Multi-period hub location problem: a review

Amir Khaleghi[1], Alireza Eydi[2*]

[1]PhD in Industrial Engineering, Faculty of Engineering, University of Kurdistan, Sanandaj, Iran, A.khaleghi@eng.uok.ac.ir

[2]Associate Professor in Faculty of Engineering, University of Kurdistan, Sanandaj, Iran, Alireza.eydi@uok.ac.ir

Abstract

In hub-and-spoke systems, due to the changes in the input parameters affecting the system over time, transportation system providers should make the right and timely decisions about hub facilities. For this reason, multi-period hub location problems are especially important. This paper presents a comprehensive review of multi-period hub location problems from 1990 up to the most recent published studies. First, we study the developed models based on some characteristics: type of planning horizon (continuous-time and discrete-time), capacity constraints (uncapacitated, capacitated, and modular), type of problem (median and covering), type of services (single-level and hierarchical), number of commodities and modes, type of hub facility (mobile and virtual), type of assignment (single and multiple), and type of parameters (deterministic and uncertain). Also, practical applications of the models are investigated. Then, we survey the proposed solution methods and real-life case studies. Finally, future suggestions are presented for researchers.

Keywords: transportation, dynamic facility location, multi-period hub location, discrete-time planning horizon, continuous-time planning horizon

1. Introduction

Hub location problem (HLP) is a subset of facility location problem. Hubs are intermediate facilities in transportation systems and are used for indirect connections between demand points. In hub systems, flows from the origin point are entered into the hub, consolidated at the hub, and then distributed at the destination point. Hubs reduce the number of connections in the transportation networks. The HLP aims to find the best location for hubs and allocate demand points to the located hubs, to minimize system costs or maximize the profits of hub system design. Classification and review of the basic models and solution methods of HLP have been investigated by researchers in Refs. [1]-[5].

The decisions in HLP are made based on the value of some parameters such as demand flows, transportation costs, and fixed costs of hubs. These parameters usually change over time. If changes in the parameters are predictable, planning for the location of facilities and other relevant decisions in the future may be desirable. In this case, instead of determining the location of facilities, the timing of locating the facilities should also be specified [6]. In such problems, a dynamic and time-dependent model must be designed. Such a model allows for decisions such as opening and closing facilities and adjustment of operational capacities (which is often more cost-effective than opening new facilities). Instead of changing in values of the parameters of the model, due to some conditions of the real world, such as budget constraints, we need to design a dynamic model.

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When the facility location decisions are made over time, the planning horizon should be defined. The planning horizon is a time frame in which, Decision-Maker (DM) plans. In the real world, usually, decisions are made in multi periods. For example, when a project is in progress and includes construction and investment phases, there is a planning horizon consisting of some time periods. During the time periods, DM can make his/her decisions. Thus, the initial configuration of the system may change. In each time period, the previous configuration of the system is updated and completed by the last period of the planning horizon.

In terms of implementation time of multi-period hub location problem decisions, the planning horizon can be categorized into two types: discrete-time planning horizon and continuous-time planning horizon. In the discrete-time planning horizon, the implementation time of problem decisions is predetermined, and the length of time periods can be specified. But in the continuous-time planning horizon, decisions can be made at any point in time in the planning horizon. The implementation time of decisions is determined in the decision-making process.

In the static hub location problem, the decisions are made once for all periods, and the network configuration may not be optimal for the future; because the values of the problem parameters may change over time. One of the advantages of the multi-period hub location problem over the static hub location problem is that the problem decisions are made according to the changes in parameters. Another advantage of the multi-period hub location is that it pays attention to the relationship between the network structures in different periods and makes the optimal decision for the whole planning horizon. But in the static hub location, the relationship between network configurations in different periods is not considered.

The models studied in this review paper include hub location-relocation models (continuous-time models) and multi-period hub location models (discrete-time models). The facility location-relocation problem and multi-period facility location problem are subsets of dynamic facility location problem [7]. In terms of facility location problem, due to the common features of these problems, in many references, they have been considered as a single problem (for example, in Ref. [8]). In this study, both problems are investigated in the context of the multi-period hub location problem.

In the published review papers in the context of facility location problem, hub location problems have been studied as a subset of facility location problems. In Ref. [9], integer programming models, dynamic programming and stochastic programming solution methods, and scenario planning techniques have been studied for strategic facility location problems. Ref. [9] only mentions one of the multi-period hub location models. This model is the first model in the context of the multi-period hub location problem proposed in Ref. [10]. It is related to the location-relocation of terminals in a fixed region with increasing demand density, considering a continuous-time planning horizon. Then, in Ref. [11], researchers categorized the proposed models and solution methods for solving dynamic facility location problems. Also, in Ref. [7], models, solution methods, and applications of static and dynamic facility location problems until 2011 were studied. In Refs. [7] and [11], multi-period hub location problems were not addressed. Then, in Ref. [3], models, solution methods, and applications of hub location problems were studied, and two works in the context of multi-period hub location problem were mentioned (Refs. [12] and [13]). Then, in Ref. [14], the proposed models, solution methods, and applications of dynamic facility location problems until 2015 were studied and pointed to the proposed models for multi-period hub location problem until 2015.

The present study provides a review of multi-period HLP models, solution approaches, real-world applications, and a gap analysis for future research.

Based on the previous research, there are some motivations for the present study:

- In Ref. [14], facility location problems have been studied, and the proposed models in the context of multi-period hub location problems until 2015 have been mentioned. Since 2015, many researchers have been studied numerous variants of the multi-period hub location problem.
- To categorize the models, we have considered various features for the models, such as type of planning horizon, capacity constraints, type of problem, type of services, number of transportation modes and commodities, type of facility, allocation rule, number of objectives, and the nature of the parameters. These characteristics are discussed comprehensively in this paper.
- In the description of the proposed models, the practical aspects of the models and the advantages they can have for DMs are mentioned.
- In the conclusion section, some subjects that need to be considered as recent trends in multi-period hub location problem are mentioned. These subjects include: paying more attention to multi-objective optimization, considering aspects of sustainability in the problem, considering continuous-time planning horizon, modeling waiting time at hubs (queueing theory), considering continuous solution space, considering uncertainty for problem parameters, modeling competition and pricing, applying the problem in healthcare systems and crisis management, and developing efficient solution methods.

Therefore, this study tries to not only provide a broader review of the analyzed papers up to 2015 but also shed light on some uncovered aspects of multi-period hub location models that have been published after 2015.

The remainder of this research is organized as follows: in the next section, we introduce the most common models in the context of multi-period hub location problem, which have been applied by the literature (discrete-time and continuous-time models). Also, the mathematical formulations related to the two basic models in the literature are presented. The first formulation is related to the continuous-time multi-period hub location problem (Ref. [10]). The second formulation is related to the discrete-time multi-period hub location problem (Ref. [12]). Then, in section 3, we classify the proposed solution approaches. Section 4 presents the applications of the problem in the real world. Finally, in section 5, conclusions and future research are presented.

2. Multi-period hub location models

2.1. Multi-period HLP with continuous-time planning horizon

The first model in the context of multi-period hub location was presented in Ref. [10]. This model is an approximation of a general carrier serving a fixed region with expanding demand density. This model is to locate new terminals, relocate existing terminals, and routing the flows during the continuous-time planning horizon. The objective function of the model is to minimize costs of transportation, locating new terminals, and relocating existing terminals. Notations used in this work are listed in **Table 1**.

Table 1. Notations used in Ref. [10]

Symbol	**Description**
s	Service region
A	Size of the service region
$K(t)$	The number of terminals at time t
$X(t) = (x_1(t), x_2(t), \ldots, x_K(t))$	Location of terminals 1 to K at time t
$M(t)$	The cumulative number of terminals at time t
$\rho(x,t)$	Demand density of location x at time t
q	Effective discount rate
$\int_{t_0}^{\infty} K(t)e^{-qt}dt$	Total discounted terminal cost
$\int_{t_0}^{\infty} [\int_s \rho(x,t)D(x,X(t))dx]e^{-qt}dt$	Total discounted transportation cost
R	Fixed cost per unit of time for relocation
r	Discounted relocating fixed cost ($r = qR$)
$\int_{t_0}^{\infty} M(t)re^{-qt}dt$	Total discounted relocation costs

Based on the notations above, the objective function of the model can be written as **Eq. (1)**.

$$C(X(t)) = \int_{t_0}^{\infty} [K(t) + \int_s \rho(x,t)D(x,X(t))dx]e^{-qt}dt + \int_{t_0}^{\infty} M(t)re^{-qt}dt \quad (1)$$

If demand has a uniform density, then $\rho(x,t) = \rho(t)$, and the cost function is transformed into **Eq. (2)**.

$$C(X(t)) = \int_{t_0}^{\infty} [K(t) + \rho(t)A\bar{D}(s,X(t))]e^{-qt}dt + \int_{t_0}^{\infty} M(t)re^{-qt}dt \quad (2)$$

In **Eq. (2)**, $\bar{D}(s,X(t))$ is the average transportation cost (for all shipments).

The author proposed three myopic strategies to get the upper and lower bounds on distribution costs. In this model, the demand points are randomly scattered in the service region. The author examines the performance of the myopic strategies for locating terminals. In these strategies, the choice of new terminal locations is based on current demand values and current terminal locations, not on predicting future parameter values. The author showed that the myopic strategies, which have been studied for both one-dimensional and two-dimensional cases, can provide upper and lower bounds on optimal transportation costs (which is unknown). Thus, the model can help the DM to determine the location of terminals, relocation of terminals, and the timing of implementing decisions in a service region with increasing demand in a continuous-time planning horizon and with near-optimal transportation costs.

2.2. Multi-period public transportation network design

In Ref. [15], the authors proposed a MIP formulation for a multi-period HLP with applications in public transportation network design. In their work, there is a finite planning horizon consisting of several time periods. In the first time period, the transportation network has a set of openable and closable hubs and hub links (initial configuration). During the planning horizon, the transportation network can be developed, new openable hubs and hub links can be opened, and closable hubs and hub links can be closed. The objective is to minimize the network's overall costs, which include transportation costs and fixed costs of establishing hubs and hub links. Decisions in this problem are the location of hubs and hub links and routing of network flows. In this model, at each period, the available budget for operational costs of hubs and hub links (costs of opening, maintenance, and closing of facilities) is limited. The remaining budget in each period is subject to interest rate and can be used in subsequent periods. Researchers proposed a greedy neighborhood search algorithm for solving more instances of the problem due to the high complexity of the problem.

This model can be applied to develop a public transportation network that has an initial configuration (current status of the network). It helps the DM to make optimal decisions regarding the hubs and hub links location and routing the network flows in each time period, given the initial configuration of the public transport network and changes of the parameters within the finite planning horizon and limited budget available in each period.

2.3. Multi-period uncapacitated hub location

An integer quadratic programming formulation for a multi-period and multi-commodity uncapacitated HLP was presented in Ref. [12]. In this formulation, set H denotes the candidate nodes for locating hubs. Set T is the set of periods. $K_1, \dots, K_{|T|}$ are sets of commodities that must be routed in each period, and W_k^t denotes the quantity of commodity k routed in time period $t \in T$. Parameters f_i^t, g_i^t, and q_i^t denote fixed costs of establishing, operating, and closing a hub at node i at time period t for a specific node $i \in H$. Hubs are fully inter-connected, and each commodity is routed on path $(o(k), i, j, d(k))$, in which $o(k)$ and $d(k)$ are origin and destination nodes, respectively and i, j are intermediate hubs, and i and j can be the same. Let d_{ij}^t denotes the distance between nodes i and j, and $e \in E$ for $E = \{L \subseteq H, 1 \leq |L| \leq 2\}$ is a hub edge. Considering a discount factor α for inter-hub connections, directed and undirected transportation costs of routing commodity k are $\hat{F}_{ijk}^t = W_k^t(d_{o(k)i}^t + \alpha d_{ij}^t + d_{jd(k)}^t)$ and $F_{ek}^t \in \min(\hat{F}_{ijk}^t, \hat{F}_{jik}^t)$, respectively. x_{ek}^t and z_i^t are decision variables. x_{ek}^t takes value 1 if commodity k uses hub edge e at time period t, 0 otherwise. z_i^t takes value 1, if node i is selected as a hub in period t, otherwise 0.

Based on the above descriptions, the integer quadratic programming model of multi-period uncapacitated HLP is formulated as **Eqs. (3)-(7)**.

$$\min \sum_{i \in H} \sum_{t \in T} f_i^t (1 - z_i^{t-1}) z_i^t + \sum_{i \in H} \sum_{t \in T} g_i^t z_i^t - \sum_{i \in H} \sum_{t \in T} q_i^t (1 - z_i^t) z_i^{t-1} + \sum_{e \in E} \sum_{k \in K^t} \sum_{t \in T} F_{ek}^t x_{ek}^t \quad (3)$$

Subject to.

$$\sum_{e \in E} x_{ek}^t = 1 \qquad \forall\, k \in K^t, t \in T \quad (4)$$

$$\sum_{\{e \in E : i \in e\}} x_{ek}^t \leq z_i^t \qquad \forall\, i \in H, k \in K^t, t \in T \quad (5)$$

$$x_{ek}^t \geq 0 \qquad \forall\, e \in E, k \in K^t, t \in T \quad (6)$$

$$z_i^t \in \{0,1\} \qquad \forall\, i \in H, t \in T \quad (7)$$

In the above formulation, **Eq. (3)** minimizes the total fixed costs and routing costs over the planning horizon. Constraint (4) ensures that each commodity is routed using a unique path at time period t. Constraint (5) stipulates that the route of each commodity has to pass hub nodes. Constraints (6) and (7) are related to the type of decision variables.

This model can be used in the field of Less-than-truckload (LTL) systems and air transportation. In the first case, terminals (hubs) can be used in the collection, transmission, and distribution of freight and to maximize the utilization of trucks. Therefore, the choice of location of hubs has a significant impact on transportation costs. This model is used when there is an initial network configuration for the LTL system, and we are going to develop it. For example, if two LTL freight companies combine, the number of terminals increases. In this new case, given the expected increase in transportation demand, the DM must determine which hubs will continue to operate and which hubs will be moved or closed in the network during the planning horizon. Another application of this model is in the field of message transmission networks. In such networks, equipment such as concentrators, switches, and multiplexers can be considered as hubs. Communication links include fiber-optics, telephone lines, satellite channels, and microwaves. Flows include data packages that must be moved from origin to destination through hubs and links. In messaging networks, which are usually developed periodically, the model helps the DM to be aware of how to optimally develop the network, given the changes in the volume of data packages and transportation costs in different time periods.

2.4. Multi-period virtual HLP

In Ref. [16], researchers proposed a mathematical model for air transportation planning when the weather is inclement. Bad weather conditions cause incapacitation in original hubs, and other predetermined underutilized facilities are needed to recover the incapacitation and improve the flexibility of the network and demand flow. These facilities are called virtual hubs. In this problem, there is an initial configuration for the transportation network, and decisions are on how to locate virtual hubs and how to route the network flows in various time periods of the planning horizon so that transportation costs and costs of opening, closing, and maintaining the virtual hubs in the network for all time periods are minimized.

In this model, researchers considered a maximum available budget for operational costs of the virtual hubs throughout the planning horizon. These costs include setup, maintenance, and closure costs. Then, researchers in Ref. [13] studied the virtual hub location problem under fuzzy uncertainty of the variation factor of hubs (related to the capacity) and hubs capacity (sum of the receiving and dispatching capacity). The authors considered these parameters as triangular fuzzy numbers. To cope with uncertainty, researchers proposed a fuzzy integer programming approach to minimize transportation costs throughout the planning horizon. The authors showed that the total system costs when the virtual hubs exist in the network are less than when these hubs do not exist in the network. In the absence of virtual hubs, the capacity of original hubs must be increased, which is more costly and time-consuming. In an air transportation system, in the case of bad weather conditions, some transportation demands cannot be allocated to the desired airports due to the incapacitation of that airports. In such cases, the DM can allocate these demand nodes to virtual hubs without cancelation the allocations. In this way, the total system costs throughout the planning horizon reduces.

2.5. Multi-period hub network design considering modular hub capacity

In Ref. [17], researchers studied a multi-period HLP in the situation where the capacity of hubs can be expanded discretely over time. In this problem, the planning horizon is finite and consists of some time periods. There are different types of modules for each hub, and each hub can receive at most one module at each period to capacity expansion. Examples of modular capacity in the real world are new terminals in airports and new sorting lines in postal systems. Each module has a specific capacity, and in each time period, the capacity of each hub is determined by the number of installed modules on that hub. Researchers proposed MIP models for both single and multiple allocation cases. Decisions are hub and hub link location, allocation, installing modules, and routing of the network flows in different time periods to minimize total transportation costs, hubs and hub links operational costs, and capacity expansion costs. The authors also proposed some valid inequalities for improving the formulation.

In addition to the incoming flows from different origins to a hub (related to the collection), the authors also considered the incoming flows from other hubs to that hub (related to the transfer) in capacity constraints. Also, the operational costs of the incoming flows related to the transfer were considered in the objective function. They developed the proposed models for the case that there is an initial configuration for the transportation network. The proposed models can be used in a public transportation system in which there is an initial network configuration, and we intend to develop it. Another application of the models is in a transportation system in which facilities are operating and according to the changes in the problem parameters in different periods, the operational capacity of these facilities can be adjusted. Using some measures, the authors showed that disregarding the multi-period nature of the problem can cause the DM to make mistakes in choosing the location of the hubs and incur additional costs.

Then, a stochastic version of the multi-period hub location problem considering modular hub capacity was proposed in Ref. [18]. In this work, transportation demand is subject to uncertainty and captured by some scenarios. Each hub receives some initial modules, and its capacity can be expanded using additional modules in the following time periods. The authors proposed a MIP formulation for multiple allocation case. Decisions are about the location of hubs, the initial hubs capacity, capacity expansion, and routing the network flows in different time periods. The objective function is to minimize total expected costs over the planning horizon. The cost items include costs of establishing hubs, installing initial and additional modules, and transportation. Because of the high complexity of the model, researchers proposed some valid inequalities to strengthen the formulation and solve more instances of the problem.

The authors showed that by increasing each module’s capacity, the average number of hubs located, the average cost of routing, and the average capacity used by hubs decrease. Also, by increasing the discount factor, the average number of hubs located and the associated costs decrease, but the average used capacity of hubs does not change significantly. Given that the proposed model is a two-stage stochastic programming model, the implementation of the resulting solution consists of two parts: The first part involves implementing of decisions about the location of hubs in different periods and determining the initial capacity of hubs. In the second part, decisions related to increasing the capacity of hubs and routing flows in different periods (according to possible scenarios) are implemented.

2.6. Multi-period hub covering location problem

In Ref. [19], the authors proposed a multi-period hub covering location problem under fuzzy possibilistic uncertainty considering two objectives. The first objective is to minimize costs of transportation, covering, opening, reopening, activating facilities located at hubs, and purchase costs of transporters. The second objective is to minimize total shipping times from origin to destination through hubs. In this problem, decisions are hub location and allocation, maximum covering radius of hubs, opening and reopening factories at hubs, the capacity allocation for facilities, usage of vehicles, and routing the flows during the planning horizon. For solving the proposed model, the authors used a two-phase approach. The possibilistic model is transformed into a crisp counterpart model in the first phase, and the crisp model is solved using three methods in the second phase.

This model makes a relationship between the hub location-allocation problem and supply chain management. It can be used in a three-level and two-echelon supply chain in which manufacturers or distributors are considered as hubs. Also, suppliers and consumers are considered as clients. The model helps the DM in supply chain management to minimize system costs and minimize the total time of transfer of goods from origin to destination through hubs in a finite planning horizon.

Also, in work [20], researchers proposed robust and fuzzy goal programming solution methods for a hub location-allocation model considering three objectives. In their model, minimization of supply chain costs, minimization of the weighted sum of service time and earliness and tardiness, and minimization of emission costs of plants located at hubs and vehicles are considered as sustainability objectives. This model helps the DM to manage a three-level and two-echelon supply chain in a finite planning horizon, considering sustainability objectives.

In Ref. [21], the authors proposed a mathematical formulation for a multi-period hub set covering location problem. In this model, the covering radius is not predetermined. There is a planning horizon consisting of several time periods, and the DM can control the covering radius variable in different time periods. Decisions in this problem are hub location and allocation, closing the hubs, routing the flows on the network, and covering radius at each time period. They supposed that movable facilities at closed hubs can transfer to a newly established hub. The number of possible movements in each time period is limited and is equal to the minimum of total located hubs and total closed hubs in that period. The objective is to minimize costs of transportation, hub establishment, covering, and closing, minus benefits of movable facilities at closed hubs. The initial model was non-linear, and the authors presented a set of linear constraints to the linearization of the model.

This model can be applied in the field of telecommunications and freight transportation networks. In these applications, the coverage radius is proportional to the number of areas covered by the hubs. Unlike models that consider the coverage radius to be constant, in this model, the appropriate coverage radius can be adjusted according to changes in parameters in different time periods. This model helps the DM to determine the appropriate coverage radius for each period, as well as make decisions about opening and closing hubs, allocation, and routing the network flows.

In Ref. [22], the authors studied an uncapacitated single allocation hub covering location problem considering periodic changes of the parameters. In the proposed model, the objectives are maximizing total covered demand and minimizing opening and closing costs of hubs during the planning horizon. In this model, the number of hubs in each period is already known. Also, in each period, the travel time from origin to destination must not exceed the coverage radius. This model can be applied in the field of air transportation network design with the aim of maximizing the covered demand of customers and minimizing the costs of the transportation system considering periodic changes of parameters.

2.7. Multi-period p-mobile HLP

In Ref. [23], researchers proposed a mathematical formulation for HLP considering the possibility of moving the hubs from their current locations to other locations using mobility infrastructures (railways) during the planning horizon. At each time period, p hubs must be located, and the hub network is fully inter-connected. Hubs mobility infrastructure is established in the first period, and it can be used in further periods. Decisions of the problem are hub location in a specific node or transferring a hub from another node to that specific node, allocation, treating hubs as immobile hubs, routing the flows on the network and establishing railway between nodes in the planning horizon. The objective is to minimize costs of transportation, establishing mobility infrastructures, establishing hubs, and establishing railways between nodes.

This model is used in situations where hubs can be moved easily. For example, this model can be applied in postal systems with mobile mail centers and in disasters, considering mobile emergency centers. In a disaster, cure centers can be easily relocated at any time period, and a low-cost cure can be provided to the injured. It can also be used in a blood supply chain to collect blood in larger volumes and with a lower collection cost.

2.8. Multi-period HLP considering budget constraints

An uncapacitated multiple allocation multi-period HLP was studied in Ref. [24]. In this work, there is an initial configuration for the transportation network before starting the planning horizon (a set of hubs and hub links). The planning horizon is finite and consists of several time periods. The available budget for opening and closing hubs and hub links is limited for each time period. The authors also considered an interest rate for the unused budget at each time period. During the planning horizon, existing elements of the network (closable hubs and hub links) can be closed, and new elements (openable hubs and hub links) can be opened. The authors proposed a MIP formulation in which decisions are determining network configuration in each time period (opening or closing hubs and hub links) and routing the flows on the network to minimize total costs. The cost items include transportation costs and the costs of opening, closing, and maintaining hubs and hub links. Due to the complexity of the proposed model, researchers proposed a local search-based procedure to obtain feasible solutions for the problem.

This model can be used to develop a transportation network that is currently operating. The model helps the DM to make optimal decisions about locating hubs and hub links and routing the network flows at each time period, given the initial configuration of the network and changes in the parameters over time, and the limited budget available in each period.

Then, in Ref. [25], the authors studied a similar problem and proposed a neighborhood search procedure and a Benders decomposition algorithm. The proposed Benders decomposition algorithm is an extension of the proposed Benders decomposition algorithm in Ref. [26] developed for a single-period problem. This model can be used in the field of maritime transport systems for medium-term hub network design, in which hub facilities (ports) are not purchased or built for a specific period but leased. In such systems, the location of the facilities can be easily changed in different periods due to changes in demand and transportation costs. This model can also be used in cross-dock problems (cross-docks leasing or ground transportation). Regarding the application of the model in the field of maritime transport, the DM must make its decisions about port lease or cancellation of the port lease contract, communication links between ports, routing the flows, and allocating demand points to ports due to budget constraints for operating of ports and communication links. Note that budget constraints are also considered in Refs. [13], [15], [16], and [27].

2.9. Multi-period hub location considering multiple capacity levels of hubs

In Ref. [28], researchers introduced a single allocation multi-period HLP considering multiple capacity levels for each hub. During the planning horizon, the size of hubs can be changed. In this problem, decisions are the hub location and allocation, selecting capacity levels for hubs, and routing the flows on the network in each time period to minimize transportation costs and costs of opening, closing, and resizing the hubs during the planning horizon. Researchers proposed four mathematical models for the problem and compared their complexity. Since the objective function of models was non-linear due to the product of binary variables, the authors linearized the objective function using standard techniques.

This model can be applied to design a postal network in which there are different levels for the capacity of postal centers. In each time period of the planning horizon, the model determines the location and allocation decisions, routing of the postal packages, and the desired level of capacity of the postal centers for the DM. This model can also be used in LTL transportation systems, messaging networks, and computer networks.

In Ref. [29], a single allocation multi-period hub location problem considering the life cycle and the possibility of reconstruction of hubs was studied. The authors assumed that after selecting a contractor to open a hub, it has a certain life cycle. The hub can be reconstructed or closed at the end of this life cycle. Researchers also incorporated multiple levels of capacity and the possibility of reconstruction in the problem modeling. This model can be used in the field of urban transportation planning, transportation of perishable products, and emergency services.

2.10. Multi-period p-hub median location problem with multiple allocations

In Ref. [30], the authors extended the *p*-hub median location problem for a dynamic case. In this problem, demand flows change during the planning horizon, and hubs and links are uncapacitated. Opening and closing of hubs can also be done in each time period. Researchers showed that their proposed model has a lower cost than the static case (single-period model), and by increasing the number of time periods, costs will be reduced.

This model is used in the field of air transportation. The DM can use this model to select the best location for airports and the best allocation of demand nodes to the airports and make decisions about reopening and closing airports in each period.

2.11. Multi-period inter-modal hub-and-spoke biomass supply chain network design

In Ref. [31], researchers proposed a cost-efficient and reliable inter-modal multi-period hub-and-spoke model with applications in biomass supply chain network design to cope with supply fluctuations of biomass and hedge against natural disasters. In this problem, there are two modes of transportation (rail and barge). They formulated the problem as a Mixed-integer Non-Linear Programming (MINLP) model. Researchers proposed a rolling horizon algorithm to solve the model. Due to the complexity of sub-problems in the proposed rolling horizon method, they also proposed an accelerated Benders decomposition method for solving the sub-problems.

2.12. Reliable multi-period HLP

In Ref. [27], the authors studied a reliable and inter-modal multi-period freight network expansion problem considering the risk of disruption in hubs and links. In this model, there is a finite planning horizon consisting of two sets of periods (namely strategic periods and routing periods). In each strategic period, the location of new inter-modal terminals (for rail and road modes), capacity expansion of terminals, and retrieving disrupted terminals and links can be done. In each routing period, flows are routed on the network. A finite set of scenarios is considered for variations in demand and disruption for each expansion period.

A probabilistic-robust model was proposed by the authors. The objective is to minimize total expansion costs and expected costs of routing the flows for all scenarios and all routing periods. Also, in the calculation of routing costs, a penalty is considered for unmet demands. Researchers used a hybrid simulated annealing algorithm for solving problem instances. The authors showed that multi-period planning compared to static planning, can reduce total transportation costs and improve the usage of capacity in inter-modal terminals. Also, considering two time scales in the problem can reduce transportation costs compared to when only a time scale is considered.

The results also showed that the available budget should be used to develop or build new terminals instead of repairing weaker links. In a bi-modal freight system (rail and road), this model helps the DM to make decisions about each time period in the event of disruption according to possible scenarios. In the above model, it is assumed that the disruptions are independent of each other.

In Ref. [32], researchers worked on a multi-period HLP on a hierarchical and multi-modal network considering dynamic disruptions. In the network of the problem, there are three levels of hierarchy (central hubs, non-central hubs, and demand nodes), two transportation modes (air and ground), and two service levels (central and non-central hubs). A star (mesh) network connects all airport hubs (ground hubs). There is a finite planning horizon consisting of several time periods. The total number of central and non-central hubs is predetermined over this planning horizon. The authors supposed that disruptions in hubs and links are time-dependent and site-dependent. They proposed a MIP formulation for two cases: one without considering disruption and one with considering disruption. In the proposed model, decisions are opening hubs and links, closing disrupted hubs and links, allocations in different levels of hierarchy, and routing the flows on the network in each time period. The objective is to minimize transportation costs, fixed costs of hubs and links, and operational costs. To solve the problem, they proposed a two-phase heuristic algorithm based on Monte Carlo simulation.

This model can be used in a transport system with two levels of service (or three levels of hierarchy) and two modes of transport in which there is a possibility of disruption of hubs and links. Network disruption affects hub location and allocation.

The model helps the DM to determine hub location and allocation at different levels according to the demand fluctuations in different periods and disruptions. Also, alternative facilities for disrupted facilities are identified. Because of network changes due to disruption, the transportation system incurs costs. These costs include repair, maintenance, temporary closing, and reopening costs. Therefore, in comparison with the case that disruption is not considered, location and allocation must be modified because the disruption in one period influences the decisions of the following periods.

A bi-objective reliable multi-period hub location problem considering the effects of flow congestion was studied in Ref. [33]. The first objective of the model includes minimizing transportation costs, opening and closing costs of hubs, and penalties for exceeding the effective capacity of hubs minus the revenue from closed hub facilities. The second objective is to maximize the minimum reliability of network routes. This model can be used to design a reliable air transportation network with congestion considerations.

2.13. Sustainable multi-period HLP

In Ref. [34], the authors presented a robust multi-objective model for a continuous-time multi-period hub location problem considering modular capacity of hubs and links and sustainability objectives. The first objective is to minimize costs of transportation, opening and closing hubs and hub links, capacity expansion of hubs and hub links, and transferring the capacity between hubs. The second objective is to minimize the emission costs at links, and the third objective is to maximize fixed and variable job opportunities caused by the opening of hubs. Problem decisions include opening and closing hubs and increasing the capacity of hubs and hub links, transferring the capacity between hubs, and routing flows on the network over the planning horizon. Also, the best times to implement decisions (breakpoints) are determined by solving the model. The researchers assumed that the transportation demand is time-dependent and uncertain (robust interval uncertainty).

To solve the problem, the researchers used the two-phase Torabi and Hassini (TH) method. In their implementation of the TH method, in the first phase, the problem is converted into a robust counterpart by using the Bertsimas and Sim's method. In the second phase, the problem is converted into a single-objective parametric linear programming problem to get the Pareto-optimal solutions. The authors showed that in the robust case, the total capacity selected for the facilities is greater than in the case where the demand function parameters select their nominal values. This model can be used in applications of multi-period hub location problems where transportation demand is time-dependent, and the change in transportation demand is linear.

3. Proposed solution methods for multi-period HLP

In this section, we present the proposed solution approaches for multi-period HLPs. There are various solution methods in the literature. Some exact algorithms such as Branch and bound and Benders decomposition algorithms have been developed for solving the problem in small and medium sizes. Due to the high complexity of the problem, even for small sizes, in some cases, many heuristic and meta-heuristic algorithms were proposed for solving different types of the problem.

In **Table 2**, notations used for determining the different types of the multi-period hub location problem are listed. The proposed solution methods are presented in **Table 3**. In **Table 3**, in the first column, the characteristics of the problem are specified. For example, 'DT-C-SA-Ex-V-SC-SO' indicates that the planning horizon is discrete-time, the capacity of hubs is limited, the allocation rule is single allocation, the number of hubs is exogenous, the problem type is covering, there is a single commodity, and only one objective is considered. In the second column, the model reference is specified. The third column shows the solution method used. Columns 4 to 6 show the number of demand nodes, the number of hubs, and the number of time periods that could be solved by the proposed solution method.

According to **Table 3**, we find that in the field of multi-period hub location problems, considering the continuous-time planning horizon, only two studies have been conducted (Refs. [10] and [34]). Between 1990 and 2021, there is no research considering the continuous-time planning horizon. One of the possible reasons for this is that considering the continuous-time planning horizon in the problem model makes the computational complexity of the model more. In Refs. [10], [12], [15], [21]- [25], and [30], the capacity of the hubs is unlimited. In the other studies in **Table 3**, the capacity constraints are considered for hubs. Among the studies in which the capacity constraints are considered for hubs, there are three studies in which the capacity of hubs is modular. These studies include Refs. [17], [18], and [34]. Refs. [17] and [32] have studied the problem in both single and multiple allocation cases. In other studies, only one of the single and multiple allocation cases has been investigated. The number of studies in both cases is the same (nine studies in each case).

The number of studies in which the number of hubs is endogenous is more than the number of studies in which the number of hubs is exogenous. The multi-period hub covering location problem has been studied in Refs. [19]-[22]. Refs. [27] and [31] have been conducted in inter-modal case and the proposed model in Ref. [32] is multi-modal. In other studies, only one mode has been considered for transportation. Only one of the researches (Ref. [12]) is multi-commodity. Other studies have been investigated in the single-commodity case. The models presented in Refs. [22], [33], and [34] are multi-objective, and in other studies, only the optimization of one objective has been considered. Multi-objective models have been examined after 2018. This shows that in recent years, the tendency to study multi-objective models has increased. In Refs. [13], [18]-[20], [27], [32], and [34], the models are subject to uncertainty. The uncertainties considered for the parameters include fuzzy, stochastic, robust, probabilistic-robust, and probabilistic uncertainties.

Refs. [25] and [31] have proposed both exact and approximation methods for solving the problem. In Refs. [12], [13], [17]- [20], [22], [28], [30], [33], and [34], only exact algorithms have been used. In the other studies in **Table 3**, heuristic solution methods have been used by researchers. In Refs. [22], [25], and [31], researchers have used the Benders decomposition algorithm. In Refs. [17], [18], [28], and [30], the Mixed-integer Programming approach has been used to solve the problem. Also, in Refs. [16], [27], and [31], hybrid algorithms have been used for solving the problem.

Table 2. Notations for different types of multi-period HLP

Capacity of hubs	Allocation rule	Type of problem	Planning horizon	Number of hubs	Transportation modes	Number of commodities	Number of objectives
Capacitated (C)	Single allocation (SA)	Median (M)	Continuous-Time (CT)	Exogenous (Ex)	Inter-Modal (IM)	Single Commodity (SC)	Single Objective (SO)
Uncapacitated (U)	Multiple allocation (MA)	Covering (V)	Discrete-Time (DT)	Endogenous (En)	Multi-Modal (MM)	Multi Commodity (MC)	Multi Objective (MO)

Table 3. Proposed solution methods for multi-period HLP

Problem	Reference	Solution approach	Efficiency (# of nodes)	# of hubs	# of periods
CT-U-Ex-SC-SO	[10]	Heuristic algorithm	50 customers	10 terminals	-
DT-U-MA-En-SC-SO	[15]	Greedy neighborhood search-based algorithm	40	-	12
DT-U-MA-En-MC-SO	[12]	Branch and bound (based on Lagrangian relaxation)	100	-	10
DT-C-MA-En-SC-SO	[16]	Hybrid simulated annealing	81	-	4
DT-C-MA-En-SC-SO	[13]	Fuzzy programming approach	20	-	4
DT-U-MA-En-SC-SO	[24]	Local search-based procedure	100	-	12
DT-C-SA-Ex-V-SC-SO	[19]	Two-phase approach	5	3	5
DT-C-SA-En-SC-SO	[28]	Mixed-integer programming	25	-	5
DT-C-MA-En-IM-SC-SO	[31]	Rolling horizon heuristic and accelerated Benders decomposition	259	-	12
DT-U-MA-Ex-SC-SO	[30]	Mixed-integer programming	32	3	4
DT-U-MA-En-SC-SO	[25]	Neighborhood search and Benders decomposition	30	-	9
DT-C-SA-Ex-V-SC-SO	[20]	Fuzzy goal programming and TH method	4	3	3
DT-C-SA & MA-En-SC-SO	[17]	Mixed-integer programming	25	-	5
DT-U-SA-En-V-SC-SO	[21]	Modified genetic algorithm	100	-	4
DT-C-MA-En-SC-SO	[18]	Mixed-integer programming	25	-	3
DT-U-SA-Ex-SC-SO	[23]	Genetic algorithm and simulated annealing	100	3	4
DT-C-MA-En-IM-SC-SO	[27]	Hybrid SA algorithm	44	-	20
DT-C-SA & MA-Ex-MM-SC-SO	[32]	Two-phase heuristic based on Monte Carlo simulation	70	21 ground hubs, 19 airports, and 5 central hubs	6
DT-U-SA-Ex-V-SC-MO	[22]	Epsilon constraint method (based on Benders decomposition)	25	5	3
DT-C-SA- En-SC-SO	[29]	Genetic algorithm	50	-	6
DT-C-SA-En-SC-MO	[33]	Epsilon constraint method	12	-	4
CT-C-SA-Ex-SC-MO	[34]	Robust optimization-TH method	8	3	3

4. Applications of multi-period HLP in the real world

This section presents applications of multi-period HLP in different industrial contexts. Application area, datasets, and real case studies are classified in **Table 4**. According to **Table 4**, the air industry and postal systems are the most studied application areas. In Refs. [13], [16]-[18], [22], [30], and [33], applications are in air transportation and in Refs. [12], [23]- [25], [28], and [29], applications are in postal systems. Also, in Refs. [27] and [31], applications are in inter-modal hub network design. The application area of Refs. [19] and [20] is related to plant location, and the application area of Refs. [21] and [34] is related to cargo delivery systems. In other applications (terminal location, public transportation, and multi-modal multi-period hub network design), only one research has been allocated to each application area (Refs. [10], [15], and [32], respectively).

Table 4. Applications of multi-period HLP

Application area	Dataset	Place	Reference
Terminal location	-	-	[10]
Public transportation	-	-	[15]
Postal systems	AP	Australia	[12]
Air transportation	CAB	U.S.	[16]
Air transportation	CAB-TR	U.S.-Turkey	[13]
Postal systems	AP	Australia	[24]
Plant location	-	-	[19]
Postal systems	AP	Australia	[28]
Biomass supply chain	-	U.S.	[31]
Air transportation	IAD	Iran	[30]
Postal systems	AP	Australia	[25]
Plant location	-	-	[20]
Airlines	CAB	U.S.	[17]
Cargo delivery system	-	Iran	[21]
Airlines	CAB	U.S.	[18]
Postal systems	AP	Australia	[23]
Inter-modal rail-road network	FAF4	U.S.	[27]
Airport-railway network	ARWN-CAB	Iran	[32]
Airlines	CAB	U.S.	[22]
Postal systems	AP	Australia	[29]
Air transportation	IAD	Iran	[33]
Cargo delivery system	TR	Turkey	[34]

5. Conclusions and future research

In this paper, a review of multi-period HLP was presented, and solution techniques and applications of this problem in the real world were studied. Based on published studies, some research fields, including multi-objective optimization, sustainability, continuous-time modeling, waiting for services (queuing theory), multi-period hub location on the plane or sphere, work on other forms of uncertainty, game theory, applications of multi-period hub location in global logistics systems, and development of efficient solution methods can be considered in future researches. Future trends in multi-period HLP are discussed as follows:

- In the context of multi-period HLP, only three models are multi-objective (Refs. [22], [33], and [34]). Considering various objective functions in the modeling process makes the problem closer to the real-world conditions, and it is necessary to use multi-objective optimization techniques for solving the problem.

- In multi-period HLPs, some strategic decisions have a considerable effect on the economy, environment, and society. Thus, it is necessary to consider sustainability aspects in modeling to reduce the negative environmental effects of decisions (such as air and noise pollution) and improve the social effects of the problem (such as employment and regional development). Only one research has been done in the context of sustainable multi-period hub location (Ref. [34]). This field is relatively new and needs further study by researchers.
- Continuous-time planning horizon has been considered in Refs. [10] and [34]. Other models are discrete-time. Continuous-time models are more real and require more attention from researchers.
- The waiting time of customers at hubs to get the services affects their decision. In real-world hub systems, the existence of queue is inevitable, and queuing theory can be used for modeling the waiting time of customers at hubs for serving the customers in less time and improving the level of service at hubs.
- Multi-period settings can be considered for other variants of the HLP, such as *p*-hub center location problem, incomplete HLP, and continuous HLP (HLP on a plane or sphere).
- In most studies, the parameters of the problem are considered deterministic. Given the uncertain nature of the multi-period hub location problem, considering other forms of fuzzy, probabilistic, and robust uncertainties and a combination of these uncertainties can help bring the problem model closer to real-world conditions.
- Multi-period HLP can be studied in the case that there is more than one system for routing demand flows. In this case, competitive and pricing models should be developed.
- The applications of multi-period HLP in real case studies such as healthcare systems and crisis management can be studied by researchers.
- Due to the high complexity of multi-period HLP, developing exact algorithms for this problem is not simple. Thus, heuristic and meta-heuristic algorithms, hybrid algorithms, and the extension of existing approaches can be applied to solve the problem.

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