fig-15: Changes in the optimal TP with variations in cost parameters – Centralized case

Analyzing the numerical results provides the following managerial insights:

- There is no doubt that decreasing the unit purchasing cost effectively enhances the profit of the system. In decentralized structure, varying c_p has no impact on the retailer's inventory system. This is what the related equations projected as well; optimizing the retailer's system is totally independent of the supplier's one. As the supplier's purchasing cost rises, Q_0 gets smaller values. Then, the breeding period needs to increase in order to meet the retailer's ordering quantity. Therefore, if the supplier can choose among different newborn-chicken providers, selecting the one with the lowest purchasing cost has two positive effects: Enhancing the profit of the system as much as possible and lowering the risk of poultry disease during growth by shortening the breeding period. As figure 8 projects, the unit purchasing cost has the most notable impact on in comparison with other two costs of the supplier.

In centralized structure, the retailer's ordering size and inventory cycle are also affected. The retailing price is not very sensitive to changes in c_p . The obtained profits of the retailer and the supplier are computed according to the total profit of the chain as well as the profit-sharing ratio (ρ). So the pattern of changes in OP_S and OP_R cannot be studied with respect to changes in c_p , isolatedly. That is because, as c_p changes, TP_S and TP_R also take different values altering the profit-sharing ratio. For instance, by 25 percent raising c_p , the total profit declines. On the other hand, the supplier's sharing ratio changes from 0.4285 to 0.4219 This amplifies the reduction in OP_S . Contrarily, an increase in OP_R is observed which conveys the fact that the reduction in ρ outweighs the decrease in TP .

- Obviously, decreasing the unit-item breeding cost raises TP_S in decentralized chain structure. Similarly, changes in c_b have no impact on the retailer's inventory system. By decreasing c_b , an increase in T_S is observed. That is because by lowering c_b , the breeding costs during the growth period decreases. So the system has the chance to lengthen the growth period. As the retailer's order size is fixed, a longer T_S would result in a lower Q_0 . Then, if the supplier confronts a serious limitation in periodic purchasing budget, it can decrease Q_0 by decreasing the breeding costs. This is possible by amending the holding and flourishment facilities as well as managing the feeding costs. In centralized structure, the

retailer's ordering size and inventory cycle are also affected. The retailing price is not really sensitive to changes in c_b . Since T_R and then Q_R increase by decreasing c_b , the reduction in Q_0 , is lower than the decentralized case.

- Analyzing A_S in decentralized case provides surprising results: The variation in supplier's ordering cost not only cannot affect the retailer's system, but also has no impact on the optimal decision variables of the supplier. Remember that the supplier's inventory system is recurred every T_R units of time and its ordering cost will be paid based on T_R . So the changes in A_S will not influence T_S . Since the retailer's order size and the supplier's breeding period remain fixed, no change in Q_0 is perceived as well. In centralized structure the patterns differ. The breeding period is independent of changes in A_S . On the other hand, as A_S takes larger values, T_R increases to diminish the ordering costs. Raising T_R results in higher Q_R , which increases Q_0 to meet the order size. Note that due to this direct relation, the percentage of changes in Q_0 and Q_R are equal. In the case of capacity or cyclic purchasing budget constraints in either of the echelons, choosing the option with lower A_S seems beneficial.
- As p_s increases, the unit profit of the supplier rises, while the retailer faces a drop in its profit. That is because for the retailer, p_s plays the same role as c_p in the supplier's system. As p_s takes larger values, the retailer has to raise his selling price. Then its demand experiences a notable reduction. That is why the changes in retailer's profit are much more than the supplier's. T_S is not sensitive to changes in p_s . By increasing p_s , the retailer's order quantity and inventory cycle get smaller values. So the reduction in the supplier's order quantity is expected. As figures 8, 9 and 14 project, p_s is the most influential parameter on changes in TP_S , TP_R and TP .

In the centralized case, the changes in the variables are not as large as the decentralized case which is the results of integration.

- Altering h reveals interesting insights. In classic EOQ, raising h decreases the ordering quantity and inventory cycle. This is not always authentic when the demand rate is price dependent. As h gets larger values, the retailer has to raise its selling price due to cost increases. This can notably decrease the demand rate. So if the impact of raising h on

demand reduction is stronger than its effect on inventory level decrease, the retailer's inventory cycle and ordering quantity will increase. This is what is observed in our problem as well. Raising Q_R , increases Q_0 to meet the order size. Note that due to this direct relation, the percentage of changes in Q_0 and Q_R are equal.

In the centralized case, the impact of inventory level decrease outweighs the effect of demand reduction, so the results are reverse.

- By increasing A_R in either of the chain structures (centralized and decentralized), the retailer's ordering quantity and inventory cycle rise. This is what we expect in classical inventory models as well. Again the changes in decentralized case are more drastic than the centralized one. Similarly, T_S is insensitive to changes in A_R .

The represented results convey two general messages: "The retailing price is not highly influenced by the most parameters of the system" and "The changes in most of the cost parameters do not alter the breeding period". These come as no surprise:

In most cases, regulations intervene in pricing of this kind of items (such as meat and chicken) and the companies face some restrictions in pricing. Moreover, as the items from different retailers are almost identical, the market is very price sensitive and a slight increase in the retailing price can shift the customers from one seller to another.

The breeding period is determined based on the growth pattern of the inventory rather than external factors. As the growth function imposes limitations on the speed of the weight increase and also the ultimate weight, the breeding period is independent of the retailer's cost parameters. Furthermore since quality standards and negative impact of overbreeding is considered in the model, the changes in the supplier's purchasing cost and even breeding cost do not change the breeding period in a severe scale in comparison with other variables.

7. Conclusion

In this paper, sustainable inventory control and pricing policies were studied in a coordinated sustainable two-echelon supply chain under the effect of carbon emission. The chain involved a class of inventory called growing items. These items were grown-up in the supplier's system, slaughtered and finally sent to the retailer. The slaughtered items were transported through a

SSSD policy. Transportation and carbon emission cost were utilized in this study. The slaughtered items experienced deterioration in the retailer's system. The problem was analyzed under decentralized and centralized chain structures. An analytic solution approach was developed and validated by experimental results. The results verified the claim that the centralized chain structure can effectively enhance the profit of the system. It was shown that the unit purchasing cost of each echelon has a major effect on the profit in comparison to the other cost factors. Moreover, sensitivity analysis on key parameters of the model provided other fruitful managerial insights.

Other growth curves and feeding functions as well as other deterioration patterns might be applied in future studies. Furthermore, backordering and uncertain demand can be embedded to the assumptions. Analyzing growth through three-level supply chain is another promising research direction.

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