DECISION AND COORDINATION ANALYSIS OF EXTENDED WARRANTY SERVICE IN A REMANUFACTURING CLOSED-LOOP SUPPLY CHAIN WITH DUAL PRICE SENSITIVITY UNDER DIFFERENT CHANNEL POWER STRUCTURES*

XIAODONG ZHU*; LIAN DING; YAJIE GUO; HUITING ZHU

Abstract. This paper studies the impact of dual price sensitivity (product price sensitivity and extended warranty service price sensitivity) on the decision of the closed-loop supply chain (CLSC) where manufacturers are responsible for recycling and providing extended warranty service under different channel power structures. First, the dynamic game model is used to describe the three channel power structure models of centralized (C), Manufacturer-led Stackelberg (MS), and Retailer-led Stackelberg (RS). Then, mathematical optimization and reverse induction are used to derive supply chain decision-making and performance levels. On this basis, we further designed a contract to coordinate the supply chain, and use numerical examples to analyze and verify the impact of remanufacturing cost savings and dual price sensitivity on the supply chain system. Research shows that channel power structures, remanufacturing cost savings and dual price sensitivity all have an important impact on the supply chain and corporate profits.

Keywords: remanufacturing closed-loop supply chain; extended warranty service; power structure; dual price sensitivity; coordination contract

1. INTRODUCTION

As we all know, the rapid economic growth has brought about some significant problems, such as resource depletion and environmental pollution. The traditional production mode has brought tremendous pressure to the ecosystem, and it will inhibit economic and social development. It is
inevitable to build a circular economy development model that combines environmental maintenance. The development model of circular economy aims to convert products at the end of their useful life into remanufactured resources to minimize the waste of resources [1]. Governments of various countries are taking measures actively to achieve this environmental goal [2]. A more common measure is the implementation of the Extended Producer Responsibility (ERP). According to this system, manufacturers are obliged to recycle and remanufacture products. Remanufacturing can not only reduce damage of environment and waste of resources, but also reduce production costs and improve economic efficiency of enterprises. Many world-renowned companies, such as HP, Dell, Canon and Xerox obtain high profits through recycling and remanufacturing [3].

The 2020 China-EU Auto Parts Remanufacturing Summit pointed out that the insufficient social awareness of remanufactured products is the main difficulty faced by the remanufacturing industry in the development process. Consumers misunderstanding of remanufactured products is the main reason that the development of remanufacturing industry is restricted severely. How to alleviate consumers concern to the quality of remanufactured products has become an important issue that needs to be solved urgently. The extended warranty policy is a service plan in which the supplier promises to provide consumers with obligations or paid services on the technical performance, use effect and maintenance of the product within a certain period of time after the expiration of the original warranty [4]. The extended warranty service is a signaling mechanism that can transmit product quality information, which can eliminate consumers distrust of product quality and increase consumers recognition of enterprises [5]. Many companies provide extended warranty services [6, 7], such as, Huawei offers a certain period of extended warranty services when selling some remanufactured products; HP and Apple sell extended warranty service at lower prices for remanufactured products. As a marketing strategy, the extended warranty service can improve consumers cognition of remanufactured product, and plays a key role in stimulating the remanufactured market.

In the past 20 years, global companies and academia have conducted in-depth research on supply chain management, but it still faces new challenges: (1) Each subject of the supply chain needs to face complex and diverse channel power structures (such as centralized decision-making, retailer-led, manufacturer-led) and proposed countermeasures. (2) Consumers have strong sensitivity to product and service prices, which in turn affects the market demand structure. (3) In a CLSC system, there is a difference in production cost between new products and remanufactured products. The fluctuation of cost difference influences the development trend of remanufacturing industry. At the same time, the channel power structure and consumers price sensitivity increase the complexity of the CLSC. This paper focuses on solving the following problems:

- Under different channel power structures, how does each subject of the supply chain determine product prices, extended warranty service prices and product recycling strategies?
- What is the impact of key parameters on the decision-making of supply chain subjects?
- How does the coordination contract be designed to eliminate marginal utility and realize Pareto improvement of supply chain?

This paper constructs a CLSC that includes one manufacturer and one retailer, and explores the impact of channel power structures, remanufacturing cost savings and dual price sensitivity on the decision-making of CLSC. The main contributions of this paper are as follows: (1) Introducing the extended warranty service to the CLSC, we provide guidance for the development of extended warranty service industry. Previous studies mostly focused on the forward supply chain
and ignored the economic and environmental benefits of providing extended warranty service for
remanufactured products. (2) By studying the influence of double price sensitivity on the profit,
we provide guidance for enterprises to formulate reasonable business strategy. Previous studies
mostly focused on the sensitivity of consumers to product price, and ignored the impact of con-
sumers’ multiple price sensitivity on the supply chain. (3) The influence of different channel power
structures on the decision-making is demonstrated, and a coordination contract is proposed to
realize the perfect coordination of the CLSC, so that the profit of each subject can achieve Pareto
improvement.

The rest of the paper is organized as follows: the next section summarizes the literature in
related fields. Section 3 introduces the assumptions and symbols of the analysis model. Section 4
solves the best strategies under different channel power structures. Section 5 proposes coordination
contracts to improve the coordination of CLSC. Section 6 verifies the conclusion of section 4, and
the sensitivity analysis of key factors is carried out. Section 7 summarizes the whole paper.

2. Literature review

This study related to three streams of literature: closed-loop supply chain (CLSC), extended war-
ranty service, and channel power structure.

2.1. Closed-loop supply chain (CLSC)

With the rapid economic development, environmental pollution and resource shortages have
become more and more serious [8]. A large number of scholars have conducted in-depth research
on product recycling and remanufacturing. Some scholars studied the differential pricing strategy
of the CLSC from the perspective of dual channels [9-11]. Yang et al. [12] studied the relationship
between the revenue and price sensitivity of remanufacturing product in the context of multiple
retailers. Wang et al. [13] studied the influence of consumers willingness to pay on the decision
of each member of the CLSC, and determined the manufacturers optimal recycling strategy. Lang et
al. [14] studied the influence of retailers dispersion and concentration on the decision-making of
CLSC. Huang and Wang [15] studied the influence of consumers behavior preferences and reman-
ufacturing costs on the three supply chain of complete remanufacturing, partial remanufacturing
and no remanufacturing. Qu et al. [16] studied the optimal decision-making and social welfare
of the green CLSC. Jian et al. [17] studied the impact of fairness concerns on economic and en-
vironmental benefits of CLSC. Hong et al. [18] studied the influence of voluntary information
disclosure and mandatory information disclosure on the operation strategy of CLSC. Liu et al.
[19] studied the impact of capacity uncertainty on recovery mode and remanufacturing strategy of
CLSC. Mirzagoltabar et al. [20] develops two hybrid heuristic algorithms to study the decision of
multi-objective CLSC with uncertain price and demand. Some scholars have designed coordina-
tion strategies of CLSC [21-24]. Existing scholars rarely introduce dual price sensitivity into the
closed-loop supply chain for in-depth analysis. At the same time, this paper designs a two-part
tariff to improve system coordination while eliminating double marginal utility.
2.2. Extended warranty service

Due to consumers concerns about the quality of products, extended warranty services have become more and more important in commercial competition. Many scholars have conducted in-depth research on extended warranty services. Some scholars studied different forms of extended warranty, such as free replacement extended warranty, combination extended warranty and proportional extended warranty, and explore the effects of different extended warranty services on supply chain [25,26]. Some scholars studied the diversity and complexity of extended warranty suppliers, such as manufacturers providing extended warranty, retailers providing extended warranty, and third-party repairers providing extended warranty [27-29]. Ranjan and Jha [30] studied the impact of consumers uncertainty of the holding time on the extended warranty. Heidary and Aghaie [31] studied the impact of consumers loss aversion on the decision-making of extended warranty. Wang et al. [32] studied the impact of three different extended warranty strategies on the benefits of members in the forward supply chain based on product performance differences. Bian et al. [33] studied the impact of traditional and new extended warranty strategies on the benefits of supply chain, and worked out the best pricing strategy. Wang et al. [34] designed the optimal extended warranty period and extended warranty price from the perspective of product failure rate. Wang et al. [35] studied the decision-making and coordination of e-commerce supply chain that are dominated by e-commerce platforms and provide extended warranty services. Most scholars focus on the research of forward supply chain but ignore that providing extended warranty service for remanufactured products in CLSC will bring great social benefits.

2.3. Channel power structure

With the rapid development of the market economy, the channel power structure of the market is characterized by diversification and complexity. Most of the current research focuses on the impact of a specific power structure on production, recycling, and pricing [36,38]. A few scholars have studied the influence of different channel power structures on the decision-making of each subject in the supply chain. Zheng et al. [38] studied the influence of the two situations of demand information symmetry and demand information asymmetry on the decision-making of manufacturers under different channel power structures. Gao et al. [39] studied the effectiveness of recycling efforts and sales efforts to expand market demand under different power structures. Chakraborty et al. [40] studied the impact on the economic benefits of the supply chain when different entities gain leadership in a three-level CLSC. Liu et al. [41] studied the impact of different channel power structures on the profits under the premise that a third party is responsible for recycling waste electronic products. Chen et al. [42] studied the impact of process innovation of remanufactured products on pricing and profit under different power structures. Mondal and Giri [43] studied the influence of subsidy and power structure on the profit of each subject under government subsidies. At present, extended warranty service for remanufactured products is still in the initial development stage, and the market power structure has the characteristics of diversity and complexity. This paper makes a comparative analysis of three different powers to provide guidance and suggestions for the development of extended warranty industry.

In summary, in previous studies, most scholars only discussed the extended warranty service in the forward supply chain. There are few literatures on the extended warranty service in the remanufacturing CLSC. In commercial practice, the extended warranty service of the CLSC has strong
market demand and business prospects. This paper studies the optimal strategy of CLSC with extended warranty service under different channel power structures, and a coordination strategy is proposed to realize the Pareto optimization of CLSC.

3. Problem statement

3.1. Problem description

This paper constructs a CLSC that includes one manufacturer and one retailer. The manufacturer incorporates the remanufacturing process of old products into its original production system. Considering the impact of different channel power structures on recycling strategies, extended warranty strategies, and product optimal pricing, the decision-making model of the CLSC includes centralized decision-making and decentralized decision-making. Decentralized decision-making includes manufacturer-led Stackelberg and retailer-led Stackelberg. The CLSC model is shown in Figure 1.

The research in this paper is based on the following assumptions.

Assumption 1: In order to describe the relationship between product demand and price succinctly, the total market demand is known as 1, and consumer demand is linear. These have been widely assumed in the literature [44-46]. So, the market demand function of productions and extended warranty are:

\[ q = 1 - \alpha p \]  
\[ q_e = q - \frac{dp_e}{t} \]

where \( \alpha \) represents the sensitivity coefficient of consumers to remanufactured product price, \( 0 < \alpha < 1 \), \( d \) represents the sensitivity coefficient of consumers to the price of extended warranty, \( 0 < d < 1 \), \( t \) represents the length of extended warranty, \( t > 0 \), \( w \) represents the wholesale price of unit remanufactured product, \( m \) represents the profit margin of unit remanufactured product, so
\[ p = w + m \] represents the retail price of unit remanufactured product \([47, 48]\), \( p_e \) represents the price of extended warranty.

**Assumption 2:** It is assumed that the manufacturer acts as the main body of recycling and extended warranty \([49]\). Since the manufacturer is responsible for the processing of remanufactured products, all recycled materials will eventually flow to the manufacturer, so this paper assumes that the manufacturer is the main body of recycling. Compared with ordinary products, the structure of remanufactured products is more complex, and the manufacturer has a higher understanding of the structure and function of the remanufactured product. Therefore, this paper assumes that the manufacturer is the main service provider for the extended warranty of remanufactured products.

**Assumption 3:** It is assumed that the actions of the participants have a sequential order, and the later actors can observe the actions selected by the first actor. That is the dynamic game between the different subjects of the CLSC \([50, 51]\). In a real market environment, information often circulates with each other. Later decision makers are often able to learn the decision information of the early decision makers. Therefore, this assumption is in line with reality.

**Assumption 4:** Based on previous studies, it is assumed that the product recovery rate is denoted by \( \tau = \sqrt{I/L} \) \([52, 53]\), where \( L \) represents remanufactured product recovery investment cost coefficient, \( I \) represents the total investment in the cycle activities of recycler. The number of failures is convex and increases with the length of extended warranty \([54, 55]\). So the number of failures denoted by \( \theta = kt^2 \), where represents the failure rate of remanufactured product. The total cost of the extended warranty service is \( c_e = \theta \gamma \), \( \gamma \) is the average cost of repair.

**Assumption 5:** In order to ensure the existence of an optimal solution under different channel power structures, namely, to ensure that the Hessian matrix of the revenue matrix is negative definite, the following conditions should be met \( 4dL - L\alpha - d\alpha \Delta^2 > 0 \) (See proof in Appendix A, Appendix B, and Appendix C.).

Based on the above assumptions, the profit models of different entities are summarized.

\[
\pi_M = (p_e - c_e) \left( -\frac{dp_e}{t} + \alpha(-(m + w)) + 1 \right) + (-c + \tau\Delta + w)(1 - \alpha(m + w)) - L\tau^2 \tag{3}
\]

The first part of the formula (3.3) is manufacturers revenue of remanufactured product, the second part is the revenue of extended warranty service, and the last part is the cost of recovery.

\[
\pi_R = m(1 - \alpha p) \tag{4}
\]

The formula (3.4) is retailers profit on sales.

\[
\pi_S = (p_e - c_e) \left( -\frac{dp_e}{t} + \alpha(-(m + w)) + 1 \right) + (1 - \alpha(m + w))(-c + (m + w) + \tau\Delta) - L\tau^2 \tag{5}
\]

The first part of the formula (3.5) is systems revenue of remanufactured products, the second part is the revenue of extended warranty service, and the last part is the cost of recovery.

### 3.2. Symbols

The various symbols used in the paper are listed and defined in Table 1.
Table 1. Notations

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>Unit production cost of the remanufactured product</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Consumers price sensitivity coefficient to remanufactured product</td>
</tr>
<tr>
<td>$k$</td>
<td>Failure rate of remanufactured product</td>
</tr>
<tr>
<td>$L$</td>
<td>Product recovery investment cost coefficient</td>
</tr>
<tr>
<td>$I$</td>
<td>Total investment in the cycle activities of recycler</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Remanufacturing cost savings</td>
</tr>
<tr>
<td>$d$</td>
<td>Consumers price sensitivity coefficient to extended warranty service</td>
</tr>
<tr>
<td>$t$</td>
<td>The length of extended warranty</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The number of failures</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The average cost of repair</td>
</tr>
<tr>
<td>$w$</td>
<td>The wholesale price of unit remanufactured product</td>
</tr>
<tr>
<td>$m$</td>
<td>The profit margin of unit remanufactured product</td>
</tr>
<tr>
<td>$c_e$</td>
<td>The total cost of the extended warranty</td>
</tr>
<tr>
<td>$p$</td>
<td>The retail price of unit remanufactured product</td>
</tr>
<tr>
<td>$p_e$</td>
<td>The price of the warranty service</td>
</tr>
<tr>
<td>$q$</td>
<td>Demand for remanufactured product</td>
</tr>
<tr>
<td>$q_e$</td>
<td>Demand for extended warranty service</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Waste recovery rate</td>
</tr>
<tr>
<td>$\pi_M$</td>
<td>Manufacturers profit</td>
</tr>
<tr>
<td>$\pi_R$</td>
<td>Retailers profit</td>
</tr>
<tr>
<td>$\pi_S$</td>
<td>Total profit</td>
</tr>
</tbody>
</table>

4. Equilibrium Analysis

4.1. Power structure of different channels

4.1.1. Centralized decision-making (C)

Under centralized decision-making mode, manufacturer and retailer cooperate with each other to jointly pursue the maximization of total profit. The optimization problem can be defined as:

$$\pi_C^S = (p_e - c_e) \left( -\frac{dp_e}{t} + \alpha(-m + w)) + 1 \right) + (1 - \alpha(m + w))(c - \alpha m + w + \tau\Delta) - L\tau^2$$  \hspace{1cm} (6)

**Proposition 1:** There is a unique optimal solution for the centralized decision-making CLSC, and the equilibrium results are as follows (See proof in Appendix A.).

$$p_C = \frac{d \left( L \left( 2ac + \alpha \gamma kt^2 + 2 \right) - \alpha \Delta^2 \right) - \alpha L t}{\alpha \left( \alpha(-d)\Delta^2 + 4dL - \alpha L t \right)}$$  \hspace{1cm} (7)

$$p_e^C = \frac{-t \left( 2L \left( ac - 2\gamma dt + \alpha \gamma kt^2 - 1 \right) + \alpha \gamma dt\Delta^2 \right)}{-2ad^2 + 8dL - 2\alpha L t}$$  \hspace{1cm} (8)
\[
\tau^C = \frac{d\Delta (-2ac + \alpha\gamma(-k)t^2 + 2)}{2(\alpha(-d)\Delta^2 + 4dL - \alpha L)^2} \\
q^C = \frac{dL (-2ac + \alpha\gamma(-k)t^2 + 2)}{\alpha(-d)\Delta^2 + 4dL - \alpha L} \\
\pi^C_S = \frac{d\left(\alpha^2\gamma^2dk^2t^3\Delta^2 - 4L\left(\alpha^2c^2 + \alpha\gamma\lambda k^2 - 2\right) + \alpha\gamma^2dk^2t^3 - \alpha\gamma k^2 + 1\right)}{4\alpha(\alpha d\Delta^2 - 4dL - \alpha L)}
\]

4.1.2. Manufacturer-led Stackelberg (MS)

Under manufacturer-led decision-making mode, manufacturer is the leader of the Stackelberg game, while retailer is the follower. In this case, the decision sequence of the supply chain is as follows. First, the manufacturer determines the wholesale price \(w\), the extended warranty price \(p_e\) and the recovery rate \(\tau\). Then, the retailer determines the retail price \(p = w + m\) based on the manufacturers decision. The decision models of manufacturer and retailer can be defined as:

\[
\pi^M_M = (p_e - c_e) \left( -\frac{dp_e}{t} + \alpha(-m + w) + 1 \right) + (-c + \tau\Delta + w)(1 - \alpha(m + w)) - L\tau^2
\]

\[
\pi^M_R = m(1 - \alpha(m + w))
\]

**Proposition 2:** There is a unique optimal solution for the manufacturer-led decision-making CLSC, and the equilibrium results are as follows (See proof in Appendix B.).

\[
m^M_MS = \frac{dL (-2ac + \alpha\gamma(-k)t^2 + 2)}{\alpha(\alpha(-d)\Delta^2 + 8dL - \alpha L)}
\]

\[
w^M_MS = \frac{d\left(L\left(2ac + \alpha\gamma\lambda k^2 + 6\right) - \alpha\Delta^2\right)}{\alpha(\alpha(-d)\Delta^2 + 8dL - \alpha L)} - \alpha L
\]

\[
p^M_MS = \frac{d\left(L\left(2ac + \alpha\gamma\lambda k^2 + 6\right) - \alpha\Delta^2\right)}{\alpha(\alpha(-d)\Delta^2 + 8dL - \alpha L)} - \alpha L
\]

\[
p_e^M_MS = -\frac{t\left(2L\left(\alpha c - 4\gamma\lambda k^2 + \alpha\gamma\lambda k^2 - 1\right) + \alpha\gamma\lambda d\Delta^2\right)}{-2\alpha d\Delta^2 + 16dL - 2\alpha L}
\]

\[
\tau^M_MS = \frac{d\Delta (-2ac + \alpha\gamma(-k)t^2 + 2)}{2(\alpha(-d)\Delta^2 + 8dL - \alpha L)}
\]

\[
q^M_MS = \frac{dL (-2ac + \alpha\gamma(-k)t^2 + 2)}{\alpha(\alpha(-d)\Delta^2 + 8dL - \alpha L)}
\]

\[
\tau^M_MS = \frac{d\left(4L\left(\alpha^2c^2 + \alpha\gamma\lambda k^2 - 2\right) + 2\alpha\gamma^2dk^2t^3 - \alpha\gamma k^2 + 1\right) - \alpha^2\gamma^2dk^2t^3\Delta^2}{4\alpha(\alpha d\Delta^2 + 8dL - \alpha L)}
\]

\[
\pi^M_R = \frac{d^2L^2\left(-2ac + \alpha\gamma(-k)t^2 + 2\right)^2}{\alpha(\alpha(-d)\Delta^2 + 8dL - \alpha L)^2}
\]

\[
\tau^M_MS = \frac{d\left(A - (8dL - da\Delta^2 - \alpha L)t\right)\left((\alpha^2 + 2\alpha)\gamma^2dk^2t^3\Delta^2 - 4L\left(\alpha^2c^2 + \alpha\gamma\lambda k^2 - 2\right) - \alpha\gamma k^2 + 1\right)}{4\alpha(8dL - da\Delta^2 - \alpha L)^2}
\]
where \( A = 4dL^2 (2 - 2ac - \alpha \gamma kt^2)^2 \).

### 4.1.3. Retailer-led Stackelberg (RS)

Under retailer-led decision-making mode, retailer is the leader of the Stackelberg game, while manufacturer is the follower. In this case, the decision sequence of the supply chain is as follows. First, the retailer determine the retail price \( p \), then, the manufacturer determines the wholesale price \( w \), the extended warranty price \( p_e \) and the recovery rate \( \tau \). The decision models of manufacturer and retailer can be defined as:

\[
\pi_{RS}^M = (p_e - c_e) \left( -\frac{dp_e}{L} + \alpha(-m+w) + 1 \right) + (-c + \tau \Delta + w)(1 - \alpha(m+w)) - L\tau^2
\]  

(23)

\[
\pi_{RS}^R = m(1 - \alpha(m+w))
\]  

(24)

**Proposition 3:** There is a unique optimal solution for the retailer-led decision-making CLSC, and the equilibrium results are as follows (See proof in Appendix C.).

\[
m_{RS} = \frac{-2ac + \alpha \gamma (-k)^2 + 2}{4\alpha}
\]  

(25)

\[
w_{RS} = \frac{d \left( 2L (6ac + 3\alpha \gamma kt^2 + 2) - \alpha \Delta^2 (2ac + \alpha \gamma kt^2 + 2) \right) - \alpha L t (2ac + \alpha \gamma kt^2 + 2)}{4\alpha (\alpha(-d)\Delta^2 + 4dL - \alpha L t)}
\]  

(26)

\[
p_{RS} = \frac{d \left( L (2ac + \alpha \gamma kt^2 + 6) - 2\alpha \Delta^2 \right) - 2\alpha L t}{2\alpha (\alpha(-d)\Delta^2 + 4dL - \alpha L t)}
\]  

(27)

\[
p_{RS}^e = \frac{t \left( L (2ac - 8\gamma dkt + 3\alpha \gamma kt^2 - 2) + 2\alpha \gamma dkt \Delta^2 \right)}{4\alpha d\Delta^2 + 16dL - 4\alpha L t}
\]  

(28)

\[
\tau_{RS} = \frac{d \Delta \left( -2ac + \alpha \gamma (-k)^2 + 2 \right)}{4(\alpha(-d)\Delta^2 + 4dL - \alpha L t)}
\]  

(29)

\[
q_{RS} = \frac{d L \left( -2ac + \alpha \gamma (-k)^2 + 2 \right)}{2\alpha d\Delta^2 + 8dL - 2\alpha L t}
\]  

(30)

\[
\pi_{RS}^M = \frac{d \left( L (4\alpha^2 c^2 - 4ac (\alpha \gamma kt^2 - 2) + \alpha \gamma^2 (-k)^2 t^3 (3\alpha t - 16d) - 4\alpha \gamma kt^2 + 4) - 4\alpha^2 \gamma^2 dk^2 t^3 \Delta^2 \right)}{16\alpha (\alpha(-d)\Delta^2 + 4dL - \alpha L t)}
\]  

(31)

\[
\pi_{RS}^R = \frac{d L \left( -2ac + \alpha \gamma (-k)^2 + 2 \right)^2}{8\alpha (\alpha(-d)\Delta^2 + 4dL - \alpha L t)}
\]  

(32)

\[
\pi_{RS}^S = \frac{d \left( L (-12\alpha^2 c^2 - 12ac (\alpha \gamma kt^2 - 2) + \alpha \gamma^2 k^2 t^3 (at - 16d) + 12\alpha \gamma kt^2 - 12) + 4\alpha^2 \gamma^2 dk^2 t^3 \Delta^2 \right)}{16\alpha (ad\Delta^2 - 4dL + \alpha L t)}
\]  

(33)
4.2. Comparative analysis of CSCL models under different channel power structures

This section mainly compares the optimal pricing and profit under three different channel power structures when the manufacturer is responsible for recycling and extended warranty, and draws some findings about centralized and decentralized decision-making. This is conducive for the government and enterprises to formulate beneficial industrial policies and corporate countermeasures.

**Proposition 4:** The recovery rate satisfies \( \tau_{MS} < \tau_{RS} < \tau_C \).

Channel power structure has an important impact on the recycling effect of waste products. The recycling effect of the manufacturer-led is inferior to the retailer-led, and the recycling effect of the decentralized decision-making is inferior to the centralized decision-making. In the CLSC, the existence of leadership is not conducive to the recycling of waste products. In the CLSC dominated by manufacturers, since manufacturers can obtain higher profit from the sales of remanufactured products in the forward supply chain, their enthusiasm for recycling waste products to reduce production costs is weakened. On the contrary, in the CLSC dominated by retailers, manufacturers are in a passive position, and sales revenue is reduced. In order to obtain higher profit, their choose to pursue the recycling strategy to reduce the comprehensive production cost.

**Proposition 5:** The wholesale price satisfies \( w_{RS} < w_{MS} \).

The channel power structure has an impact on the wholesale price. The wholesale price of CLSC dominated by manufacturers is higher than that dominated by retailers. This is because in the CLSC dominated by manufacturers, the manufacturer, as the channel leader, will deliberately increase the wholesale price of the product to profit from it. In the CLSC dominated by retailers, retailers have stronger market control and higher bargaining power, which will make manufacturers willing to reduce wholesale price.

**Proposition 6:** The retail price satisfies \( p_C < p_{RS} < p_{MS} \).

The channel power structure has an impact on product price. The retail price of the decentralized decision-making is higher than that of the centralized decision-making, and the retail price of manufacturer-led is higher than that of the retailerled. The influence of channel power structure on retailer price is mainly due to two reasons: (1) Cost transfer caused by wholesale prices. Combining with the previous inference, it can be seen that when manufacturers continue to increase their power, retailers need to increase retail price to transfer wholesale cost to consumers in order to alleviate self-generated cost pressure. (2) When the retailer occupies the dominant position in the market, to obtain more profits leads to \( p_C < p_{RS} \). However, due to the fierce market competition, the amount of cost transfer is limited, so there \( p_{RS} < p_{MS} \). Therefore, by comparing the power structure of the three channels, consumers are willing to choose a market structure without leadership.

**Proposition 7:** The profit margin of unit remanufactured product satisfies \( m_{MS} < m_{RS} \).

The channel power structure has an important impact on the unit product profit obtained by retailers. The unit product profit of CLSC dominated by retailers is higher than that of dominated by manufacturers. When retailers dominate, in order to obtain more profits, they will increase profit through pricing decision. When the manufacturer is in a dominant position, it will increase the wholesale price to increase its own income, while the retailer cannot increase the product price too much because of market competition, which will adversely affect the retailers unit product profit margin.

**Proposition 8:** The extended warranty price satisfies \( p_{eMS} < p_{eRS} < p_{eC} \).
The extended warranty price under the manufacturer-led is higher than that under the retailer-led. In the CLSC dominated by manufacturers, because manufacturers can obtain high profit from product sales in the supply chain, they weaken enthusiasm for providing extended warranty services to obtain benefits. Conversely, in the CLSC dominated by retailers, because the manufacturer is in a passive position and sales revenue is reduced, in order to obtain higher profits, the manufacturer chooses to increase profit by providing extended warranty services.

**Proposition 9**: The total profit satisfies $\pi_{MS} < \pi_{RS} < \pi_{CS}$, and the retailers profit satisfies $\pi_{RS} < \pi_{MS}$.

(i) $\pi_{RS} < \pi_{MS}$, when the consumers price sensitivity to extended warranty meets the precondition of $3\alpha/8 < d < 1$ and the manufacturer cost savings meets $0 < \Delta < \sqrt{L(8d - 3\alpha)/\sqrt{3d\alpha}}$, or the consumers sensitivity to the extended warranty price meets the precondition of $0 < d < 3\alpha/8$.

(ii) $\pi_{RS} > \pi_{MS}$, when the consumers sensitivity to the extended warranty price meets the precondition of $3\alpha/8 < d < 1$ and the manufacturer cost savings meets $\Delta > \sqrt{L(8d - 3\alpha)/\sqrt{3d\alpha}}$.

Compared with the centralized decision-making, the overall benefit of the decentralized decision-making is poor, and the CLSC is in an uncoordinated state. The degree of discoordination of the decentralized decision-making system is related to the power structure, and the manufacturer-led has the lowest degree of coordination.

5. **Enterprise coordination mechanism design**

In actual operation, it is difficult for each node enterprise to make collective decision-making with the goal of maximizing the total profit spontaneously, and it is necessary to design a coordination mechanism to achieve it. This paper designs a two-part coordinated contract. On the one hand, the manufacturer sells the remanufactured product to the retailer at a lower price $\bar{w}$. On the other hand, the manufacturer charges retailers a certain fixed fee $f$ as a profit guarantee to ensure that the manufacturers profit after coordination is greater. Fixed fee can ensure that the profit of each subject of the CLSC is greater than the original profit and improve the enthusiasm of each subject to adopt the contract. This contract improves the coordination of the CLSC and eliminates the marginal utility of decentralized decision-making. It can be seen from Proposition 9 that the manufacturer has the lowest coordination in the case of manufacturer dominance. Therefore, this section coordinates manufacturer-led CLSC. The profit model at this time is as follows:

$$\pi_{MS}^* = (p_e - c_e) \left( -\frac{dp_e}{t} + \alpha(-p) + 1 \right) + (1 - \alpha p) \left( -c + \tau \Delta + \bar{w} \right) + f - L\tau^2 \quad (34)$$

$$\pi_{RS}^* = (1 - \alpha p) \left( p - \bar{w} \right) - f \quad (35)$$

**Proposition 10**: Under the two-part tariff, the optimal solution is the same as that of the centralized decision, that is, the fixed fee $f$ has no effect on the optimal solution under the coordination contract.

$$\bar{w} = \frac{4cdL + 2\gamma dkL^2 - d\Delta^2 - Lt}{\alpha(-d)\Delta^2 + 4dL - \alpha Lt} \quad (36)$$
dominant player (the manufacturer-led Stackelberg). We compare the influence of various

When the strong manufacturers sell through the weak retailers, the manufacturer becomes the

CLSC power relationship between the retailer and manufacturer (the centralized decision-making).

consumers purchase remanufactured products in large supermarkets, which often dominate the

supply chain because of their large market share (the retailer-led Stackelberg). In the CLSC,

of the manufacturer and the retailer.

coordination of CSCL can be achieved, and the fixed cost

\[ f = \frac{8d^3L^3 (\alpha d + \alpha \gamma (-k)t^2 + 2)^2 (\alpha (d) \Delta^2 + 6dL - \alpha Lt)}{\alpha (\alpha (d) \Delta^2 + 8dL - \alphaLt)(\alpha (d) \Delta^2 + 4dL - \alpha Lt)^2} \]

Proposition 11: Under the two-part tariff, the profit of each entity are as follows:

\[ \pi^M_{MS^*} = \frac{d ((ad\Delta^2 - 4dL + \alpha Lt) (\alpha^2 \gamma^2 dk^2 t^3 \Delta^2 - 4L (\alpha^2 c^2 + ac (\alpha \gamma kt^2 - 2) + \alpha \gamma^2 dk^2 t^3 - \alpha \gamma kt^2 + 1)) - B)}{4a (4dL - d\alpha \Delta^2 - \alpha Lt)^2} + f \]

\[ \pi^R_{MS^*} = \frac{dL (\alpha 2ac + \alpha \gamma (-k)t^2 + 2)}{\alpha (\alpha (d) \Delta^2 + 4dL - \alpha Lt) - f} \]

\[ \pi^S_{MS^*} = \frac{d (\alpha^2 \gamma^2 dk^2 t^3 \Delta^2 - 4L (\alpha^2 c^2 + ac (\alpha \gamma kt^2 - 2) + \alpha \gamma^2 dk^2 t^3 - \alpha \gamma kt^2 + 1))}{4a (ad\Delta^2 - 4dL + \alpha Lt)} \]

where \( B = 4dL^2 (-2ac - k\alpha \gamma t^2 + 2)^2 \)

In this case, \( \pi^M_{MS^*} + \pi^M_{MS} = \pi^M_{MS} = \pi^C_S \) be met, the total profit of the CLSC is the same as that of the centralized decision-making, so the coordination of CSCL can be achieved. There is no guarantee that profit will rise for both manufacturers and retailers. Manufacturers and retailers are willing to cooperate to implement this contract only when the profit after coordination is more than the profit before coordination. That is to ensure \( \pi^R_{MS^*} > \pi^M_{MS^*} \) and \( \pi^R_{RS^*} > \pi^R_{RS^*} \).

Proposition 12: Under the two-part tariff, when the fixed fee \( f \) meet \( f < \bar{f} \), the perfect coordination of CSCL can be achieved, and the fixed cost \( f \) is determined by the negotiating power of the manufacturer and the retailer.

\[ \bar{f} = \frac{8d^3L^3 (\alpha 2ac + \alpha \gamma (-k)t^2 + 2)^2 (\alpha (d) \Delta^2 + 6dL - \alpha Lt)}{\alpha (\alpha (d) \Delta^2 + 8dL - \alpha Lt)(\alpha (d) \Delta^2 + 4dL - \alpha Lt)^2} \]

\[ \bar{f} = \frac{4d^3L^3 (\alpha 2ac + \alpha \gamma (-k)t^2 + 2)^2}{\alpha (ad\Delta^2 - 8dL + \alpha Lt)(\alpha (d) \Delta^2 + 4dL - \alpha Lt)^2} \]

6. Numerical analysis

The three channel power structures studied in this paper are common in CLSC. For instance, consumers purchase remanufactured products in large supermarkets, which often dominate the supply chain because of their large market share (the retailer-led Stackelberg). In the CLSC, there are also powerful manufacturers, such as Apple and Dell. Thus, there may be a balanced CLSC power relationship between the retailer and manufacturer (the centralized decision-making). When the strong manufacturers sell through the weak retailers, the manufacturer becomes the dominant player (the manufacturer-led Stackelberg) [56, 57]. We compare the influence of various
factors under different channel power structures on business decisions through sensitivity analysis. Specifically, we discuss the effects of remanufacturing cost savings and double price sensitivity on profits of CLSC. For the demonstration purpose, numerical analysis is carried out in combination with relevant literature in the field and the Canadian Aerospace Remanufacturing Corporation dataset [58, 59]. Combining theoretical research with realistic conditions, we assume: \( c = 300, t = 1, L = 100, k = 0.5, \gamma = 100, \Delta = 100, \alpha = 0.001, d = 0.002. \)

6.1. Case demonstration

According to the analysis of the optimal pricing strategy, we calculate the equilibrium solution under the three channel power structures. The results are shown in Table 2. And the change of the profit difference with the fixed fee \( f \) before and after the supply chain coordination is studied, as shown in Figure 2.

| Table 2. Optimal strategies under different channel power structures |
|---------------------------------|-----------------|-----------------|-----------------|
| parameters                      | Collective decisionmaking | Manufacturer-led Stackelberg | Retailer-led Stackelberg |
| \( m \)                         | 182.432          | 337.500         |                  |
| \( w \)                         | 635.135          | 463.971         |                  |
| \( p \)                         | 602.941          | 817.567         | 801.471          |
| \( p_e \)                       | 124.265          | 70.608          | 74.632           |
| \( q \)                         | 0.397            | 0.182           | 0.198            |
| \( \tau \)                      | 0.199            | 0.091           | 0.099            |
| \( \pi_R \)                     | 33.203           | 66.825          |                  |
| \( \pi_M \)                     | 62.663           | 34.821          |                  |
| \( \pi_S \)                     | 135.234          | 95.866          | 101.646          |

According to the optimal strategies and profit equilibrium solutions of member firms as shown in Table 2, the accuracy of the propositions in Chapter 4 can be verified.

Fig 2 shows that the fixed transfer payment factor has a certain degree of flexibility, and \( f \) is in the range \((85.219, 124.374)\), manufacturers and retailers achieve a win-win situation, and supply chain profits achieve a Pareto improvement. It can be seen from Fig 2 that the transfer payment range that manufacturers can choose is relatively large, and the coordination contract has effectively improved the profits of supply chain enterprises.

6.2. The impact of remanufacturing cost savings \( \Delta \) on decision-making model

Compare the effects of remanufacturing cost savings \( \Delta \) on the profit of manufacturer, the profit of retailer and the total profit under different decision models. Taking \( \Delta \) is valued within \([0, 300]\), as shown in Fig 3.

It can be seen from Fig 3, (1) Manufacturers profit, retailers profit, and total profit always rise with the increase of remanufacturing cost savings. Therefore, the implementation of remanufacturing is beneficial to each subject. Reducing the processing cost of remanufacturing or improving the utilization rate of old products can further improve the profit of each subject and promote the development of remanufactured product market. (2) Under different channel power structures,
the variation range of each subject's profit is different. The sensitivity of centralized decision to remanufacturing cost savings is higher than that of decentralized decision. Centralized decision can eliminate the marginal effect brought by decentralized decision and maximize the benefit brought by remanufacturing. The sensitivity of retailer-dominated to remanufacturing cost savings is always higher than that of manufacturer-dominated. Because of improving remanufacturing cost savings, manufacturers need to pay a huge investment cost, so the sensitivity of manufacturers profit to remanufacturing cost savings is low. Instead, retailers get free rider benefits. In the case of no cost input, retailers can quickly improve profits by buying remanufactured products at a lower wholesale price. (3) In the above three different channel power structures, manufacturers and retailers get the largest profits when they are market leaders, respectively. When different supply chain members gain the dominant power, they always strive for more benefits through their own position advantages.

6.3. The impact of dual price sensitivity on decision-making models

This paper analyzes the impact of the price sensitivity of product and the price sensitivity of extended warranty on profit under different decision-making models. $\alpha$ takes a value within $[0.005, 0.0015]$, and $d$ takes a value within $[0.0015, 0.0025]$. The result is shown in Fig 4.

It can be seen from Fig 4, (1) Under different channel power structures, manufacturers profit, retailers profit and total profit always decrease with the increase of dual price sensitivity. It can be seen that the increase of consumers' attention to the price of products and services will lead to the decrease of each subject's profit. In this case, manufacturers should optimize production capacity and strengthen technological innovation to reduce costs and increase profit. Retailers can improve profit by reducing consumers' price sensitivity by providing a superior user experience. (2) Consumers' price sensitivity to product is higher than that of extended warranty service. It can be seen that consumers pay more attention to the price of the product, but pay less attention to the extended warranty service after purchasing the product. On the basis of comprehensive
Figure 3. (a) The impact of remanufacturing cost savings on manufacturers profits; (b) The impact of remanufacturing cost savings on retailers profits; (c) The impact of remanufacturing cost savings on total profits.

consideration of dual price sensitivity, enterprises can reasonably plan the price of products and extended warranty service to realize the increase of profit.

7. Conclusion

This paper studies the optimal strategy of the supply chain under different channel power structures, and obtains the optimal price and the optimal recovery rate. This paper introduces the two-part coordination contract to coordinate the subjects of the CLSC, and realizes the Pareto
improvement of the CLSC. On this basis, it further analyzes the impact of remanufacturing cost savings and dual price sensitivity on the profits, and the main conclusions are as follows:

(1) Under the power structure of centralized decision-making, the total profit of the supply chain is the highest. Within a certain threshold, the profits of supply chain subjects are positively correlated with their own power. In the real market environment, one party often has the dominant power. In this case, a coordination strategy should be adopted to promote the Pareto improvement of the profit of supply chain members.

(2) Under the power structure of centralized decision-making, the recycling effect of waste products is the best. When the channel power of the supply chain system is tilted, the companys
efforts to recycling waste products decrease, and the supply chain dominance moving down is conducive to better recycling of waste products.

(3) Under different channel power structures, the increase in remanufacturing cost savings can promote the increase in the profit of each subject. Reducing the processing cost of remanufacturing or improving the utilization rate of old products will promote the development of the remanufactured product market.

(4) Under different channel power structures, manufacturers profit, retailers profit, and total profit will decrease with the increase of dual price sensitivity (the price sensitivity to product and the price sensitivity to extended warranty service) simultaneously. Consumers price sensitivity to remanufactured product is higher than that of extended warranty service. Manufacturers should optimize production capacity and strengthen technological innovation to reduce production costs and increase profit. Retailers can improve profit by reducing consumers price sensitivity by providing a superior user experience.

The directions worthy of expansion of this research are: (1) The behavior preference of consumers will influence the decision of CLSC. This paper does not consider the influence of consumer behavior on optimal strategy. (2) This paper assumes that there is no difference between new products and remanufactured products, and the difference in quality, product pricing, and extended warranty service can be considered. (3) In addition, different extended warranty models can be provided for different remanufactured products. This paper ignores the influence of extended warranty modes on subject selection.

Appendix

7.1. Appendix A. Centralized decision-making (C)

7.1.1. Proof of Proposition 1

First, the derivatives of $\pi_C$ with respect to $p$, $p_e$ and $\tau$ are obtained, respectively. According to equation (4.1), the Hessian matrix can be easily solved as follows:

$$
H(p, p_e, \tau) = \begin{bmatrix}
-2\alpha & -\alpha & -\alpha \Delta \\
\alpha & -2d/t & 0 \\
-\alpha \Delta & 0 & -2L
\end{bmatrix}
$$

(46)

According to the assumptions, $4dL - L(\alpha - d\alpha \Delta^2) > 0$ and $\alpha > 0$, then, $H_1 = -2\alpha < 0$, $H_2 = 4ad/t - \alpha^2 > 0$, and $H_3 = \alpha(2L\alpha - 8dL + 2d\alpha \Delta^2)/t < 0$. Thus, the Hessian matrices of $\pi_C$ is negative, which means that $\pi_C$ is a strictly concave function concerning $p^C$, $p_e^C$ and $\tau^C$. Then, the unique optimal solutions can be solved.

7.2. Appendix B. Manufacturer-led Stackelberg (MS)

7.2.1. Proof of Proposition 2

The backward induction was used to solve the model. First, the first-order partial derivative of with respect to is obtained. Its equation can be obtained by setting their partial derivatives equal
to 0 in parallel: \( p = (1 + wa)/2\alpha \). According to equation (4.3), the Hessian matrices can be easily solved as \( H(p) = (-2\alpha) \).

According to \( \alpha > 0 \), then, \( H_1 = (-2\alpha) < 0 \). Thus, the Hessian matrix of \( \pi_{RS}^M \) is negative, which means that \( \pi_{RS}^M \) is a strictly concave function concerning \( p \). Plugging \( p \) into equation (4.2), the derivatives of \( \pi_{RS}^M \) with respect to \( w, p_e \) and \( \tau \) are obtained, respectively. According to equation (4.2), the Hessian matrix can be easily solved:

\[
H(w, p_e, \tau) = \begin{bmatrix}
-2\alpha & -\alpha/2 & -\alpha\Delta/2 \\
\alpha/2 & -2d/t & 0 \\
-\alpha\Delta/2 & 0 & -2L
\end{bmatrix}
\]  

(47)

According to the assumptions, \( -d\alpha\Delta^2 + 4dL - \alpha Lt > 0 \) and \( \alpha > 0 \), then, \( H_1 = -\alpha < 0 \), \( H_2 = 2ad - \alpha^2 > 0 \), and \( H_3 = \frac{\alpha(\alpha\Delta^2 - 8dL + \alphaLt)}{2\alpha} < 0 \). Thus, the Hessian matrices of \( \pi_{RS}^M \) is negative, which means that \( \pi_{RS}^M \) is a strictly concave function concerning \( w, p_e \) and \( \tau \). Then, the unique optimal solutions can be solved.

### 7.3. Appendix C. Retailer-led Stackelberg (RS)

#### 7.3.1. Proof of Proposition 3

The backward induction was used to solve the model. First, the first-order partial derivative of \( \pi_{RS}^M \) with respect to \( w, p_e \) and \( \tau \) are obtained. Its equation can be obtained by setting their partial derivatives equal to 0 in parallel:

\[
w = \frac{\alphaLt(1 - \alpha m) - d (L (2ac + \alpha\gamma kt^2 - 2am + 2) + \alpha\Delta^2(\alpha m - 1))}{\alpha (ad\Delta^2 - 4dL + \alpha Lt)}
\]  

(48)

\[
p_e = -\frac{t (2L (\alpha c - 2\gamma dkt + \alpha\gamma kt^2 + \alpha m - 1) + \alpha\gamma dt\Delta^2)}{-2ad\Delta^2 + 8dL - 2\alpha Lt}
\]  

(49)

\[
\tau = \frac{d\Delta (2ac + \alpha\gamma kt^2 + 2am - 2)}{2(\alpha d\Delta^2 - 4dL + \alpha Lt)}
\]  

(50)

According to equation (4.4), the Hessian matrices can be easily solved as follows:

\[
H(w, p_e, \tau) = \begin{bmatrix}
-2\alpha & -\alpha & -\alpha\Delta \\
-\alpha & -2d/t & 0 \\
-\alpha\Delta & 0 & -2L
\end{bmatrix}
\]  

(51)

According to the assumptions, \( -d\alpha\Delta^2 + 4dL - \alpha Lt > 0 \) and \( \alpha > 0 \), then, \( H_1 = -2\alpha < 0 \), \( H_2 = \frac{4ad - \alpha^2}{t} > 0 \), and \( H_3 = \frac{\alpha(\alpha d\Delta^2 - 8dL + \alphaLt)}{t} < 0 \). Thus, the Hessian matrix of \( \pi_{RS}^M \) is negative, which means that \( \pi_{RS}^M \) is a strictly concave function concerning \( w, p_e \) and \( \tau \).

Plugging \( w, p_e \), and \( \tau \) into equation (4.5), the derivatives of \( \pi_{RS}^M \) with respect to \( m \) is obtained. According to equation (4.5), the Hessian matrix can be easily solved:

\[
H(m) = \frac{4dL\alpha}{\alpha d\Delta^2 - 4dL + \alpha Lt}
\]  

(52)
According to the assumptions, then, $H_1 < 0$. Thus, the Hessian matrices of $\pi^{RS}_S$ is negative, which means that $\pi^{RS}_R$ is a strictly concave function concerning $p$. Then, the unique optimal solutions can be solved.

Data Availability: The data used to support the findings of this study are included within the paper.

Conflicts of Interest: The authors declare that they have no conflicts of interest.

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