Manufacturing/ remanufacturing based supply chain management under advertisements and carbon emission process

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Abstract: One of the most successful ways to get the word out about a product's popularity across all types of customers is through advertising. It has a valuable direct influence on increasing product demand. The supply chain model is developed for manufacturer and retailer, where advertisements are dependent on demand. The advertisement rate has been considered a function that has enhanced at a diminishing rate concerning time, although the growth rate slowed. During the manufacturing cycle, the market's demand is a function of advertisement, and the customer's demand is a linear function of time. The production rate exceeds the demand rate during manufacturing and remanufacturing; shortages are not faced. It involves a manufacturing/remanufacturing process that quickly delivers consumer products and less waste. To keep the environment clean, the cost of carbon emissions is incorporated into the manufacturer's and supplier's holding and degrading costs. The model's primary purpose is to minimize the overall cost of manufacturing and remanufacturing. The overall cost during the manufacturing cycle is higher than that during the remanufacturing cycle. This study confirms that the increasing cost of advertising provides the continuous increasing value of the total cost. A numerical example is provided, graphical representation and sensitivity analysis determine the function's behavior and test the model.

Keywords: Advertisements; Carbon-emission; Deterioration; Remanufacturing; Supply chain management

1. INTRODUCTION

With the increasing competition in the global market, industrial players look into various methods to help them increase their sales and profits. Supply chain management (SCM) is a significant factor in any business process. It handles the production flow ranging from supply of raw material to delivery of final product out to the customer. The main element of the supply chain is planning,
source, production, delivery, and return. If the supply chain system is effective, it minimizes the production cycle's total cost, time, and wastage.

There is an increasing need to combat increasing carbon emission and environmental degradation in Today's Era. The notion of manufacturing, recycling, and reverse logistics help achieve the goal. By remanufacturing, much of carbon is used again without being emitted as gases in the atmosphere. This process promotes the use of old or broken products resulting in the reduction of waste. The decline in carbon emission leads us to a green environment. El Saadany et al. [13] analyzed an SCM model in which the product's carbon footprint determines demand. Habib et al. [16] established an optimization model that reduces the cost of overall biodiesel supply chain processes while also reducing carbon emissions during operations.

Reverse logistics refers to the activities connected after the sale of a product to capture the value added and the life cycle of a product. It includes returning the product to the producer or recycling it. It is at a time called the aftermarket supply chain. During this change, the product has to go through various processes such as recycling, waste management, warehouse management, returns management, servicing, remanufacturing products being rebuilt with old or repaired parts, and refurbishment deals with the resale of repaired parts products. Ullah et al. [50] established a closed-loop supply chain management system for reverse logistics activities with stochastic demand and returns, which increases the cumulative uncertainty in the system to investigate optimal remanufacturing strategy.

Advertisement is done to publicity products, services, or events with the help of media platforms like newspapers, magazines, radios, television. It is designed so that it immediately attracts the attention of the people. It plays a crucial part in influencing the consumers' minds, tastes, and motives. It is essential in improving the sales of commodities or services. Business firms pay massive amounts to create an advertisement that helps in increasing the demand for their product in the market. It is a play of words with witty expressions and catchy punch lines that influence society. Manna et al. [25] analyzed a model for imperfect production in which advertising influences demand.

The sale of most items is dependent on its advertising in the public realm. As a result, advertising plays an essential role in enhancing a commodity's demand. Most of the existing literature considered supply chain management systems focused on carbon emission and waste reduction manufacturing. This study considers advertising demand and carbon emissions under reverse logistics, according to the information of the authors, which has not been done in any research so far. Therefore, it fills the research gap that reverse logistics is not considered advertisement and carbon emission costs. This study looks at advertisement demand, in which the rate of advertisement increases with time passing at a slower pace to record the sale before it is destroyed by deterioration. Demand depends on advertisement and time as demand largely impacts customers. An inventory model is developed with a single manufacturer and retailer by assuming that goods that have been remanufactured are as excellent as new items. Both manufacturing and remanufacturing models are separately formulated with various parameters and constant deterioration rates. Remanufacturing in world used products and reverse Logistics. This study can be used in the textile industry, smart products, and other fields in the real world.
FIGURE 1. The flow of this study

The following is a breakdown of the work: A literature review is in Section 2, the aim of the study, assumptions, and notations are defined in part 3, and a mathematical model is formulated in Section 4; Section 5 explains the solution approach, numerical examples, and sensitivity are included in Sections 6 and 7. At last, Section 8 gives the conclusion and future research directions for the paper.

2. LITERATURE REVIEW

Over the years, researchers have agreed that inventory turns out to be a comprehensive study where inventory management is optimized. Today's problems related to EPQ include demand, deterioration of products, reverse logistics, and sustainability. Firstly, Harris developed the model with the assumption of constant demand. Later, Wilson [52] discussed this model with stock control. Silver and Meal [42] modified the model to include a variable demand rate. Donaldson [11], Lo et al. [24], and others developed a demand model under a linear trend. Time-dependent demand model developed by Goyal and Giri [15], Silver and Peterson [43]. Karimi-Nasab et al. [19] discussed a price-dependent demand inventory model. Nowadays, advertisements are significant in controlling the demand for any product in the market. Cho [7] established a model with an optimal production and promotional policy for this purpose. Hazari et al. [17] examined a model for defective production with an advertisement policy in an uncertain environment. Khan et al. [20] created models for perishable items (with and without shortages), in which demand is driven by the frequency of advertising and the selling price of the commodity. Udayakumar et al. [49] studied a model for no instantaneously decaying items that consider money inflation and time discounting under advertisement-dependent demand, with the provider offering an acceptable wait period as an alternative to price discount. A smart supply chain management about variable lead time and variance under controllable production rate and advertisement-dependent demand is proposed by Dey et al. [9].

By assuming an instantaneous repairable rate, Schardy [40] developed the rules for inventories with repairable products. The inventory model with limited storage capacity and finite repair rate was established by Nahmias and Rivera [26]. Conard [8] proposed a closed-loop supply chain
(CLSC) model to examine the effect of customer issues on cost and market. Sebatjane and Adetunji [41] proposed a three-echelon SC model in which inventory levels and expiration dates determine the demand for increasing products. The policies for a CLSM model for green products with remanufacturing were developed by Chai et al. [5]. Inderfurth et al. [18] created a model for defective products subject to rework and deterioration. Widyadana and Wee [51] proposed a declining product manufacturing model that included a river and numerous production sets. Alamri [1] created a CLSC model for decaying products with dependent demand. Richter [30] created an EOQ model based on fixed demand and the condition that old products be repaired. El Saadany and Jaber [12] developed a CLSC model in which the returning rate is influenced by price and quality. Saxena et al. [37] determined a CLSC model with remanufacturing for the buyer/supplier. Rani et al. [27] established a CLSC model for decline items that took inflation and remanufacturing into account. Rani et al. [29] created a model that considered remanufacturing and learning effects. A CLSC model with remanufacturing was established by Liu et al. [23] and Aminipour et al. [2]. Sarkar et al. [36] produced a three-tiered sustainable SC model. Sarkar et al. [34] constructed a CLSC model to overcome financial and storage constraints. Defective products may be reworked during each cycle when shortages were eliminated, and shortages can be reworked in the respective phase following the last cycle, according to Sarkar [35]. Bhuniya et al. [4] described an energy-efficient smart production system in which production is variable and defective items are produced out of control. Preventive maintenance and restoration are utilized within the smart production system to avoid the out-of-control state. Sarkar et al. [33] proposed a single-stage cleaner production system impact of random defective rates for multiple products for a non-linear constraint problem. To solve the model, a non-linear optimization technique is used. Bhuniya et al. [3] introduced a supply chain management model to avoid the backorder cost with constant and fuzzy demand. This model improves the quality of the products and reduces the vendor’s setup cost by using the Kuhn-Tucker optimization technique.

Chang et al. [6] explored a production–inventory model with a multi-stage supply chain that uses preservation technology to prevent deterioration and boost investment. In the context of preservation technology investment, Kumar et al. [21] determined the optimal policies for deteriorating artifacts. Saha et al. [32] identified the optimum dynamic marketing investment. They proved that the dynamic investment could control the market’s demand. Even though the rate of deterioration is high, efficient preservation technology can be adopted to reduce the deterioration rate. Though the deterioration rate is reduced, wastes can be generated, which cannot be zero by the preservation technology. The preservation technique only can reduce the rate of deterioration. Therefore, even though dynamic investment and preservation technology were used, the waste cannot be fully controlled. This reduction of waste in a supply chain was initiated by Yadav et al. [48]. They introduced a new waste-free SC model. They proved that only preservation technology could reduce huge amounts of waste from product deterioration without having dynamic investment. Saxena et al. [38] investigated two approaches to waste management and time replenishment. Dey et al. [10] examined an autonation policy in this model to find defective products from the production where the defective rate is random. Exponential demand is considered with safety stock and backorder. A multi-period multi-objective optimization problem of all three components of the business triad cost, quality, and time was managed by Tayyab and Sarkar [46] in a supply chain management model.

Kumar et al. [22] derived a CLSC model for smart objects using a reverse logistics system, with the retailer taking carbon emissions into account. Rani et al. [28] presented a green CLSC model
for products that do not degrade instantly. Sepehri et al. [39] created a model for degrading low-quality objects in which the pace of degradation is constant and may be controlled by investing in preservation technologies. Manufacturing processes emit carbon, which may be decreased by investing in carbon-reduction technology. Gennady et al. [14] established a logistics and SC model in the face of uncertainty. For degrading products, Singh et al. [44] established a method that combined the effects of costs of carbon emissions and programs involving trade credits. Teng et al. [47] developed a system for remanufacturing under Cap-and-Trade legislation to minimize faulty items and limit carbon emissions, which is the manufacturer's primary goal in assuring sustainability. Sarkar et al. [50] developed a sustainable development framework for a cleaner textile production system with emissions tax and allocated cap policies for manufacturing. Table 1 shows some of the author's previous work.

Table 1. (The author's previous research)

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Remanufacturing</th>
<th>Type of Production</th>
<th>Demand</th>
<th>Environmental Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sarkar et al. [34]</td>
<td>No</td>
<td>Constant</td>
<td>Quadratic function</td>
<td>Costs effective under Carbon emission</td>
</tr>
<tr>
<td>Saxena et al. [38]</td>
<td>Yes</td>
<td>Constant</td>
<td>Constant</td>
<td>No</td>
</tr>
<tr>
<td>Rani et al. [29]</td>
<td>Yes</td>
<td>Constant</td>
<td>Carbon dependent</td>
<td>Demand effective under Carbon emission</td>
</tr>
<tr>
<td>Dey et al. [9]</td>
<td>No</td>
<td>Flexible production</td>
<td>Advertisement dependent</td>
<td>Costs not effective under Carbon emission</td>
</tr>
<tr>
<td>Saxena et al. [37]</td>
<td>Yes</td>
<td>Constant</td>
<td>Constant</td>
<td>Costs not effective under Carbon emission</td>
</tr>
<tr>
<td>Bhuniya et al. [4]</td>
<td>No</td>
<td>Constant</td>
<td>Constant and fuzzy demands</td>
<td>Costs not effective under Carbon emission</td>
</tr>
<tr>
<td>Kumar et al. [22]</td>
<td>Yes</td>
<td>Linear</td>
<td>Time-dependent</td>
<td>Demand effective under Carbon emission</td>
</tr>
<tr>
<td>This study</td>
<td>Yes</td>
<td>Linear</td>
<td>Advertisement and time-dependent</td>
<td>Costs effective under Carbon emission</td>
</tr>
</tbody>
</table>

3. ASSUMPTIONS & NOTATION

The following aim of the study, assumption, and notation are used for this study:

3.1 Aim of the study

The focus of the study is on manufacturing/remanufacturing-based supply chain management in the context of advertisements and carbon emissions. During the manufacturing cycle, the market's demand is an advertisement function. To maintain the environment clean for both manufacturing and remanufacturing processes, carbon emissions costs have been added on deterioration costs and holding costs of items. This research is significant in advertising and remanufacturing procedures in the manufacturing industry. The study's flow is depicted in Figure 1.
3.2 Assumptions

1. At this time, it is clear that the sale of an item is dependent on its advertising in the public sphere. As a result, advertising plays an essential role in enhancing a global commodity demand. In this way, we have looked at the demand function for ideal quality things affected by advertising. The demand rate function of advertising is:

\[ D_m = D_0 e^{-\lambda t} + \frac{a_0}{\lambda} (1 - e^{-\lambda t}) + \frac{a_1}{a_2 - \lambda} (e^{-a_2 t} - e^{-\lambda t}). \]

As a result, the following differential equation describes the rate of change in demand for an item:

\[ D_m' = C_{ACM} - \lambda D_m \text{ with } D(0) = D_0. \]

Where \( a(t) = a_0 - a_1 e^{-a_2 t}, 0 < t < T_\ell \), where \( a_0, a_1, a_2 \) are known parameters and \( 0 < \lambda < 1 \) (Manna et al. [25]).

2. This study considered carbon emissions and energy costs in terms of environmental criteria while calculating holding costs and deterioration costs for manufacturing and the manufacturing cycle (Singh et al.[44]).

3. During manufacturing and remanufacturing, the production rate exceeds the rate of demand. In the beginning, the stock level is zero during manufacturing and remanufacturing (Rani et al.[27]).

4. During manufacturing and remanufacturing, the production cycle is single. Lead time is zero (Rani et al. [29]).

5. Remanufactured products have the exact cost and demand as they are like the new products (Saxena et al. [37] and El Saadany [12]).

6. Returned products are gathered and put through a waste-recovery procedure. Parameter \( \mu_r \) is the remanufactured product recovery rate (Rani et al. [29]).

7. The rate of deterioration is different for manufacturing parameters and retailers' manufacturing parameters (Kumar et al. [22]).

3.3 Notation

The model uses the following notation:

Manufacturing parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_m )</td>
<td>demand rate parameter: it is a function of advertisement and demand rate of market</td>
</tr>
<tr>
<td>( \alpha, \beta )</td>
<td>production rate parameters (units) for production ((\alpha + \beta t)) where ( \alpha &gt; \beta )</td>
</tr>
<tr>
<td>( C_{SCM} )</td>
<td>setup cost ($/setup)</td>
</tr>
<tr>
<td>( C_{DCM} )</td>
<td>deterioration cost ($/unit/unit time)</td>
</tr>
<tr>
<td>( C_{DCM1} )</td>
<td>carbon emissions owing to deterioration are included in the cost ($/unit/unit time)</td>
</tr>
<tr>
<td>( C_{HC} )</td>
<td>holding cost ($/unit/unit time)</td>
</tr>
<tr>
<td>( C_{HC1} )</td>
<td>costs include carbon emissions from holding items ($/unit/unit time)</td>
</tr>
<tr>
<td>( C_{CMD} )</td>
<td>production cost ($/unit)</td>
</tr>
<tr>
<td>( C_{PCM} )</td>
<td>procurement cost</td>
</tr>
<tr>
<td>( C_{ACM} )</td>
<td>advertisement cost ($/advertisement)</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>deterioration rate</td>
</tr>
<tr>
<td>( a(t) )</td>
<td>advertisement rate</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>depreciation rate</td>
</tr>
<tr>
<td>( t_{m1} )</td>
<td>time when the inventory is highest (unit time)</td>
</tr>
</tbody>
</table>
time when the inventory reaches a minimum \( t_{m2} \),

\( Q_{m1}(t) \) at time \( t \), inventory levels are at a certain level in the range \( 0 \leq t \leq t_{m1} \)

\( Q_{m2}(t) \) at time \( t \), inventory levels are at a certain level. in the range \( t_{m1} \leq t \leq t_{m2} \)

\( I_m \) highest quantity during manufacturing

**Remanufacturing parameters**

- \( D_r \) demand rate parameter
- \( \delta_r \) production rate parameter \( \text{[units/cycle]} \)
- \( C_{SCR} \) set up cost \( \text{[$/setup]} \)
- \( C_{DCR} \) deterioration cost \( \text{[$/unit/unit time]} \)
- \( C_{DCR1} \) carbon emissions owing to deterioration are included in the cost \( \text{[$/unit/unit time]} \)
- \( C_{HCR} \) holding cost \( \text{[$/unit/unit time]} \)
- \( C_{HCR1} \) costs include carbon emissions from holding items \( \text{[$/unit/unit time]} \)
- \( C_{PCR} \) procurement cost \( \text{[$/unit]} \)
- \( d_1 \) deterioration rate
- \( t_{r1} \) time when remanufacturing of inventory starts
- \( t_{r2} \) time when the inventory reaches the highest
- \( T_t \) time when the inventory reaches the minimum
- \( Q_{r1}(t) \) at time \( t \), inventory levels are at a certain level in the range \( t_{r1} \leq t \leq t_{r2} \).
- \( Q_{r2}(t) \) at time \( t \), inventory levels are at a certain level in the range \( t_{r1} \leq t \leq T_t \).
- \( I_r \) highest quantity during manufacturing.

**Collection parameters for collected products**

- \( I_c \) collected products level at \( t = t_{m2} \)
- \( T_c \) time when the stock reaches zero
- \( Q_{c1}(t) \) at time \( t \), inventory levels are at a certain level during the cycle of collection in the range \( t_{r1} \leq t \leq t_c \).
- \( Q_{c2}(t) \) at time \( t \), inventory levels are at a certain level in the range \( t_c \leq t \leq T_c \).
- \( \eta_r \) recovery rate
- \( \mu_r \) returned rate

**Retailer's manufacturing parameters**

- \( p, q \) demand rate parameter \( \text{[units]} \)
- \( \alpha, \beta \) production rate parameters \( \text{[units]} \) for production \( (\alpha + \beta t) \) where \( \alpha > \beta \).
- \( C_{PCR} \) purchasing cost \( \text{[$/unit]} \)
- \( C_{DCRM} \) deterioration cost \( \text{[$/unit/unit time]} \)
- \( C_{DCRM1} \) carbon emissions owing to deterioration are included in the cost \( \text{[$/unit/unit time]} \)
- \( C_{HCRM} \) holding cost \( \text{[$/unit/unit time]} \)
- \( C_{DCRM1} \) carbon emission cost \( \text{[$/unit/unit time]} \)
- \( C_{OCR} \) ordering cost \( \text{[$/order]} \)
\[ \begin{align*} 
\frac{d}{dt} & = \text{deterioration rate} \\
\tau & = \text{time when the inventory is zero} \\
Q_m(t) & = \text{at time t, inventory levels are at a certain level in the range } 0 \leq t \leq \tau \\
I_{rm} & = \text{initial level of quantity during retailer's cycle} \\
C_1 & = \text{number of cycles} \\
\end{align*} \]

**Retailer's remanufacturing parameters**

- \( C_{PCRR} \): purchasing cost \( \)\$/unit\)
- \( C_{DCRR} \): deterioration cost \( \)\$/unit/unit time\)
- \( C_{DCRR1} \): carbon emission cost \( \)\$/unit/unit time\)
- \( C_{HCRR} \): holding cost \( \)\$/unit/unit time\)
- \( C_{HCRR1} \): costs include carbon emissions from holding items \( \)\$/unit/unit time\)
- \( C_{OCRR} \): ordering cost \( \)\$/order\)

\[ \begin{align*} 
t & = \text{time when the inventory is zero} \\
Q_r(t) & = \text{at time t, inventory levels are at a certain level in the range } 0 \leq t \leq t_r \\
C_2 & = \text{number of cycles} \\
\end{align*} \]

**Decision variables**

- \( t_{m1} \): time when the inventory is highest \( \text{(unit time)} \)
- \( t_{m2} \): time when the inventory reaches a minimum \( \text{(unit time)} \)
- \( t_{r1} \): time when remanufacturing of inventory starts \( \text{(unit time)} \)
- \( T_t \): time when the inventory reaches the minimum \( \text{(unit time)} \)

**4. FORMULATION OF MATHEMATICAL MODEL**

This model assumes that a single manufacturing cycle is followed by a single remanufacturing cycle. Manufacturing continues until \( t_{m1} \), at which point it is stopped. Stock is reduced due to demand and degradation. The overall cycle time is time \( t_{m2} \). Items that have been used are collected and returned to the manufacturer for recycling and remanufacturing. Production runs from \( t_{m2} \) to \( t_{r1} \) during remanufacturing. Afterwards, remanufacturing is halted. It is supposed that one production cycle has \( C_1 \) retail cycles and a single remanufacturing cycle has \( C_2 \) retail cycles, resulting in a total cycle time of \( T_t \). The customer is thought to be concerned about the environment. The rate of deterioration is supposed to be constant. Figure 2 shows this inventory flow.

**4.1 Manufacturing cycle**

For Manufacturing cycle, level of inventory for suppliers at time \( t \), rate of deterioration \( d_1 \), rate of production is linear \( \alpha + \beta t \), and advertisement demand is \( D_m \).

\[ \frac{d Q_{m1}(t)}{dt} + d_1 Q_{m1}(t) = (\alpha + \beta t) - D_m, 0 \leq t \leq t_{m1} \]  
\[ \frac{d Q_{m2}(t)}{dt} + d_1 Q_{m2}(t) = -D_m, t_{m1} \leq t \leq t_{m2} \]  

From Equation (1), one can find

\[ Q_{m1}(t)e^{d_1t} = \int (\alpha + \beta t) - D_0 e^{-\lambda t} + \frac{a_0}{\lambda} (1 - e^{-\lambda t}) + \frac{a_1}{a_2 - \lambda} (e^{-a_1 t} - e^{-\lambda t}) e^{d_1 t} + C_m \]
\[ Q_{m1}(t)e^{d_1t} = \left[ \frac{e^{d_1t}}{d_1} \left( \alpha - \frac{a_0}{\lambda} - \frac{\beta}{d_1} + \beta t \right) - \left( \frac{a_1e^{(d_1-a_2)t}}{(a_2-\lambda)(d_1-a_2)} \right) \right] + C_m \] (4)

At \( t = 0, Q_{m1}(t) = 0 \)

\[ Q_{m2}(t)e^{d_1t} = - \int \left( D_0 e^{-\lambda t} + \frac{a_0}{\lambda} (1 - e^{-\lambda t}) + \frac{a_1}{a_2-\lambda} (e^{-a_2t} - e^{-\lambda t}) \right) e^{d_1t} dt + C_{m2} \] (7)

\[ Q_{m2}(t)e^{d_1t} = - \left[ \left( \frac{e^{(d_1-a_2)t}}{d_1-\lambda} \right) \left( D_0 - \frac{a_0}{a_2-\lambda} + \frac{a_0e^{d_1t}}{\lambda d_1} \right) + \frac{a_1e^{(d_1-a_2)t}}{(a_2-\lambda)(d_1-a_2)} \right] + C_{m2} \] (8)

At \( t = t_{m2}, Q_{m2}(t) = 0 \)

\[ Q_{m2}(t) = \left[ \left( \frac{1}{d_1-\lambda} \right) \left( D_0 - \frac{a_0}{a_2-\lambda} \right) e^{(d_1-a_2)t_{m2}} - e^{-\lambda t} + \frac{a_0}{\lambda d_1} (e^{d_1t_{m2}} - 1) \right] + \left( \frac{a_1}{(a_2-\lambda)(d_1-a_2)} \right) e^{(d_1-a_2)t_{m2}} - e^{-a_2t} \] (10)

Now, the following are the various supplier costs.
4.1.1 Setup cost
This is a fixed cost for establishing the manufacturing substructure. It is calculated as follows:

\[ CM_{SC} = C_{SCM} \] (11)

4.1.2 Advertisement cost
The total cost of advertising in the manufacturing organization throughout the cycle is provided by

\[ CM_{AC} = C_{ACM} \int_0^{t_{m2}} a(t) dt = C_{ACM} \left[ a_0 t_{m2} + \frac{a_1}{a_2} \left( e^{-a_2 t_{m2}} - 1 \right) \right] \] (12)

4.1.3 Production cost
All expenditures incurred by a firm due to creating a product or offering a service are referred to as production costs. The following is a formula for calculating the rate of production:

\[ CM_{PDC} = C_{PDCM} \int_0^{t_{m1}} (\alpha + \beta t) dt = C_{PDCM} \left( \alpha t_{m1} + \frac{\beta t_{m1}^2}{2} \right) \] (13)

4.1.4 Procurement Cost
This includes a variety of expenses related to the acquisition of raw materials, equipment, and other goods during the production process and is calculated as follows:

\[ CM_{PC} = C_{PCM} \int_0^{t_{m1}} Q_{m1}(t) dt \]

\[ = C_{PCM} \left[ \frac{\beta t_{m1}^2}{2d_1} + \frac{1}{d_1^2} \left( \left( \alpha - \frac{a_0}{\lambda} - \frac{\beta}{d_1} \right) \left( d_1 t_{m1} + e^{-d_1 t_{m1}} - 1 \right) \right) + \left( 1 - e^{-d_1 t_{m1}} \right) \left( \frac{1}{d_1(d_1-\lambda)} \right) \right] \]

\[ + \left( \frac{D_0}{\lambda} - \frac{a_0}{\lambda} - \frac{a_1}{d_2-\lambda} \right) + \left( e^{-d_1 t_{m1}} - 1 \right) \left( \frac{1}{\lambda(d_1-\lambda)} \right) \left( D_0 - \frac{a_0}{\lambda} - \frac{a_1}{\lambda} \right) \]

\[ + \left( 1 - e^{-d_1 t_{m1}} \right) \left( \frac{a_1}{d(a_2-\lambda)(d_1-a_2)} \right) + \left( e^{-a_2 t_{m1}} - 1 \right) \left( \frac{a_1}{d_1-a_2} \right) \] (14)

4.1.5 Deterioration Cost
The cost of deterioration includes the costs of spoilage, wear and tear, expiration, and other variables that cause the value of the stock in hand to decrease.

In this model, deteriorating cost, which includes together the conventional deteriorating cost \( C_{DCM} \) and the carbon emission cost \( C_{DCM1} \) caused by decaying products, is:

\[ CM_{DC} = (C_{DCM} + C_{DCM1}) \left[ \int_0^{t_{m1}} d_1 Q_{m1}(t) dt + \int_{t_{m1}}^{t_{m2}} d_1 Q_{m2}(t) dt \right] \]
4.1.6 Holding Cost
Rent, security, storage space, and insurance are just a few of the expenditures associated with storing inventory.

There are two components to the holding cost in this case. One component is related with product holding as \( C_{HCM} \), while the other is carbon emission as \( C_{HCM1} \). Consequently, the total cost of holding is:

\[
CM_{HCM} = (C_{HCM} + C_{HCM1}) \left[ \int_0^{t_{m1}} Q_m(t) \, dt + \int_{t_{m1}}^{t_{m2}} Q_m(t) \, dt \right]
\]

\[
= (C_{HCM} + C_{HCM1}) \left[ \frac{\beta t_{m1}^2}{2d_1} + \frac{1}{d_1^2} \left\{ \left( \alpha - \frac{a_0}{\lambda} - \frac{\beta}{d_1} \right) (d_1 t_{m1} + e^{-d_1 t_{m1}}) \right\} 
+ \left( \frac{1}{\lambda (d_1 - \lambda)} \right) \left( D_0 - \frac{a_0}{\lambda} - \frac{a_1}{\lambda - \lambda} \right) \left( \frac{1}{d_1} (1 - e^{-d_1 t_{m1}}) + (t_{m2} - t_{m1}) e^{(d_1 - \lambda) t_{m2}} \right) 
+ \left( \frac{1}{d (a_2 - \lambda) (d_1 - a_2)} \right) \left( \frac{1}{d_1} (1 - e^{-d_1 t_{m1}}) + (t_{m2} - t_{m1}) e^{(d_1 - a_2) t_{m2}} \right) 
+ \frac{a_0}{a_2} (t_{m2} - t_{m1}) (e^{d_1 t_{m2}} - 1) \right]
\]

\[(15)\]

The total average cost for a supplier is calculated as follows:

\[
(\text{TCM})_M = \frac{1}{t_{m2}} (CM_{SC} + CM_{AC} + CM_{PDC} + CM_{PC} + CM_{DC} + CM_{HCM})
\]

\[(17)\]

4.2 Retailer's inventory from manufactured products
By assuming \( d_1 \) as the rate of deterioration and demand is a linear function, then the level of inventory at time \( t \) is given as:

\[
\frac{dQ_m(t)}{dt} + d_2 Q_m(t) = -(p + q t), 0 \leq t \leq t_m
\]

\[(18)\]
At \( t = t_m \), \( Q_m(t) = 0 \)  
\[ Q_m(t)e^{d_2t} = -\int (p + qt) e^{d_2t} dt \]  
\[ Q_m(t) = \left[ \frac{e^{d_2(t-m-t)}}{d_2} \left( p + qt_m - \frac{q}{d_2} \right) - \frac{1}{d_2} \left( p + qt - \frac{q}{d_2} \right) \right] \]  
(20)  
The following are the different retailer costs:  
Ordering cost \( CR_{OCR} = C_{OCR M} \)  
Purchasing cost \( CR_{PCR} = C_{PCR M} Q_m(t = 0) \)  
\[ = C_{PCR M} \left[ \frac{e^{d_2tm}}{d_2} \left( p + qt_m - \frac{q}{d_2} \right) - \frac{1}{d_2} \left( p - \frac{q}{d_2} \right) \right] \]  
(22)  
Deterioration cost \( CR_{DCR} = (C_{DCRM} + C_{DCRM1}) \int_0^{t_m} d_2 Q_m(t)dt \)  
\[ = (C_{DCRM} + C_{DCRM1}) \left[ \frac{1}{d_2} \left( p + qt_m - \frac{q}{d_2} \right) \left( e^{d_2tm} - 1 \right) - t_m \left( p + qt_m - \frac{q}{d_2} \right) \right] \]  
(23)  
Holding cost \( CR_{HCR} = (C_{HCRM} + C_{HCRM1}) \int_0^{t_m} Q_m(t)dt \)  
\[ = (C_{HCRM} + C_{HCRM1}) \left[ \frac{1}{d_2} \left( p + qt_m - \frac{q}{d_2} \right) \left( e^{d_2tm} - 1 \right) - t_m \left( p + qt_m - \frac{q}{d_2} \right) \right] \]  
(24)  
For retailer's model, the total average cost for a retailer is calculated as follows:  
\[ (TCR)_M = \frac{1}{t_m} (CR_{OCR} + CR_{PCR} + CR_{DCR} + CR_{HCR}) \]  
(25)  
This study assumes that total cycles for retailer products are \( C_1 \). Cycle time for the retailer is \( t_m \).  
So, \( t_m \) is written as:  
\[ t_m = \frac{t_{m2}}{C_1} \]  
(26)  
As a consequence, the overall average cost of the manufacturing cycle is:  
Total Cost \([TC_M(t_{m1}, t_{m2})] = (TCM)_M + (TCR)_M \)  
\[ TC_M(t_{m1}, t_{m2}) = \frac{1}{t_{m2}} \left( CM_{SC} + CM_{AC} + CM_{PDC} + CM_{PC} + CM_{DC} + CM_{HCM} \right) + \frac{C_1}{t_{m2}} (CR_{OCR} + CR_{PCR} + CR_{DCR} + CR_{HCR}) \]  
(27)  
### 4.3 Collection inventory  
Consumers return used products to retailers, transporting them back to the manufacturer. The return rate for all products is expected to be the same. After that, the products are treated and remanufactured. During remanufacturing, assuming \( \eta_r, \mu_r \) and \( \delta_r \) as the returned rate, recovery rate, and production rate parameter, the level of collection inventory is determined by:  
\[ \frac{dQ_{C1}(t)}{dt} = \eta_r \mu_r(D_r + D_o) - \delta_r, T_{C1} \leq t \leq T_{C2} \]  
(28)  
At \( t = T_{C1}, Q_{C1}(t) = I_c \)  
\[ Q_{C1}(t) = [\eta_r \mu_r(D_r + D_o) - \delta_r](t - I_{C1}) + I_c \]  
(30)  
\[ \frac{dQ_{C2}(t)}{dt} = \eta_r \mu_r(D_r + D_o), T_{C2} \leq t \leq T_{t+t_{C1}} \]  
(31)  
At \( t = T_{C2}, Q_{C1}(t) = 0, Q_{C2}(t) = 0 \)  
\[ Q_{C2}(t) = [\eta_r \mu_r(D_r + D_o)](t - T_{C2}) \]  
(33)  
At \( t = T_t + T_{C2}, Q_{C2}(t) = I_c \)  
\[ \delta_r = [\eta_r \mu_r(D_r + D_o)] - \frac{I_c}{T_{C1-T_{C2}}} \]  
(35)
\[ T_{C1} - T_{C2} = T_t \frac{I_C}{\eta_r \mu_r (D_r + D_0)} \]  \hspace{1cm} (36)

### 4.4 Remanufacturing cycle

Using the production rate as \( \delta_r \), demand as \( D_r \) and the deterioration rate as \( d_1 \), at time \( t \), the following is the inventory level:

\[
\frac{dQ_1(t)}{dt} + d_1 Q_1(t) = \delta_r - D_r, t_{m2} \leq t \leq t_{r1} \tag{37}
\]
\[
\frac{dQ_2(t)}{dt} + d_1 Q_2(t) = -D_r, t_{r1} \leq t \leq T_t \tag{38}
\]

From Equation (37), we get

\[
Q_1(t)e^{d_1t} = \int(\delta_r - D_r)e^{d_1t}dt + C_{r1}
\]

\[
Q_1(t)cdot e^{d_1t} = \left(\frac{\delta_r - D_r}{d_1}\right)e^{d_1t} + C_{r1}
\]

At \( t = t_{m2} \), \( Q_1(t) = 0 \)

\[
Q_1(t) = \left(\frac{\delta_r - D_r}{d_1}\right)\{1 - e^{(t_{m2}-t)d_1}\}
\]

From Equation (38), we get

\[
Q_2(t)cdot e^{d_1t} = \left[-\frac{D_r}{d_1}\right]e^{d_1t} + C_{r2}
\]

At \( t = T_t \), \( Q_2(t) = 0 \)

\[
Q_2(t) = \left(\frac{D_r}{d_1}\right)e^{(T_t-t)d_1} - 1
\]

The costs of several remanufacturers are listed below.

Set up cost \( C_{rSC} = C_{SCR} \)

Production cost \( C_{rPDC} = C_{PDCR} \int_{t_{m2}}^{t_{r1}} \delta_r dt = C_{PDCR} \delta_r (t_{r1} - t_{m2}) \)

Procurement cost \( C_{rPC} = C_{PCR} \int_{t_{m2}}^{t_{r1}} Q_1(t) dt = C_{PCR} \left(\frac{\delta_r - D_r}{d_1}\right)\left\{\left(t_{r1} - t_{m2}\right) + \frac{1}{d_1}\left(e^{(t_{m2} - t_{r1})d_1} - 1\right)\right\} \)

Deterioration cost \( C_{rDC} = (C_{DCR} + C_{DCR1}) \int_{t_{m2}}^{t_{r2}} d_1 Q_1(t) dt + \int_{t_{r2}}^{T_t} d_1 Q_2(t) dt \)

\[
= (C_{DCR} + C_{DCR1})\left[\delta_r (t_{r2} - t_{m2}) - D_r (T_t - t_{m2}) + \left(\frac{D_r}{d_1}\right)\left(e^{(t_{m2} - t_{r2})d_1} - e^{(T_t - t_{r2})d_1}\right)\right] \tag{48}
\]

Holding cost \( C_{rHC} = (C_{HCR} + C_{HCR1}) \int_{t_{m2}}^{t_{r1}} Q_1(t) dt + \int_{t_{r1}}^{T_t} Q_2(t) dt \)

\[
= (C_{HCR} + C_{HCR1})\left(\frac{1}{d_1}\right)\left[\delta_r (t_{r1} - t_{m2}) + D_r (t_{m2} - T_t) + \left(\frac{D_r}{d_1}\right)\left(e^{(t_{m2} - t_{r1})d_1} - 1\right)\right] + \left(\frac{D_r}{d_1}\right)\left(e^{(T_t - t_{r1})d_1} - e^{(t_{m2} - t_{r1})d_1}\right) \tag{49}
\]

The total average cost of remanufacturing is calculated as follows:
\[(TCr)_R = \frac{1}{\tau_1-t_{m2}}(Cr_{SC} + Cr_{PDC} + Cr_{PC} + Cr_{DC} + Cr_{HC})\]

4.5 Retailer's model from remanufacturing inventory

By assuming \(d_2\) as the rate of deterioration and demand is a linear function, the level of inventory at time \(t\) is given as:

\[
\frac{dQ_r(t)}{dt} + d_2 Q_r(t) = -(p + qt), \quad 0 \leq t \leq \tau_r
\]

At \(t = \tau_r\), \(Q_r(t) = 0\)

\[
Q_r(t) = \left(\frac{a}{d_2}\right) \left[\left(\frac{a}{d_2} - p - qt\right) + e^{(\tau_r-t)d_2}\left(p + qt_r - \frac{q}{d_2}\right)\right]
\]

The following are the different retailer costs.

Ordering Cost \(CR_{OCR} = C_{OCR}\)

Purchasing Cost \(CR_{PCR} = C_{PCR}Q_r(t=0)\)

\[
= C_{PCR}\left(\frac{1}{d_2}\right) \left[\left(\frac{a}{d_2} - p\right) + e^{\tau_r d_2}\left(p + qt_r - \frac{q}{d_2}\right)\right]
\]

Deterioration Cost \(CR_{DCR} = \left(C_{DCRR} + C_{DCRR1}\right) \int_0^{\tau_r} d_2 Q_r(t)dt\)

\[
= (C_{DCRR} + C_{DCRR1}) \left[\left(\frac{a}{d_2} - p t_r - \frac{q t_r^2}{2}\right) + \frac{1}{d_2}\left(p + qt_r - \frac{q}{d_2}\right)\left(e^{\tau_r d_2} - 1\right)\right]
\]

Holding Cost \(CR_{HCR} = \left(C_{HCRR} + C_{HCRR1}\right) \int_0^{\tau_r} Q_r(t)dt\)

\[
= (C_{HCRR} + C_{HCRR1}) \left(\frac{1}{d_2}\right) \left[\left(\frac{a}{d_2} - p t_r - \frac{q t_r^2}{2}\right) + \frac{1}{d_2}\left(p + qt_r - \frac{q}{d_2}\right)\left(e^{\tau_r d_2} - 1\right)\right]
\]

The total average cost for a retailer is calculated as follows:

\[(TCR)_R = \frac{1}{\tau_r} \left(CR_{OCR} + CR_{PCR} + CR_{DCR} + CR_{HCR}\right)\]

This study assumes that the total cycles for retailer products is \(C_2\). Cycle time for the retailer is \(\tau_r\). So, \(\tau_r\) is written as:

\[
\tau_r = \frac{\tau_1-t_{m2}}{C_2}
\]

As a consequence, the overall average cost of the remanufacturing cycle is calculated as follows:

Total Cost \([TCR(t_{r1},T_r)] = (TCR)_R + (TCR)_R\)

\[
TCR(t_{r1},T_r) = \frac{1}{\tau_1-t_{m2}} (Cr_{SC} + Cr_{PDC} + Cr_{PC} + Cr_{DC} + Cr_{HC}) + \frac{C_2}{\tau_1-t_{m2}} (CR_{OCR} + CR_{PCR} + CR_{DCR} + CR_{HCR})
\]

5. SOLUTION METHODOLOGY

The prime goal of this work is to reduce the overall cost functions of manufacturing and remanufacturing in the provided equations (27A) and (60A). (See appendices)

The manufacturing part is processed in Mathematica-9 software under \(0 < t_{m1} < t_{m2}\).

Put \(t_m = \frac{t_{m2}}{C_1}\) other parameters. Therefore, find the values \(t_{m1}^\ast, \quad t_{m2}^\ast\), and minimize total cost.
The remanufacturing part is processed in Mathematica-9 software under $0 < t_{r1} < T_t$ and satisfied Hessian matrix.

This research gets the following result from Equation (60A).

$$\frac{\partial T_C}{\partial t_{r1}} = 0 \quad \text{and} \quad \frac{\partial T_C}{\partial T_t} = 0$$

It is difficult to identify the optimal values of $t_{r1}^*$ and $T_t^*$; hence, the Mathematica software is utilized to do it.

The Hessian matrix,

$$H_{RC} = \det \begin{pmatrix} \frac{\partial^2 T_C}{\partial t_{r1}^2} & \frac{\partial^2 T_C}{\partial t_{r1} \partial T_t} \\ \frac{\partial^2 T_C}{\partial t_{r1} \partial T_t} & \frac{\partial^2 T_C}{\partial T_t^2} \end{pmatrix} > 0 \quad \text{and} \quad \frac{\partial^2 T_C}{\partial t_{r1}^2} > 0, \quad \frac{\partial^2 T_C}{\partial T_t^2} > 0$$

function's sufficient condition.

The total minimum-cost function for remanufacturing is then calculated using Equation (60A).

**Algorithm**

1. First, the total cost function is determined as $T_C(t_m, t_R)$ using the value of the total cost function $t_f = \frac{T_t - t_{r1}}{c_2}$.
2. The values of $t_{r1}^*$ and $T_t^*$ are then determined by considering the required criteria $\frac{\partial T_C}{\partial t_{r1}} = 0$ and $\frac{\partial T_C}{\partial T_t} = 0$.
3. The Hessian matrix is satisfied by the sufficient condition as follows:

$$H_{RC} = \det \begin{pmatrix} \frac{\partial^2 T_C}{\partial t_{r1}^2} & \frac{\partial^2 T_C}{\partial t_{r1} \partial T_t} \\ \frac{\partial^2 T_C}{\partial t_{r1} \partial T_t} & \frac{\partial^2 T_C}{\partial T_t^2} \end{pmatrix} > 0 \quad \text{and} \quad \frac{\partial^2 T_C}{\partial t_{r1}^2} > 0, \quad \frac{\partial^2 T_C}{\partial T_t^2} > 0.$$

4. This research then determines the values of $t_{m1}, t_{m2}, t_{r1}$, and $T_t$.
5. Lastly, this research determines the total cost function of manufacturing and remanufacturing's minimum value.

**6. NUMERICAL EXAMPLE**

This section determines the optimal time and minimum total cost by utilizing Mathematica-9.0 as well as necessary input parameter values (Manna et al.[25] and Rani et al.[29]).

**Example 1.** In this study during manufacturing take value as $C_{SCM} = 92$/setup, $C_{ACM} = 81$/advertisement, $a_0 = 24$, $a_1 = 11$, $a_2 = 0.25$, $C_{PDCM} = 74$/unit, $C_{PCM} = 10$/unit, $\alpha = 77$ units, $d_1 = 0.9$, $D_0 = 34$ units, $\beta = 0.5$ units, $\lambda = 0.27$, $C_{DCM} = 15$/unit/unit time, $C_{DCM1} = 3$/unit/unit time, $C_{HC} = 77$/unit/unit time, $C_{HC1} = 3$/unit/unit time, $C_{DCRM} = 89$/order, $C_{PCRM} = 66.5$/unit, $p = 55$ units, $q = 0.1$ units, $d_2 = 0.3$, $C_{DCRM} = 44$/unit/unit time, $C_{DCRM1} = 6$, $C_{HC} = 45$/unit/unit time, $C_{HC1} = 3$/unit/unit time. The best possible value of total cost is 20562.4, $t_{m1}$ is 0.41 and $t_{m2}$ is 0.46. $t_m$ can be found out from
Equation (33) and $t_m = \frac{t_{m2}}{c_1}$. The entire numerical is examined using the Mathematica-9.0 software.

Example 2. In this study during remanufacturing take value as $C_{SCR} = 198$/setup, $C_{PDCR} = 175$/unit, $C_{PCR} = 10$/unit, $\alpha = 97$ units, $d_1 = 0.9$, $\delta_r = 89$ units/cycle, $\beta = 0.5$ units, $\lambda = 0.27$, $C_{DCR} = 36$/unit/unit time, $C_{DCR1} = 4$/unit/unit time, $C_{HCR} = 60$/unit/unit time, $C_{HCR1} = 5$/unit/unit time, $C_{PCR} = 84$/order, $C_{PCRR} = 195$/unit, $p = 55$ units, $q = 0.1$ units, $d_2 = 0.3$, $C_{DCR} = 40$/unit/unit time, $C_{DCR1} = 5$/unit/unit time, $C_{HCR} = 60$/unit/unit time, $C_{HCR1} = 4$/unit/unit time, $\eta_r = 0.92$, $\mu_r = 0.69$, $D_r = 83$, $D_0 = 34$, $l_c = 40$, $t_{m2} = 0.46$ unit. Total cost has the best possible value of 3261.21, $t_{r1}$ is 0.98, and $T_t$ is 1.14. Equation (74) and $t_r = \frac{T_t-t_{r1}}{c_2}$ can be used to calculate $t_r$. The entire numerical is evaluated using the Mathematica-9.0 software.

This study may validate numerically through the Hessian matrix.

And satisfied condition $\det \left( \frac{\partial^2 rC_R}{\partial t_1^2} \frac{\partial^2 rC_R}{\partial t_1 \partial t_t} \right) > 1.3 \times 10^8 > \frac{\partial^2 rC_2}{\partial t_1^2} > 7247.41 > 0$, $\frac{\partial^2 rC_2}{\partial T_t^2} > 22358.8 > 0$ and $\frac{\partial^2 rC_2}{\partial t_1 \partial T_t} = -5410.03$ for the optimal value $t_{r1}^*, T_t^*$.

### 7. Sensitivity Analysis

This component of the research looks at the consequences of changing organization parameters in order to conduct a sensitivity of the planned model by regard to a few parameters.

#### TABLE 2. Total cost during manufacturing and remanufacturing

<table>
<thead>
<tr>
<th>During manufacturing</th>
<th>During remanufacturing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{m1}$</td>
<td>$t_{m2}$</td>
</tr>
<tr>
<td>0.41</td>
<td>0.46</td>
</tr>
</tbody>
</table>

#### TABLE 3. Total cost change with respect to advertisement cost

<table>
<thead>
<tr>
<th>$C_{ACM}$</th>
<th>Percentage change</th>
<th>$t_{m1}$</th>
<th>$t_{m2}$</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>-10%</td>
<td>0.41</td>
<td>0.46</td>
<td>20451.8</td>
</tr>
<tr>
<td>21</td>
<td>-5%</td>
<td>0.41</td>
<td>0.46</td>
<td>20507.1</td>
</tr>
<tr>
<td>21</td>
<td>0%</td>
<td>0.41</td>
<td>0.46</td>
<td>20562.4</td>
</tr>
<tr>
<td>21</td>
<td>+5%</td>
<td>0.34</td>
<td>0.37</td>
<td>21012.6</td>
</tr>
<tr>
<td>21</td>
<td>+10%</td>
<td>0.34</td>
<td>0.37</td>
<td>21129.3</td>
</tr>
</tbody>
</table>

#### TABLE 4. Total cost change with respect to retailer's manufacturing deterioration cost
**TABLE 5.** Total cost change with respect to retailer's manufacturing purchasing cost

<table>
<thead>
<tr>
<th>Percentage change</th>
<th>$t_{m1}$</th>
<th>$t_{m2}$</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{DCRM} \ (44)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-10%</td>
<td>0.41</td>
<td>0.46</td>
<td>20543.4</td>
</tr>
<tr>
<td>-5%</td>
<td>0.41</td>
<td>0.46</td>
<td>20552.9</td>
</tr>
<tr>
<td>0%</td>
<td>0.41</td>
<td>0.46</td>
<td>20562.4</td>
</tr>
<tr>
<td>+5%</td>
<td>0.44</td>
<td>0.46</td>
<td>20571.4</td>
</tr>
<tr>
<td>+10%</td>
<td>0.34</td>
<td>0.38</td>
<td>20980.4</td>
</tr>
</tbody>
</table>

**TABLE 7.** Total cost change with respect to change in various parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{SCR}$</td>
<td>3298.36</td>
</tr>
<tr>
<td>$C_{PCR}$</td>
<td>3378.11</td>
</tr>
<tr>
<td>$C_{DCR}$</td>
<td>3246.98</td>
</tr>
</tbody>
</table>

**FIGURE 3.** Flow of manufacturer's advertisement cost, retailer's manufacturing deterioration and purchasing cost
FIGURE 4. Flow of remanufacturing’s setup, procurement and deterioration cost

FIGURE 5. Flow of remanufacturing’s holding cost, retailer’s remanufacturing purchasing cost and demand rate (p)
Observations and managerial Insights

The manufacturing cycle results are shown in Figure 3 and tables 3, 4, and 5, whereas the remanufacturing cycle results are shown in Tables 7, Figures 4, 5, and 6. The following are the results of this study:

i. Figure 4 shows that adjusting the cost parameters $C_{SCR}$ and $C_{PCR}$ by a percentage reduces the overall cost of remanufacturing cycle. As a result, as the deterioration parameter $C_{DCR}$ was modified by a percentage, the overall cost increased.

ii. Table 3 demonstrates that a very tiny drop follows a change in advertising cost in manufacturing time $t_{m1}$ and $t_{m2}$, while the total cost is increased.

iii. As shown in Figure 5, the overall cost of remanufacturing cycle increased due to percentage increases ($-10, -5, 0, +5,$ and $+10$ percent) in the output parameters $C_{HCR}, C_{PCR}$ and $p$.

iv. When the manufacturing deterioration cost of a retailer $C_{DCRM}$ is changed, the manufacturing time $t_{m1}$ and $t_{m2}$ remain unchanged or decrease, resulting in a total cost rise as shown in table 4.

v. The overall cost of remanufacturing increased when the parameters $q$ and $C_{HCRR}$ were modified by a percentage in Figure 6. The overall cost of remanufacturing cycle reduced as the deterioration cost parameter $C_{DCRR}$ moved from negative to positive by a percentage change.

vi. Figure 3 indicates that the manufacturing cycle overall cost increased when the parameter $C_{PCRM}$ was adjusted by a percentage.

8. CONCLUSION

The advertisement has a positive effect on customer demand. This method considers several facets of the Green Supply Chain, such as waste collection, reverse logistics, and remanufacturing. Items that have been remanufactured are thought to be as excellent as new products. The manufacturing and remanufacturing models are solved individually because remanufacturing has its own set of costs, such as new infrastructure and labor, and modeling involves considering the collection cycle.
A supply chain inventory framework is designed in this work to pick the optimum total cost. It also allows analysis remanufacturing separately and assess its impact. The finding confirms that as advertising costs rise, the total cost rises. There was also a significant increase in the market's worth. As a result, the overall cost throughout the manufacturing cycle is higher than the overall cost during the remanufacturing cycle, as there is an item of expenditure on the advertisement cost during the manufacturing cycle. Furthermore, the holding and deterioration costs were spent on carbon emission, which is excellent for the environment. Some parameters showed fluctuations. Thus the necessary parameters are set to a certain value to evaluate the model's feasibility. This research could be useful in the smart products and garment industries in the real world.

The sensitivity analysis and case study yielded substantial managerial insights. Advertisements have a significant beneficial effect on industries by providing immediate information to the customer. As a result, the demand for industrial items is growing rapidly. This proposed study, which primarily consists of research on different reverse logistics services and techniques in the industries, has been used for both manufacturing and remanufacturing, but it does have some limitations. This study has not taken the same decision variables for manufacturing and remanufacturing cycles. The presented model can be modified in numerous ways for further research, including (1) allowing for trade-credit in the vendor-buyer integration and (2) extending this model for stock-dependent demand, shortage and backlog, inflation, and selling price. (3) This model can be modified to include a fuzzy environment. (4) This study can be extended to achieve environmental sustainability, such as Sarkar et al. [36], Bhuniya et al. [4] and Kumar et al. [22].

Appendices:

\[
TC_m(t_{m1}, t_{m2}) = \frac{1}{t_{m2}} \left( C_{SCM} + C_{ACM} \left( \frac{1}{a_2} \left( 1 + e^{-\alpha_2 t_{m2}} \right) a_1 \right) + a_0 t_{m2} \right) + C_{PDCM} \left( \frac{\beta t_{m1}^2}{2} + \alpha t_{m1} \right) +
\]

\[
C_{PDCM} \left( \frac{\lambda}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) + \frac{\beta t_{m1}^2}{2} \left( \frac{\alpha}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) + \frac{\beta t_{m1}}{2} \left( \frac{\alpha}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) +
\]

\[
D_0 t_{m1} \left( \frac{\lambda}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) + \frac{\beta t_{m1}}{2} \left( \frac{\alpha}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) + \frac{\beta t_{m1}}{2} \left( \frac{\alpha}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) +
\]

\[
\left( \frac{\lambda}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) + \frac{\beta t_{m1}}{2} \left( \frac{\alpha}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) + \frac{\beta t_{m1}}{2} \left( \frac{\alpha}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) +
\]

\[
\left( \frac{\lambda}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) + \frac{\beta t_{m1}}{2} \left( \frac{\alpha}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) + \frac{\beta t_{m1}}{2} \left( \frac{\alpha}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) +
\]

\[
\left( \frac{\lambda}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) + \frac{\beta t_{m1}}{2} \left( \frac{\alpha}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) + \frac{\beta t_{m1}}{2} \left( \frac{\alpha}{\lambda-a_2} a_2 \left( 1 + e^{-\alpha_2 t_{m1}} \right) a_1 \right) +
\]
\[
q(t_m) + \frac{(C_{DCM}+C_{DCRM})}{d_2^2}(2(-1 + e^{d_2 t_m})q + d_2(p - e^{d_2 t_m}p + t_m(-e^{d_2 t_m}q + e^{d_2 t_m}(-q + d_2(p + q t_m))))))
\]

\[
TC_R(t_{r1},T_t) = \frac{1}{(T_t-t_{m2})}\{C_{SCR} + C_{PDCR}\delta_r(t_{r1} - t_{m2}) + C_{PCR}\frac{(-1+e^{d_1(t_{m2}-t_{r1})})}{d_1} + \frac{(-t_{m2}+t_{r1})(-D_r+\delta_r)}{d_1} + (C_{DCR} + C_{DCR1})(\frac{-1+e^{d_1(t_{r1}+T_t)}}{d_1} + D_r(t_{r1} - T_t) + \frac{(-1+e^{d_1(t_{m2}-t_{r1})})}{d_1} - (D_r) + \delta_r)) + \frac{c_2}{(T_t-t_{m2})}\{C_{OCRR} + C_{PCR}(\frac{1-e^{d_2 t_r}q}{d_2} + \frac{(-1+e^{d_2 t_r}p + e^{d_2 t_r}q t_r)}{d_2}) - \frac{(C_{DCRR}+C_{DCRR1})}{d_2^2}(2(-1 + e^{d_2 t_r})q + d_2(p - e^{d_2 t_r}p + t_r(-e^{d_2 t_r}q + e^{d_2 t_r}(-q + d_2(p + q t_r)))) + \frac{(C_{HCRR}+C_{HCRR1})}{d_2^2}(2(-1 + e^{d_2 t_r})q + d_2(p - e^{d_2 t_r}p + t_r(-e^{d_2 t_r}q + e^{d_2 t_r}(-q + d_2(p + q t_r))))})
\]

\[\text{(27A)}\]

\[\text{....(60A)}\]

**REFERENCES**


