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A NEW SYNERGISTIC STRATEGY FOR RANKING RESTAURANT LOCATIONS: A DECISION-MAKING APPROACH BASED ON THE HEXAGONAL FUZZY NUMBERS

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Abstract. This research addresses the problem of restaurant locations ranking with applications for a cosmopolitan big city like Kolkata, India. A restaurant selection is based on occasions, spending capability, environment, location, comfort, quality of the food etc. In this research paper an exhaustive set of factors and sub-factors is taken into consideration to select and rank restaurants situated at different locations in the city of Kolkata with a population of around fifteen million. The ranking of restaurants depends on complex, conflicting qualitative attributes. In the paper hexagonal fuzzy numbers (HFN) have been used to suitably depict the imprecise uncertain environment. HFN, its distance measure and defuzzification have been applied to deal with the hesitancy and impreciseness of the decision makers. Analytic hierarchy process (AHP) has been used as a Multi Criteria Decision Making (MCDM) tool to obtain factors and sub-factors weights. TOPSIS and COPRAS methods were used for ranking different restaurant locations. Using comparative analysis it is shown that HFN with the TOPSIS and COPRAS method gives better result than other fuzzy numbers. The sensitivity analysis portion also gives a direction for taking a suitable decision in different possible scenario.

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1. Introduction

In today's fast paced life, restaurant not only serves food they are a source of pleasant memories. Competition of restaurant business is increasing as there are more dining alternative. The results identified the important factor of judging the authenticity of restaurants. Further it was found that localness has scored more than authenticity and the two of them were linked [1]. The demand of restaurants are increasing throughout the globe. Customer expectations from restaurants are increasing and they choose restaurant based on their pref-

Keywords. HFN, MCDM, FAHP, FTOPSIS, FCOPRAS, Defuzzification, Decision maker (DM), Restaurant selection.

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erences. This study used mean end approach to identify customer's choice among the three different segment of restaurants *i.e.* casual restaurants, fast food restaurants and fine dining restaurants [2]. Potential Customers search information for selecting restaurant based on the desired service they want to avail. Information search survey was conducted in the city of Zaragoza in the north-east of Spain. Further the motivating factors for these activities were determined [3]. MCDM tools Fuzzy AHP (FAHP) and Fuzzy TOPSIS (FTOPSIS) were applied to evaluate the factors weight and ranking of sites [4].

There are several factors and sub-factors that influences on preferring a location while choosing restaurant. Thus Multi Criteria Decision Making (MCDM) can be applied to solve such problem. MCDM is an important branch of Operation Research. It transforms real life complex decision making to a logical conclusion. MCDM incorporates numerous qualitative, contradicting and/or quantitative factors and sub-factors to a logical conclusion that needs a consensus [5]. MCDM is most extensively used decision methodologies in several areas such as business, energy, economy, environment, production, sustainability and so on [4,6]. MCDM techniques augment the standard of decision making. There are several studies [7–9] that have exhibited the vitality of this space.

In real world situations uncertainty, vagueness, indeterminacy are prevalent. Fuzzy concept is suitable to express such situations. In this paper Fuzzy MCDM methods are used. Many researchers have used different uncertain environment based on the suitability in real life problems. The authors name, application area, number of factors and sub-factors, uncertain environmental category and MCDM techniques are described in Table 1.

Kolkata, the city under our consideration has seven locations where a cluster of restaurants are present. For different location, the attributes and qualities differs but in a particular location restaurants are homogeneous. For example China town has a cluster of restaurants providing Chinese food of similar quality. An individual willing to avail restaurant food first need to decide which location he/she will travel to have access to a restaurant. This research helps in location selection and ranking of restaurant locations. This paper attempts at the following:

(1) Selection of different locations for restaurant selection in the city of Kolkata and ranking them on the basis of multiple conflicting factors and sub-factors taking into account the evaluation of multiple decision makers (DMs).
(2) Identification of different factors and sub-factors influencing location selection for restaurant. Developing a comparison matrix using hexagonal fuzzy numbers (HFN) suitable for analytic hierarchy process (AHP). Obtaining Fuzzy HFN weight for all the factors and sub-factors.
(3) Ranking of different locations using Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) and Fuzzy Complex Proportional Assessment (Fuzzy COPRAS or FCOPRAS).
(4) To carry out sensitivity analysis and comparative analysis for checking reliability and sensitivity of our model.

A list of factors and sub-factors used in literature have been depicted in Table 2.

Various researchers integrated fuzzy numbers with MCDM tools like AHP, TOPSIS, COPRAS. In this research, Hexagonal Fuzzy Numbers with MCDM approach has been used for ranking restaurant locations and a detailed sensitivity analysis has been carried out which is a novelty of this research. Figure 1 describes the successive steps followed in this study.

The paper is arranged in the following way: Section 2 briefly describes the concept of fuzzy set and fuzzy numbers, α-cut of fuzzy number, HFN and respective arithmetic operations. Distance measure, defuzzification formulae are also represented in this section. Section 3 describe the MCDM technique AHP, Hexagonal fuzzy weight, Fuzzy TOPSIS and Fuzzy COPRAS. Description of factors and sub-factors, alternatives and numerical application are covered in Sections 4 and 5 respectively. Section 6 represents the Numerical calculation of DM‘s data. Section 7 portray about Comparative analysis. Section 8 represents sensitivity analysis. Section 9 discusses the managerial insights. Finally, conclusion and future research scope are covered in Section 10.

TABLE 1. Literature on relevant Fuzzy MCDM problems showing number of factors and sub-factors, type environment and MCDM techniques used.

Authors of the article	Application area	Numbers of factor and sub-factors	Environment category	MCDM techniques
Ghorui *et al.* [4]	Shopping mall site selection problem	7 Factors and 17 sub-factors	Triangular fuzzy numbers (TFN)	Fuzzy AHP, Fuzzy TOPSIS
Sarkar *et al.* [6]	Selecting best family car	11 Factors	Triangular fuzzy numbers (TFN)	FTOPSIS, FMARCOS, FVIKOR
Biswas *et al.* [10]	Medical representative recruitment system	4 Factors	Single valued trapezoidal neutrosophic numbers	Fuzzy TOPSIS
Ghosh *et al.* [11]	Site selection of electric vehicle charging station	4 Factors and 13 sub-factors	Hexagonal fuzzy numbers (HNF)	FAHP, FTOPSIS, FCOPRAS
Biswas *et al.* [12]	Most suitable tablet selection	6 Factors	Single valued neutrosophic number	TOPSIS
Hezam *et al.* [13]	COVID-19 vaccination priority for different groups	4 Factors and 15 sub-factors	Triangular neutrosophic fuzzy set	AHP, TOPSIS
Tanoumand *et al.* [14]	Selecting cloud computing	6 Factors	Triangular fuzzy numbers (TFN)	FAHP
Ali *et al.* [15]	Measuring the possibility of cloud adoption for software tasting	10 Factors and 70 sub-factors	Triangular fuzzy numbers (TFN)	FMCDM
Stankovi *et al.* [16]	Road traffic risk analysis	6 Factors	Triangular fuzzy numbers (TFN)	Fuzzy MARCOS
Tzeng *et al.* [17]	Restaurant location in Taipei	5 Factors and 11 sub-factors	Linguistic variable in decimal numbers	AHP and VIKOR
Timor *et al.* [18]	Fast-food restaurant site selection	7 Factors and 36 sub-factors	Linguistic variable in decimal numbers	AHP
Karasan *et al.* [19]	Residential construction site selection	4 Factors and 14 sub-factors	Hesitant fuzzy numbers	CODAS
Moatya *et al.* [20]	A site selection decision making process	6 Factors and 26 sub-factors	Linguistic variable in decimal numbers	AHP and TOPSIS
Sriniketha *et al.* [21]	Plant location selection	4 Factors and 13 sub-factors	Linguistic variable in decimal numbers	AHP and PROMETHEE
Chatterjee *et al.* [22]	Hospital location selection	3 Factors and 11 sub-factors	Linguistic variable in decimal numbers	AHP and multi factor evaluation
Sun [23]	Site selection for EVCSs	4 Factors and 19 sub-factors	Linguistic variable in decimal numbers	AHP and TOPSIS
Ramu *et al.* [24]	Airport site selection	14 Factors	Fuzzy numbers	AHP and FAHP
Wibisono *et al.* [25]	Selection of cafe location	5 Factors and 16 sub-factors	Priority analysis through cluster matrix	AHP
Chen *et al.* [26]	Sustainable selection of a teahouse location	11 Factors	Molt-Karlo simulation method	WASPAS and EDAS
In this paper	Location selection for a restaurant	5 Factors and 17 sub-factors	Hexagonal fuzzy numbers (HFN)	FAHP, FTOPSIS and FCOPRAS

2. Preliminaries

2.1. Fuzzy set

Fuzzy sets are the set whose every element has a degree of membership value. The fuzzy concept was first introduced by Zadeh [29, 30]. There are several application of Fuzzy set theory in different domains like Differential equation [31, 32], linear programming problem [33–35], non-linear programming problem [36, 37], decision making problem [38, 63–66] etc.

TABLE 2. Factors and sub-factors for restaurant location selection, source: [4,17,18,27,28].

Factors	Sub-factors
Food	* Price of food * Food quality * Variety of food * Taxes * Site and development costs
Service	* Service quality * Behavior of staff * Quick service * Pleasant physical environment * Customer satisfaction * Staff members are friendly and helpful
Image	* Past experience * Word of mouth * Online review * Brand reputation * Sales promotion * Store size * Building's condition
Location	* Area * Parking Capacity * Safety/Crime Rates * Clean and comfortable * Noise and air pollution free * Convenience of garbage disposal * Residential areas * Industrial areas * Shopping center * Sports and cultural areas * Business areas * Educational areas * Distance to nearest highway
Occasion	* Quick meal/convenience * Social occasion * Business necessity * Celebration * Development of nearby areas * Future development

Definition 1 (Fuzzy set)**.** Let Φ be a set (finite or infinite). Let $\tilde{S}$ be a set contained in Φ. A function $\mu_{\tilde{S}} : \Phi \to [0,1]$ is called a membership function. If $x \in \Phi$ then $\mu_{\tilde{S}}(x)$ the degree of membership of x in Φ.

Definition 2 (α-cut of a fuzzy set)**.** The α-cut or α-level set of the fuzzy set $\tilde{S}$ of Φ is a classical set $\tilde{S}_\alpha$ which contains all members of Φ such that membership values of ϕ ($\in \tilde{S}$) bigger than or equal to α *i.e.* $\tilde{S}_\alpha = \{\phi : \mu_{\tilde{S}}(\phi) \geq \alpha, \phi \in \Phi\}$, $\alpha \in [0,1]$.

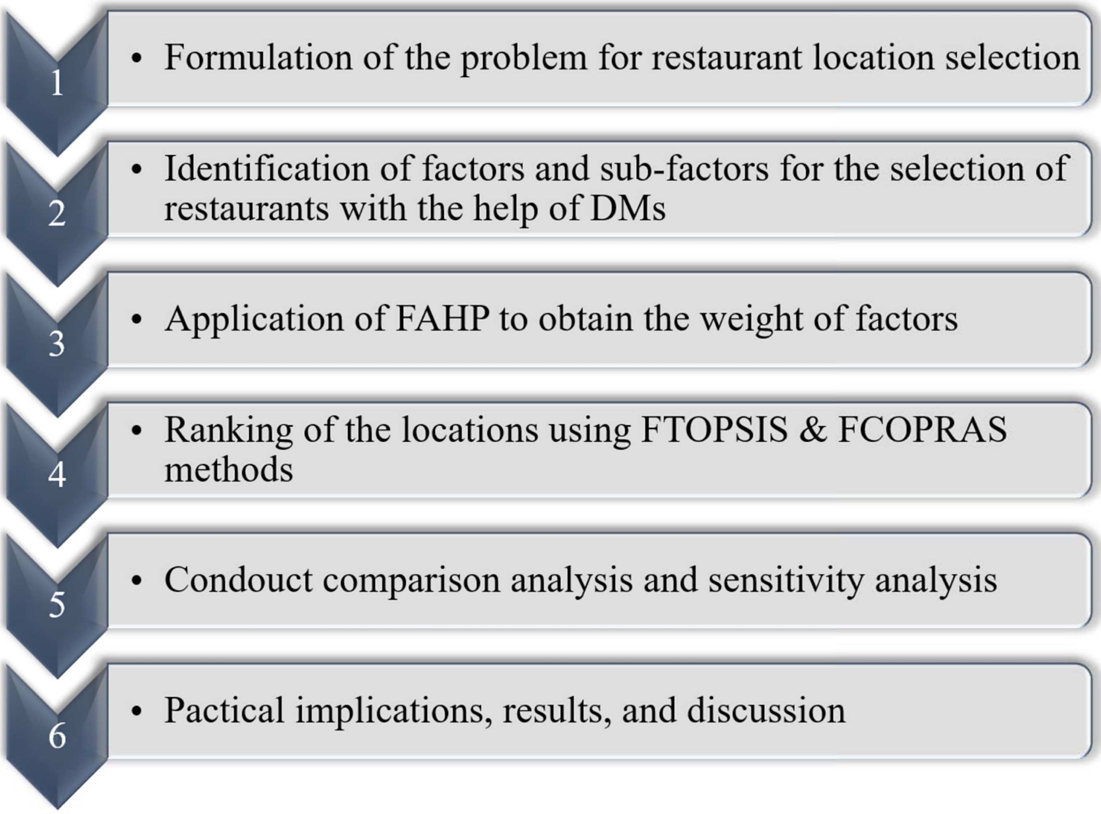

FIGURE 1. The structural framework of this paper.

Definition 3 (Strong α-cut of a fuzzy set)**.** The strong α-cut or strong α-level set of the fuzzy set $\tilde{S}$ of Φ is a classical set $\tilde{S}_{\tilde{\alpha}}$ which contains all members of Φ such that membership values of ϕ $(\in \tilde{S})$ bigger than α *i.e.* $\tilde{S}_{\tilde{\alpha}} = \{\phi : \mu_{\tilde{S}}(\phi) > \alpha, \phi \in \Phi\}$, $\alpha \in [0,1]$.

2.2. Concept of fuzzy numbers:

An extension of real number set $\mathbb{R}$ with membership function is called fuzzy number [39], *i.e.* fuzzy numbers are connected set of possible values with the membership function $\tilde{\lambda}(\in [0,1])$. The weight of the element is called membership value and the function through which the weight is assigned is called membership function.

Definition 4 (Normal fuzzy set [40])**.** A fuzzy set is called normal fuzzy set if its core (*i.e.*, the α-cut set along $\alpha = 1$) is non-empty.

Definition 5 (Convex fuzzy set [41])**.** A fuzzy set $\tilde{S}$ is said to be convex fuzzy set, if $\tilde{S}(\gamma x + (1-\gamma)y) \geq \min\{\tilde{S}(x), \tilde{S}(y)\}$ for $x, y \in \Phi$ and $\gamma \in (0,1)$.

2.3. Hexagonal fuzzy number

There are many research papers published on fuzzy numbers, that developed and used fuzzy numbers [24]. Depending on the need of the problem, researchers can use triangular fuzzy numbers (TFN) [4, 6, 14–16], trapezoidal fuzzy numbers (TrFN) [42], pentagonal fuzzy numbers (PFN) [43], hexagonal fuzzy numbers (HFN) [44] and hesitant fuzzy numbers [19]. In this paper hexagonal fuzzy numbers have been used.

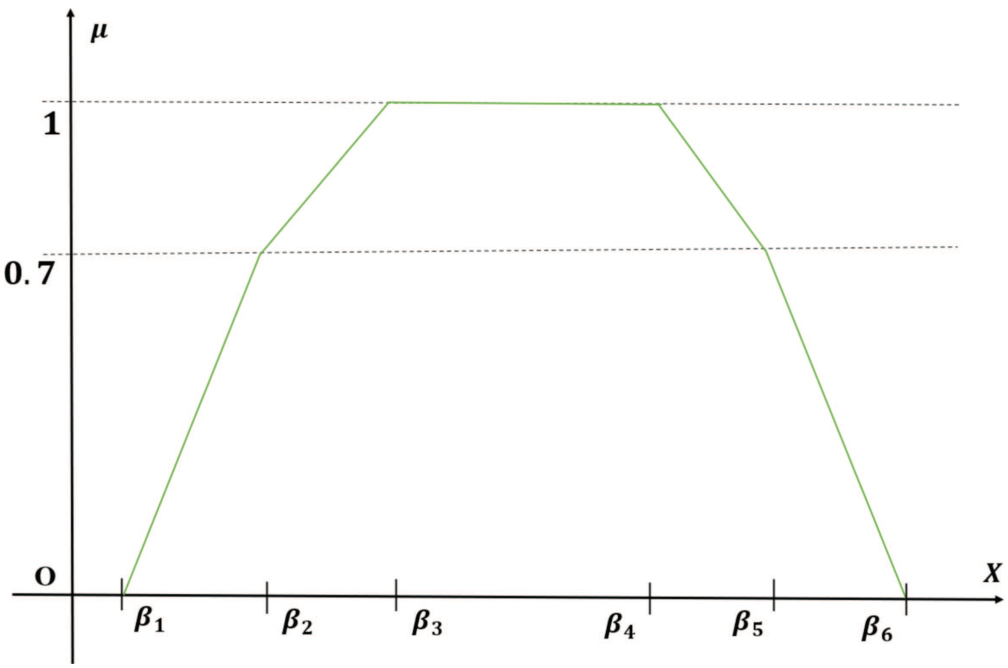

FIGURE 2. Geometric representation of hexagonal fuzzy number.

Definition 6 (Hexagonal fuzzy numbers (HFN) [44]). A fuzzy number $\tilde{H}(\phi) = \left\{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}(\phi)}\right\}$ is said to be hexagonal fuzzy number (HFN) where $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$ are real numbers with ascending order, $0 \le r, s \le 1$ and its membership function $\mu_{\tilde{H}}(\phi)$ is defined as

$$\mu_{\tilde{H}}(\phi) = \begin{cases} 0 & \text{if } \phi \le \beta_1 \\ r\frac{(\phi-\beta_1)}{(\beta_2-\beta_1)} & \text{if } \beta_1 \le \phi \le \beta_2 \\ r + (1-r)\frac{(\phi-\beta_2)}{(\beta_3-\beta_2)} & \text{if } \beta_2 \le \phi \le \beta_3 \\ 1 & \text{if } \beta_3 \le \phi \le \beta_4 \\ s + (1-s)\frac{(\beta_5-\phi)}{(\beta_5-\beta_4)} & \text{if } \beta_4 \le \phi \le \beta_5 \\ s\frac{(\beta_6-\phi)}{(\beta_6-\beta_5)} & \text{if } \beta_5 \le \phi \le \beta_6 \\ 0 & \text{if } \beta_6 \le \phi. \end{cases} \tag{1}$$

Here, $\beta_1 \le \beta_2 \le \beta_3 \le \beta_4 \le \beta_5 \le \beta_6$ with all β_i $(i = 1, 2, \dots, 6)$ real constants and $0 < r, s < 1$.

Figure 2, represents a geometric representation on a particular hexagonal fuzzy number $\tilde{H}(\phi)$ where fuzzy set is

$$\tilde{H}(\phi) = \left\{(0.5, 1, 2, 4, 5.5, 6.5; 0.5, 0.6); \mu_{\tilde{H}(\phi)}\right\}.$$

Definition 7. A fuzzy number $\tilde{H} = \left\{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}(\phi)}\right\}$ is a hexagonal fuzzy number if it's membership function $\mu_{\tilde{H}}$ satisfies following conditions $\mu_{\tilde{H}} : \mathbb{R} \to I = [0, 1]$, and defined as

(1) $\mu_{\tilde{H}}$ is upper semi-continuous;
(2) There exists real numbers β_1, β_2, β_3, β_4, β_5 and β_6 such that $\beta_1 \le \beta_2 \le \beta_3 \le \beta_4 \le \beta_5 \le \beta_6$ and
   (a) $\mu_{\tilde{H}}(\phi)$ is monotonically increasing on $[\beta_1, \beta_2]$ and $[\beta_2, \beta_3]$,

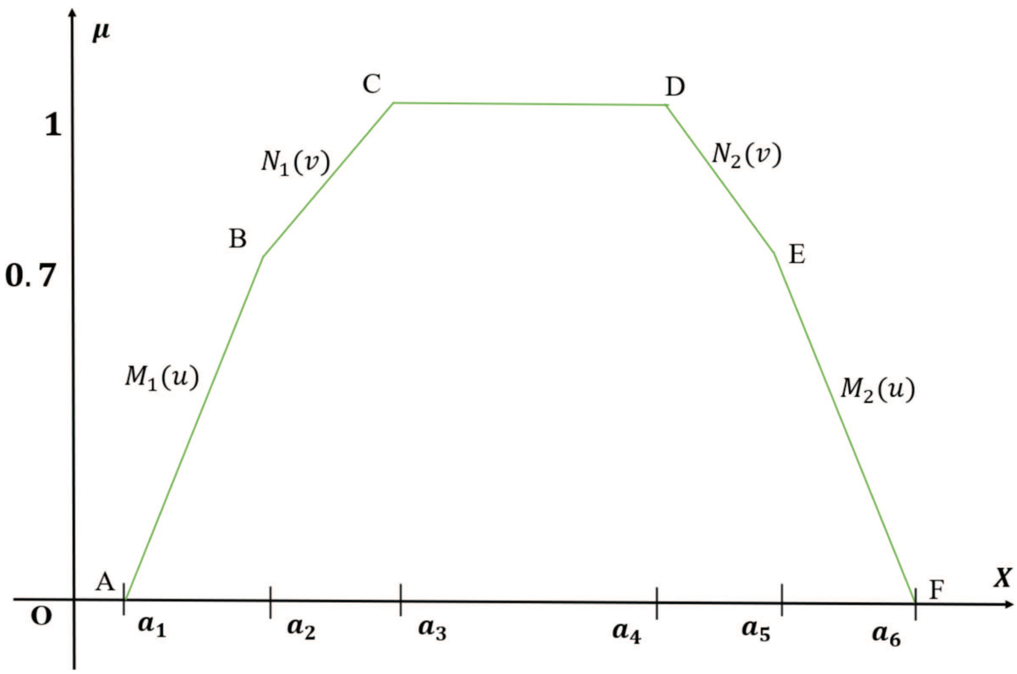

FIGURE 3. Geometric description of hexagonal fuzzy number $\tilde{H}$ where membership function $\mu_{\tilde{H}}(\phi)$ is bounded continuous and $\phi \in [0, 1]$.

(b) $\mu_{\tilde{H}}(\phi)$ is monotonically decreasing on $[\beta_4, \beta_5]$ and $[\beta_5, \beta_6]$,
(c) $\mu_{\tilde{H}}(\phi) = 1$ when $\beta_3 \leq \phi \leq \beta_4$,
(d) $\mu_{\tilde{H}}(\phi) = 0$, if ϕ lies outside the interval $[\beta_1, \beta_6]$.

2.4. α-cut of HFN

The alpha cut of $\tilde{H}$ on fuzzy set is denoted by $\tilde{H}_\alpha$ and constructed by the elements of $\tilde{H}$ whose membership value is not less than α. α-cut concept is briefly described in details [45] and [46].

Definition 8. Let $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6); \mu_{\tilde{H}}\}$ be a hexagonal fuzzy number and the membership function $\mu_{\tilde{H}}$, then α-cut set of $\tilde{H}$ is

$$\tilde{H}_\alpha = \{\phi \in \Phi | \mu_{\tilde{H}}(\phi) \geq \alpha\} = \begin{cases} [M_1(\alpha), M_2(\alpha)]; & \text{for } \alpha \in [0, 0.5] \\ [N_1(\alpha), N_2(\alpha)]; & \text{for } \alpha \in [0.5, 1] \end{cases} \tag{2}$$

Note 1. *Here, we consider membership function $\mu_{\tilde{H}}(\phi)$ are continuous function. Four functions of $\tilde{H}_\alpha$ are $M_1(u), N_1(v), N_2(v), M_2(u)$ satisfies the properties:*

(1) The non decreasing continuous bounded function $M_1(u)$ expand in $[0, 0.5]$.
(2) The non decreasing continuous bounded function $N_1(u)$ expand in $[0.5, 1]$.
(3) The non increasing continuous bounded function $N_2(u)$ expand in $[0.5, 1]$.
(4) The non increasing continuous bounded function $M_2(u)$ expand in $[0, 0.5]$.

A bounded continuous membership function $\mu_{\tilde{H}}(\phi)$ of HFN $\tilde{H}$ is graphically represented in Figure 3. Also the function $M_1(u), N_1(v), N_2(v), M_2(u)$ are described.

If we get α-cut operations on $\tilde{H}$ shall be obtained as follows for $\alpha \in [0,1]$.

Consider, $M_1(x) = r\frac{\phi-\beta_1}{\beta_2-\beta_1} = \alpha$

or,

$$\phi = \frac{\alpha}{r}(\beta_2 - \beta_1) + \beta_1. \tag{3}$$

Similarly, $M_2(x) = s\frac{\beta_6-\phi}{\beta_6-\beta_5} = \alpha$

or, $\beta_6 - \phi = \frac{\alpha}{s}(\beta_6 - \beta_5)$

or,

$$\phi = -\frac{\alpha}{s}(\beta_6 - \beta_5) + \beta_6. \tag{4}$$

This implies $[M_1(\alpha), M_2(\alpha] = \left[\frac{\alpha}{r}(\beta_2 - \beta_1) + \beta_1, -\frac{\alpha}{s}(\beta_6 - \beta_5) + \beta_6\right]$ when $\alpha \in [0, 0.5]$.

Again, $N_1(\phi) = r + (1-r)\frac{\phi-\beta_2}{\beta_3-\beta_2}$

or, $\phi - \beta_2 = \frac{\alpha-r}{1-r}(\beta_3 - \beta_2)$

or,

$$\phi = \frac{\alpha - r}{1 - r}(\beta_3 - \beta_2) + \beta_2. \tag{5}$$

Similarly, $N_2(\phi) = s + (1-s)\frac{\beta_5-\phi}{\beta_5-\beta_4} = \alpha$

or, $\beta_5 - \phi = \frac{\alpha-s}{1-s}(\beta_5 - \beta_4)$

or,

$$\phi = -\frac{\alpha - s}{1 - s}(\beta_5 - \beta_4) + \beta_5. \tag{6}$$

This implies $[N_1(\alpha), N_2(\alpha] = \left[\frac{\alpha-r}{1-r}(\beta_3 - \beta_2) + \beta_2, -\frac{\alpha-s}{1-s}(\beta_5 - \beta_4) + \beta_5\right]$ when $\alpha \in [0.5, 1]$.

Hence,

$$\tilde{H}_\alpha = \begin{cases} \left[\frac{\alpha}{r}(\beta_2 - \beta_1) + \beta_1, -\frac{\alpha}{s}(\beta_6 - \beta_5) + \beta_6\right] & \text{for } \alpha \in [0, 0.5] \\ \left[\frac{\alpha-r}{1-r}(\beta_3 - \beta_2) + \beta_2, -\frac{\alpha-s}{1-s}(\beta_5 - \beta_4) + \beta_5\right] & \text{for } \alpha \in [0.5, 1]. \end{cases} \tag{7}$$

If we consider $r = 0.5$ and $s = 0.5$, then α-cut of $\tilde{H}$ becomes

$$\tilde{H}_\alpha = \begin{cases} [2\alpha(\beta_2 - \beta_1) + \beta_1, -2\alpha(\beta_6 - \beta_5) + \beta_6] & \text{for } \alpha \in [0, 0.5] \\ [(2\alpha - 1)(\beta_3 - \beta_2) + \beta_2, -(2\alpha - 1)(\beta_5 - \beta_4) + \beta_5] & \text{for } \alpha \in [0.5, 1]. \end{cases} \tag{8}$$

Example 1. Let us consider $\tilde{H} = \{(2,3,4,7,8,9); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(4,6,8,14,16,18); \mu_{\tilde{G}}\}$ are two hexagonal fuzzy numbers and their membership functions are continuous. Then α-cut of two fuzzy numbers are $\tilde{H}$ and $\tilde{G}$ are respectively $\tilde{H}_\alpha = [2\alpha + 2, -2\alpha + 9]$ and $\tilde{G}_\alpha = [4\alpha + 4, -4\alpha + 18]$ where $\alpha \in [0,1]$.

2.5. α-cut operations

Let us consider $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6); \mu_{\tilde{G}}\}$ are two Hexagonal fuzzy numbers and membership functions of $\tilde{H}$ and $\tilde{G}$ are continuous. Also let $\tilde{H}_\alpha$ and $\tilde{G}_\alpha$ are two α-cut of HFN $\tilde{H}$ and $\tilde{G}$ respectively. α-cut operations are described on [45–47].

Then *addition* of two α-cuts $\tilde{H}_\alpha$ and $\tilde{G}_\alpha$ is $\tilde{H}_\alpha + \tilde{G}_\alpha =$

$$\begin{cases} [2\alpha(\beta_2 - \beta_1) + \beta_1, -2\alpha(\beta_6 - \beta_5) + \beta_6] \\ \quad +[2\alpha(\zeta_2 - \zeta_1) + \zeta_1, -2\alpha(\zeta_6 - \zeta_5) + \zeta_6] & \text{for } \alpha \in [0, 0.5] \\ [(2\alpha - 1)(\beta_3 - \beta_2) + \beta_2, -(2\alpha - 1)(\beta_5 - \beta_4) + \beta_5] \\ \quad +[(2\alpha - 1)(\zeta_3 - \zeta_2) + \zeta_2, -(2\alpha - 1)(\zeta_5 - \zeta_4) + \zeta_5] & \text{for } \alpha \in [0.5, 1] \end{cases} \tag{9}$$

We again consider $\tilde{H} = \{(2,3,4,7,8,9);\mu_{\tilde{H}}\}$ and $\tilde{G} = \{(4,6,8,14,16,18);\mu_{\tilde{G}}\}$ are two Hexagonal fuzzy numbers with membership functions are continuous. Then addition of α-cut of two fuzzy numbers is $\tilde{H}_\alpha + \tilde{G}_\alpha = [2\alpha + 2, -2\alpha+9] + [4\alpha+4, -4\alpha+18] = [6\alpha+6, -6\alpha+27]$ where $\alpha \in [0,1]$.

If $\alpha = 0$ then $\tilde{H}_0 + \tilde{G}_0 = [6,27]$, and if $\alpha = 0.5$ then $\tilde{H}_{0.5} + \tilde{G}_{0.5} = [9,24]$ and if $\alpha = 1$ then $\tilde{H}_1 + \tilde{G}_1 = [12,23]$. Hence $\tilde{H}_\alpha + \tilde{G}_\alpha = \{(6,9,12,23,24,27);\mu_{\tilde{H}} + \mu_{\tilde{G}}\}$, this imply that all points coincide with the sum of two hexagonal fuzzy numbers.

Then *subtraction* of two α-cuts $\tilde{H}_\alpha$ and $\tilde{G}_\alpha$ is $\tilde{H}_\alpha - \tilde{G}_\alpha =$

$$\begin{cases} [2\alpha(\beta_2-\beta_1)+\beta_1, -2\alpha(\beta_6-\beta_5)+\beta_6] \\ \quad -[2\alpha(\zeta_2-\zeta_1)+\zeta_1, -2\alpha(\zeta_6-\zeta_5)+\zeta_6] & \text{if } \alpha \in [0,0.5] \\ [(2\alpha-1)(\beta_3-\beta_2)+\beta_2, -(2\alpha-1)(\beta_5-\beta_4)+\beta_5] \\ \quad -[(2\alpha-1)(\zeta_3-\zeta_2)+\zeta_2, -(2\alpha-1)(\zeta_5-\zeta_4)+\zeta_5] & \text{if } \alpha \in [0.5,1]. \end{cases} \tag{10}$$

For same example $\tilde{H} = \{(2,3,4,7,8,9);\mu_{\tilde{H}}\}$ and $\tilde{G} = \{(4,6,8,14,16,18);\mu_{\tilde{G}}\}$ are two Hexagonal fuzzy numbers with membership functions are continuous. Then subtraction of α-cut of two fuzzy numbers is $\tilde{H}_\alpha - \tilde{G}_\alpha = [2\alpha+2, -2\alpha+9] - [4\alpha+4, -4\alpha+18] = [-2\alpha-2, 2\alpha-9]$ where $\alpha \in [0,1]$.

If $\alpha = 0$ then $\tilde{H}_0 - \tilde{G}_0 = [-2,-9]$, and if $\alpha = 0.5$ then $\tilde{H}_{0.5} - \tilde{G}_{0.5} = [-3-8]$ and if $\alpha = 1$ then $\tilde{H}_1 - \tilde{G}_1 = [-4,-7]$. Hence $\tilde{H}_\alpha - \tilde{G}_\alpha = \{(-2,-3,-4,-7,-8,-9);\mu_{\tilde{H}} - \mu_{\tilde{G}}\}$, this imply that all value are same with the subtraction of two HFNs.

Then *multiplication* of two α-cuts $\tilde{H}_\alpha$ and $\tilde{G}_\alpha$ is $\tilde{H}_\alpha \times \tilde{G}_\alpha =$

$$\begin{cases} [2\alpha(\beta_2-\beta_1)+\beta_1, -2\alpha(\beta_6-\beta_5)+\beta_6] \\ \quad \times[2\alpha(\zeta_2-\zeta_1)+\zeta_1, -2\alpha(\zeta_6-\zeta_5)+\zeta_6] & \text{if } \alpha \in [0,0.5] \\ [(2\alpha-1)(\beta_3-\beta_2)+\beta_2, -(2\alpha-1)(\beta_5-\beta_4)+\beta_5] \\ \quad \times[(2\alpha-1)(\zeta_3-\zeta_2)+\zeta_2, -(2\alpha-1)(\zeta_5-\zeta_4)+\zeta_5] & \text{if } \alpha \in [0.5,1]. \end{cases} \tag{11}$$

We again consider $\tilde{H} = \{(2,3,4,7,8,9);\mu_{\tilde{H}}\}$ and $\tilde{G} = \{(4,6,8,14,16,18);\mu_{\tilde{G}}\}$ are two Hexagonal fuzzy numbers with membership functions are continuous. Then multiplication of α-cut of two fuzzy numbers is $\tilde{H}_\alpha \times \tilde{G}_\alpha = [2\alpha+2, -2\alpha+9] \times [4\alpha+4, -4\alpha+18] = [(2\alpha+2)\times(-4\alpha+4), (-2\alpha+9)\times(-4\alpha+18)]$ where $\alpha \in [0,1]$.

If $\alpha = 0$ then $\tilde{H}_0 \times \tilde{G}_0 = [6,162]$, and if $\alpha = 0.5$ then $\tilde{H}_{0.5} \times \tilde{G}_{0.5} = [18,128]$ and if $\alpha = 1$ then $\tilde{H}_1 \times \tilde{G}_1 = [32,98]$. Hence $\tilde{H}_\alpha \times \tilde{G}_\alpha = \{(6,18,32,98,128,162);\mu_{\tilde{H}} \times \mu_{\tilde{g}}\}$, this imply that all points coincide with the multiplication of two hexagonal fuzzy numbers.

2.6. Defuzzification methods of HFN

Defuzzification is the procedure to produce a quantifiable result in crisp logic from fuzzy set and it's membership function. It is generally needed in fuzzy control systems. Defuzzification and fuzzification are opposite process to convert fuzzy set to crisp set and vice versa respectively. There exist several defuzzification methods, but the common and useful methods described as follows:

2.6.1. Centroid-based method (CBM) of a hexagonal fuzzy number:

Let $\tilde{H} = \{(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5,\beta_6);\mu_{\tilde{H}}\}$ is a hexagonal fuzzy number with $\mu_{\tilde{H}} = 0.5$. This method described and applied in the paper Ghosh *et al.* [11]. HFN is divided into four sub region; two triangle and two trapezium. Furthermore, one trapezium can be divided into three sub parts; two triangle and one rectangle. At the end, summation of all triangles and rectangles are excuted to get the centroid formulae of HFN. The method is illustrated below

- The centroid of triangle $\triangle$ AIB is $\left(\frac{\beta_1+\beta_2+\beta_3}{3}, \frac{r}{3}\right)$
- The centroid of triangle $\triangle$ FEJ is $\left(\frac{\beta_4+\beta_5+\beta_6}{3}, \frac{r}{3}\right)$

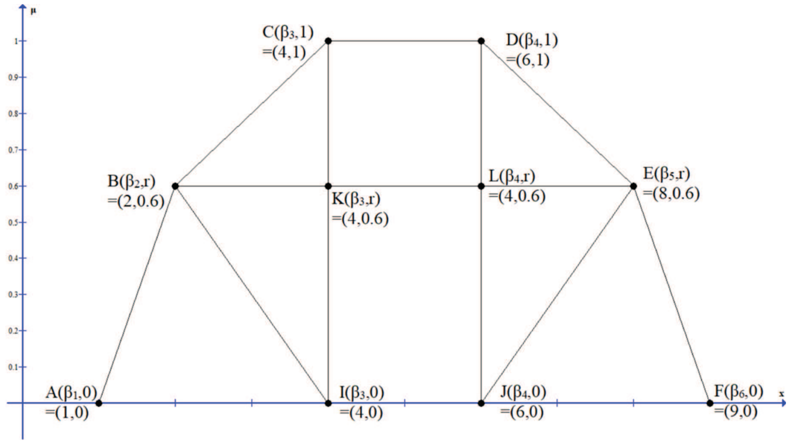

FIGURE 4. $\tilde{H}$ is a hexagonal fuzzy number (HFN), divided into six triangles and two rectangles is visualise in this picture.

– The centroid of trapezium □ IJEB is
  - The centroid of triangle △ IKB is $\left(\frac{\beta_2+2\beta_3}{3}, \frac{2r}{3}\right)$
  - The centroid of triangle △ JEL is $\left(\frac{2\beta_4+\beta_5}{3}, \frac{2r}{3}\right)$
  - The centroid of rectangle □ IJLK is $\left(\frac{\beta_3+\beta_4}{2}, \frac{r}{2}\right)$

  Therefore, the trapezium □ IJEB is $\left(\frac{2\beta_2+7\beta_3+7\beta_4+2\beta_5}{6}, \frac{11r}{6}\right)$

– The centroid of trapezium □ BEDC is $\left(\frac{2\beta_2+7\beta_3+7\beta_4+2\beta_5}{6}, \frac{11r+7}{6}\right)$ by similar way.

Therefore, the centroid of this HFN is

$$\mathrm{CBM}(\tilde{H}) = \left(\frac{3\beta_1+3\beta_2+10\beta_3+10\beta_4+5\beta_5+3\beta_6}{34}, \frac{26r+7}{6}\right). \tag{12}$$

This is the defuzzified value of the hexagonal fuzzy number $\tilde{H}$ and geometrically represents at Figure 4.

2.7. Arithmetic operation on HFN

Arithmetic operation on hexagonal fuzzy numbers (HFN) plays significant role in the theory of hexagonal fuzzy numbers and it's application [48]. Therefore it is an important branch of research. Different operations exist in the field of hexagonal fuzzy number (HFN) and it is very useful and popular. In this section, we define some important operations such as addition, subtraction, multiplication and division on HFN.

Definition 9 (Addition of HFN)**.** If $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6; t, u); \mu_{\tilde{G}}\}$ are two hexagonal fuzzy number (HFN), then the addition of $\tilde{H}$ and $\tilde{G}$ is define as $\tilde{H} \oplus \tilde{G} = \left\{(\beta_1+\zeta_1, \beta_2+\zeta_2, \beta_3+\zeta_3, \beta_4+\zeta_4, \beta_5+\zeta_5, \beta_6+\zeta_6; v, w); \mu_{\tilde{H}\oplus\tilde{G}}\right\}$ whose membership function is given by $\mu_{\tilde{H}\oplus\tilde{G}}(\phi) =$

$$\begin{cases} 0 & \text{if } \phi \in (-\infty, \beta_1+\zeta_1] \\ v\left(\frac{\phi-(\beta_1+\zeta_1)}{(\beta_2+\zeta_2)-(\beta_1+\zeta_1)}\right) & \text{if } \phi \in [(\beta_1+\zeta_1), (\beta_2+\zeta_2)] \\ v+(1-v)\left(\frac{\phi-(\beta_2+\zeta_2)}{(\beta_3+\zeta_3)-(\beta_2+\zeta_2)}\right) & \text{if } \phi \in [(\beta_2+\zeta_2), (\beta_3+\zeta_3)] \\ 1 & \text{if } \phi \in [(\beta_3+\zeta_3), (\beta_4+\zeta_4)] \\ w+(1-w)\left(\frac{(\beta_5+\zeta_5)-x}{(\beta_5+\zeta_5)-(\beta_4+\zeta_4)}\right) & \text{if } \phi \in [(\beta_4+\zeta_4), (\beta_5+\zeta_5)] \\ w\left(\frac{(\beta_6+\zeta_6)-\phi}{(\beta_6+\zeta_6)-(\beta_5+\zeta_5)}\right) & \text{if } \phi \in [(\beta_5+\zeta_5), (\beta_6+\zeta_6)] \\ 0 & \text{if } \phi \in [(\beta_6+\zeta_6), \infty) \end{cases} \tag{13}$$

where $v = r + t - rt$, $w = s + u - su$ and $\mu_{\tilde{H}\oplus\tilde{G}} = \mu_{\tilde{H}} + \mu_{\tilde{G}} - \mu_{\tilde{H}}\mu_{\tilde{G}}$.

Definition 10 (Subtraction of HFN)**.** If $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6; t, u); \mu_{\tilde{G}}\}$ are two hexagonal fuzzy number (HFN), then the subtraction of $\tilde{H}$ and $\tilde{G}$ is define as $\tilde{H} \ominus \tilde{G} = \left\{(\beta_1-\zeta_6, \beta_2-\zeta_5, \beta_3-\zeta_4, \beta_4-\zeta_3, \beta_5-\zeta_2, \beta_6-\zeta_1; v, w); \mu_{\tilde{H}\ominus\tilde{G}}\right\}$ whose membership function is given by $\mu_{\tilde{H}\ominus\tilde{G}}(\phi) =$

$$\begin{cases} 0 & \text{if } \phi \in (-\infty, \beta_1-\zeta_6] \\ v\left(\frac{\phi-(\beta_1-\zeta_6)}{(\beta_2-\zeta_5)-(\beta_1-\zeta_6)}\right) & \text{if } \phi \in [(\beta_1-\zeta_6), (\beta_2-\zeta_5)] \\ v+(1-v)\left(\frac{\phi-(\beta_2-\zeta_5)}{(\beta_3-\zeta_4)-(\beta_2-\zeta_5)}\right) & \text{if } \phi \in [(\beta_2-\zeta_5), (\beta_3-\zeta_4)] \\ 1 & \text{if } \phi \in [(\beta_3-\zeta_4), (\beta_4-\zeta_3)] \\ w+(1-w)\left(\frac{(\beta_5-\zeta_2)-\phi}{(\beta_5-\zeta_2)-(\beta_4-\zeta_3)}\right) & \text{if } \phi \in [(\beta_4-\zeta_3), (\beta_5-\zeta_2)] \\ w\left(\frac{(\beta_6-\zeta_1)-\phi}{(\beta_6-\zeta_1)-(\beta_5-\zeta_2)}\right) & \text{if } \phi \in [(\beta_5-\zeta_2), (\beta_6-\zeta_1)] \\ 0 & \text{if } \phi \in [(\beta_6-\zeta_1), \infty) \end{cases} \tag{14}$$

where $v = r + u - ru$, $w = s + t - st$ and $\mu_{\tilde{H}\ominus\tilde{G}} = \mu_{\tilde{H}} + \mu_{\tilde{G}} - \mu_{\tilde{H}}\mu_{\tilde{G}}$.

Definition 11 (Scalar multiplication of HFN)**.** If $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}}\}$ is a hexagonal fuzzy number (HFN) and k is taken as positive real constant, then the scalar multiplication of $\tilde{H}$ by k is define by

$$k\tilde{H} = \{(k\beta_1, k\beta_2, k\beta_3, k\beta_4, k\beta_5, k\beta_6; r, s); \mu_{\tilde{H}}\}. \tag{15}$$

Scalar multiplication also true for negative real constant in similar way.

Definition 12 (Multiplication of HFN)**.** If $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6; t, u); \mu_{\tilde{G}}\}$ are two hexagonal fuzzy number (HFN), then the multiplication of $\tilde{H}$ and

$\tilde{G}$ is define as $\tilde{H} \otimes \tilde{G}(\phi) =$

$$
\begin{cases}
\{(\min(\beta_1\zeta_6, \beta_6\zeta_6), \min(\beta_2\zeta_5, \beta_5\zeta_5), \min(\beta_3\zeta_4, \beta_4\zeta_4), \max(\beta_4\zeta_3, \beta_3\zeta_3), \\
\max(\beta_5\zeta_2, \beta_2\zeta_2), \max(\beta_6\zeta_1, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_6 \leq 0 \\
\{(\min(\beta_1\zeta_6, \beta_6\zeta_1), \min(\beta_2\zeta_5, \beta_5\zeta_5), \min(\beta_3\zeta_4, \beta_4\zeta_4), \max(\beta_4\zeta_3, \beta_3\zeta_3), \\
\max(\beta_5\zeta_2, \beta_2\zeta_2), \max(\beta_6\zeta_6, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_5 \leq 0, \beta_6 \geq 0 \\
\{(\min(\beta_1\zeta_6, \beta_6\zeta_1), \min(\beta_2\zeta_5, \beta_5\zeta_2), \min(\beta_3\zeta_4, \beta_4\zeta_4), \max(\beta_4\zeta_3, \beta_3\zeta_3), \\
\max(\beta_5\zeta_5, \beta_2\zeta_2), \max(\beta_6\zeta_6, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_4 \leq 0, \beta_5 \geq 0 \\
\{(\min(\beta_1\zeta_6, \beta_6\zeta_1), \min(\beta_2\zeta_5, \beta_5\zeta_2), \min(\beta_3\zeta_4, \beta_4\zeta_3), \max(\beta_4\zeta_4, \beta_3\zeta_3), \\
\max(\beta_5\zeta_5, \beta_2\zeta_2), \max(\beta_6\zeta_6, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_3 \leq 0, \beta_4 \geq 0 \\
\{(\min(\beta_1\zeta_6, \beta_6\zeta_1), \min(\beta_2\zeta_5, \beta_5\zeta_2), \min(\beta_3\zeta_3, \beta_4\zeta_3), \max(\beta_4\zeta_4, \beta_3\zeta_4), \\
\max(\beta_5\zeta_5, \beta_2\zeta_2), \max(\beta_6\zeta_6, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_2 \leq 0, \beta_3 \geq 0 \\
\{(\min(\beta_1\zeta_6, \beta_6\zeta_1), \min(\beta_2\zeta_2, \beta_5\zeta_2), \min(\beta_3\zeta_3, \beta_4\zeta_3), \max(\beta_4\zeta_4, \beta_3\zeta_4), \\
\max(\beta_5\zeta_5, \beta_2\zeta_5), \max(\beta_6\zeta_6, \beta_1\zeta_1), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_1 \leq 0, \beta_2 \geq 0 \\
\{(\min(\beta_1\zeta_1, \beta_6\zeta_1), \min(\beta_2\zeta_2, \beta_5\zeta_2), \min(\beta_3\zeta_3, \beta_4\zeta_3), \max(\beta_4\zeta_4, \beta_3\zeta_4), \\
\max(\beta_5\zeta_5, \beta_2\zeta_5), \max(\beta_6\zeta_6, \beta_1\zeta_6), rt, su), \mu_{\tilde{H}}\mu_{\tilde{G}}\} & \text{if } \beta_1 \geq 0.
\end{cases} \tag{16}
$$

Definition 13 (Division of HFN)**.** If $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6; t, u); \mu_{\tilde{G}}\}$ are two hexagonal fuzzy number (HFN) with $\tilde{H} > 0$ & $\tilde{G} > 0$, then the division of $\tilde{H}$ and $\tilde{G}$ is define as $\tilde{H} \oslash \tilde{G} = \big\{(\beta_1/\zeta_6, \beta_2/\zeta_5, \beta_3/\zeta_4, \beta_4/\zeta_3, \beta_5/\zeta_2, \beta_6/\zeta_1; v, w); \mu_{\tilde{H}\oslash\tilde{G}}\big\}$ whose membership function is given by

$$
\mu_{\tilde{H}\oslash\tilde{G}}(\phi) = \begin{cases}
0 & \text{if } \phi \in (-\infty, \beta_1/\zeta_6] \\
v\Big(\frac{\phi-(\beta_1/\zeta_6)}{(\beta_2/\zeta_5)-(\beta_1/\zeta_6)}\Big) & \text{if } \phi \in [(\beta_1/\zeta_6), (\beta_2/\zeta_5)] \\
v + (1-v)\Big(\frac{\phi-(\beta_2/\zeta_5)}{(\beta_3/\zeta_4)-(\beta_2/\zeta_5)}\Big) & \text{if } \phi \in [(\beta_2/\zeta_5), (\beta_3/\zeta_4)] \\
1 & \text{if } \phi \in [(\beta_3/\zeta_4), (\beta_4/\zeta_3)] \\
w + (1-w)\Big(\frac{(\beta_5/\zeta_2)-\phi}{(\beta_5/\zeta_2)-(\beta_4/\zeta_3)}\Big) & \text{if } \phi \in [(\beta_4/\zeta_3), (\beta_5/\zeta_2)] \\
w\Big(\frac{(\beta_6/\zeta_1)-\phi}{(\beta_6/\zeta_1)-(\beta_5/\zeta_2)}\Big) & \text{if } \phi \in [(\beta_5/\zeta_2), (\beta_6/\zeta_1)] \\
0 & \text{if } \phi \in [(\beta_6/\zeta_1), \infty)
\end{cases} \tag{17}
$$

where $v = ru$, $w = st$ and $\mu_{\tilde{H}\oslash\tilde{G}} = \mu_{\tilde{H}}\mu_{\tilde{G}}$.

If $\tilde{H} > 0$ & $\tilde{G} < 0$, then the division of $\tilde{H}$ and $\tilde{G}$ is define as $\tilde{H} \oslash \tilde{G} = \big\{(\beta_6/\zeta_1, \beta_5/\zeta_2, \beta_4/\zeta_3, \beta_3/\zeta_4, \beta_2/\zeta_5, \beta_1/\zeta_6; v, w); \mu_{\tilde{H}\oslash\tilde{G}}\big\}$ whose membership function is given by

$$
\mu_{\tilde{H}\oslash\tilde{G}}(\phi) = \begin{cases}
0 & \text{if } \phi \in (-\infty, \beta_6/\zeta_1] \\
v\Big(\frac{\phi-(\beta_6/\zeta_1)}{(\beta_5/b_2)-(\beta_6/\zeta_1)}\Big) & \text{if } \phi \in [(\beta_6/\zeta_1), (\beta_5/\zeta_2)] \\
v + (1-v)\Big(\frac{\phi-(\beta_5/\zeta_2)}{(\beta_4/\zeta_3)-(\beta_5/\zeta_2)}\Big) & \text{if } \phi \in [(\beta_5/\zeta_2), (\beta_4/\zeta_3)] \\
1 & \text{if } \phi \in [(\beta_4/\zeta_3), (\beta_3/\zeta_4)] \\
w + (1-w)\Big(\frac{(\beta_2/\zeta_5)-\phi}{(\beta_2/\zeta_5)-(\beta_3/\zeta_4)}\Big) & \text{if } \phi \in [(\beta_3/\zeta_4), (\beta_2/\zeta_5)] \\
w\Big(\frac{(\beta_1/\zeta_6)-\phi}{(\beta_1/\zeta_6)-(\beta_2/\zeta_5)}\Big) & \text{if } \phi \in [(\beta_2/\zeta_5), (\beta_1/\zeta_6)] \\
0 & \text{if } \phi \in [(\beta_1/\zeta_6), \infty)
\end{cases} \tag{18}
$$

where $v = st$, $w = ru$ and $\mu_{\tilde{H}\oslash\tilde{G}} = \mu_{\tilde{H}}\mu_{\tilde{G}}$.

Definition 14 (Inverse of HFN)**.** If $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}}\}$ is a hexagonal fuzzy number (HFN) with $0 \notin \tilde{H}$ (*i.e.*, $0 \notin [\beta_1, \beta_6]$), then inverse of $\tilde{H}$ define as

$$\tilde{H}^{-1} = \{(1/\beta_6, 1/\beta_5, 1/\beta_4, 1/\beta_3, 1/\beta_2, 1/\beta_1; v, w); \mu_{\tilde{H}^{-1}}\} \tag{19}$$

where $v = s$, $w = r$ and $\mu_{\tilde{H}^{-1}} = \mu_{\tilde{H}}$.

Also, if $0 \in \tilde{H}$ then inverse is not define.

Definition 15 (Identical Equality of two HFN)**.** Let $\tilde{H} = \{(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6; r, s); \mu_{\tilde{H}}\}$ and $\tilde{G} = \{(\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5, \zeta_6; t, u); \mu_{\tilde{G}}\}$ are two hexagonal fuzzy number (HFN), then they are identically equal if $\beta_1 = \zeta_1$, $\beta_2 = \zeta_2$, $\beta_3 = \zeta_3$, $\beta_4 = \zeta_4$, $\beta_5 = \zeta_5$, $\beta_6 = \zeta_6$, $r = t$, $s = u$ and $\mu_{\tilde{H}} = \mu_{\tilde{G}}$.

Numerical example: Numerical example has been discussed below to illustrate the arithmetic operations of Hexagonal fuzzy numbers.

Example 2. Let $\tilde{H} = \{(3, 4, 6, 7.5, 9, 10; 0.25, 0.7); 1\}$ and $\tilde{G} = \{(2, 2.5, 3.5, 5, 6, 7; 0.4, 0.5); 1\}$ are two hexagonal fuzzy numbers and $k = 2 (> 0)$ be a scalar. Then arithmetic operation on HFN gives

(1) Addition: $\tilde{H} \oplus \tilde{G} = \{(5, 6.5, , 9.5, 12.5, 15, 17; 0.55, 0.85); 1\}$
(2) Subtraction: $\tilde{H} \ominus \tilde{G} = \{(-4, -2, 1, 4, 6.5, 8; 0.625, 0.82); 1\}$
(3) Scalar multiplication: $2\tilde{H} = 2 \times \tilde{H} = \{(6, 8, 12, 15, 18, 20; 0.25, 0.7); 1\}$
(4) Multiplication: $\tilde{H} \otimes \tilde{G} = \{(6, 10, 21, 37.5, 54, 70; 0.1, 0.35); 1\}$
(5) Division: $\tilde{H} \oslash \tilde{G} = \{(3/7, 2/3, 6/5, 15/7, 18/5, 5; 0.125, 0.28); 1\}$
(6) Inverse: $\tilde{H}^{-1} = \{(1/10, 1/9, 2/15, 1/6, 1/4, 1/3; 0.7, 0.25); 1\}$

Example 3. Let $\tilde{A} = \{(1, 4, 6, 7, 8, 10; 0.6, 0.4); 0.5\}$ and $\tilde{B} = \{(-15, -13, -10, -8, -4, -2; 0.2, 0.8); 0.5\}$ are two hexagonal fuzzy numbers and $k = -2 (< 0)$ be a scalar. Then arithmetic operation on HFN gives

(1) Addition: $\tilde{A} \oplus \tilde{B} = \{(-14, -9, -4, -1, 4, 8; 0.68, 0.88); 0.75\}$
(2) Subtraction: $\tilde{A} \ominus \tilde{B} = \{(3, 8, 14, 17, 21, 25; 0.92, 0.52); 0.75\}$
(3) Scalar multiplication: $-2\tilde{B} = (-2) \times \tilde{B} = \{(4, 8, 16, 20, 26, 30; 0.2, 0.8); 0.5\}$
(4) Multiplication: $\tilde{A} \otimes \tilde{B} = \{(-150, -104, -70, -48, -16, -2; 0.12, 0.32); 0.25\}$
(5) Division: $\tilde{A} \oslash \tilde{B} = \{(-2/3, -8/13, -7/10, -3/4, -1, -1/2; 0.08, 0.48); 0.25\}$
(6) Inverse: $\tilde{B}^{-1} = \{(-1/2, -1/4, -1/8, -1/10, -1/13, -1/15; 0.8, 0.2); 0.5\}$

3. Hexagonal fuzzy multi sub criterion based decision making

3.1. Fuzzy Analytic Hierarchy Process (FAHP) Method

Analytic hierarchy process (AHP) is a famous mathematical tool of optimization of alternatives which is used in Multi Criteria Decision Making (MCDM). This method was first developed by Satty [49] and Wind and Saaty [50]. It is used explicitly for obtaining the factors and sub-factors weight. AHP gives a scientific solution to real life problems. This method help decision makers (DMs) to resolve complex problems with heuristic solution. The comparison of factors and sub-factors, by giving preference in crisp value can be considered as a complex assignment for DMs, thus FAHP methodology captures the blurriness of the problem. The determination of factors and sub-factors weights are very important for the customers in a restaurant. AHP works with a problem hierarchy, where a comparison matrix is constructed to represent subjective judgments regarding factors and sub-factors. In this paper, FAHP is taken instead of AHP, keeping in mind the fuzzy logic which allows the DMs in the evaluation of the well optimized result. The step of FAHP are given described below:

(i) Construction of comparison matrix in term of hexagonal fuzzy number (HFN) by a group of decision experts.
Let a group of 'N' decision-makers assigned for the comparison of factors and sub-factors. Let each DM express their preference in the pairwise comparison of factors and sub-factors. Thus, 'n' set of matrices are obtained, $T_n = \{t_{pqn}\}$.
Where $t_{pqn} = \left(\tilde{a}_{pqn}, \tilde{b}_{pqn}, \tilde{c}_{pqn}, \tilde{d}_{pqn}, \tilde{e}_{pqn}, \tilde{f}_{pqn}\right)$ denotes the HFN of p factor to q factor as expressed by the 'n' DM and $p = 1, 2, \ldots, i;\ q = 1, 2, \ldots, j$.

$$\begin{cases} a_{pq} = \min_{n=1,2,\ldots,N} \tilde{a}_{pqn} \\ b_{pq} = \min_{n=1,2,\ldots,N} \tilde{b}_{pqn} \\ c_{pq} = \sqrt[N]{\prod_{n=1}^{N} \tilde{c}_{pqn}} \\ d_{pq} = \sqrt[N]{\prod_{n=1}^{N} \tilde{d}_{pqn}} \\ e_{pq} = \max_{n=1,2,\ldots,N} \tilde{e}_{pqn} \\ f_{pq} = \max_{n=1,2,\ldots,N} \tilde{f}_{pqn} \end{cases} \tag{20}$$

(ii) Defuzzification of HFN:
Defuzzification of the hexagonal fuzzy number (HNF) by the centroid-based method (CBM) used this paper. Thus using equation (12), convert a fuzzy number to a crisp value.

(iii) Normalization of the defuzzied matrix:

$$\begin{cases} S_q = \sum_{p=1}^{i} V_{pq} \\ U_{pq} = \frac{V_{pq}}{S_q} \end{cases} \tag{21}$$

where $p = 1, 2, \ldots, i;\ q = 1, 2, \ldots, j$. This normalization makes the sum of the weights equal to one.

(iv) Estimation of factors and sub-factors weights:

$$E = \frac{N\text{th root value}}{\sum N\text{th root}}. \tag{22}$$

(v) To test the Consistence Index (C.I.) of the matrix:

$$C.I. = \frac{\alpha_{\max} - j}{j - 1} \tag{23}$$

where j denotes the size of the matrix.

(vi) Determination of Consistency Ratio (C.R.):

$$\text{C.R.} = \frac{\text{C.I.}}{\text{R.I.}} \tag{24}$$

where R.I. denote Random Index and its value depends on the size of the matrix n.

The assessment of C.R. ≤ 0.1 is acceptable and indicates that the weights obtained are consistent.

3.2. Determination of hexagonal fuzzy weights of factors

To obtain the HFN weight of factors and sub- factors, we refer Ghosh *et al.* [11] and the process of determining the HFN weight is discussed below:

(i) Determine the geometric mean of the HFN by using

$$a_{rt} = \left(\prod_{s=1}^{k} y_{rst}\right)^{\frac{1}{k}} \tag{25}$$

where $y_{rst} \in \tilde{H}$, $r = 1, 2, \ldots, k$; $s = 1, 2, \ldots, k$ and $t = 1, 2, \ldots, 6$.

(ii) Summing the each column of geometric mean criterion matrix.

$$S_t = \sum_{r=1}^{k} a_{rt} \tag{26}$$

where bound of r and t are same as previous.

(iii) Then find the inverse of S_t using the equation (19) and then arranging in increasing order, S'_t(let).

(iv) The hexagonal fuzzy weights of factors by the given equation:

$$W_t = a_{rt} \times S'_t. \tag{27}$$

(v) Calculation of global HFN sub-factors weight are determined by the product of factors HFN weight with the respective sub-factor HFN weight.

3.3. Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) approach

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) Multi Criteria Decision Making (MCDM) tool is a widely used technique developed by Hwang and Yoon [51]. This method also use in different field like Best Employees selection [52], Medicine selection [12], Creating priority group for Vaccine [13] etc. The TOPSIS method is taken as a distance measure technique in which the optimal alternative is nearest to the positive ideal solution (PIS) and farthest from the negative ideal solution (NIS). The linguistic terms assigned by human choice of decisions can be reflected by HFN. This approach is convenient for handling the uncertainty of the situation involving multiple factors and sub-factors. Thus, for the factor attracting the customers in a restaurant, the classical MCDM method TOPSIS is integrated with Fuzzy number *i.e.* FTOPSIS to capture the problem in an efficient way. The steps of FTOPSIS are described below:

(i) Construction of the decision matrix by the help of decision experts in terms of linguistic terms. The linguistic terms are then converted to a HFN.
Let i number of restaurants and j number of factors. Let N denotes the number of decision makers (DMs). Then evaluate the value of $\tilde{a}_{pq}$, $\tilde{b}_{pq}$, $\tilde{c}_{pq}$, $\tilde{d}_{pq}$, $\tilde{e}_{pq}$ and $\tilde{f}_{pq}$ by equation (20) and the value of r_{pq}, s_{pq} find by:

$$\begin{cases} r_{pq} = \min_{n=1,2,3,\ldots,N} r_{pqn} \\ s_{pq} = \min_{n=1,2,3,\ldots,N} s_{pqn} \end{cases} \tag{28}$$

(ii) To evaluate the normalized HFN fuzzy decision matrix:

$$\begin{aligned} \tilde{N} &= [n_{pq}]_{ij}, \qquad p = 1, 2, 3, \ldots, i;\ q = 1, 2, 3, \ldots, j \\ N_{pq}^{B} &= \left\langle \left(\frac{a_{pq}}{f^*}, \frac{b_{pq}}{f^*}, \frac{c_{pq}}{f^*}, \frac{d_{pq}}{f^*}, \frac{e_{pq}}{f^*}, \frac{f_{pq}}{f^*} \right); r_{pq}, s_{pq} \right\rangle \quad d \in \text{B.A},\ f^* = \max f_{pq} \\ N_{pq}^{NB} &= \left\langle \left(\frac{a^*}{f_{pq}}, \frac{a^*}{e_{pq}}, \frac{a^*}{d_{pq}}, \frac{a^*}{c_{pq}}, \frac{a^*}{b_{pq}}, \frac{a^*}{a_{pq}} \right); r_{pq}, s_{pq} \right\rangle \quad d \in \text{N.B.A},\ a^* = \min a_{pq} \end{aligned} \tag{29}$$

where B.A and N.B.A signifies the benefit attributes and non-benefit attributes, respectively.

(iii) To evaluate the weighted fuzzy normalized matrix, the sub-factors' fuzzy weights describe on equation (27) are multiplied with the normalized fuzzy value:

$$WN = [P_{pq}]_{ij} \tag{30}$$

where $P_{pq} = N_{pq} * \tilde{W}_q$, $p = 1, 2, 3, \ldots, i$; $q = 1, 2, 3, \ldots, j$.

(iv) Calculate the fuzzy positive ideal solution (FPIS) (PIS^+) and fuzzy negative ideal solution (FNIS) (NIS^-), where a_p^+ denotes the maximum value of a_{pq} and a_p^- denotes the minimum value of a_{pq}:

$$\left.\begin{aligned} \text{PIS}^+ &= \{a_1^+, a_2^+, \ldots, a_j^+\} = \{(\max\, a_{pq} | q \in M_B), (\min\, a_{pq} | q \in M_{NB})\} \\ \text{NIS}^- &= \{a_1^-, a_2^-, \ldots, a_j^-\} = \{(\min\, a_{pq} | q \in M_B), (\max\, a_{pq} | q \in M_{NB})\} \end{aligned}\right\} \tag{31}$$

where M_B denotes the benefit attributes and M_{NB} denotes the non-benefit attributes.

(v) Calculation of the distance measure of all alternatives from the PIS and NIS. The two Euclidean distances for individual alternatives can be calculated as follows:

$$\left.\begin{aligned} L_p^+ &= \textstyle\sum_{q=1}^{j} d(P_{pq}, q_p^+),\ p = 1, 2, 3, \ldots, i \\ L_p^- &= \textstyle\sum_{q=1}^{j} d(P_{pq}, q_p^-),\ p = 1, 2, 3, \ldots, i \end{aligned}\right\} \tag{32}$$

where $d(.,.)$ denotes the Euclidean distance between two fuzzy numbers.

(vi) Determination of the relative closeness to the ideal alternatives:

$$R_p = \frac{L_p^-}{L_p^+ + L_p^-} \tag{33}$$

where $p = 1, 2, 3, \ldots, i$.

(vii) Rank the alternatives:
The alternatives are ranked based on the score obtained by R_p. The larger value of R_p signifies the better alternatives.

3.4. Fuzzy Complex Proportional Assessment (FCOPRAS) approach

The COmplex PRoportional ASsessment (COPRAS) was first introduced by Zavadskas, Kalklauskas and Sarka [53]. An extended representation of COPRAS is Fuzzy COPRAS (FCOPRAS) which is used for ranking of the alternatives in various decision making problem [54]. This method is based on stepwise ranking and evaluation of the alternative in reference to utility degree and significance. Earlier COPRAS method were applied by Ghosh *et al.* in site selection [11], Fouladgar *et al.* in property management [55], economy by Narayanamoothy *et al.* [56], Evaluating the potential capability of air cargo sector Tolga and Durak [57], selection of optimal material for solar car by Ghose *et al.* [58]. The procedure of FCOPRAS method includes the following steps:

(i) Construction of HFN decision matrix by the opinion of decision experts. The DM's assigns linguistic terms with respect to the factor.

(ii) The normalized matrix is calculated in the same way as discussed in FTOPSIS method using equation (29).

(iii) Construction of weighted normalized matrix by multiplying the factor weights and the normalized matrix using equation (30).

(iv) Evaluation of the beneficial attributes (BA) and non-beneficial attributes (NBA) denoted as B^+ and B^- respectively as follows:

$$\begin{aligned} B^+ &= \left\{ \sum_{p=1}^{m} a_p^{\text{WN}}, \sum_{p=1}^{m} b_p^{\text{WN}}, \sum_{p=1}^{m} c_p^{\text{WN}}, \sum_{p=1}^{m} d_p^{\text{WN}}, \sum_{p=1}^{m} e_p^{\text{WN}}, \sum_{p=1}^{m} f_p^{\text{WN}} \right\} \\ B^- &= \left\{ \sum_{p=m+1}^{i} a_p^{\text{WN}}, \sum_{p=m+1}^{i} b_p^{\text{WN}}, \sum_{p=m+1}^{i} c_p^{\text{WN}}, \sum_{p=m+1}^{i} d_p^{\text{WN}}, \sum_{p=m+1}^{i} e_p^{\text{WN}}, \sum_{p=m+1}^{i} f_p^{\text{WN}} \right\} \end{aligned} \tag{34}$$

where equation (30) gives the value of $a_p^{\text{WN}}, b_p^{\text{WN}}, c_p^{\text{WN}}, d_p^{\text{WN}}.e_p^{\text{WN}}, f_p^{\text{WN}}$ and $p = 1, 2, \ldots, m$ denotes the beneficial attributes and $p = m{+}1, m{+}2, \ldots, i$ denotes the non-beneficial attributes among the alternatives. In this study, only price of the food (F_{11}) is the non-beneficial sub-factor. All others factors and sub-factors are Beneficial attributes in this research. Beneficial criteria are those criteria where enhancement of value will benefit the customers, for non-beneficial criteria it is the reverse.

(v) At the end, defuzzification of HFN is done using the centroid-based method (CBM) of a hexagonal fuzzy number. The value of Sq^+ for the beneficial attributes and Sq^- for the non-beneficial attributes are calculated.

(vi) Finally, calculation the equation:

$$Cq = Sq^+ + \frac{Sq^-_{\min} \times G}{Sq^- \times H} \tag{35}$$

where

$$\begin{cases} G = \sum_{q=1}^{j} Sq^- \\ H = \sum_{q=1}^{j} \frac{Sq^-_{\min}}{Sq^-} \end{cases}$$

and $q = 1, 2, \ldots, j$ are the alternatives.

(vii) Now, ranking the alternatives from the above data.

$$R = \frac{Cq}{Cq_{\max}} \times 100\% \tag{36}$$

where Cq denotes the qth defuzzified value and $Cq_{\max}$ denotes the maximum defuzzified value from the considered alternatives.

3.5. Pseudo code depicting the empirical study application

The research model under consideration involving "i" number of alternatives based on "j" number of factor is represented below. The input taken in our study are the preferential linguistic terms assigned by DMs. These variables are converted to HFN for obtaining the output *i.e.*, the ranking of the alternatives. i = Restaurants location as alternative j = Number of factor $i \times j$ = Size of the matrix

Input: The preferential rating matrix in terms of HFN

Output: The ranking order of the restaurants location as alternative in the TOPSIS approach

1. **for** ($p = 1$ to i, $q = 1$ to j) **do**
2. Generate HFN by DMs.

For every given criteria create a matrix and compare the given criteria with each other using linguistic terms in HFN 1–9 scale

3. Calculating criteria weight in HFN by using FAHP.
4. Use HFN AHP methodology to check whether the matrix is consistent or not.
5. If the matrix is consistent, calculate HFN-TOPSIS for ranking of restaurants location as alternatives.

Else, Go back to step 4.

6. Construct normalized values NZ_{ef}

$$\tilde{N} = [n_{pq}]_{ij}, \qquad p = 1, 2, \ldots, i;\ q = 1, 2, \ldots, j;$$

$$N^B_{pq} = \left\langle \left(\frac{\tilde{a}_{pq}}{\tilde{f}^*}, \frac{\tilde{b}_{pq}}{\tilde{f}^*}, \frac{\tilde{c}_{pq}}{\tilde{f}^*}, \frac{\tilde{d}_{pq}}{\tilde{f}^*}, \frac{\tilde{e}_{pq}}{\tilde{f}^*}, \frac{\tilde{f}_{pq}}{\tilde{f}^*}; r_{pq}, s_{pq} \right) \right\rangle, \quad d \in \text{B.A.},\ \tilde{f}^* = \max f_{pq}$$

$$N^{NB}_{pq} = \left\langle \left(\frac{\tilde{a}^*}{\tilde{f}_{pq}}, \frac{\tilde{a}^*}{\tilde{e}_{pq}}, \frac{\tilde{a}^*}{\tilde{d}_{pq}}, \frac{\tilde{a}^*}{\tilde{c}_{pq}}, \frac{\tilde{a}^*}{\tilde{b}_{pq}}, \frac{\tilde{a}^*}{\tilde{a}_{pq}}; r_{pq}, s_{pq} \right) \right\rangle, \quad d \in \text{N.B.A.},\ \tilde{a}^* = \min a_{pq}$$

7. Generate weighted normalized value $P_{pq} = \tilde{N}_{pq} \times \tilde{W}_q$;
8. Calculate (FPIS^+) and (FNIS^-)

$$\text{PIS}^+ = \{a_1^+, a_2^+, \ldots, a_j^+\} = \{(\max a_{pq} | a \in M_B), (\min a_{pq} | q \in M_{NB})\}$$

$$\text{NIS}^- = \{a_1^-, a_2^-, \ldots, a_j^-\} = \{(\min\, a_{pq}|a \in M_B), (\max\, a_{pq}|q \in M_{NB})\}$$

9. Calculate distance measure of each alternatives from (PIS^+) and (NIS^-)

$$\tilde{L}_p^+ = \sum_{q=1}^{j} d(P_{pq}, q_p^+),\ \tilde{L}_p^- = \sum_{q=1}^{j} d(P_{pq}, q_p^-)$$

10. Compute relative closeness $R_p = \frac{\tilde{L}_p^-}{\tilde{L}_p^+ + \tilde{L}_p^-}$;

11. **end for**

4. Factor and sub factor for attracting the customers in a restaurant

A restaurant not only attracts people based on the food it has to offer but also due to various other attributes/factors which are enumerated in Figure 5 and discussed below:

4.1. Food (F_1)

To attract customers in a restaurant, the food items in the menu is an important aspect. Great taste of food and consistently maintaining the quality of dishes help in making a strong relationship between customers and the restaurant.

4.1.1. Price (F_{11})

Price sensitive Customers give significance to this attribute. Keeping in mind the preference and needs of this segment of consumers, focusing on price of the food is an important sub- factor.

4.1.2. Food Quality (F_{12})

A restaurant is known by the food it serves. The quality of the food items being offered should match the profile of the target audience to whom it is served. Customer's flock into a restaurant based on various attributes of the food namely (i) Appearance (shape, size, gloss, colour etc), (ii) Oiliness of the food, (iii) Flavour, (iv) Nutritional content, (v) Ethical and sustainable raw materials used etc.

4.1.3. Variety (F_{13})

Variety of food is important as it depends upon the likes and dislikes of the customers. If foods of all variation are available *e.g.* Chinese, continental, north Indian etc. consumers will prefer more to go that specific restaurant which avail all types of cuisine.

4.2. Service (F_2)

In the restaurant industry good service is very important. It may success their overall restaurant business. Good quality service will enhance the frequency of visit by a satisfied customers.

4.2.1. Service Quality (F_{21})

Reliability, responsiveness, tangibility, empathy and assurance are the key dimensions through which service quality is measured. These dimensions of service quality are very important from customer's perspective [59].

4.2.2. Behavior (F_{22})

The first aspect which touches a customer, even before the taste of food hits the tongue is the behaviour of the staff personnel who take down the order as well as deliver the food. If their behaviour is polite then it attracts customers to keep ordering for more food items whereas impolite behaviour is often the ground for low sales of the restaurant.

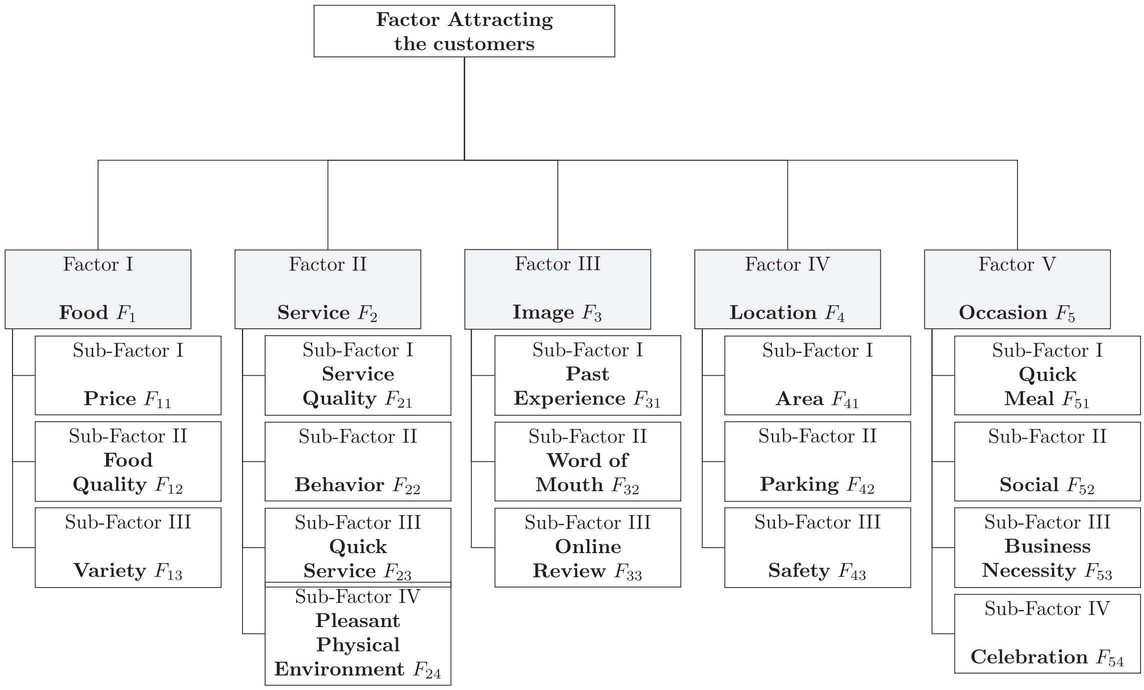

FIGURE 5. *Factor* and *sub-factor* for attracting the customers in a restaurant.

4.2.3. Quick Service (F_{23})

Customers who are in a hurry, mostly want everything in short time. When it comes to restaurants near busy road, railway stations, bus terminus customers prefer places where foods are served quickly and efficiently. The pandemic has impacted the restaurant business worldwide, thus quick service restaurant are the specific ones which serves food within minimal possible time.

4.2.4. Pleasant Physical Environment (F_{24})

Pleasant physical environment symbolises noise, soothing fragrance, music and temperature. Loud and harsh music affects the emotions of customers in restaurants. Soothing light, fragrance enhances customer's mood and emotions and thus influences food consumption. Certain temperature impacts the consumer's behaviour negatively. So, pleasant ambience is a significant factor in restaurant selection and building a long standing relation with the customers.

4.3. Image (F_3)

Image of a restaurant in the eyes of a customers help in building loyalty and new customer base. Strategic marketing, maintaining quality helps in building image.

4.3.1. Past Experience (F_{31})

Past pleasing experience always helps in retaining customers. Satisfactory service leads to a loyal customer base.

4.3.2. Word of Mouth (F_{32})

One will always prefer the restaurant which has earned reputation with time. Customers often opt for the restaurant which has gained a lot positive feedback from their past consumers.

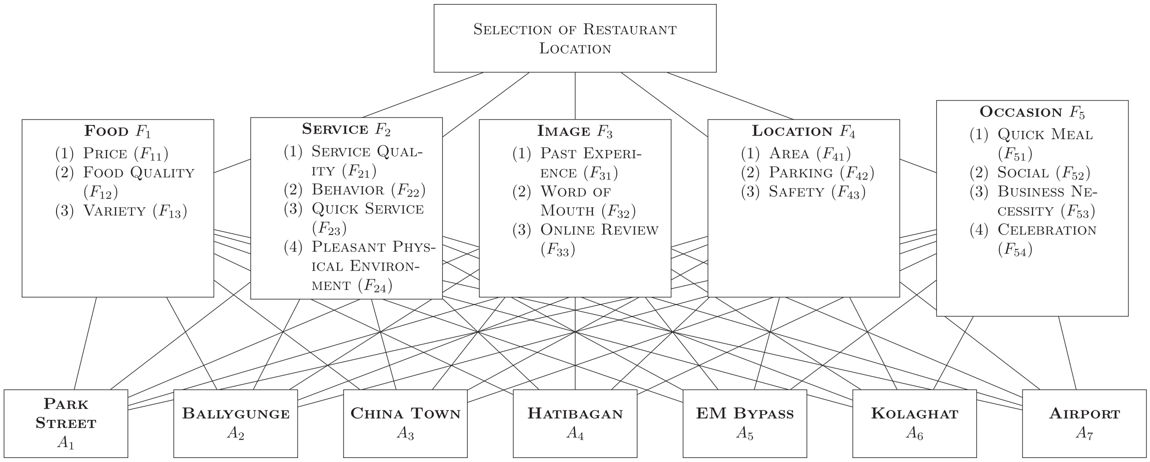

FIGURE 6. Hierarchical structure representing the factors and sub-factors with alternatives.

4.3.3. Online Review (F_{33})

In this world of technology, people use their smart phones, laptops etc. in order to check the review of the restaurant such as rating, image, menu, direction to have a clear impression on their mind for the particular restaurant. Thus, making it easier and convenient for them to select the restaurant.

4.4. Location (F_4)

The location of the restaurant is important. This location preference varies from one restaurant to the other based on their target audience. In case a restaurant is built without keeping these in mind it will not get a good number of customers.

4.4.1. Area (F_{41})

An adequate area is necessary for a restaurant layout. The consumer seating place should be at an optimal distance and properly ventilated from the restaurant kitchen, pantry and storage room such that the noise of preparation of food and the smell of cooking do not reach the consumers.

4.4.2. Parking (F_{42})

Whenever a customer with own vehicle contemplates about having food in a restaurant the first thought that strikes is about its parking options. A good parking area ensures that the customer will be able to have his meal in peace and feel contended thereby increasing the chances of his coming back to the restaurant again and again.

4.4.3. Safety (F_{43})

The safety factor inside and around a restaurant is important since the customers come to the place to relax and consume food. If there is any kind of compromise on the customer's safety then it won't be long before it goes out of business.

4.5. Occasion (F_5)

Social gatherings, celebrations, quick meals, business events are different occasions for which restaurants are availed to have food service within the venue.

TABLE 3. Alternative consider in this paper and details of those alternatives.

Location	Latitude and longitude	Location	Latitude and longitude
Park Street (A_1)	22.5768 °N; 88.3504 °E	Ballygunge (A_2)	22.5280 °N; 88.3659 °E
China Town (A_3)	22.5739 °N; 88.3556 °E	Hatibagan (A_4)	22.5975 °N; 88.3707 °E
EM Bypass (A_5)	22.4942 °N; 88.4008 °E	Kolaghat (A_6)	22.4352 °N; 87.8607 °E
Airport (A_7)	22.6531 °N; 88.4449 °E		

4.5.1. Quick Meal (F_{51})

For any type of restaurant service for quick meal is a popular strategy for attracting people specially those who are in hurry. The places like train station, airport, busy road adjacent locations, refueling stations etc. are the places where people search for quick meals.

4.5.2. Social (F_{52})

Celebration like ring ceremony, marriage like social occasions requires suitable venue. many a times suitably located restaurant are preferred for such occasions.

4.5.3. Business Necessity (F_{53})

Corporate events and business meets are arranged keeping in mind various commercial dimensions. These events are regularly held in restaurants depending on number of crowds and profile of the attendees. This segment is lucrative part of restaurant business.

4.5.4. Celebration (F_{54})

Birth day, friendship day etc like celebration may be arrange in a restaurant.

Figure 6 represents the hierarchical structure of the numerical study taken into consideration.

5. Model Set up and corresponding problem

Seven location in the state of West Bengal, India is chosen for this study. The locations are: Park Street (A_1), Ballygunge (A_2), China Town (A_3), Hatibagan (A_4), EM Bypass (A_5), Kolaghat (A_6) and Airport (A_7). Their Latitude and Longitude are given in the following Table 5. The satellite location are shown in Figure 7.

Our objective is to rank the locations of restaurant as per preference. We first measure the factors and sub-factors for each location by two decision makers (DM).

5.1. Data source for the study

Data has been collected from customers, restaurant owners and people associated with restaurant business. They were interviewed relating to the questions associated with various important attributes of restaurant and eating pattern. Information regarding factors and sub factors associated with this problem, like the ongoing competitive price associated with the food variety has been collected from two restaurateurs.

5.2. Linguistic terms expressed in HFN in different scale

Linguistic terms are expressed in 1–9 scale and HFN scale for analysing data. Relation between different scale are describe in Table 4.

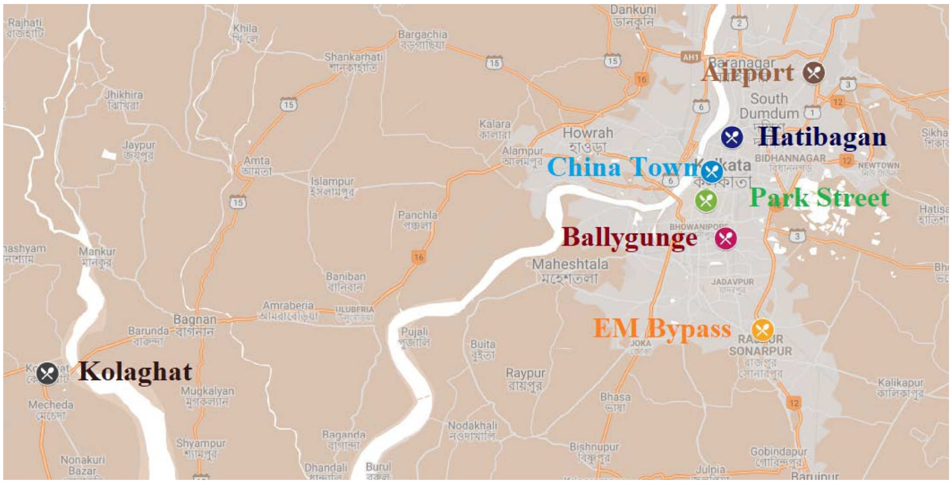

FIGURE 7. Different Restaurant Location around the city of West Bengal, India form Google my map, 2022

TABLE 4. Linguistic terms in hexagonal fuzzy number (HFN) in 1–9 scale.

Linguistic terms	1–9 Scale	Hexagonal fuzzy number (HFN)
Equally important (EI)	1	$(1, 1, 1, 1, 1, 1)$
Moderately important (MI)	3	$(2, 4, 5, 6, 6.5, 7)$
Strongly important (SI)	5	$(3.5, 5, 6, 7, 8, 8.9)$
Very strongly important (VSI)	7	$(6, 7, 7, 8, 9, 9.5)$
Absolutely important (AI)	9	$(7, 8, 9, 9, 9.5, 10)$
Moderately not important (MUI)	1/3	$(1/7, 1/6.5, 1/6, 1/5, 1/4, 1/2)$
Strongly not important (SUI)	1/5	$(1/8.9, 1/8, 1/7, 1/6, 1/5, 1/3.5)$
Very strongly not important (VSUI)	1/7	$(1/9.5, 1/9, 1/8, 1/7, 1/7, 1/6)$
Absolutely not important (AUI)	1/9	$(1/10, 1/9.5, 1/9, 1/9, 1/8, 1/7)$

TABLE 5. Factor matrix for comparison between two design makers (DMs).

Factor	Food (F_1)		Service (F_2)		Image (F_3)		Location (F_4)		Occasion (F_5)	
	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2
Food (F_1)	EI	EI	AI	AI	EI	EI	MUI	MUI	MI	MI
Service (F_2)	AUI	AUI	EI	EI	SI	SI	AI	AI	SI	SI
Image (F_3)	EI	EI	SUI	SUI	EI	EI	MUI	SUI	MI	SI
Location (F_4)	MI	MI	AUI	AUI	MI	SI	EI	EI	MI	MI
Occasion (F_5)	MUI	MUI	SUI	SUI	MUI	SUI	MUI	MUI	EI	EI

TABLE 6. Sub-factor matrix for Food (F_1)

Sub-factor	Price (F_{11})		Food Quality (F_{12})		Variety (F_{13})	
	DM1	DM2	DM1	DM2	DM1	DM2
Price (F_{11})	EI	EI	EI	EI	MUI	SI
Food Quality (F_{12})	EI	EI	EI	EI	MUI	SI
Variety (F_{13})	MI	SUI	MI	SUI	EI	EI

TABLE 7. Sub-factor matrix for Service (F_2).

Sub-factor	Service Quality (F_{21})		Behavior (F_{22})		Quick Service (F_{23})		Pleasant Physical Environment (F_{24})	
	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2
Service Quality (F_{21})	EI	EI	SI	SI	EI	EI	SUI	MUI
Behavior (F_{22})	SUI	SUI	EI	EI	VSI	SI	SI	SI
Quick Service (F_{23})	EI	EI	VSUI	SUI	EI	EI	SUI	MUI
Pleasant Physical Environment (F_{24})	SI	MI	SUI	SUI	SI	MI	EI	EI

TABLE 8. Sub-factor matrix for Image (F_3).

Sub-factor	Past Experience (F_{31})		Word of Mouth (F_{32})		Online Review (F_{33})	
	DM1	DM2	DM1	DM2	DM1	DM2
Past Experience (F_{31})	EI	EI	SI	VSI	SI	VSI
Word of Mouth (F_{32})	SUI	VSUI	EI	EI	SI	SI
Online Review (F_{33})	SUI	VSUI	SUI	SUI	EI	EI

TABLE 9. Sub-factor matrix for Location (F_4).

Sub-factor	Area (F_{41})		Parking (F_{42})		Safety (F_{44})	
	DM1	DM2	DM1	DM2	DM1	DM2
Area (F_{41})	EI	EI	VSUI	SI	VSUI	VSUI
Parking (F_{42})	VSI	SUI	EI	EI	MUI	VSUI
Safety (F_{44})	VSI	VSI	MI	VSI	EI	EI

5.3. Factor to factor comparison conducted by two DMs

Table 5 describe the opinions of two DMs in linguistic terms. All five factors Food (F_1), Service (F_2), Image (F_3), Location (F_4) and Occasion (F_5) are consider with 1st Decision Maker (DM1) and 2nd Decision Maker (DM2).

Table 6 describes the decision makers review in linguistic terms of the sub-factor Food (F_1). Similarly, Tables 7–10 describes the decision makers review in linguistic terms of the sub-factors Service (F_2), Image (F_3), Location (F_4) and Occasion (F_5) respectively.

Table 10. Sub-factor matrix for Occasion. (F_5)

Sub-factor	Quick Meal (F_{51})		Social (F_{52})		Business Necessity (F_{53})		Celebration (F_{54})	
	DM1	DM2	DM1	DM2	DM1	DM2	DM1	DM2
Quick Meal (F_{51})	EI	EI	SUI	VSI	VSUI	VSI	SUI	EI
Social (F_{52})	SI	VSUI	EI	EI	SUI	VSUI	EI	SUI
Business Necessity (F_{53})	VSI	VSUI	SI	VSI	EI	EI	SI	SI
Celebration (F_{54})	SI	EI	EI	SI	SUI	SUI	EI	EI

Table 11. Description of preference of factors in defuzzified form using CBM method.

Factors	Food (F_1)	Service (F_2)	Image (F_3)	Location (F_4)	Occasion (F_5)
Food (F_1)	1.00	8.90	1.00	0.21	5.22
Service (F_2)	0.11	1.00	6.54	8.90	6.33
Image (F_3)	1.00	0.17	1.00	0.20	5.89
Location (F_4)	5.34	0.11	6.01	1.00	5.22
Occasion (F_5)	0.21	0.17	0.20	0.21	1.00
Sum	7.67	10.34	14.74	10.54	23.66

Table 12. Representation of the normalized weight of the factors.

Factors	Food (F_1)	Service (F_2)	Image (F_3)	Location (F_4)	Occasion (F_5)
Priority Weight	0.24	0.33	0.11	0.28	0.04

Table 13. The priority of factors weight are represented.

Factors	Food (F_1)	Service (F_2)	Image (F_3)	Location (F_4)	Occasion (F_5)	**Sum**
Food (F_1)	0.24	2.89	0.11	0.06	0.22	3.53
Service (F_2)	0.03	0.33	0.73	2.47	0.27	3.82
Image (F_3)	0.24	0.05	0.11	0.06	0.25	0.71
Location (F_4)	1.30	0.04	0.67	0.28	0.22	2.50
Occasion (F_5)	0.05	0.05	0.02	0.06	0.04	0.23

6. Numerical illustration

Table 11 describes the preference of factors in defuzzified form by CBM method using the equation (12). The normalized weight of the factors are represented in Table 12 by using the equation (21).

The weight of the factors are represented in Table 13, using the equation (22). This table shows that the highest weight amongst the factor is Service (F_2), followed by Location (F_4), Food (F_1), Image (F_3) and the least weight is obtained for the factor Occasion (F_5) for selection of the best site for restaurant in Kolkata; the capital city of West Bengal, India.

In Table 14 describe the linguistic variables rating in terms of HFN for alternative rating. Table 15 shows the fuzzy weights of the factors, sub-factors and global weight (sub-factors) in HFN.

Note 2. *In HFN r and s represents the membership function of β_2 and β_5 respectively. Here $r = s = 0.5$ for the numerical application.*

TABLE 14. Linguistic variables represented using HFN for alternative rating with respect to sub-factor.

Linguistic terms	Hexagonal fuzzy number (HFN)
Extremely good (EG)/extremely high (EH)	$(6, 7, 7.5, 8, 8.5, 9)$
Very good (VG)/very high (VH)	$(5.5, 6, 7, 7.5, 8, 8.5)$
Good (G)/high (H)	$(3.5, 4.5, 5, 6.5, 7, 8)$
Poor (P)/low (L)	$(3, 4, 4.5, 5.5, 6.5, 7)$
Very poor (VP)/very low (VL)	$(1, 1.5, 2.5, 3.5, 5, 6)$

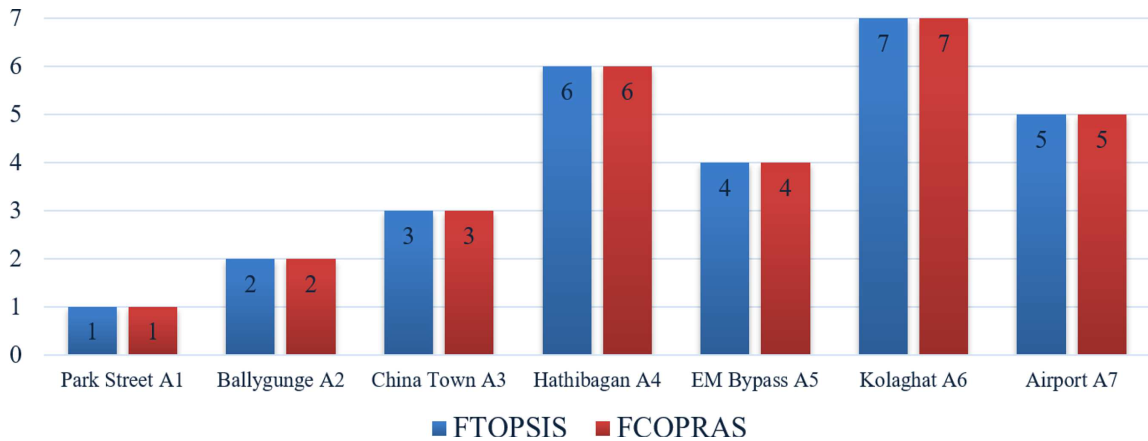

FIGURE 8. Presentation of the ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

Tables 16 and 17 describes the comparison table for rating of alternatives in linguistic variables with respect to sub-factors by two decision makers (DMs).

Table 18 describes the positive relative distance and negative relative distance between alternatives and ranking them by evaluating data of relative closeness by FTOPSIS method. Similarly, Table 19 describe the positive relative distance and negative relative distance between alternatives and ranking them by evaluating data of relative closeness by FCOPRAS method.

Remarks 1. Ranking of the best restaurant location depends on several conflicting factors and sub-factors with uncertainty of the selection problem. As, fuzzy logic handle these attributes appropriately, HFN is used in this research to capture the uncertain and imprecise data of the factors, sub-factors and alternatives. HFN is an efficient tool comparative to triangular fuzzy number (TFN), trapezoidal fuzzy number (TrFN) and pentagonal fuzzy number (PFN) which is in the comparative analysis section. The DMs uncertainty can be better captured using HFN. In real life scenario, DMs have to assign linguistic rating for factors and alternatives. These linguistic terms are then transformed to HFN. Finally, HFN is integrated with MCDM tools AHP, TOPSIS and COPRAS to yield weight and ranking of the factors and alternatives respectively. Thus, Considering the problem of this study *i.e.*, ranking of the best restaurant location, we have used HFN.

Figure 8 represented the ranking of alternatives by two MCDM techniques TOPSIS and COPRAS in HFN field.

Table 15. Fuzzy weights of factors, sub-factors and global weight (sub-factors) in HFN.

Factor weight	Sub-factor weight	Global weight
$F_1 = (0.14, 0.19, 0.22, 0.26, 0.32, 0.47)$	$F_{11} = (0.07, 0.07, 0.32, 0.37, 2, 2.07)$	$F_{11} = (0.01, 0.01, 0.07, 0.1, 0.63, 0.98)$
	$F_{12} = (0.07, 0.07, 0.32, 0.37, 2, 2.07)$	$F_{12} = (0.01, 0.01, 0.07, 0.1, 0.63, 0.98)$
	$F_{13} = (0.3, 0.03, 0.29, 0.35, 3.48, 3.66)$	$F_{13} = (0.01, 0.01, 0.06, 0.09, 1.1, 1.73)$
$F_2 = (0.18, 0.25, 0.31, 0.35, 0.44, 0.60)$	$F_{21} = (0.12, 0.16, 0.2, 0.25, 0.32, 0.48)$	$F_{21} = (0.02, 0.04, 0.06, 0.09, 0.14, 0.29)$
	$F_{22} = (0.17, 0.24, 0.32, 0.4, 0.52, 0.73)$	$F_{22} = (0.03, 0.06, 0.1, 0.14, 0.23, 0.44)$
	$F_{23} = (0.05, 0.06, 0.08, 0.09, 0.13, 0.2)$	$F_{23} = (0.01, 0.02, 0.02, 0.03, 0.05, 0.12)$
	$F_{24} = (0.13, 0.22, 0.3, 0.38, 0.5, 0.72)$	$F_{24} = (0.02, 0.05, 0.09, 0.13, 0.22, 0.43)$
$F_3 = (0.60, 0.08, 0.10, 0.12, 0.15, 0.24)$	$F_{31} = (0.37, 0.5, 0.68, 0.82, 1.09, 1.38)$	$F_{31} = (0.02, 0.04, 0.07, 0.1, 0.17, 0.34)$
	$F_{32} = (0.11, 0.14, 0.18, 0.22, 0.29, 0.42)$	$F_{32} = (0.01, 0.01, 0.02, 0.03, 0.04, 0.1)$
	$F_{33} = (0.04, 0.04, 0.05, 0.06, 0.09, 0.13)$	$F_{33} = (0.002, 0.003, 0.01, 0.01, 0.01, 0.03)$
$F_4 = (0.11, 0.20, 0.26, 0.31, 0.39, 0.53)$	$F_{41} = (0.03, 0.03, 0.1, 0.12, 0.3, 0.11)$	$F_{41} = (0.004, 0.01, 0.02, 0.04, 0.12, 0.06)$
	$F_{42} = (0.04, 0.04, 0.11, 0.13, 0.37, 0.61)$	$F_{42} = (0.004, 0.01, 0.03, 0.04, 0.14, 0.33)$
	$F_{43} = (0.35, 0.45, 0.7, 0.85, 1.23, 1.64)$	$F_{43} = (0.04, 0.09, 0.18, 0.27, 0.48, 0.87)$
$F_5 = (0.02, 0.03, 0.03, 0.04, 0.06, 0.12)$	$F_{51} = (0.02, 0.01, 0.13, 0.16, 1.31, 0.23)$	$F_{51} = (0.001, 0.001, 0.005, 0.01, 0.08, 0.03)$
	$F_{52} = (0.02, 0.01, 0.08, 0.1, 0.49, 0.62)$	$F_{52} = (0.001, 0.001, 0.003, 0.004, 0.03, 0.07)$
	$F_{53} = (0.12, 0.09, 0.43, 0.52, 2.21, 0.94)$	$F_{53} = (0.003, 0.003, 0.01, 0.02, 0.13, 0.11)$
	$F_{54} = (0.07, 0.04, 0.27, 0.32, 2.08, 2.55)$	$F_{54} = (0.001, 0.001, 0.01, 0.01, 0.12, 0.3)$

Table 16. Comparison table in linguistic variables by decision maker 1 (DM1).

	DM1	Sub-factor																
		F_{11}	F_{12}	F_{13}	F_{21}	F_{22}	F_{23}	F_{24}	F_{31}	F_{32}	F_{33}	F_{41}	F_{42}	F_{43}	F_{51}	F_{52}	F_{53}	F_{54}
Alternative	Park Street (A_1)	VH	VG	EH	EH	EH	EH	EG	EG	EG	EG	EG	P	EH	EG	EG	EH	EG
	Ballygunge (A_2)	EH	EG	VH	VH	VG	EH	EG	EG	EG	EG	EG	P	EH	P	EG	H	EG
	China Town (A_3)	H	G	H	H	G	H	G	G	G	G	P	G	H	P	G	VL	G
	Hatibagan (A_4)	L	G	L	L	G	L	P	P	G	P	P	VP	H	G	P	VL	VP
	EM Bypass (A_5)	H	VG	L	L	G	L	G	G	G	G	G	G	H	P	G	H	G
	Kolaghat (A_6)	L	G	L	H	G	L	P	G	P	P	G	G	L	G	P	VL	VG
	Airport (A_7)	L	G	L	H	G	H	G	G	P	G	G	G	L	G	P	VL	G

TABLE 17. Comparison table in linguistic variables by decision maker 2 (DM2).

	DM2	Sub-factor																
		F_{11}	F_{12}	F_{13}	F_{21}	F_{22}	F_{23}	F_{24}	F_{31}	F_{32}	F_{33}	F_{41}	F_{42}	F_{43}	F_{51}	F_{52}	F_{53}	F_{54}
Alternative	Park Street (A_1)	VH	VG	EH	EH	EH	EH	EG	EG	EG	EG	EG	P	EH	EG	EG	EH	EG
	Ballygunge (A_2)	EH	VG	VH	VH	VG	EH	VG	VG	EG	VG	VG	P	EH	P	EG	H	EG
	China Town (A_3)	H	G	H	H	G	H	G	G	G	G	P	G	H	P	G	VL	G
	Hatibagan (A_4)	L	P	L	L	G	L	P	P	G	P	P	VP	H	G	P	VL	VP
	EM Bypass (A_5)	H	G	L	L	G	L	G	G	G	G	G	G	H	P	G	H	G
	Kolaghat (A_6)	VL	G	L	H	G	L	P	G	P	P	G	G	L	G	P	VL	G
	Airport (A_7)	L	G	L	H	G	H	G	G	P	G	G	G	L	G	P	L	VG

TABLE 18. Relative distance between alternatives and ranks by evaluating data of relative closeness by FTOPSIS method.

Alternatives	L_P+	L_P-	$R_p = \frac{L_p^-}{L_p^+ + L_p^-}$	Ranking
Park Street (A_1)	0.038218	0.968535	0.962038	1
Ballygunge (A_2)	0.11621	0.916555	0.887477	2
China Town (A_3)	0.463056	0.571668	0.552483	3
Hatibagan (A_4)	0.684387	0.349598	0.338108	6
EM Bypass (A_5)	0.504143	0.523579	0.509456	4
Kolaghat (A_6)	0.908425	0.12678	0.122468	7
Airport (A_7)	0.584408	0.442658	0.430993	5

TABLE 19. Relative distance between alternatives and ranks by evaluating data of relative closeness by FCOPRAS method.

Alternatives	Sq^+	Sq^-	Cq	$R(\%)$	Ranking
Park Street (A_1)	1.501891	0.038295	1.595873	159.5873	1
Ballygunge (A_2)	1.464773	0.034311	1.569666	156.9666	2
China Town (A_3)	1.252848	0.054506	1.318877	131.8877	3
Hatibagan (A_4)	1.158175	0.062447	1.215808	121.5808	6
EM Bypass (A_5)	1.243346	0.054506	1.309376	130.9376	4
Kolaghat (A_6)	1.189432	0.161916	1.21166	121.166	7
Airport (A_7)	1.207976	0.062447	1.265609	126.5609	5

6.1. Computational complexity

In this section we describe the computational complexity for proposed fuzzy MCDM model. The computational complexity idea is not new (see [60–62]), but here we find the same for our problem. The number of calculation has been use to determine the time complexity which is denoted by T on this problem. We also denoting i as the number of factors, j as the number of sub-factors, k as the number of alternatives and N as the number of decision makers. Therefore the following steps are taking the calculating the computational complexity.

(1) Each FAHP comparison matrix is of i^2 entries, therefore the entries given by N DMs is of $N \times i^2$ entries. To find the comparison matrix need Ni^2 number operations. Then for defuzzification process i^2 operations need and for normalised the defuzzified comparison matrix also i^2 operations performed. Then for nth root and factor weight there are $2i$ operations. Factor weight calculated by i^2 operations also. There after factor

sum and sum/weight calculated by $2i$ operations. Finally consistency ratio calculated by 3 more operations. The total calculation needs $N \times i^2 + i^2 + i^2 + 2i + i^2 + 2i + 3 = (N+3)i^2 + 4i + 3$ number of operations.

(2) For Hexagonal Fuzzy Weight of factors and sub-factors, calculate geometric mean by i^2 operations. For sum, inverse and calculate factor weight we performed $i + i + i^2 = i^2 + 2i$ calculation. Total $i^2 + i^2 + 2i = 2i^2 + 2i$ calculation conducted.

For sub-factor weight, $j = j_1 + j_2 + \cdots + j_i$ number of sub-factors with N decision makers is there. So for the comparison matrix need to calculated $N(j_1^2 + j_2^2 + \cdots + j_i^2)$ operations. For geometric mean conducted $j_1^2 + j_2^2 + \cdots + j_i^2$ operations. Then for sum and inverse there are $2 \times (j_1 + j_2 + \cdots + j_i)$ operation needed. Finally, calculated sub-factor weight by $j_1^2 + j_2^2 + \cdots + j_i^2$ operations. Then total operations of sub-factor weight is $N(j_1^2 + j_2^2 + \cdots + j_i^2) + (j_1^2 + j_2^2 + \cdots + j_i^2) + 2 \times (j_1 + j_2 + \cdots + j_i) + (j_1^2 + j_2^2 + \cdots + j_i^2) = (N+2)(j_1^2 + j_2^2 + \cdots + j_i^2) + 2(j_1 + j_2 + \cdots + j_i)$.

Grand total calculation for factor and sub-factor weight calculation is $2(i^2+i) + (N+2)\big(j_1^2 + j_2^2 + \cdots + j_i^2\big) + 2(j_1 + j_2 + \cdots + j_i)$.

(3) For FTOPSIS method, decision matrix is $k \times j$ entries with N DMs, so there are Njk entries. To construct the decision matrix Njk operations. For findings normalized and weighted normalized decision matrix need to performed $2jk$ operations. For finding positive and negative ideal solution there are $2j$ number of operations conducted. Measure the distance form positive and negative ideal solution there are $2jk$ operations and for calculated total sum $2k$ numbers of operations performed. Finally compassion ratio and ranking the alternatives $2k$ operations needed. Then total $Njk + 2jk + 2j + 2jk + 2k + 2k = (N+4)jk + 2(j+k)$ number of operations performed.

(4) For FCOPRAS methodology, up to weighted normalized decision matrix $Njk + 2kj$ calculation performed. Then calculated sum of beneficial and non-beneficial attributes $2k$ number of operations performed. For defuzzification process $2k$ number of operation conducted. Then calculated Q_i values for k operations conducted. Finally k number of operation performed to rank the alternatives. Total $Njk + 2kj + kj + 2k + 2k + k + k = (N+3)jk + 6k$ number of operations conducted.

Thus the time complexity of this study T is calculated as factor $i = 5$, sub-factor $j = 17$, alternatives $k = 7$ and decision maker $N = 2$ are given as follows

- For FAHP, number of calculations are $(2+3) \times 5^2 + 4 \times 5 + 3 = 148$.
- For weight, number of operations are $2 \times 5^2 + 2 \times 5 = 60$.
- For FTOPSIS, number of operations are $(2+4) \times 17 \times 7 + 2 \times (17+7) = 762$.
- For FCOPRAS, number of calculations are $(2+3) \times 17 \times 7 + 6 \times 7 = 637$.

Then the total time complexity $T = 148 + 60 + 762 + 637 = 1607$.

7. Comparative analysis

Comparative study has been conducted to validate the consistency and robustness of the ranking. To check the efficacy of the methods used, FTOPSIS and FCOPRAS ranking tools are integrated with various uncertain environment *i.e.*, triangular fuzzy number (TFN), trapezoidal fuzzy number (TrFN) and pentagonal fuzzy number (PFN) with respect to hexagonal fuzzy number (HFN).

7.1. Comparative analysis using FTOPSIS method

Table 20 shows the consistency of alternatives ranking using FTOPSIS method inherent with FAHP in the number TFN, TrFN, PFN & HFN and Figure 9 depicts the graphical presentation of the alternatives ranking.

Remarks 2. Form Table 20 and Figure 9 we see that, the FTOPSIS ranking of the alternatives on the four fuzzy numbers are same. Therefore we can conclude that the ranking of alternatives is consistence with respect to HFN also.

TABLE 20. Ranking of the alternatives on the bases of different fuzzy numbers using FTOPSIS method.

Alternatives	TFN	TrFN	PFN	HFN
Park Street (A_1)	1	1	1	**1**
Ballygunge (A_2)	2	2	2	**2**
China Town (A_3)	3	3	3	**3**
Hatibagan (A_4)	6	6	6	**6**
EM Bypass (A_5)	4	4	4	**4**
Kolaghat (A_6)	7	7	7	**7**
Airport (A_7)	5	5	5	**5**

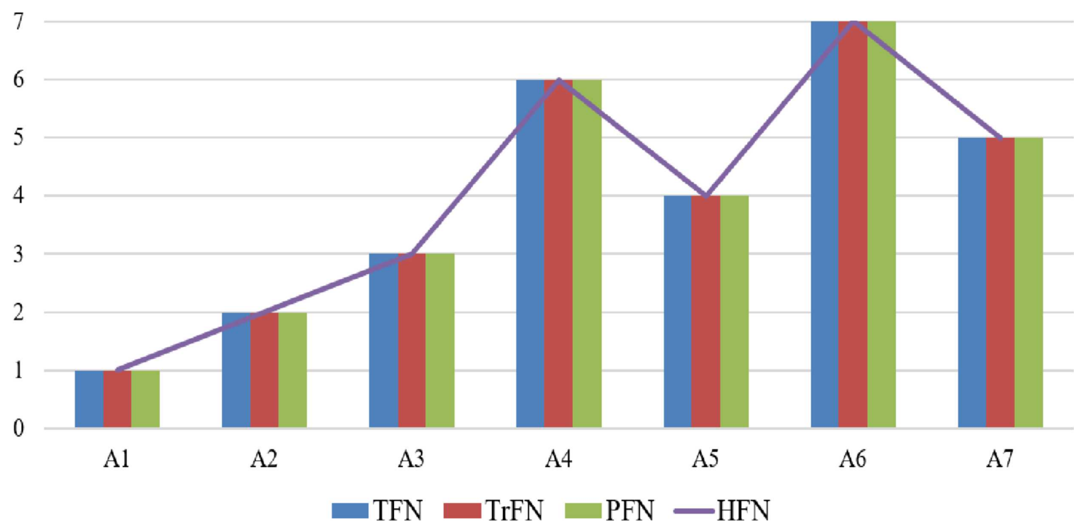

FIGURE 9. Illustration of comparative ranking of the alternatives using TFN, TrFN and PFN with HFN using FTOPSIS.

TABLE 21. The alternatives ranking on the bases of different fuzzy numbers using FCOPRAS method.

Alternatives	TFN	TrFN	PFN	HFN
Park Street (A_1)	1	1	1	**1**
Ballygunge (A_2)	2	2	2	**2**
China Town(A_3)	3	3	3	**3**
Hatibagan (A_4)	7	7	7	**6**
EM Bypass (A_5)	4	4	4	**4**
Kolaghat (A_6)	6	6	6	**7**
Airport (A_7)	5	5	5	**5**

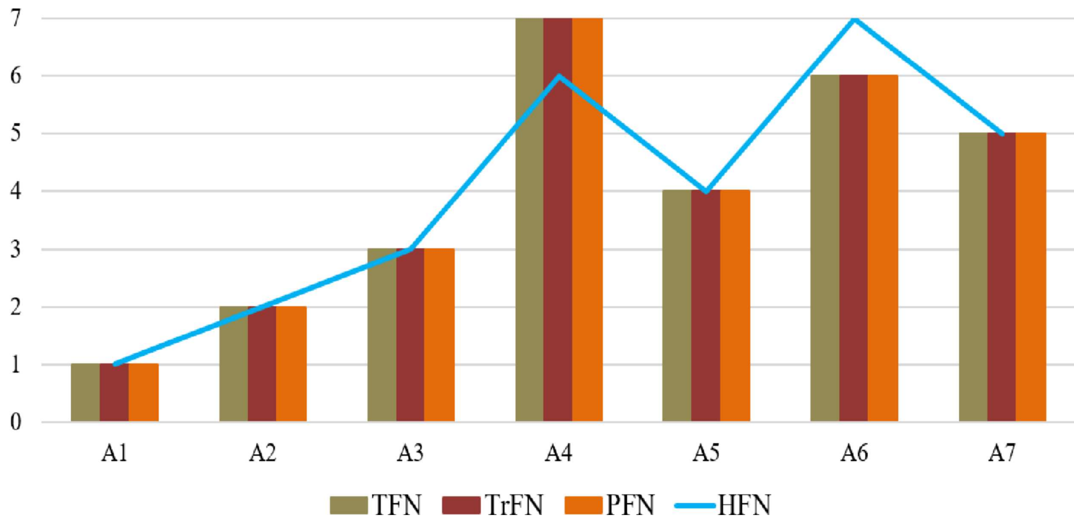

Figure 10. Demonstration of alternatives comparative ranking using TFN, TrFN and PFN with HFN using FCOPRAS.

7.2. Comparative analysis using FCOPRAS method

Table 21 exhibit the steadiness of alternatives ranking using FCOPRAS method inherent with FAHP in the number TFN, TrFN, PFN & HFN and Figure 10 depicts the representation of the alternatives ranking diagrammatically.

Remarks 3. Table 21 and Figure 10 shown that the ranking of the alternatives is same for Park Street (A_1), Ballygunge (A_2), China Town (A_3), EM Bypass (A_5) and Airport (A_7) but the position of the alternatives Hatibagan (A_4) and Kolaghat (A_6) is interchanged for fuzzy numbers TFN, TrFN and PFN with HFN respectively. Therefore, from comparative analysis, we conclude that HFN measure uncertainty of the DMs, gives most probable and effective ranking of the alternatives. HFN shows proximity to the optimal result as it is observed that FTOPSIS and FCOPRAS gives the same ranking of the alternatives.

8. Sensitivity analysis

Sensitivity analysis has been carried out to check the sensitivity of rankings based on different priorities for different segment of customers. Our interaction with customers revealed that same customers may behave differently based on the occasion and time availability. Thus the following cases of sensitivity analysis has been taken into consideration.

8.1. Interchange sub-factors Price (F_{11}) and Food Quality (F_{12})

Individual priorities for price of the food and food quality varies. There are customers sensitive to price and some are sensitive to food quality. Thus this interchange of weightage is carried out to conduct the sensitivity analysis.

Table 22 and Figure 11 represents the ranking obtained through the interchange the weights for the two sub-factors "Price (F_{11})" and "Food Quality (F_{12})". It shows the impact of interchanging weights.

TABLE 22. Ranking of two MCDM methods (FTOPSIS and FCOPRAS) by interchange two sub-factors Price (F_{11}) and Food Quality (F_{12}).

Alternatives	FTOPSIS ranking	FCOPRAS ranking
Park Street (A_1)	1	1
Ballygunge (A_2)	3	2
China Town (A_3)	4	4
Hatibagan (A_4)	7	7
EM Bypass (A_5)	6	6
Kolaghat (A_6)	2	3
Airport (A_7)	5	5

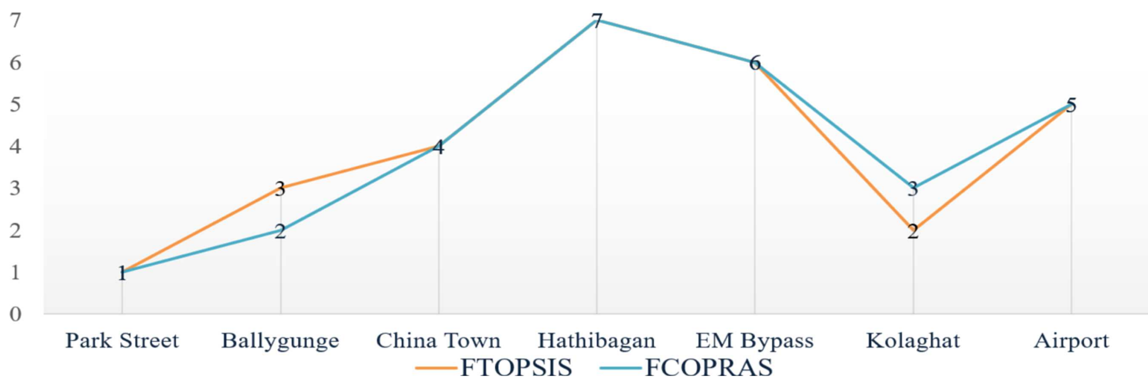

FIGURE 11. *Representation of sensitivity analysis:* Interchange two Sub-factors Price (F_{11}) and Food Quality (F_{12}) the ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

8.2. Interchange Service Quality (F_{21}) and Quick Service (F_{23})

For customers with time constraint, quick service will be priority and for customers without time constraint, may behave differently while selecting restaurant. Thus these two weights are interchanged to carry out sensitivity analysis.

8.3. Removing the sub-factor Parking (F_{42})

For a segment of customers, parking facility is not under consideration as they avail public conveyance, hired cars where no parking facility is required. Thus, for such customers parking weight will be 0.

Table 23 represents the ranking obtained through the interchange the weights for the two sub-factors "Service Quality (F_{21})" and "Quick Service (F_{23})" and removing the Sub-factor "Parking (F_{42})". It can be observe removal of Parking has not impacted the ranking significantly.

Figure 12 represent of interchange two Sub-factors "Service Quality (F_{21})" and "Quick Service (F_{23})" and removing the Sub-factor "Parking (F_{42})" effects on Ranking of alternatives by two MCDM methods.

8.4. Removing the sub-factor Price (F_{11})

For a segment of customers price is not under consideration as they have affluence to afford it. Hence food quality is getting more importance and price has no weightage for those customers.

The ranking of Fuzzy TOPSIS and Fuzzy COPRAS are calculated from the hexagonal fuzzy weightage of the sub-factors by removing the sub-factor "Price (F_{11})" are describe in Table 24 and Figure 13.

Table 23. Ranking of two MCDM methods (FTOPSIS and FCOPRAS) by interchange two sub-factors Service Quality (F_{21}) and Quick Service (F_{23}) and removing the sub-factor Parking (F_{42}).

Alternatives	FTOPSIS ranking	FCOPRAS ranking
Park Street (A_1)	1	1
Ballygunge (A_2)	2	2
China Town (A_3)	3	3
Hatibagan (A_4)	6	6
EM Bypass (A_5)	4	4
Kolaghat (A_6)	7	7
Airport (A_7)	5	5

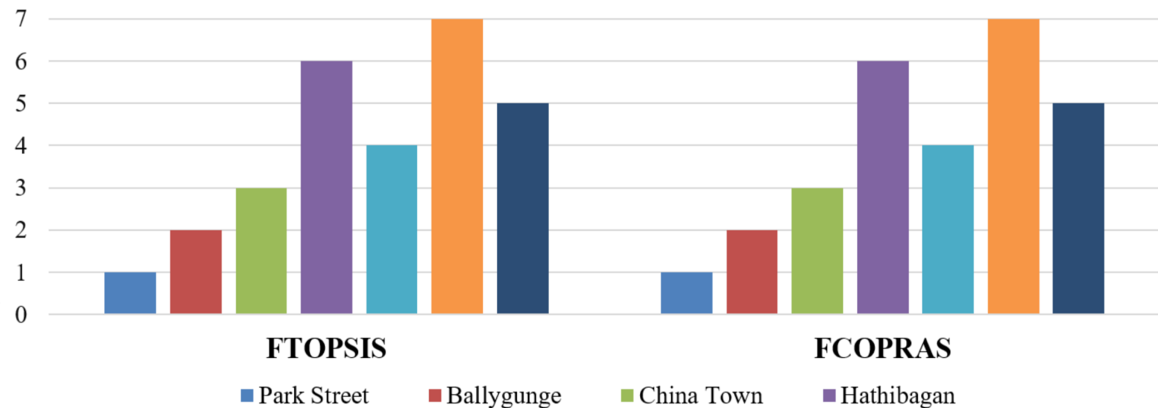

Figure 12. *Representation of sensitivity analysis:* Interchange two sub-factors Service Quality (F_{21}) and Quick Service (F_{23}) and removing the sub-factor Parking (F_{42}) give the same ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

Table 24. Ranking of two MCDM methods (FTOPSIS and FCOPRAS) by removing the sub-factor Price (F_{11}).

Alternatives	FTOPSIS ranking	FCOPRAS ranking
Park Street (A_1)	1	1
Ballygunge (A_2)	2	2
China Town (A_3)	3	3
Hatibagan (A_4)	7	7
EM Bypass (A_5)	4	4
Kolaghat (A_6)	6	6
Airport (A_7)	5	5

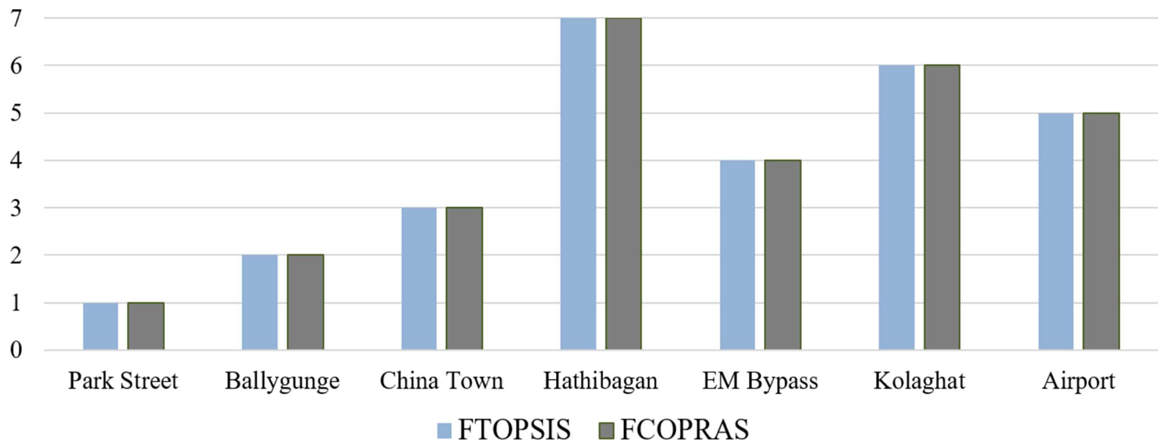

FIGURE 13. *Representation of sensitivity analysis:* Removing the sub-factor Price (F_{11}) the ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

TABLE 25. Ranking of two MCDM methods (FTOPSIS and FCOPRAS) by removing the sub-factor Quick Service (F_{23}).

Alternatives	FTOPSIS ranking	FCOPRAS ranking
Park Street (A_1)	1	4
Ballygunge (A_2)	2	1
China Town (A_3)	3	2
Hatibagan (A_4)	6	6
EM Bypass (A_5)	4	3
Kolaghat (A_6)	7	7
Airport (A_7)	5	5

8.5. Removing the sub-factor Quick Service (F_{23})

Many a times, for family outing and other occasions, the customers have plenty of time and hence quick service facility is not under consideration at all.

Table 25 and Figure 14 represents the ranking obtained through the removing the sub-factor "Quick Service (F_{23})".

8.6. Removing the sub-factors Word of Mouth (F_{32}) and Online Review (F_{33})

Customers with prior good experience are not interested in assigning any weightage to word of mouth and online review. Their own excellent experience will be the sole deciding factors for a section of customers.

The rankings using Fuzzy TOPSIS and Fuzzy COPRAS are calculated from the Hexagonal Fuzzy weightage of the sub-factors by removing the Sub-factors "Word of Mouth (F_{32})" and "Online Review (F_{33})" are describe in Table 26 and Figure 15.

8.7. Removing the sub-factor Quick Meal (F_{51})

For restaurant selection, while having plenty of time to spend customers like to enjoy the pleasant environment for longer duration, such situation reduces the quick meal weightage to 0.

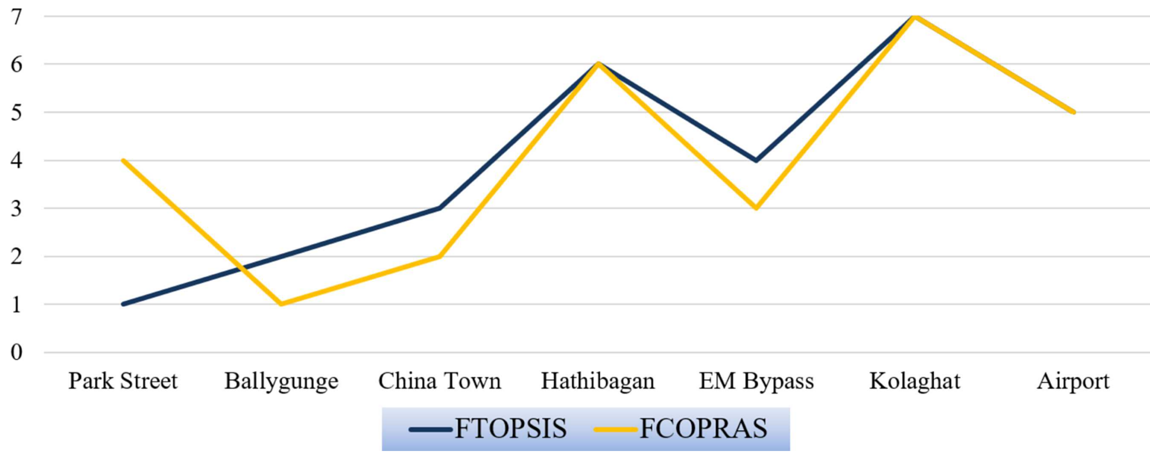

FIGURE 14. *Ranking representation of sensitivity analysis:* Removing the sub-factor Quick Service (F_{23}) the ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

TABLE 26. Ranking of two MCDM methods (FTOPSIS and FCOPRAS) by removing the sub-factors Word of Mouth (F_{32}) and Online Review (F_{33}).

Alternatives	FTOPSIS ranking	FCOPRAS ranking
Park Street (A_1)	1	1
Ballygunge (A_2)	2	2
China Town (A_3)	3	3
Hatibagan (A_4)	6	6
EM Bypass (A_5)	4	4
Kolaghat (A_6)	7	7
Airport (A_7)	5	5

8.8. Removing the sub-factor Social (F_{52})

For business and professional outing, for social outing weightage will be 0.

8.9. Removing the sub-factor Business Necessity (F_{53})

For social, family outings, the business necessity weightage will be 0.

8.10. Removing the sub-factor Celebration (F_{54})

For non-celebratory occasion like food for hunger and quick meal, the celebration weightage will be 0.

Above last four case; section 8.7, removing the sub-factor "Quick Meal (F_{51})", Section 8.8, removing the sub-factor "Social (F_{52})", Section 8.9, removing the sub-factor "Business Necessity (F_{53})" and Section 8.10, removing the sub-factor "Celebration (F_{54})" the ranking using Fuzzy TOPSIS and Fuzzy COPRAS are calculated from the hexagonal fuzzy weightage of the sub-factors are give same ranks which are described in Table 26 and Figure 15.

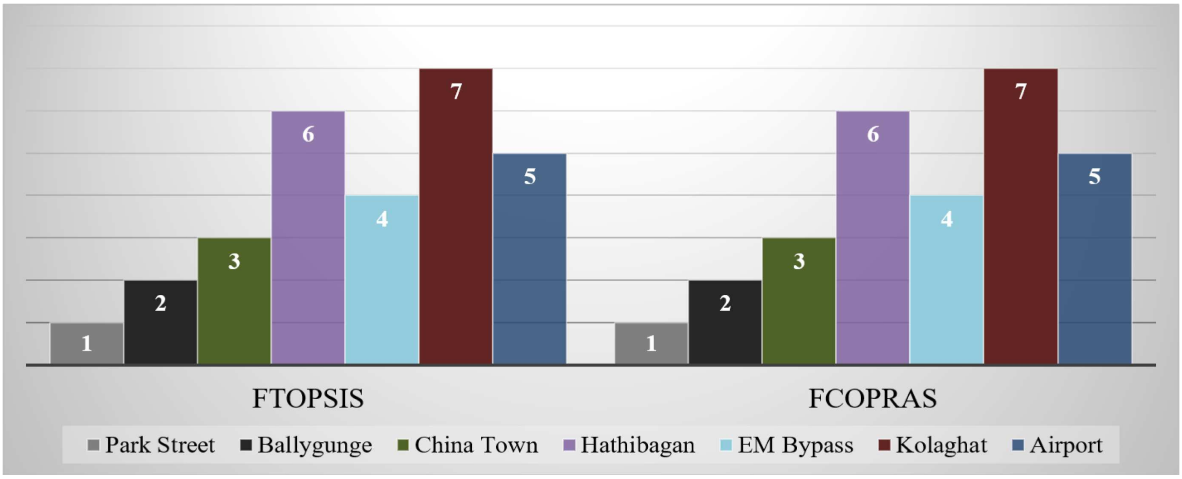

FIGURE 15. *Representation of Sensitivity Analysis:* Removing the sub-factors Word of Mouth (F_{32}) and Online Review (F_{33}) the ranking obtained by the two MCDM technique FTOPSIS and FCOPRAS.

9. Practical implications

The aim of this research is to rank different restaurant locations in the city of Kolkata. For this purpose, we took the data for different factors and sub-factors associated with the said problem by taking hexagonal fuzzy numbers (HFN). This ranking is helpful for entrepreneurs who wish to invest in food vans, food truck, mobile kitchen, mobile canteen, catering truck, food trailer, cloud kitchen and business entities who wish to invest to start restaurant business, location specific utility. It also helps food lover to chose his destination. Similar model can be applied in different cities and problems like tourism location ranking, weekend destination ranking, theme park ranking etc.

10. Conclusion and future research scope

10.1. Summary of problem and contributions

This research aims to evaluate the restaurants location selection using five factors and seventeen sub-factors. The factors and sub-factors play a significant role for selection of restaurants. Seven location of Kolkata, India are chosen as alternatives and those are Park Street, Ballygunge, China Town, Hatibagan, EM Bypass, Kolaghat and Airport. Ranking of location depends on complex and conflicting attributes, experts opinion improves the quality of the decision and help in optimal decision making. HFN is applied to deal with the hesitancy and vagueness of the decision makers (DMs). MCDM tool analytic hierarchy process (AHP) with fuzzy set theory called FAHP is applied to obtain factors and sub factors weight. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and COmplex PRoportional ASsessment (COPRAS) with Fuzzy environment called FTOPSIS and FCOPRAS respectively are used for ranking of the alternatives. Further, comparative analysis and sensitivity analysis are conducted to examine the sturdiness and robustness of the methods used.

10.2. The major findings

The results obtained through this research are discussed in this section. The ranking obtained under FTOPSIS and FCOPRAS method yields the same ranking of the alternatives. The alternative “Park Street” ranks first position followed by “Ballygunge”, “China Town”, “EM Bypass”, “Airport”, “Hatibagan” and “Kolaghat” respectively. Tables 18 and 19 represent the ranking of these methods and Figure 8 graphically demonstrates the ranking. Comparative analysis is performed based on different type of fuzzy number and check consistency and robustness of the alternatives ranking. Table 20 and Figure 9 represent the ranking of alternatives on FTOPSIS method. Similarly, Table 21 and Figure 10 describe the alternatives ranking of FCOPRAS techniques. Sensitivity analysis is conducted, discussed in Section 8 where different cases are considered in which the most sensitive sub-factors weight are interchanged. The results yield through sensitivity analysis reveals the alternative "Park Street" consistently holds the first position. The ranking so attained under sensitivity analysis are represented in Tables 22, 23, 24 and 25. Graphically it is represented in Figures 11, 12, 13 and 14. The findings so obtained using MCDM tools are rational and scientific. It provides future potential work to the researchers.

10.3. The limitations and directions for future research

This research is helpful for entrepreneurs who wish to invest in food vans, food truck, mobile kitchen, mobile canteen, catering truck, food trailer, cloud kitchen and business entities who wish to invest to start restaurant business, location specific utility. One of the limitation of this study is that it involve qualitative assessment on various sub factors like food quality, pleasantness of the environment etc. These qualitative assessments are imprecise and fuzzy in nature. In future research, we can extend the problem with different type of uncertainty settings such as intuitionistic fuzzy, neutrosophic etc. Researchers can explore different sites and find the most important factors and sub factors for a business venture. Similar model can be applied in different cities in other states in India or other countries with different number of alternatives. This study can be applied various problem like tourism location ranking, weekend destination ranking, theme park ranking etc.

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