Optimal Pricing, Production, and Intelligentization Policies for Smart, Connected Products Under Two-Level Trade Credit

Yu-Chung Tsao[1,2,3,4,5*], Nandya Shafira Pramesti[1,6], Thuy-Linh Vu[1,2], Iwan Vanany[6]

1. *Department of Industrial Management, National Taiwan University of Science and Technology, Taipei, Taiwan (*yctsao@mail.ntust.edu.tw*)*
2. *Artificial Intelligence for Operations Management Research Center, National Taiwan University of Science and Technology, Taipei, Taiwan*
3. *Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan*
4. *Department of Business Administration, Asia University, Taichung, Taiwan*
5. *Institute of Innovation and Circular Economy, Asia University, Taichung, Taiwan*
6. *Department of Industrial and Systems Engineering, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia*

Abstract

The development of technologies such as the Internet of Things has transformed traditional physical products into smart connected products (SCPs) that combine hardware, sensors, data storage, microprocessors, software, and connectivity in myriad ways. SCPs raise a new set of strategic choices for creating value and pricing products, how relationships with business partners such as channels are redefined, and what role companies should play as industry boundaries are expanded. This study develops an inventory model that considers optimal pricing, production, and intelligent policies for SCPs. In this model, customer demand is assumed to increase as the selling price decreases and the effort to improve product intelligence (i.e., intelligent effort) increases. In addition, a two-level trade credit is included in the SCPs supply chain channel. The manufacturer often receives a permissible delay-in-payment (trade credit) from the supplier while also offering a delayed payment to end customers to attract more sales. Trade credit is particularly important for SCPs as it can act as a payment plan to reduce the product's price barrier. This study aims to determine the optimal selling price, lot size, and level of intelligent effort while maximizing the manufacturer's profit under a two-level trade credit. The optimal solution is clarified, numerical examples are provided, and a sensitivity analysis is performed to illustrate the theoretical results and solution approach. The results reveal that considering the level of intelligent effort as a decision can benefit the manufacturer. Notably, as the intelligent effort coefficient increases by 55 percent, the total profit increases by 65.8 percent.

Keywords: *Inventory; Smart, Connected Products; Pricing; Lot-Sizing; Trade Credit*.

1. Introduction

The development of technology has made the world today more connected. The Internet of Things (IoT) connects objects, machines, and people. The emergence of the IoT reflects the growing number of smart connected products (SCPs) equipped with intelligence and connectivity (Porter & Heppelmann, 2014). An SCP includes three core elements: physical (e.g., engine block, tires, batteries, monitor, hardcover), smart (e.g., sensors, data storage, analytics, and software), and connected components (e.g., ports, antennae, protocols). SCP can have new functions not previously needed: sensors, software, connectivity, data storage, operating system, and analytics. For example, General Motors, Honda, Audi, and Hyundai recently joined forces to utilize Google's Android operating system for their vehicle (HBR, 2014). The system can help customers sync with their smartphone, music, and app and collect information. Another example of speaker products is SONOS, an American developer and manufacturer of audio products. The company's product lineup consists mainly of powered speakers, amplifiers, and peripherals. Recently, SONOS placed the user interface in the cloud to improve the products, enabling users to control the portable device from a smartphone (HBR, 2014). That means its products should be smarter and connected, encouraging customers to use their products and service. Thus, the increased range, capabilities, and analytical sophistication of SCPs offer value beyond the typical standard product. Responding to current technological trends is critical for companies to remain competitive. A survey of 1,013 manufacturers by the Capgemini Research Institute (2018) reported that nearly 50% of their products would be smart and connected by 2020, compared to only 15% in 2014. This trend continues to increase. To illustrate, approximately 801.5 million smart home products were shipped in 2020 and are forecast to surpass 1.4 billion by 2025 (IDC, 2021).

However, the selling price remains one of the main barriers for customers to purchase SCPs. Accenture (2016) polled 28,000 consumers in 28 countries and found that 62% of consumers believed that SCPs/IoT devices were too expensive. In addition, selling prices negatively affect customers' purchase intentions, such as smart home technology (Nikou, 2019), smart farm technology (Yoon et al., 2020), smart healthcare products (Karahoca et al., 2018), and wearable device technology (GlobalData, 2019). Nevertheless, perceived usefulness positively influences customers' intention to use SCPs (Gao et al., 2016; Nikou, 2019). Moreover, customers are willing to spend on devices when they believe there is a compelling value proposition (Accenture, 2016). The enhanced capabilities of SCPs have increased their acceptance of such products. Although, the higher capabilities of SCPs undoubtedly increase their purchase price. Customers may not want to pay for extra functionality because it may exceed what they perceive as added value and because of the increased complexity of use (Porter & Heppelmann, 2014). This trade-off between the selling price and the values of increased product capabilities may pose a strategic risk to companies. Consequently, companies must develop a pricing strategy that truly captures the enhanced capabilities and values of SCPs.

The trade-off between the selling price and product capabilities can be described as an increase in demand when the selling price decreases and product intelligence or capability increases. Product intelligence increases when the manufacturer adds a greater level of intelligent effort (i.e., smarter and connected components) to the product. According to the law of demand, the higher the product price, the lower the quantity demanded. Three common ways to quantify the decreasing demand function of the selling price P have been identified: (a) a linear pattern (Dye & Yang, 2015), (b) constant price elasticity (Urban & Baker, 1997), and (c) an exponential pattern (Feng et al., 2017). In this study, in order to deduce optimal policies for the production and sales of an SCP by a manufacturer, we developed an economic production quantity (EPQ) model that considers selling price, level of intelligent effort, and production cycle time as decision variables to maximize profit.

The traditional economic production quantity (EPQ) model assumes that the buyer must pay the seller the entire fee for the ordered item upon receipt. However, sellers often offer trade credit to buyers to attract more sales. Through this policy, the seller, as the credit provider (e.g., supplier), is willing to offer the buyer as the credit receiver (e.g., manufacturer) a certain credit period during which the buyer receives the items ordered while paying until sometime later. During the credit period, the buyer can earn revenue by selling the items and gain interest by investing in an interest-bearing account. After the credit period ends, the buyer must pay according to the purchase amount and is charged some interest if they fail to do so. More than 80% of business transactions in the United Kingdom use trade credit (Wilson & Summers, 2002). Non-financial businesses in the United States rely on an account payable strategy for 15% of their financing, whereas account receivables constitute approximately 13–40% of sales (Seifert et al., 2013). Trade credit can also attract more customers who view it as a type of price reduction or an alternative to price discounts without triggering price competition. Trade credit could also help reduce the perceived price barriers of SCPs.

This study contributes to the literature in three ways. First, it proposes an EPQ model that considers selling price, level of intelligent effort, and production cycle time as decision variables for SCP under two-level trade credit. To the best of our knowledge, this is the first study to consider the level of intelligence effort on SCPs as a decision variable and a factor influencing product demand. Accordingly, most studies on trade credit have not considered pricing strategies or the phenomena of IoT-enabled products that include SCPs (see Table 1). Second, this study utilizes an algorithm developed based on the optimality conditions allowing the determination of the optimal solution of the decision variables. The proposed algorithm can be easily applied to help manufacturers determine the optimal selling price, lot size, and level of intelligent effort while maximizing profit. Third, it provides numerical examples, and sensitivity is analyzed to illustrate the model and solution approach and provide managerial insights into the effect of SCP. The results show that the model and the proposed algorithm can be applied effectively to solve the problem.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 initially defines the notations and assumptions used in this study and subsequently develops the mathematical model. Next, Section 4 derives the theoretical results and develops an algorithm to determine the optimal solution. Section 5 provides numerical examples to illustrate the model and solution approaches. Finally, Section 7 concludes the study.

2. Literature Review

2.1 Price-dependent Demand Function in Inventory Models

In practice, the selling price is one of the critical factors affecting customer demand. Shinn (1997) used a maximization profit model to determine retailers' optimal price and replenishment cycle time, in which the supplier permits customer delay-in-payment. The demand rate is represented by a constant price elasticity function, in which the deduced selling price is influenced by the length of the credit period. Giri and Maiti (2013) developed a model in which demand is sensitive to the selling price and trade credit under a profit-sharing contract to identify the optimal selling price, cycle time, and credit period offered by the retailer. Feng et al. (2017) considered that the demand rate is affected by price, freshness, and inventory as functions of time. The objective of this study was to determine the unit price, cycle time, and end inventory level to maximize total profit. Duan et al. (2018) investigated joint dynamic pricing and production decisions for deteriorating items, where the potential demand is characterized by a stochastic process. Feng and Chan (2019) studied joint pricing and production policies for new products under two-level trade credits when the learning curve effect is most prominent, and demand depends on the selling price. Esmaeili and Nasrabadi (2021) presented an inflation-related inventory model for deteriorating items in a single-vendor, multi-retailer supply chain. In the model, demand is a function of the retail price, and the vendor offers retailers trade credit.

2.2 Trade Credit Policies

Goyal (1985) first explored a single-item EOQ model in which a supplier offers a permissible delay to customers. Huang (2007) developed an EPQ model for two-level trade credit. Teng and Chang (2009) relaxed the assumption used in the previous model, where $N < M$, and considered the case in which the manufacturer is forced to give a permissible delay to the customer while not receiving any permissible delay-in-payment from the supplier ($M = 0$). Moreover, the model amended the fact that the retailer offers N. Hence, the retailer receives revenue from N to $T + N$, not from 0 to T. Kreng and Tan (2011) established an EPQ model under two-level trade credit by incorporating defective items that were considered imperfect or scrap. This added quality-related costs to the model.

Teng et al. (2014) explored an EPQ model from a seller's perspective. Here, trade credit is provided to the buyer to increase sales, opportunity costs, and default risk. The authors consider the learning curve, assuming that demand is an exponentially increasing function of the credit period given to the buyer. Zhang and Yuan (2017) established a new EOQ model under two-level trade credit, in which the supplier offers the retailer a partial trade credit period for payments, even if the order quantity is less than a threshold quantity. Simultaneously, the retailer provides a full trade credit period to customers. Wu et al. (2018) established an inventory policy for perishable items based on trapezoidal demand under a two-level trade credit using discounted cash flows. Feng and Chan (2019) studied joint pricing and lot-sizing policies for new products under two-level trade credits when the learning curve effect is most prominent. Mandal et al. (2020) describe an unreliable production-inventory model with varying demand under two-level trade credit policies. Das et al. (2021) considered an EPQ model in which the manufacturer produces products with certain reliability, demand is an exponentially increasing function of price, and the manufacturer receives an upstream credit period from the supplier while providing a partial downstream credit period to the customer.

2.3 SCPs

The SCPs were first described by Porter and Heppelmann (2014). The authors discuss the various benefits and impacts of SCPs on business and competition. SCPs can monitor their own conditions and surroundings, remotely control their operation, optimize utilization and performance, and adapt to user preferences that allow them to service themselves and operate autonomously (Porter and Heppelmann, 2015). Alternative IoT-based terms for describing SCPs include smart products, smart things, smart objects, and digitized products (Pardo et al., 2020). Currently, SCP research mainly addresses the conceptualization of smart products, such as in Pardo et al. (2020) and Raff et al. (2020), factors of resistance and intention to adopt SCP (Gao et al., 2016), and appropriate business strategies and frameworks for SCP companies (Porter and Heppelmann, 2014 and 2015). There is yet to be a study incorporating the SCP into an EPQ model, especially under two-level trade credit.

Table 1 provides a summary and comparison of the various previous models and this study. As shown in Table 1, most studies on trade credit have not considered pricing strategies and the phenomena of IoT-enabled products that include SCPs. To close this gap, this study proposes an EPQ model that considers selling price, level of intelligent effort, and production cycle time as decision variables for SCP under two-level trade credit.

Table 1. A summary of earlier studies and our present work.

References	Demand function	Trade Credit	Inventory	Decision Variable
Chang et al. (2003)	Time	Upstream	EOQ	Lot-sizing
Teng and Chang (2009)	Constant	Two-level	EPQ	Lot-sizing
Feng et al. (2013)	Constant	Two-level	EPQ	Lot-sizing
Teng et al. (2014)	Time	Two-level	EOQ	Lot-sizing
Wu et al. (2014)	Price & stock	Upstream	EOQ	Lot-sizing & ending inventory level
Wu and Chan (2014)	Constant	Two-level	EOQ	Lot-sizing
Wu et al. (2018)	Time	Two-level	EOQ	Timing inventory level reaches zero
Feng et al. (2017)	Price & time	No	EOQ	Lot-sizing, pricing & ending stock
Chen et al. (2019)	Price & time	No	EOQ	Lot-sizing & pricing
Tiwari et al. (2018)	Price	Two-level	EOQ	Lot-sizing, pricing & timing inventory level reaches zero

References	Demand function	Trade Credit	Inventory	Decision Variable
Feng and Chan (2019)	Price	Two-level	EPQ	Lot-sizing & pricing
Zou and Tian (2020)	Constant	Two-level	EOQ	Lot-sizing & payment strategy
Li et al. (2021)	Price & downstream credit period	Two-level	EOQ	Lot-sizing, pricing & downstream credit period
This paper	*Price & intelligent effort*	*Two-level*	*EPQ*	*Lot-sizing, pricing & intelligentization*

3. Proposal Model

3.1 Assumption and Notations

This section clarifies the definitions, assumptions, and notations of the problem used in the model. In this study, a supply chain system consists of a supplier, a manufacturer, and a group of customers. The manufacturer produces SCPs and orders product components (i.e., physical, smart, and connected components) from the supplier. The manufacturer embeds smart and connected components into physical components to make SCPs. This embedment process is the manufacturer's effort to make the product smart, called intelligent effort. The manufacturer then sells the SCPs to customers based on the selling price and the level of intelligent effort. The following assumptions were made to develop the mathematical EPQ model.

1. As detailed in Feng et al. (2017), Li et al. (2017), and Feng and Chan (2019), the demand rate is assumed to be an exponential function of price. In addition, demand increases when the level of intelligent effort increases. Hence, the demand rate $D(P,S)$ is a function of the selling price P and level of intelligent effort S formulated as follows:

   $$D(P,S) = Ke^{-aP}S^{b}, \qquad (1)$$

   where $K > 0$ is the maximum number of potential customers, $a > 0$ is the coefficient of price, and $b > 0$ is the coefficient of intelligence effort. For convenience, $D(P,S)$ and D are used interchangeably in this study.

2. The level of intelligent effort S includes a set of smart and connected components that costs c_s (cost of intelligent effort) for a one-level increase in product intelligence.
3. The unit product cost consists of the physical component cost and cost of intelligent effort multiplied by the level of intelligent effort embedded in the product. The cost of the

intelligent effort is assumed to be the same for any level of intelligent effort given to the product. Hence, unit product cost is formulated as follows:

$$Unit\ product\ cost = c + c_s S \quad (2)$$

4. The manufacturer receives an upstream credit period of u years from the supplier and offers a downstream credit period of days (d) to customers. If $u \geq d$, then the sales revenue generated from customers is deposited in an interest-bearing account after time d. If $u \geq T + d$, the manufacturer receives the last payment from customers by $T + d$, accumulates all sales revenue, and pays the full purchase cost at time u without any interest charged. If $u \leq T + d$, the manufacturer pays the supplier all the units sold by $u - d$, receives the profits for other uses, and starts paying the interest charges for all the items sold after $u - d$ until payment to the final customer is made at $T + d$. Otherwise, if $u \leq d$, the manufacturer must finance the purchase cost at u and then pay off the loan from d to $T + d$.
5. Replenishment by the supplier is instantaneous and lead time is negligible.
6. Shortages are prohibited. Hence, there are no backorders or lost sales.

The following notations are used throughout the paper.

Table 2. List of notations.

Notation	Description
T	Production cycle time (years)
S	Level of intelligent effort (level/product)
P	Selling price ($/product)
T^*	Manufacturer's optimal production cycle time (years)
S^*	Manufacturer's optimal level of intelligent effort (level/product)
P^*	Manufacturer's optimal selling price ($/product)
a	Price coefficient on demand
b	Intelligent effort level coefficient on demand
c	Physical component cost ($/product)
c_s	Cost of intelligent effort ($/intelligent effort level)
d	Manufacturer's downstream credit period to customers (years)
u	Manufacturer's upstream credit period from the supplier (years)
h	Inventory holding cost excluding interest charge ($/unit/year)
p	Annual production rate that is larger than the annual demand rate (units)
X_e	Interest rate earned ($/year)
X_c	Interest rate charged ($/year)
K	Maximum number of potential customers with $p \geq K$
o	Set-up cost ($/production run)
$D(P,S)$	Annual demand rate as a function of unit selling price P and intelligent effort level S (units)
Q	Manufacturer's production lot size $Q = D(P,S)T$ (units)
$\Pi(P,S,T)$	Manufacturer's profit function ($/year)
Π^*	Manufacturer's optimal profit ($/year)

3.2 Mathematical Model

Based on the above assumptions, we formulate all revenue and relevant costs in this subsection. The manufacturer considers its revenue, production cost, setup cost, holding cost, and interest earned and charged to maximize its yearly profit $\Pi(\mathrm{P,S,T})$. The relevant revenue and cost components that form the mathematical model are as follows:

1. Sales revenues are calculated by multiplying the selling price and total customer demand: $SR = PD(P,S)$
2. The total product cost (PC) is calculated by multiplying the production cost per unit ($c + c_sS$) by the total customer demand: $PC = (c + c_sS)D(P,S)$, where the production cost increases as intelligent effort increases (see the third assumption).
3. The setup cost per year was calculated based on the number of setups ($\frac{D(P,S)}{Q}$): $SC = \frac{D(P,S)}{Q}o = \frac{o}{T}$.
4. The holding cost is the cost of holding products during storage. This is calculated by multiplying the number of holding items per cycle ($\frac{Q}{2}\left(\frac{p-D(P,S)}{p}\right)$) by the holding cost per product (h). Thus, $HC = \frac{Qh}{2}\left(\frac{p-D(P,S)}{p}\right) = \frac{hD(P,S)T}{2}\left(1 - \frac{D(P,S)}{p}\right)$.
5. The interest earned and charged is calculated based on the trade credit period and production cycle. Three possible cases can be explained through revenue versus time graphs.

+ Case 1: $u \geq d$ and $u \leq T + d$

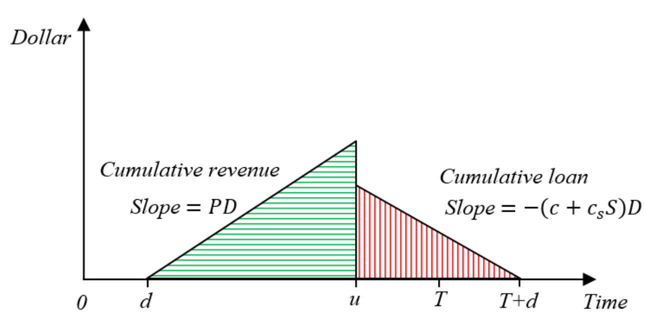

Fig. 1. Graphical representation of $u \geq d$ and $u \leq T + d$.

As shown in **Fig. 1.** Graphical representation of $u \geq d$ and $u \leq T + d$. Fig. 1, the manufacturer starts production and sells products at zero but receives revenue at d. The manufacturer saves revenue in an interest-bearing account that earns X_e per dollar per year during $u - d$. The interest earned per cycle time T is PX_e multiplied by the area of the cumulative revenue, given by

$$IE = \frac{1}{2} PDX_e(u - d)^2 \quad (3)$$

When the upstream credit period ends at u, the manufacturer must pay the supplier all the units sold in $u - d$, keep the profits from the sales, and pay for all the remaining products sold after $u - d$ at an interest rate of X_c per dollar per year. At a certain point before T, the production finishes, and the manufacturer fulfills all the demand at T. Finally, the payment made by customers who receive their products at T is at $T + d$. Therefore, the interest charged per cycle time T is $(c + c_s S)X_c$ multiplied by the area of the cumulative loan given by

$$IC = \frac{1}{2}(c + c_s S)DX_c(T + d - u)^2 \quad (4)$$

+ Case 2: $u \geq d$ *and* $u \geq T + d$

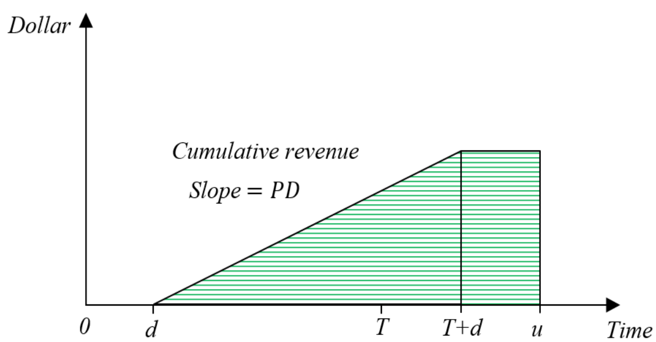

Fig. 2. Graphical representation of $u \geq d$ and $u \geq T + d$.

As shown in Fig. 2, the manufacturer starts production and sells products at 0 but starts to receive payment from customers at d. Similarly, the production finishes at a certain point before T, and the manufacturer fulfills the demand at T. However, the final payment by customers who receive their product at T is at $T + d$. Before the trade credit period given by the supplier ends at u, the manufacturer saves revenue in an interest-bearing account and can fully pay the supplier the total purchase cost by u. Therefore, the period of earning interest revenue is $u - d$. Hence, the manufacturer has no interest in the charge ($IC = 0$). The interest earned per cycle time T is then the product of PX_e and the area of cumulative revenue:

$$IE = PDX_e\left(u - d - \frac{T}{2}\right)T \tag{5}$$

+ *Case 3:* $u \leq d$

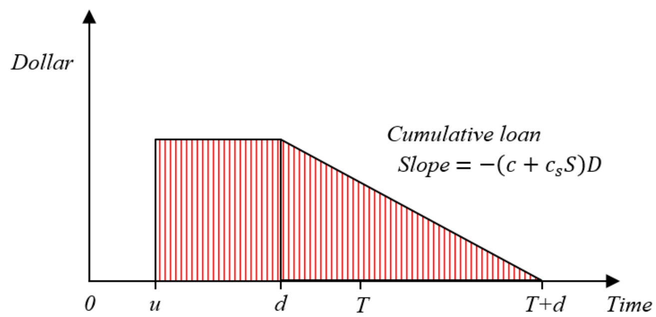

Fig. 3. Graphical representation of $u \leq d$.

As shown in Fig. 3, the manufacturer starts production and sells products at 0 but starts receiving payment from customers at d. The manufacturer does not have any revenue to pay off the supplier at u. Hence, the manufacturer must finance the purchase cost at u and start paying off the loan from d to $T + d$. The decrease in loans is shown from point d to $T + d$ as the customer starts paying for the product. $IE = 0$ and the interest charged per cycle time T is calculated by multiplying $(c + c_s S)X_c$ by the area of the cumulative loan as follows:

$$IC = (c + c_s S)DX_c\left(d - u + \frac{T}{2}\right)T \tag{6}$$

The manufacturer's annual profit can therefore be expressed as:

$\Pi(P, S, T) =$ Sales revenue, (SR) − product cost, (PC) − set-up cost, (SC) − holding cost, (HC) − interest charged (IC) + interest earned (IE)

Hence, the manufacturer's annual profit for Case 1 is as follows:

$$\Pi_1(P, S, T) = PD - (c + c_s S)D - \frac{o}{T} - \frac{hDT}{2}\left(1 - \frac{D}{p}\right) - \frac{1}{2T}(c + c_s S)DX_c(T + d - u)^2 + \frac{1}{2T}PDX_e(u - d)^2 \tag{7}$$

The manufacturer's profit in Case 2 is as follows. Note that there is no interest in the charge.

$$\Pi_2(P,S,T) = PD - (c + c_s S)D - \frac{o}{T} - \frac{hDT}{2}\left(1 - \frac{D}{p}\right) + PDX_e\left(u - d - \frac{T}{2}\right) \tag{8}$$

From (7) and (8), it is clear that

$$\Pi_1(P,S,u-d) = \Pi_2(P,S,u-d), if\ T = u - d. \tag{9}$$

For Case 3, the manufacturer's annual profit is as follows. It should be noted that no interest was received.

$$\Pi_3(P,S,T) = PD - (c + c_s S)D - \frac{o}{T} - \frac{hDT}{2}\left(1 - \frac{D}{p}\right) - (c + c_s S)DX_c\left(d - u + \frac{T}{2}\right) \tag{10}$$

4. Theoretical Results and Algorithm

In this section, a concavity analysis of the model is conducted to investigate the existence of a unique global solution for selling price P, level of intelligent effort S, and production cycle time T that maximizes the manufacturer's annual profit. The theoretical results and proposed algorithm are as follows.

4.1 Theoretical Results

The analysis was performed by fulfilling the necessary conditions for concavity. The necessary condition is obtained by taking the first-order partial derivative of $\Pi(P,S,T)$ with respect to each decision variable. A sufficient condition was fulfilled when the second-order partial derivative was less than zero. The theoretical results obtained are as follows:

Theorem 1. For any given unit selling price P and intelligent effort S, if $2o - D(u-d)^2\big(PX_e - X_c(c + c_s S)\big) > 0$, then $\Pi_1(P,S,T)$ is a strictly concave function in T.

Proof. See Appendix A.

By setting the result of the first-order partial derivative to zero, the optimal production cycle time T is as follows:

$$T_1 = \sqrt{\frac{2o - D(u-d)^2[PX_e - (c + c_s S)X_c]}{D[(c + c_s S)X_c + h(1 - \frac{D}{p})]}}, \text{ if } 2o - D(u-d)^2[PX_e - (c + c_s S)X_c] > 0 \tag{11}$$

Otherwise, the optimal replenishment cycle time does not exist in Case 1. The manufacturer's optimal production lot size is then given by

$$Q_1 = DT_1 = \sqrt{\frac{D[2o - D(u-d)^2[PX_e - (c + c_s S)X_c]]}{[(c + c_s S)X_c + h(1 - \frac{D}{p})]}} \tag{12}$$

Theorem 2. For any given unit selling price P and intelligent effort S, then $\Pi_2(P,S,T)$ and $\Pi_3(P,S,T)$ are strictly concave functions in T.

Proof. See Appendix B.

Similarly, by setting the results of the first-order partial derivatives of both cases with respect to T, equal to zero, the optimal production cycle time T is obtained as follows:

$$T_2 = \sqrt{\frac{2o}{D[PX_e + h(1 - \frac{D}{p})]}} \tag{13}$$

and

$$T_3 = \sqrt{\frac{2o}{D[X_c(c + c_s S) + h(1 - \frac{D}{p})]}} \tag{14}$$

Hence, the manufacturer's optimal production lot size is given by:

$$Q_2 = DT_2 = \sqrt{\frac{2oD}{PX_e + h(1 - \frac{D}{p})}} \tag{15}$$

and

$$Q_3 = DT_3 = \sqrt{\frac{2oD}{X_c(c + c_s S) + h(1 - \frac{D}{p})}} \tag{16}$$

Theorem 3. For any given unit selling price P and production cycle time T, if $1 \geq b$ and $c_s S + b(c + c_s S) \geq c$, then $\Pi_1(P,S,T)$, $\Pi_2(P,S,T)$ and $\Pi_3(P,S,T)$ is a strictly concave function in S.

Proof. See Appendix C.

Theorem 4. For any given unit intelligent effort S and production cycle time T, if $2 \geq aP$, then $\Pi_1(P,S,T)$, $\Pi_2(P,S,T)$ and $\Pi_3(P,S,T)$ are strictly concave functions in P.

Proof. See Appendix D.

The case of $u \geq d$ has two possible solutions, either T_1^* and T_2^*. To determine the case in which the optimal solution $T^*$is, the discrimination term $\Delta(T)$ is defined as the first-order partial derivative of (7) or (8) with respect to T as follows:

$$\Delta(T) = \frac{o}{T^2} - \frac{D(P,S)}{2}\left[PX_e + h\left(1 - \frac{D(P,S)}{p}\right)\right] \tag{17}$$

The following theoretical results can be obtained by combining the results of Theorems 1 and 2 and applying (17).

Theorem 5. For any given unit selling price P, intelligent effort S and $u \geq d$.

(a) if $\Delta(u-d) < 0$, then the manufacturer's optimal cycle time is $T^* = T_2$.

(b) if $\Delta(u-d) = 0$, then the manufacturer's optimal cycle time is $T^* = u - d$.

(c) if $\Delta(u-d) > 0$, then the manufacturer's optimal cycle time is $T^* = T_1$.

Proof. See Appendix E.

From the five derived theorems, the only closed form that can be attained is the production cycle time T. The derivations of the unit selling price P and intelligent effort S are very complex, and their closed forms are theoretically difficult to find. Therefore, an algorithm is proposed to determine the optimal decision variables based on the optimality conditions.

4.2 Solution Algorithm

Using the condition from Theorem 3 and rearranging the terms, the following equation is obtained:

$$S \geq \frac{c(1-b)}{c_s(1+b)} \tag{18}$$

The initial S is equal to that in Equation (18). The proposed algorithm is as follows:

Step 1. Set $S_{i,q=1} = C$, where C is the solution of $\frac{c(1-b)}{c_s(1+b)}$ rounded up to the nearest integer, as the initial value for each case i = 1, 2, 3, and $q = 1,2, \ldots, n$ represents the number of iterations.

Step 2. IF $u \geq d$, go to Step 2.1; otherwise, go to Step 3.

Step 2.1. Set $i = 1$.

Step 2.1.1. Calculate $T_{i,q}$ given $S_{i,q}$ from (11) to find $T_{i,q}(P_{i,q})$,

Step 2.1.2. By substituting $T_{i,q}(P_{i,q})$ and $S_{i,q}$ into equation (7), we derive the resulting $\Pi_i(P)$ to satisfy $\frac{\partial \Pi_i(P)}{\partial P} = 0$ and solve for $P_{i,q}$. IF $2o - D(u-d)^2[P_{i,q}X_e - (c + c_s S_{i,q})X_c] \geq 0$ and $P_{i,q} < \frac{2}{a}$ go to step 2.1.3, IF $2o - D(u-d)^2[P_{i,q}X_e - (c + c_s S_{i,q})X_c] \geq 0$ and $P_{i,q} \geq \frac{2}{a}$, let $P_{i,q} = \frac{2}{a}$ and go to step 2.1.3; otherwise, IF $2o - D(u-d)^2[P_{i,q}X_e - (c + c_s S_{i,q})X_c] < 0$, go to step 2.1.4.

Step 2.1.3. Substituting $P_{i,q}$ into $T_{i,q}(P_{i,q})$ to obtain $T_{i,q}$, IF $T_{i,q} \geq u - d$, and proceed to step 2.1.4.; otherwise, let $T_{i,q} = u - d$ then go to step 2.1.4.

Step 2.1.4. Set $S_{i,q+1} = S_{i,q} + \varepsilon$, where ε is a small positive value. Calculate $T_{i,q+1}(P_{i,q+1})$ from (11) given $S_{i,q+1}$,

Step 2.1.5. Substituting $T_{i,q+1}(P_{i,q+1})$ and $S_{i,q+1}$ into equation (7), we derive the resulting $\Pi_i(P)$ to satisfy $\frac{\partial \Pi_i(P)}{\partial P} = 0$ and solve for $P_{i,q+1}$. IF $2o - D(u-d)^2[P_{i,q+1}X_e - (c + c_s S_{i,q+1})X_c] \geq 0$ and $P_{i,q+1} < \frac{2}{a}$, go to step 2.1.6., IF $2o - D(u-d)^2[P_{i,q+1}X_e - (c + c_s S_{i,q+1})X_c] \geq 0$ and $P_{i,q+1} \geq \frac{2}{a}$, let $P_{i,q+1} = \frac{2}{a}$ and go to step 2.1.6, IF $2o - D(u-d)^2[P_{i,q+1}X_e - (c + c_s S_{i,q+1})X_c] < 0$, and $P_{i,q+1} < \frac{2}{a}$ go to step 2.1.4. Otherwise, IF $2o - D(u-d)^2[P_{i,q+1}X_e - (c + c_s S_{i,q+1})X_c] < 0$ and $P_{i,q+1} \geq \frac{2}{a}$, let $T_i^* = \infty$, and go to step 2.2.

Step 2.1.6. Substitute $P_{i,q+1}$ to $T_{i,q+1}(P_{i,q+1})$ to get $T_{i,q+1}$, IF $T_{i,q+1} \geq u - d$, go to step 2.1.7., otherwise let $T_{i,q+1} = u - d$ then go to step 2.1.7.

Step 2.1.7. IF $\Pi_i(P_{i,q+1}, S_{i,q+1}, T_{i,q+1}) > \Pi_i(P_{i,q}, S_{i,q}, T_{i,q})$, set $P_{i,q+1} = P_{i,q}$, $S_{i,q+1} = S_{i,q}$, $T_{i,q+1} = T_{i,q}$, then go to Step 2.1.4.; otherwise, let $T_i^* = T_{i,q}$, $P_i^* = P_{i,q}$, $S_i^* = S_{i,q}$,

Step 2.1.8. Substitute P_i^* and S_i^* to equation (17) with $T = u - d$ to get Δ_i.

Step 2.2. Set $i = 2$.

Step 2.2.1. Calculate $T_{i,q}$ given $S_{i,q}$ from (13) to find $T_{i,q}(P_{i,q})$.

Step 2.2.2. Substituting $T_{i,q}(P_{i,q})$, and $S_{i,q}$ into equation (8), we derive the resulting $\Pi_i(P)$ to satisfy $\frac{\partial \Pi_i(P)}{\partial P} = 0$ and solve for $P_{i,q}$. Check IF $P_{i,q} > \frac{2}{a}$ let $P_{i,q} = \frac{2}{a}$.

Step 2.2.3. Substituting $P_{i,q}$ into $T_{i,q}(P_{i,q})$ to obtain $T_{i,q}$, IF $T_{i,q} \leq u - d$, and proceed to step 2.2.4.; otherwise, let $T_{i,q} = u - d$ and go to step 2.2.4.

Step 2.2.4. Set $S_{i,q+1} = S_{i,q} + \varepsilon$, where ε is a small positive value. Calculate $T_{i,q+1}(P_{i,q+1})$ is calculated from (13) given $S_{i,q+1}$.

Step 2.2.5. Substituting $T_{i,q+1}(P_{i,q+1})$ and $S_{i,q+1}$ into equation (8), we derive the resulting $\Pi_i(P)$ to satisfy $\frac{\partial \Pi_i(P)}{\partial P} = 0$ and solve for $P_{i,q+1}$, Check IF $P_{i,q+1} > \frac{2}{a}$ let $P_{i,q+1} = \frac{2}{a}$.

Step 2.2.6. Substitute $P_{i,q+1}$ to $T_{i,q+1}(P_{i,q+1})$ to get $T_{i,q+1}$, IF $T_{i,q+1} \leq u - d$, go to step 2.2.7.; otherwise, let $T_{i,q+1} = u - d$ and go to step 2.2.7.

Step 2.2.7. IF $\Pi_i(P_{i,q+1}, S_{i,q+1}, T_{i,q+1}) > \Pi_i(P_{i,q}, S_{i,q}, T_{i,q})$, set $P_{i,q+1} = P_{i,q}$, $S_{i,q+1} = S_{i,q}$, $T_{i,q+1} = T_{i,q}$ then go to Step 2.2.4; otherwise, let $T_i^* = T_{i,q}$, $P_i^* = P_{i,q}$, $S_i^* = S_{i,q}$.

Step 2.2.8. IF $T_{i=1}^* = \infty$, let $T^* = T_{i=2}^*$, $P^* = P_{i=2}^*$, and $S^* = S_{i=2}^*$, and let the optimal profit of case $u \geq d$ be $\Pi^*(P^*, S^*, T^*)$. Otherwise, substitute P_i^* and S_i^* in equation (17) with $T = u - d$ to obtain Δ_i.

Step 2.3. IF $\Delta_{i=1} < 0$ and $\Delta_{i=2} < 0$, let $T^* = T_{i=2}^*$, $P^* = P_{i=2}^*$, and $S^* = S_{i=2}^*$,

IF $\Delta_{i=1} > 0$ and $\Delta_{i=2} > 0$, let $T^* = T_{i=1}^*$, $P^* = P_{i=1}^*$, and $S^* = S_{i=1}^*$,

IF $\Delta_{i=1} = 0$ and $\Delta_{i=2} = 0$, let $T^* = u - d$, $P^* = P_i^*$, where $\frac{\partial \Pi_i(P_i^*,\ S^*, u-d)}{\partial P} = 0$, and $S^* = S_i^*$ where $\frac{\partial \Pi_i(P^*, S_i^*, u-d)}{\partial S} = 0$, and let the optimal profit of case $u \geq d$ be $\Pi^*(P^*, S^*, T^*)$.

Step 3. Set $i = 3$ *for* $u \leq d$.

Step 3.1. Calculate $T_{i,q}$ given $S_{i,q}$ from Equation (14) to find $T_{i,q}(P_{i,q})$.

Step 3.2. By substituting $T_{i,q}(P_{i,q})$ and $S_{i,q}$ into equation (10), we derive the resulting $\Pi_i(P)$ to satisfy $\frac{\partial \Pi_i(P)}{\partial P} = 0$ and solve for $P_{i,q}$. IF $P_{i,q} > \frac{2}{a}$ let $P_{i,q} = \frac{2}{a}$.

Step 3.3. Substitute $P_{i,q}$ to $T_{i,q}(P_{i,q})$ to get $T_{i,q}$.

Step 3.4. Set $S_{i,q+1} = S_{i,q} + \varepsilon$, where ε is a small positive value. Calculate $T_{i,q+1}(P_{i,q+1})$ is calculated from (14) given $S_{i,q+1}$.

Step 3.5. Substituting $T_{i,q+1}(P_{i,q+1})$ and $S_{i,q+1}$ into equation (10), we derive the resulting $\Pi_i(P)$ to satisfy $\frac{\partial \Pi_{\mathrm{i}}(P)}{\partial P} = 0$ and solve for $P_{i,q+1}$. IF $P_{i,q+1} > \frac{2}{a}$ let $P_{i,q+1} = \frac{2}{a}$.

Step 3.6. Substitute $P_{i,q+1}$ to $T_{i,q+1}(P_{i,q+1})$ to get $T_{i,q+1}$.

Step 3.7 IF $\Pi_{\mathrm{i}}\left(P_{i,q+1}, S_{i,q+1}, T_{i,q+1}\right) > \Pi_{\mathrm{i}}\left(P_{i,q}, S_{i,q}, T_{i,q}\right)$, set $P_{i,q+1} = P_{i,q}$, $S_{i,q+1} = S_{i,q}$, $T_{i,q+1} = T_{i,q}$, then go to Step 3.1.4; otherwise, let $T^* = T_{i,q}$, $P^* = P_{i,q}$, and $S^* = S_{i,q}$, and let the optimal profit of case $u \leq d$ be $\Pi^*(P^*, S^*, T^*)$;

ELSE, $\Pi^*(P^*, S^*, T^*) = -\infty$.

5. Numerical Analysis

This section conducts numerical analyses to verify the theoretical results, obtain quantitative insights, and discuss the effect of the decision variables and total profit.

5.1 Numerical Examples

First, the following numerical examples were used to illustrate the proposed model. The first example illustrates the case where $u \geq d$. The second example illustrates the case of $u \leq d$. The numerical example data were taken from Feng and Chan (2019), with some additional data to adapt to our models. A sensitivity analysis was performed to evaluate the impact of changing the cost parameters.

Example 1. This example considers a supply chain system consisting of a supplier, manufacturer, and group of customers. Basically, the manufacturer produces smartphones and orders the product components, including three core elements: physical (e.g., monitor, hardcover), smart (e.g., sensors, data storage, software), and connected components (e.g., ports, antennae, protocols) from the supplier. Subsequently, the manufacturer embeds smart and connected components into physical components to make a smartphone. The manufacturer then sells smartphones to customers at a given selling price and level of intelligent effort. The following parameters are assumed in manufacturing a smartphone: component cost $c = \$35$ per unit, cost of intelligent effort $c_s = \$20$, downstream trade credit period $d = 0.08$ years, upstream trade credit period

$u = 0.25$ years, inventory holding cost $h = \$10$ per unit per year, production rate $p = 5000$ units, set-up cost $o = \$20$ per order, interest earned $X_e = 0.03$ per year, interest charged $X_c = 0.05$ per year, and demand rate $D(P,S) = 3000e^{-0.005P}S^{0.75}$ units per year.

Using MATHEMATICA 7.0, the unique optimal solution for the case of $u \geq d$ can be obtained as follows:

$P_1 = \$400$, $S_1 = 10$, $T_1 = 0.17$, $Q_1 = 388.134$, $D_1 = 2283.14$, $\Delta_1 = -19{,}209.762$ and $\Pi_1 = \$377{,}874.53$.

The solution for the case of $u \geq T + d$ is attained as follows:

$P_2 = 394.14$, $S_2 = 8$, $T_2 = 0.03357$, $Q_2 = 66.76$, $D_2 = 1988.70$, $\Delta_2 = -17{,}054.016$ and $\Pi_2 = \$398{,}840.64$.

As expected from Theorem 5, the optimal solution of **Example 1** is:

$P^* = 394.14$, $S^* = 8$, $T^* = 0.03357$, $Q^* = 66.76$, $D^* = 1988.70$, $\Pi^* = \$398{,}841.64$.

The graphical representations of $\Pi_2(P,S,T)$ are given in Fig. 4, Fig**.** , and Fig. **6**, which reveal that profit Π_2 is strictly concave in T, S and P, respectively. The dots in each figure indicate the locations of optimal solutions.

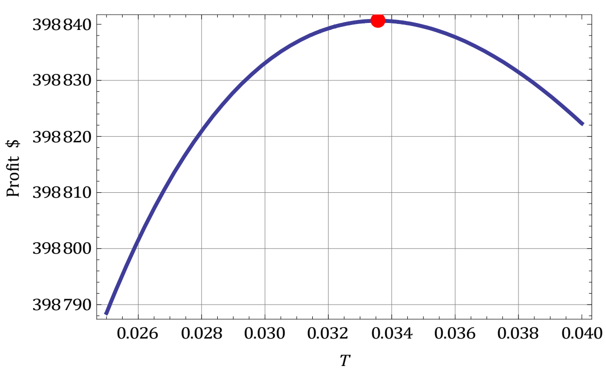

Fig. 4. Graphical representation of $\boldsymbol{\Pi_2(P^*, S^*, T)}$ of Example 1

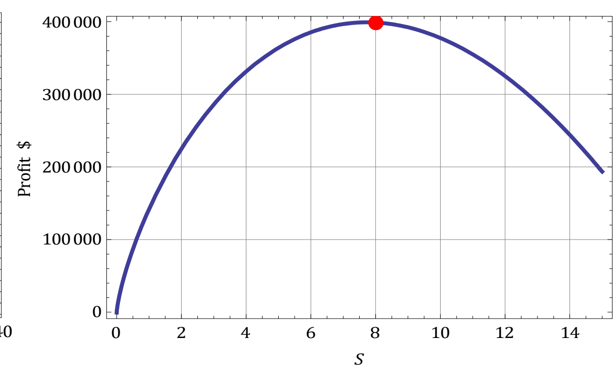

Fig. 5. Graphical representation of $\boldsymbol{\Pi_2(P^*, S, T^*)}$ of Example 1

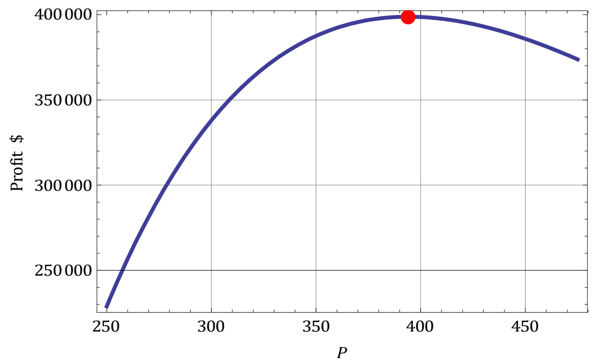

Fig. 6. Graphical representation of $\mathbf{\Pi_2}(\boldsymbol{P}, \boldsymbol{S}^*, \ \boldsymbol{T}^*)$ of Example 1

Example 2. This next example presents a case similar to Example 1. However, in this case, the manufacturer's upstream credit period from the supplier (0.16 years) is lower than or equal to the manufacturer's downstream credit period for customers (0.25 years). To illustrate the case of $u \leq d$, the following parameters are used: component cost $c = \$40$ per unit, cost of intelligent effort $c_s = \$25$, downstream trade credit period $d = 0.25$ years, upstream trade credit period $u = 0.16$ years, inventory holding cost $h = \$10$ per unit per year, production rate $p = 5000$ units, set-up cost $o = \$25$ per order, interest earned $X_e = 0.03$ per year, the interest charged $X_c = 0.05$ per year, and demand rate $D(P, S) = 3000e^{-0.005P}S^{0.75}$ units per year.

Using MATHEMATICA 7.0, the unique optimal solution for the case of $u \leq d$ can be obtained as follows:

$P^* = \ 391.14, S^* = 6, T^* = 0.04349, Q^* = 79.436, D^* = 1826.42, \Pi^* = \$324{,}712.$

The graphical representations of $\Pi_3(P, S, T)$ are shown in Fig. 7, 8, and 9, respectively, which reveal that profit Π_3 is strictly concave in T, S and P, respectively. The dots in each figure indicate the locations of optimal solutions.

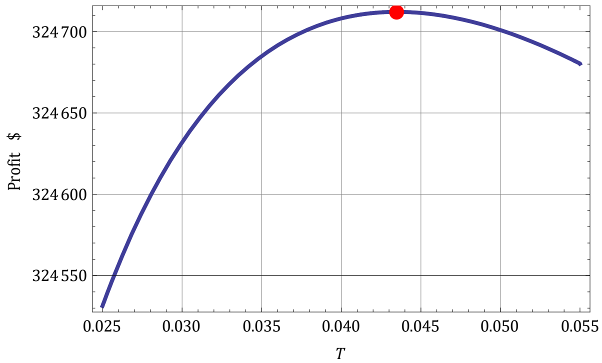

Fig. 7. Graphical representation of $\boldsymbol{\Pi_3(P^*, S^*, T)}$ of Example 2

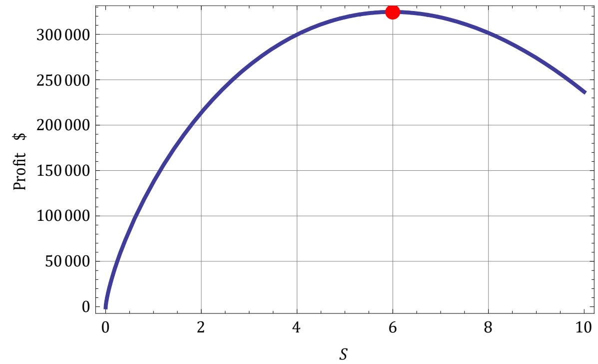

Fig. 8. Graphical representation of $\boldsymbol{\Pi_3(P^*, S, T^*)}$ of Example 2

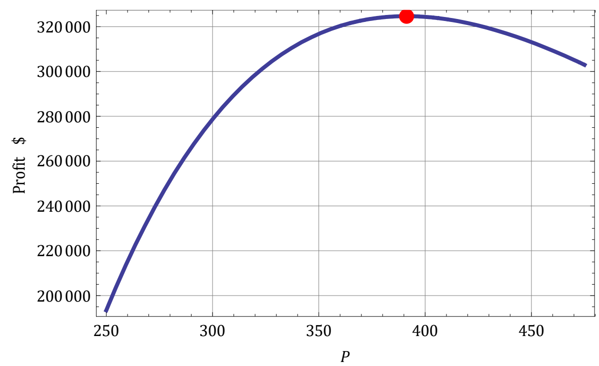

Fig. 9. Graphical representation of $\boldsymbol{\Pi_3(P, S^*, T^*)}$ of Example 2

The results in Examples 1 and 2 show that our proposed model and approach can be applied to determine the optimal value of the selling price, intelligent effort, and replenishment cycle time while maximizing the total profit. Manufacturers can practically use our model to make decisions for SCPs under two-level trade credit.

5.2 Sensitivity Analysis

To understand the possible cases of trade credits specified previously, the numerical data in Example 1 illustrate the changes in Cases 1 and 2. Sensitivity analysis was conducted to answer the question: What happens to the decision variables and objective value when the value of the system parameters changes? Our proposed model and solution approach can help manufacturers answer this question. Furthermore, we provide several management insights from the sensitivity analysis to explain the reason for changes and what we should do when the market changes. Managers or decision-makers can use the model to make decisions regarding price, cycle time, and intelligent effort whenever the market parameters change.

The optimal solution was determined by changing the parameter X_c located in Case 2 (T_2), which does not have parameter X_c, as formulated in (8)Hence, the sensitivity analysis for X_c was obtained from **Example 2.** The results of the sensitivity analysis are as follows.

Table 3. Sensitivity analysis with respect to each parameter.

Parameters	P^*	S^*	T^*	Q^*	D	Π^*
a=0.003	626.91	13	0.0238	74.54	3131.75	1,047,794.08
a=0.004	464.01	9	0.0294	71.53	2436.35	611,072.27
a=0.005	394.14	8	0.0336	66.76	1988.70	398,840.64
a=0.006	321.03	6	0.0383	64.17	1675.72	279,921.28
a=0.007	277.35	5	0.0424	61.06	1439.45	205,994.66
b=0.45	334.52	5	0.0441	51.23	1162.13	232,944.59
b=0.6	354.38	6	0.0390	58.21	1494.46	299,640.38
b=0.75	394.14	8	0.0336	66.76	1988.70	398,840.64
b=0.9	400.00	9	0.0291	85.28	2933.26	547,261.09
b=1	400.00	9	0.0273	99.74	3654.05	681,988.61
c=21	380.20	8	0.0331	70.54	2132.34	427,676.26
c=28	387.17	8	0.0333	68.61	2059.27	413,007.12
c=35	394.14	8	0.0336	66.76	1988.70	398,840.64
c=42	400.00	8	0.0338	65.26	1931.30	385,153.59
c=49	388.18	7	0.0347	64.29	1853.60	371,722.06
c_s=12	390.09	13	0.0294	85.83	2920.85	585,961.44
c_s=16	378.19	9	0.0320	75.20	2352.79	471,928.83
c_s=20	394.14	8	0.0336	66.76	1988.70	398,840.64
c_s=24	378.24	6	0.0359	62.31	1735.37	347,992.66
c_s=28	374.28	5	0.0378	58.35	1543.85	309,550.49
K=1800	394.24	8	0.0415	49.54	1192.64	239,056.12
K=2400	394.18	8	0.0367	58.42	1590.64	318,936.52
K=3000	394.14	8	0.0336	66.76	1988.70	398,840.64
K=3600	394.11	8	0.0314	74.83	2386.85	478,762.77
K=4200	394.08	8	0.0297	82.79	2785.05	558,699.53
p=3000	394.06	8	0.0364	72.38	1989.55	398,932.75
p=4000	394.11	8	0.0345	68.71	1989.00	398,874.31
p=5000	394.14	8	0.0336	66.76	1988.70	398,840.64
p=6000	394.16	8	0.0330	65.55	1988.52	398,818.71
p=7000	394.18	8	0.0326	64.73	1988.38	398,803.29

The sensitivity analysis in Tables 3~4 revealed the following:

1. The higher the price coefficient a, the higher the weight given to the same selling price P^* in reducing demand D. Consequently, the selling price P^* decreases and reduces the level of intelligent effort S^* that can be covered. The production cycle time T^* increases to

reduce the set-up cost, whereas the lot size Q^* decreases to meet the reduced demand. As a result, the total profit Π^* decreases.

2. The increase in the intelligent effort coefficient b increases the weight given to the same level of intelligent effort S^* with increasing demand D. As a result, the level of intelligent effort S^* increases, even when the selling price P^* also increases to cover the extra S^*. The production cycle time T^* decreases, and the lot size Q^* increases to meet the increased demand in a shorter time. This results in a significant increase in the total profit Π^*. As the intelligent effort coefficient increases by 55 percent, the total profit increases by 65.8 percent.

3. A higher physical component cost c first increases the selling price P^*, while the level of intelligent effort S^* is the first constant, and then decreases both P^* and S^* at $c = 49$. The maximum selling price ($P = 400$) is related to the value of a (see Theorem 4). When the selling price reaches its maximum value, the level of intelligent effort decreases because the selling price can no longer increase to cover the extra cost c. Corresponding to the reduced S^*, the selling price decreases when $c = 49$. The production cycle time T^* increases and lot size Q^* decreases to meet the decreased demand D and lower the set-up cost. This reduces the total profit Π^*.

4. The total cost of the intelligent effort varies as the level of intelligent effort S^* changes. Hence, when the cost of the intelligent effort per level c_s becomes more expensive, S^* decreases to reduce the total cost. The selling price P^* fluctuates with a decreasing trend corresponding to the level of intelligent effort and related cost. The production cycle time T^* increases, and the lot size Q^* decreases to lower setup costs and meet the reduced demand D. This significantly reduces the total profit Π^*.

5. As the number of potential customers K increases, the demand D increases. Because the optimal selling price P^* obtained in **Example 1** is already close to the maximum value allowed, the level of intelligent effort S^* cannot be increased further, as it would also increase the selling price. Hence, the selling price slightly decreases to benefit from the increased K and gain more customers. Meanwhile, the production cycle time T^* is reduced, and the lot size Q^* is increased to meet the increased demand in a shorter time. Consequently, the total profit Π^* increases.

6. The increase in production rate p could potentially increase the inventory level; hence, the production cycle time T^* shortens to prevent production from accumulating too much inventory. The lot size Q^* also decreases, which results in small, frequent batches that may increase the setup cost. Consequently, the selling price P^* increases, thus decreasing the demand D. The level of intelligent effort S^*remains constant to avoid additional costs. This results in a slight decrease in the total profit Π^*.

Table 4. Sensitivity analysis with respect to each parameter (cont')

Parameters	P^*	S^*	T^*	Q^*	D	Π^*
d=0.048	393.96	8	0.0336	66.81	1990.55	399,593.29
d=0.064	394.05	8	0.0336	66.79	1989.63	399,216.92
d=0.08	394.14	8	0.0336	66.76	1988.70	398,840.64
d=0.096	394.24	8	0.0336	66.74	1987.78	398,464.44
d=0.112	394.33	8	0.0336	66.71	1986.86	398,088.33
u=0.15	394.72	8	0.0336	66.61	1982.93	396,490.83
u=0.2	394.43	8	0.0336	66.69	1985.81	397,665.31
u=0.25	394.14	8	0.0336	66.76	1988.70	398,840.64
u=0.3	393.85	8	0.0336	66.84	1991.59	400,016.81
u=0.35	393.56	8	0.0335	66.91	1994.47	401,193.81
h=6	394.14	8	0.0361	71.78	1988.75	398,923.97
h=8	394.14	8	0.0348	69.14	1988.72	398,881.55
h=10	394.14	8	0.0336	66.76	1988.70	398,840.64
h=12	394.14	8	0.0325	64.62	1988.68	398,801.09
h=14	394.15	8	0.0315	62.67	1988.65	398,762.77
o=12	394.11	8	0.0260	51.72	1989.00	399,109.21
o=16	394.13	8	0.0300	59.72	1988.84	398,966.43
o=20	394.14	8	0.0336	66.76	1988.70	398,840.64
o=24	394.16	8	0.0368	73.13	1988.58	398,726.92
o=28	394.17	8	0.0397	78.99	1988.47	398,622.34
X_e=0.018	394.51	8	0.0392	77.76	1985.01	397,412.32
X_e=0.024	394.33	8	0.0361	71.63	1986.86	398,123.05
X_e=0.03	394.14	8	0.0336	66.76	1988.70	398,840.64
X_e=0.036	393.96	8	0.0315	62.78	1990.56	399,563.83
X_e=0.042	393.77	8	0.0298	59.45	1992.43	400,291.74
X_c=0.03	390.74	6	0.0497	80.95	1630.24	325,412.62
X_c=0.04	390.94	6	0.0463	75.35	1628.62	325,059.84
X_c=0.05	391.14	6	0.0435	70.76	1627.01	324,712.15
X_c=0.06	391.33	6	0.0412	66.92	1625.42	324,368.72
X_c=0.07	391.53	6	0.0392	63.63	1623.85	324,028.93

7. Increasing the downstream trade credit period d reduces the manufacturer's period in saving revenues in an interest-bearing account, thereby reducing interest-earned revenue. To cover the costs, the selling price P^* increases, which reduces demand D because demand only depends on P^* and S^*. The production cycle time T^* increases and lowers the set-up costs, whereas the production lot size Q^* decreases to meet the reduced demand. The level of intelligent effort S^* remained constant to avoid additional costs. Consequently, the total profit Π^* decreases.
8. Increasing the upstream trade credit period u increases the manufacturer's period of saving revenue in an interest-bearing account, adding more interest revenues. As a result, the selling price P^* can be decreased to increase demand D. On the other hand, the production cycle time T^* reduces, and with a slightly larger lot size Q^*; thus, more demand is met faster, thereby saving higher revenues and benefitting from the increased u. The level of intelligent effort S^* remained constant to avoid additional costs. Consequently, the total profit Π^*increases.
9. As the holding cost h becomes more expensive, the production cycle time T^* decreases to lower inventory costs, while the selling price P^* increases to cover the added cost, and thus decreases demand D. Hence, the lot size Q^* lowers to benefit from smaller and more frequent lot sizes. The level of intelligent effort S^* remained constant to avoid additional costs. Consequently, the total profit Π^* decreases.
10. However, as the set-up cost o becomes more expensive, the production cycle time T^* increases to reduce the number of production runs and lower the total set-up costs. The production lot size Q^* becomes larger to benefit from the large, infrequent batches. To cover the added cost, the selling price P^* increases, thereby reducing the demand D. The level of intelligent effort S^* remained constant to avoid additional costs. Consequently, the total profit Π^* decreases.
11. A higher interest rate earned X_e implies that the same value of revenues saved in an account would earn higher interest revenues. Hence, the selling price P^* can be decreased, which increases the demand D. The production cycle time T^* becomes shorter and lot size Q^* decreases as the manufacturer can order and produce less quantity and satisfy demand in a shorter time to take the benefits of trade credit more frequently. The level of intelligent

effort S^* remained constant to avoid additional costs. Consequently, the total profit Π^* increases.

12. On the other hand, a higher interest rate charged X_c means an additional cost for the same period of late payment; thus, the selling price P^* increases to cover the extra cost, which decreases demand D. The production cycle time T^* decreases and lot size Q^* decreases to meet the declined demand faster to reduce late payment. The level of intelligent effort S^* remained constant to avoid additional costs. The increase in X_c reduces the total profit Π^*.

Several managerial insights can be highlighted:

1. Based on the optimal solution in **Example 1**, it is impossible to add a higher level of intelligent effort S^* because it would increase the selling price P^* beyond the allowed value (according to the price coefficient value a and Theorem 4). Hence, the changes in K, p, d, u, h, o, X_e and X_c slightly changed P^* while S^* remained constant compared with the change in a (as this would increase or reduce the maximum selling price allowed). Increasing the intelligent effort coefficient b also increases P^*, but no more than the maximum selling price allowed (P = 400). From these results, it is reasonable to say that the maximum selling price acts as a threshold reflecting the value that is still considered attractive to customers for a given level of intelligent effort.
2. In contrast to other studies, such as Feng and Chan (2019) and Li et al. (2017, 2021), the increase in product cost does not directly increase the selling price P^*. This is because the level of intelligent effort S^* is also a decision variable and a demand factor, along with P^*. Hence, the increase in cost can also be addressed by reducing the level of intelligent effort provided to the product, which results in more flexible cost management.
3. The total profit $\Pi^*$is significantly affected by the changes in the demand parameters: price coefficient a, intelligent effort coefficient b, and the maximum number of potential customers K; hence, the manufacturer should first focus on increasing its revenues. The total profit Π^* is also affected by changes in c_s and c, although not as significantly; therefore, the manufacturer should focus on reducing these costs. The remaining parameter changes do not significantly affect Π^*.

6. Conclusion

This study developed an EPQ mathematical model to reflect: (1) demand increases when the selling price decreases and the level of intelligent effort increases; (2) the effect on the manufacturer's lot size by selling price, product intelligence, and trade credit; and (3) the situation in which the manufacturer receives an upstream trade credit period from the supplier while providing a downstream trade credit period to the customer. The necessary and sufficient conditions for obtaining the optimal solution were derived, and an explicit closed-form solution for the optimal production cycle time was obtained. Furthermore, the optimal case can be identified under certain conditions. Owing to the complexity of the problem, it is difficult to analytically obtain an explicit closed-form solution for the optimal selling price and level of intelligent effort. Hence, an algorithm was developed, and a numerical example was solved to demonstrate the proposed algorithm. The results demonstrate that the proposed model and solution approach can be applied to the data case, and the optimal annual profit for the manufacturer can be obtained.

To increase profit, the manufacturer should primarily focus on increasing revenues, for example, making use of marketing efforts to increase customers' perceived usefulness or benefits of SCP. In addition, manufacturers should focus on reducing product costs through better supplier selection or negotiation. Once costs have been reduced, the manufacturer can lower the selling price, which increases product demand. The values of a and Theorem 4 provide a price threshold reflecting the maximum price still considered attractive to customers for the given level of intelligent effort, as customers may not want to pay beyond this threshold because of the extra cost and complexity of additional product intelligence. Hence, setting the level of intelligent effort in a product as a decision variable enables more flexible product cost management. The number of intelligence levels in a product can be set to maximize profit and reduce costs.

Even so, all these models are not without limitations. In reality, demand depends not only on price and intelligence efforts, particularly for SCP. It could be expanded as a dynamic function of advertisement, downstream credit, quality, displayed stocks, lead-time, and so on. Moreover, the features of SCP enable the manufacturer to maintain relationships with customers through product updates and increased functionality, even after the product is bought. Many

SCPs offer monthly subscription plans for enhanced functionality, which can be incorporated into future research.

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Appendix A. Proof of Theorem 1

The first and second-order partial derivatives of $\Pi_1(P,S,T)$ with respect to T are:

$$\frac{\partial \Pi_1(P,S,T)}{\partial T} = -\frac{1}{2}D(h(1-\frac{D}{p}) + X_c(c+c_sS)) - (-\frac{1}{2T^2})[2o + D(u-d)^2(X_c(c+c_sS) - PX_e)] \quad \text{(A1)}$$

and

$$\frac{\partial^2 \Pi_1(P,S,T)}{\partial T^2} = -\frac{1}{T^3}\left[2o - D(u-d)^2\left(PX_e - X_c(c+c_sS)\right)\right] < 0 \quad \text{(A2)}$$

Hence, $\Pi_1(P,S,T)$ is a strictly concave function in T if $2o - D(u-d)^2\left(PX_e - X_c(c+c_sS)\right) > 0$. This completes the proof of Theorem 1.

Appendix B. Proof of Theorem 2.

For any given P and S, the first and second-order partial derivatives of $\Pi_2(P,S,T)$ and $\Pi_3(P,S,T)$ with respect to T are:

$$\frac{\partial \Pi_2(P,S,T)}{\partial T} = -\frac{1}{2}Dh(1-\frac{D}{p}) + \frac{o}{T^2} - \frac{1}{2}DPX_e \quad \text{(A3)}$$

$$\frac{\partial^2 \Pi_2(P,S,T)}{\partial T^2} = -\frac{2o}{T^3} < 0 \quad \text{(A4)}$$

and

$$\frac{\partial \Pi_3(P,S,T)}{\partial T} = -\frac{1}{2}Dh\left(1-\frac{D}{p}\right)+\frac{o}{T^2}-\frac{1}{2}DX_c(c+c_sS) \quad \text{(A5)}$$

$$\frac{\partial^2 \Pi_3(P,S,T)}{\partial T^2} = -\frac{2o}{T^3} < 0 \quad \text{(A6)}$$

Hence, $\Pi_2(P,S,T)$ and $\Pi_3(P,S,T)$ is a strictly concave function in T. This completes the proof of Theorem 2.

Appendix C. Proof of Theorem 3.

For any given P and T, the first-order partial derivatives of $\Pi_1(P,S,T)$, $\Pi_2(P,S,T)$ and $\Pi_3(P,S,T)$ with respect to S are:

$$\frac{\partial \Pi_1(P,S,T)}{\partial S} = \frac{bDP}{S}-\frac{D}{S}(c_sS+b(c+c_sS))+\frac{bDhT}{s}\left(\frac{D}{p}-\frac{1}{2}\right)-(c_sS+b(c+c_sS))\frac{D(d+T-u)^2X_c}{2TS}+\frac{bD(-d+u)^2X_e}{2TS} \quad \text{(A7)}$$

$$\frac{\partial \Pi_2(P,S,T)}{\partial S} = \frac{bDP}{S}-\frac{D}{S}(c_sS+b(c+c_sS))+\frac{bDhT}{S}\left(\frac{D}{p}-\frac{1}{2}\right)+\frac{bDPX_e}{S}\left(u-d-\frac{T}{2}\right) \quad \text{(A8)}$$

$$\frac{\partial \Pi_3(P,S,T)}{\partial S} = \frac{bD}{S}-\frac{D}{S}(c_sS+b(c+c_sS))+\frac{bDhT}{S}\left(\frac{D}{p}-\frac{1}{2}\right)-(c_sS+b(c+c_sS))\frac{DX_c}{S}\left(d+\frac{T}{2}-u\right) \quad \text{(A9)}$$

Therefore, the second-order partial derivatives are:

$$\frac{\partial^2 \Pi_1(P,S,T)}{\partial S^2} = -\frac{bDP}{S^2}(1-b)-\frac{bD}{S^2}(c_sS+b(c+c_sS)-c)+\frac{bDhT}{S^2}\left(\frac{D(2b-1)}{p}+\frac{1}{2}(1-b)\right)-\frac{bD(d+T-u)^2X_c}{2TS^2}(c_sS+b(c+c_sS)-c)-\frac{bDP(u-d)^2X_e}{2TS^2}(1-b) \quad \text{(A10)}$$

$$\frac{\partial^2 \Pi_2(P,S,T)}{\partial S^2} = -\frac{bDP}{S^2}(1-b)-\frac{bD}{S^2}(c_sS+b(c+c_sS)-c)+\frac{bDhT}{S^2}\left(\frac{D(2b-1)}{p}+\frac{1}{2}(1-b)\right)-(1-b)\frac{bDPX_e}{S^2}\left(u-d-\frac{T}{2}\right) \quad \text{(A11)}$$

$$\frac{\partial^2 \Pi_3(P,S,T)}{\partial S^2} = -\frac{bDP}{S^2}(1-b)-\frac{bD}{S^2}(c_sS+b(c+c_sS)-c)+\frac{bDhT}{S^2}\left(\frac{D(2b-1)}{p}+\frac{1}{2}(1-b)\right)-(c_sS+b(c+c_sS)-c)\frac{bD}{S^2}X_c\left(d+\frac{T}{2}-u\right) \quad \text{(A12)}$$

The only positive terms in all three cases of second derivatives are from holding cost, which generally is relatively smaller than the sales revenue and production cost. Accordingly, it is an appropriate premise that $\frac{\partial^2 \Pi(P,S,T)}{\partial S^2} \leq 0$ if $1 \geq b$ and $c_sS+b(c+c_sS) \geq c$. This completes the proof of Theorem 3.

Appendix D. Proof of Theorem 4.

Proof. Likewise, for any given S and T, the first-order partial derivatives of $\Pi_1(P,S,T)$, $\Pi_2(P,S,T)$ and $\Pi_3(P,S,T)$ with respect to P are:

$$\frac{\partial\Pi_1(P,S,T)}{\partial P} = D(1-aP) + aD(c+c_sS) - aDhT(\frac{D}{p}-\frac{1}{2}) + \frac{aD(d+T-u)^2(c+c_sS)X_c}{2T} + \frac{D(-d+u)^2X_e}{2T}(1-aP) \quad \text{(A13)}$$

$$\frac{\partial\Pi_2(P,S,T)}{\partial P} = D(1-aP) + aD(c+c_sS) - aDhT(\frac{D}{p}-\frac{1}{2}) + D(u-d-\frac{T}{2})X_e(1-aP) \quad \text{(A14)}$$

$$\frac{\partial\Pi_3(P,S,T)}{\partial P} = D(1-aP) + aD(c+c_sS) - aDhT(\frac{D}{p}-\frac{1}{2}) + aD(d+\frac{T}{2}-u)X_c(c+c_sS) \quad \text{(A15)}$$

Therefore, the second-order partial derivatives are:

$$\frac{\partial^2\Pi_1(P,S,T)}{\partial P^2} = -aD(2-aP) - a^2D(c+c_sS) + a^2DhT\left(\frac{2D}{p}-\frac{1}{2}\right) - \frac{aD(u-d)^2X_e}{2T}(2-aP) - \frac{a^2D(d+T-u)^2X_c(c+c_sS)}{2T} \quad \text{(A16)}$$

$$\frac{\partial^2\Pi_2(P,S,T)}{\partial P^2} = -aD(2-aP) - a^2D(c+c_sS) + a^2DhT\left(\frac{2D}{p}-\frac{1}{2}\right) - aD(u-d-\frac{T}{2})X_e(2-aP) \quad \text{(A17)}$$

$$\frac{\partial^2\Pi_3(P,S,T)}{\partial P^2} = -aD(2-aP) - a^2D(c+c_sS) + a^2DhT(\frac{2D}{p}-\frac{1}{2}) - a^2D(d+\frac{T}{2}-u)(c+c_sS)X_c \quad \text{(A18)}$$

The only positive terms in all three cases of second derivatives are from holding cost, which generally is relatively smaller than the sales revenue and production cost. Accordingly, it is an appropriate premise that $\frac{\partial^2\Pi(P,S,T)}{\partial P^2} \leq 0$ if $2 \geq aP$. This completes the proof of Theorem 4.

Appendix E. Proof of Theorem 5.

$\Pi_2(T)$ is strictly concave in T and from (17), it can be determined that,

$$\lim_{T\to 0}\Delta(T) = \infty, \quad \text{(A19)}$$

If $\Delta(u-d) < 0$, then by (A19) and applying the Mean-value Theorem, there exists a unique $T_2^* \in (0, u-d)$ such that $\Delta(T_2^*) = 0$. Hence, $\Pi_2(T)$ is maximized at the unique point T_2^*, which implies:

$$\Pi_2(T_2^*) \geq \Pi_2(T_2) \text{ for all } T_2 \leq u-d, \text{ and hence } \Pi_2(T_2^*) \geq \Pi_2(u-d). \quad \text{(A20)}$$

Similarly, $\Pi_1(T)$ is strictly concave in T, and,

$$\lim_{T\to\infty}\Delta(T) = -\frac{D}{2}\left[PX_e + h\left(1-\frac{D}{p}\right)\right] < 0. \quad \text{(A21)}$$

If $\Delta(u-d)<0$, by (A21) it is known that the first-order derivative of (7) with respect to T is $\Delta(T)<0$ for all $T\geq u-d$. Thus, $\Pi_1(T)$ is decreasing and maximized at $u-d$.

Hence,

$$\Pi_1(u-d)\geq\Pi_1(T_1),\text{ for all } T_1\geq u-d. \tag{A22}$$

Combining (9), (A20) and (A22), if $\Delta(u-d)<0$ then

$$\Pi_2(T_2^*)\geq\Pi_2(u-d)=\Pi_1(u-d)\geq\Pi_1(T_1),\text{ for all } T_1\geq u-d. \tag{A23}$$

This completes the proof of $\Delta(u-d)<0$.

By using an analogous argument, one can prove for $\Delta(u-d)=0$ and $\Delta(u-d)>0$. This completes the proof of Theorem 5.