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Product rollover and direct sales decisions in dual-channel supply chains

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Abstract. This study investigates product rollover and direct sales decisions in a supply chain with one fashion manufacturer and one retailer when two style generations are sequentially introduced to the market over two periods. The retailer serves as the exclusive sales channel during the introductory period of a style generation. Both the retailer and manufacturer are capable of selling the old style generation in the second period. The retailer adopts dual rollover if she sells the old style generation, and single rollover if she does not. We develop a two-stage game to explore the chain members' equilibrium decisions while accounting for the manufacturer's cost inefficiency in direct sales and consumers' mental account deficit. We find that the manufacturer's cost inefficiency has double-edged effects and can offset the negative effect of consumers' mental account deficit on the introduction of a new style generation. Furthermore, a win-win outcome can be achieved when the manufacturer with an intermediate level of cost inefficiency engages in direct sales and consumers have high valuations of the old style generation or when the manufacturer with significant cost inefficiency does not engage in direct sales.

Keywords: supply chain, product rollover, fashion product, mental account deficit, manufacturer encroachment

1. Introduction

Advancements in research and development, manufacturing and information technologies, and logistics allow firms to introduce new products into the market with shorter design-to-market lead times. Apparel, computers, consumer electronics, and software products are among the industries in which new product introductions and updates are typically observed [10, 18, 29, 34].

Although there are potential benefits of gaining market share and maintaining competitiveness, new product introductions inevitably demand prudent planning of product rollover—the introduction of new generations and displacement of old generations—taking into consideration product and market risk factors [8]. Product rollover is interlaced with a number of relevant decisions, such as pricing, new product pre-announcement, and inventory control [23, 25, 30]. However, the academic literature on product rollover has paid little attention to firms' decisions pertinent to the underlying supply chain structure, one of the product risk factors discussed by Bilington et al. [8]. As such, questions regarding how cost- and consumer-related factors influence sequential product introductions in a supply chain with multiple sales channels remain unanswered. Take the fashion industry for example. Firms such as Zara, H&M, and Mango sell their products through their brand stores, whereas other firms such as Coach, Lacoste, Nike, and Michael Kors sell their products through independent retailers and their own outlets. Clearly, in the latter supply chain structures, the displacement decisions for old generations involve not only the firms themselves but also their retail partners. Specifically, when they decide to compete directly with retailers in the market, encroachment arises and creates channel conflict between supply chain partners [4, 42]. If ill managed, channel conflict will jeopardize firms' interests if the threatened retailers retaliate against them [9]. In fact, Levi Strauss & Co.

chose to close its direct channel to resolve the channel conflict with its e-retail partners [14]. Yet, manufacturer encroachment may not always hurt retailers due to asymmetric operational efficiency [4, 19]. Whether manufacturer encroachment can lead to a win-win outcome in considerations of product rollover remains unclear.

Furthermore, supply chain partners may have unequal bargaining power, thereby leading to different power structures in a supply chain. Earlier studies revealed that power structures have a significant impact on supply chain equilibrium decisions and profits (see Xue and Zhang [46] and the references therein). Therefore, in this paper, we aim toward an understanding of the interplay between manufacturer encroachment and product rollover in a supply chain setting under various power structures. To the best of our knowledge, this paper is the first attempt to unify both manufacturer encroachment and product rollover in a dynamic pricing model with sequential introduction of two product generations. We intend to answer the following questions:

(1) Can the manufacturer's direct sales lead to a win-win outcome?
(2) What are the effects of the manufacturer's cost inefficiency in direct sales on the introduction of a newer style generation and on equilibrium outcomes?
(3) How does the power structure in the supply chain affect the manufacturer's direct sales decision and the retailer's product rollover decision?

We base our analysis on a stylized two-period model with a fashion manufacturer placing a new style generation on the market in each period. The retailer serves as the exclusive sales channel during the introductory period of a style generation [2, 19, 20, 53]. Both the manufacturer and the retailer are capable of displacing the old style generation in the second period with asymmetric cost inefficiency. Encroachment occurs when the manufacturer (he) decides to sell an old style generation on his direct channel, whereas dual rollover at the retail level takes place when the retailer (she) decides to sell both new and old style generations in the second period. We develop a two-stage game model to characterize the interactive dynamics between the manufacturer and the retailer, while accounting for consumers' mental account deficit [36, 41], and allow for different power structures under which the chain members compete in the first-stage game.

The remainder of the paper is organized as follows: In Section 2, we review the existing literature on product rollover and manufacturer encroachment. In Section 3, we lay out the supply chain setting and develop a two-stage game. In Section 4, we establish and analyze the equilibrium under different power structures in the first-stage game. In Section 5, we provide several insights based on our analysis. Finally, we conclude and discuss future research directions in Section 6.

2. Literature Review

Our study is related to two streams of the literature. The first concerns firms' product rollover strategies when they introduce new products into the market. A two-period setting is commonly adopted in the literature, with a product being introduced in the first period and a newer product being introduced in the second period [17, 23, 25, 29, 32, 57]. For example, Levinthal and Purohit [25] investigated a firm's optimal sales strategy for durable products in a two-period setting and found that the firm should phase out the old version when the new version features a modest improvement. Ferguson and Koenigsberg [17] considered a two-period setting with deteriorating units in which random demand takes place in the first period and unsold units suffer from quality degradation after period 1 and studied the firm's optimal pricing and quantity decisions for any given quality degradation. Koca et al. [23] incorporated the pre-announcement of a new product, which creates a diffusion of awareness, and found that single rollover is preferred with faster diffusion and greater improvement in the new product. Liang et al. [29] considered a two-period setting with two customer types: high-end customers and bargain hunters. They found that the firm should adopt single rollover when the new product's degree of innovation is low and a majority of high-end customers are strategic. Zhou et al. [57] examined a fashion manufacturer deciding whether to launch a new style and whether to continue selling the previous style in the second period in a market comprising myopic consumers. They incorporated mental accounting for consumers who already bought the previous style in the first period and intended to buy the new style in the second period. They found that single rollover is preferred when consumers' mental account deficit is low and the production cost is high and that dual rollover is preferred when both of them are low. Liu et al. [32] investigated a firm's rollover strategy (single or dual rollover) and pricing scheme (price skimming or price penetration) in a two-period setting with strategic consumers being offered a trade-in program. They explored the conditions under which a specific rollover strategy or a specific pricing scheme is preferred and the conditions under which provision of a trade-in program yields no benefit to a firm. Ye et al. [50] studied product rollover strategies by factoring in the impact of the innovation level of the product introduced in the second period. They found that the rollover strategy is contingent on the innovation level and the customer's discounter factor when the innovation level is low to moderate, whereas single rollover is always preferable to dual rollover when the innovation level is high. Schwarz and Tan [38] focused on the production aspect of product rollover when a new product is introduced into the market and decided the optimal sales and production rollover strategies under limited production capacity. The above referenced studies focused on two-period settings with exogenous timing for the introduction of a new product. Differing from these studies, Lim and Tang [30] studied a firm determining the timing of the introduction of a new product and that of phasing out an old product, in addition to the pricing decisions. Arslan et al. [3] also investigated the timing of new product introduction by a firm and the pricing decision for multiple product generations. Koca et al. [24] investigated the introduction of a new version of a digital product that attracts new customers and entice existing customers to upgrade and showed that the choice of the release timing of the new product can induce a sufficiently large portion of the existing customers to upgrade and hence lead single rollover to become the optimal strategy.

When introducing a new product, manufacturers may implement trade-in programs to incentivize existing customers to buy the new product. For instance, Feng et al. [16] considered implementing a trade-in program through the retail channel and the direct channel for the segment of existing consumers and found that the trade-in program aggravates the double marginalization effect if the retailer is unable to decide the trade-in rebate in the retail channel. Xiao et al. [43] also considered implementing a trade-in program through the retail channel and the direct channel for the existing customers and found that the retailer would implement the trade-in policy voluntarily in a dual-channel case when the market size of the existing customers is relatively small. Quan et al. [37] studied a two-period setting in which a manufacturer sells products through a retail channel and implements the trade-in program, and customers who purchase products in period 1 and can trade in their used products in period 2 and found that both firms usually prefer to provide the trade-in service.

The second research stream concerns manufacturer (or supplier) encroachment, i.e., the manufacturer sells directly to consumers and hence competes with the retailer. Chiang et al. [13] showed that a manufacturer's direct channel benefits itself and can benefit the retailer when the reduction in the wholesale price outweighs the reduction in the selling price due to intensified competition. Tsay and Agrawal [42] included demand-improving sales effort in both the manufacturer's direct channel and the reseller's channel and found that a wholesale price reduction can reflect the reseller's sales efforts and benefit both of them. Arya et al. [4] also found that manufacturer encroachment benefits the manufacturer and the retailer when the manufacturer's direct selling cost is relatively high. Cai

[11] showed that manufacturer encroachment increases the manufacturer's and retailer's profits when the retail channel has a sufficiently higher base demand or operational efficiency than the manufacturer's direct channel. Ha et al. [19] investigated manufacturer encroachment with endogenous product quality and found that encroachment is likely to harm the retailer's profits when product quality is endogenous and product quality differentiation is achievable. The above studies focused on encroachment under information symmetry in a single selling period. Recent studies considering encroachment under information asymmetry in a single selling period include Li et al. [27, 28], Huang et al. [22], Yang et al. [49], and Zhang et al. [52], and recent studies considering encroachment under information symmetry in two-period settings include Xiong et al. [45] and Yan et al. [48]. More recently, Chen et al. [12] studied a supplier encroachment problem in a supply chain in which the supplier and the retailer exhibit different risk attitudes and showed that in the supplier-led Stackelberg game, the combination of the supply chain members with preferred risk attitudes does not necessarily benefit the supply chain. Liu et al. [31] considered a supply chain with one supplier and multiple retailers and showed that supplier encroachment hurts the supplier when the number of retailers is sufficiently large. Li et al. [26] explored supplier encroachment in a two-period setting in which the supplier sells through a direct retail subsidiary as well as an independent retailer and the latter retailer can carry strategic inventory in the first period to counter supplier encroachment. They found that when the subsidiary decides on quantities and retail prices, both the supplier and the retailer are better off in the presence of strategic inventory. Zhang et al. [55] studied a

supplier selling products through a platform that acts as a reseller and also allows the supplier to engage in direct sales and found that the platform's investment in service can be used to influence the supplier's encroachment decision. Balasubramanian and Maruthasalam [6] examined manufacturer encroachment in a supply chain in which the manufacturer sells the national brand through the retailer and the direct channel, and the retailer sells the store brand and the manufacturer's national brand. Their analysis revealed that manufacturer encroachment could benefit the manufacturer, the retailer, and the consumer in the presence of the store brand. Hotkar and Gilbert [21] explored supplier encroachment in a supply chain in which two suppliers sell substitutable products through a non-exclusive reseller and one of them can encroach by selling through the direct channel and showed that when product substitutability is high, supplier encroachment makes the reseller worse off.

With regard to multi-channel supply chains, power structure has been reported to have great influence on individual chain members' performance and supply chain performance [33, 39, 44, 46, 47, 51, 54, 56]. This led to exploration of manufacturer encroachment under different power structures. For example, Xiao et al. [44] examined a retailer-Stackelberg supply chain in which the manufacturer decides the product variety and channel strategies and found that manufacturer encroachment is more likely to occur under the retailer-Stackelberg model than under the manufacturer-Stackelberg model if the product variety cost is sufficiently high. Zheng et al. [56] explores the effect of manufacturer encroachment in a closed-loop supply chain under different power structures and found that the manufacturer always prefers encroachment, and the retailer can benefit from a manufacturer-led supply chain in which product competition is less intense. Zhang et al. [54] studied manufacturer encroachment and retail service investing in a retailer-led supply chain and found that retail service investing can effectively discourage manufacturer encroachment and may lead to Pareto improvement for both the supply chain members and consumers. Xue and Zhang [46] considered manufacturer encroachment in manufacturer-led and retailer-led supply chains and found that in the manufacturer-led supply chain, encroachment could benefit both the manufacturer and the retailer at high quality investment efficiency.

Our paper differs from the existing literature by investigating product rollover through multiple sales channels, which makes it possible to explore the interaction between the retailer's rollover decision and the manufacturer's encroachment decision and sheds light on how the manufacturer's direct selling cost, consumers' mental account deficit, and the supply chain power structure affect such interaction. For instance, Arya et al. [4] revealed that a win-win outcome can be achieved with manufacturer encroachment when the manufacturer's direct selling cost is relatively high. Our results demonstrated that a win-win outcome with no encroachment and dual rollover through the retail channel can be achieved. Furthermore, Zhou et al. [57] showed that high levels of consumers' mental account deficit always deter the firm from selling a newer style generation in the second period. Our results complemented their finding by demonstrating that the manufacturer's cost disadvantage could offset the negative effect of consumers' mental account deficit on the introduction of a newer style generation.

Table 1 below summarizes the related literature and this study in terms of various research attributes.

Table 1. Summary of related literature

Representative papers	Two product generations	Two periods	Product rollover	Manufacturer encroachment	Mental account deficit	Effects of power structures
Levinthal and Purohit [25]	√	√	Dual			
Ferguson and Koenigsberg [17]	√	√	Dual			
Koca et al. [23]	√	√	Single & Dual			
Liang et al. [29]	√	√	Single & Dual			
Zhou et al. [57]	√	√	Single & Dual		√	
Liu et al. [32]	√	√	Single & Dual			
Ye et al. [50]	√	√	Single & Dual			
Xiao et al. [43]	√		Single	√		
Schwarz and Tan [38]	√	√	Single & Dual			
Quan et al. [37]	√	√	Single & Dual			
Chiang et al. [13]				√		
Arya et al. [4]				√		
Xiao et al. [44]				√		√
Zhang et al. [52]				√		√
Zheng et al. [56]				√		√
Our article	√	√	Single & Dual	√	√	√

3. The Model Framework

We consider a supply chain with a fashion manufacturer (male) and a retailer (female). The manufacturer sequentially introduces two style generations, g_i, of a product into the market over two periods $i = 1, 2$. Because retail exclusivity is commonly observed in the apparel industry when new products are introduced [2, 19, 20, 53], we assume that a style generation is sold exclusively by the retailer in its introductory period. By contrast, an earlier style generation can be sold through either the manufacturer's direct channel or the retail channel. As such, determining whether to make an earlier style generation available in the second period gives rise to a strategic interaction between the manufacturer and the retailer. Because operations for direct sales or product rollover must be planned ahead of price and quantity decisions, we modeled a two-stage game: In the first stage, the manufacturer decides whether to engage in direct sales and the retailer makes the product rollover decision, and in the second stage, they make price and quantity decisions in accordance with their decisions in the first stage. Specifically, in the first stage of this game, the manufacturer decides whether to sell g_1 through his direct channel in the second period, and if he does, encroachment takes place. On the other hand, the retailer decides whether to continue selling g_1 in the second period: She adopts single rollover if she does not and dual rollover if she does. For notational convenience, we use N, E to denote the manufacturer's "No encroachment" strategy and "Encroachment" strategy, respectively, and S, D the retailer's "Single rollover" strategy and "Dual rollover" strategy, respectively. A combination of both parties' strategic choices leads to four configurations SN, DN, SE, DE, as depicted in Table 2. We analyze both sequential- and simultaneous-move versions of the first-stage game in the light of different power structures.

In the second stage of the game, the decision sequence for the manufacturer and the retailer is as follows: Before the first period, the manufacturer decides the wholesale price w for either style generation in its introductory period. The retailer then determines the sales quantity q_1 for style generation g_1. Consumers arrive in period 1 and make purchasing decisions. At the beginning of period 2, the manufacturer decides the wholesale price w_d for g_1 if the retailer adopts dual

Table 2. Strategic configurations in terms of the retailer's rollover decision and the fashion manufacturer's encroachment decision

		Manufacturer	
		No encroachment (N)	Encroachment (E)
Retailer	Single rollover (S)	SN	SE
	Dual rollover (D)	DN	DE

rollover[1]; the retailer then determines the sales quantity q_2 for style generation g_2 and, if she adopts dual rollover, the sales quantity q_{r1} for g_1; subsequently, the manufacturer sets the sales quantity q_{m1} for g_1 if he engages in encroachment, and finally, consumers make purchasing decisions in period 2. In line with existing studies [4, 19, 27, 52], we assume that the manufacturer and the retailer make their quantity decisions for g_1 sold in period 2 sequentially under the DE configuration because the manufacturer can observe the retailer's quantity decision, but not vice versa, and they face the same market-clearing price for g_1 sold in period 2. We note that to achieve analytical consistency, we endogenize quantity decisions across all four configurations and use inverse demand functions to determine the selling price p for either style generation in its introductory period[2] and the selling price p_d for g_1 in the second period. Finally, we assume that the manufacturer is less efficient

[1]Adjusting the wholesale price w_d for g_1 in the second period allows the manufacturer to influence his demand through the retail channel.

[2]We assume uniform market prices for successive style generations in their introduction periods because they belong to the same product category and target the same set of customers [57].

at retail operations than the retailer and thus normalize the manufacturer's unit direct selling cost to $c \geq 0$ and the retailer's unit selling cost to zero [4, 19, 27]. Without loss of generality, we normalize the manufacturer's unit production cost to zero.

Consumers have heterogeneous valuations (or "willingness to pay") for style generations and buy at most one unit in each period. When making his or her first purchase, a consumer has the valuation v for a newly introduced style generation in either period and $\theta\, v$ for an older style generation in the second period, where v is uniformly distributed in $[0, 1]$, and $\theta \in (0, 1)$ is the discount factor reflecting the influence of trends. A uniformly distributed consumer valuation is commonly adopted in the operations management and marketing literature (e.g., 1, 7, 13, 40). Furthermore, we consider mental accounting in the utility derivation of consumers when after having bought g_1 in the first period, they decide to purchase g_2 in the second period [15, 35]. Specifically, a consumer opens a mental account for a purchased style generation [41], and the mental account keeps tracking the positive difference (mental account deficit or the mental book value) between the initial purchase price and the benefits accrued to date from consumption [36, 41]. Then, purchase of a newer style generation triggers the closing of the mental account for the older product, which will no longer be used, and hence incurs disutility from writing off the mental account deficit. In line with [57], we assume that the mental account deficit, denoted by b, is homogeneous among consumers and that consumers buy at most one unit of the same style generation over two periods. Consumers prefer purchasing to not purchasing and prefer buying earlier to buying

Table 3. Consumer segmentation over two periods

Consumer segments	Notation	Utility	Demand
In period 1,			
consumers buy g_1 in period 1.	1	$U_1 = v - p$	$Q_1 = 1 - p$
In period 2,			
consumers who buy g_1 in period 1 buy g_2 in period 2;	2	$U_2 = v - p - b$	$Q_2 = 1 - p - b$
consumers who do not buy g_1 in period 1 buy g_1 in period 2.	$N1$	$U_{N1} = \theta\, v - p_d$	$Q_{N1} = p - p_d/\theta$

later if there is a tie in the utility derived from the two purchasing decisions. Furthermore, consumers receive zero utility from not purchasing in either period. In the utility and demand functions, we use the subscripts 1, 2, and $N1$ to denote the segment of consumers who buy g_1 in period 1, the segment of consumers who buy g_1 in period 1 and g_2 in period 2, and the segment of consumers who do not buy g_1 in period 1 but buy g_1 in period 2, respectively, as depicted in Table 3.

Consumers derive their utility in each period as follows: In period 1, a consumer who buys g_1 has utility $U_1 = v - p$. We assume that consumers buy g_1 in the first period if $U_1 \geq 0$. This nonstrategic behavior is commonly observed in the fashion industry, in which fashion firms often limit their stocks, thereby making consumers less likely to wait [5, 57]. The first-period demand for g_1 is thus $Q_1 = 1 - p$. In the second period, consumers who own g_1 in the first period have utility $U_2 = v - p - b$ if they buy g_2 in the second period. That consumers with $U_2 \geq 0$ will buy g_2 in the second period leads to the demand $Q_2 = 1 - p - b$ for g_2. On the other hand, consumers who do not buy g_1 in the first period have utility $U_{N1} = \theta\, v - p_d$ if they buy g_1 in the second period. This means that consumers with $p_d/\theta \leq v < p$ will buy g_1 in the second period if it is made available, as in the DN, SE, and DE configurations. Accordingly, the second-period demand for g_1, if made available, is $Q_{N1} = p - p_d/\theta$.[3] Table 3 summarizes the utility and demand functions for these three consumer segments. We proceed to establish the inverse demand functions for Q_1 and Q_{N1} in the DE configuration in terms of the retailer's sales quantity q_1 for g_1 in period 1 and sales quantity q_{r1} for g_1 in period 2 and the manufacturer's sales quantity q_{m1} for g_1 in period 2. Setting Q_1 equal to q_1 and Q_{N1} equal to $q_{r1} + q_{m1}$ leads to the inverse demand functions (1) and (2) for Q_1 and Q_{N1}, respectively, in the DE configuration

$$p = 1 - q_1, \tag{1}$$

$$p_d = \theta\,[(1 - q_1) - (q_{r1} + q_{m1})], \tag{2}$$

and $q_2 = q_1 - b$ because Q_2 is linearly related to Q_1. Furthermore, setting $q_{r1} = 0$ or $q_{m1} = 0$ in (2) leads to the inverse demand function of Q_{N1} in the SE or DN configuration.

4. Equilibrium Analysis

We first solve the second-stage games for the four configurations in Table 2 sequentially using backward induction. We then solve the first-stage game under various power structures. We operationalize the manufacturer's unit direct selling cost c and the consumers' mental account deficit b to establish the conditions

[3]If p_d/θ were greater than p, then no consumer would buy g_1 in period 2. It is thus reasonable to focus on $p_d/\theta < p$ in equilibrium.

for the feasibility of the four configurations. These two drivers make it possible to explore how they affect the equilibrium outcome in the first-stage game. We let π_r^k and π_m^k denote the retailer's profit and the manufacturer's profit over two periods in the second-stage game, respectively, under configuration $k \in \{SN, SE, DN, DE\}$. We further use the superscript * to denote the equilibrium quantity in each second-stage game.

4.1. Second-stage games

Consider first the SN configuration in which the retailer adopts single rollover for g_1 and the manufacturer does not engage in encroachment. The manufacturer's profit π_m^{SN} consists of the first-period profit $w\,q_1$ and the second-period profit $w\,q_2$, whereas the retailer's profit π_r^{SN} consists of the first-period profit $(p-w)\,q_1$ and the second-period profit $(p-w)\,q_2$. With the inverse function in (1) and $q_2 = q_1 - b$, the manufacturer's profit and the retailer's profit over two periods can be organized as π_m^{SN} and π_r^{SN}, respectively, in (3):

$$\pi_m^{SN} = w\,(2\,q_1 - b),\ \pi_r^{SN} = (1 - q_1 - w)(2\,q_1 - b). \tag{3}$$

Lemma 4.1 summarizes the results for the second-stage game under this configuration.

Lemma 4.1. *Under the SN configuration, the manufacturer's equilibrium wholesale price w^{SN*} and the retailer's equilibrium sales quantities q_1^{SN*}, q_2^{SN*} are*

$$w^{SN*} = \frac{2-b}{4},\ q_1^{SN*} = \frac{2+3\,b}{8},\ q_2^{SN*} = \frac{2-5\,b}{8}; \tag{4}$$

the manufacturer's equilibrium profit π_m^{SN*} *and the retailer's equilibrium profit* π_r^{SN*} *are*

$$\pi_m^{SN*} = \frac{(2-b)^2}{16},\ \pi_r^{SN*} = \frac{(2-b)^2}{32}, \tag{5}$$

and the equilibrium selling price p^{SN*} *is*

$$p^{SN*} = \frac{6-3\,b}{8}. \tag{6}$$

The proofs of Lemma 4.1 and subsequent lemmas and corollaries are included in the appendix.

Next, we consider the DN configuration in which the retailer adopts dual rollover and the manufacturer does not encroach. The manufacturer's profit π_m^{DN} consists of the first-period profit $w\,q_1$ and the second-period profit $w\,q_2 + w_d\,q_{r1}$, totaling $w\,(q_1+q_2) + w_d\,q_{r1}$. The retailer's profit π_r^{DN} consists of the first-period profit $(p-w)\,q_1$ and the second-period profit $(p-w)\,q_2 + (p_d - w_d)\,q_{r1}$, totaling $(p-w)(q_1+q_2) + (p_d - w_d)\,q_{r1}$. Factoring the inverse demand functions in (1) and (2) with $q_{m1} = 0$ and $q_2 = q_1 - b$ into the manufacturer's profit and the retailer's profit yields π_m^{DN} and π_r^{DN}, respectively, in (7):

$$\pi_m^{DN} = w\,(2\,q_1 - b) + w_d\,q_{r1},\ \pi_r^{DN} = (1-q_1-w)(2\,q_1-b) + [\theta\,(1-q_1-q_{r1}) - w_d]\,q_{r1}. \tag{7}$$

Lemma 4.2. *Under the DN configuration, the manufacturer's equilibrium wholesale prices* w^{DN*}, w_d^{DN*} *and the retailer's equilibrium sales quantities* q_1^{DN*}, q_2^{DN*},

q_{r1}^{DN*} *are*

$$w^{DN*} = \frac{2-b}{128}\left(\frac{512}{16-\theta}-\theta\right),\ w_d^{DN*} = \frac{\theta\,(2-b)(48-\theta)}{16\,(16-\theta)}, \tag{8}$$

$$q_1^{DN*} = \frac{6+b}{8}-\frac{4\,(2-b)}{16-\theta},\ q_2^{DN*} = \frac{6-7\,b}{8}-\frac{4\,(2-b)}{16-\theta},\ q_{r1}^{DN*} = \frac{(2-b)(48-\theta)}{32\,(16-\theta)}, \tag{9}$$

the manufacturer's equilibrium profit π_m^{DN*} *and the retailer's equilibrium profit* π_r^{DN*} *are*

$$\pi_m^{DN*} = \frac{(2-b)^2\,(16+\theta)^2}{256\,(16-\theta)},\ \pi_r^{DN*} = \frac{(2-b)^2\,[8192+\theta\,(768-\theta\,(128-7\,\theta))]}{1024\,(16-\theta)^2}, \tag{10}$$

and the equilibrium selling prices p^{DN*}, p_d^{DN*} *are*

$$p^{DN*} = \frac{(2-b)(48-\theta)}{8\,(16-\theta)},\ p_d^{DN*} = \frac{3\,\theta\,(2-b)(48-\theta)}{32\,(16-\theta)}. \tag{11}$$

We proceed to examine the SE configuration in which the retailer adopts single rollover and the manufacturer sells g_1 through his direct channel in the second period. The manufacturer's profit π_m^{SE} consists of the first-period profit $w\,q_1$ and the second-period profit $w\,q_2 + (p_d - c)\,q_{m1}$, whereas the retailer's profit π_r^{SE} consists of the first-period profit $(p-w)\,q_1$ and the second-period profit $(p-w)\,q_2$. Inclusion of the inverse demand functions in (1) and (2) with $q_{r1}=0$ and $q_2 = q_1 - b$ in the manufacturer's profit and the retailer's profit yields

$$\pi_m^{SE} = w\,(2\,q_1-b)+[\theta\,(1-q_1-q_{m1})-c]\,q_{m1},\ \pi_r^{SE} = (1-q_1-w)(2\,q_1-b). \tag{12}$$

Lemma 4.3 below summarizes the results for the second-stage game under the SE configuration.

Lemma 4.3. *Under the SE configuration, the manufacturer's equilibrium wholesale price w^{SE*} and sales quantity q_{m1}^{SE*} and the retailer's equilibrium sales quantities q_1^{SE*}, q_2^{SE*} are*

$$w^{SE*} = \frac{(2-b)(8+\theta)-4\,c}{2\,(16-\theta)},\ q_{m1}^{SE*} = \frac{3\,\theta\,(2-b)-8\,c}{\theta\,(16-\theta)}, \tag{13}$$

$$q_1^{SE*} = \frac{4+6\,b-\theta+c}{16-\theta},\ q_2^{SE*} = \frac{4-b\,(10-\theta)-\theta+c}{16-\theta}; \tag{14}$$

the manufacturer's equilibrium profit π_m^{SE*} and the retailer's equilibrium profit π_r^{SE*} are

$$\pi_m^{SE*} = \frac{\theta\,(2+\theta)\,(2-b)^2-6\,c\,\theta\,(2-b)+8\,c^2}{2\,\theta\,(16-\theta)},\ \pi_r^{SE*} = \frac{((2-b)(4-\theta)+2\,c)^2}{2\,(16-\theta)^2}, \tag{15}$$

and the equilibrium selling prices p^{SE*}, p_d^{SE*} are

$$p^{SE*} = \frac{6\,(2-b)-c}{16-\theta},\ p_d^{SE*} = \frac{3\,\theta\,(2-b)+c\,(8-\theta)}{16-\theta}. \tag{16}$$

Finally, we investigate the DE configuration in which the retailer adopts dual rollover and the manufacturer sells g_1 in the second period. The manufacturer's profit π_m^{DE} consists of the first-period profit $w\,q_1$ and the second-period profit $w\,q_2 + w_d\,q_{r1} + (p_d - c)\,q_{m1}$, totaling $w\,(q_1+q_2) + w_d\,q_{r1} + (p_d-c)\,q_{m1}$. The retailer's profit π_r^{DE} consists of the first-period profit $(p-w)\,q_1$ and the second-period profit $(p-w)\,q_2 + (p_d-w_d)\,q_{r1}$, totaling $(p-w)(q_1+q_2)+(p_d-w_d)\,q_{r1}$. With the inverse demand functions in (1) and (2) with $q_2 = q_1 - d$, the manufacturer's profit and the retailer's profit can be organized as

$$\pi_m^{DE} = w\,(2\,q_1-b) + w_d\,q_{r1} + [\theta\,(1-q_1-q_{m1}-q_{r1})-c]\,q_{m1}, \tag{17}$$

$$\pi_r^{DE} = (1-q_1-w)(2\,q_1-b) + [\theta\,(1-q_1-q_{m1}-q_{r1})-w_d]\,q_{r1}. \tag{18}$$

Lemma 4.4 details the results for the second-stage game under configuration DE.

Lemma 4.4. *Under the DE configuration, the manufacturer's equilibrium wholesale prices* w^{DE*}, w_d^{DE*} *and sales quantity* q_{m1}^{DE*} *are*

$$w^{DE*} = \frac{(2-b)(8+\theta)-4\,c}{2\,(16-\theta)},\ w_d^{DE*} = \frac{9\,\theta\,(2-b)-c\,(8+\theta)}{3\,(16-\theta)}, \tag{19}$$

$$q_{m1}^{DE*} = \frac{6\,(2-b)-c}{2\,(16-\theta)} - \frac{5\,c}{6\,\theta}; \tag{20}$$

the retailer's equilibrium sales quantities q_1^{DE*}, q_2^{DE*}, q_{r1}^{DE*} *are*

$$q_1^{DE*} = \frac{4+6\,b-\theta+c}{16-\theta},\ q_2^{DE*} = \frac{4-b\,(10-\theta)-\theta+c}{16-\theta},\ q_{r1}^{DE*} = \frac{2\,c}{3\,\theta}; \tag{21}$$

the manufacturer's equilibrium profit π_m^{DE*} *and the retailer's equilibrium profit* π_r^{DE*} *are*

$$\pi_m^{DE*} = \frac{3\,\theta\,(2+\theta)\,(2-b)^2 - 18\,c\,\theta\,(2-b) + 2\,c^2\,(28-\theta)}{6\,\theta\,(16-\theta)}, \tag{22}$$

$$\pi_r^{DE*} = \frac{9\,\theta\,(4-\theta)^2\,(2-b)^2 + 36\,c\,\theta\,(2-b)(4-\theta) + 4\,c^2\,(256-23\,\theta+\theta^2)}{18\,\theta\,(16-\theta)^2}, \tag{23}$$

and the equilibrium selling prices p^{DE*}, p_d^{DE*} *are*

$$p^{DE*} = \frac{6\,(2-b)-c}{16-\theta},\ p_d^{DE*} = \frac{9\,\theta\,(2-b)+2\,c\,(4-\theta)}{3\,(16-\theta)}. \tag{24}$$

We observe that the equilibrium wholesale price w^{DE*} in (19) and the equilibrium sales quantities q_1^{DE*} and q_2^{DE*} in (21) are identical to w^{SE*} in (13) and q_1^{SE*} and q_2^{SE*} in (14), respectively. This is because the manufacturer's best response for w_d neutralizes the effect of the sales quantity q_1 on the retailer's second-period profit generated from the sales of g_1, as shown in the proof of Lemma 4.4. (The manufacturer's best response for w_d, however, does not neutralize this effect under the DN configuration.) As a result, the retailer's best response for q_1 takes the same form as her best response for q_1 under the SE configuration, which in turn leads to the manufacturer's wholesale price w^{DE*} being equal to his wholesale price w^{SE*} under the SE configuration.

We base our analysis on a framework where a manufacturer introduces two style generations sequentially over two periods. However, introducing the newer style generation in the second period may cause it to lose favor in light of consumers' mental account deficit and the manufacturer's cost inefficiency. We now take a closer look at the effects of these two drivers on the introduction of a newer style generation. It is evident from (4) in Lemma 4.1 and (9) in Lemma 4.2 that the manufacturer's cost inefficiency has no impact on the sales quantities q_2^{SN*} and q_2^{DN*}. We can show that introducing a newer style generation brings value to the considered supply chain if the mental account deficit $b < 2/5$ in the SN configuration or $b < \beta_2^{DN*}$ in the DN configuration, where $\beta_2^{DN*} = (32-6\,\theta)/(80-7\,\theta)$. Note that $2/5 > \beta_2^{DN*}$. It is expected that the mental account deficit will be bounded from above because closing a mental account while experiencing a greater deficit (larger b) will discourage consumers from buying a newer style generation in the second period, hence yielding no benefit to introducing it into the market. On the other hand, when the manufacturer encroaches, his cost inefficiency is correlated with consumers' mental account deficit in terms of determining whether a newer style generation is introduced in the second period, as seen in (14) in Lemma 4.3

and in (21) in Lemma 4.4. We can show that introducing a newer style generation benefits the supply chain in either the SE or DE configuration under the condition $b < \beta_2^{E*}$, where $\beta_2^{E*} = (4 - \theta + c)/(10 - \theta)$. When the manufacturer has low cost inefficiency (small c), this condition is tighter than the earlier condition $b < \beta_2^{DN*}$ for configuration DN. This suggests that consumers' mental account deficit should be lower to justify the introduction of a newer style generation when the manufacturer has low cost inefficiency and decides to encroach. On the other hand, when the manufacturer has significant cost inefficiency (large c), the condition is less tight than $b < \beta_2^{DN*}$. This means that while introducing a newer style generation does not benefit the supply chain when the retailer adopts dual rollover under no manufacturer encroachment, it may do so when the retailer adopts single rollover and a manufacturer with large cost inefficiency is encroaching.

In subsequent analysis, we focus on cases in which a newer style generation introduced in the second period benefits the supply chain under all configurations. To facilitate exposition, we define a set Ω^*

$$\Omega^* = \{(c,b) |\ (c,b) \text{ is such that } b < \beta_2^{DN*} \text{ and } b < \beta_2^{E*}\}, \tag{25}$$

and let L_2^{DN*} and L_2^{E*} denote the lines $b - \beta_2^{DN*} = 0$ and $b - \beta_2^{E*} = 0$ on the c-b plane, respectively, as illustrated in Figure 1. Our next task is to investigate how the manufacturer's cost inefficiency and consumers' mental account deficit affect the feasibility of the configurations on the basis of which the manufacturer and the retailer choose actions in the first-stage game.

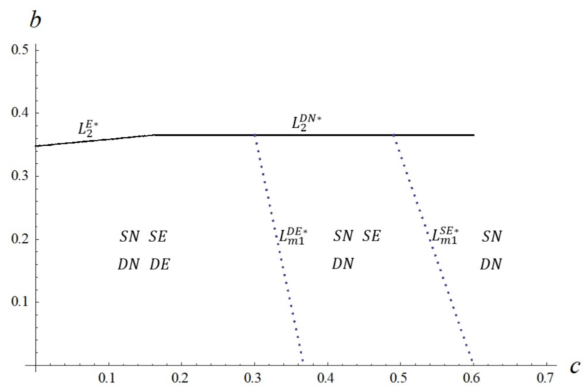

FIGURE 1. The feasible regions of (c, b) for different configurations, where $\theta = 0.8$.

Corollary 4.5. *(i) Configurations SN and DN are feasible when $(c, b) \in \Omega^*$ in (25). (ii) Configuration SE is feasible when $(c, b) \in \Omega^*$ and $b < \beta_{m1}^{SE*}$, where $\beta_{m1}^{SE*} = 2 - (8\,c)/(3\,\theta)$. (iii) Configuration DE is feasible when $(c, b) \in \Omega^*$ and $b < \beta_{m1}^{DE*}$, where $\beta_{m1}^{DE*} = (18\,\theta - c\,(40 - \theta))/(9\,\theta)$, and $\beta_{m1}^{DE*} < \beta_{m1}^{SE*}$.*

For notational convenience, we let L_{m1}^{SE*} and L_{m1}^{DE*} denote the lines $b - \beta_{m1}^{SE*} = 0$ and $b - \beta_{m1}^{DE*} = 0$ on the c-b plane, respectively, where β_{m1}^{SE*} and β_{m1}^{DE*} are defined in Corollary 4.5. With the aid of lines L_2^{DN*}, L_2^{E*}, L_{m1}^{SE*}, and L_{m1}^{DE*}, we identify the regions on the c-b plane in which certain configurations are feasible, as illustrated in Figure 1.

4.2. First-stage game

To shed light on whether the power structure affects the equilibrium outcome, we allow the manufacturer and the retailer to engage in either a sequential-move version or a simultaneous-move version of the first-stage game. In the sequential-move version of the first-stage game, we examine the case of the manufacturer acting as the Stackelberg leader and the case of the retailer acting as the Stackelberg leader. We next order the retailer's and the manufacturer's preferences.

Corollary 4.6. *From the retailer's perspective, (i)* $\pi_r^{DN*} > \pi_r^{SN*}$*; (ii)* $\pi_r^{DE*} > \pi_r^{SE*}$ *in the feasible region for the DE configuration; (iii)* $\pi_r^{SE*} < \pi_r^{SN*}$ *in the feasible region for the SE configuration; and (iv)* $\pi_r^{DE*} < (>)\, \pi_r^{DN*}$ *in the feasible region for the DE configuration if* $b < (>)\, \beta_r^*$ *and* $\theta > 0.7326$*, where* π_r^{SN*}*,* π_r^{DN*}*,* π_r^{SE*}*, and* π_r^{DE*} *are given in (5), (10), (15), and (23), respectively, and*

$$\beta_r^* \quad = \quad \frac{6\,(\rho_1 - 512\,c\,(4-\theta)) - 32\,c\,\sqrt{2\,\rho_2}}{3\,\rho_1}; \tag{26}$$

$\rho_1 = \theta\,(4864 - 640\,\theta + 7\,\theta^2)$*, and* $\rho_2 = 1318912 - 312576\,\theta + 25984\,\theta^2 - 801\,\theta^3 + 7\,\theta^4$.

Corollary 4.7. *From the manufacturer's perspective, (i)* $\pi_m^{SE*} > \pi_m^{SN*}$ *in the feasible region for the SE configuration; (ii)* $\pi_m^{DE*} > \pi_m^{DN*}$ *in the feasible region for the DE configuration; (iii)* $\pi_m^{DE*} > \pi_m^{SE*}$ *in the feasible region for the DE configuration; and (iv)* $\pi_m^{SE*} > (<)\, \pi_m^{DN*}$ *in the feasible region for the SE configuration if* $b < (>)\, \beta_m^*$*, where* π_m^{SN*}*,* π_m^{DN*}*,* π_m^{SE*}*, and* π_m^{DE*} *are given in (5), (10), (15), and (22), respectively, and*

$$\beta_m^* \quad = \quad \frac{2\,\theta\,(96-\theta) - c\,(384 + \sqrt{48+\theta})}{\theta\,(96-\theta)}, \tag{27}$$

and β_m^* *is less than* β_r^* *in (26) for* $c > 0$.

We let L_r^* and L_m^* denote the lines $b = \beta_r^*$ and $b = \beta_m^*$, respectively, on the c-b plane, where β_r^* is given in (26), and β_m^* is given in (27). Note that L_r^* is present only when θ is not small ($\theta > 0.7326$), and L_r^* is to the left of L_{m1}^{DE*}, with the distance between these two lines widening as θ increases. Superimposing L_r^* and L_m^* on the feasible regions in Figure 1 characterized by L_2^{E*}, L_2^{DN*}, L_{m1}^{DE*}, and L_{m1}^{SE*} leads to Figure 2, in which regions I, II, and III correspond to the feasible regions for all four configurations; region IV corresponds to the feasible region for the SN, DN, and SE configurations; and region V corresponds to the feasible regions for the SN and DN configurations. We observe from Figure 2 that region III is relatively small in size compared to the other two regions I and II, even with larger values of θ. With the aid of Corollaries 4.6 and 4.7, we summarize both firms' configuration preferences in each region of Figure 2 in Table 4. The first column of Table 4 indicates the regions in Figure 2, and the second column depicts the feasible configurations organized in a similar way to the 2x2 game table in Table 2, with "—" meaning N/A. The third and fourth columns of Table 4 lay out the preferences for the manufacturer and the retailer.

We are now ready to solve the first-stage game.

Proposition 4.8. *In settings with the introduction of a newer style generation in the second period, benefiting the supply chain, (i) under the condition $b < \beta_r^*$ when $\theta > 0.7326$ or the condition $b < \beta_{m1}^{DE*}$ when $\theta < 0.7326$, the manufacturer encroaches, and the retailer adopts dual rollover, regardless of the underlying power structure, and only the manufacturer achieves his preferred outcome; (ii) under the*

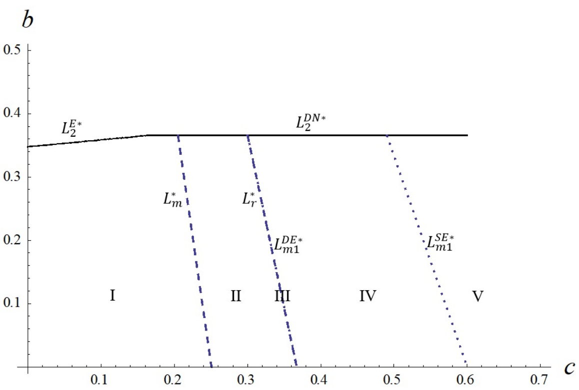

FIGURE 2. Partitioning of the feasible regions for different configurations, where $\theta = 0.8$.

TABLE 4. Chain members' preferences in different regions of (c, b) in Figure 2

Region	Structures	Retailer's preferences	Manufacturer's preferences
I	SN, SE DN, DE	$\pi_r^{DN*} > \pi_r^{SN*}, \pi_r^{DE*} > \pi_r^{SE*}$	$\pi_m^{DE*} > \pi_m^{SE*} > \pi_m^{DN*} > \pi_m^{SN*}$
II	SN, SE DN, DE	$\pi_r^{DN*} > \pi_r^{SN*}, \pi_r^{DE*} > \pi_r^{SE*}$	$\pi_m^{DE*} > \pi_m^{DN*} > \pi_m^{SE*} > \pi_m^{SN*}$
III	SN, SE DN, DE	$\pi_r^{DE*} > \pi_r^{DN*} > \pi_r^{SN*} > \pi_r^{SE*}$	$\pi_m^{DE*} > \pi_m^{DN*} > \pi_m^{SE*} > \pi_m^{SN*}$
IV	SN, SE DN, —	$\pi_r^{DN*} > \pi_r^{SN*} > \pi_r^{SE*}$	$\pi_m^{DN*} > \pi_m^{SE*} > \pi_m^{SN*}$
V	SN, — DN, —	$\pi_r^{DN*} > \pi_r^{SN*}$	$\pi_m^{DN*} > \pi_m^{SN*}$

condition $\beta_r^* \leq b < \beta_{m1}^{DE*}$ when $\theta > 0.7326$, the manufacturer encroaches, and the retailer adopts dual rollover, regardless of the underlying power structure, and both firms achieve the preferred outcome; and (iii) under the condition $b \geq \beta_{m1}^{DE*}$, there *is no manufacturer encroachment, and the retailer's dual rollover is an achievable win-win outcome for both firms.*

Proposition 4.8 can be obtained via a standard best response analysis, with the aid of Table 4, and thus its proof is omitted for brevity. Proposition 4.8(i) is based on the conditions for b and c in regions I and II of Figure 2. It is clear from Table 4 that in these two regions, the retailer's adoption of dual rollover for g_1 benefits both firms but that they have asymmetric preferred configurations. When they engage in a sequential-move version of the first-stage game, the equilibrium outcome DE is invariant, with either firm acting as the first mover. This is because each firm has a dominant strategy and acts independently of the other's choice. This also means that both firms will arrive at the DE outcome in equilibrium when engaging in a simultaneous-move version of the first-stage game.

Proposition 4.8(ii) refers to the conditions for b and c in region III of Figure 2. Unlike Proposition 4.8(i), the retailer's preferred configuration is now aligned with the manufacturer's in region III. Because both firms have dominant strategies, as revealed in Table 4, they are able to achieve the preferred outcome DE in a sequential-move version of the first-stage game with either firm leading or in a simultaneous-move version of the first-stage game. These results suggest that when consumers have higher valuations of an older style in the second period ($\theta > 0.7326$) and the manufacturer's cost inefficiency is at an intermediate level, encroachment benefits both firms and leads to a win-win outcome.

Finally, Proposition 4.8(iii) refers to the conditions for b and c in regions IV and V of Figure 2. With the aid of Table 4, we find that both firms achieve the same and the best outcome DN with either leading in a sequential-move version of the first-stage game in regions IV and V. On the other hand, in a simultaneous-move version of the first-stage game, there are two equilibria DN and SE in region IV and one equilibrium DN in region V. Because both firms prefer DN to SE, the former is more likely to become a focal point and emerge as the equilibrium outcome in region IV.

4.3. Discussion

In light of the second-stage games, our analysis revealed that consumers' mental account deficit (b) should be lower to justify the introduction of a newer style generation, being consistent with the finding in the work of Zhou et al. [57] (see Figure 1). Our analysis also showed that while introducing a newer style generation does not benefit the supply chain under dual rollover and manufacturer encroachment, it could benefit the supply chain under single rollover and manufacturer encroachment. We further found that for a given level of mental account deficit on the part of consumers, an increase in the cost inefficiency (larger c) is likely to decrease the number of configurations available to both the manufacturer and retailer, and at certain levels of cost inefficiency, the number of configurations available to both firms is also contingent on consumers' mental account deficit, as governed by the negative-sloped lines L_{m1}^{SE*} and L_{m1}^{DE*} in Figure 1. These findings enable the manufacturer and the retailer to properly choose their actions in the first-stage game while accounting for the available configurations.

With regard to the first-stage game, we found that both firms have dissimilar preferences in different regions of c and b, as summarized in Table 4. These preferences lead to the equilibrium at which the retailer adopts dual rollover and the manufacturer encroaches when the manufacturer has low cost inefficiency (in regions I and II of Figure 2). The equilibrium remains unchanged with both firms having an equal bargaining power or with either firm having a higher bargaining power. Such an invariant property certainly benefits the manufacturer because only he can achieve the preferred outcome at this equilibrium. Furthermore, we found that when the manufacturer's cost inefficiency is at an intermediate level (region III of Figure 2), both firms achieve the preferred outcome, i.e., a win-win outcome, if consumers have higher valuations of an older style in the second period. Again, this win-win outcome is invariant under different power structures. On the other hand, if consumer valuation for an older style is low, a win-win outcome is unattainable. This finding enriches our understanding of the extent to which manufacturer encroachment allows a win-win outcome in the considered supply chain. Finally, when the manufacturer's cost inefficiency is at higher levels (regions IV and V of Figure 2), encroachment no longer benefits the manufacturer. In this case, both firms prefer the outcome at which the retailer adopts dual rollover and the manufacturer does not encroach (see the last two rows in Table 4). However, they shall be aware that the power structure could affect whether they can attain this win-win outcome.

5. Managerial insights

Our analyses offer the following insights:

First, consumers' mental account deficit should be sufficiently low to justify the introduction of a newer style generation when the manufacturer has low cost inefficiency and decides to encroach. This requirement is relaxed when the manufacturer has significant cost inefficiency. As a result, while introducing a newer style generation does not benefit the supply chain with the retailer adopting dual rollover under no manufacturer encroachment, it may do so when the retailer adopts single rollover and a manufacturer with large cost inefficiency is encroaching. The important insight here is that the manufacturer's cost disadvantage has double-edged effects that could offset the negative effect of consumers' mental account deficit upon the introduction of a newer style generation. These results also complement the finding of Zhou et al. [57] that a high mental account deficit always deters the firm from selling a newer style generation in period 2.

Second, with the aid of Proposition 4.8, we find that when the manufacturer has low cost inefficiency, he can benefit from direct sales, regardless of the underlying power structure in the supply chain. In this case, only the manufacturer achieves the preferred outcome. This insight suggests that when direct sales is manageable from a long-term perspective, the manufacturer should exert efforts constantly to improve his direct sales operations. When a manufacturer with an intermediate level of cost inefficiency engages in direct sales and consumers have high valuations of the old style generation or when a manufacturer with significant cost inefficiency does not engage in direct sales, both the manufacturer and the retailer can obtain their preferred outcomes. Clearly, such a win-win outcome helps sustain the relationship between the manufacturer and the retailer. Because consumers' perceived value plays a role in attaining the win-win outcome when the manufacturer has an intermediate level of cost inefficiency, both the manufacturer and the retailer are incentivized to enhance consumers' perceived value, e.g., through marketing efforts.

Third, in equilibrium, the retailer always sells both new and old style generations in the second period, and the manufacturer engages in direct sales only when he has a small or intermediate selling cost. This implies that when the manufacturer has low direct sales operational efficiency, he should focus on selling products through the retailer unless direct sales operational efficiency can be improved within a foreseeable time frame.

6. Summary and Conclusion

In this paper, we investigated the interplay between a manufacturer and a retailer on the displacement of an older style generation when a newer style generation is introduced in a two-period model in which two style generations are sequentially introduced into the market. We formulated a two-stage game to explore the chain members' equilibrium decisions while considering the manufacturer's cost inefficiency in direct sales and consumers' mental account deficit under different power structures in the first-stage game. We showed that the manufacturer's cost inefficiency has double-edged effects and could offset the negative effect of the mental account deficit of consumers on the introduction of a newer style generation. We found that in settings where the introduction of a newer style generation benefits the supply chain, the manufacturer always achieves his preferred outcome, and the retailer achieves her preferred outcome only when a manufacturer with an intermediate level of cost inefficiency engages in direct sales and consumers have high valuations of the older style generation in the second period or when a manufacturer with significant cost inefficiency does not engage in direct sales. We also found that the power structures underlying the first-stage game do not affect the equilibrium outcome; the retailer always sells both the new and old style generations in the second period, and the manufacturer engages in direct sales only when he has a small or intermediate selling cost.

Our analysis nevertheless has some limitations. We assumed retail exclusivity in the introduction of a new style generation to mitigate channel conflict. Thus, the manufacturer can engage in direct sales only in the second period. In future research, it is of interest to understand the implications of direct selling if the manufacturer is allowed to engage in direct sales in either period. A second limitation is the fact that we focused on horizontally differentiated products in our analysis. A possible extension to our study is therefore to allow for vertically differentiated products and explore how quality differentiation between channels and/or between product generations influences the interactive dynamics between the supply chain members. Finally, we did not consider trade-in programs when the manufacturer introduces a new product generation. As trade-in programs are applicable to certain product categories, it would be worthwhile to extend the current setting to include trade-in programs in new product introduction and to explore how trade-in programs would influence the interplay between manufacturer encroachment and product rollover in a supply chain.

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Appendix. Proofs of the Lemmas and Corollaries

Proof of Lemma 4.1. We adopt backward induction to obtain the equilibrium decisions for the second-stage game under the SN configuration. Solving the first-order condition of the retailer's profit π_r^{SN} in (3) for q_1 yields the best response $\tilde{q}_1^{SN} = (2 - 2\,w - b)/4$ because $\partial^2 \pi_r^{SN}/\partial\, q_1^2 = -4 < 0$. Substituting $q_1 = \tilde{q}_1^{SN}$ into the manufacturer's profit π_m^{SN} in (3) yields $\tilde{\pi}_m^{SN} = w\,(2 - 2\,w - b)/2$. Then, solving the first-order condition of $\tilde{\pi}_m^{SN}$ for w yields the equilibrium wholesale price $w^{SN*} = (2 - b)/4$ because $\partial^2 \tilde{\pi}_m^{SN}/\partial\, w_1^2 = -2 < 0$. Accordingly, the retailer's equilibrium sales quantities are $q_1^{SN*} = \tilde{q}_1^{SN}|_{w=w^{SN*}} = (2 + 3\,b)/8$ and $q_2^{SN*} = q_1^{SN*} - b = (2 - 5\,b)/8$. It then follows that the manufacturer's equilibrium profit π_m^{SN*} is $\pi_m^{SN*} = \pi_m|_{w=w^{SN*}, q_1=q_1^{SN*}} = (2 - b)^2/16$, and the retailer's equilibrium profit π_r^{SN*} is $\pi_r^{SN*} = \pi_r|_{w=w^{SN*}, q_1=q_1^{SN*}} = (2 - b)^2/32$. With the inverse function in (1), the equilibrium selling price p^{SN*} is $p^{SN*} = 1 - q_1^{SN*} = (6 - 3\,b)/8$ in (6). □

Proof of Lemma 4.2. Because the second derivative of π_r^{DN} in (7) is $\partial^2 \pi_r^{DN}/\partial\, q_{r1}^2 = -2\,\theta < 0$, solving the first-order condition of π_r^{DN} with respect to q_{r1} leads to the best response $\tilde{q}_{r1}^{DN} = (\theta\,(1 - q_1) - w_d)/(2\,\theta))$. Let $\tilde{\pi}_m^{DN} = \pi_m^{DN}|_{q_{r1}=\tilde{q}_{r1}^{DN}}$, where π_m^{DN} is given in (7). Because $\partial^2 \tilde{\pi}_m^{DN}/\partial\, w_d^2 = -1/\theta < 0$, solving the first-order condition of $\tilde{\pi}_m^{DN}$ with respect to w_d yields the manufacturer's best response $\tilde{w}_d^{DN} = \theta\,(1-q_1)/2$. After anticipating this best response $\tilde{w}_d^{DN}$, the retailer now has profit $\tilde{\pi}_r^{DN} = \pi_r^{DN}|_{q_{r1}=\tilde{q}_{r1}^{DN}, w_d=\tilde{w}_d^{DN}}$. Again, solving the first-order condition of $\tilde{\pi}_r^{DN}$ with respect to q_1 yields the retailer's best response $\tilde{q}_1^{DN} = 1-(8\,(2+2\,w-b))/(32-\theta)$ because $\partial^2\tilde{\pi}_r^{DN}/\partial\,q_1^2 = -4+\theta/8 < 0$. Finally, let $\hat{\pi}_m^{DN} = \tilde{\pi}_m^{DN}|_{q_1=\tilde{q}_1^{DN}}$, and because $\partial^2\hat{\pi}_m^{DN}/\partial\,w^2 = -128\,(16-\theta)/(32-\theta)^2 < 0$, solving the first-order condition of $\hat{\pi}_m^{DN}$ with respect to w gives the equilibrium wholesale price w^{DN*} in (8). Substituting the equilibrium wholesale price w^{DN*} into the best responses $\tilde{q}_1^{DN}$, $\tilde{w}_d^{DN}$, and $\tilde{q}_{r1}^{DN}$ in a recursive manner leads to the equilibrium sales quantity q_1^{DN*}, q_2^{DN*} in (9), the equilibrium wholesale price w_d^{DN*} in (8), and the equilibrium sales quantity q_{r1}^{DN*} in (9). Accordingly, we obtain the manufacturer's equilibrium profit π_m^{DN*} and the retailer's equilibrium profit π_r^{DN*} in (10), as well as the equilibrium selling prices p^{DN*} and p_d^{DN*} in (11). □

Proof of Lemma 4.3. Because $\partial^2\pi_m^{SE}/\partial\,q_{m1}^2 = -2\,\theta < 0$, solving the first-order condition of π_m^{SE} with respect to q_{m1} yields the manufacturer's best response $\tilde{q}_{m1}^{SE} = (\theta\,(1-q_1)-c)/(2\,\theta)$, where π_m^{SE} is given in (12). Substituting $q_{m1} = \tilde{q}_{m1}^{SE}$ into the retailer's profit π_r^{SE} in (12) yields $\tilde{\pi}_r^{SE} = (1-q_1-w)(2\,q_1-b)$. Because $\partial^2\tilde{\pi}_r^{SE}/\partial\,q_1^2 = -4 < 0$, solving the first-order condition of $\tilde{\pi}_r^{SE}$ with respect to q_1 yields the best response $\tilde{q}_1^{SE} = (2-2\,w+b)/4$. Finally, let $\tilde{\pi}_m^{SE} = \pi_m^{SE}|_{q_{m1}=\tilde{q}_{m1}^{SE}, q_1=\tilde{q}_1^{SE}}$. Because $\partial^2\tilde{\pi}_m^{SE}/\partial\,w^2 = -(16-\theta)/8 < 0$, solving the first-order condition of $\tilde{\pi}_m^{SE}$ with respect to w yields the equilibrium price $w^{SE*} = ((2-b)(8+\theta)-4\,c)/(2\,(16-\theta))$ in

(13). Then, by using the best responses $\tilde{q}_1^{SE}$, $\tilde{q}_{m1}^{SE}$, we obtain the retailer's equilibrium sales quantities q_1^{SE*}, q_2^{SE*} in (14) and the manufacturer's equilibrium sales quantity q_{m1}^{SE*} in (13). At these equilibrium wholesale price and sales quantities, we obtain the manufacturer's equilibrium profit π_m^{SE*} and the retailer's equilibrium profit π_r^{SE*} in (15), as well as the equilibrium selling prices p^{SE*}, p_d^{SE*} in (16). □

Proof of Lemma 4.4. Again, we adopt backward induction to obtain the equilibrium decisions. Because $\partial^2 \pi_m^{DE}/\partial q_{m1}^2 = -2\,\theta < 0$, where π_m^{DE} is given in (17), solving $\partial \pi_m^{DE}/\partial q_{m1} = 0$ for q_{m1} yields the manufacturer's best response $\tilde{q}_{m1}^{DE} = (\theta\,(1-q_1-q_{r1})-c)/(2\,\theta)$. Substituting $q_{m1} = \tilde{q}_{m1}^{DE}$ into the retailer's profit π_r^{DE} in (18) yields $\tilde{\pi}_r^{DE} = (1-q_1-w)(2\,q_1-b)+(q_{r1}/2)(\theta\,(1-q_1-q_{r1})+c-2\,w_d)$. Because $\partial^2 \tilde{\pi}_r^{DE}/\partial q_{r1}^2 = -\theta < 0$, the retailer's best response is $\tilde{q}_{r1}^{DE} = (\theta\,(1-q_1)-2\,w_d+c)/(2\,\theta)$, which satisfies $\partial \tilde{\pi}_r^{DE}/\partial q_{r1} = 0$. Let $\tilde{\pi}_m^{DE}$ denote the value of π_m^{DE} at $q_{m1} = \tilde{q}_{m1}^{DE}$ and $q_{r1} = \tilde{q}_{r1}^{DE}$. Because $\partial^2 \tilde{\pi}_m^{DE}/\partial w_d^2 = -3/(2\,\theta) < 0$, solving $\partial \tilde{\pi}_m^{DE}/\partial w_d = 0$ for w_d yields the best response $\tilde{w}_d^{DE} = (3\,\theta\,(1-q_1)-c)/6$. By anticipating the best response $\tilde{w}_d^{DE}$, the retailer's profit becomes $\hat{\pi}_r^{DE} = \tilde{\pi}_r^{DE}|_{w_d=\tilde{w}_d^{DE}} = (1-q_1-w)(2\,q_1-b)+2\,c^2/(9\,\theta)$. Note that the manufacturer's best response $\tilde{w}_d^{DE}$ leads the retailer's second-period profit component $[\theta\,(1-q_1-q_{m1}-q_{r1})-w_d]\,q_{r1}$, which stems from the sales of g_1, to a constant $2\,c^2/(9\,\theta)$ and her second-period sales of g_1 to $(2\,c)/(3\,\theta)$ and hence neutralizes the effect of the sales quantity q_1 on her second-period profit and her second-period sales of g_1. This means that the retailer's best response $\tilde{q}_1^{DE}$ will have the same form as her best response $\tilde{q}_1^{SE}$ in the proof of Lemma 4.3. Because $\partial^2 \hat{\pi}_r^{DE}/\partial q_1^2 = -4 < 0$, $\hat{\pi}_r^{DE}$ is concave in

q_1, and the retailer's best response is thus $\tilde{q}_1^{DE} = (2 - 2\,w + b)/4$, which satisfies $\partial\,\hat{\pi}_r^{DE}/\partial\,q_1 = 0$. Let $\hat{\pi}_m^{DE}$ denote the value of $\tilde{\pi}_m^{DE}$ at $w_d = \tilde{w}_d^{DE}$ and $q_1 = \tilde{q}_1^{DE}$. Because $\partial^2\,\hat{\pi}_m^{DE}/\partial\,w^2 = -2 + \theta/8 < 0$, solving the first-order condition of $\hat{\pi}_m^{DE}$ with respect to w yields the equilibrium wholesale price w^{DE*} in (19). Furthermore, by using the best responses $\tilde{q}_1^{DE}$, $\tilde{w}_d^{DE}$, $\tilde{q}_{r1}^{DE}$, and $\tilde{q}_{m1}^{DE}$, we can obtain the equilibrium values q_1^{DE*} and q_2^{DE*} in (21), w_d^{DE*} in (19), q_{r1}^{DE*} in (21), and q_{m1}^{DE*} in (20). Substituting these equilibrium values into the manufacturer's profit yields π_m^{DE*} in (22), and substituting them into the retailer's profit yields π_r^{DE*} in (23). Finally, substituting these values into the inverse demand functions (1) and (2) yields the equilibrium selling prices p^{DE*} and p_d^{DE*} in (24). □

Proof of Corollary 4.5. Because configurations SN and DN do not involve the manufacturer's direct sales, we find that the equilibrium values in Lemmas 4.1 and 4.2 are positive for all $(c, b) \in \Omega^*$. Next, with regard to the SE configuration, we find that $q_1^{SE*} > 0$ always holds and that $q_{m1}^{SE*} > 0$ is equivalent to $b < \beta_{m1}^{SE*}$, where $\beta_{m1}^{SE*} = 2 - 8\,c/(3\,\theta)$. When $b < \beta_{m1}^{SE*}$ holds, $p_d^{SE*} > c$ and $p^{SE*} > w^{SE*} > 0$ also hold. We already know that $q_2^{SE*} > 0$ holds for $(c, b) \in \Omega^*$ and therefore conclude that the SE configuration is feasible when $(c, b) \in \Omega^*$ and $b < \beta_{m1}^{SE*}$. Finally, in the light of the DE configuration, we find that q_1^{DE*} in (21) increases with b and that q_1^{DE*} at $b = 0$ is $(4 - \theta + c)/(16 - \theta) > 0$, indicating that $q_1^{DE*} > 0$ always holds. Clearly, $q_{r1}^{DE*} > 0$ and $q_2^{DE*} > 0$ hold for $(c, b) \in \Omega^*$. Because q_{m1}^{DE*} in (20) decreases with b, solving $q_{m1}^{DE*} = 0$ for b leads to the upper bound $\beta_{m1}^{DE*} = (18\,\theta - c\,(40 - \theta))/(9\,\theta)$, which is less than β_{m1}^{SE*} because $\beta_{m1}^{DE*} - \beta_{m1}^{DE*} =$

$-c\,(16-\theta)/(9\,\theta) < 0$. We further find that $p_d^{DE*} > c$ and $p^{DE*} > w^{DE*} > w_d^{DE*}$ when $b < \beta_{m1}^{DE*}$, and $p_d^{DE*} > w_d^{DE*} > 0$ always holds, thus concluding that the DE configuration is feasible when $(c, b) \in \Omega^*$ and $b < \beta_{m1}^{DE*}$. □

Proof of Corollary 4.6. This proof consists of four parts. In Part (i), we find that $\pi_r^{DN*} > \pi_r^{SN*}$ because $\pi_r^{DN*} - \pi_r^{SN*} = \theta\,(2-b)^2\,(1792 - 160\,\theta + 7\,\theta^2)/(1024\,(16-\theta)^2) > 0$. In Part (ii), we find that $\pi_r^{DE*} > \pi_r^{SE*}$ because $\pi_r^{DE*} - \pi_r^{SE*} = 2\,c^2/(9\,\theta) > 0$. In Part (iii), to prove $\pi_r^{SE*} < \pi_r^{SN*}$ for $b < \beta_{m1}^{SE*}$, we let $\Delta_1 = \pi_r^{SE*} - \pi_r^{SN*}$. Because $\partial^2\,\Delta_1/\partial\,b^2 = -3\,\theta\,(32-5\,\theta)/(16\,(16-\theta)^2) < 0$, Δ_1 is concave in b. Furthermore, because $\partial\,\Delta_1/\partial\,b|_{b=\beta_{m1}^{SE*}} = 2/(32-2\,\theta) > 0$, Δ_1 increases with b for $b < \beta_{m1}^{SE*}$ and is bounded above by $\Delta_1|_{b=\beta_{m1}^{SE*}} = 0$. We thus conclude that $\pi_r^{SE*} < \pi_r^{SN*}$ for $b < \beta_{m1}^{SE*}$. Finally, in Part (iv), we let $\Delta_2 = \pi_r^{DE*} - \pi_r^{DN*}$. Because $\partial^2\,\Delta_2/\partial\,b^2 = -(4864\,\theta - 640\,\theta^2 + 7\,\theta^3)/(512\,(16-\theta)^2) < 0$ for $0 < \theta < 1$, Δ_2 is concave in b. Because $\partial\,\Delta_2/\partial\,b|_{b=\beta_{m1}^{SE*}} = c\,(157696 - 21248\,\theta + 920\,\theta^2 - 7\,\theta^3)/(4608\,(16-\theta)^2) > 0$, Δ_2 increases with b for $b < \beta_{m1}^{DE*}$. We find that if $\theta > \theta^*$, $\Delta_2|_{b=\beta_{m1}^{DE*}} > 0$, where θ^* satisfies $-114688 + 178176\,\theta - 30400\,\theta^2 + 1200\,\theta^3 - 7\,\theta^4 = 0$ and has a numerical value 0.7326. In this case, let β_r^* denote the left root of $\Delta_2 = 0$, where β_r^* is given in (26). Then, if $b < \beta_r^*$, $\Delta_2 < 0$, and $\pi_r^{DE*} < \pi_r^{DN*}$, and if $\beta_r^* < b < \beta_{m1}^{DE*}$, $\pi_r^{DE*} > \pi_r^{DN*}$. □

Proof of Corollary 4.7. This proof consists of four parts. In Part (i), we find $\pi_m^{SE*} > \pi_m^{SN*}$ because $\pi_m^{SE*} - \pi_m^{SN*} = (8\,c - 3\,\theta\,(2-b))^2/(16\,\theta\,(16-\theta)) > 0$. In Part (ii), we let $\Delta_3 = \pi_m^{DE*} - \pi_m^{DN*}$. Δ_3 is convex in b because $\partial^2\,\Delta_3/\partial\,b^2 = \theta\,(96-\theta)/(128\,(16-$

$\theta)) > 0$. Because Δ_3 has the minimum $c^2\,(960-124\,\theta+\theta^2)/(3\,\theta\,(16-\theta)(96-\theta)) > 0$ at $b = 2 - (384\,c)/(\theta\,(96-\theta))$, which satisfies the first-order condition of Δ_3, we conclude that $\pi_m^{DE*} > \pi_m^{DN*}$. In Part (iii), we find $\pi_m^{DE*} > \pi_m^{SE*}$ because $\pi_m^{DE*} - \pi_m^{SE*} = c^2/(3\,\theta) > 0$. Finally, in Part (iv), we let $\Delta_4 = \pi_m^{DN*} - \pi_m^{SE*}$, and we find that Δ_4 is concave in b because $\partial^2\,\Delta_4/\partial\,b^2 = -\theta\,(96-\theta)/(128\,(16-\theta)) < 0$. Furthermore, because $\partial\,\Delta_4/\partial\,b|_{b=\beta_{m1}^{DE*}} = c\,(384 - 136\,\theta + \theta^2)/(1152\,(16-\theta)) > 0$, $\Delta_4|_{b=\beta_{m1}^{DE*}} = c^2\,(39936 + 2368\,\theta - 176\,\theta^2 + \theta^3)/(20736\,\theta\,(16-\theta)) > 0$, and $\Delta_4|_{b=\beta_{m1}^{SE*}} = c^2\,(48+\theta)/(36\,\theta\,(16-\theta)) > 0$, Δ_4 increases with b and intersects the horizontal axis at $b = \beta_m^*$, which is the left root of $\Delta_4 = 0$ such that $\beta_m^* < \beta_{m1}^{DE*}$, as given in (27). We then conclude that $\pi_m^{DN*} < \pi_m^{SE*}$ if $b < \beta_m^*$ and $\pi_m^{DN*} > \pi_m^{SE*}$ if $\beta_m^* < b < \beta_{m1}^{SE*}$. We can further show that β_r^* in (26) is greater than β_m^* for all $c > 0$ because the difference $\beta_r^* - \beta_m^*$ increases with c and $\beta_r^* - \beta_m^* = 0$ at $c = 0$.

□