The choice of cooperative technology innovation strategies in a supply chain under governmental subsidy

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Abstract

Governments all over the world usually establish the policy of subsidies to stimulate firms' technology innovation behaviors. The participating firms may share the high risk of expense through cooperative technology innovation. Different forms of governmental subsidies may have a significant impact on the choice of firms' cooperative innovation strategies. This paper investigates the effect of government subsidies on firms' technology innovation strategies. We consider two modes of cooperative technology innovation (technology transfer or joint innovation) in a two-level supply chain including an upstream manufacturer (UM) and a downstream manufacturer (DM) in the presence of two forms of governmental subsidies (a per-unit production subsidy or an innovation subsidy). We find that in the presence of either form of governmental subsidy, technology transfer mode is better off for the UM than joint innovation mode when the UM's distribution power is greater than a threshold, otherwise joint innovation mode is better off. In the presence of a given form of governmental subsidy, the DM's response strategy is influenced by the interaction of different values of the proportion of revenue and the fraction of innovation cost. In the presence of a per-unit production subsidy, the social welfare is always more under technology transfer mode than under joint innovation mode, while in the presence of an innovation subsidy, the opposite is true. We also show that under a given cooperative innovation mode, both the UM and DM expect a per-unit production subsidy if the per-unit tax credit is high, and they expect an innovation subsidy if the proportion of governmental subsidy is high. Finally, we discuss the robustness of the theoretical results.

Keywords: Technology innovation; Governmental subsidy; Technology transfer; Joint innovation; Supply chain

1 Introduction

Technology innovation is an effective measure taken by firms all over the world in the face of fierce competition. In order to save the cost of technology innovation, many firms become willing to cooperate with other innovators in a supply chain to deliver technology. Especially for those technology-intensive firms, they

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could pursue time and cost reductions, better product design through technology innovation cooperation (Zhang et al., 2018). As a response to conduct technology innovation, many governments usually implement subsidies to stimulate firms' innovation behaviors (Australian Government, 2013; Government of Canada, 2017). In turn, the support of government policies also affects cooperative innovation behaviors of firms in a supply chain. Therefore, studying the interaction between governmental subsidies and firms' cooperation innovation behaviors is a research issue worth exploring.

In practice, a government may offer subsidies to promote innovation behaviors of firms, and the common forms are per-unit production subsidy and innovation effort subsidy. The former, per-unit production subsidy, is a tax credit that the government returns tax to the firms by first collecting and then refunding, or collecting and refunding, which is a kind of government subsidy granted in the form of tax incentives. For example, the Government of India Ministry of Power, through the Unnat Jyoti By Affordable LEDs for All program, subsidized the procurement of LED lightbulbs in the country.[1] Alternatively, innovation effort subsidy, is a subsidy that the government directly subsidizes a certain percentage of technological innovation cost to innovative firms. For instance, part of the American Recovery and Reinvestment Act of 2009 allocated 400 million dollars to Advanced Research Projects Agency-Energy to fund innovation of energy technologies.[2] One aim of this paper is to explicate how firms should respond to governmental subsidies to choose technology innovation strategies.

Two modes of cooperative technology innovation commonly occur in a supply chain, one is technology transfer mode and the other is joint innovation mode. The former, technology transfer, refers to the transfer of systematic knowledge about manufacturing products, applying production methods or providing services (Feld, 1974). In this mode, only the technology exporter will receive government subsidies, because only it is directly involved in innovation. Furthermore, one firm transfers technology to the other firm in a supply chain to obtain a certain percentage of revenue. At the same time, for the technology importing firm, it has to sacrifice a certain percentage of its own profit in exchange for acquiring technology. With the increase in the degree of cooperation among members in the supply chain, it has gradually evolved into a long-term and stable cooperative alliance, that is, the latter, a joint innovation mode. In this mode, one firm motivates the other firm in a supply chain to exert effort by sharing the innovation effort cost, and the two firms also divide government subsidies according to this sharing ratio. The existence of government subsidies may make the firms entangled in whether to obtain more subsidies, or to adopt optimal cooperation strategy to pursue more cost reduction for technology innovation.

Motivated by the above considerations, we consider a cooperative innovation strategy choice problem in a supply chain under governmental subsidy considering the impact of firms' cooperative innovation strategies and forms of governmental subsidies on the equilibrium decisions, the profits of firms and social welfare. In particular, our research attempts to address the following questions: (1) How do the proportion of revenue and the fraction of innovation cost affect firms' profits? (2) In the presence of a given form of governmental subsidy, which mode of cooperative innovation are firms' choices and governmental expectation? (3) Under a given mode of cooperative innovation, which form of governmental subsidy is more effective?

To answer these questions, we consider a supply chain including an UM and a DM that jointly develops an innovative product. The UM produces a key component and sells it to the DM, and the DM processes the component into the final product and sells the product to the market. We study the innovation efforts and the profits of the UM and DM under the two cooperative innovation modes (technology transfer, and joint innovation) and two forms of governmental subsidies (a per-unit production subsidy and an innovation subsidy). The UM is the chain's leader, while the DM is the follower. We concentrate on manufacturers' cooperative behaviours on the basis of their own profits and social welfare, and find the following results.

First, the UM's profit increases with the proportion of revenue or decreases with the fraction of innovation cost, but the DM's profit might not decrease with the proportion or increase with the fraction. Second,under either form of governmental subsidy, technology transfer mode is better off for the UM than joint innovation mode when the UM's distribution power is greater than a threshold, otherwise joint innovation mode is better off; under a given form of governmental subsidy, the DM's response strategy is influenced by the interaction of different values of the proportion of revenue and the fraction of innovation cost; in the presence of a per-unit production subsidy, the social welfare is always more under technology transfer mode than under joint innovation mode, while in the presence of an innovation subsidy, the opposite is true. Third, under a given cooperative innovation mode, both the UM and DM expect a per-unit production subsidy if the per-unit tax credit is high, and they expect an innovation subsidy if the proportion of governmental subsidy is high; for the government, he always prefers a per-unit production subsidy under technology transfer mode, but prefers an innovation subsidy under joint innovation mode.

The rest of this paper is organized as follows. We first outline the literature related to our work, following which, we present our model and analysis. We then discuss our findings and key insights before offering our conclusions and suggestions for future research. All the proofs are presented in the Appendix.

2 Literature review

The literature related to this paper mainly comes from two streams of research, one on cooperative innovation and the other on governmental subsidies for technology innovation. We briefly review the literature for each.

2.1 Cooperative innovation

Cooperative innovation refers to an innovative act of joint research and development, production and sales between firms and firms to implement technology innovation (Krishnan and Ulrich, 2001). Generally, cooperative innovation literature can be divided into horizontal cooperative innovation (D'Aspremont and Jacquemin, 1988; Kamien et al., 1992; Amir, 2000; Ge and Hu, 2008; Erkal and Piccinin, 2010) and vertical cooperative innovation. Our study belongs to the area of vertical cooperative innovation. Actually, Riggs and Von Hippel (1994) show that it is very critical for a firm to cooperate closely with its vertical partners in innovation activities. Moreover, Arranza and Arroyabe (2008) discover that vertical technology cooperation with firms occurs more frequently in practice than horizontal cooperation with competitors. Specifically, for an upstream firm, a major motivation for technology cooperation with a downstream firm is to determine market requirements, because market requirements are often unable to be accurately forecasted by the upstream firm itself (Zipkin, 2001). In contrast, a downstream firm may obtain significant benefits from cooperating early with an upstream firm (Bidault et al. 1998). The reader is referred to Krishnan and Loch (2005) for a comprehensive review of this literature. In this study, we analyze the motivation and profit pursuit of upstream and downstream firms in a supply chain under two modes of cooperative innovation, which enriches the research connotation of upstream and downstream vertical cooperation.

Some researchers have focused on the effects of different forms of innovative cooperation on firm's behaviors. In general, the common forms of firms' cooperative innovation include technology transfer, joint innovation and so on. Technology transfer, according to Takahashi (2005) and Davenport (2013), is the act of transferring a technology, and refers to the process of sharing and absorption of it. Kotabe et al. (2003) examine the effects of two forms of knowledge exchange together with the prior duration of the buyer-supplier relationship. Blalock and Gertler (2008) use a panel dataset of Indonesian manufacturing establishments to test the hypothesis that multinational firms operating in emerging markets transfer technology to local suppliers to increase their productivity and to lower input prices. Savva and Taneri (2014) develop a model to explain the mixed-use of equity, royalty and fixed fee in university technology transfer. Silva et al. (2018) review technology transfer in the supply chain oriented to industry 4.0, focusing on the supply, manufacturing industry and final consumer stages. Alternatively, firms in joint innovation share perfectly their innovation technology but decide independently their innovation investments (Ge et al., 2014). Xiao and Xu (2012) study the effect of royalty revision on incentives and profits in research and development alliance. Ge et al. (2014) investigate how knowledge spillovers and cartelization as two basic means of cooperation affect firms' behaviors. Ghosh and Shah (2015) explore the impact of cost sharing contract on the key decisions of supply chain players undertaking green initiatives. Fu et al. (2018) introduce risk attitude into contract design in research and development alliance, and also explored how the marketer designs and chooses optimal contract between royalty contract and milestone contract. Zhou et al. (2020) study cooperative innovation decisions in a two-level supply chain with knowledge spillovers and uncertain technology efficiency to investigate the effects of knowledge spillovers and cartelization. Wei and Wang (2021) use differential game methods to study the interaction between carbon reduction technology innovation and government intervention under decentralized decision with cost sharing. Liu et al. (2021) study green strategy implementation of a retailer working with a supplier introducing a new green product into the market of a congeneric non-green product. In this study, based on game theory, we explore the choice of innovation strategies (including technology transfer and joint innovation) of upstream and downstream firms in the supply chain. However, current literature on technology transfer is more based on empirical methods, and the literature on joint innovation is based on the integration of other factors.

Our study is most related to the research of Chen et al. (2019) in exploring innovation behaviors between supply chain players; however, we study a supply chain consisting of an upstream manufacturer and a downstream manufacturer forming two representative innovation modes (i.e., technology transfer or joint innovation), while they consider a supply chain consisting of a manufacturer and a retailer conducting a research joint venture. Furthermore, we explore how government subsidies affect the firms' innovation strategies.

2.2 Governmental subsidies for technology innovation

Within the field of government subsidy, one stream close to our work is studying technology innovation. Among the literature of this area, scholars have studied government subsidies for technology innovation from two perspectives, i.e., a government (i.e., an endogenous government subsidy) and a non-governmental organization (i.e., an exogenous government subsidy). From the perspective of a government, for example, Atasu and Wassenhove (2012) and Ozdemir et al. (2012) study how the government uses take-back or recycling subsidies when incorporating externality of the operations to maximize social welfare. Krass et al. (2013) and Raz and Ovchinnikov (2015) examine the use of a government subsidy for firms producing public interest goods and analyze the government's ability. Jung and Feng (2020) investigate the government's subsidy design for firms' green technology development in an evolving industry and the subsidy's impact on environment and social welfare. From the perspective of a non-governmental organization, for example, Cohen et al. (2016) study how demand uncertainty impacts green technology adoption and a manufacturer's production and pricing decisions when designing per-unit government subsidies directly to end consumers. Our research is from the perspective of a non-governmental organization. To further understand the effect of government subsidy policies on technology innovation in a supply chain, our research specifically considers two distinct forms of subsidies, a per-unit production subsidy and an innovation effort subsidy.

For a per-unit production subsidy, for example, Li et al. (2018) study the impacts of government's consumption subsidy (i.e., a per-unit production subsidy) and replacement subsidy towards environmental-friendly products in a dual-channel supply chain. Huang et al. (2019) study a government's optimal subsidies for energy-efficient products in a market with two competing firms, where the government designs a subsidy scheme specifying a qualification standard and subsidy amount per unit of qualified product sold (i.e., a per-unit production subsidy). For an innovation effort subsidy, for example, Liu et al. (2019) establish a three-stage Stackelberg game model that consists of the government, a dominant retailer and some suppliers, where the government subsidizes the dominant retailer according to its level of effort (i.e., an innovation effort subsidy).

Our study is most related to the research of Wang et al. (2017), Li et al. (2018), Chen et al. (2019) in exploring government subsidies for technology innovation; however, we study a supply chain consisting of an upstream manufacturer and a downstream manufacturer jointly developing an innovative product under two forms of government subsidies, i.e., per-unit production subsidy and innovation effort subsidy, while Wang et al. (2017) consider two forms of subsidies, i.e., innovation subsidy and insurance subsidy; Li et al. (2018) address the effectiveness of green loan and government subsidy on promoting clean production; Chen et al. (2019) consider two forms of RJV formation (retailer and manufacturer initiated) and three types of subsidy (per-unit production subsidy, innovation effort subsidy and both types of subsidy).

To summarize, our research contributes to the literature by exploring firms' innovation strategies under government subsidies in a supply chain. We study two innovation modes, i.e., technology transfer, and joint innovation. Technology transfer, in particular, is relatively rare in the existing literature. In addition, we explore the multi-factor combination of technology innovation, subsidies and cooperation contracts, and the existing literature generally considers the combination of innovation, subsidies and insurance or loan or innovation initiation order.

3 Model Description

Consider a supply chain including an upstream manufacturer (UM) and a downstream manufacturer (DM) that jointly develops an innovative product. The UM produces a key component and sells it to the DM with a wholesale price w, and the DM processes the component into the final product which is sold to the market with a retail price p. For simplicity, we assume that each final product requires only one component. In order to encourage innovation, the government provides subsidies to the manufacturers participating in technology innovation in the form of per-unit production subsidy or innovation subsidy. Usually, the manufacturers have the incentive to conduct technology innovation in order to obtain subsidies or expand the market. However, independent innovation may incur investment risk due to huge innovation cost. Therefore, the UM often conducts cooperative innovation with the DM to share the risk brought by innovation cost. Accordingly, the DM also has the incentive to cooperate with the UM for technology innovation in order to expand the market or obtain governmental subsidies. In our study, we focus on two modes of cooperation between the UM and DM: technology transfer or joint innovation. The notations involved in this research are given in Table 1.

Table 1: Summary of notations

w	Wholesale price of the component set by the UM
p	Retail price of final product
D	Demand for the product
a	The basic scale of demand
c	Per-unit production cost
e	Innovation effort
β	Increase in per-unit production cost due to innovation effort
k	Proportion of revenue shared by the UM under a technology transfer contract
r	Fraction of innovation effort cost and the governmental subsidy shared by the UM
Φ	Subsidy provided by the government
t	Per-unit tax credit for each produced unit
α	Proportion of governmental subsidy for the cost of innovation
π_u	The UM's profit
π_d	The DM's profit
π_g	Social welfare

3.1 Customer demand

In general, customer demand is negatively related to the price of product, and positively related to innovation effort. In particular, we assume that the demand is set to follow a linear, downward-sloping function, i.e., $D = a - p + e$, where a is the basic scale of demand, and e the innovation effort. In our study, innovation effort refers to firms' effective innovation activities that modify a product attribute and render the older product obsolete, which in turn is thought of as important criteria for customer purchasing decisions. The innovation effort can be evaluated by the level of innovation of a product, and the consumers can perceive the effect of innovation effort from the upgrading of products. For example, compared with the A14 chip used in iPhone 12, the A15 chip is used in iPhone 13[3], and the users experience Apple's innovation effort in chip development through the upgrading of mobile phones. Accordingly, the innovation effort increases demand, for example, customers may have higher utility consuming the new innovation product, so their willingness-to-pay increases, leading to greater demand. Generally, the innovation effort has not an upper bound, which means that the more innovation effort manufacturers make, the more popular their products are in the market. Similar to Chen et al. (2019) and Li et al. (2020), in this demand curve we assume that, like p, e has a leading coefficient of 1 (i.e., effort sensitivity of demand equals 1). In fact, the results for arbitrary sensitivity coefficient can be obtained through a scaling of the equilibrium effort and corresponding adjustments of other related coefficients. Furthermore, to explore the impacts of a flexible sensitivity on the results, we extend the demand function to a more realistic form with a variable price elasticity and an excitability of innovative product in Section 6, and present how the sensitivity coefficients of p and e affect the equilibrium strategies.

3.2 Technology innovation

We assume that the total cost of technology innovation comprises two major components: the fixed cost during design and manufacturing system development, and the per-unit variable manufacturing cost. Actually, innovation products, especially for technology-intensive products, the cost usually includes not only fixed cost (such as equipment, software, movies, etc.) but also per-unit variable cost (for example, the labor adjustment cost that responds to the new equipment or software) (Chen et al., 2019). For the fixed cost of innovation, to capture the decreasing marginal effect of effort, it can be assumed to be convexly increased with the effort. Similar to Cabon-dhersin (2008), Stepanova and Tesoriere (2011), Shibata (2014), and Ge et al. (2014), we assume that the effort cost of innovation is of the quadratic form he^2, in which $h > 0$ is the coefficient of the effort cost. Without loss of generality, we further assume that the coefficient h is normalized to one, which is a common assumption in the literature (see, for example, Cabon-dhersin, 2008; Stepanova and Tesoriere, 2011; Ge et al., 2014; Shibata, 2014; Zhou et al., 2020). Furthermore, we assume that the innovation effort may also affect the per-unit production cost, $c = c_0 + \beta e$ with the base cost, c_0, normalized to zero for no effort exerted, where $0 < \beta < 1$ denotes the case in which innovation effort leads to an increase in per-unit production cost (Chen et al., 2019). Such treatment is common in actual production practice, for example, environmentally friendly production processes increase the cost of coffee production by about 30%.[4]

Regarding technology innovation in a supply chain, two main modes exist between firms, i.e., technology transfer mode (Mode T), and joint innovation mode (Mode J). For Mode T, Technology transfer is a relatively easy mode of cooperation, which refers to sharing a technology such as documents, reports and patents between two firms according to a transfer contract (Davenport, 2013; Silva et al., 2018). In Mode T, the UM transfers the technology to the DM by sharing a proportion $1-k$ of the DM's profit. For example, Coca-Cola, KFC and their partner firms are such a cooperation mode of technology transfer. Specifically, these well-known multinational companies transfer their prescriptions or trademarks to franchise stores around the world to expand their scales and thus benefit more profits. With the deepening of cooperation among firms in the supply chain, the technology transfer mode is unable to meet the needs of developing at a high level. Accordingly, joint innovation among firms can promote the industrialization of innovation. In Mode J, the UM conducts cooperative innovation with the DM by sharing a fraction r of fixed innovation cost to the DM under a cost-sharing contract (Ghosh and Shah, 2015; Chen et al., 2019). For example, as the world's largest solar-boat manufacturer, AltEn forms a joint venture with an Indian firm to make solar boats for the Indian market in which the firm procures its critical power components from AltEn (Srinivasan, 2016).

3.3 Governmental subsidy

In order to encourage technology innovation, the government might provide subsidies to the two manufacturers. In our study, the government provides two forms of subsidies, i.e., a per-unit production subsidy, and an innovation subsidy, collectively referred to as Φ. Regarding the first form of subsidy, the government returns tax to the firms by first collecting and then refunding, or collecting and refunding, which is a kind of governmental subsidy granted in the form of tax incentives. We assume that the government may provide a tax credit for each unit produced, i.e., $\Phi = tD$, where $t \geq 0$ is the per-unit tax credit. For simplicity, we assume that all units produced are sold. The second form of subsidy, is a sum of money offered by the government to innovative manufacturers to cover part of the total fixed investment in technology innovation. This form of subsidy can be denoted by $\Phi = \alpha e^2$, where $0 < \alpha < 1$ represents the proportion of governmental subsidy with respect to the fixed cost of innovation. At this time, the effective fixed cost of innovation reduces to $(1-\alpha)e^2$. Note that this innovation cost $(1-\alpha)e^2$ is an increasing and convex function which reflects how innovation effort has come about through manufacturers making the initial changes in products and processes easily, with subsequent improvements being more difficult with diminishing returns. This treatment is consistent with practice and is commonly used in some literature (for example, Chen et al., 2019; Li et al., 2020). In Mode T, the UM enjoys the whole governmental subsidy. In Mode J, the governmental subsidy is shared between the two manufacturers with r fraction going to the UM and the remaining amount, $1-r$ to the DM.

3.4 Profits of the members

In our study, there are two modes of technology innovation, and two forms of governmental subsidies. Accordingly, the profits of the members in the supply chain can be described as the following two modes.

(1) Mode T. The UM independently engages in technology innovation, thus obtains governmental subsidy. Afterwards, the UM transfers technology to the DM to share a certain percentage of revenue under a transfer contract.

The profits of the UM and the DM are as follows

$$\pi_u(e, w) = wD - \beta eD - e^2 + k(p - w)D + \Phi \tag{1}$$

and

$$\pi_d(p) = (1 - k)(p - w)D. \tag{2}$$

(2) Mode J. The UM initiates technology innovation. The UM motivates the DM to exert effort by sharing the fixed innovation cost and the governmental subsidy. In this mode, the two manufacturers jointly cooperate to maximize their respective profits.

The profits of the UM and the DM are as follows

$$\pi_u(e, w) = wD - \beta eD - re^2 + r\Phi \tag{3}$$

and

$$\pi_d(p) = (p - w)D - (1 - r)e^2 + (1 - r)\Phi. \tag{4}$$

By incorporating positive externality from technology innovation, subsidy, the profits of the UM and DM, and consumer surplus, as commonly defined in the literature (Raz and Ovchinnikov, 2015; Chen et al., 2019; Jung and Feng, 2020). The government's objective function (i.e., social welfare) can be expressed as follows

$$\pi_g = \tau e - \Phi + \pi_u + \pi_d + \frac{D^2}{2}. \tag{5}$$

The first term corresponds to the external benefits of technology innovation effort, τ is a constant marginal benefit in effort level, for simplicity, we consider it as 1 (Raz and Ovchinnikov, 2015; Meng et al., 2021). The second term is the total subsidy provided to the UM and DM. The remaining terms are the social aspect, which includes the profit of the UM and DM and the consumer surplus. Consumer surplus is generally defined in the economics literature as the area under the demand curve above the given price (Raz and Ovchinnikov, 2015), which can be calculated by the integral formula (Liu et al., 2019, Chai and Xiao, 2019): $\int_{p_{\min}}^{p_{\max}} D dp = \int_{a-D+e}^{a+e} (a - p + e) dp = \frac{D^2}{2}$, consistent with previous literature such as Atasu et al. (2009), Raz and Ovchinnikov (2015) and Chen et al. (2019).

In the two modes, after the government determines the form of subsidies, the UM and DM play a two-stage game. The UM is the leader and the DM is the follower. In the first stage, the UM firstly determines the effort level e and the wholesale price w. In the second stage, the DM decides the retail price p. Next, a backward induction is used to solve the game, and the equilibrium solutions are discussed further.

4 The optimal decisions

In this section, we analyze the game between the UM and the DM under the two innovation modes in the presence of two forms of governmental subsidies. For ease of exposition, we define indexes i and j, where $i \in \{P, I\}$ represents the government provides a per-unit production subsidy ($i = P$) or an innovation subsidy ($i = I$), and $j \in \{T, J\}$ denotes the UM chooses technology transfer mode ($j = T$), or joint innovation mode ($j = J$). For example, index PJ represents the case where the government provides a per-unit production subsidy to encourage the manufacturers to innovate, and the UM chooses joint innovation with the DM.

All four cases are a two-stage game, and backward induction can be used to solve them. Herein, we take Case PT as an example to carry out formal analysis. The similar analysis and the equilibrium solutions of the other three cases (IT, PJ, IJ) are also available.

We solve the equilibrium of Case PT by a backward induction method in detail.

Since π_d^{PT} is concave in p, solving the first-order optimality condition for p, $\frac{\partial \pi_d^{PT}}{\partial p} = 0$ yields the optimal retail price

$$p^*(e, w) = \frac{a + e + w}{2}. \tag{6}$$

By substituting $p^*(e, w)$ into π_u, then π_u^{PT} is jointly concave in e and w. Solving the first-order optimality condition for e and w, the optimal innovation effort and the wholesale price can be obtained by maximizing π_u

$$e^* = \frac{(a + t)(1 - \beta)}{8 - (1 - \beta)^2 - 4k}, \qquad w^* = a - \frac{(a + t)(3 + \beta)}{8 - (1 - \beta)^2 - 4k}. \tag{7}$$

Thus, the retail price p can be expressed as

$$p^* = a - \frac{(a + t)(\beta + 1)}{8 - (1 - \beta)^2 - 4k}. \tag{8}$$

Substituting p, e and w into π_u and π_d, we can obtain the profits of the UM and DM:

$$\pi_u^{PT} = \frac{(a + t)^2}{8 - (1 - \beta)^2 - 4k}, \tag{9}$$

$$\pi_d^{PT} = \frac{4(a + t)^2(1 - k)}{(8 - (1 - \beta)^2 - 4k)^2}. \tag{10}$$

Therefore, the social welfare can be written as

$$\pi_g^{PT} = \frac{(a + t)(a + 1 - \beta - t)}{A - 4k} - \frac{2(a + t)^2(2k - 3)}{(A - 4k)^2}. \tag{11}$$

The equilibrium of Cases IT, PJ and IJ by a backward induction method is presented in the Appendix. The optimal decisions and corresponding profits of all four cases are summarized as follows.

Lemma 1. *The optimal solutions to the retail price, the innovation effort, the wholesale price, the demand and the corresponding profits of the UM, the DM and the social welfare are summarized in Table 2.*

Table 2: The optimal decisions, corresponding profits and social welfare

	Case PT	Case IT	Case PJ	Case IJ
p^*	$a-\frac{(a+t)(\beta+1)}{A-4k}$	$a-\frac{a(\beta-2\alpha+1)}{A-4(2\alpha+k-\alpha k)}$	$a-\frac{(a+t)(\beta+2r-1)}{A+8(r-1)}$	$\frac{a(B+6r(1-\alpha))}{A+8(r-\alpha r-1)}$
e^*	$\frac{(a+t)(1-\beta)}{A-4k}$	$\frac{a(1-\beta)}{A-4(2\alpha+k-\alpha k)}$	$\frac{(a+t)(1-\beta)}{A+8(r-1)}$	$\frac{a(1-\beta)}{A+8(r-\alpha r-1)}$
w^*	$a-\frac{(a+t)(3+\beta)}{A-4k}$	$\frac{a(4+B)-4a(\alpha+k-\alpha k)}{A-4(2\alpha+k-\alpha k)}$	$\frac{(a+t)(4r+B)}{A+8(r-1)}-rt$	$\frac{a(B+4r(1-\alpha))}{A+8(r-\alpha r-1)}$
D	$\frac{2(a+t)}{A-4k}$	$\frac{2a(1-\alpha)}{A-4(2\alpha+k-\alpha k)}$	$\frac{2r(a+t)}{A+8(r-1)}$	$\frac{2a(\beta+r-\alpha r)}{A+8(r-\alpha r-1)}$
π_u	$\frac{(a+t)^2}{A-4k}$	$\frac{a^2(1-\alpha)}{A-4(2\alpha+k-\alpha k)}$	$\frac{r(a+t)^2}{A+8(r-1)}$	$\frac{a^2r(1-\alpha)}{A+8(r-\alpha r-1)}$
π_d	$\frac{4(a+t)^2(1-k)}{(A-4k)^2}$	$\frac{4a^2(1-\alpha)^2(1-k)}{(A-4(2\alpha+k-\alpha k))^2}$	$\frac{(a+t)^2(4r^2-C)}{(A+8(r-1))^2}$	$\frac{a^2(1-\alpha)(4r^2(1-\alpha)-C)}{(A+8(r-\alpha r-1))^2}$
π_g	$\frac{(a+t)(a+1-\beta-t)}{A-4k}-\frac{2(a+t)^2(2k-3)}{(A-4k)^2}$	$\frac{a(a+1-\beta)}{A-4(2\alpha+k-\alpha k)}-\frac{2a^2(1-\alpha)(7\alpha+2k-4\alpha k-3)}{(A-4(2\alpha+k-\alpha k))^2}$	$\frac{(a+t)(a-\beta+t-2rt+1)}{A+8(r-1)}+\frac{2r(a+t)^2(7r-4)}{(A+8(r-1))^2}$	$\frac{7a^2}{32}+\frac{a(7a-16\beta-6a\beta+7a\beta^2+16)}{16(A+8(r-\alpha r-1))}+\frac{a^2(7\beta^4-12\beta^3+42\beta^2+52\beta-25)}{32(A+8(r-\alpha r-1))^2}$

* $A=8-(1-\beta)^2$, $B=\beta(1-\beta)$, $C=(1-r)(1-\beta)^2$

Lemma 1 shows that there exists a unique optimal solution to the retail price, the innovation effort, the wholesale price and the demand under four different cases.

To facilitate the hereafter discussion, we define $\frac{r}{1-r}$ as the UM's distribution power, which represents the power of the UM to allocate the governmental subsidy under Mode J. When the power is large, it means that the UM shares more of the governmental subsidy. Alternatively, when the power is small, it means that the UM shares less governmental subsidy. In particular, from the Eqs. (3) and (4), the parameter r denotes not only the fraction of the governmental subsidy shared by the UM, but also his share of fixed innovation cost under Mode J. Accordingly, the UM's distribution power represents not only the power of the UM to allocate the governmental subsidy, but also the power of the UM to allocate the innovation cost.

For the optimal solutions and corresponding profits for all four cases, the following corollary can be obtained.

Corollary 1. *The impact of per-unit tax credit t, the proportion of revenue k, the proportion of governmental subsidy α and the fraction of innovation cost r on the optimal solutions to the retail price p, the innovation effort e, the wholesale price w, the demand D and the UM's profit π_u and the DM's profit π_d are summarized in Table 3.*

Corollary 1 reveals that in Case PT, an increase in k reduces the retail price and the wholesale price, and increases the effort, the demand and the UM's profit, but leads to the DM's profit that firstly increases and then decreases. Intuitively, a larger k means more profit for the UM, thus prompting him to lower wholesale price, which leads the DM to lower retail price. The effort and demand also increase at this time because of an increase in marginal revenue. The reason for the DM's profit to increase firstly and then decrease is that, when k starts to increase, the DM's profit increase caused by the increase of marginal revenue and demand prevails over the DM's profit decrease caused by the profit sharing to the UM; as k continues to increase, the decrease in the DM's profit outweighs the increase. Moreover, an increase in t reduces the retail price and wholesale price, and increases the effort, the demand, the UM's profit and the DM's profit. This finding is clear. A larger t means more governmental subsidies, prompting the UM and DM to lower wholesale price

Table 3: The analytical sensitivity analysis towards p, e, w, D, π_u and π_d

Mode T	$k\uparrow$	$r\uparrow$	$t\uparrow$	$\alpha\uparrow$	Mode J	$k\uparrow$	$r\uparrow$	$t\uparrow$	$\alpha\uparrow$
p^{PT}	$\downarrow$	-	$\downarrow$	-	p^{PJ}	-	$\downarrow$	$\downarrow$	-
e^{PT}	$\uparrow$	-	$\uparrow$	-	e^{PJ}	-	$\downarrow$	$\uparrow$	-
w^{PT}	$\downarrow$	-	$\downarrow$	-	w^{PJ}	-	$\downarrow$	$\uparrow$	-
D^{PT}	$\uparrow$	-	$\uparrow$	-	D^{PJ}	-	$\downarrow$	$\uparrow$	-
π_u^{PT}	$\uparrow$	-	$\uparrow$	-	π_u^{PJ}	-	$\downarrow$	$\uparrow$	-
π_d^{PT}	$\uparrow\downarrow$	-	$\uparrow$	-	π_d^{PJ}	-	$\uparrow\downarrow$	$\uparrow$	-
p^{IT}	$\downarrow$	-	-	$\uparrow$	p^{IJ}	-	$\downarrow$	-	$\uparrow$
e^{IT}	$\uparrow$	-	-	$\uparrow$	e^{IJ}	-	$\downarrow$	-	$\uparrow$
w^{IT}	$\downarrow$	-	-	$\uparrow$	w^{IJ}	-	$\downarrow$	-	$\uparrow$
D^{IT}	$\uparrow$	-	-	$\uparrow$	D^{IJ}	-	$\downarrow$	-	$\uparrow$
π_u^{IT}	$\uparrow$	-	-	$\uparrow$	π_u^{IJ}	-	$\downarrow$	-	$\uparrow$
π_d^{IT}	$\downarrow$	-	-	$\uparrow\downarrow$	π_d^{IJ}	-	$\uparrow$	-	$\downarrow$

$\uparrow$ means the desicion (such as e) increases with the parameter (such as k); $\downarrow$ means the desicion decreases with the parameter; '-' denotes 'no effect'; $\uparrow\downarrow$ means the desicion initially increases and then decreases with the parameter; $\downarrow\uparrow$ means the desicion initially decreases and then increases with the parameter.

and retail price, respectively. A larger t has a positive effect on innovation effort and demand, thus more profits of the UM and DM.

In Case IT, an increase in k reduces the retail price, the wholesale price and the DM's profit, and increases the effort, the demand and the UM's profit. In line with Case PT, a larger k prompts the UM and DM to lower wholesale price and retail price, respectively, but leads to greater effort and demand. Actually, the increase of k means that the UM shares more revenue, so the DM shares less revenue. Therefore, the UM's profit increases with k, while the DM's profit decreases with k. Moreover, an increase in α increases the retail price, the effort, the wholesale price, the demand and the UM's profit, but leads to the DM's profit that firstly increases and then decreases. In fact, a larger α means more governmental subsidies and less innovation cost for the UM, leading to greater effort and demand, which results in greater profit of the UM. The reason for the DM's profit to firstly increase and then decrease is that when α starts to increase, the DM's profit increase caused by the increase of demand prevails over the DM's profit decrease caused by the decrease of marginal revenue (although both wholesale price and retail price increase, marginal revenue may be decrease); as α continues to increase, the decrease in the DM's profit outweighs the increase.

In Case PJ, an increase in r reduces the retail price, the effort, the wholesale price, the demand and the UM's profit, but leads to the DM's profit that firstly increases and then decreases. Actually, this observation can be explained. The higher the fraction r of innovation cost may damage the enthusiasm of the UM, resulting in less effort and smaller demand. Meanwhile, the higher the fraction r of governmental subsidies may prompt the UM to lower wholesale price, which leads to lower retail price. Accordingly, the UM's profit will decrease. Interestingly, a larger r leads to firstly increase and then decrease in the DM's profit. The reason for this fact is that when r starts to increase, in the short term, the governmental subsidies shared by the DM is larger and she gets more profits; however, as r continues to increase, the governmental subsidies received by the DM greatly decreases, and her profit naturally falls. Moreover, an increase in t reduces the retail price, and increases the effort, the wholesale price, the demand and the UM's profit and the DM's profit. In fact, a larger t can bring more governmental subsidies to the DM, so she sets a lower retail price. However, the UM raises wholesale price because the DM shares government subsidies. Both the UM and DM invest in innovation and obtain governmental subsidies in Case PJ, and they conduct more effort to produce more product in order to obtain more governmental subsidies. Accordingly, both the UM and DM are beneficial.

In Case IJ, an increase in r reduces the retail price, the effort, the wholesale price, the demand and the UM's profit, and increases the DM's profit. The retail price, the effort, the wholesale price, the demand and the UM's profit all decrease with r in line with Case PJ. Accordingly, the reason is also similar to Case PJ. The reason why the DM's profit increases is that the DM will bear less innovation cost with the increase of r. Moreover, an increase in α increases the retail price, the effort, the wholesale price, the demand and the UM's profit, and reduces the DM's profit. The retail price, the effort, the wholesale price, the demand and the UM's profit all increase with α in line with Case IT. Accordingly, the reason is also similar to Case IT. The reason why the DM's profit decreases is that, although a larger α can bring more governmental subsidies to the DM, the superimposed effect of the decrease of marginal profit (the increase of wholesale price exceeds the increase of the retail price) and the increase of innovation cost finally leads to the decrease of the DM's profit.

5 Comparison and implications

In this section, we compare the innovation efforts and the profits of the UM, the DM and the social welfare under the two innovation modes in the presence of two forms of subsidies, and observe a few surprising results and summarize them in the following Theorems.

5.1 Innovation effort comparison

In this subsection, we examine the effect of the two cooperative innovation modes on a given type of subsidy, i.e., comparisons between iT and iJ, $i \in \{P, I\}$. Theorem 1 studies the impacts of the parameters k and r on the innovation effort e.

Theorem 1. *For any given form of governmental subsidy, $e^{iT} > e^{iJ}$ holds if $k > 2 - 2r$, and $e^{iT} < e^{iJ}$ holds if $k < 2 - 2r$.*

Theorem 1 shows that, if the proportion of revenue by the UM is sufficiently high, the innovation effort under Mode T is greater than that in Mode J, otherwise Mode T is less than Mode J. This finding is intuitive. A higher k means that the fraction of revenue shared by the UM is greater than that shared by the DM.

Recall that in Mode T, the UM independently invests in more innovation effort. Therefore, the UM will have the willingness to invest in more innovation effort under a transfer contract in order to enhance his revenue. On the other hand, a higher r indicates that the fraction of innovation effort cost shared by the UM is larger than that shared by the DM. Accordingly, the UM will have the willingness to invest in more innovation effort under a cost-sharing contract.

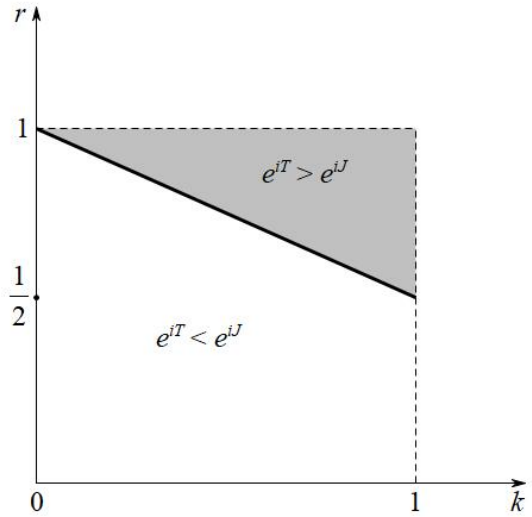


Figure 1: Comparison of e^{iT} and e^{iJ}

We present the effort comparison in Figure 1. It can be found that the innovation effort $e^{iT} > e^{iJ}$ holds in the shaded region, and $e^{iT} < e^{iJ}$ holds in the other blank region. Observantly, the shaded area is a quarter of the entire feasible region, and the boundary between the two regions decreases in the proportion of revenue shared by the UM (k). Moreover, for Mode T and Mode J, innovation effort is likely to be larger when the UM and DM are under a cost-sharing contract, specifically, the effort under a cost-sharing contract is three times the effort under a transfer contract. This finding is interesting. With respect to the innovation effort, joint innovation is more popular than technology transfer. The reason for this fact is that joint innovation is a deeper form of cooperation than technology transfer, and the UM and DM are willing to put more effort into deep cooperation.

The managerial insight revealed by Theorem 1 is that, regardless of which form of subsidy the government provides, the UM and DM always negotiate a larger fraction of revenue such that they would conduct more of innovation effort than if a smaller fraction of revenue was identified. Moreover, the UM and DM are more likely to make more efforts under joint innovation mode than under technology transfer mode.

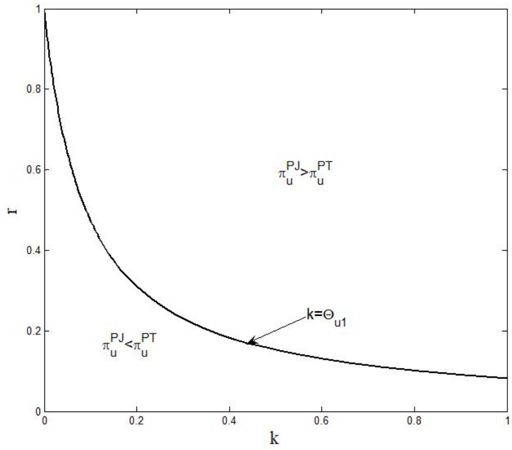
5.2 Profit comparison

In this subsection, we compare the profits of the UM, the DM and the social welfare under different cooperative innovation modes and different forms of governmental subsidies.

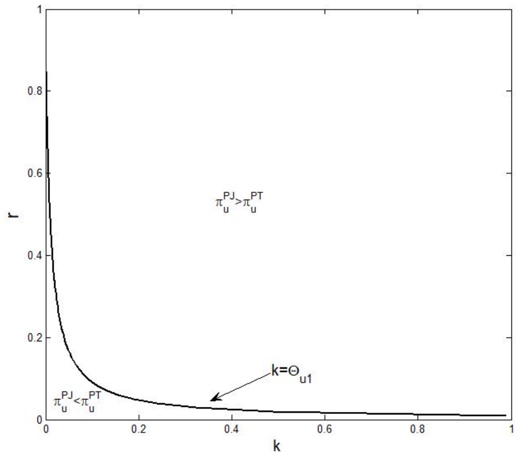
5.2.1 Effects of cooperative innovation modes

In the presence of a given form of subsidy, how the UM, the DM and the government choose cooperative innovation strategies is the question studied. Theorem 2 indicates the strategic choices of the UM, the DM and the government under a per-unit production subsidy, and Theorem 3 indicates the strategic choices of the UM, the DM and the government under an innovation subsidy.

Theorem 2. *In the presence of a per-unit production subsidy, (a) For the profit of the UM, $\pi_u^{PT} > \pi_u^{PJ}$ holds if the UM's distribution power $\frac{r}{1-r} > \frac{(1-\beta)^2}{4k}$, $\pi_u^{PT} < \pi_u^{PJ}$ holds if the UM's distribution power $\frac{r}{1-r} < \frac{(1-\beta)^2}{4k}$; (b) For the profit of the DM, when $0 < k < \frac{(1-\beta)-(1-\beta)^2}{4}$ satisfies, $\pi_d^{PT} < \pi_d^{PJ}$ if $\Theta_{d1} < r < \Theta_{d2}$, and $\pi_d^{PT} > \pi_d^{PJ}$ if $r < \Theta_{d1}$ or $r > \Theta_{d2}$; when $\frac{(1-\beta)-(1-\beta)^2}{4} < k < 1$ satisfies, $\pi_d^{PT} > \pi_d^{PJ}$ if $\Theta_{d1} < r < \Theta_{d2}$, and $\pi_d^{PT} < \pi_d^{PJ}$ if $r < \Theta_{d1}$ or $r > \Theta_{d2}$; (c) For the social welfare, $\pi_g^{PT} > \pi_g^{PJ}$ always holds.*



(a) $\beta = 0.4$



(b) $\beta = 0.8$

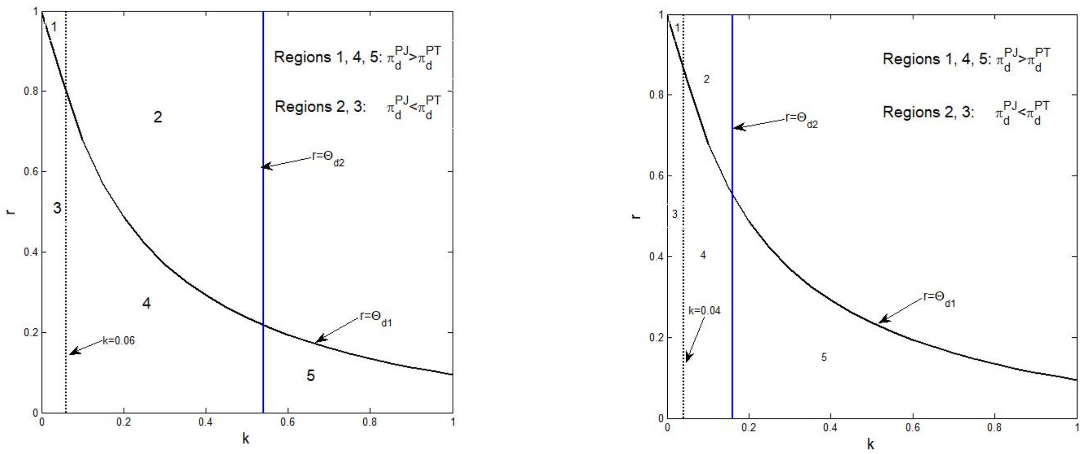
Figure 2: Comparison of π_u^{PT} and π_u^{PJ} when $\beta = 0.4$ (a) and $\beta = 0.8$ (b)

Theorem 2(a) indicates that, once the government provides a per-unit production subsidy, the UM under Mode T is better off than in Mode J when the UM's distribution power $\frac{r}{1-r}$ is greater than a threshold $\frac{(1-\beta)^2}{4k}$, and Mode T is worse off than in Mode J when the UM's distribution power $\frac{r}{1-r}$ is less than the threshold. In fact, from the perspective of the UM, the profit of the UM under a transfer contract will be larger than under a cost-sharing contract when he has a larger distribution power. On the other hand, the profit of the UM under a cost-sharing contract will be larger than under a transfer contract when he has a smaller distribution power. This finding is intuitive. The distribution power is the power of the UM to allocate the innovation cost under the joint innovation mode. When the power is large, the UM bears more innovation cost, and thus the UM is inclined to the technology transfer mode. When the power is small, the UM bears less innovation cost, and thus the UM is inclined to the joint innovation mode.

We capture the comparison of π_u^{PT} and π_u^{PJ} in Figure 2. As in Figure 2, $\pi_u^{PJ} < \pi_u^{PT}$ holds above the line $k = \Theta_{u1}$, and $\pi_u^{PJ} > \pi_u^{PT}$ holds below the line. On the line, a larger k requires a smaller r, i.e., r increases with decreasing k. Furthermore, when β is larger, the region of $\pi_u^{PJ} < \pi_u^{PT}$ is larger, and the region of

$\pi_u^{PJ} > \pi_u^{PT}$ is smaller. Therefore, the UM benefits more with a larger β under a cost-sharing contract than under a transfer contract. Note that a larger increase in β encourages the UM to conduct a deeper innovation cooperation.

Theorem 2(b) gives a comparison of the DM's profits in the two modes in the presence of a per-unit production subsidy. When the proportion of revenue k is relatively smaller, the DM under Mode J is better off than in Mode T as the fraction of innovation cost r is between the thresholds Θ_{d1} and Θ_{d2}, and Mode J is worse off than in Mode T as r is smaller than the thresholds Θ_{d1} or greater than the threshold Θ_{d2}. On the other side, when k is relatively larger, the DM under Mode T is better off than in Mode J as r is between the thresholds Θ_{d1} and Θ_{d2}, and Mode T is worse off than in Mode J as r is smaller than the thresholds Θ_{d1} or greater than the threshold Θ_{d2}. These findings are interesting. When the proportion of revenue shared by the UM is sufficiently low, it is more profitable for the DM to choose a cost-sharing contract as the fraction of innovation cost shared by the UM is moderate (rather than small enough); otherwise, the DM is willing to choose a transfer contract as the fraction is small enough or large enough. The possible reason for this happening is that if the UM bears a moderate fraction of innovation cost in Mode J, he will not receive too much subsidies, which will benefit DM instead at this time; on the contrary, if the UM bears a sufficiently small fraction of innovation cost in Mode J, it will receive too little subsidies, which will not benefit DM; if the UM bears a sufficiently large fraction of innovation cost in Mode J, he will also bear more cost at this time. As a result, the DM will still choose a cost-sharing contract. The interpretation of the second part of Theorem 2(b) is similar to that of the first part. In fact, in the presence of a per-unit production subsidy, both the proportion of revenue in Mode T and the fraction of innovation effort cost in Mode J play significant roles in the choice of cooperation modes of the DM. Therefore, a one-sided situation does not appear.



(a) $\beta = 0.4$

(b) $\beta = 0.8$

Figure 3: Comparison of π_d^{PT} and π_d^{PJ} when $\beta = 0.4$ (a) and $\beta = 0.8$ (b)

We present a comparison of π_d^{PT} and π_d^{PJ} in Figure 3. As in Figure 3, $\pi_d^{PT} < \pi_d^{PJ}$ holds in the regions 1, 4 and 5, while $\pi_d^{PT} > \pi_d^{PJ}$ holds in the regions 2 and 3. Observantly, a change in β causes a change in

k, which leads to a change in the area of these regions. This finding is intuitive. We recall Corollary 1, in which π_d^{PT} firstly increases and then decreases with k, and π_d^{PJ} firstly increases and then decreases with r. As a result, the changes in k and r have an alternating effect on the changes in π_d^{PT} and π_d^{PJ}. This leads to the relationship between the sizes of the profits of UM and DM does not remain unchanged in the presence of a per-unit subsidy.

A numerical experiment is used to verify Theorem 2(c). In fact, we find that the social welfare in different cases are linked to several parameters from their expressions in Table 2. We focus on the influence of the per-unit tax credit on the social welfare, and the values of other parameters set as: $a = 100, \beta = 0.5, k = 0.5, r = 0.5$.

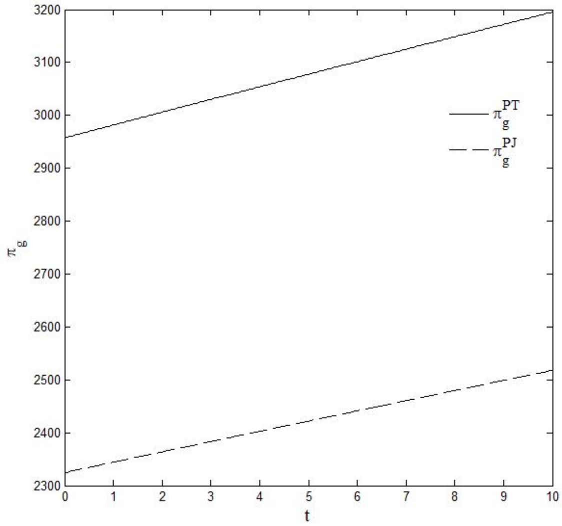


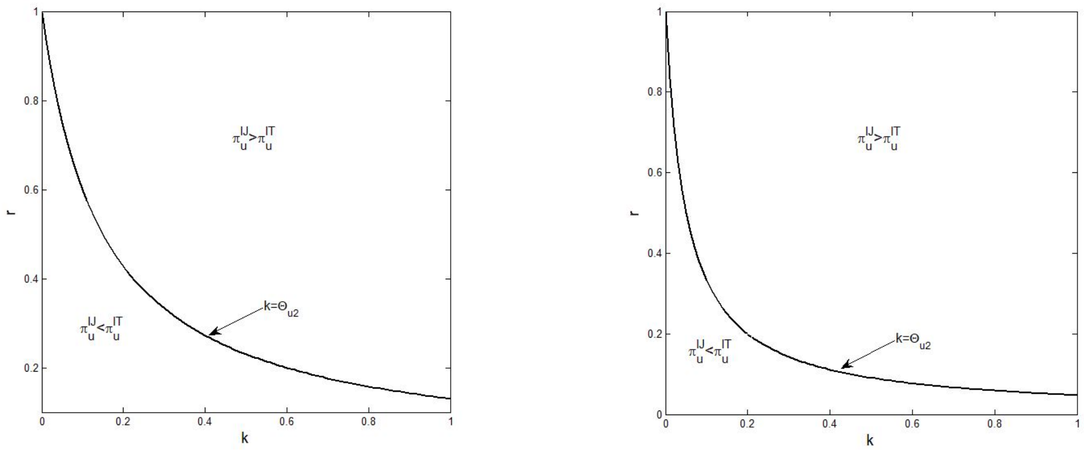
Figure 4: Comparison of social welfares under a per-unit production subsidy

Figure 4 compares the social welfares in the presence of a per-unit production subsidy. From Figure 4, we find that in both cooperation modes, the social welfares increase with the per-unit tax credit. Moreover, the social welfares are always more under technology transfer mode than under joint innovation mode. Recalling Corollary 1, the innovation effort, the demand and the profits of the UM and DM increase with the per-unit tax credit, and thus the social welfares increase with the per-unit tax credit in Mode T and Mode J. The increase of the per-unit tax credit can encourage the UM and DM to produce more products to obtain more government subsidies. Actually, joint innovation mode can promote the quantity of products to be more than that of technology transfer mode, which means that the government needs to provide more subsidies under joint innovation mode. As a consequence, the social welfare would be more under technology transfer mode.

From the proof of Theorem 2, an interesting fact can be found that the parameter t does not appear in the expressions of the thresholds $\frac{(1-\beta)^2}{4k}$, Θ_{d1} and Θ_{d2}. It indicates that the UM and DM's innovation strategies are independent of the per-unit tax credit in the presence of a per-unit production subsidy. From Corollary 1, the innovation efforts (e^{PT} and e^{PJ}), the demands (D^{PT} and D^{PJ}), the UM's profits (π_u^{PT} and π_u^{PJ}) and the DM's profits (π_d^{PT} and π_d^{PJ}) all increase with the per-unit tax credit t, indicating that the key decision variables and the profits of manufacturers depend on t. However, when comparing the manufacturers' profits under the two modes, the factor $(a+t)^2$ involving t cancels out that leads to the manufacturers' innovation mode choice decisions are independent of t. The possible reason for this phenomenon is that an exogenous t causes per-unit production subsidy tD to be regarded as part of aD (can be considered as the revenue from basic market). The manufacturers' innovation mode choice decisions should be intuitively related to t, but our finding is irrelevant to t. Accordingly, this fact means we have an interesting result.

Theorem 3. *In the presence of an innovation subsidy, (a) For the profit of the UM, $\pi_u^{PT} > \pi_u^{PJ}$ holds if the UM's distribution power $\frac{r}{1-r} > \frac{(1-\beta)^2}{4k(1-\alpha)}$, $\pi_u^{PT} < \pi_u^{PJ}$ holds if the UM's distribution power $\frac{r}{1-r} < \frac{(1-\beta)^2}{4k(1-\alpha)}$; (b) For the profit of the DM, $\pi_d^{IT} > \pi_d^{IJ}$ if $\Theta_{d3} < r < \Theta_{d4}$, and $\pi_d^{IT} < \pi_d^{IJ}$ if $r < \Theta_{d3}$ or $r > \Theta_{d4}$; (c) For the social welfare, $\pi_g^{IT} < \pi_g^{IJ}$ always holds.*

From Theorem 3(a), we can find that the relationship between the profits of the UM in the two modes in the presence of an innovation subsidy is almost the same as that in the case of a per-unit production subsidy except for the different thresholds of the UM's distribution power.

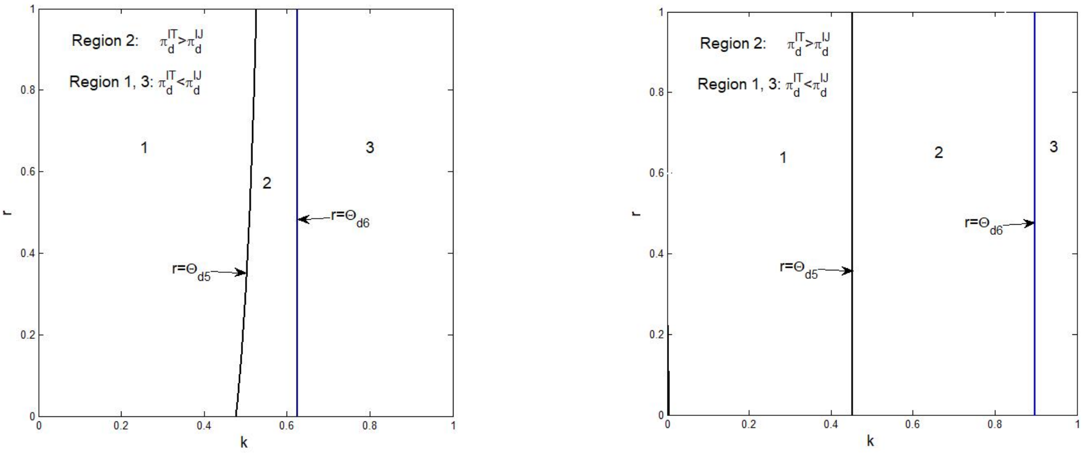


(a) $\alpha = 0.4$, $\beta = 0.4$ (b) $\alpha = 0.8$, $\beta = 0.8$

Figure 5: Comparison of π_u^{IT} and π_u^{IJ} when $\alpha = 0.4$, $\beta = 0.4$ (a) and $\alpha = 0.8$, $\beta = 0.8$ (b)

We capture a comparison of π_u^{IT} and π_u^{IJ} in Figure 5. As in Figure 5, $\pi_u^{IJ} > \pi_u^{IT}$ holds above the line $k = \Theta_{u2}$, $\pi_u^{IJ} < \pi_u^{IT}$ holds below the line. On the line, a larger k requires a smaller r, i.e., r increases with decreasing k. Furthermore, when both β and α are larger, the region of $\pi_u^{IJ} < \pi_u^{IT}$ is smaller, and the region of $\pi_u^{IJ} > \pi_u^{IT}$ is larger. Therefore, the UM benefits more with a larger β and a larger α under a cost-sharing contract than under a transfer contract. Note that a larger increase in β and α encourage the UM to conduct a deeper innovation cooperation.

Theorem 3(b) gives a comparison of the profits of the DM in the two modes in the presence of an innovation subsidy. When the fraction of innovation cost shared by the UM is moderate, it is more profitable for the DM to choose a transfer contract; otherwise, it is better for the DM to choose a cost-sharing contract. When the fraction of innovation cost shared by the UM is moderate, it is more profitable for the DM to choose a transfer contract; otherwise, it is better for the DM to choose a cost-sharing contract. The reason is that if the UM bears less innovation cost, he will receive less subsidies, which harms the interests of the UM. Accordingly, it is not beneficial to cooperation in the presence of an innovation subsidy since the subsidy is positively related to the innovation cost. Therefore, if the fraction of innovation cost is at a moderate level, it is more attractive for the DM to choose Mode T.



(a) $\alpha = 0.4$, $\beta = 0.4$ (b) $\alpha = 0.8$, $\beta = 0.8$

Figure 6: Comparison of π_d^{IT} and π_d^{IJ} when $\alpha = 0.4$, $\beta = 0.4$ (a) and $\alpha = 0.8$, $\beta = 0.8$ (b)

We present a comparison of π_d^{IT} and π_d^{IJ} in Figure 6. As in Figure 6, when both β and α are larger, the region 2 is larger. This means that the DM benefits more with a larger β and a larger α under a transfer contract than under a cost-sharing contract. Note that a larger increase in β and α encourages the DM to conduct a shallower innovative cooperation.

A numerical experiment is also used to verify Theorem 3(c). Similar to Theorem 2(c), we focus on the influence of the proportion of the government subsidy for innovation on the social welfares, and the values of other parameters set the same as the proof of Theorem 2(c).

Figure 7 compares the social welfares in the presence of an innovation subsidy. From Figure 7, we find that in both cooperation modes, the social welfares increase with α. Furthermore, the social welfares are always more under joint innovation mode than under technology transfer mode. Recalling Corollary 1, the innovation effort, the demand and the profit of the UM increase with the proportion of the government subsidy for innovation, and the DM's profit does not always increase with the proportion of the government subsidy for innovation. Since the increase in the proportion of the government subsidy for innovation causes positive effect to overwhelm negative effect, and the social welfares increase with the proportion of the government subsidy for innovation in Mode T and Mode J. Accordingly, the increase of the proportion of the government subsidy for innovation can encourage the UM and DM to conduct more effort to obtain more government subsidies. Actually, joint innovation mode can promote the effort to be more than that of technology transfer mode, which leads to more external benefits of technology innovation effort for the

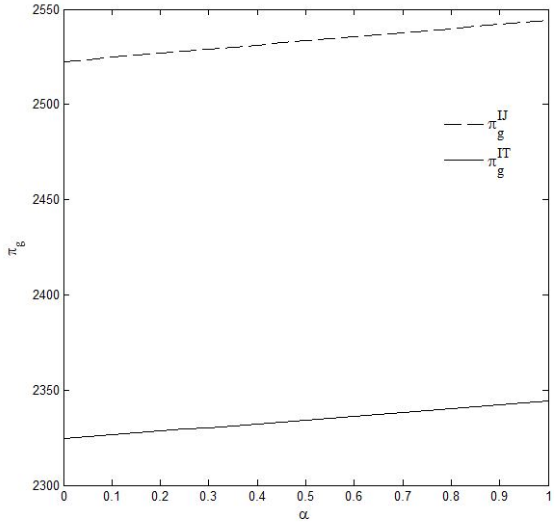


Figure 7: Comparison of social welfares under an innovation subsidy

government under joint innovation mode. Therefore, the social welfare would be more under joint innovation mode.

The managerial insight revealed by Theorems 2 and 3 is that, if the government provides a per-unit production subsidy, for the UM, if he has a large distribution power in joint innovation mode, he tends to choose technology transfer mode; otherwise, he tends to choose joint innovation mode. For the DM, if she has a high (low) proportion of revenue-sharing under technology transfer mode and a moderate (sufficiently large or sufficiently small) fraction of innovation cost-sharing under joint innovation mode, she tends to choose joint innovation mode; otherwise, she tends to choose technology transfer mode. For the government, he always hopes that UM and DM carry out cooperation in technology transfer mode. In addition, once the government provides a per-unit production subsidy, the UM and DM in the supply chain do not care about the per-unit tax credit when making innovation decisions. If the government provides an innovation subsidy, for the UM, his strategies are the same as a per-unit production subsidy, that is, if he has a large distribution power in joint innovation mode, he tends to choose technology transfer mode; otherwise, he tends to choose joint innovation mode. For the DM, if she has a sufficiently large or sufficiently small fraction of innovation cost-sharing under joint innovation mode, she tends to choose joint innovation mode; if she has a moderate fraction of innovation cost-sharing under joint innovation mode, she tends to choose technology transfer mode. For the government, he always hopes that UM and DM carry out technology under joint innovation mode. In addition, both the UM and DM are worse off in the presence of a per-unit production subsidy or an innovation subsidy, so technology transfer mode or joint innovation mode is never Pareto optimal to all parties in the supply chain.

5.2.2 Effects of forms of governmental subsidy

Under a given cooperative mode, which form of governmental subsidies is more effective for the UM, the DM and the government is studied. Theorem 4 describes the willingness of the UM, the DM and the government to subsidize under a transfer contract, and Theorem 5 indicates the willingness of the UM, the DM and the government to subsidize under a cost-sharing contract.

Theorem 4. *Under the Mode T, (a) For the profit of the UM, $\pi_u^{PT} > \pi_u^{IT}$ holds if $t > \Gamma_{u1}$, and $\pi_u^{PT} < \pi_u^{IT}$ holds if $0 < t < \Gamma_{u1}$; (b) For the profit of the DM, $\pi_d^{PT} > \pi_d^{IT}$ holds if $t > \Gamma_{d1}$, and $\pi_d^{PT} < \pi_d^{IT}$ holds if $0 < t < \Gamma_{d1}$; (c) For the social welfare, $\pi_g^{PT} > \pi_g^{IT}$ always holds.*

As shown in Theorem 4, (a) and (b), under a transfer contract, for both the UM and DM, two undifferentiated points of per-unit tax credit exist, i.e., Γ_{u1} and Γ_{d1}. On the left side of the undifferentiated points, the effect of an innovation subsidy is better, and on the right side of the undifferentiated points, the effect of a per-unit production subsidy is better. Both of the UM and DM are willing to receive a per-unit production subsidy as the per-unit tax credit is sufficiently large, otherwise they are willing to receive an innovation subsidy. This finding is intuitive. The larger per-unit tax credit provides more credit to the the UM and DM, and thus they prefer a per-unit production subsidy from the government. In addition, the threshold Γ_{d1} is larger than the threshold Γ_{u1}. This finding indicates that, compared with the threshold of the UM, only when the per-unit tax credit is greater than a larger threshold, the DM is willing to choose a per-unit production subsidy. This is because a per-unit production subsidy can bring more governmental subsidies at this time, the DM can still obtain more profit under a per-unit production subsidy than an innovation subsidy after the UM gets some profit.

A numerical experiment is used to verify Theorem 4(c). We focus on the influence of the proportion of shared revenue k on the social welfares, and the values of other parameters set as: $a = 100, t = 5, \beta = 0.5, \alpha = 0.5$.

Figure 8 compares the social welfares under the Mode T. From Figure 8, we find that in the presence of the two forms of subsidies, the social welfares increase with the proportion of revenue k. Moreover, the social welfare are always more in the presence of a per-unit production subsidy than an innovation subsidy. Recalling Corollary 1, both the innovation effort and the demand increase with the proportion of revenue in the presence of two forms of subsidies. As a result, the social welfares increase with the proportion of revenue. In addition, the UM's profit firstly increases and then decreases in the presence of a per-unit production subsidy, while his profit keeps decreasing in the presence of an innovation subsidy. Accordingly, the social welfare in the presence of a per-unit production subsidy exceeds that of an innovation subsidy.

Theorem 5. *Under the Mode J, (a) For the profit of the UM, $\pi_u^{PJ} > \pi_u^{IJ}$ holds if $t > \Gamma_{u2}$, and $\pi_u^{PJ} < \pi_u^{IJ}$ holds if $0 < t < \Gamma_{u2}$; (b) For the profit of the DM, $\pi_d^{PJ} > \pi_d^{IJ}$ holds if $t > \Gamma_{d2}$, and $\pi_d^{PJ} < \pi_d^{IJ}$ holds if $0 < t < \Gamma_{d2}$; (c) For the social welfare, $\pi_g^{PJ} < \pi_g^{IJ}$ always holds.*

As shown in Theorem 5, (a) and (b), under a cost-sharing contract, for both the UM and DM, two other undifferentiated points of per-unit tax credit exist, i.e., Γ_{u2} and Γ_{d2}. On the left side of the undifferentiated

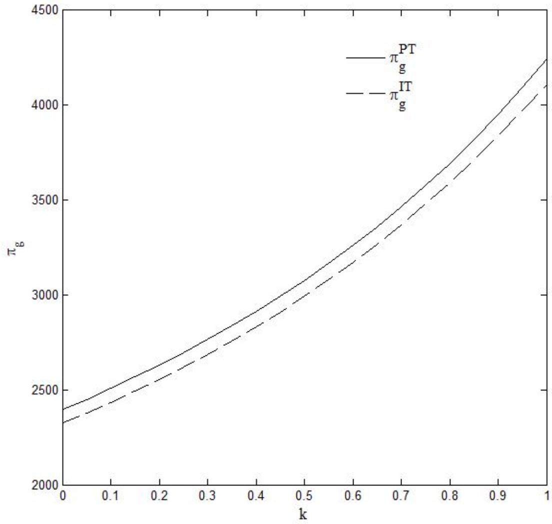


Figure 8: Comparison of social welfares under the Mode T

points, the effect of an innovation subsidy is better, and on the right side of the undifferentiated points, the effect of a per-unit production subsidy is better. Both of the UM and DM are willing to receive a per-unit production subsidy as the per-unit tax credit is sufficiently large, otherwise they are willing to receive an innovation subsidy. This finding is the similar with Theorem 4 except for two undifferentiated points, and is also intuitive.

A numerical experiment is used to verify Theorem 5(c). We focus on the influence of the fraction of innovation effort cost r on the social welfares, and the values of other parameters set as: $a = 100, t = 5, \beta = 0.5, \alpha = 0.5$.

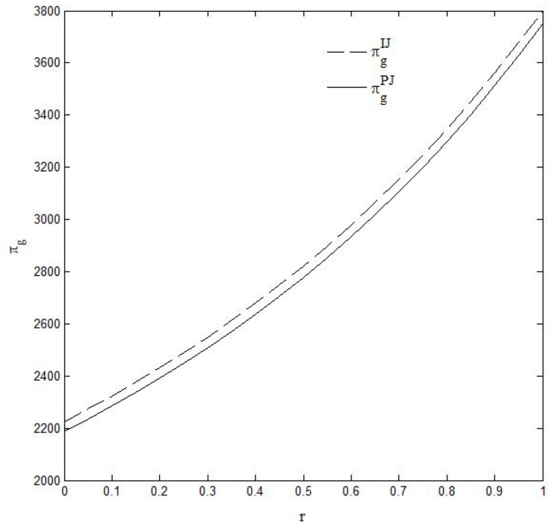


Figure 9: Comparison of social welfares under the Mode J

Figure 9 compares the social welfares under the Mode J. From Figure 9, we find that in the presence of the two forms of subsidies, the social welfares increase with the fraction of innovation effort cost r. Moreover, the social welfare are always more in the presence of an innovation subsidy than a per-unit production subsidy. In fact, the increase of the fraction of innovation cost means the deepening of the cooperation between the UM and DM, which directly leads to the increase of the social welfares. In addition, the supply chain produces more products under the Mode J. If the government subsidizes products based on the quantity of products (a per-unit production subsidy), it is not as cost-effective as subsidizing innovation cost (an innovation subsidy). Accordingly, the social welfare in the presence of an innovation subsidy exceeds that of a per-unit production subsidy.

The managerial insight revealed by Theorems 4 and 5 is that, for the UM and DM, they make decisions based on the forms and coefficients of government subsidies, that is, if the per-unit tax credit is high they expect a per-unit production subsidy, and if the fraction for innovation cost is high they expect an innovation subsidy; for the government, he always prefers a per-unit production subsidy under technology transfer mode, but prefers an innovation subsidy under joint innovation mode. This conclusion is a supplement to the research on government subsidy policy when supply chain adopts joint innovation mode in Chen et al. (2019). Note that the subsidy decisions made by the government under a given innovation mode are consistent with the governmental expectation of innovation modes for supply chain in the presence of given form of subsidy in Theorems 2 and 3.

6 Extensions

In this section, we will relax some assumptions and evaluate the robustness of our results.

6.1 A variable price elasticity

In Subsection 3.1, we assume that the coefficient of the price is 1 in the demand function. In reality, the demand may have some flexibility with respect to the product price. Therefore, we change the price-sensitive parameter from one to a general value. That is, the demand function becomes

$$D = a - bp + e, \tag{12}$$

where $b \in (0, 1)$ denotes the degree of price-sensitive, which is one in the base model.

A numerical example for the four cases, i.e., Case PT, Case IT, Case PJ and Case IJ, is conducted based on the following parameters: $a = 100, \beta = 0.5$. The results of sensitivity analysis with respect to t, α, k, r and b are shown in Table 4.

From the numerical results in Table 4, we can conclude that when the parameter to be investigated changes, and the other parameters are moderate, the results are consistent with the results of Corollary 1 or Table 3. For example, in order to analyze the sensitivity of the parameter α in Mode T, we respectively fix the values of the parameters t, k, r and b at 1, 0.5, 0.5 and 0.5, and change the value of the parameter α from 0.4 to 0.7. Accordingly, we can obtain the results in the lines 6–9 of Table 4. Moreover, the retail price, the innovation effort, the wholesale price and the profits of the UM and DM depend on the price-sensitive

Table 4: Sensitive analysis on the decisions and the profits with respect to t, α, k, r and b

Mode T		p^{PT}	e^{PT}	w^{PT}	π_u^{PT}	π_d^{PT}	p^{IT}	e^{IT}	w^{IT}	π_u^{IT}	π_d^{IT}
t	0.5	179.44	30.85	97.18	4123.1	1691.5	-	-	-	-	-
	1	179.38	30.92	96.92	4143.7	1700.0	-	-	-	-	-
	1.5	179.33	31.00	96.67	4164.3	1708.4	-	-	-	-	-
	2	179.28	31.08	96.41	4185.0	1716.9	-	-	-	-	-
α	0.4	-	-	-	-	-	224.24	60.61	127.27	4848.5	2350.8
	0.5	-	-	-	-	-	253.33	80.0	146.67	5333.3	2844.4
	0.6	-	-	-	-	-	309.80	117.65	184.31	6274.5	3936.9
	0.7	-	-	-	-	-	466.67	222.22	288.89	8888.9	2901.2
k	0.4	180.95	28.58	104.74	3829.5	1742.3	248.19	72.29	151.81	4819.3	2787.1
	0.5	179.38	30.92	96.92	4143.7	1799.0	243.33	80.00	146.67	5970.1	2744.4
	0.6	177.54	33.69	87.71	4514.1	1614.0	239.70	89.55	140.30	6779.7	2651.4
	0.7	175.34	36.99	76.69	4957.2	1459.8	237.80	101.69	132.20	8333.3	2557.8
r	0.4	-	-	-	-	-	-	-	-	-	-
	0.5	-	-	-	-	-	-	-	-	-	-
	0.6	-	-	-	-	-	-	-	-	-	-
	0.7	-	-	-	-	-	-	-	-	-	-
b	0.4	250.00	45.64	135.91	5727.4	2603.3	428.57	142.86	250.00	8928.6	6377.6
	0.5	179.38	30.92	96.92	4143.7	1700.0	253.33	80.00	146.67	5333.3	2844.4
	0.6	139.71	22.64	75.02	3254.1	1255.6	179.39	53.44	103.05	3816.8	1748.1
	0.7	114.30	17.33	60.98	2684.4	949.9	138.60	38.75	78.99	2980.6	1243.8
Mode J		p^{PJ}	e^{PJ}	w^{PJ}	π_u^{PJ}	π_d^{PJ}	p^{IJ}	e^{IJ}	w^{IJ}	π_u^{IJ}	π_d^{IJ}
t	0.5	234.87	52.30	165.38	3495.7	1063.9	-	-	-	-	-
	1	234.76	52.43	165.54	3513.1	1069.2	-	-	-	-	-
	1.5	234.04	52.57	165.71	3530.6	1074.5	-	-	-	-	-
	2	233.13	52.70	165.87	3548.2	1079.9	-	-	-	-	-
α	0.4	-	-	-	-	-	341.18	117.65	247.06	4705.9	2760.8
	0.5	-	-	-	-	-	428.57	171.43	314.29	5714.3	1816.3
	0.6	-	-	-	-	-	563.16	315.79	494.74	8421.1	1761.8
	0.7	-	-	-	-	-	641.06	412.67	607.20	9234.4	1536.9
k	0.4	-	-	-	-	-	-	-	-	-	-
	0.5	-	-	-	-	-	-	-	-	-	-
	0.6	-	-	-	-	-	-	-	-	-	-
	0.7	-	-	-	-	-	-	-	-	-	-
r	0.4	267.81	72.65	190.91	3894.1	764.2	663.16	315.79	494.74	6421.1	734.0
	0.5	234.76	52.43	165.54	3513.1	1069.2	428.57	171.43	314.29	5714.3	816.3
	0.6	216.41	41.02	151.18	3298.0	1480.8	341.18	117.65	247.06	4705.9	1660.9
	0.7	204.49	33.69	141.91	3159.9	1336.7	295.52	89.55	211.94	4179.1	2290.0
b	0.4	354.38	83.67	250.50	5250.1	1875.0	1187.5	500.00	875.00	7625.0	1438.0
	0.5	234.76	52.43	165.54	3513.1	1069.2	428.57	171.43	314.29	5714.3	816.3
	0.6	175.45	36.87	123.27	2649.3	984.8	260.56	98.59	190.14	3521.1	745.5
	0.7	139.83	27.53	97.98	2132.6	876.8	186.70	66.50	135.55	2557.5	526.1

parameter. Specifically, the retail price, the innovation effort, the wholesale price and the profits of the UM and DM in all four cases decrease with increasing b. This finding is intuitive. Since a larger sensitivity coefficient of the price causes the market demand to decrease with retail price, which consequently leads to lower retail price, innovation effort and wholesale price, thus lower profits of the UM and DM.

In summary, this extension shows that even a variable price elasticity does not qualitatively change the theoretical results obtained with the coefficient of the price 1 in the demand function.

6.2 An excitability of innovative product

Recall that the coefficient of innovative effort is 1 in the demand function, as characterized in Subsection 3.1. In practice, the demand may have some flexibility with respect to the innovative product. Thus, we also change the innovation-excitability parameter from one to a general value. That is, the demand function becomes

$$D = a - p + \gamma e, \tag{13}$$

where $\gamma \in (0, 1)$ denotes the excitability degree of innovative product, which is one in the base model.

A numerical example for the four cases, i.e., Case PT, Case IT, Case PJ and Case IJ, is conducted based on the following parameters: $a = 100, \beta = 0.5$. The results of a sensitivity analysis with respect to t, α, k, r and γ are shown in Table 5.

From the numerical results in Table 5, we can conclude that when the parameter to be investigated change, and the other parameters are moderate, the results are consistent with the results of Corollary 1 or Table 3. For example, in order to analyze the sensitivity of the parameter k, we respectively fix the values of the parameters t, α, r and γ at 1, 0.5, 0.5 and 0.5, and change the value of the parameter k from 0.4 to 0.7. Accordingly, we can obtain the results in the lines 10–13 of Table 5. Moreover, the retail price, the innovation effort, the wholesale price and the profits of the UM and DM depend on the innovation-excitability parameter. Specifically, the retail price, the innovation effort, the wholesale price and the profits of the UM and DM in all four cases increase with increasing γ. This finding is intuitive. Since a larger coefficient of innovation effort causes the market demand to increase with innovation effort, which consequently leads to higher retail price, innovation effort and wholesale price, thus higher profits of the UM and DM.

Therefore, taking both the variable price elasticity and excitability of innovative product into consideration, we find that our theoretical results are robust.

7 Conclusions

In this paper, we studied a supply chain including an UM and a DM that jointly developed an innovative product. We considered two cooperative innovation modes and two forms of governmental subsidies and formed four cases. Furthermore, we focused on firms' and governmental behaviours based on the profits they can obtain. We revealed the following results. First, the UM's profit increases with the proportion of revenue, but the DM's profit might not decrease with it; the UM's profit decreases with the fraction of innovation

Table 5: Sensitive analysis on the decisions and the profits with respect to t, α, k, r and γ

Mode T		p^{PT}	e^{PT}	w^{PT}	π_u^{PT}	π_d^{PT}	p^{IT}	e^{IT}	w^{IT}	π_u^{IT}	π_d^{IT}
t	0.5	67.96	3.37	34.24	1683.4	561.1	-	-	-	-	-
	1	67.80	3.39	33.91	1700.2	566.7	-	-	-	-	-
	1.5	67.64	3.41	33.58	1717.0	572.3	-	-	-	-	-
	2	67.48	3.42	33.25	1734	578	-	-	-	-	-
α	0.4	-	-	-	-	-	69.10	5.62	35.39	1671.3	558.7
	0.5	-	-	-	-	-	69.59	6.76	35.81	1672.2	559.3
	0.6	-	-	-	-	-	70.34	8.47	36.44	1673.6	560.2
	0.7	-	-	-	-	-	71.59	11.36	37.50	1676.0	551.8
k	0.4	69.83	3.18	38.07	1593.9	597.7	71.52	6.33	39.87	1562.5	585.9
	0.5	67.80	3.39	33.91	1700.2	666.7	69.59	6.76	35.81	1666.7	555.6
	0.6	65.49	3.63	29.15	1821.6	520.5	67.39	7.25	31.16	1785.7	510.2
	0.7	62.81	3.91	23.66	1961.7	452.7	64.84	7.81	25.78	1923.1	443.8
r	0.4	-	-	-	-	-	-	-	-	-	-
	0.5	-	-	-	-	-	-	-	-	-	-
	0.6	-	-	-	-	-	-	-	-	-	-
	0.7	-	-	-	-	-	-	-	-	-	-
γ	0.4	66.95	1.69	33.23	1693.0	563.6	67.89	3.34	34.45	1662.2	551.3
	0.5	67.80	3.39	33.91	1700.2	566.7	69.59	6.76	35.81	1666.7	555.6
	0.6	68.90	5.13	34.72	1703.0	568.6	71.82	10.31	37.46	1672.2	559.3
	0.7	70.25	6.92	35.66	1711.6	574.4	74.65	14.08	39.44	1689.2	570.7
Mode J		p^{PJ}	e^{PJ}	w^{PJ}	π_u^{PJ}	π_d^{PJ}	p^{IJ}	e^{IJ}	w^{IJ}	π_u^{IJ}	π_d^{IJ}
t	0.5	77.16	5.08	52.03	1262.5	631.1	-	-	-	-	-
	1	77.05	5.10	52.04	1275.1	637.6	-	-	-	-	-
	1.5	76.93	5.13	52.05	1287.8	643.9	-	-	-	-	-
	2	76.82	5.15	52.06	1300.5	650.3	-	-	-	-	-
α	0.4	-	-	-	-	-	78.81	8.47	53.39	1255.2	625.99
	0.5	-	-	-	-	-	79.59	10.20	54.08	1256.3	624.98
	0.6	-	-	-	-	-	80.77	12.83	55.13	1257.9	624.97
	0.7	-	-	-	-	-	82.76	17.24	56.90	1260.5	624.96
k	0.4	-	-	-	-	-	-	-	-	-	-
	0.5	-	-	-	-	-	-	-	-	-	-
	0.6	-	-	-	-	-	-	-	-	-	-
	0.7	-	-	-	-	-	-	-	-	-	-
r	0.4	77.63	6.39	52.65	1279.1	635.6	80.77	12.82	55.13	1257.9	621.0
	0.5	77.05	5.10	52.04	1278.3	637.6	79.09	10.20	54.08	1256.3	625.0
	0.6	76.66	4.24	51.06	1277.8	638.4	78.81	8.47	53.39	1255.2	626.7
	0.7	76.38	3.63	51.25	1277.4	637.9	78.26	7.25	52.90	1254.5	627.6
γ	0.4	75.70	2.53	50.89	1268.3	637.6	76.88	5.03	51.76	1246.3	625.0
	0.5	77.05	5.10	52.04	1275.1	639.1	79.59	10.20	54.08	1250	628.8
	0.6	78.82	7.75	53.49	1278.3	642.8	83.25	15.71	7.07	1256.3	634.9
	0.7	81.06	10.52	55.26	1288.0	645.5	88.04	21.74	60.87	1275.5	644.7

cost, but the DM's profit might not increase with it. Second, under either form of governmental subsidy, for the UM, technology transfer mode is better off than joint innovation mode when the UM's distribution power is greater than a threshold, and technology transfer mode is worse off than joint innovation mode when the distribution power is less than the threshold; under a given form of governmental subsidy, the DM's response strategy is influenced by the interaction of different values of the proportion of revenue and the fraction of innovation cost; in the presence of a per-unit production subsidy, the social welfare is always more under technology transfer mode than under joint innovation mode, while in the presence of an innovation subsidy, the social welfare is always more under joint innovation mode than under technology transfer mode. Third, under a given cooperative innovation mode, for both the UM and DM, if the tax credit for each produced unit is high, they expect a per-unit production subsidy, and if the proportion of governmental subsidy is high, they expect an innovation subsidy; for the government, he always prefers a per-unit production subsidy under technology transfer mode, but prefers an innovation subsidy under joint innovation mode. When we extended the model to involve a variable price elasticity and an excitability of innovative product, we find that the profits of the two firms depend on the variable price elasticity and excitability of innovative product, and the theoretical results hold. We also find that our theoretical results are robust.

Some management implications can be derived based on the above results. One insight that government policy makers may take away from our results is that when upstream and downstream firms in a supply chain choose technology transfer mode, government incentive policies provide a per-unit production subsidy; when governments use innovation subsidy, joint innovation mode is preferable for upstream and downstream firms in a supply chain. Another group that may benefit from our work is firm's leaders looking to respond to government incentive programs by undertaking research in cooperative innovation modes. No matter what government policies are, the decision-making of innovation mode of innovation-led upstream firms depends on their distribution power, while that of downstream firms depends on her proportions of revenue-sharing and cost-undertaking. Specifically, when governments provide a per-unit production subsidy, if innovation-led upstream firms have a large distribution power in joint innovation mode, they tend to technology transfer mode; otherwise, they tend to joint innovation mode; if downstream firms have a high (low) proportion of revenue-sharing under technology transfer mode and a moderate (sufficiently large or sufficiently small) fraction of innovation cost-sharing under joint innovation mode, they tend to joint innovation mode; otherwise, they tend to technology transfer mode. When governments provide an innovation subsidy, innovation-led upstream firms' strategies are the same as a per-unit production subsidy; if downstream firms have a sufficiently large or sufficiently small fraction of innovation cost-sharing under joint innovation mode, they tend to joint innovation mode; if downstream firms have a moderate fraction of innovation cost-sharing under joint innovation mode, they tend to choose technology transfer mode. Finally, many firms have lobbyists to help present their views to government. These lobbyists may request that governments provide subsidies in order to help firms satisfy various government policies. One insight that lobbyists may glean from our analysis is that cooperative innovation mode of supply chain partners is expected to be technology transfer when governments use per-unit production subsidy; and their cooperative innovation mode is expected to be joint innovation when a per-unit production subsidy is present. Moreover, supply chain partners expect a per-unit production subsidy when governments provide a higher unit tax credit, and they expect an innovation subsidy if governments provide a higher proportion for innovation effort cost.

Finally, we propose some ideas for future work. This paper considers a deterministic demand model, but the demand for some products may be uncertain. In addition, future research should examine the innovation behaviour of firms when designing other cooperative innovation contracts.

Notes

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Appendix

Solving the equilibrium of Case IT.

Since π_d^{IT} is concave in p, solving the first-order optimality condition for p, $\frac{\partial \pi_d^{IT}}{\partial p} = 0$ yields the optimal retail price

$$p^*(e, w) = \frac{a + e + w}{2}. \tag{14}$$

By substituting $p^*(e,w)$ into π_u, then π_u^{IT} is jointly concave in e and w. Solving the first-order optimality condition for e and w, the optimal innovation effort and wholesale price can be obtained by maximizing π_u

$$e^* = \frac{a(1-\beta)}{8-(1-\beta)^2-4(2\alpha+k-\alpha k)}, \quad w^* = \frac{a[4+\beta(1-\beta)]-4a(\alpha+k-\alpha k)}{8-(1-\beta)^2-4(2\alpha+k-\alpha k)}. \tag{15}$$

Thus, the retail price p can be expressed as

$$p^* = a - \frac{a(\beta-2\alpha+1)}{8-(1-\beta)^2-4(2\alpha+k-\alpha k)}. \tag{16}$$

Substituting p, e and w into π_u and π_d, we can obtain the profits of the UM and DM:

$$\pi_u^{IT} = \frac{a^2(1-\alpha)}{8-(1-\beta)^2-4(2\alpha+k-\alpha k)}, \tag{17}$$

$$\pi_d^{IT} = \frac{4a^2(1-\alpha)^2(1-k)}{[8-(1-\beta)^2-4(2\alpha+k-\alpha k)]^2}. \tag{18}$$

Therefore, the social welfare can be written as

$$\pi_g^{IT} = \frac{a(a+1-\beta)}{A-4(2\alpha+k-\alpha k)} - \frac{2a^2(1-\alpha)(7\alpha+2k-4\alpha k-3)}{(A-4(2\alpha+k-\alpha k))^2}. \tag{19}$$

Solving the equilibrium of Case PJ.

Since π_d^{PJ} is concave in p, solving the first-order optimality condition for p, $\frac{\partial \pi_d^{PJ}}{\partial p} = 0$ yields the optimal retail price

$$p^*(e,w) = \frac{a+e+w-t(1-r)}{2}. \tag{20}$$

By substituting $p^*(e,w)$ into π_u, then π_u^{PJ} is jointly concave in e and w. Solving the first-order optimality condition for e and w, the optimal innovation effort and wholesale price can be obtained by maximizing π_u

$$e^* = \frac{(a+t)(1-\beta)}{8-(1-\beta)^2+8(r-1)}, \quad w^* = \frac{(a+t)(4r+\beta(1-\beta))}{8-(1-\beta)^2+8(r-1)} - rt. \tag{21}$$

Thus, the retail price p can be expressed as

$$p^* = a - \frac{(a+t)(\beta+2r-1)}{8-(1-\beta)^2+8(r-1)}. \tag{22}$$

Substituting p, e and w into π_u and π_d, we can obtain the profits of the UM and DM:

$$\pi_u^{PJ} = \frac{r(a+t)^2}{8-(1-\beta)^2+8(r-1)}, \tag{23}$$

$$\pi_d^{PJ} = \frac{(a+t)^2(4r^2-(1-r)(1-\beta)^2)}{(8-(1-\beta)^2+8(r-1))^2}. \tag{24}$$

Therefore, the social welfare can be written as

$$\pi_g^{PJ} = \frac{(a+t)(a-\beta+t-2rt+1)}{A+8(r-1)} + \frac{2r(a+t)^2(7r-4)}{(A+8(r-1))^2}. \tag{25}$$

Solving the equilibrium of Case IJ.

Since π_d^{IJ} is concave in p, solving the first-order optimality condition for p, $\frac{\partial \pi_d^{IJ}}{\partial p} = 0$ yields the optimal retail price

$$p^*(e, w) = \frac{a + e + w}{2}. \tag{26}$$

By substituting $p^*(e, w)$ into π_u, then π_u^{IJ} is jointly concave in e and w. Solving the first-order optimality condition for e and w, the optimal innovation effort and wholesale price can be obtained by maximizing π_u

$$e^* = \frac{a(1-\beta)}{8-(1-\beta)^2+8(r-\alpha r-1)}, \quad w^* = \frac{a(\beta(1-\beta)+4r(1-\alpha))}{8-(1-\beta)^2+8(r-\alpha r-1)}. \tag{27}$$

Thus, the retail price p can be expressed as

$$p^* = \frac{a(\beta(1-\beta)+6r(1-\alpha))}{8-(1-\beta)^2+8(r-\alpha r-1)}. \tag{28}$$

Substituting p, e and w into π_u and π_d, we can obtain the profits of the UM and DM:

$$\pi_u^{IJ} = \frac{a^2 r(1-\alpha)}{8-(1-\beta)^2+8(r-\alpha r-1)}, \tag{29}$$

$$\pi_d^{IJ} = \frac{a^2(1-\alpha)(4r^2(1-\alpha)-(1-r)(1-\beta)^2)}{(8-(1-\beta)^2+8(r-\alpha r-1))^2}. \tag{30}$$

Therefore, the social welfare can be written as

$$\pi_g^{IJ} = \frac{7a^2}{32} + \frac{a(7a-16\beta-6a\beta+7a\beta^2+16)}{16(A+8(r-\alpha r-1))} + \frac{a^2(7\beta^4-12\beta^3+42\beta^2+52\beta-25)}{32(A+8(r-\alpha r-1))^2}. \tag{31}$$

Proof of Corollary 1.

In Case PT, the fact that p^{PT} and w^{PT} decrease with t, and e^{PT}, D^{PT}, π_u^{PT} and π_d^{PT} increase with t can be easily derived from their expressions. The fact that p^{PT} and w^{PT} decrease with k, and e^{PT}, D^{PT} and π_u^{PT} increase with k can be also derived from their expressions. We only prove the monotonicity of π_d^{PT} with respect to k.

Solving the derivative of π_d^{PT} for k, we can get

$$\frac{\partial \pi_d^{PT}}{\partial k} = \frac{4(a+t)^2(8-A-4k)}{(A-4k)^3}, \tag{32}$$

where $A = -\beta^2 + 2\beta + 7 = -(\beta-1)^2 + 8 \in (7, 8)$, $\beta \in (0, 1)$, thus, $A - 4k > 0$; As k increases from 0 to 1, the sign of $8 - A - 4k$ changes from positive to negative. Specifically, $8 - A - 4k > 0$ as $0 < k < 2 - \frac{A}{4}$, and $8 - A - 4k < 0$ as $2 - \frac{A}{4} < k < 1$. Then, the sign of $\frac{\partial \pi_d^{PT}}{\partial k}$ also varies from positive to negative. Therefore, π_d^{PT} firstly increases and then decreases as k increases.

In Case IT, the fact that p^{IT} decreases with k, and e^{IT}, D^{IT}, π_u^{IT} and π_d^{IT} increase with k can be easily derived from their expressions. The fact that e^{IT} increases with α can be also derived from their expressions. We only prove the monotonicity of w^{IT} with respect to k, and p^{IT}, w^{IT}, D^{IT}, π_u^{IT} and π_d^{IT} with respect to α.

First, solving the derivative of w^{IT} for k, we can get

$$\frac{\partial w^{IT}}{\partial k} = \frac{-4a(1-\alpha)(3+4\alpha+\beta)}{(A-4(2\alpha+k-\alpha k))^2}, \tag{33}$$

It can be easily found that the sign of $\frac{\partial w^{IT}}{\partial k}$ is always negative. Therefore, w^{IT} decreases with k.

Second, solving the derivative of p^{IT} for α, we can get

$$\frac{\partial p^{IT}}{\partial \alpha} = \frac{2a(1-\beta)(3-2k+\beta)}{(A-4(2\alpha+k-\alpha k))^2}, \tag{34}$$

It can be easily found that the sign of $\frac{\partial p^{IT}}{\partial \alpha}$ is always positive. Therefore, p^{IT} increases with α.

Third, solving the derivative of w^{IT} for α, we can get

$$\frac{\partial w^{IT}}{\partial \alpha} = \frac{4a(1-\beta)(1-k+\beta)}{(A-4(2\alpha+k-\alpha k))^2}, \tag{35}$$

It can be easily found that the sign of $\frac{\partial w^{IT}}{\partial \alpha}$ is always positive. Therefore, w^{IT} increases with α.

Fourth, solving the derivative of D^{IT} for α, we can get

$$\frac{\partial D^{IT}}{\partial \alpha} = \frac{2a(1-\beta)^2}{(A-4(2\alpha+k-\alpha k))^2}, \tag{36}$$

It can be easily found that the sign of $\frac{\partial D^{IT}}{\partial \alpha}$ is always positive. Therefore, D^{IT} increases with α.

Fifth, solving the derivative of π_u^{IT} for α, we can get

$$\frac{\partial \pi_u^{IT}}{\partial \alpha} = \frac{a^2(1-\beta)^2}{(A-4(2\alpha+k-\alpha k))^2}, \tag{37}$$

It can be easily found that the sign of $\frac{\partial \pi_u^{IT}}{\partial \alpha}$ is always positive. Therefore, π_u^{IT} increases with α.

Sixth, solving the derivative of π_d^{IT} for α, we can get

$$\frac{\partial \pi_d^{IT}}{\partial \alpha} = \frac{8a^2(1-k)(1-\alpha)(1-\beta)^2}{[A-4(2\alpha+k-\alpha k)]^3}, \tag{38}$$

where $1-\alpha > 0$, $1-k > 0$; As α increases from 0 to 1, the sign of $A-4(2\alpha+k-\alpha k)$ changes from positive to negative. Specifically, $A-4(2\alpha+k-\alpha k) > 0$ as $0 < \alpha < \frac{A-4k}{8-4k}$, and $A-4(2\alpha+k-\alpha k) < 0$ as $\frac{A-4k}{8-4k} < \alpha < 1$. Thus, the sign of $\frac{\partial \pi_d^{IT}}{\partial \alpha}$ also varies from positive to negative. Therefore, π_d^{IT} firstly increases and then decreases as α increases.

In Case PJ, the fact that p^{PJ}, e^{PJ}, w^{PJ}, D^{PJ}, π_u^{PJ} and π_d^{PJ} increase with t can be easily derived from their expressions. The fact that e^{PJ}, D^{PJ} and π_u^{PJ} increase with r can be also derived from their expressions. We only prove the monotonicity of p^{PJ}, w^{PJ} and π_d^{PJ} with respect to r.

First, solving the derivative of p^{PJ} for r, we can get

$$\frac{\partial p^{PJ}}{\partial r} = \frac{-2(a+t)(1-\beta)(3+\beta)}{(A+8(r-1))^2}, \tag{39}$$

It can be easily found that the sign of $\frac{\partial p^{PJ}}{\partial r}$ is always negative. Therefore, p^{PJ} decreases with r.

Second, solving the derivative of w^{PJ} for r, we can get

$$\frac{\partial w^{PJ}}{\partial r} = \frac{-4(a+t)(1-\beta^2)}{(A+8(r-1))^2}, \tag{40}$$

It can be easily found that the sign of $\frac{\partial w^{PJ}}{\partial r}$ is always negative. Therefore, w^{PJ} decreases with r.

Third, solving the derivative of π_d^{PJ} for r, we can get

$$\frac{\partial \pi_d^{PJ}}{\partial r} = \frac{-(a+t)^2(1-\beta)^2(16r+(1-\beta)^2)}{[A+8(r-1)]^3}, \tag{41}$$

where $(a+t)^2 > 0$, $(1-\beta)^2 > 0$, $(16r+(1-\beta)^2) > 0$; As r increases from 0 to 1, the sign of $A+8(r-1)$ changes from negative to positive. Specifically, $A+8(r-1) < 0$ as $0 < r < 1-\frac{A}{8}$, and $A+8(r-1) > 0$ as $1-\frac{A}{8} < r < 1$. Then, the sign of $\frac{\partial \pi_d^{PJ}}{\partial k}$ also varies from positive to negative. Therefore, π_d^{PJ} firstly increases and then decreases as r increases.

In Case IJ, the fact that e^{IJ} increases with α, and π_d^{IJ} decreases with α can be easily derived from their expressions. The fact that e^{IJ} increases with r, and π_d^{IJ} increases with r can be also derived from their expressions. We only prove the monotonicity of p^{IJ}, w^{IJ}, D^{IJ} and π_u^{IJ} with respect to α and r.

First, solving the derivative of p^{IJ} for α, we can get

$$\frac{\partial p^{IJ}}{\partial \alpha} = \frac{2ar(1-\beta)(3+\beta)}{(A+8(r-\alpha r-1))^2}, \tag{42}$$

It can be easily found that the sign of $\frac{\partial p^{IT}}{\partial \alpha}$ is always positive. Therefore, p^{IJ} increases with α.

Second, solving the derivative of w^{IJ} for α, we can get

$$\frac{\partial w^{IJ}}{\partial \alpha} = \frac{4ar(1-\beta^2)}{(A+8(r-\alpha r-1))^2}, \tag{43}$$

It can be easily found that the sign of $\frac{\partial w^{IT}}{\partial \alpha}$ is always positive. Therefore, w^{IJ} increases with α.

Third, solving the derivative of D^{IJ} for α, we can get

$$\frac{\partial D^{IJ}}{\partial \alpha} = \frac{2ar[(1-\beta)^2+8\beta]}{(A+8(r-\alpha r-1))^2}, \tag{44}$$

It can be easily found that the sign of $\frac{\partial D^{IT}}{\partial \alpha}$ is always positive. Therefore, D^{IJ} increases with α.

Fourth, solving the derivative of π_u^{IJ} for α, we can get

$$\frac{\partial \pi_u^{IJ}}{\partial \alpha} = \frac{a^2r(1-\beta)^2}{(A+8(r-\alpha r-1))^2}, \tag{45}$$

It can be easily found that the sign of $\frac{\partial \pi_u^{IJ}}{\partial \alpha}$ is always positive. Therefore, π_u^{IJ} increases with α.

Fifth, solving the derivative of p^{IJ} for r, we can get

$$\frac{\partial p^{IJ}}{\partial r} = \frac{-2a(1-\alpha)(1-\beta)(3+\beta)}{(A+8(r-\alpha r-1))^2}, \tag{46}$$

It can be easily found that the sign of $\frac{\partial p^{IT}}{\partial r}$ is always negative. Therefore, p^{IJ} decreases with r.

Sixth, solving the derivative of w^{IJ} for r, we can get

$$\frac{\partial w^{IJ}}{\partial r} = \frac{-4a(1-\alpha)(1-\beta^2)}{(A+8(r-\alpha r-1))^2}, \tag{47}$$

It can be easily found that the sign of $\frac{\partial w^{IT}}{\partial r}$ is always negative. Therefore, w^{IJ} decreases with r.

Seventh, solving the derivative of D^{IJ} for r, we can get

$$\frac{\partial D^{IJ}}{\partial r}=\frac{-2a(1-\alpha)[(1-\beta)^2+8\beta]}{(A+8(r-\alpha r-1))^2}, \tag{48}$$

It can be easily found that the sign of $\frac{\partial D^{IJ}}{\partial r}$ is always negative. Therefore, D^{IJ} decreases with r.

Eighth, solving the derivative of π_u^{IJ} for r, we can get

$$\frac{\partial \pi_u^{IJ}}{\partial r}=\frac{-a^2(1-\alpha)(1-\beta)^2}{(A+8(r-\alpha r-1))^2}, \tag{49}$$

It can be easily found that the sign of $\frac{\partial \pi_u^{IJ}}{\partial r}$ is always negative. Therefore, π_u^{IJ} decreases with r.

This completes the proof of Corollary 1.

Proof of Theorem 1.

First, we compare e^{PT} and e^{PJ}. Subtracting them, we can get

$$e^{PT}-e^{PJ}=\frac{(a+t)(1-\beta)}{A-4k}-\frac{(a+t)(1-\beta)}{A-8(1-r)}. \tag{50}$$

In order to make $e^{PT}>e^{PJ}$, it requires $4k>8(1-r)$, i.e., $k>2(1-r)$; otherwise, it requires $k<2(1-r)$.

Second, we compare e^{IT} and e^{IJ}. Subtracting them, we can get

$$e^{IT}-e^{IJ}=\frac{a(1-\beta)}{A-8\alpha-4k(1-\alpha)}-\frac{a(1-\beta)}{A-8(1+r\alpha-r)}. \tag{51}$$

In order to make $e^{IT}>e^{IJ}$, it requires $8\alpha+4k(1-\alpha)>8(1+r\alpha-r)$, i.e., $k>2(1-r)$; otherwise, it requires $k<2(1-r)$.

This completes the proof of Theorem 1.

Proof of Theorem 2.

a) We compare π_u^{PT} and π_u^{PJ}. If we let $\pi_u^{PT}>\pi_u^{PJ}$, then

$$\frac{(a+t)^2}{A-4k}>\frac{r(a+t)^2}{A-8(1-r)} \tag{52}$$

$$\Leftrightarrow A(1-r)>8(1-r)-4kr \tag{53}$$

$$\Leftrightarrow k>\frac{(1-r)(8-A)}{4r} \tag{54}$$

$$\Leftrightarrow k>\frac{(1-r)(1-\beta)^2}{4r}\triangleq\Theta_{u1} \tag{55}$$

$$\Leftrightarrow \frac{r}{1-r}>\frac{(1-\beta)^2}{4k}. \tag{56}$$

Therefore, $\pi_u^{PT}>\pi_u^{PJ}$ holds if $\frac{r}{1-r}>\frac{(1-\beta)^2}{4k}$, $\pi_u^{PT}<\pi_u^{PJ}$ holds if $\frac{r}{1-r}<\frac{(1-\beta)^2}{4k}$.

b) We compare π_d^{PT} and π_d^{PJ}. If we let $\pi_d^{PT}=\pi_d^{PJ}$, then

$$\frac{4(1-k)(a+t)^2}{(A-4k)^2}=\frac{(a+t)^2[4r^2-(1-r)(1-\beta)^2]}{[A-8(1-r)]^2} \tag{57}$$

$$\Leftrightarrow (256(1-k)-4(A-4k)^2)r^2+(64A(1-k)-512(1-k)-(A-4k)^2(1-\beta)^2)r \tag{58}$$

$$+4(1-k)A^2-64A(1-k)+256(1-k)+(A-4k)^2(1-\beta)^2=0. \tag{59}$$

For the coefficient of r^2, $256(1-k)-4(A-4k)^2>0$ if $0<k<\frac{(1-\beta)-(1-\beta)^2}{4}$, and $256(1-k)-4(A-4k)^2<0$ if $\frac{(1-\beta)-(1-\beta)^2}{4}<k<1$.

We denote the roots of Eq.

$$\frac{(1-\beta)^2[A(A-8k)+16(2-k)^2]-(1-\beta)(A-4k)\sqrt{A^2(8k-A)+24A(A-8)+16k^2(24-A)+512(1-k)}}{[8(64(1-k)-(A-4k)^2)]} \triangleq \Theta_{d1}. \tag{60}$$

and

$$\frac{(1-\beta)^2[A(A-8k)+16(2-k)^2]+(1-\beta)(A-4k)\sqrt{A^2(8k-A)+24A(A-8)+16k^2(24-A)+512(1-k)}}{[8(64(1-k)-(A-4k)^2)]} \triangleq \Theta_{d2}. \tag{61}$$

Then, when $0<k<\frac{(1-\beta)-(1-\beta)^2}{4}$ satisfies, $\pi_d^{PT}<\pi_d^{PJ}$ if $\Theta_{d1}<r<\Theta_{d2}$, and $\pi_d^{PT}>\pi_d^{PJ}$ if $r<\Theta_{d1}$ or $r>\Theta_{d2}$; when $\frac{(1-\beta)-(1-\beta)^2}{4}<k<1$ satisfies, $\pi_d^{PT}>\pi_d^{PJ}$ if $\Theta_{d1}<r<\Theta_{d2}$, and $\pi_d^{PT}<\pi_d^{PJ}$ if $r<\Theta_{d1}$ or $r>\Theta_{d2}$.

This completes the proof of Theorem 2.

Proof of Theorem 3.

a) We compare π_u^{IT} and π_u^{IJ}. If we let $\pi_u^{IT}<\pi_u^{IJ}$, then

$$\frac{a^2(1-\alpha)}{A-8\alpha-4k(1-\alpha)}<\frac{a^2r(1-\alpha)}{A-8(r\alpha+1-\alpha)} \tag{62}$$

$$\Leftrightarrow (A-8)(1-r)<4kr(\alpha-1) \tag{63}$$

$$\Leftrightarrow k<\frac{(1-r)(8-A)}{4r(1-\alpha)} \tag{64}$$

$$\Leftrightarrow k<\frac{(1-r)(1-\beta)^2}{4r(1-\alpha)}=\frac{\Theta_{u1}}{1-\alpha}\triangleq\Theta_{u2} \tag{65}$$

$$\Leftrightarrow \frac{r}{1-r}>\frac{(1-\beta)^2}{4k(1-\alpha)}. \tag{66}$$

Therefore, $\pi_u^{PT}>\pi_u^{PJ}$ holds if $\frac{r}{1-r}>\frac{(1-\beta)^2}{4k(1-\alpha)}$, $\pi_u^{PT}<\pi_u^{PJ}$ holds if $\frac{r}{1-r}<\frac{(1-\beta)^2}{4k(1-\alpha)}$.

b) We compare π_d^{IT} and π_d^{IJ}. If we let $\pi_d^{IT}=\pi_d^{IJ}$, then

$$\frac{4a^2(1-\alpha)^2(1-k)}{(A-8\alpha-4k(1-\alpha))^2}=\frac{a^2(1-\alpha)(4r^2(1-\alpha)-(1-r)(8-A)))}{(A-8(\alpha r+(1-r)))^2} \tag{67}$$

$$\Leftrightarrow 4(1-\alpha)(64(1-\alpha)^2(1-k)-(A-4(2\alpha+k(1-\alpha)))^2)r^2+(64A(1-\alpha)^2(1-k) \tag{68}$$

$$-512(1-\alpha)^2(1-k)-(8-A)(A-4(2\alpha+k(1-\alpha)))^2)r+4(1-k)(1-\alpha)A^2 \tag{69}$$

$$-64A(1-\alpha)(1-k)+256(1-\alpha)(1-k)+(8-A)(A-4(2\alpha+k(1-\alpha)))^2=0. \tag{70}$$

For the coefficient of r^2, $4(1-\alpha)(256(1-\alpha)^2(1-k)-4(A-4(2\alpha+k(1-\alpha)))^2)<0$. In fact,

$$4(1-\alpha)(256(1-\alpha)^2(1-k)-4(A-4(2\alpha+k(1-\alpha)))^2) \tag{71}$$

$$=64k^2(\alpha-1)^3+32k(A-8)(\alpha-1)^2+4(\alpha-1)(A-8)(A-16\alpha+8). \tag{72}$$

We can find the discriminant of this quadratic trinomial of k is that $16384(\alpha-1)^5(1-\beta)^2$ is always less than 0. In addition, the coefficient of k^2, i.e., $64(\alpha-1)^3<0$. Thus, the coefficient of r^2 is always negative.

We denote the roots of Eq.

$$\frac{\begin{array}{r}(512(1-\alpha)^2(1-k)+(8-A)(A-4(2\alpha+k(1-\alpha)))^2\\ -64A(1-\alpha)^2(1-k))-(A-8\alpha-4k+4\alpha k)\sqrt{E}\end{array}}{8(1-\alpha)(64(1-\alpha)^2(1-k)-(A-4(2\alpha+k(1-\alpha)))^2)} \triangleq \Theta_{d3}. \tag{73}$$

and

$$\frac{\begin{array}{r}(512(1-\alpha)^2(1-k)+(8-A)(A-4(2\alpha+k(1-\alpha)))^2\\ -64A(1-\alpha)^2(1-k))+(A-8\alpha-4k+4\alpha k)\sqrt{E}\end{array}}{8(1-\alpha)(64(1-\alpha)^2(1-k)-(A-4(2\alpha+k(1-\alpha)))^2)} \triangleq \Theta_{d4}. \tag{74}$$

where $E=(A-8)(A^3+8A^2\alpha k-32A^2\alpha-8A^2k+8A^2+16A\alpha^2k^2-384A\alpha^2k+512A\alpha^2-32A\alpha k^2+640A\alpha k-512A\alpha+16Ak^2-256Ak+192A-256\alpha^3k^2+640\alpha^2k^2+3072\alpha^2k-4096\alpha^2-512\alpha k^2-5632\alpha k+6144\alpha+128k^2+2560k-2560)$

Then, $\pi_d^{IT}>\pi_d^{IJ}$ if $\Theta_{d3}<r<\Theta_{d4}$, and $\pi_d^{IT}<\pi_d^{IJ}$ if $r<\Theta_{d3}$ or $r>\Theta_{d4}$.

This completes the proof of Theorem 3.

Proof of Theorem 4.

a) We compare π_u^{PT} and π_u^{IT}. If we let $\pi_u^{PT}>\pi_u^{IT}$, then

$$\frac{(a+t)^2}{A-4k} > \frac{a^2(1-\alpha)}{A-8\alpha-4k(1-\alpha)} \tag{75}$$

It is easily found that when $A-8\alpha-4k(1-\alpha)>0$, i.e., $0<\alpha<\frac{A-4k}{8-4k}$, $\pi_u^{IT}>0$ holds. Accordingly, we obtain $t>a(\sqrt{\frac{A-4k-\alpha(A-4k)}{A-4k-\alpha(8-4k)}}-1)\triangleq\Gamma_{u1}$.

Therefore, $\pi_u^{PT}>\pi_u^{IT}$ holds if $t>\Gamma_{u1}$, $\pi_u^{PT}<\pi_u^{IT}$ holds if $0<t<\Gamma_{u1}$.

b) We compare π_d^{PT} and π_d^{IT}. If we let $\pi_d^{PT}>\pi_d^{IT}$, then

$$\frac{4(a+t)^2(1-k)}{(A-4k)^2} > \frac{4a^2(1-\alpha)^2(1-k)}{[A-8\alpha-4k(1-\alpha)]^2} \tag{76}$$

It is easily found that when $0<\alpha<\frac{A-4k}{8-4k}$, $A-8\alpha-4k(1-\alpha)>0$ holds. Accordingly, we obtain $t>a(\frac{A-4k-\alpha(A-4k)}{A-4k-\alpha(8-4k)}-1)\triangleq\Gamma_{u2}$.

Therefore, $\pi_d^{PT}>\pi_d^{IT}$ holds if $t>\Gamma_{u2}$, $\pi_d^{PT}<\pi_d^{IT}$ holds if $0<t<\Gamma_{u2}$.

This completes the proof of Theorem 4.

Proof of Theorem 5.

a) We compare π_u^{PJ} and π_u^{IJ}. If we let $\pi_u^{PJ}>\pi_u^{IJ}$, then

$$\frac{r(a+t)^2}{A-8(1-r)} > \frac{a^2r(1-\alpha)}{A-8(1-r)-8r\alpha} \tag{77}$$

It is easily found that when $A-8(1-r)-8r\alpha>0$, i.e., $0<\alpha<\frac{A-8(1-r)}{8r}$, $\pi_u^{IJ}>0$ holds. At this time, $A-8(1-r)>0$, or $\pi_u^{PJ}>0$ clearly holds. Accordingly, we obtain $t>a(\sqrt{\frac{(1-\alpha)(A-8(1-r))}{A-8(1-r)-8r\alpha}}-1)\triangleq\Gamma_{u2}$.

Therefore, $\pi_u^{PJ}>\pi_u^{IJ}$ holds if $t>\Gamma_{u2}$, $\pi_u^{PJ}<\pi_u^{IJ}$ holds if $0<t<\Gamma_{u2}$.

b) We compare π_d^{PJ} and π_d^{IJ}. If we let $\pi_d^{PJ}>\pi_d^{IJ}$, then

$$\frac{(a+t)^2(4r^2-C)}{(A-8(1-r))^2} > \frac{a^2(1-\alpha)^2(4r^2(1-\alpha)-C)}{[A-8(1-r)-8r\alpha]^2} \tag{78}$$

It is easily found that when $0 < \alpha < \frac{A-8(1-r)}{8r}$, $A - 8(1-r) - 8r\alpha > 0$ holds. In addition, when $0 < \alpha < 1 - \frac{(1-r)(1-\beta)^2}{4r^2}$, $4r^2(1-\alpha) - C > 0$ holds. At this time, $4r^2 - C > 0$ clearly holds. Accordingly, we obtain $t > a(\frac{A-8(1-r)}{A-8(1-r)-8r\alpha}\sqrt{\frac{(1-\alpha)(4r^2(1-\alpha)-C)}{4r^2-C}} - 1) \triangleq \Gamma_{d2}$.

Therefore, $\pi_d^{PJ} > \pi_d^{IJ}$ holds if $t > \Gamma_{d2}$, and $\pi_d^{PJ} < \pi_d^{IJ}$ holds if $0 < t < \Gamma_{d2}$.

This completes the proof of Theorem 5.