RAIRO Operations Research
Will be set by the publisher

OPERATING EFFICIENCY ASSESSMENT OF COMMERCIAL BANKS WITH COOPERATIVE-STACKELBERG HYBRID TWO-STAGE DEA

Jianfeng Ma[1] and Tianmingdi Zhao[2]

Abstract. The two-stage Data Envelopment Analysis (DEA) is widely applied to assess the efficiency of commercial banks in recent years. Even though this approach well simulates the sequence of banks production process, the independent operations within sub-stages are generally ignored, and the cooperative or non-cooperative relations between sub-stages are usually investigated separately. Commercial banking production system, however, has complex internal structure within which parallel and series structure can co-exist, and cooperative relations may concurrently occur with non-cooperative ones. In this paper, we develop a hybrid two-stage DEA to consider simultaneously the series-parallel internal structure and the cooperative-Stackelberg relations between sub-stages. The data of 19 Chinese listed commercial banks are used to show the abilities of the proposed models. This approach represents a powerful and flexible efficiency measurement implement that can be applied when the system in question has a complex internal structure in terms of both sub-systems features and sub-systems relations.

Mathematics Subject Classification. 90B30, 91-10

Introduction

Introduced by Charnes *et al.* [8], Data Envelopment Analysis (DEA) is a non-parametric mathematical method for assessing the relative efficiency of a set of homogenous Decision Making Units (DMUs). The banking sector is a popular field for methodological and applied researches involving DEA techniques [15]. The study of Sherman and Gold [36] is one of the first studies that apply the CCR model [8] to evaluate operating efficiency of banks. The authors point out that DEA results provide meaningful insights not available from other techniques. Over the last three decades, the banking industry has been probably becoming the most heavily studied by DEA approach of all business sectors [32].

The traditional CCR model [8] treats DMU as a black box of converting initial inputs to final outputs. Without considering the internal structure, the black box approach could not give specific information regarding the sources of inefficiency within DMUs [22], and tends to produce inaccurate efficiency scores or misleading results [16]. In classic two-stage DEA, DMU is considered as a system having two-stage internal structure, in which the initial inputs are transformed into intermediate measures [11] through the first stage, and then the intermediate measures are developed into final output in the second stage. Two-stage DEA simulates a general internal structure of the production process and provides the possibility to assess the efficiency of the whole system and to decompose the overall efficiency into the efficiency of each sub-stage [18], [9], [33].

The significant applications of two-stage DEA in banking industry have been rising since the late 1990s. For example, Athanassopoulos [5] develop a two-stage DEA model to assess the operating efficiency and the quality of provided services of 68 commercial bank branches in Greece. Seiford and Zhu [34] propose a well-known two-stage DEA model that separates profitability and marketability to examine the performance of the top 55 U.S. commercial banks and indicate that the developed model is suitable to identify areas for improved

---

...

[1] School of Economics and Management, University of Science and Technology Beijing,Beijing 100083, P. R. China

[2] School of Economics and Management, University of Science and Technology Beijing,Beijing 100083, P. R. China

This provisional PDF is the accepted version. The article should be cited as: RAIRO: RO, doi: 10.1051/ro/2021152

bank performance over the production process. Similarly, Luo [27] evaluates the profitability and marketability performances of 245 large banks in the U.S..

The earlier studies have explicitly described the basic two-stage internal structure as deposit-producing stage and profit-earning stage linked by deposits generated from the first one. Two-stage DEA has been considerably developed in the applications to the banking efficiency measurement after 2010. Table 1 and Table 2 (Continued Table) summarize some of these studies.

Table 1. Summary of two-stage DEA applications to the efficiency assessment of bank industry

Authors	Models	Inputs	Intermediate measures,	Outputs
Zha and Liang [45]	Two-stage DEA with shared inputs.	Employees, Assets, Equity.	Revenues, Profits.	Market value, Earnings, Returns.
Tsolas [40]	Independent two-stage DEA.	Personnel and Rental expenses, Other operational expenses, Depreciation.	Interest and non-interest income, Loans, Commission.	Net income.
Fukuyama and Matousek [13]	Slacks-based two-stage DEA.	Labor, Capital.	Deposits.	Loans, Securities.
Ashrafi and Jaafar [4]	Two-stage DEA with undesirable input-outputs.	Personnel, Expenditures, Depreciation.	Resources.	Income, Usages, Receivable.
Avkiran and McCrystal [6]	Two-stage DEA with add. input and intermediate measures.	Capital, Labor, Customer service training.	Number of referrals, Number of transactions.	Number of referral sales.
Akther *et al.* [1]	Two-stage DEA with undesirable outputs.	Employees, Capital, Equity, Bad loans.	Deposits.	Loans, Bad loans, Investments.
Zhou *et al.* [47]	Nash bargaining two-stage DEA.	Employees, Fixed assets, Expenses.	Credit, Interbank loans.	Loans, Profits.
Wang *et al.* [42]	Additive two-stage DEA.	Labor, Fixed assets.	Deposits.	Interest income, Bad loans.
Wanke and Barros [41]	Centralized two-stage DEA.	Number of branches, Employees.	Administrative Personnel expenses.	Permanent assets, Equity.
Kwon and Lee [20]	Two-stage DEA-neural network.	Employees, Equity, Expenses.	Deposits, Loans, Investments.	Profit.
Liu *et al.* [25]	Two-stage DEA with undesirable in-inter.-output.	Employees, Fixed assets, Operating expenses.	Profits, Loans, Non-performing loans.	Market value, Earnings per share, Volatility.
Amirteimoori *et al.* [2]	Two-stage DEA with shared inputs and intermediate measures.	Fund from customers, Number of check accounts, Operating costs.	Deposits.	Number of transaction, Loans, Profits, Deposits.
Chen *et al.* [10]	Envelopment and multiplier-based two-stage DEA.	Employees, Equality.	Revenues, Profits.	Returns.
Shi *et al.* [37]	Two-stage cost efficiency DEA model.	Fixed assets, Labor and other operating expense.	Deposits.	Non-interest income, Interest income.

Lozano [26] summarized more than twenty network DEA applications in the efficiency measurement of banks or bank branches, and the author indicated that most studies consider two-stage systems in series. Nevertheless,

Table 2. Summary of two-stage DEA applications to the efficiency assessment of bank industry (Continued Table)

Authors	Models	Inputs	Intermediate measures,	Outputs
Fukuyama and Matousek [14]	Two-stage DEA with jointly and non-jointly good and bed outputs.	Labor, Capital.	Deposits.	Loans, non-performing loans, Securities investments.
An *et al.* [3]	Two-stage DEA fair efficiency decomposition.	Employees, Expenses. Fixed assets.	Credits, Interbank loans.	Loans, Profits.
Kwon *et al.* [21]	BPNN approach with two-stage DEA.	Operational expenses, Equity.	Deposits.	Loans, Investments, Revenue.
Kourtzidis *et al.* [19]	Two-stage DEA with Weight Assurance model.	Employees, Total assets.	Deposits.	Loans, Securities.
Tavakoli and Mostafaee [38]	Two-stage DEA in Free Disposal Hull (FDH).	Human resources, Fixed assets, No. of branches.	Deposits.	Profits.
Mehdizadeh, *et al.* [29]	Two-stage DEA with stochastic data.	Employees, Fixed assets, Expenses.	Deposits, Interbank Deposits.	Loan, Profits.
Xu and Zhou [43]	Two-stage AR-DEA model.	Total assets, Labors, Operating expenses.	Deposits.	Interest and non-incomes, Non performing loans.
Shahbazifar, *et al.* [35]	Group efficiency of two-stage DEA.	Personnel costs, Paid interests.	Raised funds, related to currency transactions.	Loans, Common incomes.
An *et al.* [3]	Two-stage closest target DEA model.	Operation costs, Interest costs, Labors.	Deposits.	Interest income, Non-interest income.

this structural feature presents certain insufficiencies in the banks performance evaluation. For example, if the operations within a sub-stage are also ordered in series, the problem can be mathematically formulated to a three-stage DEA modelling such as a network DEA framework proposed by Matthews [28] or to a more general multi-stage systems case studied by Kao [17]; but, if the internal operations of a sub-stage are organized in parallel and effectuated independently, how to evaluate and decompose the efficiency of the system with respecting the different efficiency formation mechanisms?

Indeed, there are a few studies concerning the above issues. For example, Naini *et al.* [30] consider the production process of bank as a two-stage system where the second stage includes two parallel sub-stages sharing intermediate measures. Ebrahimnejad *et al.* [12] propose a three-stage DEA model to evaluate the efficiencies of a banking system where the two first independent parallel stages are linked to the third stage. However, a common inadequacy in the two studies is that they do not offer any measures to assess the overall efficiency of system, so the overall efficiency formation mechanism is not involved in their modelling. Moreover, the relation between the deposit-producing stage and profit-earning stage is usually considered as cooperative. That is, the two stages are supposed to optimize the efficiencies simultaneously in the condition that the overall efficiency of the production system is maximized [24], and the intermediate measures are therefore to be given the same weights in both stages. However, the decision makers may prefer to give priority to one of the two stages, which will implicate potential conflicts between the two stages arising from the intermediate measures [34]. The latter situation is studied by the leader-follower or Stackelberg game EDA [23]. So, how to measure the overall efficiency and decompose it into the efficiency of each sub-system, if the cooperative and non-cooperative relations coexist in a parallel-series production system? What are the strategic implications for managers according to the cooperative situation, non-cooperative situation, and different leader-follower positions of two sub-stages?

With these questions in mind, we attempt to make up for the insufficiency of the existing two-stage DEA approaches by developing a set of cooperative-Stackelberg hybrid two-stage DEA models with consideration of complex internal structure and relations. The remainder of this paper is organized as follows. The methodology section presents the hybrid two-stage DEA models with its efficiency assessment and decomposition procedures in cooperation and cooperation-Stackelberg situations. In the illustrative application section, our proposed approach is applied to the data of 19 Chinese listed commercial banks. Section 4 outlines conclusions and future research directions.

1. METHODOLOGY

The primary task of commercial bank is to transform savings into investments and get profits as efficiently as possible. A certain sequence exists in collecting savings deposits and granting loans in the production process. Besides, the savings of commercial banks are derived mainly from household and business, and the exploitations of these two sources are usually conducted independently. According to these procedural characteristics, we propose to consider banking production process as a two-stage system comprised of three sub-systems as shown in Fig 1.

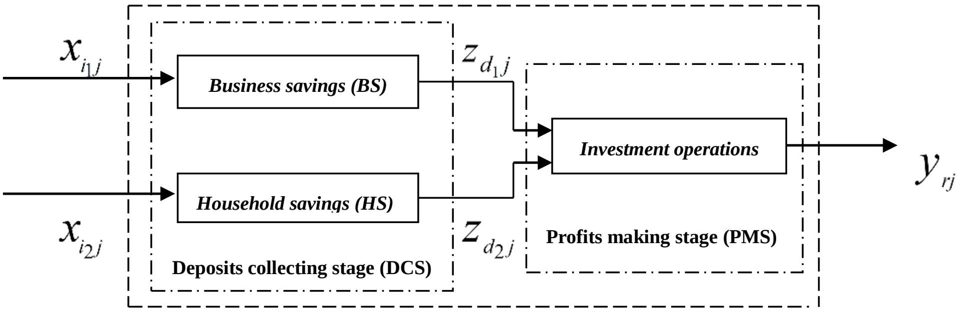

FIGURE 1. Structure of bank's hybrid two-stage production system

More specifically, we regard the commercial bank having a hybrid two-stage production system as a set of homogeneous DMUs, denoted as $\text{DMU}_j\,(j = 1, ..., n)$. For any commercial bank $j, (j = 1, ..., n)$, the deposits collecting stage (DCS) is comprised of two independent sub-systems in parallel where the business savings services system (BS) consumes I_1 operational costs $x_{i_1j}, (i_1 = 1, ..., I_1)$ to generate D_1 business deposits $z_{d_1j}, (d_1 = 1, ..., D_1)$, and the household savings services system (HS) uses I_2 operational costs $x_{i_2j}, (i_2 = 1, ..., I_2)$ to yield D_2 household deposits $z_{d_2j}, (d_2 = 1, ..., D_2)$.

The DCS is then linked to profits making stage (PMS) in series by the outputs from DCS $z_{d_1j}, (d_1 = 1, ..., D_1)$ and $z_{d_2j}, (d_2 = 1, ..., D_2)$, referred to as intermediate measures [11] or links [39]. PMS employs both household and business savings to yield S final outputs $y_{rj}, (r = 1, ..., S)$.

Let v_{i_1} and v_{i_2} denote the weights associated with the inputs $x_{i_1j}, (i_1 = 1, ..., I_1)$ and $x_{i_2j}, (i_2 = 1, ..., I_2)$, respectively. Since the intermediate measures play a dual role in DCS and PMS, we denote $u^1_{d_1}$ and $u^1_{d_2}$ as the weights on the outputs flowing from DCS, and $u^2_{d_1}$ and $u^2_{d_2}$ as the weights on the intermediate measures entering PMS, respectively. The weight u_r is given to $y_{rj}, (r = 1, ..., S)$.

1.1. COOPERATIVE HYBRID TWO-STAGE DEA APPRAOCH

1.1.1. *Cooperative hybrid two-stage DEA models*

We start with the CCR efficiency estimation of BS and HS for DMU_0 by the following models (1) and (2), respectively.

$$\begin{aligned} \theta_0^{1.1} = \quad & \max \frac{\sum_{d_1=1}^{D_1} u^1_{d_1} z_{d_10}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_10}} \\ & \text{s.t. } \frac{\sum_{d_1=1}^{D_1} u^1_{d_1} z_{d_1j}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1j}} \leq 1,\ j = 1, ..., n \\ & \quad u^1_{d_1},\ v_{i_1} \geq 0,\ d_1 = 1, ..., D_1,\ i_1 = 1, ..., I_1. \end{aligned} \tag{1}$$

$$
\begin{aligned}
\theta_0^{1.2} = \quad & \max \frac{\sum_{d_2=1}^{D_2} u_{d_2}^1 z_{d_2 0}}{\sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 0}} \\
& \text{s.t. } \frac{\sum_{d_2=1}^{D_2} u_{d_2}^1 z_{d_2 j}}{\sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 j}} \le 1,\ j = 1, ..., n \\
& u_{d_2}^1,\ v_{i_2} \ge 0,\ d_2 = 1, ..., D_2,\ i_2 = 1, ..., I_2.
\end{aligned} \tag{2}
$$

According to the assumptions about the internal structure, BS and HS are organized in parallel and operate independently. Since there are no trade-offs between BS and HS, we propose to define the efficiency of DCS, denoted as θ_0^1, as a weighted sum of efficiencies of BS and HS, denoted as $\theta_0^{1.1}$ and $\theta_0^{1.2}$, respectively.

$$
\theta_0^1 = \max \left(w_1 \theta_0^{1.1} + w_2 \theta_0^{1.2} \right) \tag{3}
$$

Where w_1 and w_2 are the associated weights to $\theta_0^{1.1}$ and $\theta_0^{1.2}$ such that $w_1 + w_2 = 1$. In Chen *et al.* [9], these weights are specified as the proportions of total weighted resources devoted to the respective sub-systems.

$$
w_1 = \frac{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 0}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 0} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 0}} \text{ }, and\ w_2 = \frac{\sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 0}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 0} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 0}}. \tag{4}
$$

Therefore, viewed from the point of internal resources allocation, w_1 and w_2 reasonably represent the relative contribution or importance of the efficiencies of BS and HS to the performance of DCS. With models (1) and (2), and formulas (3) and (4), the efficiency of DCS can be evaluated by model (5):

$$
\begin{aligned}
\theta_0^1 = \quad & \max \frac{\sum_{d_1=1}^{D_1} u_{d_1}^1 z_{d_1 0} + \sum_{d_2=1}^{D_2} u_{d_2}^1 z_{d_2 0}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 0} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 0}} \\
& \text{s.t. } \frac{\sum_{d_1=1}^{D_1} u_{d_1}^1 z_{d_1 j}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 j}} \le 1,\ j = 1, ..., n \\
& \frac{\sum_{d_2=1}^{D_2} u_{d_2}^1 z_{d_2 j}}{\sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 j}} \le 1,\ j = 1, ..., n \\
& u_{d_1}^1,\ u_{d_2}^1,\ v_{i_1},\ v_{i_2} \ge 0,\ d_1 = 1, ..., D_1,\ d_2 = 1, ..., D_2, \\
& i_1 = 1, ..., I_1,\ i_2 = 1, ..., I_2.
\end{aligned} \tag{5}
$$

Note that the definition and procedure mentioned above are based on the thoughts of additive DEA method [9], which is developed originally to evaluate the efficiency of a two-stage system in series. However, considering that the intermediate measures are concerned in view of resources rather than its link role, it is appropriate to apply the additive method to estimate the efficiency of DCS.

The efficiency of PMS for DMU$_0$, denoted as θ_0^2, can be calculated via the following fractional model (6):

$$
\begin{aligned}
\theta_0^2 = \quad & \max \frac{\sum_{r=1}^{S} u_r y_{r0}}{\sum_{d_1=1}^{D_1} u_{d_1}^2 z_{d_1 0} + \sum_{d_2=1}^{D_2} u_{d_2}^2 z_{d_2 0}} \\
& \text{s.t. } \frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{d_1=1}^{D_1} u_{d_1}^2 z_{d_1 j} + \sum_{d_2=1}^{D_2} u_{d_2}^2 z_{d_2 j}} \le 1,\ j = 1, ..., n \\
& u_r,\ u_{d_1}^2,\ u_{d_2}^2 \ge 0,\ r = 1, ..., S,\ d_1 = 1, ..., D_1,\ d_2 = 1, ..., D_2.
\end{aligned} \tag{6}
$$

Suppose that relations among all sub-systems within the production system are cooperative. That is, with given initial inputs $\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 j}$ and $\sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 j}$, the DCS will yield optimal $\sum_{d_1=1}^{D_1} u_{d_1} z_{d_1 j}$ and $\sum_{d_2=1}^{D_2} u_{d_2} z_{d_2 j}$, to maximize the overall efficiency. Therefore, the weights associated with intermediate measures should be identical no matter they play the role of output from DCS or input to PMS, i.e., $u_{d_1}^1 = u_{d_1}^2 = u_{d_1}$ and $u_{d_2}^1 = u_{d_2}^2 = u_{d_2}$ [18], [23]. We define then the overall efficiency, denoted as $\theta_0^{overall}$, as the product of efficiencies of DCS and PMS according to the thoughts of multiplicative DEA method [18].

$$
\theta_0^{overall} = \max \theta_0^1 \cdot \theta_0^2 \tag{7}
$$

Thus, the overall efficiency of DMU_0 can be obtained by model (8):

$$
\begin{aligned}
\theta_0^{overall} = \ & \max \frac{\sum_{r=1}^{S} u_r y_{r0}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 0} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 0}} \\
& \text{s.t. } \frac{\sum_{d_1=1}^{D_1} u_{d_1} z_{d_1 j}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 j}} \le 1, \ j = 1, ..., n \\
& \frac{\sum_{d_2=1}^{D_2} u_{d_2} z_{d_2 j}}{\sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 j}} \le 1, \ j = 1, ..., n \\
& \frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{d_1=1}^{D_1} u_{d_1} z_{d_1 j} + \sum_{d_2=1}^{D_2} u_{d_2} z_{d_2 j}} \le 1, \ j = 1, ..., n \\
& u_r, \ u_{d_1}, \ u_{d_2}, \ v_{i_1}, \ v_{i_2} \ge 0, \ r = 1, ..., S, d_1 = 1, ..., D_1, \ d_2 = 1, ..., D_2, \\
& i_1 = 1, ..., I_1, \ i_2 = 1, ..., I_2.
\end{aligned}
\tag{8}
$$

By using the Charnes-Cooper transformation [7], model (8) can be transformed into program (9):

$$
\begin{aligned}
\theta_0^{overall} = \ & \max \sum_{r=1}^{S} \mu_r y_{r0} \\
& \text{s.t. } \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \le 0, \ j = 1, ..., n \\
& \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \le 0, \ j = 1, ..., n \\
& \sum_{r=1}^{S} \mu_r y_{rj} - \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} \le 0, \ j = 1, ..., n \\
& \sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} + \sum_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 0} = 1 \\
& \mu_r, \ \mu_{d_1}, \ \mu_{d_2}, \ \nu_{i_1}, \ \nu_{i_2} \ge 0, \ r = 1, ..., S, \ d_1 = 1, ..., D_1, \ d_2 = 1, ..., D_2, \\
& i_1 = 1, ..., I_1, \ i_2 = 1, ..., I_2.
\end{aligned}
\tag{9}
$$

The overall efficiency and the efficiencies of sub-systems can be calculated with the optimal solution to model (9). However, the uniqueness of optimal solution for model (9) is not guaranteed, and the decomposition of the overall efficiency as the product of the efficiencies of DCS and PMS should not be necessarily unique in consequence.

Moreover, it is also uncertain to obtain a uniqueness of optimal solution for model (5), which will make the unique efficiency decomposition for all sub-systems more complicated. As we mentioned above, sub-stages and sub-systems of the banks production system will collaborate to achieve a maximal overall efficiency in cooperative case. That is, the cooperative situation implicates that efficiency scores of sub-stage or sub-system should be at the same level whether the decision is in favour of themself or the others. In view of these, we propose a procedure of addressing the unique efficiency decomposition issues in the next sub-section.

1.1.2. *Efficiency decomposition procedure*

With the optimal overall efficiency obtained from model (9), denoted as $\theta_0^{overall*}$, we calculate either DCS efficiency θ_0^{1+} or PMS efficiency θ_0^{2+} in the most favorable manner to the stage respectively, and then derive the minimal efficiencies of the corresponding stage θ_0^{2-} or θ_0^{1-} accordingly. If $\theta_0^{1+} = \theta_0^{1-}$ or $\theta_0^{2+} = \theta_0^{2-}$, unique efficiency decomposition for DCS and PMS is achieved, i.e. $\theta_0^{1+} = \theta_0^{1-} = \theta_0^{1*}$ and $\theta_0^{2+} = \theta_0^{2-} = \theta_0^{2*}$. where the " * " signifies the optimal efficiency level.

After that, we will determine either the largest efficiency score of BS $\theta_0^{1.1+}$ or that of HS $\theta_0^{1.2+}$, and derive the minimal $\theta_0^{1.2-}$ or $\theta_0^{1.1-}$ within DCS. Finally, we have unique efficiency decomposition for all stages and sub-systems if $\theta_0^{1.1+} = \theta_0^{1.1-} = \theta_0^{1.1*}$ and $\theta_0^{1.2+} = \theta_0^{1.2-} = \theta_0^{1.2*}$.

Suppose firstly that the DCS is to be given pre-emptive priority, its maximal efficiency can be calculated by solving program (10):

$$
\begin{aligned}
\theta_0^{1+} = \quad & \max \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} \\
& \text{s.t. } \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \le 0,\ j = 1, ..., n \\
& \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \le 0,\ j = 1, ..., n \\
& \sum\nolimits_{r=1}^{S} \mu_r y_{rj} - \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} \le 0,\ j = 1, ..., n \\
& \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} + \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 0} = 1 \\
& \sum\nolimits_{r=1}^{S} \mu_r y_{r0} = \theta_0^{overall*} \\
& \mu_r,\ \mu_{d_1},\ \mu_{d_2},\ \nu_{i_1},\ \nu_{i_2} \ge 0,\ r = 1, ..., S,\ d_1 = 1, ..., D_1,\ d_2 = 1, ..., D_2, \\
& i_1 = 1, ..., I_1,\ i_2 = 1, ..., I_2.
\end{aligned}
\tag{10}
$$

The minimal efficiency of PMS can be then calculated as $\theta_0^{2-} = \frac{\theta_0^{overall*}}{\theta_0^{1+}}$ according to formula (7). Note that the efficiency of PMS θ_0^{2-} denoted with " $^{2-}$ " is to indicate that this stage is not given the pre-emptive priority.

Instead, the maximal efficiency of PMS can be evaluated by model (11), when pre-emptive priority is to be given this stage.

$$
\begin{aligned}
\theta_0^{2+} = \quad & \max \sum\nolimits_{r=1}^{S} \mu_r y_{r0} \\
& \text{s.t. } \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \le 0,\ j = 1, ..., n \\
& \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \le 0,\ j = 1, ..., n \\
& \sum\nolimits_{r=1}^{S} \mu_r y_{rj} - \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} \le 0,\ j = 1, ..., n \\
& \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} = 1 \\
& \sum\nolimits_{r=1}^{S} \mu_r y_{r0} - \theta_0^{overall*} \cdot \left(\sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} + \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 0} \right) = 0 \\
& \mu_r,\ \mu_{d_1},\ \mu_{d_2},\ \nu_{i_1},\ \nu_{i_2} \ge 0,\ r = 1, ..., S,\ d_1 = 1, ..., D_1,\ d_2 = 1, ..., D_2, \\
& i_1 = 1, ..., I_1,\ i_2 = 1, ..., I_2.
\end{aligned}
\tag{11}
$$

The corresponding efficiency of DCS will be $\theta_0^{1-} = \frac{\theta_0^{overall*}}{\theta_0^{2+}}$ according to formula (7). In other words, θ_0^{1-} is the minimal efficiency of DCS with given $\theta_0^{overall*}$ and θ_0^{2+}. As mentioned above, we have a unique efficiency decomposition for DCS and PMS if $\theta_0^{1+} = \theta_0^{1-}$ or $\theta_0^{2+} = \theta_0^{2-}$. If this is the case, we can decompose the optimal efficiency of DCS θ_0^{1*} into the efficiencies of BS and HS.

Within DCS, if BS is to be given pre-emptive priority, model (12) can determine its maximal efficiency $\theta_0^{1.1+}$, while maintaining the efficiency of DCS at θ_0^{1*}.

$$
\begin{aligned}
\theta_0^{1.1+} = \quad & \max \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} \\
& \text{s.t. } \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \le 0,\ j = 1, ..., n \\
& \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \le 0,\ j = 1, ..., n \\
& \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} - \theta_0^{1*} \cdot \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 0} = \theta_0^{1*} \\
& \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} = 1 \\
& \mu_{d_1},\ \mu_{d_2},\ \nu_{i_1},\ \nu_{i_2} \ge 0,\ d_1 = 1, ..., D_1,\ d_2 = 1, ..., D_2, \\
& i_1 = 1, ..., I_1,\ i_2 = 1, ..., I_2.
\end{aligned}
\tag{12}
$$

According to formula (3), the corresponding efficiency of HS is calculated as $\theta_0^{1.2-} = \frac{\theta_0^{1*} - w_1^* \cdot \theta_0^{1.1+}}{w_2^*}$, where w_1^* and w_2^* represent optimal weights obtained from model (9) by way of (4). That is, these weights guarantee the optimal efficiency of the whole system and permit BS to have a maximal efficiency. When we give pre-emptive priority to HS, its maximal efficiency can be obtained by model (13).

$$\begin{aligned}\theta_0^{1.2+} = \; & \max \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} \\ & \text{s.t. } \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \le 0, \; j = 1, ..., n \\ & \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \le 0, \; j = 1, ..., n \\ & \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} - \theta_0^{1*} \cdot \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} = \theta_0^{1*} \\ & \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 0} = 1 \\ & \mu_{d_1}, \; \mu_{d_2}, \; \nu_{i_1}, \; \nu_{i_2} \ge 0, \; d_1 = 1, ..., D_1, \; d_2 = 1, ..., D_2, \\ & i_1 = 1, ..., I_1, \; i_2 = 1, ..., I_2. \end{aligned} \tag{13}$$

The minimal efficiency of BS can be calculated as $\theta_0^{1.1-} = \frac{\theta_0^{1*} - w_2^* \cdot \theta_0^{1.2+}}{w_1^*}$, where w_1^* and w_2^* represent optimal weights obtained from model (9) by way of (4). Finally, if $\theta_0^{1.1+} = \theta_0^{1.1-}$ or $\theta_0^{1.2+} = \theta_0^{1.2-}$, unique efficiency decomposition of DCS is obtained. We denote then the optimal efficiency of BS and HS as $\theta_0^{1.1*}$ and $\theta_0^{1.2*}$, respectively. With this procedure, we realize the unique efficiency decomposition of the overall efficiency of the banks production system into efficiencies of the BS, HS, and PMS, as well as that of DCS and PMS.

1.2. COOPERATIVE-STACKELBERG HYBRID TWO-STAGE DEA APPROACH

1.2.1. *Non-cooperative model in deposits collecting stage leader case*

Based on the thoughts of Stackelberg game or leader-follower DEA [24], [23], we develop a cooperative-Stackelberg hybrid two-stage DEA approach to evaluate and decompose the efficiency of series-parallel banking production system.

Consider again the system as illustrated in Fig.1, we suppose that there is a preference for DCS or for PMS to managers. The stage with preference is the leader stage for which the efficiency maximization is more preferable. We firstly assume that DCS is the leader. To evaluate the efficiency of the leader stage, we propose model (14) with reference to model (5).

$$\begin{aligned}\theta_0^{1(L)*} = \; & \max \frac{\sum_{d_1=1}^{D_1} u_{d_1} z_{d_1 0} + \sum_{d_2=1}^{D_2} u_{d_2} z_{d_2 0}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 0} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 0}} \\ & \text{s.t. } \frac{\sum_{d_1=1}^{D_1} u_{d_1} z_{d_1 j}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 j}} \le 1, \; j = 1, ..., n \\ & \frac{\sum_{d_2=1}^{D_2} u_{d_2} z_{d_2 j}}{\sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 j}} \le 1, \; j = 1, ..., n \\ & u_{d_1}, \; u_{d_2}, \; v_{i_1}, \; v_{i_2} \ge 0, \; d_1 = 1, ..., D_1, \; d_2 = 1, ..., D_2, \\ & i_1 = 1, ..., I_1, \; i_2 = 1, ..., I_2. \end{aligned} \tag{14}$$

Where " $^{1(L)*}$ " signifies that DCS is the leader. By using the Charnes-Cooper transformation [7], we can transform model (14) into model (15).

$$\begin{aligned}\theta_0^{1(L)*} = \; & \max \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} \\ & \text{s.t. } \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \le 0, \; j = 1, ..., n \\ & \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \le 0, \; j = 1, ..., n \\ & \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} + \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 0} = 1 \\ & \mu_{d_1}, \; \mu_{d_2}, \; \nu_{i_1}, \; \nu_{i_2} \ge 0, \; d_1 = 1, ..., D_1, \; d_2 = 1, ..., D_2, \\ & i_1 = 1, ..., I_1, \; i_2 = 1, ..., I_2. \end{aligned} \tag{15}$$

As the follower, PMS will maximize its efficiency by taking the DCS efficiency $\theta_0^{1(L)*}$ as a constraint. That is, the PMS will consider the optimal weights on intermediate measures $u_{d_1}^*$ and $u_{d_2}^*$ that maintain $\theta_0^1 = \theta_0^{1(L)*}$. Considering non-cooperative relations between DCS and PMS, we assume that $u_{d_1} = q \cdot \mu_{d_1}$ and $u_{d_2} = q \cdot \mu_{d_2}$.

That is, the role and position of the intermediate measures are different in the two stages, but the business deposits and the household deposits are equally important to profits making activities in the second stage. We develop then model (16) to evaluate the efficiency of PMS.

$$
\begin{aligned}
\theta_0^{2(S)*} = \quad & \max \frac{\sum_{r=1}^{S} u_r y_{r_0}}{q\left(\sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0}\right)} \\
& \text{s.t. } \frac{\sum_{r=1}^{S} u_r y_{r_j}}{q\left(\sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} + \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j}\right)} \leq 0,\ j = 1, ..., n \\
& \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \leq 0,\ j = 1, ..., n \\
& \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \leq 0,\ j = 1, ..., n \\
& \sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} + \sum_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 0} = 1 \\
& \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} = \theta_0^{1(L)*} \\
& q,\ \mu_r,\ \mu_{d_1},\ \mu_{d_2},\ \nu_{i_1},\ \nu_{i_2} \geq 0,\ r = 1, ..., S,\ d_1 = 1, ..., D_1,\ d_2 = 1, ..., D_2, \\
& i_1 = 1, ..., I_1,\ i_2 = 1, ..., I_2.
\end{aligned}
\tag{16}
$$

Where " ${}^{2(S)*}$ " signifies that PMS is the follower. Let $\mu_r = \frac{u_r}{q}$, $r = 1, ..., S$, model (16) is then equivalent to model (17).

$$
\begin{aligned}
\theta_0^{2(S)*} = \quad & \max \frac{\sum_{r=1}^{S} u_r y_{r_0}}{\theta_0^{1(L)*}} \\
& \text{s.t. } \sum_{r=1}^{S} \mu_r y_{rj} - \left(\sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} + \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j}\right) \leq 0,\ j = 1, ..., n \\
& \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \leq 0,\ j = 1, ..., n \\
& \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \leq 0,\ j = 1, ..., n \\
& \sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} + \sum_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 0} = 1 \\
& \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} = \theta_0^{1(L)*} \\
& \mu_r,\ \mu_{d_1},\ \mu_{d_2},\ \nu_{i_1},\ \nu_{i_2} \geq 0,\ r = 1, ..., S,\ d_1 = 1, ..., D_1,\ d_2 = 1, ..., D_2, \\
& i_1 = 1, ..., I_1,\ i_2 = 1, ..., I_2.
\end{aligned}
\tag{17}
$$

Note that $\theta_0^{1(L)*} \cdot \theta_0^{2(S)*} = \sum_{r=1}^{S} u_r^* y_{r_0}$ at optimality with $\sum_{i_1=1}^{I_1} \nu_{i_1}^* x_{i_1 0} + \sum_{i_2=1}^{I_2} \nu_{i_2}^* x_{i_2 0} = 1$. That is, $\theta_0^{1(L)*} \cdot \theta_0^{2(S)*} = \max \frac{\sum_{r=1}^{S} u_r^* y_{r0}}{\sum_{i_1=1}^{I_1} v_{i_1}^* x_{i_1 0} + \sum_{i_2=1}^{I_2} v_{i_2}^* x_{i_2 0}}$ with the optimal solutions to model (17).

1.2.2. *Non-cooperative model in profit making stage leader case*

If we assume that PMS is the leader, the efficiency of DCS is computed subject to the requirement that the efficiency of PMS is fixed at the optimal level. We calculate the efficiency of PMS with the following standard CCR model (18).

$$
\begin{aligned}
\theta_0^{2(L)*} = \quad & \max \frac{\sum_{r=1}^{S} u_r y_{r0}}{\sum_{d_1=1}^{D_1} u_{d_1} z_{d_1 0} + \sum_{d_2=1}^{D_2} u_{d_2} z_{d_2 0}} \\
& \text{s.t. } \frac{\sum_{r=1}^{S} u_r y_{rj}}{\sum_{d_1=1}^{D_1} u_{d_1} z_{d_1 j} + \sum_{d_2=1}^{D_2} u_{d_2} z_{d_2 j}} \leq 1,\ j = 1, ..., n \\
& u_r,\ u_{d_1},\ u_{d_2} \geq 0, r = 1, ..., S,\ d_1 = 1, ..., D_1,\ d_2 = 1, ..., D_2.
\end{aligned}
\tag{18}
$$

Where " $^{2(L)*}$ " signifies that PMS is the leader. The linear model will be the following model (19) by using Charnes-Cooper transformation [7].

$$
\begin{aligned}
\theta_0^{2(L)*} = \ & \max \sum_{r=1}^{S} \mu_r y_{r0} \\
& \text{s.t.} \ \sum_{r=1}^{S} \mu_r y_{rj} - \left(\sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} + \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} \right) \leq 0, \ j = 1, ..., n \\
& \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} = 1 \\
& \mu_r, \ \mu_{d_1}, \ \mu_{d_2} \geq 0, \ r = 1, ..., S, \ d_1 = 1, ..., D_1, \ d_2 = 1, ..., D_2.
\end{aligned}
\tag{19}
$$

When $^{2(L)*}$ is obtained, the DCS efficiency can be evaluated via model (20).

$$
\begin{aligned}
\theta_0^{1(S)*} = \ & \max \frac{q \left(\sum_{d_1=1}^{D_1} u_{d_1} z_{d_1 0} + \sum_{d_2=1}^{D_2} u_{d_2} z_{d_2 0} \right)}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 0} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 0}} \\
& \text{s.t.} \ \frac{q \sum_{d_1=1}^{D_1} u_{d_1} z_{d_1 j}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 j}} \leq 1, \ j = 1, ..., n \\
& \frac{q \sum_{d_2=1}^{D_2} u_{d_2} z_{d_2 j}}{\sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 j}} \leq 1, \ j = 1, ..., n \\
& \sum_{r=1}^{S} \mu_r y_{rj} - \left(\sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} + \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} \right) \leq 0, \ j = 1, ..., n \\
& \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} = 1 \\
& \sum_{r=1}^{S} \mu_r y_{r_0} = \theta_0^{2(L)*} \\
& q, \ \mu_r, \ \mu_{d_1}, \ \mu_{d_2}, \ \nu_{i_1}, \ \nu_{i_2} \geq 0, \ r = 1, ..., S, \ d_1 = 1, ..., D_1, \ d_2 = 1, ..., D_2, \\
& i_1 = 1, ..., I_1, \ i_2 = 1, ..., I_2.
\end{aligned}
\tag{20}
$$

Where " $^{1(S)*}$ " signifies that DCS is the follower. Let $\nu_{i_1} = \frac{v_{i_1}}{q}$, $i_1 \in I_1$ and $\nu_{i_2} = \frac{v_{i_2}}{q}$, $i_2 \in I_2$, model (20) is then equivalent to the following linear model (21).

$$
\begin{aligned}
\frac{1}{\theta_0^{1(S)*}} = \ & \min \sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} + \sum_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 0} \\
& \text{s.t.} \ \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \leq 0, \ j = 1, ..., n \\
& \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \leq 0, \ j = 1, ..., n \\
& \sum_{r=1}^{S} \mu_r y_{rj} - \left(\sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} + \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} \right) \leq 0, \ j = 1, ..., n \\
& \sum_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} = 1 \\
& \sum_{r=1}^{S} \mu_r y_{r0} = \theta_0^{2(L)*} \\
& \mu_r, \ \mu_{d_1}, \ \mu_{d_2}, \ \nu_{i_1}, \ \nu_{i_2} \geq 0, \ r = 1, ..., S, \ d_1 = 1, ..., D_1, \ d_2 = 1, ..., D_2, \\
& i_1 = 1, ..., I_1, \ i_2 = 1, ..., I_2.
\end{aligned}
\tag{21}
$$

Note that from model (21), we have $\theta_0^{1(S)*} \cdot \theta_0^{2(L)*} = \frac{\sum_{r=1}^{S} u_r^* y_{r0}}{\sum_{i_1=1}^{I_1} v_{i_1}^* x_{i_1 0} + \sum_{i_2=1}^{I_2} v_{i_2}^* x_{i_2 0}}$ at optimality, and we observe that $\theta_0^{1(L)*} \cdot \theta_0^{2(S)*} = \frac{\sum_{r=1}^{S} u_r^* y_{r0}}{\sum_{i_1=1}^{I_1} v_{i_1}^* x_{i_1 0} + \sum_{i_2=1}^{I_2} v_{i_2}^* x_{i_2 0}}$ in model (17). That is, the leader-follower game DEA models imply an efficiency decomposition approach for the parallel-series system. The overall efficiency of the parallel-series banking production system is thus the product of DCS efficiency and PMS efficiency.

Note further that, $\theta_0^{1(L)*}$ and $\theta_0^{2(S)*}$ in DCS leader case, and $\theta_0^{1(S)*}$ and $\theta_0^{2(L)*}$ in PMS case, are all optimal values to linear programs. Therefore, the efficiency decomposition is unique, and the result is not affected by possible multiple optimal solutions. However, the efficiency decomposition of deposits collecting stage may not be the same in the DCS leader case and the PMS leader case.

1.2.3. *Efficiency decomposition of deposits collecting stage in cooperative-Stackelberg case*

Note that DCS is composed of BS and HS. We suppose that the two sub-systems operate cooperatively, or there is no priority between different sources of deposits collecting. We propose the following procedure of addressing the efficiency decomposition of the DCS.

We develop model (22) by reference to model (12) to evaluate the optimal efficiency of BS, which is to be given pre-emptive priority, while maintaining the efficiency of DCS at $\theta_0^{1(L)*}$, *i.e.*, in the DCS leader case.

$$\begin{aligned}\theta_0^{1.1(1(L))+} = \; & \max \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} \\ & \text{s.t.} \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \le 0, \; j=1,...,n \\ & \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \le 0, \; j=1,...,n \\ & \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} - \theta_0^{1(L)*} \cdot \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 0} = \theta_0^{1(L)*} \\ & \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} = 1 \\ & \mu_{d_1}, \; \mu_{d_2}, \; \nu_{i_1}, \; \nu_{i_2} \ge 0, \; d_1=1,...,D_1, \; d_2=1,...,D_2, \\ & i_1=1,...,I_1, \; i_2=1,...,I_2.\end{aligned} \tag{22}$$

According to formulas (3), the minimal efficiency of HS can be determined as $\theta_0^{1.2(1(L))-} = \frac{\theta_0^{1(L)*} - w_1^{1(L)*} \cdot \theta_0^{1.1(1(L))+}}{w_2^{1(L)*}}$, where $w_1^{1(L)*}$ and $w_2^{1(L)*}$ represent optimal weights obtained from model (15) by way of (4). These weights guarantee the optimal efficiency of DCS as leader and permit BS to have maximal efficiency.

If we give the pre-emptive priority to HS, the optimal efficiency of HS can be obtained from model (23).

$$\begin{aligned}\theta_0^{1.2(1(L))+} = \; & \max \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} \\ & \text{s.t.} \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \le 0, \; j=1,...,n \\ & \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \le 0, \; j=1,...,n \\ & \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} - \theta_0^{1(L)*} \cdot \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} = \theta_0^{1(L)*} \\ & \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} = 1 \\ & \mu_{d_1}, \; \mu_{d_2}, \; \nu_{i_1}, \; \nu_{i_2} \ge 0, \; d_1=1,...,D_1, \; d_2=1,...,D_2, \\ & i_1=1,...,I_1, \; i_2=1,...,I_2.\end{aligned} \tag{23}$$

Similarly, the corresponding minimal efficiency of BS can be calculated as $\theta_0^{1.1(1(L))-} = \frac{\theta_0^{1(L)*} - w_2^{1(L)*} \cdot \theta_0^{1.2(1(L))+}}{w_1^{1(L)*}}$, where $w_1^{1(L)*}$ and $w_2^{1(L)*}$ represent optimal weights obtained from model (15) by way of (4). If $\theta_0^{1.1(1(L))+} = \theta_0^{1.1(1(L))-}$ or $\theta_0^{1.2(1(L))+} = \theta_0^{1.2(1(L))-}$, unique decomposition of DCS is obtained. We denote the optimal efficiency scores of BS and HS as $\theta_0^{1.1(1(L))*}$ and $\theta_0^{1.2(1(L))*}$, respectively. With this procedure, the unique efficiency decomposition of DCS into the efficiencies of BS and HS is realized in the DCS leader case.

When PMS is the leader, we develop model (24) and (25) to calculate the efficiencies of BS and HS, with pre-emptive priority given respectively.

$$\begin{aligned}\theta_0^{1.1(2(L))+} = \; & \max \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} \\ & \text{s.t.} \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \le 0, \; j=1,...,n \\ & \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \le 0, \; j=1,...,n \\ & \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} - \theta_0^{1(S)*} \cdot \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 0} = \theta_0^{1(S)*} \\ & \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} = 1 \\ & \mu_{d_1}, \; \mu_{d_2}, \; \nu_{i_1}, \; \nu_{i_2} \ge 0, \; d_1=1,...,D_1, \; d_2=1,...,D_2, \\ & i_1=1,...,I_1, \; i_2=1,...,I_2.\end{aligned} \tag{24}$$

$$\begin{aligned}
\theta_0^{1.2(2(L))+} = \quad & \max \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} \\
& \text{s.t. } \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 j} - \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 j} \leq 0,\ j = 1, ..., n \\
& \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 j} - \sum\nolimits_{i_2=1}^{I_2} \nu_{i_2} x_{i_2 j} \leq 0,\ j = 1, ..., n \\
& \sum\nolimits_{d_1=1}^{D_1} \mu_{d_1} z_{d_1 0} + \sum\nolimits_{d_2=1}^{D_2} \mu_{d_2} z_{d_2 0} - \theta_0^{1(S)*} \cdot \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} = \theta_0^{1(S)*} \\
& \sum\nolimits_{i_1=1}^{I_1} \nu_{i_1} x_{i_1 0} = 1 \\
& \mu_{d_1},\ \mu_{d_2},\ \nu_{i_1},\ \nu_{i_2} \geq 0,\ d_1 = 1, ..., D_1,\ d_2 = 1, ..., D_2, \\
& i_1 = 1, ..., I_1,\ i_2 = 1, ..., I_2.
\end{aligned} \tag{25}$$

According to formulas (3), the efficiencies of HS and BS can be determined as $\theta_0^{1.2(2(L))-} = \frac{\theta_0^{1(S)*} - w_1^{1(S)*} \cdot \theta_0^{1.1(2(L))+}}{w_2^{1(S)*}}$ and $\theta_0^{1.1(2(L))-} = \frac{\theta_0^{1(S)*} - w_2^{1(S)*} \cdot \theta_0^{1.2(2(L))+}}{w_1^{1(S)*}}$, respectively, where $w_1^{1(S)*}$ and $w_2^{1(S)*}$ represent optimal weights obtained from model (15) by way of (4). Finally, if $\theta_0^{1.1(2(L))+} = \theta_0^{1.1(2(L))-}$ or $\theta_0^{1.2(2(L))+} = \theta_0^{1.2(2(L))-}$, unique decomposition of DCS is obtained. We denote then the optimal efficiency scores of BS and HS as $\theta_0^{1.1(2(L))*}$ and $\theta_0^{1.2(2(L))*}$, respectively. With this procedure, the unique efficiency decomposition of DCS into the efficiencies of BS and HS is realized in the PMS leader case.

2. Illustrative application

After formulating the methodological framework in the previous section, this section illustrates the proposed cooperative and cooperative-Stackelberg hybrid two-stage DEA models on the datasets of 19 main Chinese commercial banks listed in the China Stock Market.

Paradi *et al.* [31] proposed three approaches for variables selection in the banking performance evaluation: production approach, intermediary approach and profitability approach. From the viewpoint of production process, we regard the commercial bank using operational and capital costs [44], [46], [3] to produce standard outputs like incomes, profits, loans, and investment returns, via deposits collected from individual consumers and business consumers. The selection of input-output variable in this study is consistent with the main literature.

As illustrated in Fig.1, for any commercial bank $j, (j = 1, ..., 19)$ in the sample, the DCS is comprised by BS and HS in parallel, which uses Business banking operational costs ($x_{1_1 j}$) and Business banking capital costs ($x_{2_1 j}$) to generate Business deposits (z_{1j}), and uses Consumer banking operational costs ($x_{1_2 j}$) and Consumer banking capital costs ($x_{2_2 j}$) to generate Consumer deposits (z_{2j}), respectively. Then, PMS takes z_{1j} and z_{2j} as inputs to yield Net interest income (y_{1j}), Net commissions and fees income (y_{2j}), Net profit (y_{3j}), Earnings per share (y_{4j}), Total loans and advances (y_{5j}), and Investment returns (y_{6j}), which are considered as final outputs of the banking production system.

The data is derived from the 2017 annual reports of 19 Chinese listed commercial banks, including Bank of Beijing (BBJ), Changshu Bank (CSB), Bank of Chengdu (BCD), Industrial and Commercial Bank of China (ICBC), China Everbright Bank (CEB), Bank of Guiyang (BGY), Bank of Hangzhou (BHZ), China Construction Bank (CCSB), Bank of Jiangsu (BJS), Jiangsu Jiangyin Rural Commercial Bank (JRCB), Bank of Communications (BCM), China Minsheng Bank (CMSB), Bank of Shanghai (BSH), Wujiang Rural Commercial Bank (WRCB), Bank of Changsha (BCS), China Merchants Bank (CMB), Bank of Zhengzhou (BZZ), Bank of China (BOC) and China Citic Bank (CCB). Table 3 shows the data set.

We apply the developed cooperative and cooperative-Stackelberg hybrid two-stage DEA models to the dataset. The models are coded by Matlab2014a, and the results are reported in Table 4, Table 5 and Table 6.

The second column of Table 4 reports the overall efficiency scores (Column 2) along with the efficiency decomposition results of the overall system when DCS takes priority (Column 3 and 4), the efficiency decomposition results of the overall system when PMS takes priority (Column 5 and 6), the efficiency decomposition results of DCS when BS is to be given pre-emptive priority (Column 7 and 8), the efficiency decomposition results of DCS when HS is to be given pre-emptive priority (Column 9 and 10), and the optimum weights associated with the efficiencies of BS and HS (Column 11 and 12), respectively.

It can be seen from Table 4 that we have a unique decomposition of overall efficiency into the efficiencies of DCS and PMS for all the 19 banks. These results arise from the fact that models (10) and (11) yield identical efficiency scores for DCS or PMS whatever the priority is assigned to one of the two stages. The unique efficiency

Table 3. Inputs, intermediate measures and outputs of 19 commercial banks

Banks	x_{1_1}	x_{2_1}	x_{1_2}	x_{2_2}	z_1	z_2	y_1	y_2	y_3	y_4	y_5	y_6
BBJ	19679	2905	3389	515	950473	250360	39376	10579	18882	0.99	1077101	395
CSB	1505	53	1282	69	41042	52244	4997	4324	1322	0.57	77811	107
BCD	1483	40	821	18	199181	100732	7463	393	3913	1.2	143589	1694
IBCB	190048	15794	148665	12964	9547107	8068894	522078	139625	287451	0.79	14233448	11927
CEB	26950	2635	21531	1652	1800948	471717	60950	30774	31611	0.59	2032056	212
BGY	1459	284	443	85	221114	65917	10861	1414	4588	1.97	7600	109
BHZ	2549	138	838	32	352637	66488	12267	1617	4550	1.24	12927	825
CCSB	166087	5110	98382	7974	8430224	7078489	452456	117798	243651	0.96	12903441	6411
BJS	16109	297	1668	243	641999	187693	27815	5779	12016	1.03	747289	229
JRCB	669	236	118	38	37419	36326	2100	53	758	0.46	55853	331
BCM	54044	8667	40575	18172	3257639	1541597	127366	40551	70233	0.91	4456914	4264
CMSB	37278	977	33064	735	2446634	523454	86552	47742	50922	1.35	2804307	2715
BSH	10356	274	3449	218	651551	205268	19117	6256	15337	1.96	664022	9637
WRCB	1496	191	204	28	37937	28824	2525	69	739	0.5	47463	120
BCS	3321	475	2178	168	240319	84395	11120	1094	3985	1.28	154487	149
CMB	66231	2930	59968	4494	2725823	1338522	144852	64018	70638	2.78	3565044	6205
BZZ	1272	358	472	134	168365	67563	8106	1865	4334	0.8	128456	547
BOC	130015	4266	78168	4719	7383774	5831228	338389	88691	184986	0.56	10896558	12155
CCB	66337	3309	34067	1981	2874198	533838	99645	46858	42878	0.84	3196887	6988

Table 4. Overall efficiency and efficiency decomposition results of cooperative models

Banks	$\theta_0^{overall}$	θ_0^{1+}	θ_0^{2-}	θ_0^{1-}	θ_0^{2+}	$\theta_0^{1.1+}$	$\theta_0^{1.2-}$	$\theta_0^{1.1-}$	$\theta_0^{1.2+}$	w_1^*	w_2^*
BBJ	0.3473	0.3473	1.0000	0.3473	1.0000	0.3283	0.3641	0.3283	0.3641	0.467614	0.532386
CSB	0.2862	0.2862	1.0000	0.2862	1.0000	0.2012	0.2862	0.2012	0.2862	0.000020	0.999980
BCD	0.8625	1.0000	0.8625	1.0000	0.8625	1.0000	1.0000	1.0000	1.0000	0.726298	0.273702
ICBC	0.3527	0.3527	1.0000	0.3527	1.0000	0.3561	0.3330	0.3561	0.3330	0.851342	0.148658
CEB	0.3237	0.3525	0.9183	0.3525	0.9183	0.4691	0.1400	0.4691	0.1400	0.645681	0.354319
BGY	0.9177	0.9177	1.0000	0.9177	1.0000	1.0000	0.6539	1.0000	0.6539	0.762206	0.237794
BHZ	0.7470	0.8074	0.9251	0.8074	0.9252	1.0000	0.5978	1.0000	0.5978	0.521179	0.478821
CCSB	0.3938	0.3972	0.9915	0.3972	0.9915	0.3763	0.4520	0.3763	0.4520	0.724530	0.275470
BJS	0.4912	0.4912	1.0000	0.4912	1.0000	0.4341	0.5653	0.4341	0.5653	0.565066	0.434934
JRCB	0.7319	0.8793	0.8324	0.8793	0.8324	0.3691	1.0000	0.3691	1.0000	0.191378	0.808622
BCM	0.2932	0.4060	0.7220	0.4059	0.7222	0.4064	0.1234	0.4064	0.1234	0.998750	0.001250
CMSB	0.3672	0.3680	0.9977	0.3680	0.9977	0.5029	0.1288	0.5029	0.1288	0.639421	0.360579
BSH	0.4775	0.4775	1.0000	0.4775	1.0000	0.4775	0.4019	0.4775	0.4017	0.999949	0.000051
WRCB	0.6054	0.7289	0.8306	0.7289	0.8306	0.1746	0.7289	0.1748	0.7289	0.000022	0.999978
BCS	0.3025	0.3517	0.8601	0.3517	0.8601	0.4933	0.2472	0.4933	0.2472	0.424832	0.575168
CMB	0.2531	0.2711	0.9337	0.2711	0.9337	0.3007	0.1436	0.3007	0.1436	0.811664	0.188336
BZZ	0.8002	0.8002	1.0000	0.8002	1.0000	0.8734	0.5035	0.8734	0.5035	0.802269	0.197731
BOC	0.4223	0.4474	0.9437	0.4474	0.9438	0.4201	0.5095	0.4201	0.5095	0.694451	0.305549
CCB	0.2528	0.2611	0.9683	0.2611	0.9684	0.3146	0.1080	0.3146	0.1080	0.740825	0.259175
Mean	0.4857	0.5233	0.9361	0.5233	0.9361	0.4999	0.4362	0.4999	0.4632	0.608816	0.391184

decomposition permits the cooperative hybrid two-stage DEA models to bring further to light the source of inefficiency.

We find out that the average efficiency of PMS is 0.9361, which is much higher than that of DCS (0.5233). That is to say, the commercial banks are more efficient in investment operations in general and the lower performance of DCS is probably the source of inefficiency. This result is consistent with the findings of [42] and [46], but the latters did not go deeper into the inefficient sub-stage to get more information. Therefore, it will be meaningful to move the focus of analysis to the inside of DCS. We decompose the efficiency of DCS into BS efficiency and HS efficiency. The decomposition results are reported by columns 7 to 10 in Table 4.

It seems that BCD, BGY, and BHZ are estimated as efficient in BS, but only two banks, BCD and JRCB, are efficient in HS. The average efficiency of BS activity is slightly higher than that of HS, and we must observe with care the individual efficiency of the banks. In fact, JRCB and WRCB are both local rural commercial banks for Chinas small cities, where household savings are relatively more important than those big banks. While for the big banks, for example, ICBC, CCSB, and BOC, the efficiencies are somewhat at a similar level for the two stages.

For most banks, the efficiency of BS appears to be the driver of DCS efficiency, because the weights associated with the efficiencies of business service operations are dominant in most cases (except CSB, JRCB, WRCB). This finding is intuitive based on the importance of business banking relative to consumer banking for most commercial banks, *i.e.*, BS are generally accepted to have more influence on the performance of deposits collecting.

However, it should be noticed that individual banks (JRCB and WRCB) having relatively low efficiency scores of the BS are evaluated with high efficiency scores of DCS. This result is not contradictive in the view of efficiency formation, because the efficiency contribution of HS is not significant in the efficiency of DCS for the two banks. Meanwhile, this result helps us to understand why a bank having inefficient sub-systems can be identified to be efficient in a particular stage even in the overall performance.

The detailed efficiency scores of sub-stage and sub-systems provide rich information about the sources of inefficiency. That is, the operation inefficiency comes from DCS for most of the commercial banks, and the efficiency of BS generally plays an important role in the efficiency formation of the DCS. Such information can help decision makers to meliorate more precisely the local or overall performance of the banking production system.

The results are obtained from our proposed cooperative approach with the assumption that none of the two stages has priority in operation efficiency maximization. Beyond these, our approach provides a set of cooperative-Stackelberg DEA models to evaluate and decompose the efficiency of the same banking production system in a non-full cooperation case. The efficiency scores are presented in Table 5 and Table 6.

Table 5. Results of Cooperative-Stackelberg DEA model (DCS as leader)

Banks	$\theta_0^{1(L)*}$	$\theta_0^{2(S)*}$	$\theta_0^{overall(1L)*}$	$\theta_0^{1.1(1(L))+}$	$\theta_0^{1.2(1(L))-}$	$\theta_0^{1.1(1(L))-}$	$\theta_0^{1.2(1(L))+}$	$w_1^{1(L)*}$	$w_2^{1(L)*}$
BBJ	0.3640	0.8842	0.3219	0.3283	0.3641	0.3283	0.3641	0.000290	0.999710
CSB	0.2862	1.0000	0.2862	0.2011	0.2862	0.2012	0.2862	0.000020	0.999980
BCD	1.0000	0.8625	0.8625	1.0000	1.0000	1.0000	1.0000	0.692946	0.307054
ICBC	0.3561	0.9358	0.3332	0.3561	0.3330	0.3561	0.3332	0.997577	0.002423
CEB	0.4690	0.5954	0.2792	0.4691	0.1400	0.4691	0.1400	0.999663	0.000337
BYG	1.0000	0.6442	0.6442	1.0000	0.6539	1.0000	0.6519	0.999990	0.000010
BHZ	1.0000	0.4276	0.4276	1.0000	0.5978	1.0000	0.6001	0.999989	0.000011
CCSB	0.4518	0.3748	0.1694	0.3763	0.4520	0.3763	0.4520	0.002240	0.997760
BJS	0.5653	0.8132	0.4597	0.4328	0.5653	0.4341	0.5653	0.000148	0.999852
JRCB	1.0000	0.6823	0.6823	0.3705	1.0000	0.3691	1.0000	0.000010	0.999990
BCM	0.4060	0.7221	0.2932	0.4064	0.1234	0.4064	0.1234	0.998750	0.001250
CMSB	0.5028	0.6465	0.3250	0.5029	0.1288	0.5029	0.1284	0.999594	0.000406
BSH	0.4775	1.0000	0.4775	0.4775	0.4017	0.4775	0.4017	0.999949	0.000051
WRCB	0.7289	0.8306	0.6054	0.1758	0.7289	0.1746	0.7289	0.000022	0.999978
BCS	0.4933	0.5148	0.2539	0.4933	0.2472	0.4933	0.2475	0.999966	0.000034
CMB	0.3005	0.8088	0.2431	0.3007	0.1436	0.3007	0.1436	0.999068	0.000932
BZZ	0.8734	0.8235	0.7192	0.8734	0.5035	0.8734	0.5042	0.999987	0.000013
BOC	0.5094	0.3418	0.1741	0.4201	0.5095	0.4201	0.5095	0.001757	0.998243
CCB	0.3145	0.6155	0.1936	0.3146	0.1080	0.3146	0.1078	0.999506	0.000494
Mean	0.5841	0.7118	0.4080	0.4999	0.4362	0.4999	0.4362	0.615341	0.384659

In Table 5 and Table 6, the first three columns report the efficiency scores of DCS and PMS along with the efficiency of the overall system in the cases of DCS as the leader and PMS as the leader, respectively. The last six columns show the efficiency decomposition of DCS and the optimum weights for BS calculated by cooperative-Stackelberg DEA models. We primarily notice that there are no overall efficient banks no matter which stage is the leader.

Table 6. Results of Cooperative-Stackelberg DEA model (PMS as leader)

Banks	$\theta_0^{1(S)*}$	$\theta_0^{2(L)*}$	$\theta_0^{overall(2L)*}$	$\theta_0^{1.1(2(L))+}$	$\theta_0^{1.2(2(L))-}$	$\theta_0^{1.1(2(L))-}$	$\theta_0^{1.2(2(L))+}$	$w_1^{1(L)*}$	$w_2^{1(L)*}$
BBJ	0.3473	1.0000	0.3473	0.3283	0.3641	0.3283	0.3641	0.467604	0.532396
CSB	0.2862	1.0000	0.2862	0.2011	0.2862	0.2011	0.2862	0.000001	0.999999
BCD	1.0000	0.8625	0.8625	1.0000	1.0000	1.0000	1.0000	0.726315	0.273685
ICBC	0.3527	1.0000	0.3527	0.3561	0.3330	0.3561	0.3330	0.853895	0.146105
CEB	0.3249	0.9504	0.3087	0.4691	0.1400	0.4691	0.1400	0.561716	0.438284
BGY	0.9177	1.0000	0.9177	1.0000	0.6539	1.0000	0.6539	0.762224	0.237776
BHZ	0.7360	1.0000	0.7360	1.0000	0.5978	1.0000	0.5978	0.343616	0.656384
CCSB	0.3972	0.9961	0.3938	0.3763	0.4520	0.3763	0.4520	0.724506	0.275494
BJS	0.4912	1.0000	0.4912	0.4341	0.5653	0.4341	0.5653	0.565062	0.434938
JRCB	0.5754	1.0000	0.5754	0.3690	1.0000	0.3691	0.9999	0.672934	0.327066
BCM	0.2668	1.0000	0.2668	0.4064	0.1234	0.4064	0.1234	0.506616	0.493384
CMSB	0.3666	1.0000	0.3666	0.5029	0.1288	0.5029	0.1288	0.635491	0.364509
BSH	0.4775	1.0000	0.4775	0.4775	0.4017	0.4776	0.4017	0.999999	0.000001
WRCB	0.5532	1.0000	0.5532	0.1746	0.7289	0.1746	0.7289	0.316988	0.683012
BCS	0.3154	0.9230	0.2911	0.4933	0.2472	0.4933	0.2472	0.277190	0.722810
CMB	0.2464	1.0000	0.2464	0.3007	0.1436	0.3007	0.1436	0.654142	0.345858
BZZ	0.8002	1.0000	0.8002	0.8734	0.5035	0.8734	0.5035	0.802286	0.197714
BOC	0.4474	0.9438	0.4223	0.4201	0.5095	0.4201	0.5095	0.694642	0.305358
CCB	0.2525	1.0000	0.2525	0.3146	0.1080	0.3146	0.1080	0.699178	0.300822
Mean	0.4818	0.9827	0.4710	0.4999	0.4362	0.4999	0.4362	0.592863	0.407130

As Table 5 and Table 6 show, when DCS is the leader, the average overall efficiency is 0.4080, which is lower than that (0.4710) in PMS as leader case. The main reason given for this is that PMS is significantly more efficient when it is the leader (14 efficient banks), even if the average efficiency of DCS is lowered.

Besides, the means of overall efficiency in the cooperative-Stackelberg case with different leaders are both lower than those of overall efficiency in the cooperative approach. These results suggest that in our application, the performance of PMS is relatively dominant in the overall efficiency formation for most of the commercial banks, and non-cooperation relations between DCS and PMS will probably decrease the overall efficiency of the banking systems.

We also notice that BS and HS are carried out in parallel and undertaken independently within the DCS. This fact leads to the result that the efficiency decompositions of DCS are largely the same in the cooperative approach and in the cooperative-Stackelberg approach.

Nevertheless, testing the last two columns in Table 5 and Table 6, we find out that the weights associated with the efficiencies of BS and those of HS are quietly different. The mean of the weights associated with BS efficiency is evidently greater than the mean of the weights associated with the efficiency of HS in the cooperative-Stackelberg approach. This result is consistent with that in the cooperative situation, which verifies the dominant position of business saving services in DCS for commercial banks.

Several managerial insights into the operating efficiency management of commercial banks are summarized as follows. Firstly, the source of operating inefficiency exists in DCS for most of the commercial banks, and the managers should pay more attention to improve the efficiency of this stage. Secondly, the efficiency of BS system seems to be the driver of DCS, that is to say, the BS system are relatively more important than those of HS system from the perspective of operating efficiency management.

Finally, it should be noted that, when PMS is the leader, the overall efficiency scores of the commercial banks evaluated by the cooperative-Stackelberg approach are closest to the results in the cooperative approach, which are at a relatively optimal level. This result implies that the commercial banks should prefer to a cooperative operating system, which permits to achieve the optimal overall efficiency, and they should give priority to PMS when there is conflict between DCS and PMS.

2.1. Conclusion

Based on the series structure of the banks production process, the classic two-stage DEA model has a good fit for efficiency evaluation and decomposition. But this approach has generally lost sight of the performance of parallel operations inside sub-stages, and relations between two stages are normally limited to be cooperative or competitive. The results and information issued from classic two-stage DEA models are abundant but not rich enough.

In this paper, we develop a set of cooperative and cooperative-Strackelberg hybrid two-stage DEA models to assess the operating efficiency of commercial banks. The core points of this study and the main differences of our models with those that proposed previously is that, we believe that DEA models could reveal more useful information if we go deep into the internal structure of the system, and that the relations of sub-systems would affect the efficiency performance of banks operating.

The proposed approach contributes in the following points to the efficiency evaluation and decomposition of system with complex internal structure. Firstly, we present a very general case where the series-parallel internal structure and the cooperative-Stackelberg game relations coexist in a two-stage production system. Secondly, we adopt a combination of additive and multiplicative DEA thoughts to calculate and decompose efficiencies of the system, which can portray efficiency formation mechanisms for systems with complex internal structure and relations. Thirdly, the cooperative game and the leader-follower game theories are incorporated, which can be applied in different scenarios of games between the sub-stages of the production system at once.

The application of our proposed DEA models to the datasets of 19 main Chinese commercial banks listed in the China Stock Market provides a comprehensive picture of the banks production system performance in cooperative and non-cooperative situations. The results reveal that the sources of inefficiency exist in DCS for most banks. There is no significant difference between the efficiency levels of BS and HS, but the efficiency of BS dominants the efficiency of DCS in most cases. The efficiency of PMS is significantly higher than that of DCS no matter in the cooperative situation or the cooperative-Stackelberg situation. The overall efficiency of the banking production system will achieve a relatively higher score when all the sub-systems operate cooperatively, however, non-cooperation relation between sub-stages will probably decrease the overall efficiency performance.

There are several possible future research directions that can be drawn from this study. Firstly, the proposed approach is developed under the constant returns to scale (CRS) assumption. Future work may consider how to build the models under the variable returns to scale (VRS) assumption while tanking the complex internal structure of production system. Secondly, the undesirable outputs and uncertainty are not discussed in the proposed approach. It would be very interesting and significant to dig deeper into these aspects. Finally, the complexity of the production system can be extended to the problems of resource allocations to sub-stages within the operating system. Therefore, it becomes natural to think about how the resource allocations within the system would influence the efficiency of operating activities in the future research.

Acknowledgement

The authors wish to thank the anonymous reviewers for their constructive comments and suggestions. This work was supported by the National Natural Science Foundation of China (NSFC) under Grant 71673022.

REFERENCES

[1] S. Akther, H. Fukuyame and W.L. Weber, Estimating two-stage network slacks-based inefficiency: an application to Bangladesh banking. *Omega* 41 (2013) 88-96.

[2] A. Amirteimoori, S. Kordrostami and H. Azizi, Additive models for network data envelopment analysis in the presence of shared resources. *Transport. Res. D.* 48 (2016) 411-424.

[3] Q. An, Q. Wu, X. Zhou and X. Chen, Closest target setting for two-stage network system: An application to the commercial banks in China. *Expert Syst. Appl.* 175 (2021) 1144799.

[4] A. Ashrafi and A.B. Jaafar, Performance measurement of two-stage production systems with undesirable factors by data envelopment analysis. *J. Appl. Sci.* 11 (2011) 3515-3519.

[5] A.D. Athanassopoulos, Service quality and operating efficiency synergies for management control in the provision of financial services: Evidence form Greek bank branches. *Eur.J. Oper. Res.* 98 (1997) 300-313.

[6] N.K. Avkiran and A. McCrystal, Sensitivity analysis of network DEA: NSBM versus NRAM. *Appl. Math. Comput.* 218 (2012) 11226-11239.

[7] A. Charnes and W.W. Cooper, Programming with linear fractional functional. *Nav. Res. Log.* 9 (1962) 181-186.

[8] A. Charnes, W.W. Cooper and E. Rhodes, Measuring the efficiency of decision making units. *Eur.J. Oper. Res.* 2 (1978) 429-444.

[9] Y. Chen, W.D. Cook, N. Li and J. Zhu, Additive efficiency decomposition in two-stage DEA. *Eur.J. Oper. Res.* 196 (2009) 1170-1176.

[10] Y. Chen, Y.J. Li, L. Liang, A. Salo and H. Wu, Frontier projection and efficiency decomposition in two-stage processes with slacks-based measures. *Eur.J. Oper. Res.* 250 (2016) 543-554.

[11] Y. Chen and J. Zhu, Measuring information technologys indirect impact on firm performance. *Inform. Technol. Manag.* 5 (2004) 9-22.

[12] A. Ebrahimnejad, M. Tavana, F.H. Lotfi, R. Shahverdi and M. Yousefpour, A three-stage Data Envelopment Analysis model with application to banking industry. *Measurement* 49 (2014) 308-319.

[13] H. Fukuyama and R. Matousek, Efficiency of Turkish banking: two-stage network system: Variable returns to scale model. *J. Int. Financ. Mark. I.* 21 (2011) 75-91.

[14] H. Fukuyama and R. Matousek, Modeling bank performance: A network DEA approach. *Eur.J. Oper. Res.* 259 (2017) 721-732.

[15] M. Hemmati, S.A. Dalghandi and H. Nazari, Measuring relative performance of banking industry using a DEA and TOPSIS. *Manage. Sci. Letters.* 3 (2013) 499-503.

[16] C. Kao, Network data envelopment analysis: a review. *Eur.J. Oper. Res.* 239 (2014a) 1-16.

[17] C. Kao, Efficiency decomposition for general multi-stage systems in data envelopment analysis. *Eur.J. Oper. Res.* 232 (2014b) 117-124.

[18] C. Kao and S.N. Hwang, Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. *Eur.J. Oper. Res.* 185 (2008) 418-429.

[19] S.A. Kourtzidis, R. Matousek and N.G. Tzeremes, Productivity growth in network models: An application to banking during the financial crisis. *J. Oper. Res. Soc.* 7 (2019) 111-124.

[20] H.B. Kwon and J. Lee, Two-stage production modelling of large U.S. banks: A DEA-neural network approach. *Expert Syst. Appl.* 42 (2015) 6758-6766.

[21] H.B. Kwon, J. Lee and K.N. White Davis, Neural network modeling for a two-stage production process with versatile variables: Predictive analysis for above-average performance. *Expert Syst. Appl.* 100 (2018) 120-130.

[22] H.F. Lewis and T.R. Sexton, Network DEA: Efficiency analysis of organization with complex internal structure. *Comput. Oper. Res.* 31 (2004) 1365-1410.

[23] L. Liang, W.D. Cook and J. Zhu, DEA models for two-stage processes: Game approach and efficiency decomposition. *Nav. Res. Log.* 55 (2008) 643-653.

[24] L. Liang, F. Yang, W.D. Cook and J. Zhu, DEA models for supply chain efficiency evaluation. *Ann. Oper. Res.* 145 (2006) 35-49.

[25] W. Liu, Z. Zhou, C. Ma, D. Liu and W. Shen, Two-stage DEA models with undesirable input-intermediate-outputs. *Omega* 56 (2015) 74-87.

[26] S. Lozano, Slacks-based inefficiency approach for general network with bad outputs: An application to the banking sector. *Omega* 60 (2016) 73-84.

[27] X. Luo, Evaluating the profitability and marketability efficiency of large banks C an application of data envelopment analysis. *J. Bus. Res.* 56 (2003) 627-635.

[28] K. Matthews, Risk management and managerial efficiency in Chinese banks: A network DEA framework. *Omega* 41 (2013) 207-215.

[29] S. Mehdizadeh, A. Amirteimoori, V. Charles, M.H. Behzadi and S. Kordrostami, Measuring the efficiency of two-stage network processes: A satisficing DEA approach. *J. Oper. Res. Soc.* 72 (2020) 354-366.

[30] S.G.J Naini, A. Moini and M.J. Rezaee, Nash bargaining game model for two parallel stages process evaluation with shared inputs. *Int. J. Adv. Manuf. Tech.* 67 (2013) 475-484.

[31] J.C. Paradi, S. Rouatt and H. Zhu, Two-stage evaluation of bank branch efficiency using data envelopment analysis. *Omega* 39 (2011) 99-109.

[32] J.C. Paradi and H. Zhu, A survey on bank branch efficiency and performance research with data envelopment analysis. *Omega* 41 (2013) 61-79.

[33] B.K. Sahoo, J. Zhu, K. Tone and B.M. Klemen, Decomposing technical efficiency and scale elasticity in two-stage network DEA. *Eur.J. Oper. Res.* 233 (2014) 584-594.

[34] L.M. Seiford and J. Zhu, Profitability and marketability of the top 55US commercial banks. *Manage. Sci.* 45 (1999) 1270-1288.

[35] M.S. Shahbazifar, R.K. Matin, M. Khounsiavash and F. Koushki, Group ranking of two-stage production units in network data envelopment analysis. *Rairo-Oper. Res.* 55 (2021) 1825-1840.

[36] G. Sherman and F. Gold, Bank branch operating efficiency C evaluation with data envelopment analysis. *J. Bank. Finance.* 9 (1985) 297-315.

[37] X. Shi, Y. Li, A. Emrouznejad, J. Xie and L. Liang, Estimation of potential gains from bank mergers: A novel two-stage cost efficiency DEA model. *J. Oper. Res. Soc.* 68 (2017) 1045-1055.

[38] I.M. Tavakoli and A. Mostafaee, Free disposal hull efficiency scores of units with network structures. *Eur.J. Oper. Res.* 277 (2019) 1027-1036.

[39] K. Tone and M. Tsutsui, Network DEA: A slacks-based measure approach. *Eur.J. Oper. Res.* 197 (2009) 243-252.

[40] I.E. Tsolas, Modeling bank branch profitability and effectiveness by means of DEA. *Int. J. Product. Perfor.* 59 (2010) 432-451.

[41] P. Wanke and C. Barros, Two-Stage DEA: An application to major Brazilian banks, *Expert Syst. Appl.* 41 (2014) 2337-2344.

[42] K. Wang, W. Huang, J. Wu and Y.M. Liu, Efficiency measures of the Chinese commercial banking system using an additive two-stage DEA. *Omega* 44 (2014) 5-20.

[43] G. Xu and Z. Zhou, Assessing the efficiency of financial supply chain for Chinese commercial banks: A two-stage AR-DEA model. *Ind. Manage. Data Syst.* 121 (2020) 894-920.

[44] C. Yang and H.M. Liu, Managerial efficiency in Taiwan bank branches: A network DEA, *Econo. Mod.* 29 (2012) 450-461.

[45] Y. Zha and L. Liang, Two-stage cooperation model with input freely distributed among the stages. *Eur.J. Oper. Res.* 205 (2010) 332-338.

[46] Y. Zha, N. Liang, M. Wu and Y. Bian, Efficiency evaluation of banks in China: A dynamic two-stage slacks-based measure approach. *Omega* 60 (2016) 60-72.

[47] Z. Zhou, L. Sun, W. Yang, W. Liu and C. Ma, A bargaining game model for efficiency decomposition in the centralized model of two-stage systems. *Comput. Ind. Eng.* 64 (2013) 103-108.