Network data envelopment analysis with two-level maximin strategy

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Abstract

Network data envelopment analysis (NDEA), one of the most important branches of recent DEA developments, has been developed for examining the decision making units (DMUs) of a system with complex and internal component divisions. In this study we apply a maximin strategy to network DEA at two levels. At individual DMU level, we evaluate the system's performance by maximizing the minimum of the divisions efficiencies, which is based on the weak-link approach. At all DMUs level, we evaluate the system's performance by maximizing the minimum of the DMUs' efficiencies, which is based on the maximin ratio efficiency model. With such two-level maximin strategy, we propose the two-level maximin NDEA model to evaluate efficiencies of all divisions as well as all DMUs at the same time. The model will provide unique and unbiased efficiency scores for all divisions in a system and improve incomparable efficiency scores and weak discrimination power of traditional DEA models. In addition, we discuss the cross efficiency evaluation based on the two-level maximin NDEA model. The proposed models are applied to efficiency evaluation of supply chains for illustrations.

Keywords: Data envelopment analysis; Weak-link approach; Maximin efficiency ratio model; Cross efficiency; Supply chains.

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1. Introduction

Data envelopment analysis (DEA), initiated by Charnes et al. (1978), is a nonparametric approach for measuring the relative efficiency of independent peer decision making units (DMU) incorporating multiple inputs and outputs (Cook and Seiford, 2009). Generally, DEA estimates the efficiency of a DMU by calculating the ratio of its weighted sum of inputs and weighted sum of outputs through a set of weights. In its basic form, DEA allows each DMU under evaluation to arbitrarily choose its favorable weights for inputs and outputs such that it obtains its optimally maximized efficiency score. For its effectiveness in identifying the best-practice frontier and ranking the DMUs, DEA has been applied to many activities in many sectors for various purposes (An et al., 2017; Karsak and Dursun, 2014; Li et al., 2018), such as agricultures (Vlontzos and Pardalos, 2017; Pishgar-Komleh et al., 2020; Abbas et al., 2020), banks (Fukuyama and Matousek, 2017; Kourtzidis et al., 2021; Zhao et al., 2021), supply chains (Izadikhah and Saen, 2016; Yang et al., 2015; Wang et al., 2020).

In conventional DEA, DMUs are considered as a whole unit, in that component structures are generally ignored, and the performance of a DMU is assumed to be a function of the chosen inputs and outputs. However, most systems are composed of many divisions operating interdependently via the intermediate products that are created by some divisions and consumed by some others within the system. Ignoring the operations of the component divisions may cause misleading results in efficiency evaluation (Castelli, et al., 2010). Network DEA models are thus developed to address such issue, in which efficiencies of competent divisions and the whole system are evaluated simultaneously (Färe and Grosskopf, 1996; Kao and Hwang, 2008). In the network DEA models, three kinds of system structures are often investigated, including series, parallel and mixed structures (Kao and Hwang, 2010). The series structure refers to several processes are linked by intermediate measures in sequence. For example, the operations of a bank are divided into two serial processes to measure profitability and marketability (Seiford and Zhu,1999). The parallel structure is composed of parallel processes that operate independently. University is an example of a parallel system, where the individual stages are the departments that operate parallel and separately inside the university (Kao and Hwang, 2010). The mixed structure is more complex than the series and parallel ones, which is a mixture of serial and parallel structures (Kao, 2017).

In network DEA, the efficiency is a multi-dimensional measure, as one has to consider the efficiency of the component divisions as well as the overall system efficiency. For those comparable component divisions across all DMUs, divisional efficiencies can be defined as the ratio of the weighted sum of inputs and weighted sum of outputs. According to whether the overall system or the divisional efficiencies is given priority for optimization, two broad approaches can be classified, namely the top-down approach and the bottom-up approach (Sotiros et al., 2019). In the top-down approach, the overall system efficiency is optimized first, and then the divisional efficiencies are obtained from offspring from the optimal solution that maximizes the system efficiency. This approach mainly includes the additive efficiency decomposition method (Chen et al., 2009; Cook et al., 2010) and the relational model (Kao and Hwang, 2008; Kao, 2009). On the contrary, in the bottom-up approach, the divisional efficiencies are measured first and the system efficiency is achieved ex post. Representative methods of the bottom-up approach are the additive aggregation method (Ang and Chen, 2016; Guo et al., 2017), multiplicative aggregation method (Zha and Liang, 2010; Li et al., 2012), the min-max method (Despotis et al., 2016b) and the weak-link method (Despotis et al. 2016a; Koronakos et al., 2019; Sahoo et al., 2021; Despotis and Kuchta, 2021). The weak-link approach, first presented by Despotis et al. (2016a), is inspired by the weak-link notion in supply chains and the maximal flow-minimal cut problem in networks. In the weak-link approach, the system efficiency is obtained by maximizing the minimum of the stage efficiencies. This approach provides unique and unbiased efficiency scores of the divisions and identifies adequately the source of system inefficiency.

On the other hand, a prominent characteristic of the traditional DEA models is that allows each DMU to select the most desirable weights in calculating its efficiency score. Therefore, in the classical DEA model, this flexible weight selection prevents DMUs from comparing the efficiencies under the same baseline. Further, weak discrimination is often found in classical DEA models, since many of the DMUs are estimated as efficient. To overcome the abovementioned limitations, common set of weight (CSW) DEA models have been developed by many researchers. The common weights method indicates that each DMU applies the same benchmark for computing efficiency. Several methods are proposed for finding CSW including separation vector (Chiang et al., 2011; Kiaei and Matin, 2020), cross efficiency (Saati and Nayebi, 2015; Angiz et al., 2013), ideal point method (Jahanshahloo. et al., 2010; Khalili-Damghani and Fadaei, 2018), goal programming (Gharakhani et al., 2018; Mavi et al., 2019), and evaluation of a subset of units (Troutt, 1997; Toloo, 2019; Troutt and Leung, 2003; Pendharkar, 2020). The maximin efficiency ratio model is a significant method for evaluation of DMUs. In the maximin efficiency ratio model, CSW is determined by maximizing the minimum efficiency ratio across a set of all DMUs (Troutt, 1997; Troutt and Leung, 2003). This method not only can surmount the issues in classical DEA models which include incomparable efficiency scores and weak discrimination power but also can find the minimum efficiency unit between all DMUs.

In the field of DEA, the property of units invariance indicates that efficiency results are independent of the units in which the observed inputs and outputs are measured so long as the units are the same for every DMU (Charnes and Cooper, 1984), is a desirable property of an ideal efficiency measure (Mehdiloozad et al., 2014). Lovell and Pastor (1995) discussed that Charnes, Cooper and Rhodes's CCR and Banker, Charnes and Cooper' BCC models are units invariant with respect to the radial measure, but not to the slack component. Cooper et al. (1999) claimed the range adjusted measure (RAM) model is units invariant. Tone (2001) noted that the slacks-based measure (SBM) model satisfies the property of units invariant. The property of units invariance is considered important in designing the measures in DEA. We will discuss units invariance of the proposed models in the study.

Almost all previous studies on the maximin efficiency ratio model focused on the single-stage system, only one exception is Wu et al. (2014). Wu et al. (2014) developed a maximin efficiency multistage supply chain model which considers the internal structure of DMUs to assess the supply chain performance. In their approach, the overall efficiency of the supply chain is defined as a weighted sum efficiency of its individual divisions. Although they looked into the internal stages of the DMUs, they did not consider the efficiency of the minimum efficiency stage in multi-stage system. In this study, we establish a two-level maximin network data envelopment analysis (NDEA) model for the efficiency evaluation of network production processes. The proposed model combines weak-link approach which can identify adequately the minimum efficiency stage inside the DMU and maximin efficiency ratio model which can find the minimum efficiency unit between all DMUs. Thus, our model exhibits an advantage in enabling decision makers to identify the stages that is the minimum efficiency stage in the minimum efficiency unit and effectively improve the performance of these systems. Moreover, the proposed model can provide unique and unbiased efficiency scores for the divisions. Otherwise, this model can surmount incomparable efficiency scores and weak discrimination power. Then, we extend cross efficiency evaluation method to the multi-stage system based on the two-level maximin NDEA model. The new cross efficiency evaluation method does not require secondary objective functions and can provide unique efficiency results.

Main contributions of this study are as follows. First, we are the first to develop a two-level maximin NDEA model. In the depicted approach, we explicitly provide the measures of overall efficiency and stage efficiencies. In addition, we discuss units invariance of the two-level maximin NDEA model. Last, we develop cross efficiency model based on the two-level maximin NDEA model for network system, which is firstly to extend the cross efficiency evaluation to a network system.

The remainder of the paper is organized as follows. Section 2 introduces the CCR ratio model, maximin efficiency ratio model, and cross efficiency evaluation method. Section 3 develops the two-level maximin NDEA model and cross efficiency model based on the two-level maximin NDEA model for the efficiency evaluation of the network system. Section 4 applies these models to an empirical study with 8 three-stage supply chains. Finally, the last section concludes the paper.

2. Preliminary

2.1 The CCR ratio model

Suppose that there are *n* DMUs, and each DMU_j $(j = 1, \dots, n)$ products *s* outputs y_{rj} $(r = 1, \dots, s)$ by *m* inputs x_{ij} $(i = 1, \dots, m)$. For any given DMU_d, its CCR ratio efficiency is defined as a ratio of the weighted sum of outputs to weighted sum of inputs (Charnes et al., 1978):

$$E_d = \frac{\Sigma_{r=1}^{s} u_{rd} y_{rd}}{\Sigma_{i=1}^{m} v_{id} x_{id}} \tag{1}$$

where v_i and u_r are the weights given respectively to the *i*-th input and the *r*-th output. The CCR ratio efficiency can be formulated by the following model.

$$\begin{aligned} & Max\, E_d = \frac{\Sigma_{r=1}^{s} u_{rd} y_{rd}}{\Sigma_{i=1}^{m} v_{id} x_{id}} \\ & s.t. \frac{\Sigma_{r=1}^{s} u_r y_{rj}}{\Sigma_{i=1}^{m} v_i x_{ij}} \le 1, j = 1, \dots, n \\ & v_i, u_r \ge 0, i = 1, \dots, m; r = 1, \dots, s \end{aligned} \tag{2}$$

Model (2) is equivalent to the following linear programming after the Charnes-Cooper transformation (Charnes and Cooper, 1962).

$$\begin{aligned} & Max\, E_{dd} = \Sigma_{r=1}^{s} u_{rd} y_{rd} \\ & s.t. \Sigma_{r=1}^{s} u_r y_{rj} - \Sigma_{i=1}^{m} v_i x_{ij} \le 0, j = 1, \dots, n \\ & \Sigma_{i=1}^{m} v_{id} x_{id} = 1 \\ & v_i, u_r \ge 0, i = 1, \dots, m; r = 1, \dots, s \end{aligned} \tag{3}$$

2.2 The maximin efficiency ratio model

The maximin efficiency ratio was proposed to enable DEA analysis along with a subset of DMUs (Wu et al., 2014). The maximin efficiency ratio model (Troutt, 1997) can be given as the following.

$$\begin{aligned} & Max\, min \frac{\Sigma_{r=1}^{s} u_r y_{rj}}{\Sigma_{i=1}^{m} v_i x_{ij}} \\ & s.t. \frac{\Sigma_{r=1}^{s} u_r y_{rj}}{\Sigma_{i=1}^{m} v_i x_{ij}} \le 1, j = 1, \dots, n \\ & \Sigma_{r=1}^{s} u_r = 1 \\ & v_i, u_r \ge 0, i = 1, \dots, m; r = 1, \dots, s. \end{aligned} \tag{4}$$

It is not easy to derive an optimal solution by transforming the nonlinear model (4) into a linear one. Fortunately, Bolzano (bisection) search procedure can be used to get an approximate solution for model (4).

2.3 Cross efficiency evaluation in DEA

The cross efficiency method measures each DMU through self-evaluation and peer-evaluation two processes. For a DMU_d, its self-evaluation efficiency score is computed by using model (3).

Solving model (3), we obtain a set of optimal weights (v_{id}^*, u_{rd}^*). Then, the cross efficiency of DMU_j $(j = 1, \dots, n)$ is formulated with the weights of DMU_d obtained in peer-evaluation process

$$E_{dj} = \frac{\Sigma_{r=1}^{s} u_{rd}^* y_{rj}}{\Sigma_{i=1}^{m} v_{id}^* x_{ij}} \tag{5}$$

Now each DMU_j $(j = 1, \dots, n)$ obtains one self-evaluation efficiency E_{dd} and $n-1$ peer-evaluation efficiencies E_{dj}. Then, the cross efficiency scores of DMU_j $(j = 1, \dots, n)$ can be computed by the following

$$\bar{E}_d = \frac{1}{n} \Sigma_{j=1}^{n} E_{dj} \tag{6}$$

3. Model development

A serial multi-stage structure is one of the most representatives network systems (Kao, 2014). We next present the formulation of the new model under the serial multi-stage structure. In a general series multi-stage production system, several processes are connected in series, as shown in Figure 1. In this system, any process p $(p = 1, \dots, q)$ utilizes exogenous inputs $X_i^{(p)}(i = 1, \dots, m;\ p = 1, \dots, q)$ and intermediate products $Z_r^{(p-1)}(r = 1, \dots, g;\ p = 1, \dots, q)$ created by its preceding process, to create exogenous outputs $Y_f^{(p)}(f = 1, \dots, s;\ p = 1, \dots, q)$ and intermediate products $Z_r^{(p)}(r = 1, \dots, g;\ p = 1, \dots, q)$ for the succeeding process to use. For the first process, there are no intermediate products from other processes are utilized for the first process, and for the last process q there are no intermediate products are generated for other processes. The notation that will be used throughout the paper is listed in Table 1.

Table 1 Notation used in the paper.

Categories	Notation	Meaning
General parameters	n	Number of DMUs
	q	Number of stages
	m	Number of inputs
	g	Number of intermediate outputs
	s	Number of exogenous outputs
Data parameters	$X_i^{(p)}$	$i(i = 1, \dots, m)$ inputs of stage p $(p = 1, \dots, q)$
	$Z_r^{(p)}$	$r(r = 1, \dots, g)$ intermediate outputs of stage p $(p = 1, \dots, q)$
	$Y_f^{(p)}$	$f(f = 1, \dots, s)$ exogenous outputs of stage p $(p = 1, \dots, q)$

Decision variables	$v_i^{(p)}$	Weight attached to the $i(i=1,\dots,m)$ inputs of stage p $(p=1,\dots,q)$
	$\pi_r^{(p)}$	Weight attached to the $r(r=1,\dots,g)$ intermediate outputs of stage p $(p=1,\dots,q)$
	$u_f^{(p)}$	Weight attached to the $f(f=1,\dots,s)$ exogenous outputs of stage p $(p=1,\dots,q)$

Typically, the ratio efficiencies of process p $(p=1,\dots,q)$ of DMU_j are defined as follows:

$$E_j^p=\frac{\Sigma_{f=1}^{s}u_f^{(p)}Y_{fj}^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}Z_{rj}^{(p)}}{\Sigma_{i=1}^{m}v_i^{(p)}X_{ij}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}Z_{rj}^{(p-1)}},p=1,\dots,q \tag{7}$$

where $\varphi=\begin{cases}1, & if\ p=1,\dots,q-1\\0, & if\ p=q\end{cases}$ and $\delta=\begin{cases}1, & if\ p=2,\dots,q\\0, & if\ p=1\end{cases}$.

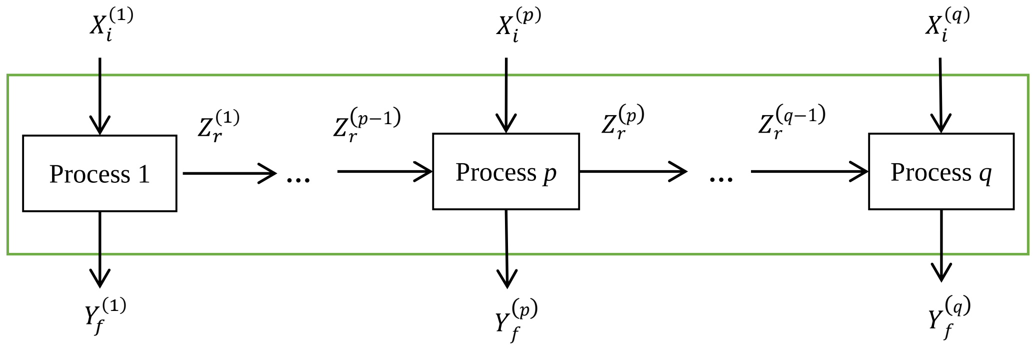

Figure 1 General multi-stage series production system.

3.1 The two-level maximin NDEA model

In our model, the maximin strategy will be applied at two levels. At the individual DMU level, the system efficiency of each DMU in the model is defined as the minimum of the stage efficiencies. Formally, the system efficiency of DMU_j is defined as

$$E_j=min\left\{E_j^1,\dots,E_j^p,\dots,E_j^q\right\} \tag{8}$$

where E_j^{p} is defined in (7).

At the all DMU level, we evaluate all the DMUs simultaneously by using the maximin efficiency ratio model for multistage as follows.

$$\begin{aligned}&Max\ min\left\{\frac{\Sigma_{f=1}^{s}u_f^{(p)}Y_{fj}^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}Z_{rj}^{(p)}}{\Sigma_{i=1}^{m}v_i^{(p)}X_{ij}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}Z_{rj}^{(p-1)}}\ ,p=1,\dots,q\right\}\\&s.t.\frac{\Sigma_{f=1}^{s}u_f^{(p)}Y_{fj}^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}Z_{rj}^{(p)}}{\Sigma_{i=1}^{m}v_i^{(p)}X_{ij}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}Z_{rj}^{(p-1)}}\le 1,p=1,\dots,q;j=1,\dots,n\\&\Sigma_{f=1}^{s}u_f^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}=1,p=1,\dots,q\end{aligned} \tag{9}$$

$$u_f^{(p)}, v_i^{(p)}, \pi_r^{(p)} \geq 0, f = 1, \ldots, s; i = 1, \ldots, m; r = 1, \ldots, g.$$

In model (9), the objective function seeks to maximize the minimum of the stage efficiencies. Let $\alpha = min\left\{\frac{\Sigma_{f=1}^{s} u_f^{(p)} Y_{fj}^{(p)} + \varphi \Sigma_{f=1}^{s} \pi_f^{(p)} Z_{fj}^{(p)}}{\Sigma_{i=1}^{m} v_i^{(p)} X_{ij}^{(p)} + \delta \Sigma_{f=1}^{s} u_f^{(p-1)} Y_{fj}^{(p-1)}}\right\}$, then the model (9) becomes

$$\begin{aligned}
&Max\, \alpha \\
&s.t. \frac{\Sigma_{f=1}^{s} u_f^{(p)} Y_{fj}^{(p)} + \varphi \Sigma_{r=1}^{g} \pi_r^{(p)} Z_{rj}^{(p)}}{\Sigma_{i=1}^{m} v_i^{(p)} X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_r^{(p-1)} Z_{rj}^{(p-1)}} \geq \alpha, p = 1, \ldots, q; j = 1, \ldots, n \\
&\frac{\Sigma_{f=1}^{s} u_f^{(p)} Y_{fj}^{(p)} + \varphi \Sigma_{r=1}^{g} \pi_r^{(p)} Z_{rj}^{(p)}}{\Sigma_{i=1}^{m} v_i^{(p)} X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_r^{(p-1)} Z_{rj}^{(p-1)}} \leq 1, p = 1, \ldots, q; j = 1, \ldots, n \\
&\Sigma_{f=1}^{s} u_f^{(p)} + \varphi \Sigma_{r=1}^{g} \pi_r^{(p)} = 1, p = 1, \ldots, q \\
&u_f^{(p)}, v_i^{(p)}, \pi_r^{(p)} \geq 0, f = 1, \ldots, s, i = 1, \ldots, m, r = 1, \ldots, g
\end{aligned} \tag{10}$$

which is further transformed into a parametric linear programming with parameter α:

$$\begin{aligned}
&Max\, \alpha \\
&s.t. \alpha \left(\Sigma_{i=1}^{m} {v'}_i^{(p)} X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_r'^{(p-1)} Z_{rj}^{(p-1)}\right) - (\Sigma_{f=1}^{s} {u'}_f^{(p)} Y_{fj}^{(p)} \\
&\qquad + \varphi \Sigma_{r=1}^{g} \pi_r'^{(p)} Z_{rj}^{(p)}) \leq 0, p = 1, \ldots, q; j = 1, \ldots, n \\
&\Sigma_{f=1}^{s} u_f^{(p)} Y_{fj}^{(p)} + \varphi \Sigma_{r=1}^{g} \pi_r^{(p)} Z_{rj}^{(p)} - (\Sigma_{i=1}^{m} v_i'^{(p)} X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_r'^{(p-1)} Z_{rj}^{(p-1)}) \\
&\qquad \leq 0, p = 1, \ldots, q; j = 1, \ldots, n \\
&\Sigma_{f=1}^{s} u_f'^{(p)} + \varphi \Sigma_{r=1}^{g} \pi_r'^{(p)} = 1, p = 1, \ldots, q \\
&u_f'^{(p)}, v_i'^{(p)}, \pi_r'^{(p)} \geq 0, f = 1, \ldots, s, i = 1, \ldots, m, r = 1, \ldots, g
\end{aligned} \tag{11}$$

We provide the following bisection algorithm to solve the model (11).

Step 1: Set $l = 1$ and denote $\left(\alpha_1'^{*(p)}, u_{1f}'^{*(p)}, v_{1i}'^{*(p)}, \pi_{1r}'^{*(p)}\right)$ as an optimal solution to model (11). The set of nq processes is then divided into two groups J_1 and J_2 as follows:

$$J_1 = \left\{p \middle| \alpha_1'^{*(p)} \left(\Sigma_{i=1}^{m} v_{1i}'^{*(p)} X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_{1r}'^{*(p-1)} Z_{rj}^{(p-1)}\right) - \Sigma_{f=1}^{s} u_{1f}'^{*(p)} Y_{fj}^{(p)} - \varphi \Sigma_{r=1}^{g} \pi_{1r}'^{*(p)} Z_{rj}^{(p)} = 0\right\}$$

and

$$J_2 = \left\{p \middle| \alpha_1'^{*(p)} \left(\Sigma_{i=1}^{m} v_{1i}'^{*(p)} X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_{1r}'^{*(p-1)} Z_{rj}^{(p-1)}\right) - \Sigma_{f=1}^{s} u_{1f}'^{*(p)} Y_{fj}^{(p)} - \varphi \Sigma_{r=1}^{g} \pi_{1r}'^{*(p)} Z_{rj}^{(p)} > 0\right\}$$

If the set J_2 is empty, $J_2 = \emptyset$ or $|J_2| = 0$, then the procedure exits; otherwise go to Step 2.

Step 2: Set $l = l + 1$, solve the following general model:

$$\begin{aligned}
&\alpha_l^* = Max\, \alpha \\
&s.t.\, \alpha_1'^{*(p)} \left(\Sigma_{i=1}^{m} v'^{(p)}_i X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_r'^{(p-1)} Z_{rj}^{(p-1)}\right) - (\Sigma_{f=1}^{s} u'^{(p)}_f Y_{fj}^{(p)} \\
&\qquad\qquad + \varphi \Sigma_{r=1}^{g} \pi_r'^{(p)} Z_{rj}^{(p)}) = 0, p \in J_1 \\
&\alpha_2'^{*(p)} \left(\Sigma_{i=1}^{m} v'^{(p)}_i X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_r'^{(p-1)} Z_{rj}^{(p-1)}\right) - (\Sigma_{f=1}^{s} u'^{(p)}_f Y_{fj}^{(p)} \\
&\qquad\qquad + \varphi \Sigma_{r=1}^{g} \pi_r'^{(p)} Z_{rj}^{(p)}) = 0, p \in J_3 \\
&\ldots \\
&\alpha_{l-1}'^{*(p)} \left(\Sigma_{i=1}^{m} v'^{(p)}_i X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_r'^{(p-1)} Z_{rj}^{(p-1)}\right) - (\Sigma_{f=1}^{s} u'^{(p)}_f Y_{fj}^{(p)} \\
&\qquad\qquad + \varphi \Sigma_{r=1}^{g} \pi_r'^{(p)} Z_{rj}^{(p)}) = 0, p \in J_{2l-3} \\
&\alpha \left(\Sigma_{i=1}^{m} v'^{(p)}_i X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_r'^{(p-1)} Z_{rj}^{(p-1)}\right) - (\Sigma_{f=1}^{s} u'^{(p)}_f Y_{fj}^{(p)} \\
&\qquad\qquad + \varphi \Sigma_{r=1}^{g} \pi_r'^{(p)} Z_{rj}^{(p)}) \leq 0, p \in J_{2l-2} \\
&\Sigma_{f=1}^{s} u_f'^{(p)} Y_{fj}^{(p)} + \varphi \Sigma_{r=1}^{g} \pi_r'^{(p)} Z_{rj}^{(p)} - (\Sigma_{i=1}^{m} v_i'^{(p)} X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_r'^{(p-1)} Z_{rj}^{(p-1)}) \\
&\qquad\qquad \leq 0, p = 1, \ldots, q; j = 1, \ldots, n \\
&\Sigma_{f=1}^{s} u_f'^{(p)} + \varphi \Sigma_{r=1}^{g} \pi_r'^{(p)} = 1, p = 1, \ldots, q \\
&u_f'^{(p)}, v_i'^{(p)}, \pi_r'^{(p)} \geq 0, f = 1, \ldots, s, i = 1, \ldots, m, r = 1, \ldots, g
\end{aligned} \tag{12}$$

We can obtain $\left(\alpha_l'^{*(p)}, u_{lf}'^{*(p)}, v_{li}'^{*(p)}, \pi_{lr}'^{*(p)}\right)$ from model (12).

Step 3: J_{2l-2} can be divided into two subsets:

$$J_{2l-1} = \left\{p \middle| \alpha_l'^{*(p)} \left(\Sigma_{i=1}^{m} v_{li}'^{*(p)} X_{ij}^{(p)} + \delta \Sigma_{f=1}^{s} u_{lf}'^{*(p-1)} Y_{fj}^{(p-1)}\right) - \Sigma_{f=1}^{s} u_{lf}'^{*(p)} Y_{fj}^{(p)} - \varphi \Sigma_{r=1}^{g} \pi_{lr}'^{*(p)} Z_{rj}^{(p)} = 0\right\}$$

and

$$J_{2l} = \left\{p \middle| \alpha_l'^{*(p)} \left(\Sigma_{i=1}^{m} v_{li}'^{*(p)} X_{ij}^{(p)} + \delta \Sigma_{f=1}^{s} u_{lf}'^{*(p-1)} Y_{fj}^{(p-1)}\right) - \Sigma_{f=1}^{s} u_{lf}'^{*(p)} Y_{fj}^{(p)} - \varphi \Sigma_{r=1}^{g} \pi_{lr}'^{*(p)} Z_{rj}^{(p)} > 0\right\}$$

If the set J_{2l} is empty, $J_{2l} = \emptyset$ or $|J_{2l}| = 0$, then the procedure exits; otherwise iterate from Step 2.

Let $W^{*p} = \left(u_f^{\prime *(p)}, v_i^{\prime *(p)}, \pi_r^{\prime *(p)}\right)$ be the solution to the problem using the maximin principle. The DEA-efficiency of p process and DMU_d can be denoted as follow:

$$E_d^p = \frac{\Sigma_{f=1}^{s} u_f^{\prime *(p)} Y_{fd}^{(p)} + \varphi \Sigma_{r=1}^{g} \pi_r^{\prime *(p)} Z_{rd}^{(p)}}{\Sigma_{i=1}^{m} v_i^{\prime *(p)} X_{id}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_r^{\prime *(p-1)} Z_{rd}^{(p-1)}}$$

and

$$E_d = min\{E_d^1, \dots, E_d^p, \dots, E_d^q\}$$

The bisection algorithm has the following characteristics.

Theorem 1. The bisection algorithm converges.

Proof. $\left(\alpha_l^{\prime *(p)}, u_{lf}^{\prime *(p)}, v_{li}^{\prime *(p)}, \pi_r^{\prime *(p)}\right)$ is a feasible solution of Model (11). We can obtain the inequation below.

$$0 \le \alpha_l^{\prime *(p)} \le \frac{\Sigma_{f=1}^{s} u_{lf}^{\prime *(p)} Y_{fj}^{(p)} + \varphi \Sigma_{r=1}^{g} \pi_{lr}^{\prime *(p)} Z_{rj}^{(p)}}{\Sigma_{i=1}^{m} v_{li}^{\prime *(p)} X_{ij}^{(p)} + \delta \Sigma_{r=1}^{g} \pi_{lr}^{\prime *(p-1)} Z_{rj}^{(p-1)}} \le 1$$

Apparently, $\left(\alpha_{l-1}^{\prime *(p)}, u_{l-1f}^{\prime *(p)}, v_{l-1i}^{\prime *(p)}, \pi_{l-1r}^{\prime *(p)}\right)$ is a feasible solution of Model (12). In addition, $\left(\alpha_l^{\prime *(p)}, u_{lf}^{\prime *(p)}, v_{li}^{\prime *(p)}, \pi_r^{\prime *(p)}\right)$ is an optimal solution of Model (12). Therefore, we have

$$\alpha_{l-1}^{\prime *(p)} \le \alpha_l^{\prime *(p)}$$

Thus, the below inequation is obtained.

$$0 \le \alpha_{l-1}^{\prime *(p)} \le \alpha_l^{\prime *(p)} \le 1$$

The bisection algorithm converges. Q.E.D.

In the field of DEA, the property of units invariance, which is in fact an application of a general mathematical property known as "dimensionless" (Lovell and Pastor, 1995), is that efficiency results is independent of the units in which the observed inputs and outputs are measured so long as the units are the same for every DMU (Charnes and Cooper, 1984). Units invariance is a desirable property (Sahoo et al., 2014). The following theorem shows the proposed model (9) is not units invariance.

Theorem 2. The model given in (9) is not units invariance.

Proof. By substitution in the model (9) we then have

$$Max\,min\left\{\frac{\Sigma_{f=1}^{s}u_f^{(p)}Y_{fj}^{(p)\prime}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}Z_{rj}^{(p)\prime}}{\Sigma_{i=1}^{m}v_i^{(p)}X_{ij}^{(p)\prime}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}Z_{rj}^{(p-1)\prime}}\ ,\ p=1,\dots,q\right\}$$

$$s.t.\frac{\Sigma_{f=1}^{s}u_f^{(p)}Y_{fj}^{(p)\prime}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}Z_{rj}^{(p)\prime}}{\Sigma_{i=1}^{m}v_i^{(p)}X_{ij}^{(p)\prime}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}Z_{rj}^{(p-1)\prime}}\le 1, p=1,\dots,q; j=1,\dots,n \quad (13)$$

$$\Sigma_{f=1}^{s}u_f^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}=1, p=1,\dots,q$$

$$u_f^{(p)}, v_i^{(p)}, \pi_r^{(p)}\ge 0, f=1,\dots,s; i=1,\dots,m; r=1,\dots,g.$$

Let $X_{ij}^{(p)\prime}=k_iX_{ij}^{(p)}, Z_{rj}^{(p)\prime}=c_rZ_{rj}^{(p)}, Y_{fj}^{(p)\prime}=a_fY_{fj}^{(p)}$, where k_i, c_r, a_f are any collection of positive constants. Model (13) becomes the following model:

$$Max\,min\left\{\frac{\Sigma_{f=1}^{s}u_f^{(p)}a_fY_{fj}^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}c_rZ_{rj}^{(p)}}{\Sigma_{i=1}^{m}v_i^{(p)}k_iX_{ij}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}c_rZ_{rj}^{(p-1)}}\ ,\ p=1,\dots,q\right\}$$

$$s.t.\frac{\Sigma_{f=1}^{s}u_f^{(p)}a_fY_{fj}^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}c_rZ_{rj}^{(p)}}{\Sigma_{i=1}^{m}v_i^{(p)}k_iX_{ij}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}c_rZ_{rj}^{(p-1)}}\le 1, p=1,\dots,q; j=1,\dots,n \quad (14)$$

$$\Sigma_{f=1}^{s}u_f^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}=1, p=1,\dots,q$$

$$u_f^{(p)}, v_i^{(p)}, \pi_r^{(p)}\ge 0, f=1,\dots,s; i=1,\dots,m; r=1,\dots,g.$$

Let $v_i^{(p)\prime}=v_i^{(p)}k_i, \pi_r^{(p)\prime}=\pi_r^{(p)}c_r, u_f^{(p)\prime}=u_f^{(p)}a_f$, Model (14) can be converted to the following model:

$$Max\,min\left\{\frac{\Sigma_{f=1}^{s}u_f^{(p)\prime}Y_{fj}^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)\prime}Z_{rj}^{(p)}}{\Sigma_{i=1}^{m}v_i^{(p)}X_{ij}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}Z_{rj}^{(p-1)}}\ ,\ p=1,\dots,q\right\}$$

$$s.t.\frac{\Sigma_{f=1}^{s}u_f^{(p)\prime}Y_{fj}^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)\prime}Z_{rj}^{(p)}}{\Sigma_{i=1}^{m}v_i^{(p)\prime}X_{ij}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)\prime}Z_{rj}^{(p-1)}}\le 1, p=1,\dots,q; j=1,\dots,n \quad (15)$$

$$\frac{\Sigma_{f=1}^{s}u_f^{(p)\prime}}{a_f}+\frac{\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}}{c_r}=1, p=1,\dots,q$$

$$u_f^{(p)\prime}, v_i^{(p)\prime}, \pi_r^{(p)\prime}\ge 0, f=1,\dots,s; i=1,\dots,m; r=1,\dots,g.$$

Comparing model (9) and model (15), we can find that the constraints have changed. Therefore, the solutions to model (15) are different from that of model (9) and the model is not units invariant. Q.E.D.

To obtain the unit invariance property, we replace the constraint $\Sigma_{f=1}^{s}u_f^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}=1, p=1,\dots,q$ with $\Sigma_{i=1}^{m}v_i^{(p)}X_{id}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}Z_{rd}^{(p-1)}=1\ ,\ p=1,\dots,q$ to the model (9). Then, we have the following model：

$$Max\ min\left\{\frac{\Sigma_{f=1}^{s}u_f^{(p)}Y_{fj}^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}Z_{rj}^{(p)}}{\Sigma_{i=1}^{m}v_i^{(p)}X_{ij}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}Z_{rj}^{(p-1)}}\ ,\ p=1,\dots,q\right\}$$

$$s.t.\frac{\Sigma_{f=1}^{s}u_f^{(p)}Y_{fj}^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)}Z_{rj}^{(p)}}{\Sigma_{i=1}^{m}v_i^{(p)}X_{ij}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}Z_{rj}^{(p-1)}}\leq 1,p=1,\dots,q;j=1,\dots,n \tag{16}$$

$$\Sigma_{i=1}^{m}v_i^{(p)}X_{id}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}Z_{rd}^{(p-1)}=1\ ,\ p=1,\dots,q;$$

$$u_f^{(p)},v_i^{(p)},\pi_r^{(p)}\geq 0,f=1,\dots,s;i=1,\dots,m;r=1,\dots,g.$$

Theorem 3. The model given in (16) is units invariance.

Proof. A similar proof of Theorem 2 for model (16), we have

$$Max\ min\left\{\frac{\Sigma_{f=1}^{s}u_f^{(p)\prime}Y_{fj}^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)\prime}Z_{rj}^{(p)}}{\Sigma_{i=1}^{m}v_i^{(p)}X_{ij}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)}Z_{rj}^{(p-1)}}\ ,\ p=1,\dots,q\right\}$$

$$s.t.\frac{\Sigma_{f=1}^{s}u_f^{(p)\prime}Y_{fj}^{(p)}+\varphi\Sigma_{r=1}^{g}\pi_r^{(p)\prime}Z_{rj}^{(p)}}{\Sigma_{i=1}^{m}v_i^{(p)\prime}X_{ij}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)\prime}Z_{rj}^{(p-1)}}\leq 1,p=1,\dots,q;j=1,\dots,n \tag{17}$$

$$\Sigma_{i=1}^{m}v_i^{(p)\prime}X_{id}^{(p)}+\delta\Sigma_{r=1}^{g}\pi_r^{(p-1)\prime}Z_{rd}^{(p-1)}=1\ ,\ p=1,\dots,q;$$

$$u_f^{(p)\prime},v_i^{(p)\prime},\pi_r^{(p)\prime}\geq 0,f=1,\dots,s;i=1,\dots,m;r=1,\dots,g.$$

Comparing model (16) and model (17), we can find that the two models are equivalent. Thus, the original model is units invariant. Q.E.D.

Theorem 4. The model given in (16) has a feasible solution.

Proof. Consider a solution where $u_f^{(p)}=0(\forall f,\forall p)$, $\pi_r^{(p)}=0(\forall f,\forall p)$, $v_i^{(p)}=0(\forall i\neq i_1,\forall p\neq p_1)$, and $v_{i_1}^{(p_1)}=\frac{1}{\Sigma_{j=1}^{n}X_{i_1j}^{(p_1)}}$. This solution is a feasible solution to model (16). Thus, the model given in (16) has feasible solution. Q.E.D.

The model (16) is units invariance. However, the model (16) cannot gain a common set of weight. To surmount incomparable efficiency scores and weak discrimination power, we next present cross efficiency model based on the two-level maximin NDEA model.

3.2 Cross efficiency model based on the two-level maximin NDEA model

In model (16), when we change the evaluated DMU_d, different optimal solutions of $u_{fd}^{\prime*(p)},v_{id}^{\prime*(p)}$ and

$\pi_{rd}^{\prime*(p)}$ can be obtained. Using the same solution procedure described in model (9), the solution of the model (16) are obtained.

Let $W_d^{*p} = \left(u_{fd}^{\prime*(p)}, v_{id}^{\prime*(p)}, \pi_{rd}^{\prime*(p)}\right)$ be the solution to the problem using the maximin principle for DMU_d. The DEA efficiencies of p process and DMU_d are

$$E_{dd}^p = \frac{\Sigma_{f=1}^s u_{fd}^{\prime*(p)} Y_{fd}^{(p)} + \varphi \Sigma_{r=1}^g \pi_{rd}^{\prime*(p)} Z_{rd}^{(p)}}{\Sigma_{i=1}^m v_{id}^{\prime*(p)} X_{id}^{(p)} + \delta \Sigma_{r=1}^g \pi_{rd}^{\prime*(p-1)} Z_{rd}^{(p-1)}}$$

and

$$E_{dd} = min\left\{E_{dd}^1, \dots E_{dd}^p, \dots, E_{dd}^q\right\}$$

Then, the peer-evaluation efficiencies given to DMU_j when DMU_d is under evaluation corresponding to the p process are defined as follows:

$$E_{dj}^p = \frac{\Sigma_{f=1}^s u_{fd}^{*p} Y_{fj}^{(p)} + \varphi \Sigma_{r=1}^g \pi_{rd}^{\prime*(p)} Z_{rj}^{(p)}}{\Sigma_{i=1}^m v_{id}^{\prime*(p)} X_{ij}^{(p)} + \delta \Sigma_{r=1}^g \pi_{rd}^{\prime*(p-1)} Z_{rj}^{(p-1)}}, j \neq d, j = 1, \dots, n$$

In this manner, the cross efficiency scores of p process and DMU_j are defined as follows:

$$CE_j^p = \frac{1}{n} \Sigma_{d=1}^n E_{dj}^p, j = 1, \dots, n$$

and

$$CE_j = min\left\{CE_j^1, \dots, CE_j^p\right\}, j = 1, \dots, n.$$

Note that all obtained efficiency scores $E_{dj}^p (j = 1, \dots, n)$ are unique. This is due to that all efficiency scores $E_{dj}^p (j = 1, \dots, n)$ are directly determined in the objective function in our modeling optimization. This is different from those secondary goals in DEA cross efficiency evaluation methods that are optimized for obtaining a set of optimal weights for the next efficiency score calculation. Then, CE_j^p and CE_j can be unique. Thus, Model (16) can obtain the unique cross efficiency.

The new models for parallel structure are similar to serial structures. In a parallel production system, at the individual DMU level, we define the system efficiency as the minimum of the subsystem efficiencies. Then, at the all DMU level, we evaluate all the DMUs simultaneously by using the maximin efficiency ratio model. For network structure, by utilizing dummy processes, a network system can be represented by a series structure where each stage in the series is of a parallel structure composed of a set of processes (Kao, 2009). Therefore, the proposed approach can be applied to any network structure.

4. A numerical illustration

In this section, we use the dataset of supply chain systems from Wu et al. (2014) to illustrate the usefulness and the validity of the proposed models. The supply chain system is a network system that can be divided into three processes: the supplier, the manufacturer, and the retailer. The supplier consumes various inputs such as labor (X_{11}) and operating cost (X_{12}) to produce revenue (Y_{11}), which then becomes the input cost to the downstream manufacturer. The manufacturer utilizes manufacturing cost (X_{21}) and lead time (X_{22}) to absorb fill rate (Y_{21}) and quantity of products (Y_{22}). Products are then shipped to the retailer. The retailer bear inventory cost (X_{31}) and backorders (X_{32}) in inventory to earn profits (Y_{31}).

Table 2 Data for the example.

DMU	Supplier			Manufacturer				Retailer		
	Inputs(X_1)		Intermediates(Y_1)	Inputs(X_2)		Intermediates(Y_2)		Inputs(X_3)		Outputs(Y_2)
	X_{11}	X_{12}	Y_{11}	X_{21}	X_{22}	Y_{21}	Y_{22}	X_{31}	X_{32}	Y_{31}
1	125	90	35	218	5	0.70	800	90	10.0	3000
2	150	80	25	190	4	0.90	700	100	11.0	2200
3	115	110	23	180	3	0.78	750	80	13.0	3200
4	155	100	24	205	5	0.88	500	70	12.5	2300
5	180	100	27	185	2	0.73	650	85	14.0	3300
6	145	85	25	180	3	0.95	550	77	13.5	2400
7	155	95	28	190	6	0.89	650	78	12.5	3500
8	180	125	35	180	2	0.87	600	90	15.5	3800

Table 2 reports this dataset. The results obtained by the model (9) and the supply chain model of Wu et al. (2014) are provided in Table 3.

In Table 3, the second and fifth columns report the efficiencies of the supply chain, the supplier, the manufacturer, and the retailer which are obtained by the model (9). The sixth to ninth columns describe the efficiencies of the supply chain, the supplier, the manufacturer, and the retailer which are got by the Wu et al.'s method. From Table 3, it can be concluded that the overall efficiency scores from the model (9) are all much less than those from Wu et al.'s method. This is due to the fact that the overall efficiency scores from the model (9) are the efficiencies of the weak-link of the supply chain and those from Wu et al.'s method are the weighted sum efficiency of stages. These differences indicate that the efficiency of the supply chain systems may be overestimated due to applying the weighted sum model.

Table 3 Results for two-level maximin NDEA model approach.

DMU	Weak-link efficiency				Weighted sum efficiency			
	e	e1	e2	e3	e	e1	e2	e3

1	0.6657(2)	0.8407	0.6657	0.7128	0.8978	1.0000	0.7569	0.9366
2	0.5918(6)	0.7457	0.8407	0.5918	0.7784	0.7115	0.9498	0.6740
3	0.5918(6)	0.5918	0.9954	0.8407	0.8153	0.5918	1.0000	0.8541
4	0.5918(6)	0.5918	0.7018	0.7018	0.6830	0.5918	0.7569	0.7004
5	0.6657(2)	0.6657	0.8284	0.8407	0.7722	0.6260	0.8238	0.8667
6	0.6657(2)	0.7018	0.8284	0.6657	0.7509	0.6975	0.8813	0.6740
7	0.7128(1)	0.7128	0.7457	0.9343	0.8593	0.7119	0.8660	1.0000
8	0.6657(2)	0.7018	0.6657	0.9343	0.8051	0.7103	0.7651	0.9398

Note that our model can show the bottleneck in the production process that is critical to the total efficiency of the system. For example, the overall efficiency of DMU_3 (0.8153) is higher than this of DMU_5 (0.7722) in Wu et al.'s method. However, the overall efficiency of DMU_3 (0.5918) is lower than this of DMU_5 (0.6657) in our method. This is attributed to the efficiency scores of the second stage and the third stage of DMU_3 is significantly higher than those of DMU_5, while the efficiency of the first stage of DMU_3 is lower than this of DMU_5. This infers that the efficiency of partial stages conceals the overall inefficiency in the weighted sum model.

The cross efficiency scores are reported in Table 4. It is shown that the efficiencies of the supply chain which are obtained by the model (9). This finding indicates that the cross efficiency model has stronger discriminating power in ranking DMUs and is able to produce more representative results.

Table 4 Results for cross efficiency model based on the two-level maximin NDEA model.

DMU	e	e1	e2	e3
1	0.6380(5)	0.9048	0.6380	0.8193
2	0.6375(6)	0.7637	0.7850	0.6375
3	0.5918(7)	0.5918	0.8496	0.8458
4	0.5918(7)	0.5918	0.6833	0.7191
5	0.6652(2)	0.6652	0.7282	0.8617
6	0.6652(2)	0.7189	0.7375	0.6652
7	0.6997(1)	0.7240	0.6997	0.9481
8	0.6652(2)	0.7028	0.6652	0.9639

5. Conclusions

In this study we introduce the two-level maximin strategy in DEA-based efficiency evaluation for network systems. We develop the two-level maximin NDEA model and cross efficiency model based on the two-level maximin NDEA model and measured the efficiency of eight supply chains with three processes. In the two-level maximin NDEA model, we combine the maximin efficiency ratio model with the weak-link approach in multi-stage systems. Based on the two-level maximin NDEA model, we build cross efficiency model for multi-stage systems.

In this paper, the proposed model was found to have the following advantages. First, the proposed model can help decision makers to identify the stage that is the minimum efficiency stage in the minimum efficiency unit and effectively improve the performance of these systems. Second, this approach provides unique and unbiased efficiency scores for the divisions. Third, the proposed cross efficiency model does not require secondary objective functions and provides unique results.

Based on the results of the application, our conclusions are summarized as follows. First, compared with the traditional multi-stage weighted sum model, our model can show the bottleneck in the production process that is critical for the total efficiency of the system can overcome that the efficiency of partial stages conceals the overall inefficiency. Second, cross efficiency model based on the two-level maximin NDEA model has stronger discriminating power in ranking DMUs, and is able to produce more representative results.

Looking at extensions to this research, in this study, all inputs, intermediate products, and outputs are considered desirable. Further studies can develop the two-level maximin NDEA model for network systems in presence of undesirable factors. For example, if available, CO_2 emissions could be incorporated into the research when the models are applied to the industrial production process. In addition, in the study, the evaluated DMUs are completely homogeneous. However, the perfect homogeneity is difficult to be found in DMUs. Some non-homogeneous factors may block the accuracy and rationality of evaluation, if they are neglected (Chen et al., 2021). Thus, one may consider non-homogeneous DMUs and evaluate their efficiency. Moreover, it would be interesting for future research to apply our methods to more real-world applications such as banks, hotels, and educational institutions. Finally, with the increase in the number of processes and DMUs, the computation time will increase. Therefore, it might be interesting to study how to optimize the algorithm to reduce the computation time.

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