Suppliers’ online channel structure strategies under product innovation effect and spillover effect

Jie Liu\textsuperscript{*}, Xiaoli Wu\textsuperscript{†}, Bai Yang\textsuperscript{‡} and Shanxue Yang\textsuperscript{§}

Abstract

In this paper, we investigate how spillovers from online sales to offline sales and product innovation jointly affect suppliers’ optimal online channel structure strategies. By comparing equilibrium outcomes of the game between a supplier, an offline retailer and an online retailer in different scenarios, including the scenario without product innovation, the scenario with exogenous product innovation and the scenario with endogenous product innovation, we obtain some novel management implications. There exists a threshold curve such that when the supplier’s marginal operating cost is below the threshold curve, the supplier is better off establishing a direct online channel, otherwise, the supplier should introduce an independent online channel. Nonetheless, the threshold curve is not a monotonic function of the spillover coefficient, but a function that decreases first and then increases with the the spillover coefficient. Exogenous product innovation does not change the supplier’s optimal online channel structure strategy qualitatively, it leads to some quantitative changes, shifting the threshold curve upward. However, endogenous product innovation changes the position and shape of the threshold curve significantly and gives the supplier the flexibility to establish the direct online channel. This paper reveals an underlying trade-off between online channel operational efficiency and channel coordination, providing suppliers managerial suggestions on online channel structure strategies.

Mathematics Subject Classification: 90B06.

Key words: game theory; dual-channel supply chain; spillovers; online channel structure; product innovation

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1 Introduction

BESTORE, the leading brand of snacks in China, attaches great importance to product research and development. In 2021, the R&D investment of BESTORE is 39.67 million yuan, an increase of 17.64% over the same period last year and a compound growth rate of 18.46% from 2018 to 2021. In 2021, BESTORE developed and mastered 19 advanced technologies in the field of food nutrition and health, including new puffing, plant meat production and processing, three reduction (reducing oil, sugar and salt), sugar-free, oil-free baking and microcapsule embedding, among which 13 technologies have been applied to newly launched products. BESTORE not only puts emphasis on product R&D, but also on brand image promotion. Through cross-border IP for product empowerment, BESTORE deepens the emotional connection with consumers. For example, for three consecutive years, BESTORE have joined hands with Dunhuang Research Institute to create a joint gift box of BESTORE and Dunhuang Research Institute. During the Spring Festival, BESTORE completed the emotional communication between brands and consumers through two advertising films, Good Products on the Table is the Year and Golden year open Golden Mouth, which respectively focused on the warmth of the Spring Festival and the social fear scene of the Spring Festival. These series of product innovation activities including product R&D and brand image promotion, make BESTORE in just 15 years to become one of the top snack brands in China.

BESTORE not only attaches importance to product innovation, but also to the development of online channels. BESTORE’s online channel business revenue has risen steadily in recent years. In 2021, the online revenue of BESTORE increased by 21.42% year on year. However, the expansion and development of online business has eroded BESTORE’s offline channels sales. In 2021, the offline revenue of BESTORE only increased by 16.31% year on year, which was significantly lower than the growth rate of online sales. Moreover, the proportion of offline sales of BESTORE is also declining year by year. In 2019, the proportion of offline income of BESTORE was 51.42%, more than half, but in 2020, the proportion was 49.32%, and in 2021, the proportion was 46.87%. These facts confirm the existence of the spillover from online sales to offline sales. Nonetheless, such spillover is also widely observed in other industries [5; 17; 39; 28; 20]. For example, empirical studies on apparel and computers suggest that sales in online distribution channels have a negative effect on sales in traditional physical channels [5; 17]. While in media and entertainment industries, such as movies and music, the e-channel not only gives access to new consumers but also has a strong stimulation effect on sales in the traditional channel [39; 28]. And for books, Hilton and Wiley [20] suggest that effect of e-channel sales on traditional channel can be positive but small. Therefore, when making decisions with regard to online channels, the supplier such as BESTORE
should take into account the spillover from online sales to offline sales.

When making online channel structure decision, BESTORE may introduce an independent online channel (IOC) or establishes a direct online channel (DOC). The IOC, such as Tmall or JD.com, has a higher channel efficiency due to professionalism and economies of scale, but it diverts channel profits. While the DOC has a lower channel efficiency, but is more conducive to channel coordination, and can reduce profit diversion. Whereas, in the presence of the spillover effect, product innovation activities may complicate the tradeoff for BESTORE on online channel structure. Product innovation can increase customer demand in online and offline channels, that is, the product innovation effect, which directly affects the equilibrium of the game between the supplier and the retailers, thus influencing the supplier’s online channel structure strategy. Furthermore, product innovation also has an impact on the spillover effect. On the one hand, product innovation amplifies the spillover effect because it boosts customer demand in online channels. On the other hand, product innovation mitigates the impact of the cannibalization spillover because it promotes customer demand in offline channels. However, it is not clear how the spillover effect and the product innovation effect jointly affect a supplier’s optimal online channel structure strategy.

Based on the above observation, in this paper, we intends to investigate the supplier’s optimal online channel structure strategy under spillover effect and product innovation effect. Concretely, we propose the following research questions. (1) How do the spillover and online channel structure affect the equilibrium outcomes of the game between the supplier and retailers? (2) What is the supplier’s optimal online channel strategy under the spillover effect? (3) What is the supplier’s optimal online channel structure strategy under the combined effect of exogenous product innovation and the spillover? (4) How does endogenous product innovation affect the supplier’s optimal online structure strategy?

To address the above research questions, we consider a two-tier dual-channel supply chain consisting of a monopolist supplier, an independent offline retailer and an independent online retailer, in which the supplier sells a product to offline consumers via the offline retailer and to online consumers via the online retailer or his own direct online channel. Nonetheless, the supplier has a disadvantage in operating online channels, compared with the online retailer, and needs to make decision on online channel structure. To expand product sales, the supplier innovates the product or the product image through product R&D, celebrity endorsement, IP empowerment, advertising and so on. Moreover, sales in the online channel have a spillover effect on sales in the offline channel, promoting or reducing offline sales. We model the following three scenarios. (1) Scenario without product innovation; (2) Scenario with exogenous product innovation; and
(3) Scenario with endogenous product innovation. Through comparing and analyzing equilibrium outcomes, we obtain the following novel managerial implications.

First, with the IOC, the spillover effect does not affect the equilibrium online product quantity, whereas a greater value of the spillover coefficient lowers the equilibrium offline product quantity. With the DOC, as the spillover coefficient increases, the equilibrium online product quantity decreases if the supplier’s marginal operating cost is low or medium, and first decreases then remains constant if the cost is high. Accordingly, as the spillover coefficient increases, the equilibrium offline product quantity decreases if the supplier’s marginal operating cost is low, and first decreases then increases if the cost is medium, and first decreases then increases and finally remains constant if the cost is high. This is because, as the decision-maker of the online product quantity, the supplier needs to consider the spillover effect from online sales to offline sales, thus reducing online product quantity to mitigate the cannibalisation spillover. Particularly, when the marginal operating cost is high and the spillover effect is cannibalistic, the supplier will abandon the DOC.

Second, there exists a threshold curve such that when the supplier’s marginal operating cost is below the threshold curve, the supplier is better off establishing the DOC, otherwise, the supplier is better off introducing the IOC. Nonetheless, the threshold curve is not a monotonic function of the spillover coefficient, but a function that decreases first and then increases with the spillover coefficient. This conclusion is determined by two countervailing forces: the operational disadvantage which promotes the supplier to introduce the IOC to improve the efficiency of online channels, and the spillover effect which drives the supplier to establish the DOC to better control the impact of online sales on offline sales. A stronger spillover effect, whether stimulative or cannibalistic, increases the supplier’s incentive to establish the DOC. Therefore, with the increase of spillover coefficient, the willingness of the supplier to establish the DOC first decreases and then increases. Moreover, the offline retailer and the supplier may have the same preference for online channel structure, thus leading to a win-win situation for the supplier and the offline retailer, and we present conditions under which they archive a win-win situation.

Third, exogenous product innovation does not change the supplier’s optimal online channel structure qualitatively. There still exists an threshold curve, such that when the marginal operating cost is below the threshold curve, the supplier is better off establishing the DOC, otherwise, the supplier should introduce the IOC. However, it leads to some quantitative changes. For example, the threshold curve shifts upward as the level of product innovation increases, which implies that product innovation increases the supplier’s incentive to establish the DOC.

Finally, in scenario with endogenous product innovation, the supplier’s optimal online channel
structure strategy is significantly different from that in scenario with exogenous product innovation. Endogenous product innovation endows suppliers with the ability to leverage product innovation to enhance the power of channel coordination. By strategically setting the product innovation level, the supplier can weaken the impact of online channel efficiency and amplify the impact of channel coordination power. As a result, endogenous product innovation endows the supplier the flexibility to establish the DOC. For the offline retailer, on the other hand, we find that the DOC would be more advantageous in almost every case. This shows that in the scenario with endogenous product innovation, the DOC can better alleviate channel friction between online and offline channels. Moreover, counterintuitively, when the the spillover is cannibalistic and the supplier’s marginal operating cost is high, the IOC structure leads to a higher quality level, compared with the DOC structure.

The contribution of this work is mainly threefold. First, spillovers from online sales to offline sales which are widely observed in various industries, along with the product innovation, affect suppliers’ online channel structure strategies. Thus, it is significant to examine the combined affect of spillovers and product innovation on suppliers’ online channel structure strategies. This paper contribute to the literature on dual-channel management by investigating how spillovers from online sales to offline sales and product innovation jointly affect suppliers’ online channel structure strategies. To the best of our knowledge, this paper is the first one to discuss such a problem in the literature. Second, this paper reveals an underlying trade-off between the online channel operation efficiency and channel coordination when suppliers make decisions on online channel structure, which can guide supplier to make optimal online channel structure decisions and to develop online retailing in future. Third, this study uncovers that endogenous product innovation endows suppliers with the ability to leverage product innovation to enhance the power of channel coordination, thus giving suppliers the the flexibility to establish the DOC. This founding emphasizes the importance of strategically setting the level of product innovation under the spillover effect, and provides guidance for supplier supply chain management.

The rest of the paper is organized as follows. In Section 2, related literature is reviewed briefly. Section 3 introduces the models. In Section 4, we derive the equilibrium outcomes to models and analyze the equilibrium outcomes. Section 5 explores the scenario with endogenous product innovation. Section 6 provides some numerical examples to check the validity of our theoretical results. Section 7 gives conclusions.
2 Literature review

This paper is related to the literature on the interaction between the online and offline channels. Previous literature considers a manufacturer establishing a direct online channel to complement the traditional offline channel, through investigates pricing strategies [13; 48; 9; 21; 8; 10; 26] or quantity strategies [2; 15; 19; 23; 24] to maximize the manufacturer’s profit. Some literature also considers a retailer setting up a direct online channel in addition to its traditional channel [4; 21; 19; 19], and examines pricing or quantity strategies to maximize the retailer’s profit. Beside optimizing a supply chain member’s profit, lots of work studies the cooperation of online and offline channels, proposing various types of coordination contracts [42; 3; 6; 29; 7; 10; 15]. These work highlights the impact of pricing/quantity strategies on the interaction between the online and offline channels, however, the impact of non-price features are not taken into consideration. Different from the above literature, this paper incorporates the product innovation into the study on dual-channel supply chain management. We investigate both the impact of exogenous product innovation and that of endogenous product innovation on suppliers’ online channel structure decisions.

Since this paper considers the impact of product innovation on suppliers’ online channel structure decisions, it is also related to literature on dual-channel supply management that considers the impact of non-price features. A large body of literature on dual-channel supply management considers the impact of non-price features. For example, Dumrongsiri et al. [16] study a dual-channel supply chain in which a manufacturer sells product to consumers via a retailer and a direct channel, in which consumers make purchase decisions based on price and service qualities. Chen et al. [12] study a manufacturer’s channel choice decision and pricing decision, when his direct online sales channel competes with a third-party physical retail channel in service. Ofek et al. [33] study competing retailers that can operate both online and offline channels and investigate how pricing strategies and physical store assistance levels change as a result of the additional online channel. Dan et al. [14] evaluate the impacts of retail services and the degree of customer loyalty to the retail channel on the manufacturer and retailer’s pricing behaviors. Xia et al. [45] study the distribution channel decision of a manufacturer who considers whether to add an direct online channel to its independent traditional retailer who faces the opportunity invests in store assistance to help consumers purchase. Ha et al. [18] investigate a supply chain in which the manufacturer establish a direct channel and product quality is endogenous and customers have heterogeneous preferences for quality. They find that direct channel establishment always makes the retailer worse off in a large variety of scenarios. Chen et al. [11] investigate price and quality decisions in dual-channel supply chains and demonstrate that quality improvement can be realized when a new channel is introduced.
and the supply chain performance could be improved due to a new channel augmented. Pal and Sarkar [35] examine the effect of green improvement in a dual-channel supply chain consisting of two manufacturers and a retailer. Sarkar and Pal [36] investigate pricing and service strategies in a dual-channel supply chain under return-refund policy. Other interesting work includes the studies on dual-channel closed-loop supply chain ([35; 34]). The above literature considers the impact of non-price features, such as product quality, green improvement and service quality on the channel decision in the context of dual-channel supply chain. Similarly, this paper considers a non-price feature, product innovation, in a dual-channel supply chain setting. Differently, we take into account both the product innovation effect and the spillover effect in a dual-channel supply chain, and investigates the combined impact of the spillover effect and product innovation effect on suppliers’ online channel structure strategies. To the best of our knowledge, this paper is the first one to discuss such an issue in the literature.

This paper is also related to the literature on dual-channel supply management that considers the impact of spillover effect from online sales to offline sales. Abhishek et al. [1] discuss a online retailer’s format preference between the reseller and the marketplace, and show that when sales in the online channel lead to a negative effect on demand in the traditional channel, the online retailer prefers agency selling, whereas when sales in the online channel lead to substantial stimulation of demand in the traditional channel, the online retailer prefer reselling. Yan et al. [46] investigate whether and under what conditions the marketplace channel should be introduced in addition to the reseller channel by measuring the combined effects of the online spillover, the platform fee and the manufacturer’s retailing inefficiency. Nie et al. [30] study two traditional brick-and-mortar retailers’ distribution strategies, who consider adding a new online channel to supplement their retail channels, with taking into account the cross-channel spillover effect. This paper differs from the above paper in that we consider the impact of product innovation, including the impact from exogenous product innovation and the impact from endogenous product innovation which is rarely considered in the prior literature, and explore the combined effect of spillovers and product innovation on suppliers’ online channel structure decisions.

3 Models

We consider a two-tier dual-channel supply chain consisting of a monopolist supplier (he), an offline retailer (she) and an online retailer (it), in which the supplier sells a product to offline consumers via the offline retailer and to online consumers via the online retailer or his own online channel. We employ an analytical game model which is developed according to a representative
customer’s utility. Following Singh and Vives [38], the customers’ utility function can be formulated as follows,

\[ U(q_1, q_2) = a_1 q_1 + a_2 q_2 - (b_1 q_1^2 + 2\tau q_1 q_2 + b_2 q_2^2)/2 - p_1 q_1 - p_2 q_2, \]

where \( a_i \) stands for the basic market demand in channel \( i \), \( q_i \) is the selling quantity in channel \( i \), and \( b_i \) denotes the rate of change of marginal utility in channel \( i \), \( i = 1, 2 \), where \( i = 1 \) represents the offline channel and \( i = 2 \) represents the online channel. Moreover, parameter \( \tau \) is the intensity of competition. By solving the first-order conditions of \( U(q_1, q_2) \), we obtain the inverse demand functions, \( p_1 = a_1 - b_1 q_1 - \tau q_2 \), \( p_2 = a_2 - b_2 q_2 - \tau q_1 \). Following prior literature (e.g., [41; 44; 31]), we assume that \( a_1 = a_2 = 1, b_1 = b_2 = 1 \). Thus, the inverse demand functions are

\[ p_1 = 1 - q_1 - \tau q_2, \quad p_2 = 1 - q_2 - \tau q_1. \]

(3.1)  (3.2)

Due to the rapid development of e-commerce, online sales have a significant impact on offline sales. For example, in China’s book market, the proportion of online sales increased from 28% in 2012 to 64.1% in 2018 (Iresearch, 2019). On the other hand, due to the convenience of online shopping, consumers’ shopping habits are slowly shifting from physical store shopping to online shopping, which also leads to the impact of online sales on offline sales is significantly higher than that of offline sales on online sales. Therefore, we assume that \( \tau = 0 \) in Equation (3.2). Similar assumption is widely used in previous literature (e.g., [43; 1; 32]). Consequently, the inverse demand function in the online channel is given by

\[ p_2 = 1 - q_2. \]

(3.3)

Moreover, we allow that \( \tau \) in Equation (3.1) can be a negative, which implies a stimulation across-channel spillover from online sales to offline sales. Specifically, \( \tau < 0 \) captures a positive across-channel effect (stimulation spillover) from online sales to offline sales, i.e., every sale in the online channel leads to \( \tau \) units of increased sales in the offline channel. The factor driving this increase could be word-of-mouth, complementarity of the products sold in two channels and so on. For example, when a consumer bought an e-book on Apple’s iBook store or Amazon Kindle, he may recommend it to his peers, who may become interested and buy a print version at a physical store [20]. On the other hand, \( \tau > 0 \) captures a negative cross-channel effect (cannibalisation spillover) from online sales to offline sales, i.e., every sale in the online channel leads to \( \tau \) units of decreased sales in the offline channel. Such cannibalisation spillover can be explained by quantity competition. Similar assumptions are widely adopted in the prior literature (e.g., [1; 46; 30]).
We consider two scenarios, scenario without product innovation and scenario with exogenous product innovation. In this paper, product innovation refers to all the means provided by suppliers to improve the value of products, including product R&D, brand image promotion activities, such as star endorsement, IP empowerment, advertising and so on. Product innovation expands product demand, following prior literature (e.g., [40; 25; 47]), we assume that the basic demand is expanded to be 1 + $\theta$, where $\theta$ represents the product innovation level. For each scenario, the supplier can either introduce the IOC or establishes a DOC (see Figure 1). Hence, there are four cases:

(1) Case i. The supplier produces a product without product innovation and introduces the IOC. Then, the supplier’s, the offline retailer’s and the online retailer’s profit are as follows,

$$\pi_s = w (q_1 + q_2), \quad (3.4)$$
$$\pi_1 = (1 - q_1 - \tau q_2 - w) q_1, \quad (3.5)$$
$$\pi_2 = (1 - q_2 - w) q_2. \quad (3.6)$$

Here, $w$ represents wholesale price charged to the two retailers.

(2) Case d. The supplier produces a product without product innovation and establishes the DOC. Then, the supplier’s and the offline retailer’s profit are given by

$$\pi_s = w q_1 + (1 - q_2 - c) q_2 \quad (3.7)$$
$$\pi_1 = (1 - q_1 - \tau q_2 - w) q_1. \quad (3.8)$$

Here, $c$ represents the supplier’s marginal operating cost of the direct online channel. Note that the online retailer’s marginal operating cost is normalized to zero. Such assumption can capture the operational disadvantages of the supplier.

Figure 1: Online channel structure
(3) Case I. The supplier innovates the product and introduces the IOC. Then, the inverse demand functions are given by

\[ p_1 = 1 - q_1 + \tau q_2 + \theta, \]  
\[ p_2 = 1 - q_2 + \theta. \]  

Consequently, the supplier’s, the offline retailer’s and the online retailer’s profit are as follows,

\[ \pi_s = w (q_1 + q_2), \]  
\[ \pi_1 = (1 - q_1 + \tau q_2 + \theta - w)q_1, \]  
\[ \pi_2 = (1 - q_2 + \theta - w)q_2. \]  

(4) Case D. The supplier innovates the product and establishes the DOC. Then, the supplier’s and the offline retailer’s profit are given by

\[ \pi_s = w q_1 + (1 - q_2 + \theta - c)q_2, \]  
\[ \pi_1 = (1 - q_1 + \tau q_2 + \theta - w)q_1. \]  

The event sequences are as follows (see Figure 2).

(1) The supplier makes decision on online channel structure.

(2) If the supplier introduces the IOC, the supplier decides the wholesale price charged to the two retailers. Then, based on the supplier’s decision, the offline retailer and the online retailer decide their product quantities, simultaneously. If the supplier establishes the DOC, the supplier decides the wholesale prices charged to the offline retailer and the online channel product quantity. Then, based on the supplier’s decisions, the offline retailer decides the offline channel product quantity.

Figure 2: The event sequences

In Section 5, we will investigate the scenario with endogenous product innovation.

The notations used in this paper are summarized in Table 1.
Table 1: Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_1$</td>
<td>Product quantity in the offline channel</td>
</tr>
<tr>
<td>$q_2$</td>
<td>Product quantity in the online channel</td>
</tr>
<tr>
<td>$w$</td>
<td>Wholesale price</td>
</tr>
<tr>
<td>$p_1$</td>
<td>Retail price in the offline channel</td>
</tr>
<tr>
<td>$p_2$</td>
<td>Retail price in the online channel</td>
</tr>
<tr>
<td>$\pi_1$</td>
<td>Profit of the offline retailer</td>
</tr>
<tr>
<td>$\pi_2$</td>
<td>Profit of the online retailer</td>
</tr>
<tr>
<td>$\pi_s$</td>
<td>Profit of the supplier</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Product innovation level</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Spillover effect coefficient</td>
</tr>
<tr>
<td>$c$</td>
<td>Supplier’s marginal operating cost of the direct online channel</td>
</tr>
</tbody>
</table>

4 Analysis

In this section, we first consider scenario without product innovation to investigate the impact of spillover on the supplier’s online channel structure strategy, then scenario with product innovation to examine the combined impact of the spillover effect and product innovation on the supplier’s online channel structure strategy. We use superscripts $j$ to denote the equilibrium outcomes of case $j$, where $j \in \{i, d, I, D\}$.

4.1 Scenario without product innovation

In this subsection, we investigate the scenario without product innovation. We start by presenting the equilibrium outcomes which are derived by back induction. The following two lemmas provide equilibrium outcomes in cases $i$ and $d$, respectively.

Lemma 4.1. The equilibrium outcomes in Case $i$ are given by

\[
\begin{align*}
     w^i &= \frac{1}{2}, \\
     q_1^i &= \frac{2-\tau}{8}, \\
     q_2^i &= \frac{1}{4}, \\
     \pi_s^i &= \frac{4-\tau}{16}, \\
     \pi_1^i &= \left(\frac{2-\tau}{8}\right)^2, \\
     \pi_2^i &= \frac{1}{16}.
\end{align*}
\]

Lemma 4.2. The equilibrium outcomes in Case $d$ are given as follows.

(a) When $c \in \left(0, \frac{3}{4}\right)$, or $c \in \left[\frac{3}{4}, 1\right)$ and $\tau \in (-1, 4(1-c))$, $w^d = \frac{2(c-\tau+\frac{2}{3})}{8-\tau^2}, 
     q_1^d = \frac{\tau^2-\tau-2}{\tau^2}, 
     q_2^d = \frac{4c+\tau-4}{\tau^2-8}, 
     \pi_s^d = \frac{c\tau-4c+2c^2+3}{8-\tau^2}, 
     \pi_1^d = \frac{(c\tau-\tau+2)^2}{(\tau^2-8)^2}.

(b) When $c \in \left[\frac{3}{4}, 1\right)$ and $\tau \in [4(1-c), 1), 
     w^d = \frac{1}{2}, 
     q_1^d = \frac{1}{4}, 
     q_2^d = 0, 
     \pi_s^d = \frac{1}{8}, 
     \pi_1^d = \frac{1}{16}. 

Lemmas 4.1 and 4.2 imply that, the online retailer has an incentive to enter the market due to its operational advantage. Whereas, when the suppler’s operational disadvantage is significant and the spillover effect is cannibalistic, the supplier may abandon the DOC. This is because, under the cannibalisation spillover, when the marginal operating cost is large, revenue generated from the online channel may be not enough to compensate for the loss in the offline sales. Thus, the supplier has no incentive to establish the DOC.

The following corollary examines how the online channel structure and the spillover effect affect the equilibrium product quantity decisions.

**Corollary 4.3.** (a) With the IOC, \( \frac{\partial q_2}{\partial \tau} = 0, \frac{\partial q_1}{\partial \tau} < 0. \)

(b) With the DOC,

(b.1) when \( c \in (0, \frac{3}{4}), \frac{\partial q_1}{\partial \tau} < 0; \) when \( c \in [\frac{3}{4}, 1), \frac{\partial q_1}{\partial \tau} < 0 \) if \( \tau \in (-1, \tilde{\tau}), \) and \( \frac{\partial q_1}{\partial \tau} = 0 \) if \( \tau \in (\tilde{\tau}, 1). \)

(b.2) When \( c \in (0, \frac{5}{9}], \frac{\partial q_1}{\partial \tau} < 0; \) when \( c \in (\frac{5}{9}, \frac{3}{4}), \frac{\partial q_1}{\partial \tau} < 0 \) if \( \tau \in (-1, \tilde{\tau}), \) and \( \frac{\partial q_1}{\partial \tau} > 0 \) if \( \tau \in (\tilde{\tau}, \hat{\tau}), \) and \( \frac{\partial q_1}{\partial \tau} = 0 \) if \( \tau \in (\hat{\tau}, 1). \)

Here, \( \tilde{\tau} = \frac{2\sqrt{-2c^2 + 4c - 1} - 2}{c - 1} \) and \( \hat{\tau} = 4(1 - c). \)

Corollary 4.3 (a) shows that with the IOC, as the spillover coefficient increases, the offline channel product quantity decreases but the online channel product quantity remains unchanged. As the spillover coefficient increases, either the stimulation spillover is weakened or the cannibalisation spillover is enhanced. The offline retailer thus has to reduce product quantity to avoid more losses. Whereas, since the spillover effect discussed is unidirectional, the online retailer does not need to consider the impact of the spillover effect, the online channel product quantity thus is not affected by the spillover effect.

Corollary 4.3 (b) demonstrates that with the DOC, as the spillover coefficient increases, the online channel product quantity decreases if the supplier’s marginal operating cost is low or medium, and first decreases then remains constant if the cost is high. Unlike the online retailer, as the decision-maker of the online channel product quantity, the supplier needs to consider the spillover effect from online sales to the offline sales. As the spillover coefficient increases, the supplier will reduce the online product quantity to mitigate the negative impact on offline sales to maximize his total profit. Whereas, when the marginal operating cost is high and the spillover effect is cannibalistic, as showed in Lemma 4.2, the supplier will abandon the DOC, and the online product quantity will remain zero. Moreover, as the spillover coefficient increases, the offline product quantity decreases if the supplier’s marginal operating cost is low, and first decreases then increases...
if the cost is medium, and first decreases then increases and finally remains constant if the cost is large. As the spillover coefficient increases, the offline retailer will reduce product quantity to avoid more losses. However, when the supplier’s marginal cost is medium or high, the supplier gains a low marginal profit from online sales, thus has strong incentive to expand offline sales through reducing online product quantity when the spillover is cannibalistic. In this case, the offline product quantity will increase with the spillover coefficient. Particularly, when the marginal cost is high and the spillover is strong cannibalistic, as mentioned above, the supplier will abandon the DOC and the offline product quantity remains unchanged.

Figure 3 presents the product quantity curves for some representative settings. With the DOC, we present the the online channel product quantity curves of $c = 0.4$ and $c = 0.9$, and the the offline channel product quantity curves of $c = 0.4$, $c = 0.7$ and $c = 0.9$. From Figure 3, we can further find that with the IOC or DOC, both the offline product quantity and the online product quantity decrease with the marginal operating cost. This is because a larger marginal operating cost leads to a higher price for the online channel, thus reducing the online product quantity. While, due to the spillover effect, the offline product quantity also decreases.

![Figure 3: Product quantity curves in the scenario without product innovation.](image)

The following corollary investigates how the online channel structure and the spillover effect affect the equilibrium profits.

**Corollary 4.4.** (a) With the IOC, $\frac{\partial \pi_i}{\partial \tau} < 0$, $\frac{\partial \pi_i}{\partial \tau} < 0$ and $\frac{\partial \pi_i}{\partial \tau} = 0$.

(b) With the DOC,

(b.1) when $c \in (0, \frac{3}{4})$, $\frac{\partial \pi_d}{\partial \tau} < 0$; when $c \in (\frac{3}{4}, 1)$, $\frac{\partial \pi_d}{\partial \tau} < 0$ if $\tau \in (-1, \hat{\tau})$, and $\frac{\partial \pi_d}{\partial \tau} > 0$ if
Corollary 4.4 (a) indicates that with the IOC, both the offline retailer’s and the supplier’s profit decrease with the spillover coefficient, but the online retailer’s profit remains unchanged. As indicated in Corollary 4.3, as the spillover coefficient increases, the online channel product quantity remains constant and the offline channel product quantity decreases, thus, the online retailer’s profit holds unchanged, and both the offline retailer’s and the supplier’s profit decrease.

Corollary 4.4 (b) demonstrates that with the DOC, as the spillover coefficient increases, the offline retailer’s profit decreases if the supplier’s marginal operating cost is low, and first decreases then increases if the cost is medium, and first decreases then increases and finally remains constant if the cost is high. These results are consistent with Corollary 4.3, since the offline retailer’s profit comes from offline sales merely. Whereas, as the spillover coefficient increases, the supplier’s profit decreases if the supplier’s marginal operating cost is low or medium, and first decreases then remains constant if the cost is high. Note that the supplier’s profit curves are inconsistent with the online channel product quantity curves. This is because the supplier’s profit comes from not only the online channel but also the offline channel.

In Figure 4, we present profit curves for some representative settings. With the DOC, we present the supplier’s and offline retailer’s profit curves of \( c = 0.2, c = 0.6 \) and \( c = 0.9 \). As a benchmark, we also provide the supplier’s and offline retailer’s profit curves in the case with the IOC. From Figure 4(a), we find that with the DOC, the supplier’s profit decreases with its marginal operating cost, that is, a lower marginal operating cost leading to a higher profit. Figure 4(b) demonstrates that a high marginal operating cost of the supplier may benefit the offline retailer if the spillover is cannibalistic and may hurt the offline retailer if the spillover is stimulative.

In the following proposition, we compare the supplier’s profit and the offline retailer’s profit under the IOC structure with those under the DOC structure.

**Proposition 4.5.** (a) For the supplier,
\[
(a.1) \text{ when } c \in (0, 1 - \frac{\sqrt{2} \sqrt{(\tau^2 - 8)(\tau - 2)}}{8} - \frac{\tau}{4}), \pi_s^d \geq \pi_s^i; \\
(a.2) \text{ when } c \in (1 - \frac{\sqrt{2} \sqrt{(\tau^2 - 8)(\tau - 2)}}{8} - \frac{\tau}{4}, 1), \pi_s^d < \pi_s^i.
\]
(b) For the offline retailer,
\[
(b.1) \text{ when } \tau \geq 0, \pi_1^d \geq \pi_1^i; \\
(b.2) \text{ when } \tau < 0, \pi_1^d \geq \pi_1^i \text{ if } c \in (0, \frac{\tau^2 - 2\tau}{8}], \text{ and } \pi_1^d < \pi_1^i \text{ if } c \in (\frac{\tau^2 - 2\tau}{8}, 1).
\]
Proposition 4.5 (a) reveals that there exists a threshold curve such that when the marginal operating cost is below the threshold curve, the supplier is better off establishing the DOC, otherwise, the supplier is better off introducing the IOC (see Figure 5). Intuitively, when the marginal operating cost is low, the supplier can obtain a higher marginal profit from the DOC, thus having more incentives to establish the DOC. Nonetheless, the threshold curve is not a monotonic function of the spillover coefficient, but a function that decreases first and then increases with the spillover coefficient. This conclusion is determined by two countervailing forces: the operational disadvantage which promotes the supplier to introduce the IOC to improve the efficiency of online channels, and the spillover effect which drives the supplier to establish the DOC to better control the impact of online sales on offline sales. A stronger spillover effect, whether stimulative or cannibalistic, increases the supplier’s incentive to establish the DOC. Therefore, with the increase of spillover coefficient, the willingness of the supplier to establish the DOC first decreases and then increases.

Proposition 4.5 (b) examines the offline retailer’s preference for online channel structure. In the presence of stimulation spillover, the DOC structure is more favorable to the offline retailer when the supplier’s marginal operating cost is low and is more unfavorable to the offline retailer when the cost is high, compared with the IOC structure. The result is owing to the tension between the operational efficiency advantage of the online retailer and the channel adjustment power of the supplier. The channel adjustment power infers to the ability of the supplier to coordinate the supply chain through the DOC. When the operational disadvantage of the supplier is not significant, that is, supplier’s marginal operating cost is low, the the channel adjustment power
of the supplier dominates the operational efficiency advantage of the online retailer. In this case, DOC is beneficial to the retailer. By contrast, when the operation disadvantage of the supplier is significant, that is, supplier’s marginal operating cost is high, the channel adjustment power of the supplier is dominated by the operational efficiency advantage of the online retailer. Therefore, IOC is more beneficial to offline retailers. Whereas in the presence of cannibalisation spillover, the DOC structure is always more favorable to the offline retailer than the IOC structure. In this case, the improvement of online channel efficiency will damage offline sales, while channel adjustment of the supplier will effectively protect offline sales. Therefore, the DOC is always more beneficial to the offline retailer.

Figure 5: The supplier’s and the offline retailer’s preference over IOC with DOC.

Figure 5 graphically depicts the supplier’s and the offline retailer’s preference over IOC with DOC. A two-dimensional vector \((i, j)\) is used to represent the preferences of the supplier and the offline retailer with regard to online channel structure, where \(i, j \in \{IOC, DOC\}\), \(i\) represents the supplier’s preference, and \(j\) represents the offline retailer’s preference. From Figure 5, we find that when the supplier’s marginal operating cost is low and the stimulation spillover is strong (Region C), or when the supplier’s marginal operating cost is small and the spillover is cannibalistic (Region E), the offline retailer and the supplier has the same preference for online channel structure, that is, the DOC structure can leads to a win-win situation. Whereas when the supplier’s marginal operating cost is large and the spillover is stimulative (Region A), the IOC structure can leads to a win-win situation for the supplier and the offline retailer. Otherwise, the offline retailer will suffer (Regions B and D).
4.2 Scenario with exogenous product innovation

In this subsection, we consider the scenario with exogenous product innovation to examine the impact of product innovation on equilibrium outcomes and the supplier’s optimal online channel structure strategy. We first present the equilibrium outcomes in cases I and D.

Lemma 4.6. The equilibrium outcomes in Case I are given by

\[ w^I = \frac{1+\theta}{2}, \quad q_1^I = \frac{(2-\tau)(\theta+1)}{8}, \quad q_2^I = 1+\theta, \quad \pi_s^I = \frac{4(1-\tau)(\theta+1)^2}{64}, \quad \pi_1^I = \frac{(\theta-2)^2(\theta+1)^2}{64}, \quad \pi_2^I = \frac{(\theta+1)^2}{64}. \]

Lemma 4.7. The equilibrium outcomes in Case D are as follows.

(a) When \( c \in (0, \frac{3}{4}) \), or \( c \in [\frac{3}{4}, 1) \) and \( \theta > \frac{4}{3}c - 1 \), or \( c \in [\frac{3}{4}, 1) \) and \( \theta < \frac{4}{3}c - 1 \) and \( \tau \in (-1, \hat{\tau}) \),

\[ w^D = -\frac{2(2\theta - \tau + c + \tau - \theta + 2)}{\tau^2 - 8}, \quad q_1^D = -\frac{2\theta - \tau + c + \tau - \theta + 2}{\tau^2 - 8}, \quad q_2^D = 4\tau + 4\theta + \theta - 4, \quad \pi_1^D = \frac{2\theta - \tau + c + \tau - \theta + 2}{(\tau^2 - 8)^2}, \quad \pi_2^D = \frac{(1+\theta)^2}{64}. \]

(b) When \( c \in [\frac{3}{4}, 1) \) and \( \theta < \frac{4}{3}c - 1 \) and \( \tau \in (\hat{\tau}, 1) \), \( w^D = \frac{1+\theta}{2}, \quad q_1^D = \frac{1+\theta}{4}, \quad q_2^D = 0, \quad \pi_s^D = \frac{(1+\theta)^2}{8}. \)

Here, \( \hat{\tau} = 4(1 - \frac{c}{1+\theta}) \).

Lemmas 4.6 and 4.7 provide equilibrium outcomes in the scenario with exogenous product innovation. Similar to the scenario without product innovation, the online retailer has an incentive to enter the market. However, differently, in the context with the cannibalisation spillover and product innovation, the supplier still has an incentive to establish the DOC when the product innovation level is high. In other words, product innovation provides the supplier an additional incentive to establish the DOC. Additionally, if the product innovation level is low, the supplier may abandon the idea of establishing the DOC.

In Corollary 4.8, we examine how exogenous product innovation changes the impact of the online channel structure and the spillover effect on equilibrium product quantities.

Corollary 4.8. (a) With the IOC, \( \frac{\partial q_1^D}{\partial \tau} = 0 \) and \( \frac{\partial q_1^D}{\partial \tau} < 0 \).

(b) With the DOC,

(b.1) when \( c \in (0, \frac{3}{4}) \), or \( c \in [\frac{3}{4}, 1) \) and \( \theta > \frac{4}{3}c - 1, \quad \frac{\partial q_1^D}{\partial \tau} < 0 \). When \( c \in [\frac{3}{4}, 1) \) and \( \theta < \frac{4}{3}c - 1, \quad \frac{\partial q_1^D}{\partial \tau} < 0 \).

(b.2) When \( c \in (0, \frac{3}{4}) \), \( -\frac{\partial q_1^D}{\partial \tau} < 0 \) for \( \tau \in (-1, 1) \). When \( c \in [\frac{3}{4}, 1) \), and \( \theta > \frac{4}{3}c - 1, \quad \frac{\partial q_1^D}{\partial \tau} < 0 \) for \( \tau \in (-1, 1) \). When \( c \in [\frac{3}{4}, 1) \) and \( 0 < \theta < \frac{4}{3}c - 1, \quad \frac{\partial q_1^D}{\partial \tau} < 0 \) if \( \tau \in (-1, \hat{\tau}) \), and \( \frac{\partial q_1^D}{\partial \tau} > 0 \) if \( \tau \in (\hat{\tau}, 1) \).

When \( c \in [\frac{3}{4}, 1) \) and \( 0 < \theta < \frac{4}{3}c - 1, \quad \frac{\partial q_1^D}{\partial \tau} < 0 \) if \( \tau \in (-1, \hat{\tau}) \), and \( \frac{\partial q_1^D}{\partial \tau} > 0 \) if \( \tau \in (\hat{\tau}, 1) \). When \( c \in [\frac{3}{4}, 1) \) and \( \theta < \frac{4}{3}c - 1, \quad \frac{\partial q_1^D}{\partial \tau} < 0 \) if \( \tau \in (-1, \hat{\tau}) \), and \( \frac{\partial q_1^D}{\partial \tau} > 0 \) if \( \tau \in (\hat{\tau}, 1) \).

Here, \( \hat{\tau} = 4(1 - \frac{c}{1+\theta}) \) and \( \hat{\tau} = \frac{2\theta - 2\sqrt{-2\theta^2 + 4\theta + 4\theta - \theta^4 - 2\theta + 1}}{3\theta - \theta^2 - 1} \).
Corollary 4.8 demonstrates, with the IOC, exogenous product innovation does not distort the results in Corollary 4.3. However, with the DOC, exogenous product quality enhancement leads to some quantitative changes. Recall that in scenario without product innovation, the offline product quantity decreases only if the supplier’s marginal operating cost is low. In scenario with exogenous product innovation, when the product innovation level exceeds a threshold, the offline channel product quantity always decreases with the spillover coefficient, regardless of the supplier’s marginal operating cost. This is because when the product innovation level is high, the demand-boost effect of product innovation is dominant over the spillover effect. Due to the demand-boost effect of product innovation, the supplier can gain a high marginal profit form online sales, thus has less incentive to expand the offline sales. Thus, the offline channel product quantity in always decreases with the spillover coefficient. By contrast, when the product innovation level is low, the spillover effect is dominant over the demand-boost effect of product innovation. As indicated in Corollary 4.3, as the spillover coefficient increases, the offline product quantity may first decrease then increase, or first decrease then remain constant.

Figure 6 provides graphical illustrations of Corollary 4.8 (b) with Figure 6(a) presenting the online product quantity curves in some representative contexts ($c = 0.4, \theta = 0.1$, $c = 0.4, \theta = 0.3$, $c = 0.9, \theta = 0.1$ and $c = 0.9, \theta = 0.3$) and Figure 6, (b) (c) and (d) presenting the offline product quantity curves in some representative contexts ($c = 0.4$, $c = 0.7$ and $c = 0.9$). For each given marginal operating cost, we provide the offline product quantity curves with different product quality enhancement level ($\theta = 0.1$, $\theta = 0.3$ and $\theta = 0.7$). From Figure 6, we find that in each structure (IOC or DOC), both offline product quantity and online product quantity increase with the product quality enhancement level. This result is intuitive, product innovation promotes demand, thus increasing product quantity in each channel.

The following corollary investigates how the exogenous product innovation changes the impact of the online channel structure and the spillover effect on equilibrium profits.

**Corollary 4.9.** (a) With the IOC, $\frac{\partial s^I}{\partial \tau} < 0$, $\frac{\partial n^I}{\partial \tau} < 0$ and $\frac{\partial \sigma^I}{\partial \tau} = 0$.

(b) With the DOC,

(b.1) when $c \in (0, \frac{\theta}{3})$, or $c \in \left[\frac{3}{4}, 1\right]$ and $\theta > \frac{4}{3}c - 1$, $\frac{\partial n^P}{\partial \tau} < 0$. When $c \in \left[\frac{3}{4}, 1\right]$ and $\theta < \frac{4}{3}c - 1$, $\frac{\partial s^P}{\partial \tau} < 0$ if $\tau \in (-1, \tau)$, and $\frac{\partial \sigma^P}{\partial \tau} = 0$ if $\tau \in (\tau, 1)$.

(b.2) When $c \in (0, \frac{\theta}{5})$, $\frac{\partial n^P}{\partial \tau} < 0$ for $\tau \in (-1, 1)$. When $c \in \left[\frac{5}{9}, \frac{5}{3}\right]$, and $\theta < \frac{9}{5}c - 1$, $\frac{\partial s^P}{\partial \tau} < 0$ for $\tau \in (-1, \tau)$, and $\frac{\partial \sigma^P}{\partial \tau} > 0$ if $\tau \in (\tau, 1)$. When $c \in \left[\frac{5}{9}, \frac{5}{3}\right]$ and $0 < \theta < \frac{9}{5}c - 1$, $\frac{\partial s^P}{\partial \tau} < 0$ if $\tau \in (-1, \tau)$, and $\frac{\partial \sigma^P}{\partial \tau} = 0$ if $\tau \in (\tau, 1)$.
Corollary 4.9 demonstrates with the IOC, exogenous product innovation does not change the result in Corollary 4.4. However, differently, with the DOC, exogenous product innovation leads to some quantitative changes. To be specific, when the product innovation level exceeds a threshold, both the supplier’s and the offline retailer’s profits always decrease with the spillover coefficient. As indicated in Corollary 4.8, in this situation, the demand-boost effect of product quality enhancement is dominant over the spillover effect. As the spillover coefficient increases, offline channel product quantity decreases. Therefore, both the supplier’s and the offline retailer’s profits decrease. By contrast, when the product innovation level is low, spillover effect is dominant over the demand-boost effect of product innovation. As mentioned in Corollary 4.8, as the spillover coefficient increases, offline channel product quantity first decreases then increases. Consequently, both the supplier’s and the offline retailer’s profits first decrease then increase.

Figure 7 provides an graphic illustration of Corollary 4.9, with Figure 7(a) providing the supplier’s profit curves in some representative contexts ($c = 0.4, \theta = 0.1$, $c = 0.9, \theta = 0.1$ and $c = 0.9, \theta = 0.3$) and Figure 7, (b) (c) and (d) providing the offline retailer profit curves in some
representative contexts \( (c = 0.4, c = 0.7 \text{ and } c = 0.9) \). For each given marginal operating cost, we provide offline product quantity curves with different product innovation level \( (\theta = 0.1, \theta = 0.3 \text{ and } \theta = 0.7) \). From Figure 7, we find that in each structure (IOC or DOC), both supplier’s profit and the online retailer’s profit increase with the product quality enhancement level. Intuitively, product innovation promotes demand, thus increasing product quantity in each channel and raising both the supplier’s and the offline retailer’s profit.

In Proposition 4.10, we compare the supplier’s profit and the offline retailer’s profit under the IOC structure with those under the DOC structure.

**Proposition 4.10.** (a) For the supplier,

(a.1) When \( c \in \left(0, (1 + \theta) \left(1 - \frac{\sqrt{2}}{8} \sqrt{(\tau^2 - 8)(\tau - 2)} - \frac{\tau}{4}\right)\right) \), \( \pi_s^D \geq \pi_s^I \).

(a.2) When \( c \in \left((1 + \theta) \left(1 - \frac{\sqrt{2}}{8} \sqrt{(\tau^2 - 8)(\tau - 2)} - \frac{\tau}{4}\right), 1\right) \), \( \pi_s^D < \pi_s^I \).

(b) For the offline retailer,

(b.1) when \( \tau \geq 0, \pi_r^D \geq \pi_r^I \);
when $\tau < 0$, $\pi_1^d \geq \pi_1^i$ if $c \in \left(0, \frac{(\tau^2 - 2\tau)(1 + \theta)}{8}\right]$, and $\pi_1^d < \pi_1^i$ if $c \in \left(\frac{(\tau^2 - 2\tau)(1 + \theta)}{8}, 1\right)$.

Figure 8: The supplier’s and the offline retailer’s preference over IOC with DOC in scenario with exogenous product quality enhancement.

To better illustrate the impact of exogenous product innovation on the online channel structure preferences of the supplier and the offline retailer, we provide Figure 8 as an illustration of Proposition 4.10, where the solid line corresponds to case without exogenous product quality enhancement, the dotted solid line represents the case with exogenous product quality enhancement level of $\theta = 0.1$, and the dotted line represents the case with exogenous product quality enhancement level of $\theta = 0.2$.

Proposition 4.10 and Figure 8 reveal that exogenous product innovation does not change the result in Proposition 4.5 qualitatively. For the supplier, there still exists an threshold curve, such that when the marginal operating cost is below the threshold curve, the supplier is better off establishing the DOC, otherwise, the supplier is better off introducing the IOC. However, it leads to some quantitative changes. Visually, as the level of product innovation increases, Regions A and B shrink, Regions C and E expand, and Region D obliquely shifts upward. Moreover, the threshold curve shifts upward as the level of product innovation increases, which implies that product innovation increases the supplier’s incentive to establish the DOC. This is because product innovation raises online demand, the impact of online channel operational efficiency thus is weakened. Therefore, as the level of product innovation increases, so does the willingness of the supplier to establish the DOC. For the offline retailer, similar to the results in Proposition 4.5, under the cannibalisation...
spillover, the DOC is still more beneficial to the offline retailer than the IOC. Whereas under the
stimulation spillover, there exists a threshold curve, such that when the supplier’s marginal oper-
ating cost is below the threshold curve, the DOC structure is more beneficial to the offline retailer,
otherwise, the IOC structure is more beneficial. However, a higher product innovation level shifts
this threshold curve upward obliquely. This is because that product innovation increases online
demand, weakening the impact of online channel efficiency. The channel adjustment power of the
supplier thus becomes dominant. Consequently, the DOC is more beneficial to offline retailers than
the IOC in this case.

5  Endogenous product innovation

In this part, we consider the scenario with endogenous product innovation, where the supplier
innovates the product endogenously, that is, the supplier will strategically determine the product
innovation level to maximize his own profit. Product innovation incurs a one-time cost with the
understanding of a higher levels of product innovation generate a higher costs. The cost is usually
has noting to do with the capacity of the product. For example, in order to enhance the image of
products, BESTORE invites some public figures as their image spokespersons, and pay one-time
endorsement fees. Following the previous literature (e.g.,[22; 49; 27]), we assume that the cost
related to the product innovation is a quadratic function of the product innovation level, i.e., \( \frac{b\theta^2}{2} \)
and we normalize \( b = 1 \) for simplicity. There are two cases in the scenario with endogenous product
innovation.

(1) Case I*. The supplier introduces the IOC and innovates the product endogenously. Then,
the supplier’s, the offline retailer’s and the online retailer’s profit are given by

\[
\pi_s = w(q_1 + q_2) - \frac{\theta^2}{2}, \tag{5.1}
\]

\[
\pi_1 = (1 - q_1 - \tau q_2 + \theta - w)q_1, \tag{5.2}
\]

\[
\pi_2 = (1 - q_2 + \theta - w)q_2. \tag{5.3}
\]

(2) Case D*. The supplier establishes the DOC and innovates the product endogenously.
Then, the supplier’s and the offline retailer’s profit are given by

\[
\pi_s = wq_1 + (1 - q_2 + \theta - c)q_2 - \frac{\theta^2}{2}, \tag{5.4}
\]

\[
\pi_1 = (1 - q_1 - \tau q_2 + \theta - w)q_1. \tag{5.5}
\]

The event sequences are as follows (see Figure 9). (1) The supplier makes decide on online
channel structure (IOC or DOC). (2) If the supplier introduces the IOC, the supplier decides
the product innovation level and the wholesale price charged to both retailers. Then, based on the supplier’s decisions, the offline retailer and the online retailer decide their quantities, simultaneously. If the supplier establishes the DOC, the supplier decides the product innovation level, the wholesale price and the online channel product quantity. Then, based on the supplier’s decisions, the offline retailer decides the offline channel product quantity.

Figure 9: The event sequences in scenario with endogenous product innovation

Back induction is used to derive the equilibrium outcomes of each case. To grantee \( \pi_s \) is concave, we assume that \( \tau \in (1 - \sqrt{3}, 1) \) in Case \( D^* \).

**Lemma 5.1.** The equilibrium outcomes in Case \( I^* \) are given by

- \( g^{I*} = \frac{4-\tau}{\tau+4}, \ w^{I*} = \frac{4}{\tau+4}, \ q^{I*}_1 = \frac{2-\tau}{\tau+4}, \ q^{I*}_2 = \frac{2}{\tau+4}, \)
- \( \pi^{I*}_s = \frac{4-\tau}{2(\tau+4)}, \ \pi^{I*}_1 = \frac{(\tau-2)^2}{(\tau+4)^2}, \ \pi^{I*}_2 = \frac{4}{(\tau+4)^2}. \)

**Lemma 5.2.** The equilibrium outcomes in Case \( D^* \) are given by

- \( g^{D*} = \frac{4c+2\tau-c\tau-6}{7\tau^2-2\tau-2}, \ w^{D*} = \frac{2(c+\tau-c\tau-2)}{\tau^2-2\tau-2}, \ q^{D*}_1 = \frac{c+\tau-c\tau-2}{\tau^2-2\tau-2}, \ q^{D*}_2 = \frac{3c+\tau-4}{\tau^2-2\tau-2}, \)
- \( \pi^{D*}_s = \frac{-2(c\tau-2\tau-8+3c^2+6)}{2(\tau^2-2\tau-2)}, \ \pi^{D*}_1 = \frac{(c\tau-2\tau+2)^2}{(\tau^2-2\tau-2)^2}. \)

Lemma 5.1 shows that in the scenario with endogenous product innovation, the online retailer has an incentive to enter the market, which is consistent with the result in scenario with exogenous product innovation. Lemma 5.2 indicates that in the scenario with endogenous product innovation, the supplier always has an incentive to establish the DOC, which is different from the conclusion of Lemma 4.7. It implies that endogenous product innovation will give the supplier the flexibility to establish the DOC. That is, the supplier can gain more profit from the DOC than the IOC by strategically deciding product innovation level.

The following proposition compares the supplier’s and the offline retailer’s profit under the DOC structure with those under the IOC structure, in the scenario with exogenous product innovation level.
Proposition 5.3. In the scenario with exogenous product innovation,

(a) for the supplier,

(a.1) \( D_s^* \geq I_s^* \) if \( c \in (0, -\frac{2\sqrt{3-14\tau^2+12\tau+16}+\tau^2-16}{3\tau+12}] \),

(a.2) \( D_s^* < I_s^* \) if \( c \in (\frac{2\sqrt{3-14\tau^2+12\tau+16}+\tau^2-16}{3\tau+12}, 1) \);

(b) for the offline retailer,

(b.1) when \( \tau < 0 \), \( \pi_1^D^* \geq \pi_1^I^* \).

(b.2) when \( \tau > 0 \), \( \pi_1^D^* < \pi_1^I^* \) if \( c \in (\frac{\tau^3-3\tau^2+4\tau-4}{\tau^2+3\tau-4}, 1) \), and \( \pi_1^D^* \geq \pi_1^I^* \) if \( c \in (0, \frac{\tau^3-3\tau^2+4\tau-4}{\tau^2+3\tau-4}) \).

We further provide graphic illustration of Proposition 5.3, see Figure 10.

![Figure 10: The supplier’s and the offline retailer’s preference over IOC with DOC in scenario with endogenous product innovation.](image)

Proposition 5.3 and Figure 10 uncover that, in scenario with endogenous product innovation, the supplier’s optimal online channel structure strategy is significantly different from that in scenario with exogenous product innovation. Similarly, there exists a threshold curve such that when the marginal operating cost is below the threshold curve, the supplier is better off establishing the DOC, otherwise, the supplier is better off introducing the IOC. However, compared with the threshold curve in scenario with exogenous product innovation, the position and shape of the threshold curve have changed significantly. Most obviously, the position of the threshold curve moves up, which implies that, in the case of low marginal operating cost of the supplier, no matter what spillover effects, the DOC is the optimal strategy of the supplier on online channel structure. In addition, the slope of the curve has increased, which means that the supplier should establishes the DOC in the presence of strong and medium stimulative spillover. To conclude, endogenous product innovation endows the supplier the flexibility to establish the DOC. This is because endogenous product innovation
innovation endows suppliers with the ability to use product innovation to enhance the power of channel adjustment. By strategically setting the product innovation level, the supplier can weaken the impact of online channel efficiency and amplify the impact of channel adjustment powers. For the offline retailer, on the other hand, we find that the DOC would be more advantageous in almost every case. This shows that in the scenario with endogenous product innovation, the DOC can better alleviate channel friction between online and offline channels.

In the following proposition, we explore how the online channel structure affects product quality innovation level in the scenario with endogenous product quality innovation.

**Proposition 5.4.** In scenario with endogenous product quality innovation,

(a) When \( \tau \in (1 - \sqrt{3}, 0) \), \( \theta_{D*} > \theta_{I*} \);

(b) When \( \tau \in (0, 1) \), \( \theta_{D*} > \theta_{I*} \) if \( c \in (0, \frac{\tau^3 - 4 \tau^2 + 8 \tau - 16}{\tau^2 - 1b}) \), and \( \theta_{D*} < \theta_{I*} \) if \( c \in (\frac{\tau^3 - 4 \tau^2 + 8 \tau - 16}{\tau^2 - 1b}, 1) \).

![Figure 11: Comparison of optimal product quality innovation levels.](image)

Proposition 5.4 indicates that when the spillover is stimulative, or when the spillover is cannibalistic but the supplier’s marginal operating cost is not too high, the DOC structure leads to a higher product innovation level. Otherwise, when the the spillover is cannibalistic but the supplier’s marginal operating cost is high, the IOC structure leads to a higher product innovation level (see Figure 11). The explanation is as follows. With the DOC structure, the supplier consider not only online sales, but also the impact of online sales on offline sales. In the presence of stimulation spillover, online sales promote offline sales. Thus the supplier has more incentive to raise the product innovation level. Whereas in the presence of cannibalisation spillover, online sales press offline sales. If the marginal operating cost is high, the supplier will lower the online product quantity to mitigate the negative impact of spillovers to maintain profit. As a result, the supplier has less
incentive to raise the product innovation level.

According to the traditional wisdom, the DOC leads to a higher product innovation level than the IOC, since the DOC is a more effective distribution structure. However, Proposition 5.4 demonstrates that the existence of spillover may distort such result, which provides new insight in research on product innovation decision.

6 Numerical example

In this section, we provide numerical examples to verify our main theoretic results. For given value of related parameter, we calculate the supplier’s and the offline retailer’s profit under the IOC and the DOC structure. Additionally, we also calculate the optimal product quality innovation level under the IOC and the DOC structure in the scenario with endogenous product quality enhancement.

Numerical example 1. (a) \( c = 0.2, \tau = -0.5, 0.5 \); (b) \( c = 0.6, \tau = -0.5, 0.5 \).

Table 2: The supplier’s profit and the offline retailer’s profit in Numerical example 1

<table>
<thead>
<tr>
<th>( c = 0.2 )</th>
<th>( \tau = -0.5 )</th>
<th>( \tau = 0.5 )</th>
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</tr>
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</tr>
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</tr>
<tr>
<td>( \pi_s^i )</td>
<td>0.0352</td>
<td>0.2188</td>
</tr>
</tbody>
</table>

When \( c = 0.2, \tau = -0.5, \pi_s^d > \pi_s^i, \pi_s^d < \pi_s^i \); when \( c = 0.2, \tau = 0.5, \pi_s^d > \pi_s^i, \pi_s^d > \pi_s^i \); when \( c = 0.6, \tau = -0.5, \pi_s^d < \pi_s^i, \pi_s^d < \pi_s^i \); when \( c = 0.6, \tau = 0.5, \pi_s^d < \pi_s^i, \pi_s^d > \pi_s^i \).

The results in Table 2 are consistent with Proposition 4.5.

Numerical example 2. (a) \( \theta = 0.1, c = 0.1, \tau = -0.5, -0.1, 0.5 \); (b) \( \theta = 0.1, c = 0.4, \tau = -0.5, -0.1, 0.5 \); (c) \( \theta = 0.3, c = 0.1, \tau = -0.5, -0.1, 0.5 \); (d) \( \theta = 0.3, c = 0.4, \tau = -0.5, -0.1, 0.5 \).

When \( \theta = 0.1, c = 0.1, \tau = -0.5, \pi_s^D > \pi_s^I, \pi_s^I > \pi_s^i \); when \( \theta = 0.1, c = 0.1, \tau = -0.1, \pi_s^D > \pi_s^s, \pi_s^D < \pi_s^i \); when \( \theta = 0.1, c = 0.1, \tau = 0.5, \pi_s^D > \pi_s^I, \pi_s^D > \pi_s^i \); when \( \theta = 0.1, c = 0.4, \tau = -0.5, \pi_s^I < \pi_s^I, \pi_s^D < \pi_s^i \); when \( \theta = 0.1, c = 0.4, \tau = -0.1, \pi_s^D < \pi_s^I, \pi_s^D < \pi_s^i \); when \( \theta = 0.1, c = 0.4, \tau = 0.5, \pi_s^D < \pi_s^I, \pi_s^D < \pi_s^i \); when \( \theta = 0.3, c = 0.1, \tau = -0.5, \pi_s^D > \pi_s^I, \pi_s^D > \pi_s^i \); when \( \theta = 0.3, c = 0.1, \tau = -0.1, \pi_s^D > \pi_s^I, \pi_s^D > \pi_s^i \); when \( \theta = 0.3, c = 0.1, \tau = 0.5, \pi_s^D > \pi_s^I, \pi_s^D > \pi_s^i \); when \( \theta = 0.3, c = 0.4, \tau = -0.5, \pi_s^D > \pi_s^I, \pi_s^D < \pi_s^i \); when \( \theta = 0.3, c = 0.4, \tau = -0.1, \pi_s^D > \pi_s^I, \pi_s^D < \pi_s^i \); when \( \theta = 0.3, c = 0.4, \tau = 0.5, \pi_s^D > \pi_s^I, \pi_s^D > \pi_s^i \); when \( \theta = 0.3, c = 0.4, \tau = -0.1, \pi_s^D > \pi_s^I, \pi_s^D > \pi_s^i \); when \( \theta = 0.3, c = 0.4, \tau = 0.5, \pi_s^D > \pi_s^I, \pi_s^D > \pi_s^i \).
The results in Table 3 are consistent with Proposition 4.10.

**Numerical example 3.** (a) $c = 0.4$, $\tau = -0.2, 0.1, 0.5$; (b) $c = 0.7$, $\tau = -0.2, 0.1, 0.5$ for the supplier and $c = 0.99$, $\tau = -0.2, 0.1, 0.5$ for the offline retailer.

Table 4: The supplier’s profit and the offline retailer’s profit in Numerical example 3

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\theta = 0.1$</th>
<th>$\theta = 0.3$</th>
<th>$\theta = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c = 0.4$</td>
<td>$\pi_s^D = 0.4852$</td>
<td>$\pi_s^D = 0.3323$</td>
<td>$\pi_s^D = 0.6903$</td>
</tr>
<tr>
<td>$\pi_s^I = 0.3403$</td>
<td>$\pi_s^I = 0.2737$</td>
<td>$\pi_s^I = 0.4753$</td>
<td></td>
</tr>
<tr>
<td>$\pi_s^D = 0.4155$</td>
<td>$\pi_s^D = 0.2837$</td>
<td>$\pi_s^D = 0.5915$</td>
<td></td>
</tr>
<tr>
<td>$\pi_s^I = 0.3101$</td>
<td>$\pi_s^I = 0.3101$</td>
<td>$\pi_s^I = 0.4331$</td>
<td></td>
</tr>
<tr>
<td>$\pi_s^D = 0.3432$</td>
<td>$\pi_s^D = 0.2329$</td>
<td>$\pi_s^D = 0.4890$</td>
<td></td>
</tr>
<tr>
<td>$\pi_s^I = 0.2647$</td>
<td>$\pi_s^I = 0.2647$</td>
<td>$\pi_s^I = 0.3697$</td>
<td></td>
</tr>
</tbody>
</table>

When $c = 0.7, \tau = 0.7$, $\theta = 0.7, \tau = 0.7, \theta = 0.7$; when $c = 0.99, \tau = 0.7, \theta = 0.7$; when $c = 0.99, \tau = 0.7, \theta = 0.7$.

The results in Table 4 are consistent with Proposition 5.3.

**Numerical example 4.** (a) $c = 0.5$, $\tau = 0.2, 0.7$; (b) $c = 0.8$, $\tau = 0.2, 0.7$.

When $c = 0.5, \tau = 0.2, \theta^D > \theta^I$; when $c = 0.5, \tau = 0.7, \theta^D > \theta^I$; when $c = 0.8, \tau = 0.2, \theta^D > \theta^I$; when $c = 0.8, \tau = 0.7, \theta^D > \theta^I$.

The results in Table 5 are consistent with Proposition 5.4.
7 Conclusions

In this paper, we investigate how spillovers from online sales to offline sales and product innovation jointly affect suppliers’ optimal online channel structure strategies. To this end, we consider a two-tier dual-channel supply chain consisting of a monopolist supplier, an independent offline retailer and an independent online retailer, where the supplier sells a product to offline consumers via the offline retailer and to online consumers via the online retailer or his own direct online channel. We model the following three scenarios. (1) Scenario without product innovation; (2) Scenario with exogenous product innovation; and (3) Scenario with endogenous product innovation.

The main results of this paper are as follows. First, with the IOC, the spillover effect does not affect the equilibrium online product quantity, whereas a greater value of the spillover coefficient lowers the equilibrium offline product quantity. With the DOC, as the spillover coefficient increases, the equilibrium online product quantity decreases if the supplier’s marginal operating cost is low or medium, and first decreases then remains constant if the cost is high. Second, there exists a threshold curve such that when the supplier’s marginal operating cost is below the threshold curve, the supplier is better off establishing the DOC, otherwise, the supplier is better off introducing the IOC. Nonetheless, the threshold curve is not a monotonic function of the spillover coefficient, but a function that decreases first and then increases with the the spillover coefficient. Moreover, the offline retailer and the supplier may have the same preference for online channel structure, thus leading to a win-win situation for the supplier and the offline retailer, and we present the condition under which they have the same preference. Third, exogenous product innovation does not change the supplier’s optimal online channel structure qualitatively. However, it leads to some quantitative changes. For example, the threshold curve shifts upward as the level of product innovation increases, which implies that product innovation increases the supplier’s incentive to establish the DOC. Finally, in scenario with endogenous product innovation, the supplier’s optimal online channel structure strategy is significantly different from that in scenario with exogenous product innovation. Endogenous product innovation endows the supplier the flexibility to establish the DOC. Moreover, counterintuitively, when the the spillover is cannibalistic and the supplier’s
marginal operating cost is high, the IOC structure leads to a higher quality level, compared with the DOC structure.

Based on our theoretical results, our work provides some managerial implications and suggestions for suppliers. First, when suppliers expand online sales, the spillover effect from online sales to offline sales should be taken into account, which, along with the demand-boost effect, affect suppliers’ aggregate profits. This study shows that when the online operating disadvantage of suppliers is significant, they should introduce the IOC, however, when the spillover effect is strong, whether stimulative or cannibalistic, they are better off establishing the DOC. Second, exogenous product innovation, including product technology innovation and product image upgrading, can increase the motivation of suppliers to establish the DOC, but endogenous product innovation can endorses suppliers the flexibility to establish the DOC. That is, suppliers can gain more profit from establishing the DOC by strategically setting the level of product innovation. This result indicates that product innovation and the DOC can be used as a powerful lever for suppliers to control and coordinate the dual channels and mitigate the negative impact of spillovers. The result has implications on the recent practice of BESTORE, who not only focuses on product innovation, but also commits to the establishment of the DOC.

In what follows, we provide directions for future research. First, in this paper, the IOC is assumed to be a reseller, but other forms of cooperation between suppliers and online retailers are not considered. However, online retailers offer many other forms of cooperation, such as marketplaces, which have been widely adopted by Amazon, Sears, JD.COM and Taobao (Bonfils, 2012). Amazon offers its some upstream parters various alternative cooperation modes, such as Associate, Advantage, and Web-Store (Yan et al., 2018). Extensions to other cooperation modes represent another direction deserving of future study. Second, the manufacturer is assumed to be the leader in the game, but online giants such as Amazon, JD.COM, and Taobao are actually market leaders. Studying cooperation between a supplier and an online giant is another interesting direction for future research.

Appendix

Proof of Lemma 4.2. Consider the case that both channels stay in the market. Back induction is used to identify the equilibrium and the equilibrium outcomes are as follows, \( w^d = \frac{2(c-\tau+2)}{8-\tau}, \)
\( q_1^d = \frac{c-\tau-2}{\tau^2-8}, q_2^d = \frac{4c+\tau-4}{\tau^2-8}. \)
\( \pi_s = \frac{c^2-\tau^2+2c^2/3}{8-\tau^2}, \)
\( \pi_1^d = \frac{(c-\tau+2)^2}{(y^2-8)y^2}. \)
Note that this equilibrium exists if and only if when the online product quantity and the offline product quantity are both positive. That is, when \( c \in (0, \frac{3}{2}) \), or \( c \in [\frac{3}{2}, 1) \) and \( \tau \in (-1, 4(1-c)) \), there exist an equilibrium, where
both channels stay in the market. Otherwise, when \( c \in [\frac{3}{4}, 1) \) and \( \tau \in [4(1 - c), 1) \), the online product quantity is zero, that is, the online channel is out of the market. In this case, there exist an equilibrium, where only the offline channel is active. The equilibrium outcomes are as follows, \( w^d = \frac{1}{2}, q^d_1 = \frac{1}{2}, q^d_2 = 0, \pi^d_s = \frac{1}{8}, \pi^d_1 = \frac{1}{16} \).

**Proof of Corollary 4.3.** (b.1) When \( c \in (0, \frac{3}{4}) \), or \( c \in [\frac{3}{4}, 1) \) and \( \tau \in (-1, 4(1 - c)) \), \( \frac{\partial q^d_1}{\partial \tau} = \frac{-8c\tau - 8\tau^2 + 8\tau + 8}{(\tau^2 - 8)^2} \). The sign of \( \frac{\partial q^d_1}{\partial \tau} \) is determined by \(-8c\tau - 8\tau^2 + 8\tau + 8 < 0\). Note that \( c \in (0, 1) \) and \( \tau \in (-1, 1) \), \(-8c\tau - 8\tau^2 + 8\tau + 8 < 0 \), and thus \( \frac{\partial q^d_1}{\partial \tau} < 0 \). When \( c \in [\frac{3}{4}, 1) \) and \( \tau \in [4(1 - c), 1) \), \( \frac{\partial q^d_1}{\partial \tau} = 0 \). Therefore, when \( c \in (0, \frac{3}{4}) \), \( \frac{\partial q^d_1}{\partial \tau} < 0 \); when \( c \in [\frac{3}{4}, 1) \), \( \frac{\partial q^d_1}{\partial \tau} < 0 \) if \( \tau \in [4(1 - c), 1) \), and \( \frac{\partial q^d_1}{\partial \tau} = 0 \) if \( \tau \in [4(1 - c), 1) \).

(b.2) When \( c \in (0, \frac{3}{4}) \), or \( c \in [\frac{3}{4}, 1) \) and \( \tau \in (-1, 4(1 - c)) \), \( \frac{\partial q^d_2}{\partial \tau} = \frac{(c e^2 - c e^2 + 4c e - 4)}{(\tau^2 - 8)^2} \). Note that \( 0 < c < 1 \) and \(-1 < \tau < 1 \). It is not hard to see that \( (c \tau - \tau + 2) > 0 \).

Thus, the sign of \( \frac{\partial q^d_2}{\partial \tau} \) depends on \((4c + \tau - 4)\). Thus, when \( c \in (0, \frac{3}{4}) \), \( 4c + \tau - 4 < 0 \) for \( \tau \in (-1, 1) \), and when \( c \in [\frac{3}{4}, 1) \) and \( \tau \in (-1, \hat{\tau}) \), \( 4c + \tau - 4 < 0 \). On the other hand, when \( c \in [\frac{3}{4}, 1) \) and \( \tau \in [\hat{\tau}, 1) \), \( \frac{\partial q^d_2}{\partial \tau} = 0 \). Therefore, when \( c \in (0, \frac{3}{4}) \), \( \frac{\partial q^d_2}{\partial \tau} < 0 \); when \( c \in [\frac{3}{4}, 1) \), \( \frac{\partial q^d_2}{\partial \tau} < 0 \) if \( \tau \in (-1, \hat{\tau}) \), and \( \frac{\partial q^d_2}{\partial \tau} > 0 \) if \( \tau \in (\hat{\tau}, 1) \).

(b.2) When \( c \in (0, \frac{3}{4}) \), or \( c \in [\frac{3}{4}, 1) \) and \( \tau \in (-1, 4(1-c)) \), \( \frac{\partial \pi^d_s}{\partial \tau} = \frac{(2c e^2 - 2e^2 + 4c e - 4)}{(\tau^2 - 8)^2} \). Note that \( 0 < c < 1 \) and \(-1 < \tau < 1 \). It is not hard to see that \( (2c e^2 - 2e^2 + 4c e - 4) > 0 \). Thus, the sign of \( \frac{\partial \pi^d_s}{\partial \tau} \) depends on \((4c + \tau - 4)\). According to the proof of Corollary 1, we have the following: When \( c \in (0, \frac{5}{9}) \), \( \frac{\partial \pi^d_s}{\partial \tau} < 0 \); when \( c \in [\frac{5}{9}, \frac{3}{4}) \), \( \frac{\partial \pi^d_s}{\partial \tau} < 0 \) if \( \tau \in (-1, \hat{\tau}) \), and \( \frac{\partial \pi^d_s}{\partial \tau} > 0 \) if \( \tau \in (\hat{\tau}, 1) \), when \( c \in [\frac{3}{4}, 1) \), \( \frac{\partial \pi^d_s}{\partial \tau} < 0 \) if \( \tau \in (-1, \hat{\tau}) \), \( \frac{\partial \pi^d_s}{\partial \tau} > 0 \) if \( \tau \in (\hat{\tau}, 1) \), and \( \frac{\partial \pi^d_s}{\partial \tau} = 0 \) if \( \tau \in (\hat{\tau}, 1) \).

**Proof of Lemma 4.7.** Consider the case that both channels stay in the market. Back induction is used to identify the equilibrium and the equilibrium outcomes are as follows, \( w^d = \frac{1}{2}, q^d_1 = \frac{1}{2}, q^d_2 = 0, \pi^d_s = \frac{1}{8}, \pi^d_1 = \frac{1}{16} \).
online product quantity and the offline product quantity are both positive. That is, when \( c \in (0, \frac{3}{4}) \), or \( c \in (\frac{3}{4}, 1) \) and \( \theta > \frac{3}{4} c - 1 \), or \( c \in [\frac{3}{4}, 1) \) and \( \theta < \frac{3}{4} c - 1 \) and \( \tau \in (-1, \hat{\tau}) \), there exist an equilibrium, where both channels stay in the market. Otherwise, when \( c \in (\frac{3}{4}, 1) \) and \( \theta < \frac{3}{4} c - 1 \) and \( \tau \in (\hat{\tau}, 1) \), the online product quantity is zero, that is, the online channel is out of the market. In this case, there exist an equilibrium, where only the offline channel remains in the market. The equilibrium outcomes are as follows, \( w^D = \frac{1+\theta}{2}, q^D_1 = \frac{1+\theta}{4}, q^D_2 = 0, \pi^D_2 = 0, \pi^D_1 = \frac{(1+\theta)^2}{8} \).

Proof of Corollary 4.8. (b.1) When \( c \in (0, \frac{3}{4}) \), or \( c \in [\frac{3}{4}, 1) \) and \( \theta > \frac{3}{4} c - 1 \), or \( c \in (\frac{3}{4}, 1) \) and \( \theta < \frac{3}{4} c - 1 \) and \( \tau \in (-1, \hat{\tau}) \), \( \frac{\partial q^D}{\partial \tau} = -\frac{8 \theta (1-\tau)+8(1-\theta+c\tau+\tau^2)\theta+\tau^2}{(\tau^2-8\theta)^2} \). Recall that \( c \in (0,1) \), \( \tau \in (-1,1) \) and \( \theta > 0 \), it is not hard to see that \( \frac{\partial q^D}{\partial \tau} < 0 \). When \( c \in (\frac{3}{4}, 1) \) and \( \theta < \frac{3}{4} c - 1 \) and \( \tau \in (\hat{\tau}, 1) \), \( \frac{\partial q^D}{\partial \tau} = 0 \). Therefore, when \( c \in (0, \frac{3}{4}) \), or \( c \in [\frac{3}{4}, 1) \) and \( \theta > \frac{3}{4} c - 1 \), \( \frac{\partial q^D}{\partial \tau} < 0 \). When \( c \in (\frac{3}{4}, 1) \) and \( \theta < \frac{3}{4} c - 1 \), \( \frac{\partial q^D}{\partial \tau} < 0 \) if \( \tau \in (-1, \hat{\tau}) \), and \( \frac{\partial q^D}{\partial \tau} = 0 \) if \( \tau \in (\hat{\tau}, 1) \).

(b.2) When \( c \in (0, \frac{2}{3}) \), or \( c \in [\frac{3}{4}, 1) \) and \( \theta > \frac{3}{4} c - 1 \), or \( c \in (\frac{3}{4}, 1) \) and \( \theta < \frac{3}{4} c - 1 \) and \( \tau \in (-1, \hat{\tau}) \), \( \frac{\partial q^D}{\partial \tau} = \frac{(c-\theta -1)\tau^2+(\theta+4)\tau+8c-8\theta-8}{(\tau^2-8\theta)^2} \). The sign of \( \frac{\partial q^D}{\partial \tau} \) depends on \((c-\theta -1)\tau^2+(4\theta+4)\tau+8c-8\theta-8 \). Let \( h(\tau) = (c-\theta -1)\tau^2+(4\theta+4)\tau+8c-8\theta-8 \). Then \( h(\tau) \) has two roots \( \hat{\tau} = \frac{2\theta+2\sqrt{-2c^2+4\theta+4c-8\theta-8}}{8c-8\theta-8} \), and \( \hat{\tau}' = \frac{2\theta+2\sqrt{-2c^2+4\theta+4c-8\theta-8}}{8c-8\theta-8} \). Since \( \Delta = (-16(4c-2c^2-\theta^2+\theta(4c-2)-1) \). \( \Delta(\theta) = -16(4c-2c^2-\theta^2+\theta(4c-2)-1) \). Then \( \Delta(\theta) > 0 \) if \( \theta < \frac{9\theta}{4} - 1 \) and \( \hat{\tau}' > 1 \) if \( \theta > \frac{9\theta}{4} - 1 \). When \( c \in [\frac{3}{4}, 1) \) and \( \theta < \frac{3}{4} c - 1 \) and \( \tau \in (\hat{\tau}, 1) \), \( \frac{\partial q^D}{\partial \tau} = 0 \).

Case 1: \( c \in (0, \frac{2}{3}) \). When \( \theta > (2+\sqrt{2})c - 1 \), \( \Delta < 0 \), thus \( \frac{\partial q^D}{\partial \tau} < 0 \). When \( \theta < (2+\sqrt{2})c - 1 \), \( \Delta > 0 \) and \( \hat{\tau} > 1 \), thus \( \frac{\partial q^D}{\partial \tau} < 0 \) for \( \tau \in (-1, \hat{\tau}) \).

Case 2: \( c \in (\frac{5}{9}, \frac{2}{3}) \). When \( \theta > (2+\sqrt{2})c - 1 \), \( \Delta < 0 \), thus \( \frac{\partial q^D}{\partial \tau} < 0 \). When \( \frac{9}{5}c - 1 < \theta < (2+\sqrt{2})c - 1 \), \( \Delta > 0 \) and \( \hat{\tau} > 1 \), thus \( \frac{\partial q^D}{\partial \tau} < 0 \) for \( \tau \in (-1, \hat{\tau}) \) and \( \frac{\partial q^D}{\partial \tau} > 0 \) for \( \tau \in (\hat{\tau}, 1) \).

Case 3: \( c \in [\frac{3}{4}, 1) \). When \( \theta > (2+\sqrt{2})c - 1 \), \( \Delta < 0 \), thus \( \frac{\partial q^D}{\partial \tau} < 0 \). When \( \frac{9}{5}c - 1 < \theta < (2+\sqrt{2})c - 1 \), \( \Delta > 0 \) and \( \hat{\tau} > 1 \). Thus \( \frac{\partial q^D}{\partial \tau} < 0 \) for \( \tau \in (-1, \hat{\tau}) \) and \( \frac{\partial q^D}{\partial \tau} > 0 \) for \( \tau \in (\hat{\tau}, 1) \). When \( \theta < \frac{9}{5}c - 1 \), \( \Delta > 0 \) and \( \hat{\tau} < 1 \). Thus \( \frac{\partial q^D}{\partial \tau} < 0 \) for \( \tau \in (-1, \hat{\tau}) \) and \( \frac{\partial q^D}{\partial \tau} > 0 \) for \( \tau \in (\hat{\tau}, 1) \).

Proof of Proposition 5.3. (b) \( \tau_1^{D*} - \tau_1^{f*} = \frac{3c\tau-4c+c^2+5\tau^2+3\tau^3+12}{(\tau+4)^2(\tau^2+4\tau+13)} \). Recall that \( c \in (0,1) \) and \( \tau \in (1-\sqrt{3}, 1) \), \( (3c\tau-4c+c^2+5\tau^2+3\tau^3+12) > 0 \). Hence, the sign of \( \tau_1^{D*} - \tau_1^{f*} \) depends on \((3c\tau-4c+c^2+5\tau^2+3\tau^3+12) > 0 \). Let \( h(c) = 3c\tau-4c+c^2+3\tau^2-3\tau^3+4 \). When \( \tau \in (1-\sqrt{3}, 0) \), \( h(c) > 0 \) (recall that \( c \in (0,1) \)). When \( \tau \in (0,1) \), \( h(c) \geq 0 \) if \( c \leq \hat{c} \) and \( h(c) < 0 \) if \( c < \hat{c} \). Here, \( \hat{c} = \frac{3\tau^2+4\tau-4}{\tau^2+3\tau-4} \) is the root of \( h(c) \).

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References


