Encroachment Strategy and Revenue-sharing Contract for Product Customization

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Abstract: This paper investigates the optimal encroachment strategy of product customization and the revenue-sharing contract in a two-stage supply chain consisting of a contract manufacturer (CM) and an original equipment manufacturer (OEM). In addition to producing and wholesaling standard products for OEM, CM has the motive to manufacture customized products under store brand and encroach the end consumer market. Stackelberg game models with different strategies (encroachment or no-encroachment) under both decentralized and centralized supply chains are explored. Models analyzing CM's encroachment with product customization are rare. Besides, this paper characterizes both vertical partnership and horizontal competitive relationships between supply chain members. The findings show that it is unprofitable to encroach on the retailing market for CM when the acceptance degree of store brand is low. There is a threshold value of customization level that can gain positive demand. Interestingly, as the Stackelberg leader, OEM always suffers from the encroachment. Then a revenue-sharing contract is designed that can fully integrate the decentralized supply chain and obtain *a contract-implementing Pareto zone*. Furthermore, a numerical example is developed, demonstrating the validity of the obtained analytical results. On this basis, some suggestions for industry managers are discussed in the form of managerial insights.

Keywords: Customization; Encroachment; Hotelling model; Revenue-sharing contract.

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1 Introduction

The words, delegate the specialized work to the professionals, reflect the significance of cooperation, especially in the manufacturing industry. Many original equipment manufacturers (OEMs) are just dedicated to the research and development (R&D) and outsource the production of standard products to the professionals, contract manufacturers (CMs). Then, these standard products are sold by OEM under its national brand. Such cooperation benefits both firms in decreasing production costs. However, a CM with a large-scale production capacity may not be satisfied with the thin profit difference between wholesale price and production costs. An increasing number of CMs begin encroaching on the end consumer market and selling products under their own label or self-brand [2, 31] . For simplicity, this paper defines the own brand as store brand to correspond to the national brand of OEM. Though such encroachment behavior may make CM earn an enormous profit from retailing products than from wholesaling, something will not always be as good as it seems.

Shenzhou International Group Holdings Limited is an OEM giant who provides products for many national brands, such as Uniqlo, Adidas, Nike, and Puma [37]. The store brand, Maxwin, was introduced in 2010 and sold in 2016 [38] because of its lower acceptance degree leading to the accumulated loss in retail business. In contrast, Redcollar, a store brand established by Qingdao Kutesmart Co., Ltd., has been famous for its tailored suits. People who want to buy the goods can choose the color, button shape, waistband form, and other aspects [24]. Customization enables a firm to satisfy consumers' heterogeneous tastes by providing optional characteristics to their ideal preferences based on the flexible production line [19], which in turn makes the company more attractive to consumers and enhances its competitiveness[11, 22]. Comparing the two examples of CM encroachment, an exciting problem arises. Do all CMs successfully encroach end consumer market when they produce customized products under store brand? It seems to be Yes because CM can benefit from wholesale and retail businesses. Additionally, the store brand's existence also enhances his bargaining power [5]. However, the disruption of the customization manufacture program introduced by Levi Strauss (Levi's), named Original Spin, violates this general thinking. Motivated by these phenomena, this paper would like to explore the following problems.

(1) As discussed above, does manufacturing and selling customized products under store brand always benefit CM who produces standard products for OEM? If not, what is CM's optimal strategy?

(2) How should CM determine the optimal customization level and the corresponding retail price if he sells customized goods with the store brand?

(3) What is the impact of CM's encroachment with product customization on OEM's profit and How should OEM respond to such influence?

To address these questions, the Hotelling model [15] is adopted. We consider a market with an OEM and a CM, where OEM wholesales standard product from CM and sells it under the national brand to consumer and CM produces and sells customized products under store brand (if he chooses encroachment strategy). We assume that both products have the same quality and perform horizontal differentiation, which is captured by the difference in OEM's and CM's locations in our model. In addition, this model also uses consumers' location to demonstrate their heterogeneous ideal preferences for product characteristics. Based on this, we investigate CM's different strategies (encroachment or no-encroachment) under a decentralized and centralized supply chain and highlight its impact on firms' performance.

This study makes the following novel contributions to CM's encroachment and product customization. Firstly, CM's encroachment strategy under different supply chain settings is discussed, where the customized product performs as horizontal differentiation. This phenomenon is normal in real world, such as the Apple's Airpods with and without customized lettering. While existing research [39] assumes CM's product is different from the incumbent in quality (vertical difference). Secondly, this paper takes the point that offering customized products can satisfy consumers' demand and reduce misfit risk, which is different from the current literature [34] that a service facility can help consumers select goods without risk and enhance demands in a remanufacturing system. Thirdly, this paper considers an outsource supply chain consisting of an OEM and a CM, which differs from the research of [10]. Scholars focus on the problem of an online platform cooperates with a new manufacturer to produce tailored products in addition to selling standard product from an incumbent manufacturer. In our model, there are both vertical partnership and horizontal competitive relationship in the consumer market between OEM and CM, which is common in reality.

The research findings show that not all market conditions are available for providing customized products. There is no demand for CM's store brand when the customization level is lower than a certain threshold. Second, although the product customization can make CM obtain positive markets, the profit is not always higher than that only produces standard products for OEM. Only when the acceptance degree of store brand is not too low, CM benefits from the retail business. From the perspective of CM and the whole supply chain, encroachment under a centralized setting is the optimal equilibrium. Last but not least, OEM always suffers from CM's entry behavior even though she is the leader, owning prior power in making-decision in the decentralized supply chain. However, our results also state that the proposed revenue-sharing contract can change such an unsatisfied situation for OEM. Then we get a contract-implementing Pareto zone, in which both OEM and CM can earn higher profits and the system reach optimally.

The remainder of this study is designed as below. The relevant literature is reviewed in Section 2. Followed by it, the basic model and some assumptions are introduced in Section 3. The equilibrium outcomes with CM's different strategies under different supply chain structures are explored in Section 4. Moreover, this paper discusses the impacts of CM's entry in the product customization market on firms' performance and then obtains the optimal entry strategy in Section 5. Based on this, we propose a revenue-sharing contract to improve the efficiency of decentralized supply chain. In Section 6, we introduce numerical analysis to illustrate the analytical findings. We also examine how CM's decision and supply chain structure affects consumer surplus and social welfare in Section 7. Lastly, the conclusion and some management insights are proposed in Section 8. All proofs are summarized in Appendix.

2 Literature Review

This paper focuses on the optimal encroachment strategy of product customization under different supply chain settings and its impact on OEM and CM's profits. Therefore, the related literature will be reviewed to have a comprehensive understanding of our work and present the main contributions of this paper. We illustrate the research gap between our work and the existing research in Table 1.

The first stream of literature is on the contract manufacturer encroachment. The current research on encroachment is developed mainly from three perspectives, manufacturer's channel encroachment [27, 42, 44, 45, 47], retailer's store brand encroachment (or introduction), and the interaction between the above strategies [18, 20, 26, 46]. This paper is most relevant to the second one, but not the same. Scholars investigate this problem from the following aspects, motivation for the introduction [30, 33], store brand position [4, 23], and quality [13] issues. In addition to exploring the problem of store brand encroachment in a manufacturer-retailer supply chain, there is literature focusing on such problem in a outsource supply chain, where OEM wholesales products from CM and then sells them to consumers. Based on this cooperative between OEM and CM, the paper [6] pointed out CM always has the incentive to encroach on OEM's final market when there is no quality improvement room for the incumbent product. Different from the assumption that CM can imitate OEM's product without any quality investment cost in literature [6], scholars [39] considered a quality-related cost would incur if CM adopted encroachment strategy and defined it as a quadratic function. Then the optimal encroachment strategy and quality level were investigated under both scenarios that CM is dominant, and OEM is dominant. Apart from encroaching retail market and competing directly with OEM's products, CM also can encroach the wholesale market. Therefore, the paper [2] considered a three-level supply chain which is consisted of a CM, an OEM, and a retailer and not only studied whether to provide own-label products but also discussed which market CM should enter. Scholars [8] analyzed that whether OEM can source from a new non-competitive CM with limited capacity when facing CM's own-brand product encroachment. They found that adopting this strategy and abandoning his original CM was preferable for OEM when the market competition became more intense. In contrast to the above literature, our study assumes that the product sold by CM is of equal quality as that produced for OEM because they are all manufactured by CM. The horizontal difference between them is that the former can offer consumers personalized options while the latter without such characteristics.

Our work is also related to the literature on a firm's product customization strategy. The paper [7] analyzed how should a monopoly seller adopt a product mix strategy (produce both standard and customized products) according to information collection cost and production flexibility cost and found that the retail prices of both products increased with the efficiency of customization technology. Based on this, literature [40] extended the market to a case of competition and revealed the impacts of product mix strategy on firms' competition and profits. Generally, customization level and the corresponding cost may directly affect a firm's profit. Besides that, research [19] thought the cost of consumer involvement in customization influenced the product demands, which impacted the revenue. Considering this factor, shcolars explored manufacturer's optimal customization level, and pricing strategy. With the development of e-commerce, manufacturers' engaging in dual-channel structures, selling the same products through traditional retail and online channels, may result in channel competition and conflict. Thus, scholars [25] studied the impacts of distributing customized and standard products on online and offline channels respectively on firms' profits. In the setting of an e-tailer and n retailers, the paper [43] investigated the optimal customization strategies of supply chain members and found that the customization scope only depended on its cost efficiency and was irrelative to other companies' decisions. The online retailer also can sell different products from different manufacturers. Research [10] pointed out that introducing a customized product from a new manufacturer could hurt the profit of the incumbent one while always benefit the online retailer. Overall, scholars focus on the issue of customization from the perspective of competition with standard products without or with channel differentiation. However, the brand difference is not considered. In practice, people will also compare brand images among different goods except whether the product meets their ideal preferences when they shop. Product customization is a manufacturing decision. Though literature [36] discussed such production problem on innovation green products, competitors' strategy interaction was not considered. Our work investigates how the competition in retailing market influences CM's encroachment behavior. Moreover, the demand function is obtained by analyzing customers' consuming utility, which is different from the literature [29] that regarded demand as an imprecise one and adopted a fuzzy model to characterize such uncertainty.

Research on supply chain coordination is also relevant. Interest conflict exists in a supply chain if a channel member makes decision on maximizing his profits without considering the impact on others' member [12]. On the other hand, the discards in a supply chain, such as the wastage of food products, have polluted the environment and reduced the supply chain efficiency. The work [35] investigated how to use such resources in an efficient model with a circular economy, which is on the management of products. In contrast, our work mainly focuses on the coordination between players through designing appropriate contracts and then increasing supply chain performance. Scholars [17] found that channel members could obtain higher yield under coordination mechanism, although it was difficult to achieve because each member had an incentive to deviation. Moreover, they also pointed out that the profit-sharing mechanism and quantity discounts could coordinate the supply chain. Under a decentralized setting, the paper [1] revealed the channel preference priority of channel members were diverse and designed an easily implemented coordination mechanism, revenue sharing contract, reaching the contract-implementing Pareto zone. In an online dual-channel supply chain system, scholars [28] discussed how the two types of contracts, a single of contract and a menu of contracts, could inspire retailers to share private information to the upstream supplier and then realize supply chain coordination. Besides these research, sales rebate contracts [41], and buy back contracts [14] were also presented and confirmed could realize a win-win outcome for all members. Two-part tariffs and revenue-sharing contract were designed in literature [16] because they found the manufacturer would also suffer from the retailer's introduction of store brand although the retailer offered the same service for the two brand products. Unlike this, our work concentrates on the optimal entry strategy of product customization and discusses its impact on supply chain performance. Then the revenue-sharing contract is designed to eliminate the negative effect of CM's encroachment behavior on OEM and optimal the supply chain system.

Table. 1. Comparison between our work with relevant research.

Article	Who encroaches	Product difference	Brand difference	Customization strategy	Channel coordination
[25]	*M*	*H*	*N*	*Y*	*N*
[6]	*CM*	*V*	*N*	*N*	*N*
[39]	*CM*	*V*	*Y*	*N*	*N*
[19]	*N*	*H*	*N*	*Y*	*N*
[44]	*M*	*H*	*N*	*N*	*N*
[27]	*M*	*V*	*N*	*N*	*Y*
[43]	*N*	*H*	*N*	*Y*	*N*
[46]	*R*	*V*	*Y*	*N*	*N*
[10]	*R*	*H*	*Y*	*Y*	*N*
This paper	*CM*	*H*	*Y*	*Y*	*Y*

Notes: *M*-"Manufacturer", *R*-"Retailer", *CM*-"Contract manufacturer", *N*-"Not considering this factor", *Y*-"Considering this factor".

3 The Basic Model

This paper adopts the Hotelling model to capture the relationship among OEM's standard product under national brand, consumers with different aesthetic preferences, and CM's customized product under store brand if he encroaches the end consumer market. The consumer market is conceptualized as a straight line, where OEM is located at the left point (i.e., zero point) because she only provides the standard product. Moreover, when CM manufactures store brand product with customization level l, its location is at point l between point 0 and 1. Otherwise, it will not appear. The consumer is identified by x, where x denotes the consumer's ideal aesthetic preference and is uniformly distributed on the line interval $[0, 1]$. This paper normalizes the consumer market scale 1. Therefore, according to the above assumption, the straight line can stand for consumers' preference for products and be considered a production line where CM can determine the customization level.

We assume that each consumer can purchase at most one product subject to a perceived valuation V. When a consumer with x ideal aesthetic preference buys a standard product without any customization, it will generate disutility from not matching the taste completely. Such disutility can be expressed as $t(x-0)$. Thus, the consumer will incur a disutility of $t(x-l)$ if the CM's store brand product with customization level l is purchased. The parameter t means the consumer's sensitivity to the unmatched aesthetic preference, reflecting the unit misfit cost. In addition, CM's store brand often has a lower valuation compared with the national brand. Therefore, a product worth valuation V from OEM's national brand will bring a utility of μV to a consumer when it is marked with CM's store brand, where $\mu \in [0,1]$. Parameter μ reflects consumers' acceptance of store brand related to the brand image. We use subscript $i = n \; or \; s$ to represent the standard product under national brand or customized product under store brand. Then the prices of the two types of products are p_n and p_s, respectively. Generally, a higher price will result in lower utility. In summary, the consumer's net utility from buying one unit standard product under national brand (denoted as U_n) or customized product under store brand (denoted as U_s) can be written as:

$$U_n = V - p_n - tx, \tag{1}$$

$$U_s = \begin{cases} \mu V - p_s & \text{if } 0 \leq x \leq l, \\ \mu V - p_s - t(x-l) & \text{if } l < x \leq 1. \end{cases} \tag{2}$$

Equation (2) indicates that if consumer's ideal aesthetic preference is completely stratified by the customized product whose customization level is l (i.e., $x \in [0,l]$), such consumer will not incur disutility from unmatched bias. Otherwise, the disutility of $t(x-l)$ may arise.

Furthermore, to ensure that the equilibrium results are meaningful in all scenarios, for example, the customization level l is restrained between 0 and 1, the total demand should not exceed consumer market scale (i.e., 1) and so on, this paper assumes that the perceived valuation V must satisfy such constraint condition $0 < V < \min\left\{\frac{4kt-t^2}{2k\mu},\ 4t\right\}$.

In this paper, we assume that the production process of customized products is based on that of standard products, using common material [9]. While the difference between them is that the former provides more options for consumers to cater to individual taste, such as the color, size, and symbol picture needed to draw on the product. For example, Apple offers personalized engraving services for iPad products, which indicates that the customized product and the standard one have indifferent quality and perform as horizontal differentiation. Therefore, their marginal costs are the same and normalized to zero without loss of generality. However, other than that, providing customized options also requires additional investments, such as collecting and analyzing consumers' purchase behavior data, buying special equipment, employing more workers, and so on. The higher the customization level, the higher production flexibility is needed, and then the higher the additional cost is incurred. This paper assumes the additional cost (cost of production flexibility) is kl^2, which suggests diminishing returns on the investments. Here, the coefficient k represents the amount of the investment cost for providing a specific customized option.

We also assume CM adopts a uniform price for all customized goods as long as the customization level is determined. Same assumption can be found in papers [25, 40]. This shows that all the consumers whose aesthetic preferences are different are charged the same price regardless of the requested customization variety and whether they are located in the customized range or not. In practice, the price of customized sport shoes of Dunk series in Nike.com is 899RMB no matter what material is chosen.

The channel structure is as shown in Fig. 1. The game sequence is as follows: Firstly, the CM decides whether to encroach the end consumer market, then the customization level l is determined if CM adopts the encroachment strategy. Otherwise, this step will be omitted. Secondly, as the Stackelberg leader, OEM decides on the profit margin m_n of the national brand product. At last, CM simultaneously decides the wholesale price w_n for national brand product and the sale price p_s for store brand product if he provides tailored products.

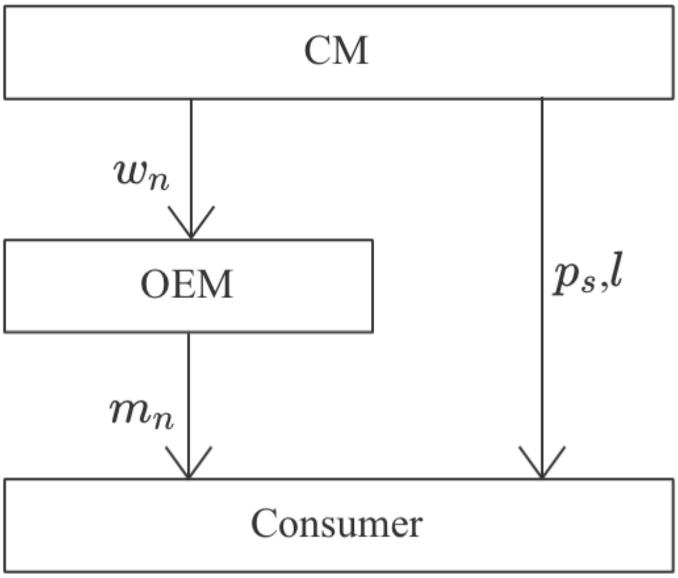


Fig. 1. Supply chain structure of the research problem.

4 Equilibrium Outcomes under Different Scenarios

This section examines the equilibrium outcomes of CM's encroachment strategy of product customization under different supply chain settings. We use the letter D to represent a decentralized supply chain in which CM and OEM make decisions dependently. Then, the letter C means a centralized supply chain in which CM and OEM operate as a whole. For CM, the strategy space is encroachment or no-encroachment (denoted as E or N). Consequently, combing the supply chain setting and CM's entry behavior, there are four different subgames represented by the superscript $TT = \{DN, DE, CN, CE\}$, as shown in Table 2. For example, superscript DE means the case that CM encroaches on the end consumer market under a decentralized supply chain. The others are similar. Moreover, the subscript $b = c$ or o represents the firm CM or OEM. We first analyze consumers' behavior and then discuss firms' decisions for the above four scenarios.

Table. 2. Four scenarios based on CM's entry strategy and supply chain setting.

	No-encroachment (N)	Encroachment (E)
Decentralized structure (D)	Scenario DN	Scenario DE
Centralized structure (C)	Scenario CN	Scenario CE

4.1 Scenario *DN*

In this section, CM does not offer customized products, and there are only standard products with national brand. CM produces according to the OEM's requirements and wholesales to the OEM at a price w_n^{DN}, which is based on the profit margin m_n^{DN} decided by OEM. In line with Equation (1), the consumer with x ideal aesthetic preference is indifferent between buying one unit standard product and leaving the market when $U_n = 0$. Then the indifference point is $x_n^{DN} = \frac{V - p_n^{DN}}{t}$. Consumer makes purchases only when the utility is larger than zero. Therefore, consumers with ideal aesthetic preferences on the left side of point x_n^{DN} will buy national brand products. The demand for OEM would be:

$$d_n^{DN} = \frac{V - p_n^{DN}}{t}, \tag{3}$$

where $p_n^{DN} = w_n^{DN} + m_n^{DN}$.

Then the profit of CM and OEM are as follows:

$$\pi_c^{DN} = w_n^{DN} d_n^{DN}, \tag{4}$$

$$\pi_o^{DN} = m_n^{DN} d_n^{DN}. \tag{5}$$

Based on the game sequence, the equilibrium outcome is solved by backward induction and is given in Lemma 1.

Lemma 1. *When CM does not encroach the end consumer market, the equilibrium wholesale price w_n^{DN*}, profit margin m_n^{DN*}, retail price p_n^{DN*}, demand d_n^{DN*}, and firms' profits π_c^{DN*} and π_o^{DN*} are given in Table 3.*

4.2 Scenario *DE*

When CM has enough capacity, he may not only produce for OEM but also manufacture and sell customized products under store brand. Then CM makes decisions on the customization level firstly. Because this is a strategic level decision and the customization level cannot be changed easily for referring to large investments on production line adjustment. The equilibrium outcome is also abstracted by backward induction.

We firstly analyze how the market share allocates when there are both standard products under national brand and customized products under store brand. Consumers compare the utility from buying the two products and take away the one that gives them a higher and positive utility. Based on this, we can obtain three marginal consumers. The first is indifferent between buying a standard product and nothing, denoted as x_n. Let $U_n = 0$ and yields $x_n = \frac{V-p_n}{t}$. The second is indifferent between buying a customized product and nothing, denoted as x_s. Let $U_s = 0$, and yields $x_s = \frac{\mu V - p_s + tl}{t}$ if $l < x < 1$, and $p_s < \mu V$ if $0 < x < l$. The third is indifferent between buying the two types products, denoted as x_{ns}. Let $U_n = U_s$ and yields $x_{ns} = \frac{(1-\mu)V+p_s-p_n}{t}$. We find that although CM encroaches the end consumer market, he does not always have positive market share. The condition under which makes the encroachment unmeaningful is given in Lemma 2.

Lemma 2. *When CM encroaches the end consumer market and provides the customized product under store brand, if the customization level is less than* $x_{ns} = \frac{(1-\mu)V+p_s-p_n}{t}$ *(i.e.,* $l < \frac{(1-\mu)V+p_s-p_n}{t}$*), there is no demand.*

According to the utility function of consuming a customized product, although the misfit cost reduces, a lower store brand image makes the perceived valuation is less than that of the national brand (i.e., $\mu V < V$). Lemma 2 indicates that the utility from buying a standard product will be higher if the customization level is too low. Thus, all consumers choose OEM's national brand product as long as their utilities are positive. It conveys that if CM wants to obtain more profits from both retail and wholesale businesse, he should set the customization level higher than the threshold x_{ns}. Otherwise, he will not gain market share and loss the investment in customization production line.

Therefore, to develop the followed analysis, this paper assumes that the customization level is not less than $x_{ns} = \frac{(1-\mu)V+p_s-p_n}{t}$ as long as CM adopts the encroachment strategy. Based on this assumption, comparing the three indifferent points, the relationship among them always holds, that is, $x_{ns} < x_n < x_s$. Consequently, when consumers' ideal aesthetic preferences are fully covered by the customized product (i.e., $x \in [0, l]$), these consumers are located at interval $[0, x_{ns}]$ will buy the standard product and located at $[x_{ns}, l]$ will buy the customized one.

When consumers' ideal aesthetic preferences are partly covered (i.e., $x \in (l, 1]$), the utility from buying a customized product is always larger than that from consuming a standard product, because of $U_s > U_n$ given $l > x_{ns}$. Hence, these consumers may take away CM's store brand products as long as their utilities are positive. It means that the market segment $[l, x_s]$ belongs to CM. In summary, the demand functions are as followed:

$$d_n^{DE} = x_{ns} - 0 = \frac{(1-\mu)\,V + p_s^{DE} - p_n^{DE}}{t}, \tag{6}$$

$$d_s^{DE} = (x_{ns} - l) + (x_s - l) = l^{DE} + \frac{(2\mu - 1)\,V + p_n^{DE} - 2p_s^{DE}}{t}. \tag{7}$$

The profit functions of CM and OEM are:

$$\pi_c^{DE} = w_n^{DE} d_n^{DE} + p_s^{DE} d_s^{DE} - k(l^{DE})^2, \tag{8}$$

$$\pi_o^{DE} = m_n^{DE} d_n^{DE}. \tag{9}$$

The same as in Section 4.1, the equilibrium outcome is given in Lemma 3 by adopting backward induction.

Lemma 3. *When CM encroaches the end consumer market, the equilibrium wholesale price w_n^{DE*}, profit margin m_n^{DE*}, retail price p_n^{DE*} , p_s^{DE*}, demand d_n^{DE*}, d_s^{DE*}, and firm profits π_c^{DE*} and π_o^{DE*} are given in Table 3.*

4.3 Scenario *CN*

Under a centralized supply chain, OEM and CM operate as an integrated firm and make decisions jointly to maximize the profit of the whole supply chain.

If CM does not manufacture the customized product, OEM makes decision on p_n^{CN} to maximize the profit of whole supply chain. The profit function is

$$\Pi^{CN} = p_n^{CN} d_n^{CN}, \tag{10}$$

where $d_n^{CN} = \frac{V - p_n^{CN}}{t}$ is obtained similar to that in Section 4.1.

Then the equilibrium results are $p_n^{CN*} = \frac{V}{2}$, $d_n^{CN*} = \frac{V}{2t}$, *and* $\Pi^{CN*} = \frac{V^2}{4t}$.

Table. 3. Equilibrium outcomes under scenarios DN and DE.

	Scenarios DN	Scenarios DE
w_n^{TT*}	$\frac{V}{4}$	$\frac{V[t(-1+\mu)+4k(1+\mu)]}{4(4k-t)}$
m_n^{TT*}	$\frac{V}{2}$	$\frac{V(1-\mu)}{2}$
p_n^{TT*}	$\frac{3V}{4}$	$\frac{V[3t(-1+\mu)-4k(-3+\mu)]}{4(4k-t)}$
p_s^{TT*}	N/A	$\frac{2kV\mu}{4k-t}$
l^{TT*}	N/A	$\frac{V\mu}{4k-t}$
d_n^{TT*}	$\frac{V}{4t}$	$\frac{V(1-\mu)}{4t}$
d_s^{TT*}	N/A	$\frac{V[t(-1+\mu)+4k(1-3\mu)]}{4t(-4k+t)}$
Td^{TT*}	$\frac{V}{4t}$	$\frac{2k\mu V}{t(4k-t)}$
π_o^{TT*}	$\frac{V^2}{8t}$	$\frac{V^2(1-\mu)^2}{8t}$
π_c^{TT*}	$\frac{V^2}{16t}$	$\frac{V^2[-t(-1+\mu)^2+4k(1+\mu(-2+5\mu))]}{16t(4k-t)}$
Π_t^{TT*}	$\frac{3V^2}{16t}$	$\frac{V^2[4k(3+\mu(7\mu-6))-3t(\mu-1)^2]}{16t(4k-t)}$

Notes:The parameter constraint condition is $\left\{(k,t,\mu)|\frac{4(1-\mu)}{3\mu+1} < \frac{t}{k} < 2, \frac{1}{5} < \mu < 1, t > 0\right\}$ to guarantee the equilibrium outcome significant under a decentralized supply chain.

4.4 Scenario *CE*

Suppose CM manufactures both customized and standard products under a centralized setting. In that case, the supply chain can be seen as a firm selling standard products with national brand and customized products with a new construct sub-brand whose brand image is not larger than the national brand's. For example, the mobile Phone branded *Huawei* and *U-MAGIC*, the latter is a new brand that *Huawei* has partnered with a network operator named *Unicom*. For the sake of consistency, we also call it the store brand.

It is noted that consumer behavior is same as that in scenario DE. Then the market share d_n^{CE} $(or\ d_s^{CE})$ has the common expression structure as d_n^{DE} $(or\ d_s^{DE})$. Therefore, the profit of the whole supply chain can be expressed as:

$$\Pi^{CE} = p_n^{CE}d_n^{CE} + p_s^{CE}d_s^{CE} - k^{CE}(l^{CE})^2. \tag{11}$$

Given the customization level, OEM maximizes $\Pi^{CE}(p_n^{CE}, p_s^{CE}, l^{CE})$ with respect to p_n^{CE} and p_s^{CE}, simultaneously. Then she will maximize $\Pi^{CE}(l^{CE})$ with respect to l^{CE}. The equilibrium outcomes are shown in Table 4.

Table. 4. Equilibrium outcomes under scenario CN and scenario CE.

	Scenario CN	Scenario CE
p_n^{TT*}	$\frac{V}{2}$	$\frac{V(4k-t+t\mu)}{2(4k-t)}$
p_s^{TT*}	N/A	$\frac{2kV\mu}{4k-t}$
l^{TT*}	N/A	$\frac{\mu V}{4k-t}$
d_n^{TT*}	$\frac{V}{2t}$	$\frac{V(1-\mu)}{2t}$
d_s^{TT*}	N/A	$\frac{V[t(-1+\mu)+4k(1-2\mu)]}{2t(-4k+t)}$
Π^{TT*}	$\frac{V^2}{4t}$	$\frac{V^2[4k(1+2\mu(-1+\mu)-t(-1+\mu)^2)]}{4(4k-t)t}$

Notes: The parameter constraint condition is $\left\{(\mu, k, t) | \frac{1}{3} < \mu < 1, \frac{4-4\mu}{1+\mu} < \frac{t}{k} < 2, t > 0\right\}$ to guarantee the equilibrium outcome significant under a centralized setting.

5 Equilibrium Analysis and Channel Coordination with Revenue-sharing Contract

This section first discusses how CM's entry in the product customization market affects supply chain members by comparing the equilibrium outcomes under different scenarios. Then, we investigate the dominant strategy and design a revenue-sharing contract to obtain the contract-implementing Pareto zone where both CM and OEM earn higher profits, and the whole supply chain reaches optimally.

5.1 The Effects of Customized Product Encroachment

We examine how the retail prices, demands, and firms' profits change with CM's market entry behavior in a decentralized and centralized supply chain, respectively.

Proposition 1. *Comparing the equilibrium outcomes under scenario DN and DE, some key insights about price are summarized as follows:*

i. $\frac{\partial w_n^{DE*}}{\partial \mu} = \frac{V(4k+t)}{4(4k-t)} > 0$ *and* $\frac{\partial m_n^{DE*}}{\partial \mu} = \frac{-V}{2} < 0$;

ii. $w_n^{DE*} > w_n^{DN*}$ *and* $m_n^{DE*} < m_n^{DN*}$ *always hold; And* $\left|w_n^{DE*} - w_n^{DN*}\right| > \left|m_n^{DE*} - m_n^{DN*}\right|$ *when* $\frac{t}{k} > \frac{4}{3}$. *Otherwise,* $\left|w_n^{DE*} - w_n^{DN*}\right| < \left|m_n^{DE*} - m_n^{DN*}\right|$;

iii. $p_n^{DE*} > p_n^{DN*}$ *when* $\frac{1}{5} < \mu < \frac{1}{3}$ *and* $\frac{4-4\mu}{1+3\mu} < \frac{t}{k} < 2$, *or* $\frac{1}{3} < \mu < 1$ *and* $\frac{4}{3} < \frac{t}{k} < 2$; $p_n^{DE*} < p_n^{DN*}$ *when* $\frac{1}{3} < \mu < 1$ *and* $\frac{4-4\mu}{1+3\mu} < \frac{t}{k} < \frac{4}{3}$.

Proposition 1(i) and (ii) means that the wholesale price increases with the consumer's acceptance degree of store brand (i.e., μ) while the margin profit of

OEM decreases with it. As consumers' acceptance of store brand increases, CM's customized product has higher competitiveness, which makes CM more powerful in making decisions. Thus, the wholesale price under scenario DE is larger than that under scheme DN and increases with parameter μ. On the other hand, higher acceptance of store brand indicates fiercer competition. To enhance the demand for standard products under the national brand, OEM must reduce the margin profit to lower the retail price. Such reduction is more apparent when the acceptance degree is larger. Moreover, the increase in wholesale price is greater than the decrease in marginal profit when $\frac{t}{k} > \frac{4}{3}$. For a given investment cost, higher sensitiveness to the misfit of ideal preference means that CM's customized product has more tremendous flexibility advantages. It makes CM adopt a more aggressive strategy of raising more wholesale prices.

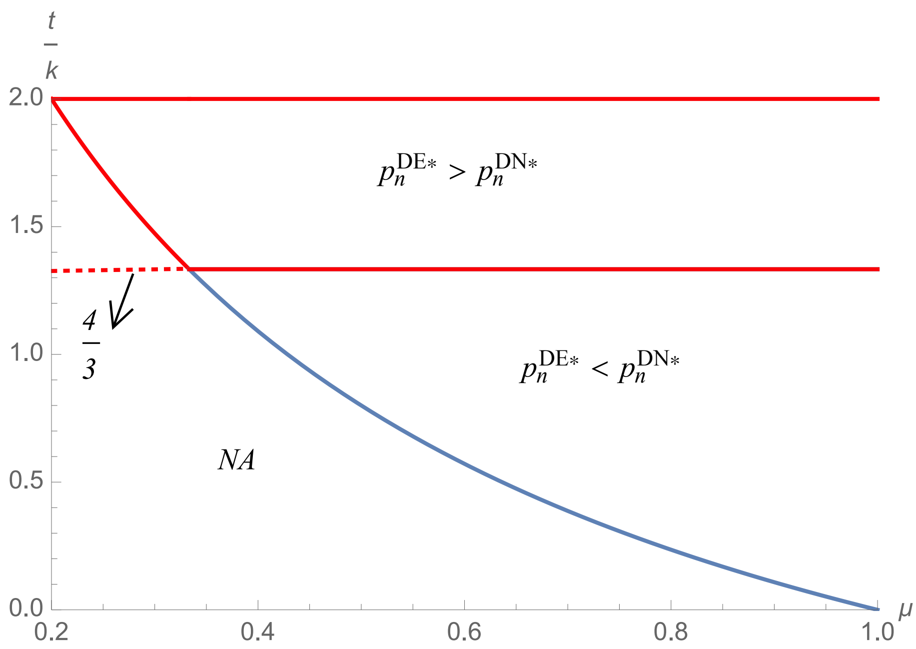


Fig. 2. Comparison of the retail price p_n.[1]

Proposition 1(iii) describes how the retail price of standard products changes when CM also sells customized products under store brand, as shown in Fig. 2. The retail price of standard products is greater than that under scenario DN when the ratio is large enough (i.e., $\frac{t}{k} > \frac{4}{3}$) regardless of the consumers' acceptance degree of store brand. This is consistent with Proposition 1(ii). The increasement

[1] NA identifies the infeasible domain of parameter combinations, which is appropriate for other figures.

of wholesale price is larger than the reduction in margin profit when $\frac{t}{k} > \frac{4}{3}$, which increases the retail price (because of $p_n = w_n + m_n$). For the second part of Proposition 1(iii), CM's store brand has strong attractiveness when its acceptance degree is large. Meanwhile, lower sensitiveness to misfit indicates lower horizontal differentiation, leading to more intense competition. Thus, both factors make lower retail price of standard product when CM provides customized products.

Proposition 2. *The comparisons of equilibrium demand for the standard product and the whole supply chain system with and without contract manufacturer encroachment are as follows:*

i. $d_n^{DE*} < d_n^{DN*}$;

ii. $Td^{DE*} > Td^{DN*}$ *when* $\frac{1}{4} < \mu < \frac{1}{3}$ *and* $4 - 8\mu < \frac{t}{k} < 2$, *or* $\frac{1}{3} < \mu < 1$ *and* $\frac{4-4\mu}{1+3\mu} < \frac{t}{k} < 2$; $Td^{DE*} < Td^{DN*}$ *when* $\frac{1}{5} < \mu < \frac{1}{4}$ *and* $\frac{4-4\mu}{1+3\mu} < \frac{t}{k} < 2$, *or* $\frac{1}{4} < \mu < \frac{1}{3}$ *and* $\frac{4-4\mu}{1+3\mu} < \frac{t}{k} < 4 - 8\mu$.

From Proposition 2(i), it can be seen that CM's entry in the product customization market always negatively influences the demand for national brand products. This is obvious when $p_n^{DE*} > p_n^{DN*}$. Because higher retail prices lead to lower demand. However, an interesting finding is that although the retail price is lower compared with that under scenario DN (i.e., $p_n^{DE*} < p_n^{DN*}$), which holds only when the acceptance degree of the store brand is not too small (i.e., $\frac{1}{3} < \mu < 1$), the demand still decreases. The implied reasons are as follows. As discussed above, a higher acceptance degree of store brand means CM's customized product has stronger competitiveness and consumers have not too small valuation for it. Although the lower price may result in higher demand for a standard product, intuitively, more powerful attractiveness competes for more market share, which causes lower demand for the national brand product.

For Proposition 2(ii), it indicates that the whole supply chain demand may increase or decrease and is jointly influenced by parameter μ and $\frac{t}{k}$, which is more explicitly demonstrated in Fig. 3. When $\frac{t}{k}$ is large, the number of these consumers that buying standard products can generate positive utilities reduces because of $p_n^{DE*} > p_n^{DN*}$. On the other hand, the store brand cannot obtain enough market share because of its lower acceptance degree (i.e., $\mu < \frac{1}{3}$). Therefore, the whole supply chain demand decreases because of the above two pressures on demand for standard and customization products.

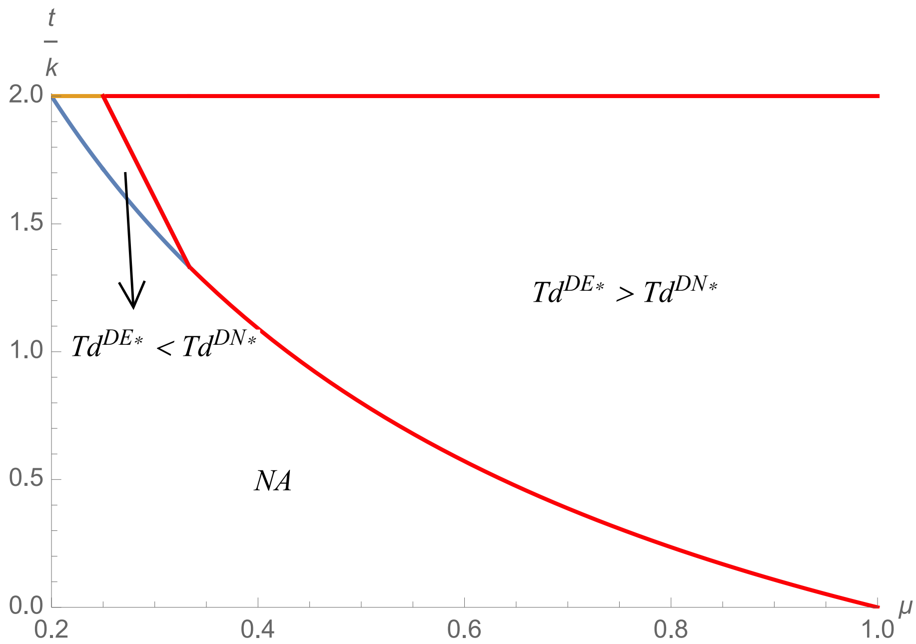


Fig. 3. Comparison of the total demand Td.

Proposition 3. *The profits earned by firms are changed as followed:*

i. $\pi_o^{DE*} < \pi_o^{DN*}$;

ii. $\pi_c^{DE*} < \pi_c^{DN*}$ *when* $\frac{1}{5} < \mu < \frac{2}{9}$ *and* $\frac{4-4\mu}{1+3\mu} < \frac{t}{k} < 2$, *or* $\frac{2}{9} < \mu < \frac{1}{4}$ *and* $\frac{4-4\mu}{1+3\mu} < \frac{t}{k} < \frac{-8+20\mu}{-2+\mu}$; $\pi_c^{DE*} > \pi_c^{DN*}$ *when* $\frac{2}{9} < \mu < \frac{1}{4}$ *and* $\frac{-8+20\mu}{-2+\mu} < \frac{t}{k} < 2$, *or* $\frac{1}{4} < \mu < 1$ *and* $\frac{4-4\mu}{1+3\mu} < \frac{t}{k} < 2$.

Proposition 3(i) shows that OEM suffers from CM's encroachment under a decentralized supply chain. The reason is that both the demand and margin profit of OEM's national brand product decrease, which leads to a reduction in the total profit. Some interesting insights are shown in part(ii). Intuitively, CM will benefit from producing both standard and customized products because he has an additional income compared to the case of wholesale products to OEM only. However, this is not the truth. The selling of customized products may harm CM's profit under some situations, such as low acceptance degree (i.e., μ) and a great ratio of sensitiveness to misfit to investment cost (i.e., $\frac{t}{k}$), as shown in the upper left area of Fig. 4. We find that the earn from retail business (i.e., $\frac{kV^2\mu(3\mu-1)}{2(4k-t)t}$)

is negative when $\mu < \frac{1}{3}$. The lower acceptance degree of store brand seizes a lower market share of customized products, which leads to the retail revenue not being able to compensate for the investment in the customization production line. Therefore, such a negative effect makes the total profit of operating retail and wholesale businesses lower than that from only producing standard products for OEM (i.e., $\pi_c^{DE*} < \pi_c^{DN*}$). The case that no one gains at the expense of others arises.

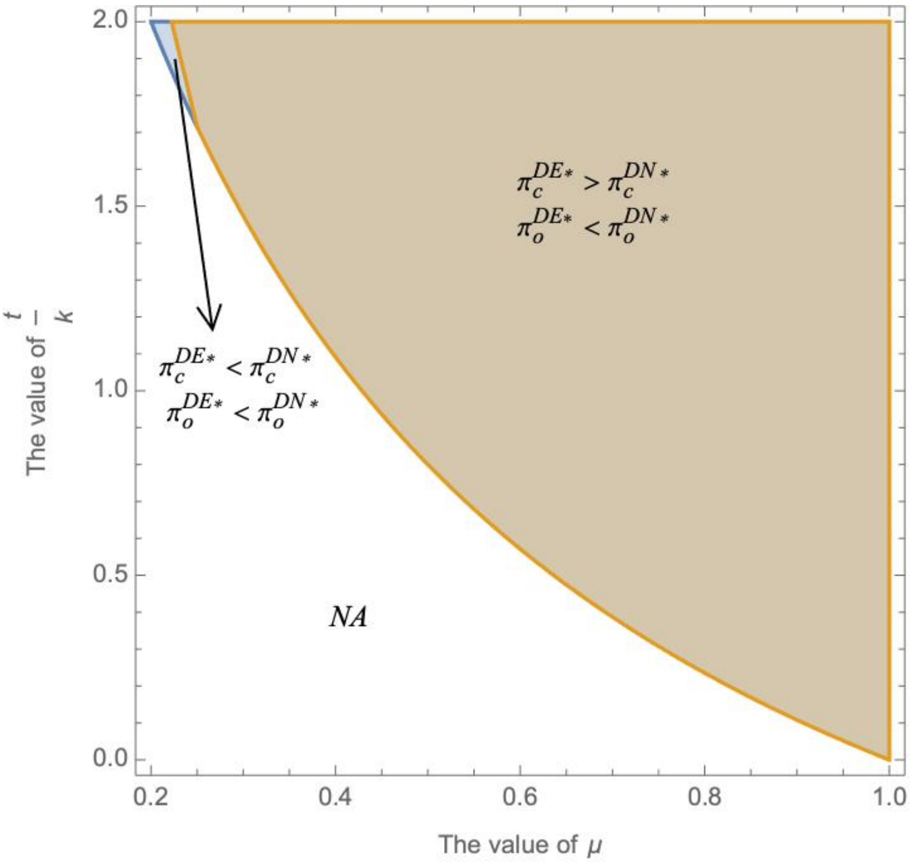


Fig. 4. The comparisons of firms' profits.

Comparing the profit of the whole supply chain under different settings, some primary results are summarized in Proposition 4.

Proposition 4. *From the perspective of overall supply chain profitability, there are such relationships hold in the feasible domain:*

i. $\Pi^{CN*} > \Pi_t^{DN*}$, *and* $\Pi^{CE*} > \Pi_t^{DE*}$;

ii. $\Pi^{CN*} > \Pi^{CE*}$ *when* $\frac{1}{3} < \mu < \frac{2}{3}$ *and* $\frac{4-4\mu}{1+\mu} < \frac{t}{k} < 2$, *or* $\frac{2}{3} < \mu < 1$ *and* $\frac{4-4\mu}{1+\mu} < \frac{t}{k} < \frac{-8+8\mu}{-2+\mu}$; $\Pi^{CN*} < \Pi^{CE*}$ *when* $\frac{2}{3} < \mu < 1$ *and* $\frac{-8+8\mu}{-2+\mu} < \frac{t}{k} < 2$.

The first part of Proposition 4 is intuitive. Whether CM produces customized production or not, the whole supply chain profit under a centralized setting is larger than that under a decentralized one. That is, coordination can alleviate the double marginal effect.

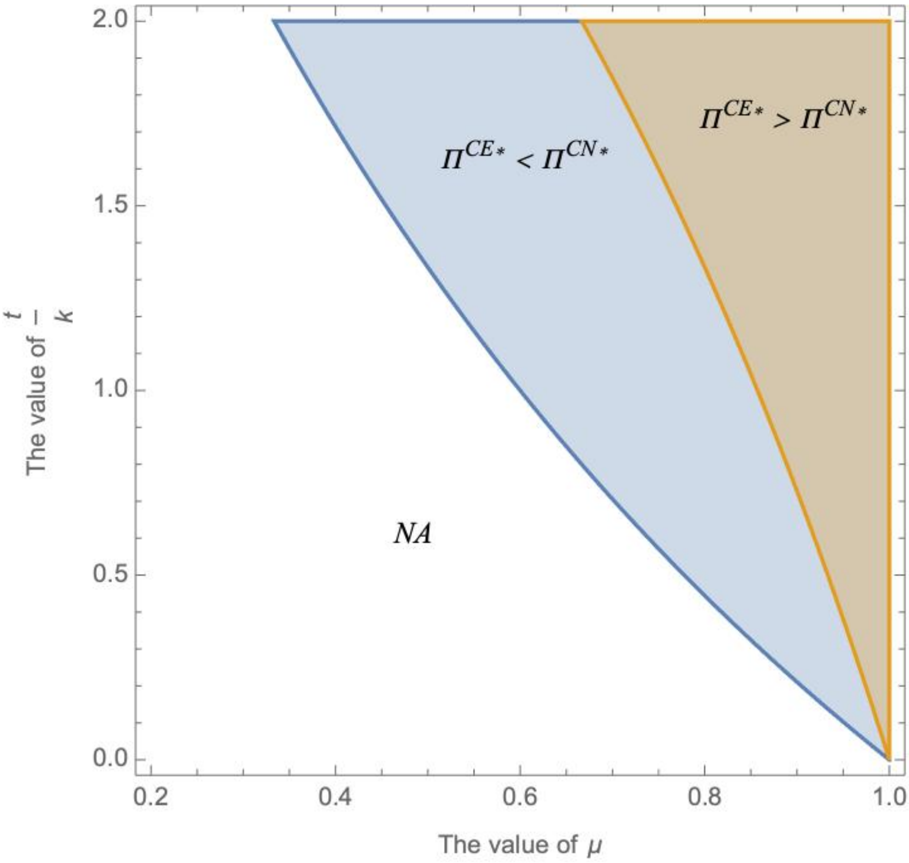


Fig. 5. The profit comparison under centralized supply chain.

Proposition 4(ii) conveys that the co-existence of two products with horizontal differentiation in the supply chain is not always preferable under a centralized supply chain, as shown in Fig. 5. It seems counter-intuitive. Usually, a firm offering various products can satisfy different consumers' needs and yield higher profits. In fact, however, such an interesting thing will firstly happen when the acceptance degree of store brand is not large (i.e., $\frac{1}{3} < \mu < \frac{2}{3}$). Under such conditions, the customized product cannot obtain enough market share to meet the investment cost. Moreover, though the profit from retail business is optimistic with the promotion of store brand image (i.e., $\frac{2}{3} < \mu < 1$). If the ratio $\frac{t}{k}$ is small (i.e., $\frac{t}{k} < \frac{-8+8\mu}{-2+\mu}$, low misfit cost t or high investment cost k), the competition between the two brands becomes intense. It makes a larger reduction in the profit from standard products than the increment in profit from customized products. Thus, the whole supply chain profit decreases. The management insight is that if a company intends to cultivate a sub-brand and expand manufacture, he should propagandize the new brand, such as advertising, to increase its brand valuation. On the other hand, differentiating the products under different brands to reduce the competition is also necessary.

5.2 Equilibrium Strategy

According to the above investigation, the boundary values of μ and $\frac{t}{k}$ are different in the centralized and decentralized supply chains, as shown in Fig. 6. We can find that CM always benefits from entry in the consumer market in area $A_1,\ A_2,\ and\ A_3$, no matter what the supply chain modes would be. Furthermore, the whole supply chain profits under scenario CE are larger than that under scenario CN in area A_1. Thus, providing customized products under centralized supply chain (i.e., scenario CE) is the dominant strategy where CM and the supply chain system perform better.

5.3 Revenue-Sharing Contract

Propositions 3 and 4 demonstrate that CM's encroachment behavior always hurts OEM but can improve the whole supply chain profit under a centralized setting in a particular situation. Then an interesting question arises: Can the revenue-sharing contract eliminate the negative effect on OEM and enhance the performance of the decentralized supply chain when CM develops the retail business?

Under the decentralized supply chain, if OEM and CM reach an agreement to manufacture customized and standard products based on the revenue-sharing contract, CM wholesales products at a cost price, and OEM returns CM a percentage λ $(0 < \lambda < 1)$ of its net earnings. We use superscript I_2 to represent such integrated supply chain. The game sequence is as follows. CM first determines the customization level based on its production capacity. Then OEM decides on retail prices of the two products. It is noted that OEM undertakes the investment cost on the customization production line. Therefore, firms' profits are as below,

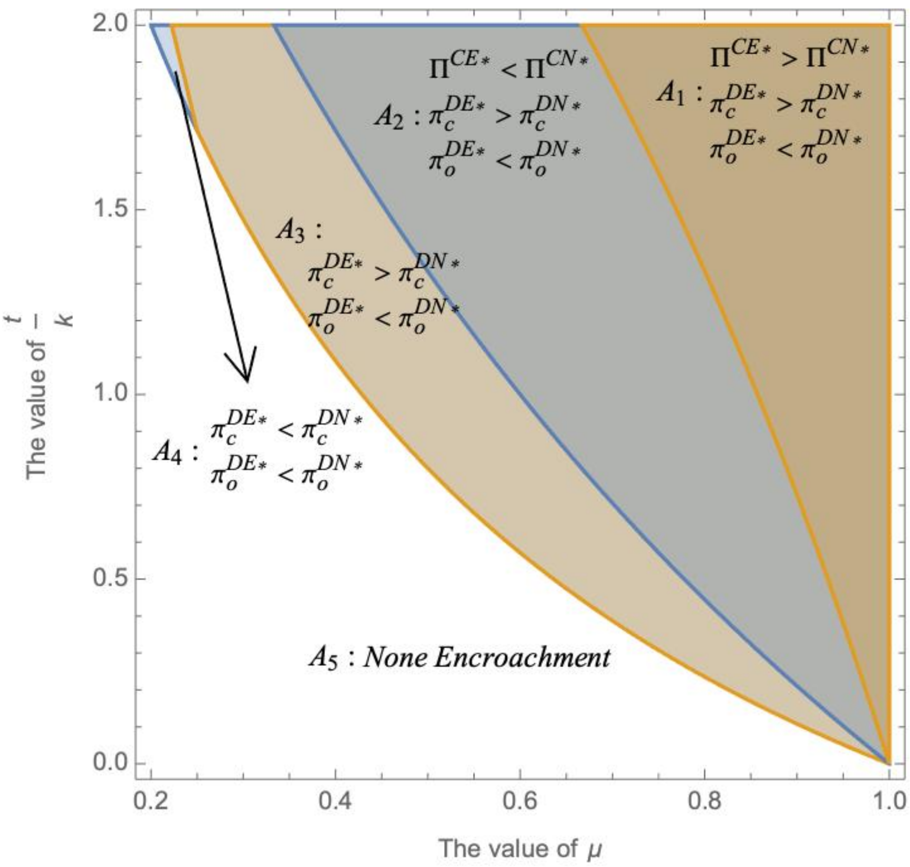


Fig. 6. The profit comparison under different supply chain settings.

respectively.

$$\pi_c^{I_2} = \lambda(d_n^{I_2} p_n^{I_2} + d_s^{I_2} p_s^{I_2} - k(l^{I_2})^2), \tag{12}$$

$$\pi_o^{I_2} = (1-\lambda)(d_n^{I_2} p_n^{I_2} + d_s^{I_2} p_s^{I_2} - k(l^{I_2})^2). \tag{13}$$

Where the expression of $d_n^{I_2}$ $(or\ d_s^{I_2})$ can be obtained by replacing all the superscript DE in $d_n^{DE}(or\ d_s^{DE})$ as I_2 in Section 4.2.

According to the discussion in Section 5.2, the equilibrium strategy CE happens in area A_1 : $\left(\frac{2}{3} < \mu < 1\ and\ \frac{-8+8\mu}{-2+\mu} < \frac{t}{k} < 2\right)$, in which $\Pi^{CE*} > \Pi^{CN*}$, $\pi_c^{DE*} > \pi_c^{DN*}$ and $\pi_o^{DE*} < \pi_o^{DN*}$. The revenue-sharing contract is better for OEM if her profit is larger than that under scenario DE. Moreover, the profit allocated to CM should not be less than that under scenario DE (i.e., $\pi_c^{I_2*} > \pi_c^{DE*}$). Otherwise, CM will not accept the revenue-sharing contract because he obtains a lower profit. Therefore, using backward induction, the equilibrium outcome is listed in Table 5. It is easy to find that the equilibrium retail price $p_n^{I_2*}$, $p_s^{I_2*}$, the customization level l^{I_2*}, and the demand $d_n^{I_2*}$, $d_s^{I_2*}$ under the revenue-sharing contract are all the same as those in the centralized supply chain, which shows that the proposed contract can fully coordinate the supply chain. As discussed above, the profit allocation mechanism should guarantee that OEM has an incentive to provide it and make sure CM will accept it. That is, $\pi_c^{I_2*} > \pi_c^{DE*}$ *and* $\pi_o^{I_2*} > \pi_o^{DE*}$. With $\pi_c^{I_2*}$ *and* $\pi_o^{I_2*}$ in Table 5, and π_c^{DE*} *and* π_o^{DE*} in Table 3, the profit-sharing ratio λ is given in Proposition 5.

Table. 5. The equilibrium outcomes with revenue-sharing contract.

	Equilibrium outcome
$p_n^{I_2*}$	$\frac{V(4k-t+t\mu)}{2(4k-t)}$
$p_s^{I_2*}$	$\frac{2kV\mu}{4k-t}$
l^{I_2*}	$\frac{\mu V}{4k-t}$
$d_n^{I_2*}$	$\frac{V(1-\mu)}{2t}$
$d_s^{I_2*}$	$\frac{V[t(-1+\mu)+4k(1-2\mu)]}{2t(-4k+t)}$
$\pi_c^{I_j*}$	$\frac{V^2\lambda[4k(1+2\mu(-1+\mu)-t(-1+\mu)^2)]}{4t(4k-t)}$
$\pi_o^{I_2*}$	$\frac{V^2(1-\lambda)[4k(1+2\mu(-1+\mu)-t(-1+\mu)^2)]}{4t(4k-t)}$
Π^{I_2*}	$\frac{V^2[4k(1+2\mu(-1+\mu)-t(-1+\mu)^2)]}{4t(4k-t)}$
λ	$(\frac{f-12\mu^2}{4f}, \frac{f-4\mu^2}{2f})$

Notes: $f = \frac{t}{k}(\mu-1)^2 + 8\mu(1-\mu) - 4$.

Proposition 5. *The revenue-sharing contract that OEM, the Stackelberg leader, shares* $\lambda \in (\underline{\lambda}, \overline{\lambda})$ *percent of her net earnings to CM under an integrated setting can be designed to obtain the contract-implementing Pareto zone, as shown in Fig. 7, where* $\underline{\lambda} = \frac{f-12\mu^2}{4f}$ *and* $\overline{\lambda} = \frac{f-4\mu^2}{2f}$, $f = \frac{t}{k}(\mu-1)^2 + 8\mu(1-\mu) - 4$.

Proposition 5 shows how to allocate the revenue when OEM collaborates with CM who manufactures customized and standard products. If the revenue-sharing contract is achieved, it is easy to find $\pi_c^{I_2*} > \pi_c^{DE*}$, $\pi_o^{I_2*} > \pi_o^{DE*}$, and $\Pi^{I_2*} = \Pi^{CE*}$ according to Table 5. It indicates that the proposed revenue-sharing contract can not only improve firms' profits, but also eliminate the double marginalization effect, making the supply chain as efficient as a centralized system. Then a contract-implementing Pareto zone arises, as area A shown in Fig. 7.

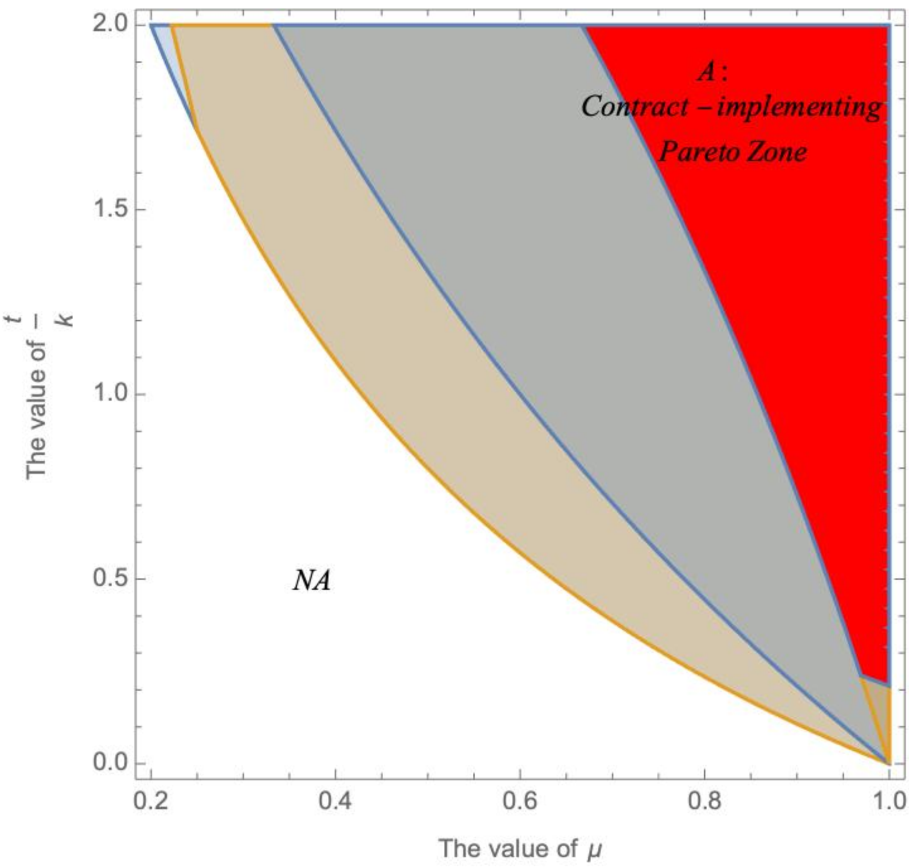


Fig. 7. The contract-implementing Pareto zone of revenue-sharing contract.

Furthermore, Proposition 5 demonstrates that the revenue-sharing rate is relevant to the acceptance degree of store brand, the unit misfit cost, and the investment cost. The lower bound $\underline{\lambda}$ represents the lowest revenue required by CM to benefit from accepting the contract. The upper bound $\overline{\lambda}$ means the highest revenue that OEM would like to share with CM. We can find that the lower and upper bounds increase with the acceptance degree of store brand. In general, the revenue-sharing rate reflects the negotiation power between OEM and CM. Then a higher acceptance degree of store brand makes CM more power over requiring revenue from OEM, resulting in a larger revenue-sharing percentage. Moreover, the growth rate of the former is larger than that of the latter (i.e., $\frac{\partial \underline{\lambda}}{\partial \mu} > \frac{\partial \overline{\lambda}}{\partial \mu} > 0$), which is vividly illustrated in Fig. 7. The contract-implementing Pareto zone shrinks with the increase of μ.

Additionally, as shown in Fig. 7, area A increases with the percentage $\frac{t}{k}$ for a given acceptance degree of store brand. A small ratio $\frac{t}{k}$ requires a lower t or a larger k. If we fix the investment cost k, a smaller t means that the competition between customized and standard products becomes more intense, resulting in lower retail prices, which mitigates the double marginalization effect in a decentralized supply chain. Therefore, the contract-implementing Pareto zone narrows.

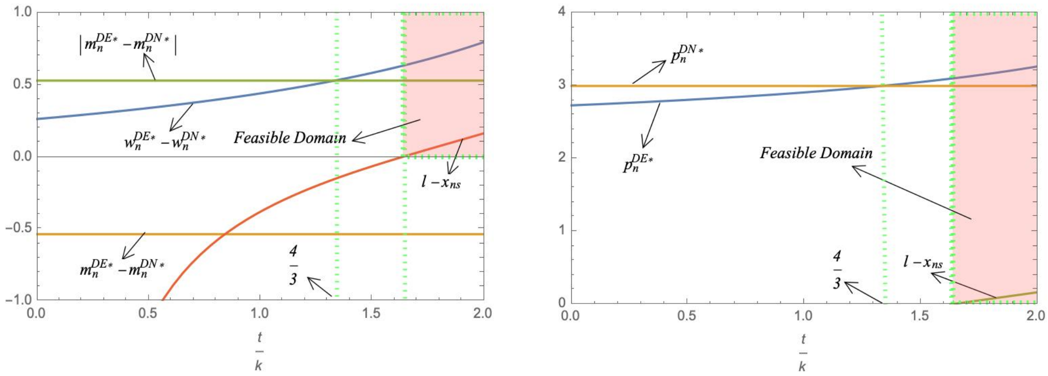
In summary, although the customized products under store brand are always harmful to OEM in the decentralized supply chain, a revenue-sharing contract can be adopted to improve the supply chain performance. Then the contract-implementing Pareto zone arises where both firms have higher revenue than scenario DE. Besides, the whole supply chain profit is the same as that in the centralized setting, realizing the optimal supply chain system.

6 Numerical Analysis

In this section, we utilize numerical analysis to demonstrate the analytical findings on the impacts of CM's entry into the end-consumer market on supply chain members presented in Proposition 1. We take the value of the parameter μ as $\frac{4}{15}$ and $\frac{9}{15}$, respectively. Adopting different values can make the numerical analysis more generally.

Firstly, what describes in Proposition 1(i) is straightforward. It is omitted here. Secondly, Fig. 8 depicts how the CM's customized products influence the wholesale price and marginal profits of the standard products with national brand when the acceptance degree of store brand is low (i.e., μ=$\frac{4}{15}$). As Fig. 8(a) shows, the encroachment behavior always has negative (positive) effects on the marginal profits (wholesale price), which present differently and are conditional on the value of $\frac{t}{k}$. It is consistent with Proposition 1(ii). Significantly, the increase in wholesale price always dominates the decrease in marginal profits under the feasible domain. The reason is that a larger $\frac{t}{k}$ means the customization is efficient with lower cost (i.e., a small k) and the flexibility advantage of customized products is large with a higher sensitiveness to the misfit of ideal preference (i.e., a large t). Consequently, the store brand is more competitive leading CM pricing powerfully. Combing the above two opposite impacts, the sale price of national brand becomes higher than that under scenario DN, as shown in Fig. 8(b).

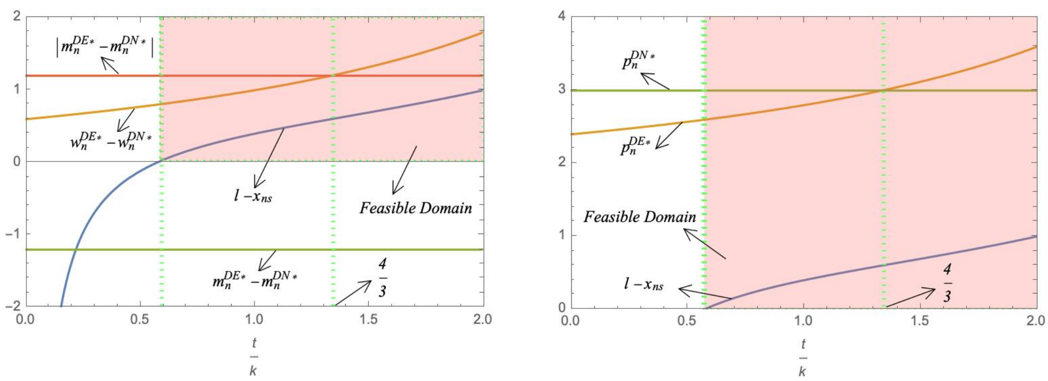
On the other hand, we also verify the results under an inclusive market situation with a high acceptance degree of store brand (i.e., μ=$\frac{9}{15}$). It shows that the



(a) (b)

Fig. 8. The impact of encroachment on marginal profits and prices under a decentralized setting with $\mu=\frac{4}{15}$.

feasible domain expands compared to the one under a low parameter value (i.e., $\mu=\frac{4}{15}$). Moreover, the feature that CM's entry has an opposite influence on supply chain members still holds. Fig. 9(a) indicates that the reduction in marginal profits is not always lower than the increment in wholesale price. Though introducing a store brand may enhance CM's bargaining power, he would not set a too high price when the value of $\frac{t}{k}$ is less than $\frac{4}{3}$. Otherwise, CM will be more aggressive. It demonstrates the analytical results in Proposition 1(ii). Affected by this inverse pressure, the change rule of retail price shown in Fig. 9(b) further verifies the conclusions in Proposition 1(iii).



(a) (b)

Fig. 9. The impact of encroachment on marginal profits and prices under a decentralized setting with $\mu=\frac{9}{15}$.

7 Extension

This section examines the impacts of customized products and supply chain structure from other views, especially the consumer surplus and the social welfare.

7.1 Consumer Surplus

Consumer surplus, denoted as CS and superscripted with the encroachment strategy and supply chain structure being used, is obtained according to the utility from consuming a standard or tailored product, as expressed in Equation (1) and (2). Therefore, consumer surplus are $CS^{TT*} = \int_0^{x_n^{TT*}} (V - p_n^{TT*} - tx)dx$ ($TT = DN$ or CN), and $CS^{TT*} = \int_0^{x_{ns}^{TT*}} (V - p_n^{TT*} - tx)dx + \int_{x_{ns}^{TT*}}^{l^{TT*}} (\mu V - p_s^{TT*})dx + \int_{l^{TT*}}^{x_s^{TT*}} (\mu V - p_s^{TT*} - t(x - l^{TT*}))dx$ ($TT = DE$ or CE), respectively. We summarize these specific results in Table 6. Based on this, we analyze how the entry in product customization market in diverse supply chain structures influences consumer surplus in Proposition 6.

Table. 6. Consumer surplus and social welfare under four scenarios.

	Consumer Surplus	Social Welfare
DN	$\frac{V^2}{32t}$	$\frac{7V^2}{32t}$
DE	$\frac{V^2[t^2(1-2\mu-15\mu^2)-8kt(1-\mu)^2+16k^2(1-2\mu+5\mu^2)]}{32t(t-4k)^2}$	$\frac{V^2[(7-14\mu)(t-4k)^2+\mu^2(304k^2-88kt-9t^2)]}{32t(t-4k)^2}$
CN	$\frac{V^2}{8t}$	$\frac{3V^2}{8t}$
CE	$\frac{V^2[t^2(1-2\mu-3\mu^2)-8kt(1-\mu)^2+16k^2(1-2\mu+2\mu^2)]}{8t(t-4k)^2}$	$\frac{V^2[(3-6\mu)(t-4k)^2+\mu^2(96k^2-32kt-t^2)]}{8t(t-4k)^2}$

Proposition 6. *Comparing the consumer surplus under different subgames, the following relationships always holds in the feasible domain:*

i. $CS^{CN*} > CS^{DN*}$ *and* $CS^{CE*} > CS^{DE*}$*;*

ii. $CS^{DE*} > CS^{DN*}$ *when* $(\mu, \frac{t}{k}) \in B_1$*, otherwise,* $CS^{DE*} < CS^{DN*}$*;*
$CS^{CE*} > CS^{CN*}$ *when* $(\mu, \frac{t}{k}) \in B_2$*, otherwise,* $CS^{CE*} < CS^{CN*}$*,*
where $B_1 \equiv \left\{(\mu, \frac{t}{k}) | \frac{1}{3} < \mu < 1,\ \frac{4-4\mu}{1+3\mu} < \frac{t}{k} < -\frac{4(-2+\mu)}{2+15\mu} + 8\sqrt{\frac{-6\mu+19\mu^2}{(2+15\mu)^2}}\right\}$,
$B_2 \equiv \left\{(\mu, \frac{t}{k}) | \frac{6}{7} < \mu < 1, -\frac{4(-2+\mu)}{2+3\mu} - 4\sqrt{\frac{-6\mu+7\mu^2}{(2+3\mu)^2}} < \frac{t}{k} < -\frac{4(-2+\mu)}{2+3\mu} + 4\sqrt{\frac{-6\mu+7\mu^2}{(2+3\mu)^2}}\right\}$.

Consumers benefit from the centralized supply chain regardless of CM's entry strategy. Because the centralization can eliminate the double marginalization effect, resulting in a lower retail price. Consequently, consumers obtain more from purchasing. The first part of Proposition 6(ii) indicates that CM's encroachment is better for consumers when the ratio $\frac{t}{k}$ is not too large, and the store brand is famous in a decentralized supply chain. For a given investment cost, a lower $\frac{t}{k}$ requires a smaller t, which means a fiercer competition. Then the decrease in retail price is favorable for consumers. Fig. 10 describes the preferable scenario for consumers under different conditions. As $\frac{t}{k}$ reduces, CE becomes superior when the acceptance degree of store brand is enough large.

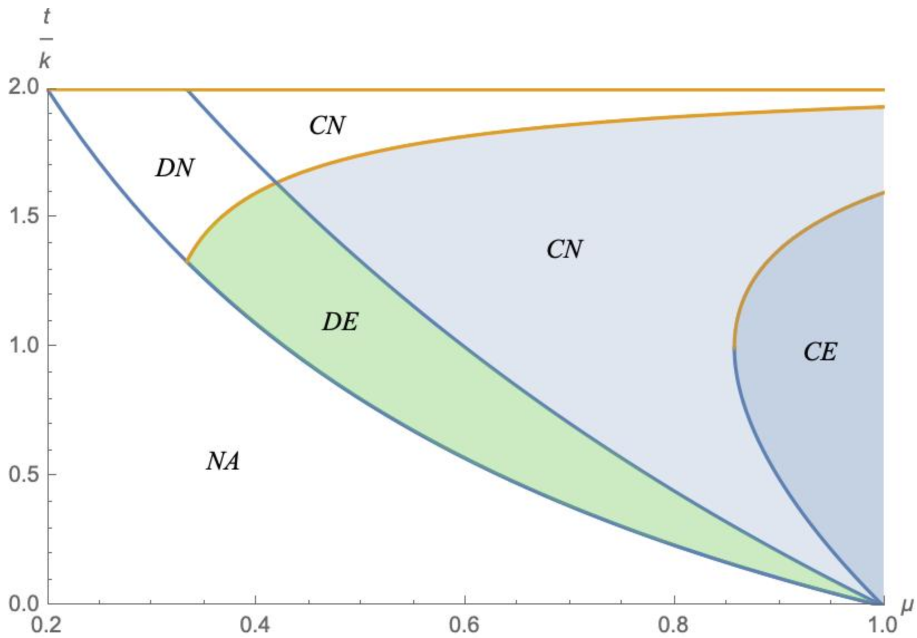


Fig. 10. Comparison with respect to consumer surplus among four subgames.

7.2 Social Welfare

This section further investigates the impacts of encroachment strategy in the view of social welfare, which means the sum of whole supply chain profits and the corresponding consumer surplus, denoted as SW. The results under different scenarios are exhibited in Table 6. In addition, we plot the dominant subgame that gives rise to the highest social welfare for each feasible combination of μ and $\frac{t}{k}$ in Fig. 11.

The region in which DE dominates shrinks compared with that in Fig. 10. Generally, SW is a comprehensive performance metric consisting of firms' profits and consumer utility. As discussed in Section 4.2, OEM always suffers from CM's encroachment. Therefore, the increase in CM's profit cannot offset the decrease in OEM's revenue when the acceptance degree of store brand is minor, resulting in lower social welfare than the one in scenario DN.

An intriguing change is that the area of CE dominance has become more prominent in contrast to Fig. 10. If $\frac{t}{k}$ is large enough, consumers suffer from more misfit costs and obtain a lower utility. And then they are worse off under scenario CE. On the other hand, in the view of total supply chain profits, we have verified that CE is the optimal strategy in area A_1 in Fig. 6, and the dominant region increases with $\frac{t}{k}$. Then, compared with scenario CN, the whole supply chain gains more from the entry of customized products for a larger ratio $\frac{t}{k}$. The reason is that a higher t indicates the horizontal difference more obvious and consequently a less intense competition. Therefore, the centralized supply chain is better off. In summary, the positive effect of customized product encroachment on the whole supply chain outweighs the negative one on consumer surplus under a centralized setting, which makes CE dominant from social welfare.

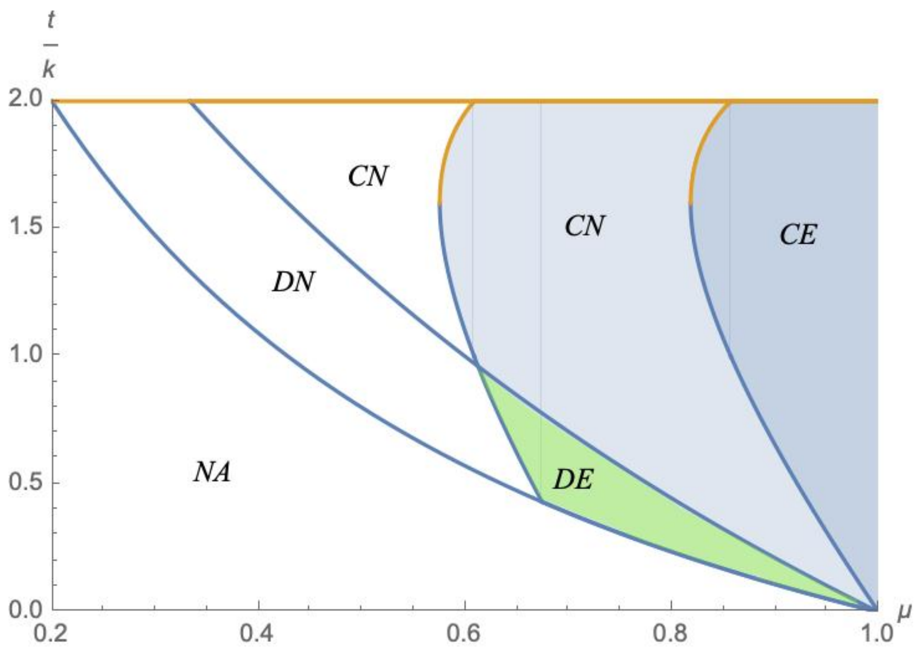


Fig. 11. Comparison with respect to social welfare among four subgames.

8 Conclusions and Managerial Insights

In this paper, we investigated the problem of CM's encroachment strategy of product customization and supply chain coordination mechanism in a two-echelon supply chain. The equilibrium strategy CE was obtained by comparing the equilibrium outcomes under four different scenarios, i.e., no-encroachment under decentralized supply chain (DN), encroachment under decentralized supply chain (DE), no-encroachment under centralized supply chain (CN), and encroachment under centralized supply chain (CE). We also proposed a revenue-sharing contract and got the contract-implementing Pareto zone where firms' profits and the performance of the whole supply chain were optimal. Additionally, we discussed how CM's entry in the retail market affected consumer surplus and social welfare. The primary findings are summarized as follows.

Firstly, there is a threshold on customization level that may gain positive market share for CM. It indicates that the encroachment is insignificant if the customization level is too low except for losing investment in the customization production line. Moreover, encroachment is beneficial only when the acceptance degree of the store brand is not too small though CM can acquire market demand for his store brand product. The equilibrium strategy is encroachment under a centralized supply chain, which benefits CM and the supply chain system. Thirdly, OEM suffers from CM's customized product under store brand. With the flexibility advantage, customized products always carve up some market share of standard products. Combing the reduction of margin profit, OEM gains less profit. The research findings also demonstrate that the revenue-sharing contract can change such a negative situation and improve supply chain members' performance.

Additionally, we can summarize several managerial insights through the research results.

(1) A higher brand acceptance degree makes the encroachment beneficial more easily. To enhance the store brand image, managers should be concerned to promote the brand concept and provide a comfortable shopping experience [3]. Moreover, to a certain extent, a lower customization cost and a higher sensitivity to the misfit of ideal preference can compensate for the disadvantage of brand value. Thus, a CM who plans to develop a new retailing business should optimize the

manufacturing process to improve customization efficiency. Besides, advertising the superiority of customized products to stimulate consumers' shopping desire is necessary.

(2) Brand competition becomes more intense when the horizontal difference lowers, reducing the whole supply chain profit. It is not always preferable to utilize such a multi-brand strategy for a firm. Thus, managers should pay attention to product differences when introducing a new sub-brand.

(3) The cooperation between supply chain members can alleviate the double marginalization effect and increase their profits, consumer surplus, and social welfare. The regulators should promote collaboration, such as a revenue-sharing contract, which can improve each party's performance and optimize the entire system. Then, a win-win outcome may arise.

This paper also has some research limitations and can be further investigated in future works. First, instead of horizontal difference, one can extend this work by considering the store brand product with lower quality [39] and then discuss how both vertical and horizontal difference influence CM's decision. Moreover, an interesting topic is whether OEM could seek a new contract manufacturer [8] and improve her performance when facing CM's encroachment behavior. Also, future work can design other coordination mechanisms except the revenue-sharing contract to optimize the supply chain system. The assessment of the popularity of customization is a new research direction. It is best to survey and predict customized products' market situation before making a manufacturing decision for the CM. Then the fuzzy regression approaches [21] can be applied to evaluate prediction results. Considering the noise and uncertainty in market data of customized products, the robust optimization [32] on prediction is also worth investigating.

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Conflict of Interest Statement

All authors declares no possible conflict of interest.

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Appendix

Proof of Lemma 1

We work out the equilibrium outcomes through backward induction. Given m_n^{DN}, according to the first-order condition (FOC) $\frac{\partial \pi_C^{DN}}{\partial w_n^{DN}} = 0$, we have $w_n^{DN}(m_n^{DN}) = \frac{V - m_n^{DN}}{2}$. Plugging $w_n^{DN}(m_n^{DN})$ into π_o^{DN}, we have OEM's decision $m_n^{DN*} = \frac{V}{2}$ based on FOC. Last, plugging m_n^{DN*} into $w_n^{DN}(m_n^{DN})$, we have $w_n^{DN*} = \frac{V}{4}$. Then the equilibrium outcome is $p_n^{DN*} = \frac{3V}{4}, d_n^{DN*} = \frac{V}{4t}, \pi_c^{DN*} = \frac{V^2}{16t}, \pi_o^{DN*} = \frac{V^2}{8t}$.

□

Proof of Lemma 2

We assume the customization level is less than the threshold $x_{ns}, l < \frac{(1-\mu)V + p_s - p_n}{t}$.

The first condition is $x \in [0, l]$. Then the utility from buying a standard product and a customized product is $U_n = V - p_n - tx \ and \ U_s = \mu V - p_s$, respectively. Comparing U_n with U_s, the condition $U_n > U_s$ always holds. It means that consumers who are located in interval $[0, l]$ will not buy the customized product.

The second condition is $x \in (l, 1]$. The utility from consuming a customized product is $U_s = \mu V - p_s - t(x - l)$. Comparing it with U_n, the condition $U_n > U_s$ always holds when $l < \frac{(1-\mu)V + p_s - p_n}{t}$. It means that consumers who are located in interval $(l, 1]$ will not buy the customized product.

Therefore, $U_n > U_s$ always holds under $l < \frac{(1-\mu)V + p_s - p_n}{t}$, which indicates no one buy the customized product in the end consumer market.

□

Proof of Lemma 3

In the last stage of the game, given $l^{DE} \ and \ m_n^{DE}$, CM maximizes π_c^{DE} with respect to $w_n^{DE} \ and \ p_s^{DE}$ simultaneously. We have $p_s^{DE}(l^{DE}, m_n^{DE}) = \frac{l^{DE} t + \mu V}{2}$ and $w_n^{DE}(l^{DE}, m_n^{DE}) = \frac{l^{DE} t - m_n^{DE} + V}{2}$ per FOCs. In the second stage, plugging $p_s^{DE}(l^{DE}, m_n^{DE})$ and $w_n^{DE}(l^{DE}, m_n^{DE})$ into equation (9), OEM maximizes π_o^{DE} with m_n^{DE}. According to FOC, we have $m_n^{DE}(l^{DE}) = \frac{V - \mu V}{2}$. Then CM maximizes $\pi_c^{DE}(l^{DE})$. The second condition satisfies $\frac{\partial^2 \pi_c^{DE}(l^{DE})}{\partial (l^{DE})^2} = \frac{-4k+t}{2} < 0 \ if \ \frac{t}{k} < 4$. Thus, supposing

$\frac{t}{k} < 4$, we have the optimal customization level is $l^{DE*} = \frac{\mu V}{4k-t}$ according to FOC. Last, plugging l^{DE*}, we have the equilibrium outcome is $m_n^{DE*} = \frac{V-\mu V}{2}, w_n^{DE*} = \frac{V[t(-1+\mu)+4k(1+\mu)]}{4(4k-t)}, p_n^{DE*} = \frac{V[3t(-1+\mu)-4k(\mu-3)]}{4(4k-t)}, p_s^{DE*} = \frac{2kV\mu}{4k-t}, d_n^{DE*} = \frac{V(1-\mu)}{4k-t}, d_s^{DE*} = \frac{V[t(-1+\mu)+4k(1-3\mu)]}{4t(-4k+t)}, \pi_c^{DE*} = \frac{V^2[-t(-1+\mu)^2+4k(1+\mu(5\mu-2))]}{16(4k-t)t}, \pi_o^{DE*} = \frac{V^2(-1+\mu)^2}{8t}$.

Noting that the market size is normalized to 1, then parameters should satisfy $0 < d_n^{DE*} + d_s^{DE*} \leq 1$. Moreover, the assumptions $l > \frac{(1-\mu)V+p_s-p_n}{t}$ also should be satisfied. Thus, we constrain parameters to $\left\{(k,t,\mu) | \frac{4(1-\mu)}{3\mu+1} < \frac{t}{k} < 2,\ \frac{1}{5} < \mu < 1, t > 0\right\}$.

□

Proof of Proposition 1

i. $\frac{\partial w_n^{DE*}}{\partial \mu} = \frac{V(4k+t)}{4(4k-t)} > 0$ *and* $\frac{\partial m_n^{DE*}}{\partial \mu} = \frac{-V}{2} < 0$ are obvious;

ii. Comparing the wholesale prices under scenario DN and DE, we have $w_n^{DE*} - w_n^{DN*} = \frac{\mu V(4k+t)}{4(4k-t)} > 0$. Comparing OEM's margin profit, we have $m_n^{DE*} - m_n^{DN*} = \frac{-\mu V}{2} < 0$.

   Comparing the magnitude of wholesale price and margin profit changes $\frac{|w_n^{DE*} - w_n^{DN*}|}{|m_n^{DE*} - m_n^{DN*}|} = \frac{4k+t}{2(4k-t)} > 1$ if $\frac{4}{3} < \frac{t}{k} < 2$.

iii. For the retail price of the standard product, we have

$$p_n^{DE*} - p_n^{DN*} = \frac{-\mu V(4k-3t)}{4(4k-t)}. \tag{A1}$$

   The sign of equation (A1) is opposite to that of formula $(4k-3t)$. Obviously, $(4k-3t) < 0$ if $\frac{t}{k} > \frac{4}{3}$, otherwise, $(4k-3t) > 0$.

   On the other hand, the condition $\frac{4-4\mu}{1+3\mu} > \frac{4}{3}$ holds when $\frac{1}{5} < \mu < \frac{1}{3}$, and $\frac{4-4\mu}{1+3\mu} < \frac{4}{3}$ holds when $\frac{1}{3} < \mu < 1$.

   Thus, $p_n^{DE*} - p_n^{DN*} > 0$ when $\frac{1}{5} < \mu < \frac{1}{3}$ *and* $\frac{4-4\mu}{1+3\mu} < \frac{t}{k} < 2$, or $\frac{1}{3} < \mu < 1$ *and* $\frac{4}{3} < \frac{t}{k} < 2$. Otherwise, $p_n^{DE*} - p_n^{DN*} < 0$ when $\frac{1}{3} < \mu < 1$ *and* $\frac{4-4\mu}{1+3\mu} < \frac{t}{k} < \frac{4}{3}$.

□

Proof of Proposition 2

i. It is apparent that $d_n^{DE*} < d_n^{DN*}$;

ii. We derive the whole supply chain demand

$$Td^{DE*} - Td^{DN*} = \frac{kV(-4 + \frac{t}{k} + 8\mu)}{4t(4k - t)}. \tag{A2}$$

Since the denominator is positive, the sign of equation (A2) will match the sign of numerator. Moreover, we have $kV > 0$. If $Td^{DE*} - Td^{DN*} > 0$, it implies $\frac{t}{k} > 4 - 8\mu$. According to the proof of Lemma 3, constrain parameters to $\left\{(k, t, \mu, V)|\frac{4(1-\mu)}{3\mu+1} < \frac{t}{k} < 2,\ \frac{1}{5} < \mu < 1, 0 < V < \frac{4t(k-t)}{2k\mu}\right\}$. Therefore, $\frac{4(1-\mu)}{3\mu+1} < 4 - 8\mu < 2$ when $\frac{1}{4} < \mu \leq \frac{1}{3}$; $4 - 8\mu < \frac{4(1-\mu)}{3\mu+1} < 2$ when $\frac{1}{3} < \mu < 1$. Thus $Td^{DE*} - Td^{DN*} > 0$ when $4 - 8\mu < \frac{t}{k} < 2$ and $\frac{1}{4} < \mu \leq \frac{1}{3}$; or $\frac{4(1-\mu)}{3\mu+1} < \frac{t}{k} < 2$ and $\frac{1}{3} < \mu < 1$. The derivation of $Td^{DE*} - Td^{DN*} < 0$ is similar.

□

Proof of Proposition 3

i. It is straightforward to verify $\pi_o^{DE*} = \frac{V^2}{8t} < \pi_o^{DN*} = \frac{V^2(-1+\mu)^2}{8t}$;

ii. Comparing CM's profit under scenarios DE and DN, we have

$$\pi_c^{DE*} - \pi_c^{DN*} = \frac{V^2 k\mu(-8 + \frac{2t}{k} + 20\mu - \frac{t\mu}{k})}{16t(4k - t)}. \tag{A3}$$

Because the denominator and $V^2 k\mu$ are positive, the sign of equation (A3) is identical to the sign of $(-8 + \frac{2t}{k} + 20\mu - \frac{t\mu}{k})$. It is positive when $\frac{t}{k} > \frac{8-20\mu}{2-\mu}$. We find that $\frac{4(1-\mu)}{3\mu+1} < \frac{8-20\mu}{2-\mu} < 2$ when $\frac{2}{9} < \mu < \frac{1}{4}$; $\frac{8-20\mu}{2-\mu} < \frac{4(1-\mu)}{3\mu+1} < 2$ when $\frac{1}{4} < \mu < 1$. Therefore, equation (A3) is positive when $\frac{8-20\mu}{2-\mu} < \frac{t}{k} < 2$ and $\frac{2}{9} < \mu < \frac{1}{4}$; or $\frac{4(1-\mu)}{3\mu+1} < \frac{t}{k} < 2$ and $\frac{1}{4} < \mu < 1$.

Suppose $-8 + \frac{2t}{k} + 20\mu - \frac{t\mu}{k} < 0$, then we have $\frac{t}{k} < \frac{8-20\mu}{2-\mu}$. In addition, $\frac{4(1-\mu)}{3\mu+1} < 2 < \frac{8-20\mu}{2-\mu}$ holds if $\frac{1}{5} < \mu < \frac{2}{9}$. And $\frac{4(1-\mu)}{3\mu+1} < \frac{8-20\mu}{2-\mu} < 2$ holds if $\frac{2}{9} < \mu < \frac{1}{4}$. Otherwise, $\frac{8-20\mu}{2-\mu} < \frac{4(1-\mu)}{3\mu+1}$. Thus, $\pi_c^{DE*} - \pi_c^{DN*} < 0$ when $\frac{4(1-\mu)}{3\mu+1} < \frac{t}{k} < 2$ and $\frac{1}{5} < \mu < \frac{2}{9}$; or $\frac{4(1-\mu)}{3\mu+1} < \frac{t}{k} < \frac{8-20\mu}{2-\mu}$ and $\frac{2}{9} < \mu < \frac{1}{4}$.

□

Proof of Proposition 4

i. Comparing the profit of the whole supply chain under different setting, we have $\Pi^{CN*} - \Pi_t^{DN*} = \frac{V^2}{16t} > 0$, and $\Pi^{CE*} - \Pi_t^{DE*} = \frac{V^2(-1+\mu)^2}{16t} > 0$;

ii. To examine the impact of selling customized products on profits of the whole supply chain, we have

$$\Pi^{CE*} - \Pi^{CN*} = \frac{V^2 k\mu(-8 + \frac{2t}{k} + 8\mu - \frac{t\mu}{k})}{4t(4k-t)}. \tag{A4}$$

Obviously, the sign of equation (A4) is consistent with that of $\left(-8 + \frac{2t}{k} + 8\mu - \frac{t\mu}{k}\right)$. Then, $\Pi^{CE*} - \Pi^{CN*} > 0$ holds if $\frac{t}{k} > \frac{8(1-\mu)}{2-\mu}$, otherwise, $\Pi^{CE*} - \Pi^{CN*} < 0$. Moreover, the feasible domain for providing customized products under a centralized supply chain is $\left\{(\mu, k, t)|\frac{1}{3} < \mu < 1,\ \ \frac{4-4\mu}{1+\mu} < \frac{t}{k} < 2, t > 0\right\}$ as shown in section 5.1. Note that $\frac{4(1-\mu)}{1+\mu} < \frac{8(1-\mu)}{2-\mu} < 2$ holds if $\frac{2}{3} < \mu < 1$, and $\frac{4(1-\mu)}{1+\mu} < 2 < \frac{8(1-\mu)}{2-\mu}$ holds if $\frac{1}{3} < \mu < \frac{2}{3}$. Thus, we have if $\frac{8(1-\mu)}{2-\mu} < \frac{t}{k} < 2$ and $\frac{2}{3} < \mu < 1$, such that $\Pi^{CE*} > \Pi^{CN*}$; if $\frac{4(1-\mu)}{1+\mu} < \frac{t}{k} < 2$ and $\frac{1}{3} < \mu < \frac{2}{3}$; or $\frac{4(1-\mu)}{1+\mu} < \frac{t}{k} < \frac{8(1-\mu)}{2-\mu}$ and $\frac{2}{3} < \mu < 1$, such that $\Pi^{CE*} < \Pi^{CN*}$.

□

Proof of Proposition 5

We first solve the equilibrium outcome under revenue-sharing contract for producing both tailored and standard products by backward induction. The firms' profits are

$$\pi_c^{I_2} = \lambda(d_n^{I_2} p_n^{I_2} + d_s^{I_2} p_s^{I_2} - k(l^{I_2})^2), \tag{A5}$$

$$\pi_o^{I_2} = (1-\lambda)(d_n^{I_2} p_n^{I_2} + d_s^{I_2} p_s^{I_2} - k(l^{I_2})^2), \tag{A6}$$

respectively. Where $d_n^{I_2} = \frac{(1-\mu)V + p_s^{I_2} - p_n^{I_2}}{t}$ and $d_s^{I_2} = l^{I_2} + \frac{(2\mu-1)V + p_n^{I_2} - 2p_s^{I_2}}{t}$.

In the last stage of the game, given the customization level, OEM decides on retail prices of the two products. According to the FOCs, we have $p_n^{I_2}(l^{I_2}) = \frac{l^{I_2}t+V}{2}$ and $p_s^{I_2}(l^{I_2}) = \frac{l^{I_2}t+V\mu}{2}$. Plugging $p_n^{I_2}(l^{I_2})$ and $p_s^{I_2}(l^{I_2})$ into equation (A5), CM maximizes $\pi_c^{I_2}(l^{I_2})$ with respect to l^{I_2}. We have $l^{I_2^*} = \frac{\mu V}{4k-t}$ per FOC. Lastly, by plugging $p_n^{I_2}(l^{I_2})$ and $p_s^{I_2}(l^{I_2})$, we can obtain the equilibrium outcome is $p_n^{I_2^*} = \frac{V(4k-t+t\mu)}{2(4k-t)}$, $p_s^{I_2^*} = \frac{2kV\mu}{4k-t}$, $d_n^{I_2^*} = \frac{V(1-\mu)}{2t}$, $d_s^{I_2^*} = \frac{V[t(-1+\mu)+4k(1-2\mu)]}{2t(-4k+t)}$, $\pi_c^{I_2^*} = \frac{\lambda V^2[-t(-1+\mu)^2+4k(1+2\mu(-1+\mu))]}{4t(4k-t)}$, $\pi_o^{I_2^*} = \frac{V^2(1-\lambda)[4k(1+2\mu(-1+\mu))-t(-1+\mu)^2]}{4(4k-t)t}$.

In area A_1, the parameter constraint condition is $\left\{(\mu,k,t,V)|\frac{2}{3} < \mu < 1, \frac{-8+8\mu}{-2+\mu} < \frac{t}{k} < 2,\ 0 < V < \frac{4kt-t^2}{2k\mu}\right\}$. According to the criteria for achieving the revenue-sharing contract, $\pi_c^{I_2*} > \pi_c^{DE*}$ *and* $\pi_o^{I_2*} > \pi_o^{DE*}$, we have

$$\pi_c^{I_2*} - \pi_c^{DE*} = \frac{V^2[4\lambda(4k-t-8k\mu+2t\mu+8k\mu^2-t\mu^2)-(4k-t-8k\mu+2t\mu+20k\mu^2-t\mu^2)]}{16t(4k-t)} > 0$$

holds if $\frac{4k-t-8k\mu+2t\mu+20k\mu^2-t\mu^2}{4(4k-t-8k\mu+2t\mu+8k\mu^2-t\mu^2)} < \lambda < 1$.

And

$$\pi_o^{I_2*} - \pi_o^{DE*} = \frac{V^2[2\lambda(4k-t-8k\mu+2t\mu+8k\mu^2-t\mu^2)-(4k-t-8k\mu+2t\mu+12k\mu^2-t\mu^2)]}{8t(4k-t)} > 0$$

holds if $0 < \lambda < \frac{4k-t-8k\mu+2t\mu+12k\mu^2-t\mu^2}{2(4k-t-8k\mu+2t\mu+8k\mu^2-t\mu^2)}$.

Moreover, we can derive $\frac{4k-t-8k\mu+2t\mu+20k\mu^2-t\mu^2}{4(4k-t-8k\mu+2t\mu+8k\mu^2-t\mu^2)} < \frac{4k-t-8k\mu+2t\mu+12k\mu^2-t\mu^2}{2(4k-t-8k\mu+2t\mu+8k\mu^2-t\mu^2)}$ always holds in area A_1.

Denoted $\underline{\lambda} = \frac{4k-t-8k\mu+2t\mu+20k\mu^2-t\mu^2}{4(4k-t-8k\mu+2t\mu+8k\mu^2-t\mu^2)} = \frac{f-12\mu^2}{4f}$, and $\overline{\lambda} = \frac{4k-t-8k\mu+2t\mu+12k\mu^2-t\mu^2}{2(4k-t-8k\mu+2t\mu+8k\mu^2-t\mu^2)} = \frac{f-4\mu^2}{2f}$, where $f = \frac{t}{k}(\mu-1)^2 + 8\mu(1-\mu) - 4$, we have $\pi_c^{I_2*} > \pi_c^{DE*}$, $\pi_o^{I_2*} > \pi_o^{DE*}$, *and* $\Pi^{I_2*} = \Pi^{CE*}$ when $\underline{\lambda} < \lambda < \overline{\lambda}$ in area A_1.

Therefore, the contract-implementing Pareto zone arises with the constraint considiton $\underline{\lambda} < \lambda < \overline{\lambda}$, as area A shown in Fig. 7.

□

The proof of Proposition 6 is similar to Proposition 4, which is omitted.