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A BI-OBJECTIVE ROBUST POSSIBILISTIC COOPERATIVE GRADUAL MAXIMAL COVERING MODEL FOR RELIEF SUPPLY CHAIN WITH UNCERTAINTY

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Abstract. The occurrence of natural and artificial disasters due to their unexpected nature requires precise planning and management in the relief supply chain. A major measure in times of crisis is to assist the damaged points. Due to the limitations in the relief process at the time of the accident, relief centers should be opened in appropriate locations that cover the needs of the damaged points in the shortest possible time. Initially, a nonlinear two-level cooperative gradual maximal covering model in relief supply chain is proposed first. The chain includes supply centers, relief, and damaged points under uncertainty of some key parameters. The major goal is to locate the relief centers and determine the allocations and transfer of goods between the two levels. The bi-objective model minimizes the high logistical costs and maximizes damaged points' coverages with uncertain costs. Different robust possibilistic programming approaches have utilized the given approaches' performances, and some suitable recommendations are given. The robust possibilistic model provides the best results among all models. The results show that the robust possibilistic programming model outperforms the possibilistic programming model.

Mathematics Subject Classification. 90B06, 90B05.

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1. Introduction

Over the past years, natural and artificial disasters like droughts, earthquakes, massive fires, floods, wars, nuclear explosions, and epidemics such as COVID-19 have increased due to various factors like increased population growth and changes in the earth's atmosphere and ecosystem degradation [1]. The occurrence of disasters causes the loss of life from the economic, social, and environmental aspects [2]. As a result, crisis management and formulating effective preparedness programs are important. One of the most effective measures in crisis

Keywords. Relief supply chain, cooperative gradual maximal covering, network design model, robust possibilistic programming, uncertainty.

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management is the proper design of the relief supply chains (RSCs), which accounts for a considerable volume of all activities in relief operations. This chain is important to send basic goods such as water bottles, canned foods, blankets, tents, medicine, and the required services within the shortest time [3]. An RSC usually includes international relief organizations, governments, militaries, and the private sector. Each addressed organization seeks to decrease the damage caused by the disaster by noting its service capacity and available information [4].

1.1. Necessity of this study

Designing a logical RSC network is very important before a disaster. There is no certainty on disasters' severity, timing, and location, and RSC needs high flexibility [5]. Integrated management among the RSC members and access to reliable information on the disaster severity after the disaster find much more important in relief operations [6]. Some other disasters occur during relief operations and cause new disasters [7]. For example, following an earthquake measuring nine on the Richter scale in Japan in 2011, a huge tsunami killed many people and disappeared some others. As a result, the widespread damage caused by disasters has led researchers to pay more attention to the RSC.

The structure and mission of the relief and commercial supply chains are identical from the viewpoint of accurately giving suitable goods. On the other hand, there are many differences between these two chains. The existence of these differences causes many challenges in the RSC. It shows that the concepts and approaches of commercial supply chains could not be directly applied to the RSC because only for its profit concern.

1.2. Research gap in the literature

There is a big gap between the main conceptual ideas between supply chain management (SCM) and relief supply chain management (RSCM). Main aim of the supply chain is to reduce the total cost or to maximize profit. In contrast, the objective of the RSC is the best optimum solution to save human beings during any disaster. There are several steps for a SCM to obtain more profit like, the number of players and their mutual strategies, information, and others whereas for the RSC, the preparedness, strategy, and safety are much more important than an SCM. This study contributes in the direction of RSCM with several distribution centers for relief products distributions, high level transportation or logistic costs and the concerned maximum areas of damage areas. Several research studies considered only single area of convergence, whereas this study considers several area of coverages with the increasing number of relief centers, added continuously until the finalization of the total relief. This study covers the area of maximum covering location problem (MLCP), cooperative maximal covering location problem (CMCLP), gradual maximal covering location problem (GMCLP), and cooperative gradual maximal covering location problem (CGMCLP).

1.3. Contribution of the study

In this study, an RSC structure comprises supply centers, relief distribution centers (DCs), and damaged points to minimize high logistical costs and maximize the damaged points' demand coverage. Some new relief DCs may be added to the existing centers. Regarding coverage, the maximum covering location model (MCLP) considers gradual and cooperative coverages of the damaged points. Figure 1 shows the coverage of most of the damaged points considering gradual and cooperative coverages. As the given figure, if only cooperative coverage is considered, damage point 1 is partially covered by one of the two facilities. Still, considering the cooperative and gradual coverages simultaneously, two facilities can fully cover it. The differences between various coverages are described in the next sections.

1.4. Orientation of the study

Section 2 provides a literature review to show the research gap. In contrast, Section 3 explains the problem description, some basic location models, the primary two-objective model of the problem, and its uncertain versions. In Section 4, numerical examples and sensitivity analysis are conducted. Eventually, Section 5 concludes the findings and further research.

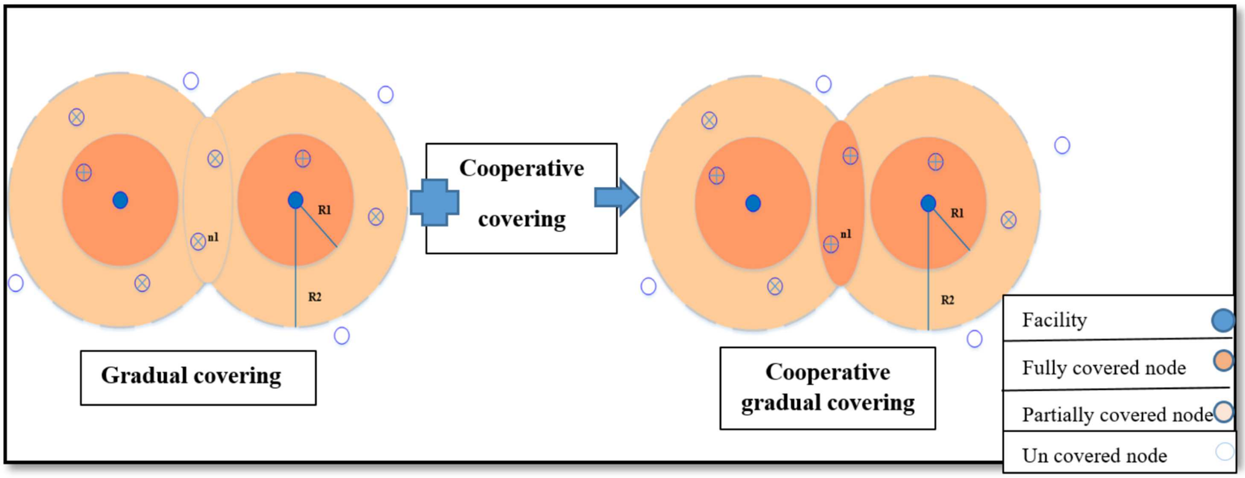

FIGURE 1. Representation of the coverage of damaged points considering gradual and cooperative coverages.

2. Literature review

Depending on the subject, in this section, we initially review the research on supply chain model, covering models with the simultaneous presence of cooperative and gradual coverages, and design models in the RSC. Finally, the existing research gaps are presented to justify the research in this paper.

2.1. Supply chain management perspective

The main aim or goal of a supply chain is to obtain the maximum profit for the supply chain players [8]. Though the profit maximization or cost minimization is continuously going on for the players, but the optimization techniques may not be traditional [9] always, it may need some uncertain or fuzzy optimization [10], robust optimization [11], game strategies optimization [12], ARDL methods [13], or classical method [14]. Some supply chain managements utilized technological development to maintain their own profits [15, 16], but these all studies are really different from RSCM. Because in RSCM, the main aim is to save human's life without the matter of cost minimization or profit maximization even though the resource optimization is there, subject to the matter that the maximum human life, wealth, property, and others can be saved properly. Governments and other non-governmental organizations together utilizes the technological support to prepare for the safety and recover the people after any disaster.

2.2. Covering models with cooperative and gradual coverages

Covering models are among the classic models of facility location problems whose purpose is to locate service facilities at a certain distance (called coverage radius) from demand points to cover them. Covering problems are generally divided into a set covering location problems (SCLP) and MCLP. In SCLP, the decision-maker (DM) must cover all demand points (*i.e.*, at least one service facility at a distance less than the coverage radius) with the minimum number of facilities. While in MCLP, the DM should locate an endless number of service facilities to cover the maximum number of damaged points. Classic covering models have three main assumptions: binary coverage, individual coverage, and fixed coverage radius [17]. Due to the inconsistency of classical covering problems with the real world, the change in these three main assumptions has led to gradual covering, cooperative covering, and covering models with dynamic coverage radius. On the other hand, the binary coverage assumption is violated in the gradual covering location problem (GCLP). In contrast, the cooperative covering location problem (CCLP) violates the individual coverage assumption.

Cooperative and gradual coverage models have been used in many real-world issues. Berman *et al.* [17] first discussed the issue of cooperative coverage, which assumes that all facilities will work together to cover demand points. Baghernejad *et al.* [18] presented a locating model with maximum covering and a cooperative gradual maximal covering model (CGMCM). The model sought to maximize the demand coverage of damaged points. Karatas and Eriskin [19] proposed a minimal coverage problem with distance constrain. Fathali [20] introduced an algorithm for the inverse single-facility location problem with variable weights. The study results showed that simultaneous consideration of cooperative and gradual coverages results in higher demand coverage than cooperative and gradual coverages are considered alone. This study only focused on the coverage level without considering its logistical costs.

2.3. Network design models in the RSC

The transportation of relief goods (RG) between existing warehouses was developed by Barbarosoglu and Arda [21]. Snyder and Daskin [22] developed a two-objective model built on the P-medium problem, considering the possibility of failure and the unavailability of some crisis RCs. A multi-criteria model that considered people's cost, response time, and satisfaction was established by Tzeng *et al.* [23]. A bi-objective model during floods with uncertainty was studied by Chang *et al.* [24]. The objectives were minimizing the distance of damaged points from RCs, and total costs.

Beamon and Balcik [25] proposed a model for locating RCs for rapid response in times of crisis. They used the maximal covering model to locate RCs. Moreover, demand and stored goods in RCs were assumed to be uncertain. Beraldi and Bruni [26] developed a two-stage stochastic model to find the optimal locations of RCs. A multi-transportation policy-based supply chain was designed by Sarkar *et al.* [27]. They aimed to minimize the total supply chain cost by transporting products through any policy but not discussed relief supply chain. Focusing on transportation network, a cross-dock model was studied by Mukherjee *et al.* [28]. They aimed to reduced material handling cost for cross-docking without any emergency planning. A two-stage stochastic model for improving the RGs delivery system was developed by Rawls and Turnquist [29], locating RCs under the uncertainty of some parameters. In this model, different scenarios were defined for parameters with uncertainty. Minimizing the costs of transporting RGs and maximizing justice among demand points are the objectives of this model. Canbolat and Von Massow [30] proposed an RSC model with random location demand points. The objective function of this model was minimizing the distances between demand points and RCs.

Afshar and Haghani [31] developed an RSC model which tackles the flows of RGs from the supplier to the damaged points by considering the routing problem. Product flow through a supply chain management was studied by Padiyar *et al.* [32] under inflation. But, they did not discuss about buffer plans for product damage.. Furthermore, the possibility of breakdown and unavailability of relief and supply centers in a crisis was considered and analyzed. A MIP model for finding medical centers and delivering different RGs was studied by Jeong *et al.* [33]. Wang *et al.* [34] proposed a scenario-based stochastic programming model (SBSPM) for locating medical centers in crisis and utilizing innovative solutions.

Tofighi *et al.* [35] examined an RSC network design problem, which included several central warehouses that fed some local DCs. A two-stage SBSPM to tackle the uncertainties in the model. Zokaee *et al.* [36] considered a three-tier RSC, including suppliers, DCs for reliefs, and uncertain damaged zones. In dealing with the uncertainties raised in this network, robust programming was used. A supply chain network design problem was studied by Javadian *et al.* [1] that included local DCs and central warehouses under uncertainty. An SBSPM was used to tackle the uncertainties. A locating model was developed by Hazrati *et al.* [37] under the possibility of not having access to RCs in a crisis. RCs were grouped into unreliable and reliable centers, in which the reliable ones support unreliable centers. Li *et al.* [38] developed a cooperative maximal covering model (CMCM) for locating RCs under the uncertainty of the demand parameter with three different scenarios. The developed model aimed at maximizing demand points coverages.

TABLE 1. Brief feature of the reviewed research in the RSC.

	Modeling			Type of model parameters			Type of model variables			Programming			Objective		Solving method			Type of facility				RG	
Authors	One-stage	Two-stages	Multi-stage	Definite	Fuzzy	Probabilistic	Integer	Continuous	Fuzzy	Stochastic	Robust	Possibilistic-robust	Single	Multi	Exact	Heurist	Metaheuristic	Central warehouse	DC	Medical center/Shelter	Other	Single	Multi
Javadian *et al.* [1]	*			*		*	*	*		*				*			*	*	*			*	
Mansoori *et al.* [5]	*			*	*		*				*			*	*					*			*
Tzeng *et al.* [23]	*			*			*							*	*				*				*
Beraldi and Bruni [26]		*		*		*	*			*			*		*	*			*			*	
Rawls and Turnquist [29]		*		*		*	*				*			*		*			*			*	
Afshar and Haghani [31]	*			*			*	*					*			*			*				*
Jeong *et al.* [33]	*			*		*	*			*			*		*				*				*
Wang *et al.* [34]	*			*		*	*				*			*	*					*		*	
Tofighi *et al.* [35]		*		*	*	*	*	*		*	*			*			*	*	*				*
Zokaee *et al.* [36]	*			*	*		*				*		*		*				*				*
Hazrati *et al.* [37]	*			*		*	*			*			*		*				*			*	
Li *et al.* [38]	*			*		*	*			*			*		*				*				*
Shavarani [39]	*			*		*	*	*		*			*				*		*		*	*	
Beiki *et al.* [40]	*			*		*	*			*				*	*				*	*		*	
Cheng *et al.* [41]	*			*	*		*	*			*			*	*				*				*
Vosooghi *et al.* [42]		*			*		*			*				*	*		*			*			*
Shokr *et al.* [43]		*		*						*				*	*								*
Sarkar *et al.* [50]	*				*			*	*			*	*			*					*		
This Research	*			*	*		*					*		*	*				*				*

Notes. (*)Indicates the contribution is satisfied; blank space in Table 1 indicates the contribution does not satisfy. DC: distribution centers; RSC: relief supply chain; RG: relief goods.

Mansoori *et al.* [5] studied a multi-objective RSC model that considered the uncertainty of demand and travel times in an earthquake. One major objective of this model was minimizing injuries and homeless people. Two robust counterpart models had been employed in dealing with the uncertainties considering the uncertainty sets in ellipsoid, boxes, and polyhedral. Shavarani [39] proposed a model to locate relief DCs and used drones to transport RGs to the damaged areas. The demand for damaged points was assumed to follow the Poisson process. Beiki *et al.* [40] simultaneously examined the two stages of preparedness, responded in crisis management, and analyzed the crisis management decisions. They considered vehicle routing for distributing RGs and transporting injured people to medical centers. Cheng *et al.* [41] studied humanitarian relief food distribution, how food is received from different suppliers and their fair distribution to needy people. Vosooghi *et al.* [42] developed a network design for RSCM with humanitarian constraint. Shokr *et al.* [43] designed a collaborative relief supply chain for disaster management. For this research, a robust programming model was presented considering the uncertainties in the amount of food needed.

2.4. Research gap analysis

Based on the reviewed research (Table 1), there are the following gaps in the RSC network design.

- No study has been done on RSC design, considering cooperative and gradual coverages. This can cause higher coverage and improve the important ambition of the RSC design.
- In the only existing research conducted in cooperative and gradual coverage, all parameters were definite. Due to the uncertain nature of disasters, historical data in the relief chain may not be reliable. As a result, the model has uncertain parameters that should be tackled robustly.

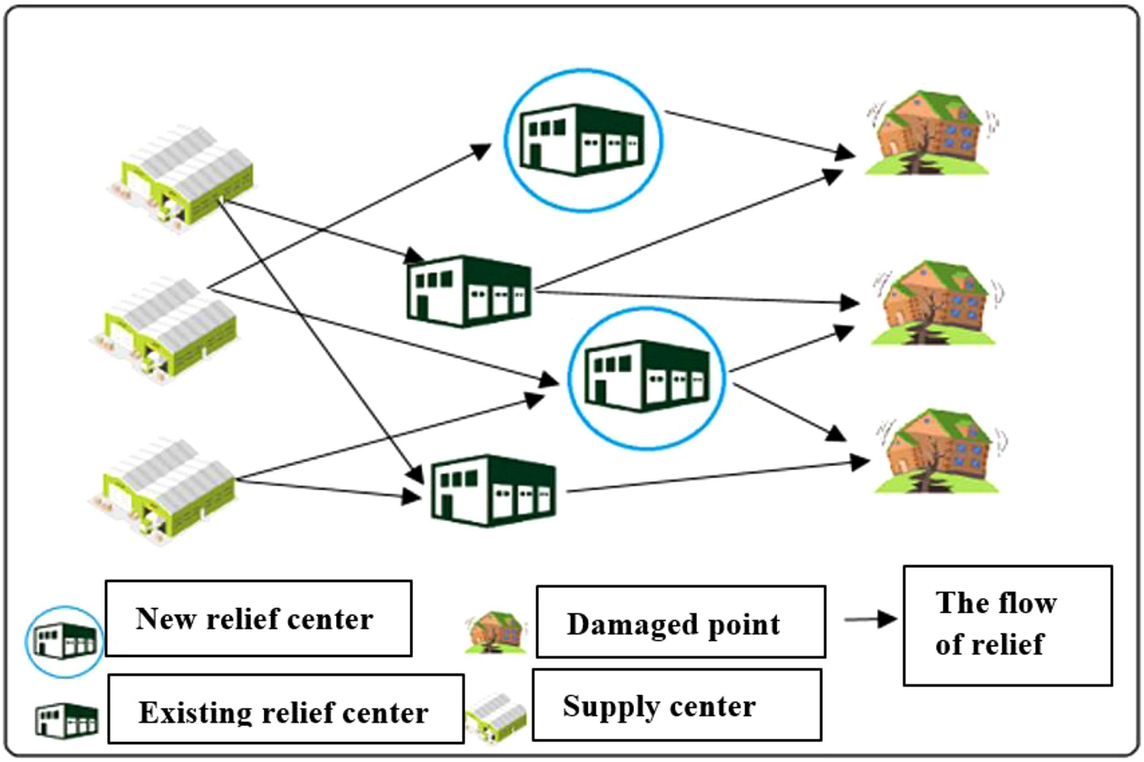

FIGURE 2. The structure of the understudy three-level RSC.

– None of the literature research considers cost minimization and coverage maximization.

Considering the three research gaps, a two-objective CGMCM is proposed under uncertainty of demand and capacity in RSC design. A possibilistic programming (PP) model and different approaches to robust possibilistic programming (RPP) are developed for the basic model. The addressed model aims to minimize total costs against maximizing demand coverage.

3. Problem description and model formulation

The notation and problem formulation are given here.

3.1. Problem description

The most important decisions in RSCs are the allocation of relief goods and services, the coverage level of damaged points, and the location of RCs. A trade-off between two conflicting goals of logistical cost reduction and maximal coverage of the damaged points is essential. The construction of many RCs can increase the coverage level, but on the other side, such a decision involves higher logistical costs and operating costs [44]. In this study, we aim to design an RSC which makes a logical trade-off between the two addressed goals considering uncertainty in some key parameters of the model. The addressed RSC consists of supply centers, RCs, and damaged points, as shown in Figure 2. Some RCs are already located, and some are only candidate locations that may be opened as new RCs.

In this problem, the locations of new RCs, the flows of RGs from the supply centers to the RCs, and then to the damaged points are determined. The model has two objectives: (a) minimizing the total costs, including

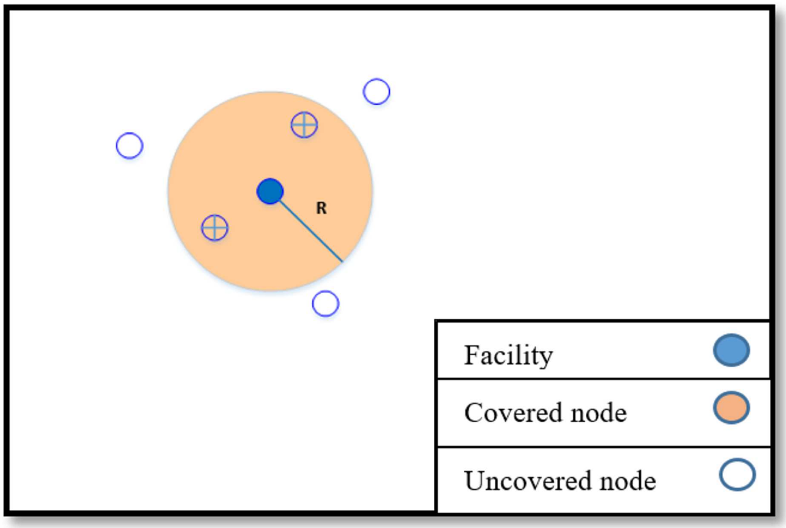

FIGURE 3. Covered and uncovered nodes in MCLP.

opening and operating RCs, transportation of relief materials and services, and inventory carrying costs, and (b) maximization of demand coverage, considering cooperative and gradual coverages simultaneously. The model has the following presumptions:

– Locations of the supply centers are known.
– There may be different quantities and types of goods in the supply centers; these quantities are uncertain.
– RCs are grouped into two categories: (a) RCs which already exist and may be used for serving the damaged points; (b) new RCs, which are candidate centers to be located; the total number of these centers is known.
– RCs have different capacities.
– Priority is given to sending RGs from the existing RCs to the damage points.
– The demand of each damaged point for relied goods may differ from others and is considered uncertain.

The coverage level of each damaged point may be complete coverage, partial coverage, and no coverage due to the simultaneous existence of cooperative and gradual coverages.

– The carrying costs of RGs in the RCs and the transportation costs of the RGs are considered uncertain.

We review the four following basic location problems to make a better introduction to our problem.

– Maximal covering location problem (MCLP).
– Cooperative maximal covering location problem (CMCLP).
– Gradual maximal covering location problem (GMCLP).
– Cooperative gradual maximal covering location problem (CGMCLP).

3.1.1. Maximal covering location problem (MCLP)

Many researchers have studied MCLP. The model covers a demand point with at least one facility. A coverage level is a binary number [45]. Figure 3 shows the coverage of some nodes (*i.e.*, demand points) in this model.

3.1.2. Cooperative maximal covering location problem (CMCLP)

In MCLP, it is assumed that each demand point is covered by only one service facility within its fixed coverage radius; Suppose that each demand point is covered by one facility (or sometimes it is said that the facility sends a signal to the demand point whose value varies with variation in the square of the distance between demand

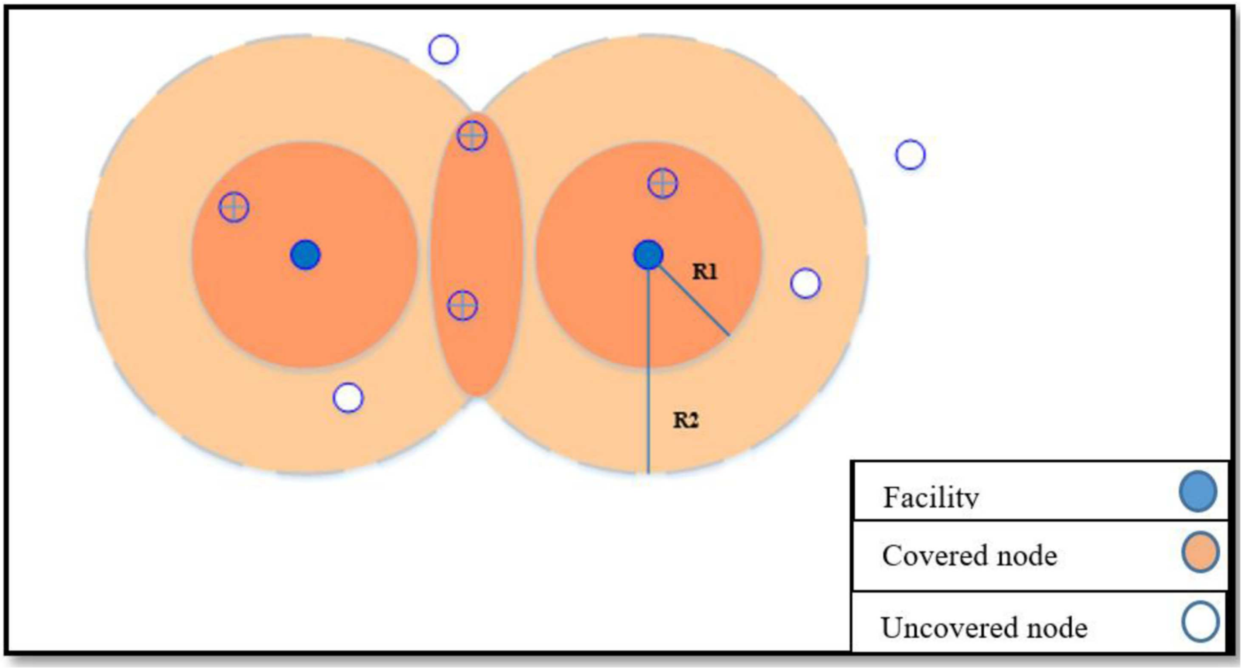

FIGURE 4. The concept of CMCLP.

point and server facility). If a total of the signals received by the demand point from all facilities surpasses the verge of a pre-determined coverage, then only the demand point is covered [45]. In this problem, a certain number of facilities cover each demand point, provided that the coverage exceeds the given coverage threshold. A simple demonstration of the cooperative coverage is depicted in Figure 4. There are some points located within the coverage radius R_1 from each facility. These points are assumed to be covered. On the other side, there are some points within the coverage radius R_2; these points are covered too, provided that the sum of received signals from the two facilities at these points exceeds the coverage threshold.

3.1.3. Gradual maximal covering location problem (GMCLP)

As mentioned earlier, the MCLP's demand points are fully covered only if they are within a facility's coverage radius. A function of the distance between each demand point and server facility (*i.e.*, gradual coverage) is that the coverage level reduces from one to zero as the addressed distance increases from a lower value to an upper value [18].

Figure 5 depicts that the coverage of the nodes (*i.e.*, demand points) is a problem. If the demand point is located in a coverage radius R_1, it is fully covered by the facility. With the increment in the distance from the facility and the coverage radius R_2 with the demand points placed within it, the coverage level takes partial values between one and zero. The points that are placed on the exterior of the coverage radius R_2 are in no way covered. In the GMCLP, we aim to maximize the demand coverage rooted in the gradual coverage of demand points by a limited number of service facilities.

3.1.4. Cooperative gradual maximal covering location problem (CGMCLP)

In this problem, cooperative and gradual coverage are considered simultaneously for the MCLP; thus, in CGMCLP, one or more facilities cover a demand point. It covers a demand point fully and partially. The goal is to maximize demand points' coverage by some given service facilities as much as possible [18]. This problem is illustrated in Figure 6. The coverage of the demand points in the dark zone is complete. That is, all the demand

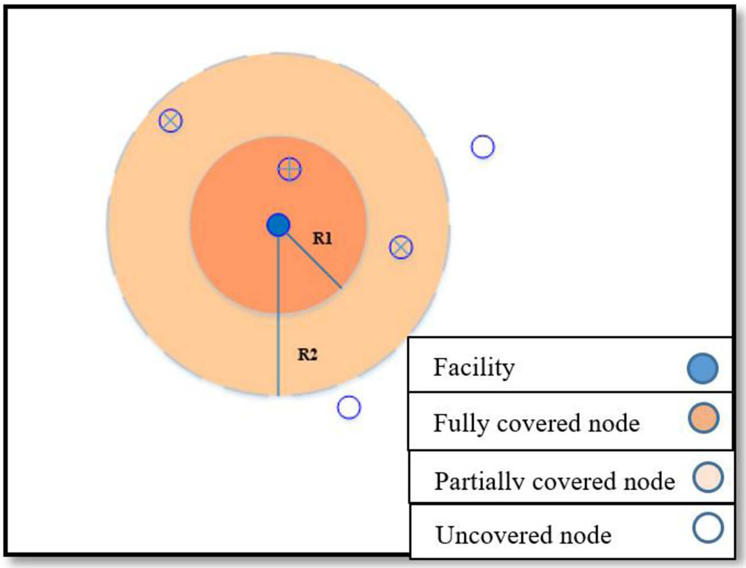

Figure 5. The concept of GMCLP.

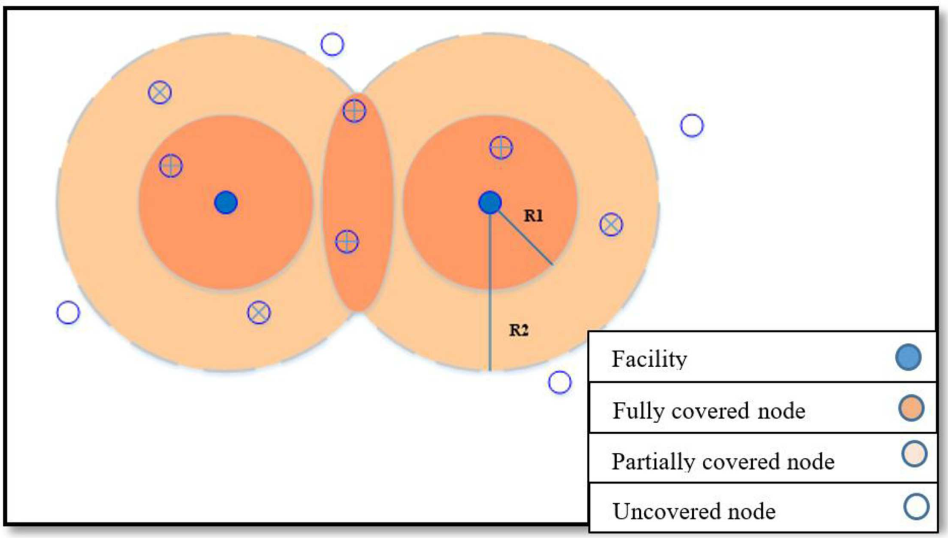

Figure 6. The concept of CGMCLP.

in these points is provided. Demand points in the bright areas are partially covered, and those in the colorless regions are not covered.

3.2. Model formulation

The elucidation of the variables and mathematical templates for the problem in the definite and random states are given.

3.2.1. Notation and problem formulation

Indices

k	index of demand points ($k \in K$)
j	index of potential locations for new RCs ($j \in J$)
r	index of existing RCs ($r \in R$)
i	index of supply centers ($i \in I$)
m	index of RGs ($m \in M$)

Parameters

d_{jk}	distance between demand point k and RC j (calculated by using Euclidean criterion) (kilometer)
s_1	primary coverage radius (kilometer)
s_2	secondary coverage radius (kilometer)
d_{rk}	distance between demand point k and RC r (calculated by using Euclidean criterion) (kilometer)
$f(d_{jk})$	gradual coverage value of demand point k by RC j obtained from equation (1)
$f(d_{rk})$	gradual coverage value of demand point k by RC r obtained from equation (2)

$$f(d_{jk}) = \frac{s_2 - d_{jk}}{s_2 - s_1}, \quad d_{jk} \in [s_1, s_2] \tag{1}$$

$$f(d_{rk}) = \frac{s_2 - d_{rk}}{s_2 - s_1}, \quad d_{rk} \in [s_1, s_2] \tag{2}$$

$\emptyset_{jk}$	signal strength sent from RC j to demand point k obtained from equation (3)
$\emptyset_{rk}$	signal strength sent from RC r to demand point k obtained from equation (4)

$$\emptyset_{jk} = \begin{cases} 1 & d_{jk} \leq s_1 \\ \frac{s_2 - d_{jk}}{s_2 - s_1} & s_1 \leq d_{jk} \leq s_2 \\ 0 & d_{jk} \geq s_2 \end{cases} \tag{3}$$

$$\emptyset_{rk} = \begin{cases} 1 & d_{rk} \leq s_1 \\ \frac{s_2 - d_{rk}}{s_2 - s_1} & s_1 \leq d_{rk} \leq s_2 \\ 0 & d_{rk} \geq s_2 \end{cases} \tag{4}$$

a_{jk}	if demand point k is within the primary or secondary coverage radius of RC j, it equals 1; otherwise, it equals 0
a_{rk}	if demand point k is within the primary or secondary coverage radius of RC r, it equals 1; otherwise, it equals 0
N_{mi}	if m type goods are available at supply center i, it equals 1; otherwise, it equals 0
u_r	if RC r is available, it equals 1; otherwise, it equals 0
C_j	capacity of RC j (units)
C_r	capacity of RC r (units)
P	number of new RCs to be located
V_m	volume of m type goods (units)
ρ_m	priority of m type goods

Random variables

$\widetilde{S}_{mi}$	amount of m type goods stored at supply center i (units)
$\widetilde{d}_{mk}$	demand of m type goods at demand point k (units)
$\widetilde{F}_r$	fixed operating cost of RC r ($)
$\widetilde{C}_{mij}$	transportation cost of each unit of m type goods from supply center i to RC j ($/shipment)
$\widetilde{F}_j$	fixed cost of opening new RC j ($/center)

$\widetilde{C}_{mir}$	cost of each unit of m type goods for transportation from supply center i to RC r (\$/unit)
$\widetilde{C}_{mjk}$	cost of each unit of m type goods for transportation from RC j to demand point k (\$/unit)
$\widetilde{C}_{mrk}$	cost of each unit of m type goods transportation from RC r to demand point k (\$/unit)
$\widetilde{h}_{mr}$	carrying cost of each unit of m type goods in RC r (\$/unit/unit time)
$\widetilde{h}_{mj}$	carrying cost of each unit of m type goods in RC j (\$/unit/unit time)

Decision variables

x_{mij}	amount of m type goods moved from supply center i to RC j (units)
x_{mir}	amount of m type goods carried from supply center i to RC r (units)
y_{mjk}	amount of m type goods transported from RC j to demand point k (units)
y_{mrk}	amount of m type goods transported from RC r to demand point k (units)
t_k	if demand point k is covered, it takes the value 1, else 0
Q_j	if the location of RC is at candidate location j, takes the value 1, else 0
Z_r	if RC r is utilized, it equals 1; otherwise, it equals 0

The mathematical model can be stated as in equations (5)–(17):

$$\begin{aligned}\text{Min } f_1 = &\sum_j \widetilde{F}_j Q_j + \sum_r \widetilde{F}_r Z_r + \sum_{m,i,j} \widetilde{h}_{mj} x_{mij} + \sum_{m,i,r} \widetilde{h}_{mr} x_{mir} + \sum_{i,j,m} \widetilde{C}_{mij} x_{mij} + \sum_{i,r,m} \widetilde{C}_{mir} x_{mir} \\ &+ \sum_{k,j,m} \widetilde{C}_{mjk} y_{mjk} + \sum_{k,r,m} \widetilde{C}_{mrk} y_{mrk}\end{aligned} \tag{5}$$

$$\text{Max } f_2 = \sum_{k,m} \rho_m \left(\sum_j y_{mjk} + \sum_r y_{mrk} \right) \tag{6}$$

such that

$$\sum_j Q_j = P \tag{7}$$

$$\sum_j a_{jk} Q_j + \sum_r a_{rk} Z_r \geq t_k \qquad \forall k \in K \tag{8}$$

$$\begin{aligned}\sum_j y_{mjk} + \sum_r y_{mrk} \leq \min\Bigg(\Bigg(&\sum_j \varnothing(d_{jk}) Q_j \\ &+ \sum_r \varnothing(d_{rk}) Z_r\Bigg) \cdot t_k, t_k\Bigg) \cdot \widetilde{d}_{mk} \qquad \forall m \in M,\ \forall k \in K\end{aligned} \tag{9}$$

$$\sum_j x_{mij} + \sum_r x_{mir} \leq N_{mi} \widetilde{s}_{mi} \qquad \forall m \in M,\ \forall i \in I \tag{10}$$

$$\sum_{i,m} V_m x_{mij} \leq C_j Q_j \qquad \forall j \in J \tag{11}$$

$$\sum_{i,m} V_m x_{mir} \leq C_r Z_r \qquad \forall r \in R \tag{12}$$

$$\sum_k y_{mjk} \leq \sum_i x_{mij} \qquad \forall m \in M,\ \forall j \in J \tag{13}$$

$$\sum_k y_{mrk} \leq \sum_i x_{mir} \qquad \forall m \in M,\ \forall r \in R \tag{14}$$

$$Z_r \leq u_r \qquad \forall r \in R \tag{15}$$
$$t_k, Z_r, Q_j \in \{0,1\} \qquad \forall k \in K,\ \forall r \in R,\ \forall j \in J \tag{16}$$
$$x_{mij}, x_{mir}, y_{mjk}, y_{mrk} \geq 0 \qquad \forall m \in M,\ \forall j \in J,\ \forall r \in R,\ \forall k \in K. \tag{17}$$

The archetype is a nonlinear model with two objectives. Firstly, the function given in equation (5) seeks to minimize total costs, including the opening costs of new RCs, the operational costs of utilizing the existing RCs, the carrying costs of goods in RCs, the costs of goods for transportation from supply centers to RCs and then to the demand points. The second objective function in equation (6) seeks to cover all possible demand points. Restriction (7) indicates the number of opened new RCs. Restraint (8) guarantees that each demand points within the coverage radius s_1 or s_2 of an RC can be covered. Constraint (9) ensures that once the corresponding RCs find the full or partial coverage of demand points, the number of goods sent to those demand points must be less than or equal to the amount of its required demand. The expression $\min\Big(\Big(\sum_j \emptyset(d_{jk})Q_j + \sum_r \emptyset(d_{rk})Z_r\Big)\cdot t_k, t_k\Big)$ in this constraint may take one of the following values due to the cooperative and gradual coverage.

If $\min\Big(\Big(\sum_j \emptyset(d_{jk})Q_j + \sum_r \emptyset(d_{rk})Z_r\Big)\cdot t_k, t_k\Big) = 1$; The coverage level is full.

If $\min\Big(\Big(\sum_j \emptyset(d_{jk})Q_j + \sum_r \emptyset(d_{rk})Z_r\Big)\cdot t_k, t_k\Big) < 1$; The coverage level is partial.

If $\min\Big(\Big(\sum_j \emptyset(d_{jk})Q_j + \sum_r \emptyset(d_{rk})Z_r\Big)\cdot t_k, t_k\Big) = 0$; Demand point is not covered.

Constraint (10) guarantees that if m-type goods are available at supply center i, the sum of m-type goods delivered from supply center i to all RCs is equal to or less than its storage capacity. Constraints (11) and (12) control the capacities of opened and existing RCs. Constraints (13) and (14) ensure that the number of output goods of type m from opened and existing RCs equals or less than the number of input goods of type m to opened and existing RCs. Constraint (15) ensures that if an RC is available, it can be used. Constraints (16) and (17) show the status of the model decision variables.

3.2.2. Linearization of the archetype

The proposed archetype is nonlinear due to the min expression in constraint (9). An auxiliary variable n_k is defined to linearize the addressed constraint as in equation (18).

$$n_k = \min\left(\left(\sum_j \emptyset_{jk}Q_j + \sum_r \emptyset_{rk}z_r\right)t_k, t_k\right). \tag{18}$$

Since there are still nonlinear terms in the given equation, two auxiliary variables of m_{jk} and mm_{rk} are used to represent $Q_j \cdot t_k$ and $z_r \cdot t_k$, respectively; thus, constraints (19)–(26) are added to the initial model given by (5)–(17).

$$m_{jk} = Q_j t_k \tag{19}$$
$$mm_{rk} = z_r t_k \tag{20}$$
$$Q_j \geq m_{jk} \qquad \forall j, k \tag{21}$$
$$t_k \geq m_{jk} \qquad \forall j, k \tag{22}$$
$$m_{jk} \geq Q_j + t_k - 1 \qquad \forall j, k \tag{23}$$
$$z_r \geq mm_{rk} \qquad \forall r, k \tag{24}$$
$$t_k \geq mm_{rk} \qquad \forall r, k \tag{25}$$
$$mm_{rk} \leq z_r + t_k - 1 \qquad \forall r, k. \tag{26}$$

Now, by putting the equivalent values in equations (19) and (20) in constraint (18) and defining a new binary variable i_k and a very large positive number M, constraint (18) can be replaced with constraints (27)–(30).

$$n_k \leq t_k \qquad \forall k \tag{27}$$

$$n_k \leq \sum_j \emptyset_{jk} m_{jk} + \sum_r \emptyset_{rk} mm_{rk} \qquad \forall k \tag{28}$$

$$n_k \geq t_k - (Mi_k) \qquad \forall k \tag{29}$$

$$n_k \geq \sum_j \emptyset_{rk} m_{jk} + \sum_r \emptyset_{rk} mm_{rk} - M(1 - i_k) \qquad \forall k. \tag{30}$$

Constraints (29) and (30) guarantee that n_k takes exactly one of the given values in the minterm.

Note that in addition to constraints (19)–(30), the two following constraints should be considered as in (31) and (32).

$$i_k, m_{jk}, mm_{rk} \in \{0, 1\} \qquad \forall k \in K,\ \forall r \in R,\ \forall j \in J \tag{31}$$

$$n_k \in [0, 1] \qquad \forall k \in K. \tag{32}$$

In all, we should replace constraint (9) with the following constraint given in (33) together with constraints (19)–(32).

$$\sum_j y_{mjk} + \sum_r y_{mrk} \leq\ n_k \cdot \widetilde{d}_{mk}. \tag{33}$$

We can summarize the model as equations (5), (6) subject to constraints (7), (8) and (10)–(17) and (19)–(33), noting that constraints (19)–(33) are the linear equivalent of constraint (9).

3.3. Development of robust possibilistic programming (RPP) models

The robustness of a mathematical model is important when there is uncertainty in the model. Robustness means that the model's output should not be too sensitive to the exact values of the input parameters [46]. Robustness should be of two types in mathematical models: feasibility, robustness and optimality robustness. In the first one, the solution space should remain as feasible as possible for different values of uncertain parameters. In contrast, in the second one, different optimal solutions for different values of uncertain parameters should remain as close to a given value as possible. PP and RPP are two approaches for tackling uncertainties, but the results of possibilistic programming are not always reliable [47, 48].

Pishvaee *et al.* [48] proposed six different models for RPP as follows:

– Robust possibilistic programming 1 (RPP-I) model with an expected cost value and a coefficient of the difference in values between the maximum and the minimum cost as the objective function plus the added values due to deviations from fuzzy constraints.
– Robust possibilistic programming 2 (RPP-II) model with an expected value of cost and a coefficient of the difference in values of the maximum and the expected values of cost as the objective function plus the added values due to deviations from fuzzy constraints.
– Robust possibilistic programming 3 (RPP-III) model with an expected value of cost as the objective function and a coefficient of the maximum cost value plus the added values due to deviations from fuzzy constraints.
– Modified robust possibilistic programming (MRPP) model with an expected value of cost as the objective function and a coefficient of the difference in values of the maximum and expected values of cost plus the less conservative added values due to deviations from fuzzy constraints.
– Hard worst-case robust possibilistic programming (HWRPP) model with the worst-case values of their uncertain parameters as the objective function and constraints.
– Soft worst-case robust possibilistic programming (SWRPP) model, which is similar to RPP1 except for replacing the maximum value of cost with the expected cost value and a coefficient of the gap between the maximum and the minimum values of the cost of RPP1 [48].

In this study, due to the given uncertainties on some parameters (including carrying cost, transportation cost, and opening and operation costs of RCs), demand, and stored goods in supply centers, they are considered fuzzy in the proposed model. Since the uncertainty parameters are considered fuzzy triangular numbers, we can define them as

$$
\begin{aligned}
\widetilde{d}_{mk} &= \left(d_{mk(1)},\ d_{mk(2)}, d_{mk(3)}\right)\\
\widetilde{s}_{mi} &= \left(s_{mi(1)},\ s_{mi(2)}, s_{mi(3)}\right)\\
\widetilde{c}_{mij} &= \left(c_{mij(1)},\ c_{mij(2)}, c_{mij(3)}\right)\\
\widetilde{c}_{mir} &= \left(c_{mir(1)},\ c_{mir(2)}, c_{mir(3)}\right)\\
\widetilde{c}_{mjk} &= \left(c_{mjk(1)},\ c_{mjk(2)}, c_{mjk(3)}\right)\\
\widetilde{c}_{mrk} &= \left(c_{mrk(1)},\ c_{mrk(2)}, c_{mrk(3)}\right)\\
\widetilde{h}_{mj} &= \left(h_{mj(1)},\ h_{mj(2)}, h_{mj(3)}\right)\\
\widetilde{h}_{mr} &= \left(h_{mr(1)},\ h_{mr(2)}, h_{mr(3)}\right)\\
\widetilde{F}_{j} &= \left(F_{j(1)},\ F_{j(2)}, F_{j(3)}\right)\\
\widetilde{F}_{r} &= \left(F_{r(1)},\ F_{r}(2), F_{r(3)}\right).
\end{aligned}
$$

The compact form of the bi-objective CGMCLP in the given RSC is stated as in (34).

$$
\begin{aligned}
&\text{Min } f_1 = F\cdot y + C\cdot x\\
&\text{Max } f_2 = h.x\\
&\text{such that}\\
&\qquad A\cdot y = P\\
&\qquad B\cdot y \geq 0\\
&\qquad G\cdot x \leq d\cdot x\\
&\qquad J\cdot x \leq N\cdot s\\
&\qquad E\cdot x \leq T\cdot y\\
&\qquad M\cdot x \leq 0\\
&\qquad y\in\{0,1\}, x\geq 0.
\end{aligned}
\tag{34}
$$

The vectors x and y correspond to the model's decision variables, which are continuous and binary, respectively. Vector F corresponds to the opening and operation costs of the RCs, vector C represents the carrying and transportation costs, and vector h represents the coefficients of the objective coverage function in the given model. P is a scalar representing the number of RCs to be opened; vector d is the demand vector; vector s corresponds to the number of stored goods in the supply centers. Vector T is the capacity of the RCs. N is the binary parameter representing the availability of goods in the supply centers. Finally, A, B, G, J, E, and M are the coefficient matrices in the given constraints.

Similar to previous research on RPP, we give a fundamental possibilistic chance constraint programming (BPCCP) model. The close-packed form of the addressed model is stated as (35) [49]

$$
\begin{aligned}
&\text{Min } f_1 = E\left[\widetilde{F}\right]y + E[\widetilde{C}]x\\
&\text{Max } f_2 = h\cdot x\\
&\text{such that}\\
&\qquad A\cdot y = P\\
&\qquad B\cdot y \geq 0\\
&\qquad \text{Nec}\left\{G\cdot x\ \leq \widetilde{d}\cdot x\right\} \geq \beta
\end{aligned}
$$

$$\begin{aligned}
&\mathrm{Nec}\{J \cdot x \leq N \cdot \widetilde{s}\} \geq \alpha \\
&E \cdot x \leq T \cdot y \\
&M \cdot x \leq 0 \\
&y \in \{0,1\}, x \geq 0.
\end{aligned} \tag{35}$$

α and β in the above model represent the minimum degree of satisfaction of the possibilistic constraints. A given DM controls these two parameters.

If r is a real number and $\widetilde{a} = (a_1, a_2, a_3)$ is a triangular fuzzy number, then, the necessity of fuzzy constraints is defined as in (36)–(38).

$$\mathrm{Nec}(\widetilde{a} \leq r) \geq \alpha \rightarrow r \geq (1-\alpha)a_2 + \alpha a_3 \tag{36}$$

$$\mathrm{Nec}(\widetilde{a} \geq r) \geq \alpha \rightarrow r \leq \alpha a_1 + (1-\alpha)a_2 \tag{37}$$

$$\mathrm{Nec}(\widetilde{a} = r) \geq \alpha \rightarrow \begin{cases} r \leq \frac{\alpha}{2}a_2 + \left(1-\frac{\alpha}{2}\right)a_3 \\ r \geq \frac{\alpha}{2}a_2 + \left(1-\frac{\alpha}{2}\right)a_1. \end{cases} \tag{38}$$

Eventually, according to the relation (37), the BPCCP can be stated as in (39).

$$\begin{aligned}
\text{Min } f_1 = &\sum_j \left(\frac{F_{j(1)} + 2F_{j(2)} + F_{j(3)}}{4}\right) Q_j + \sum_r \left(\frac{F_{r(1)} + 2F_{r(2)} + F_{r(3)}}{4}\right) Z_r \\
&+ \sum_{m,i,j} \left(\frac{h_{mj(1)} + 2h_{mj(2)} + h_{mj(3)}}{4}\right) x_{mij} + \sum_{m,i,r} \left(\frac{h_{mr(1)} + 2h_{mr(2)} + h_{mr(3)}}{4}\right) x_{mir} \\
&+ \sum_{i,j,m} \left(\frac{c_{mij(1)} + 2c_{mij(2)} + c_{mij(3)}}{4}\right) x_{mij} + \sum_{i,r,m} \left(\frac{c_{mir(1)} + 2c_{mir(2)} + c_{mir(3)}}{4}\right) x_{mir} \\
&+ \sum_{k,j,m} \left(\frac{c_{mjk(1)} + 2c_{mjk(2)} + c_{mjk(3)}}{4}\right) y_{mjk} + \sum_{k,r,m} \left(\frac{c_{mrk(1)} + 2c_{mrk(2)} + c_{mrk(3)}}{4}\right) y_{mrk} \\
\text{Max } f_2 = &\ h \cdot x \\
\text{such that}& \\
&A \cdot y = P \\
&B \cdot y \geq 0 \\
&G \cdot x \leq ((1-\beta)d_2 + \beta d_1) \cdot x \\
&J \cdot x \leq N \cdot ((1-\alpha)s_2 + \alpha s_1) \\
&E \cdot x \leq T \cdot y \\
&M \cdot x \leq 0 \\
&y \in \{0,1\}, x \geq 0.
\end{aligned} \tag{39}$$

The DM determines this model's lowest confidence levels (α, β). Possible deviations in the objective function and constraints are denied; thus, using this model risks the DM. Due to the risks of the given possibilistic programming model, different approaches to RPP are proposed in the next section.

3.3.1. Basic robust possibilistic programming (RPP) models

The RPP model generally has three main approaches RPP-I, RPP-II, and RPP-III [47,48]. In this sub-section, the implementation of the three approaches on the compact form of cooperative gradual maximal covering model is stated.

RPP-I model is stated as in (40).

$$
\begin{aligned}
&\text{Min } (f_1) = \left(\frac{F_1+2F_2+F_3}{4}\right)y + \left(\frac{C_1+2C_2+C_3}{4}\right)x + \partial((C_3x+F_3y)-(C_1x+F_1y))\\
&\qquad\qquad + \gamma((1-\alpha)s_2+\alpha s_1-s_1)\cdot N + \delta((1-\beta)d_2+\beta d_1-d_1)\cdot x\\
&\text{Max } (f_2) = h\cdot x - \theta((1-\beta)d_2+\beta d_1-d_1)\cdot x\\
&\text{such that}\\
&\qquad A\cdot y = P\\
&\qquad B\cdot y \geq 0\\
&\qquad G\cdot x \leq ((1-\beta)d_2+\beta d_1)\cdot x\\
&\qquad J\cdot x \leq N\cdot((1-\alpha)s_2+\alpha s_1)\\
&\qquad E\cdot x \leq T\cdot y\\
&\qquad M\cdot x \leq 0\\
&\qquad y\in\{0,1\}, x\geq 0\\
&\qquad 0.5\leq \alpha,\beta\leq 1. \qquad\qquad (40)
\end{aligned}
$$

The first and second terms represent the expected value of the cost objective function in the first objective function of the model. The term numbered three represents the additional cost due to the objective function's deviation between the maximum and minimum values (*i.e.*, the optimality of robustness), where ∂ is the significance coefficient of these deviations. The fourth and fifth terms represent the additional costs due to deviations of demand and supply centers constraints (*i.e.*, the feasibility robustness); the parameters of γ and δ are the significant coefficients of these deviations, respectively.

The second objective function is only directly affected by the deviations of the demand parameter. The parameter θ corresponds to the significance coefficient of demand deviation. In this model, α and β, which were input parameters in the BPCCP model, should be determined as decision variables of the model in RPP.

In the third constraint, and the two given objective functions of this model, there is non-linearity as the multiplication of two continuous variables of x by β. Using the McCormick method, by definition of $w = x\cdot\beta$ and considering $\beta_{\min} = 0.5$, $\beta_{\max} = 1$, $x_{\min} = 0$ and $x_{\max} = 1$, the equivalent linear model of (40) can be stated as in (41)

$$
\begin{aligned}
&\text{Min } (f_1) = \left(\frac{F_1+2F_2+F_3}{4}\right)y + \left(\frac{C_1+2C_2+C_3}{4}\right)x + \partial((C_3x+F_3y)-(C_1x+F_1y))\\
&\qquad\qquad + \gamma((1-\alpha)s_2+\alpha s_1-s_1)\cdot N + \delta((x-w)d_2+wd_1-d_1\cdot x)\\
&\text{Max } (f_2) = h\cdot x - \theta((x-w)d_2+wd_1-d_1\cdot x)\\
&\text{such that}\\
&\qquad A\cdot y = P\\
&\qquad B\cdot y \geq 0\\
&\qquad G\cdot x \leq (x-w)d_2+wd_1\\
&\qquad w \geq \beta_{\min}\cdot x + x_{\min}\cdot\beta - \beta_{\min}\cdot x_{\min}\\
&\qquad w \geq \beta_{\max}\cdot x + x_{\max}\cdot\beta - \beta_{\max}\cdot x_{\max}\\
&\qquad w \leq \beta_{\max}\cdot x + x_{\min}\cdot\beta - \beta_{\max}\cdot x_{\min}\\
&\qquad w \leq \beta_{\min}\cdot x + x_{\max}\cdot\beta - \beta_{\min}\cdot x_{\max}\\
&\qquad J\cdot x \leq N\cdot((1-\alpha)s_2+\alpha s_1)\\
&\qquad E\cdot x \leq T\cdot y
\end{aligned}
$$

$$
\begin{aligned}
& M \cdot x \leq 0 \\
& y \in \{0,1\}, x, w \geq 0 \\
& 0.5 \leq \alpha, \beta \leq 1.
\end{aligned} \tag{41}
$$

RPP-II model is the same as RPP-I, except that in the term numbered three in the first objective function, instead of the objective function deviation between the maximum and minimum values, the objective function deviation between the maximum and expected values should be considered in the model. The mathematical model in (42) gives the RPP-II model noting that F represents the feasible region of the RPP-I model.

$$
\begin{aligned}
\text{Min } (z_1) = & \left(\frac{F_1+2F_2+F_3}{4}\right)y + \left(\frac{C_1+2C_2+C_3}{4}\right)x \\
& + \partial\left((F_3 y + C_3 x) - \left(\left(\frac{F_1+2F_2+F_3}{4}\right)y + \left(\frac{C_1+2C_2+C_3}{4}\right)x\right)\right) \\
& + \gamma((1-\alpha)s_2 + \alpha s_1 - s_1) \cdot N + \delta((x-w)d_2 + wd_1 - d_1 \cdot x) \\
\text{Max } (z_2) = & \; h \cdot x - \theta((x-w)d_2 + wd_1 - d_1 \cdot x) \\
& \text{such that } x, y, \alpha, \beta \in F.
\end{aligned} \tag{42}
$$

In the RPP-III, the term numbered three in the first objective function is considered only sensitive to its maximum value. The model in (43) states the RPP-III model, noting that F represents the feasible region of the RPP-I model.

$$
\begin{aligned}
\text{Min } (f_1) = & \left(\frac{F_1+2F_2+F_3}{4}\right)y + \left(\frac{C_1+2C_2+C_3}{4}\right)x + \partial(F_3 y + C_3 x) + \gamma((1-\alpha)s_2 + \alpha s_1 - s_1) \cdot N \\
& + \delta((x-w)d_2 + wd_1 - d_1 \cdot x) \\
\text{Max } (f_2) = & \; h \cdot x - \theta((x-w)d_2 + wd_1 - d_1 \cdot x) \\
& \text{such that } x, y, \alpha, \beta \in F.
\end{aligned} \tag{43}
$$

3.3.2. Modified robust possibilistic programming (MRPP) model

In the previous models, the deviations of the possibilistic constraints were calculated through a conservative approach. In the MRPP model, the DM has a less traditional approach; for this purpose, the variables of β and α in the deviations terms are multiplied by δ and γ. The model given in (44) provides the MRPP noting that F represents the feasible region the RPP-I model is based [48].

$$
\begin{aligned}
\text{Min } (f_1) = & \left(\frac{F_1+2F_2+F_3}{4}\right)y + \left(\frac{C_1+2C_2+C_3}{4}\right)x \\
& + \partial\left((F_3 y + C_3 x) - \left(\left(\frac{F_1+2F_2+F_3}{4}\right)y + \left(\frac{C_1+2C_2+C_3}{4}\right)x\right)\right) \\
& + \gamma\alpha((1-\alpha)s_2 + \alpha s_1 - s_1) \cdot N + \delta\beta((x-w)d_2 + wd_1 - d_1 \cdot x) \\
\text{Max } (f_2) = & \; h \cdot x - \theta\beta((x-w)d_2 + wd_1 - d_1 \cdot x) \\
& \text{such that } x, y, \alpha, \beta \in F.
\end{aligned} \tag{44}
$$

Although the MRPP model has some benefits, this model falls into nonlinear programming models. This model can be linearized using the McCormick method, as given earlier.

3.3.3. Hard worst-case robust possibilistic programming (HWRPP) model

Model (45) gives the HWRPP model based on Pishvaee *et al.* [48]. The DM tries to have a conservative and risk-averse approach in this model. HWRPP is not sensitive to the possibilistic distribution of uncertainty parameters; in this mode, the worst case of each uncertain parameter is considered in the objective function or constraints.

$$\begin{aligned}
&\text{Min } (f_1) = \sup(f_1)\\
&\text{Max } (f_2) = h \cdot x\\
&\text{such that}\\
&\quad A \cdot y = P\\
&\quad B \cdot y \geq 0\\
&\quad G \cdot x \leq \inf\left(\widetilde{d}\right) \cdot x\\
&\quad J \cdot x \leq N \cdot \inf(\widetilde{s})\\
&\quad E \cdot x \leq T \cdot y\\
&\quad M \cdot x \leq 0\\
&\quad y \in \{0,1\}, x \geq 0.
\end{aligned} \tag{45}$$

Considering the uncertain parameter as trapezoidal, the HWRPP model is stated as in (46).

$$\begin{aligned}
&\text{Min } (f_1) = F_3 y + C_3 x\\
&\text{Max } (f_2) = h \cdot x\\
&\text{such that}\\
&\quad A \cdot y = P\\
&\quad B \cdot y \geq 0\\
&\quad G \cdot x \leq d_1 \cdot x\\
&\quad J \cdot x \leq N \cdot s_1\\
&\quad E \cdot x \leq T \cdot y\\
&\quad M \cdot x \leq 0\\
&\quad y \in \{0,1\}, x \geq 0.
\end{aligned} \tag{46}$$

3.3.4. *Soft worst-case robust possibilistic programming (SWRPP) model*

The SWRPP model is a rigorous conservative approach that is less conservative than the HWRPP model. Model (47) gives the SWRPP model, where F represents the feasible region. The RPP-I model is based on Pishvaee *et al.* [48].

$$\begin{aligned}
&\text{Min } (f_1) = (F_3 y + C_3 x) + \gamma((1-\alpha)s_2 + \alpha s_1 - s_1) \cdot N + \delta((x-w)d_2 + wd_1 - d_1 \cdot x)\\
&\text{Max } (f_2) = h \cdot x - \theta((x-w)d_2 + wd_1 - d_1 \cdot x)\\
&\qquad \text{such that } x, y, \alpha, \beta \in F.
\end{aligned} \tag{47}$$

4. Numerical examples and sensitivity analysis

This section has designed some numerical examples for testing the given model's performance and studying the effects of different RPP models. The numerical example is solved in Lingo 11 software in a Pentium i9 computer with 64 GB RAM, 2 TB HDD. The uncertain parameters are represented using fuzzy triangular numbers, and Table 2 illustrates how the values of the model parameters are generated.

TABLE 2. The values of parameters in the numerical examples.

Parameters	Values	Parameters	Values
$\|I\|$	7	$\widetilde{d}_{mk}$	Uniform $(250, 400)$
$\|J\|$	15	$\widetilde{F}_j$	Uniform $(90, 170)$ (Million Rials)
$\|R\|$	3	$\widetilde{F}_r$	Uniform $(40, 100)$ (Million Rials)
$\|K\|$	15	$\widetilde{h}_{mj}$	Uniform $(50, 170)$ (Thousand Rials)
$\|M\|$	3	$\widetilde{h}_{mr}$	Uniform $(40, 170)$ (Thousand Rials)
$\|P\|$	6	$\widetilde{s}_{mi}$	Uniform $(165, 400)$
$\widetilde{c}_{mij}, \widetilde{c}_{mir}$	Uniform $(200, 330)$ (Thousand Rials)	$\widetilde{c}_{mjk}, \widetilde{c}_{mrk}$	Uniform $(180, 330)$ (Thousand Rials)

Furthermore, the signal strength sent from RC j and r to demand point k is given as in (48) and (49).

$$\emptyset_{jk} = \begin{cases} 1 & d_{jk} < 100 \\ 0.7 & 100 \leq d_{jk} < 150 \\ 0.3 & 150 \leq d_{jk} < 200 \\ 0 & d_{jk} \geq 200 \end{cases} \tag{48}$$

$$\emptyset_{rk} = \begin{cases} 1 & d_{rk} < 100 \\ 0.7 & 100 \leq d_{rk} < 150 \\ 0.3 & 150 \leq d_{rk} < 200 \\ 0 & d_{rk} \geq 200. \end{cases} \tag{49}$$

We have utilized the ε-constraint method for solving the model. This method can provide a set of acceptable Pareto-optimal solutions to the DM. Suppose that $A(x)$ represents the constraints in each of the possibilistic models; the model (50) shows the general form of the ε-constraint model to reach Pareto-optimal solutions. The first objective function (cost) is considered the principal objective function (f_1) and the second objective function, which represents the coverage (f_2) lies in the constraints section. Changing the value of ε generates different solutions

$$\begin{aligned} &\text{Min } f_1 \\ &\text{such that } f_2 \geq \epsilon \\ &\qquad\qquad x \in A(x). \end{aligned} \tag{50}$$

Ten Pareto-optimal solutions are obtained from the PP model (*i.e.*, BPCCP model) and each RPP model (*i.e.*, RPP-I, RPP-II, RPP-III, MRPP, HWRPP, SWRPP) considering different minimum confidence levels (*i.e.*, 0.5, 0.6, and 0.7). Tables 3–5 show these solutions. Table 3 gives the Pareto-optimal solutions for the BPCCP model with confidence levels. As the minimum confidence levels increase, the coverage level and the total cost decrease. Tables 4 and 5 give the Pareto-optimal solutions for the six addressed RPP models. The results confirm the conflict between the objective functions because increasing the demand coverage increases total costs and *vice versa*.

After generating the Pareto-optimal solutions for different RPP and PP model approaches, we evaluate the robustness of the solutions given by the addressed models. For this purpose, ten instances of the problem are solved for random values of the uncertain parameters of the model. For solving each instance, uncertain parameters of the model (including the objective functions' coefficients and constraints), denoted by fuzzy triangular numbers (*e.g.*, $\widetilde{F_o} = \left(F_{o(1)}, F_{o(2)}, F_{o(3)}\right)$), are considered, and a random value (called the nominal value of the fuzzy number) is generated uniformly between the extreme points of fuzzy the number (*i.e.*, $F_{\text{real}} = \left[F_{o(1)}, F_{o(3)}\right]$). By using the addressed nominal value, the decision variables of the model *i.e.*, x^* and y^* are acquired by solving the given PP and RPP models given in (39), (41)–(43), and (47). After obtaining

TABLE 3. Pareto-optimal solutions for BPCCP model with confidence levels (*i.e.*, 0.5, 0.6 and 0.7).

Pareto-optimal solutions ($\alpha, \beta = 0.5$)	f_1 (Rials)	f_2	Pareto-optimal solutions ($\alpha, \beta = 0.6$)	f_1 (Rials)	f_2	Pareto-optimal solutions ($\alpha, \beta = 0.7$)	f_1 (Rials)	f_2
1	8.293e+8	232	1	8.223e+8	219	1	8.146e+8	207
2	9.682e+8	463	2	9.533e+8	439	2	9.383e+8	414
3	1.164e+8	695	3	1.142e+8	658	3	1.119e+8	621
4	8.514e+12	927	4	6.676e+12	878	4	4.801e+12	828
5	1.721e+13	1158	5	1.489e+13	1097	5	1.256e+13	1035
6	2.591e+13	1390	6	2.314e+13	1316	6	2.033e+13	1242
7	3.458e+13	1622	7	3.135e+13	1536	7	2.809e+13	1449
8	4.328e+13	1854	8	3.960e+13	1755	8	3.585e+13	1656
9	5.198e+13	2085	9	4.781e+13	1975	9	4.361e+13	1866
10	6.064e+13	2317	10	5.603e+13	2194	10	5.138e+13	2070

TABLE 4. Pareto-optimal solutions for RPP-I, RPP-II, and RPP-III models.

Pareto-optimal solutions (RPP-I)	f_1 (Rials)	f_2	Pareto-optimal solutions (RPP-II)	f_1 (Rials)	f_2	Pareto-optimal solutions (RPP-III)	f_1 (Rials)	f_2
1	1.071e+9	229	1	1.004e+9	229	1	1.464e+9	229
2	1.236e+9	459	2	1.154e+9	459	2	1.680e+9	459
3	1.409e+9	688	3	1.311e+9	688	3	1.907e+9	688
4	8.176e+12	918	4	4.089e+12	918	4	8.177e+12	918
5	1.680e+13	1147	5	8.402e+12	1147	5	1.680e+13	1147
6	2.539e+13	1377	6	1.269e+13	1377	6	2.539e+13	1377
7	3.401e+13	1606	7	1.701e+13	1606	7	3.401e+13	1606
8	4.260e+13	1836	8	2.130e+13	1836	8	4.260e+13	1836
9	5.123e+13	2065	9	2.561e+13	2065	9	5.123e+13	2065
10	5.985e+13	2295	10	2.992e+13	2295	10	5.985e+13	2295

the optimal solutions from each model, x^* and y^* are replaced with the corresponding values in (51), a linear programming model that generates cost value with minimum deviation for all PP and RPP model solutions. The terms R^d and R^s represent the deviations of the chance constraints of the models. The coefficients θ, λ, and μ are penalty values for the addressed deviations. In (51), R^d and R^s are the decision variables in this model. The penalty values are given in Table 6.

$$
\begin{aligned}
&\text{Min } f_1 = f_{\text{real}} y^* + c_{\text{real}} x^* + \theta R^d + \lambda R^s \\
&\text{Max } f_2 = h x^* - \mu R^d \\
&\text{such that} \\
&\qquad A \cdot y^* = P \\
&\qquad B \cdot y^* \geq 0 \\
&\qquad G \cdot x^* \leq d \cdot x^* + R^d \\
&\qquad J \cdot x^* \leq N \cdot s + R^s
\end{aligned}
$$

Table 5. Pareto-optimal solutions for MRPP, HWRPP, and SWRPP models.

Pareto-optimal solutions (MRPP)	f_1 (Rials)	f_2	Pareto-optimal solutions (HWRPP)	f_1 (Rials)	f_2	Pareto-optimal solutions (SWRPP)	f_1 (Rials)	f_2
1	1.004e+9	229	1	8.407e+8	169	1	1.091e+9	229
2	1.154e+9	459	2	9.614e+8	340	2	1.253e+9	459
3	1.311e+9	688	3	1.084e+9	510	3	1.417e+9	688
4	1.089e+9	918	4	1.206e+9	680	4	1.582e+12	918
5	1.402e+12	1147	5	1.330e+9	849	5	1.748e+12	1147
6	1.269e+12	1377	6	1.454e+9	1019	6	1.915e+13	1377
7	1.703e+13	1606	7	1.580e+9	1189	7	2.180e+13	1606
8	2.130e+13	1836	8	1.753e+9	1359	8	2.291e+13	1836
9	2.511e+13	2065	9	1.932e+9	1529	9	2.528e+13	2065
10	2.692e+13	2295	10	2.287e+9	1699	10	2.797e+13	2295

Table 6. The values of penalties in the model (51).

Penalty levels	1	2	3	4	5
θ	500 000	750 000	1 000 000	1 250 000	1 500 000
λ	500 000	750 000	1 000 000	1 250 000	1 500 000
μ	1	1	1	1	1

$$\begin{aligned}
&E \cdot x^* \leq T \cdot y^* \\
&M \cdot x^* \leq 0 \\
&R^d, R^s \geq 0.
\end{aligned} \tag{51}$$

The rest assess the performances of PP and different RPP models utilizing the model given in (51).

4.1. Performance of PP model

This section compares PP models with different minimum confidence levels. The average and standard deviation are two useful indicators for this comparison. Considering the minimum confidence levels of 0.5, 0.6, and 0.7, Figures 7 and 8 show the variations of average and standard deviation indicators when solving each penalty value's ten problem instances. As the confidence level increases, both indicators decrease. This decrease in average cost can be justified by coverage decreases for different values of confidence levels.

4.2. Performance of the RPP models (RPP-I, RPP-II, RPP-III, and SWRPP)

In solving different approaches to RPP models, the improved McCormick method was used as an approximate method to linearize the multiplication of two continuous variables. On the other hand, the approximate McCormick method was used twice in the MRPP model (given in (44)). As a result, this model does not have the desired accuracy, and the solution obtained from solving this method cannot be considered compared to other RPP models from the viewpoint of robustness. Furthermore, the HWRPP model (given in (46)) utilizes the most conservative values for the fuzzy numbers; thus, it is not used as a reference model in comparisons. The four RPP models of RPP-I, RPP-II, RPP-III, and SWRP are used.

As Figures 9 and 10 show, with the increase of penalty values for different RPP models, the average cost and standard deviation of cost generally have an upward trend.

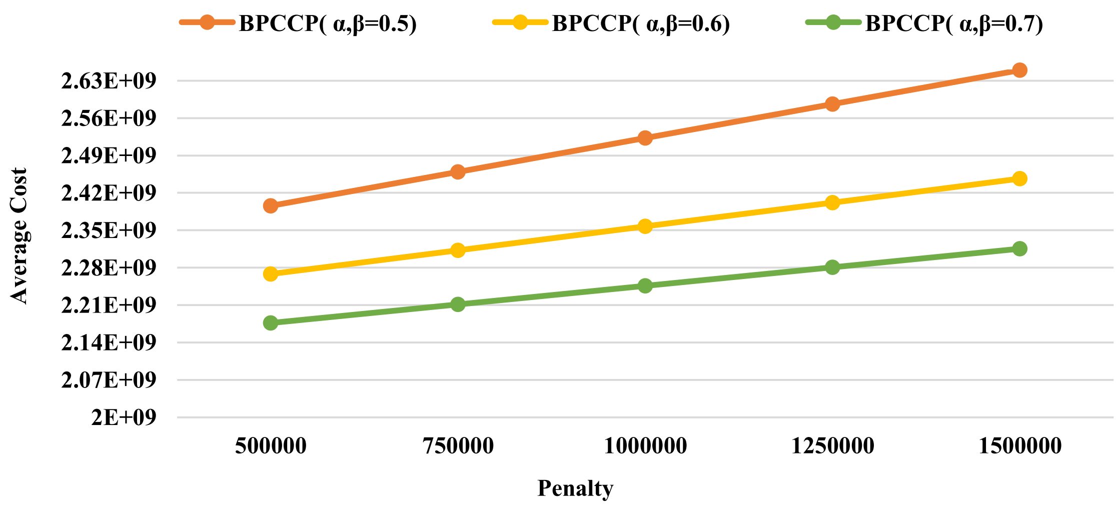

FIGURE 7. Average cost variations against the penalty values for the PP model with ten instances.

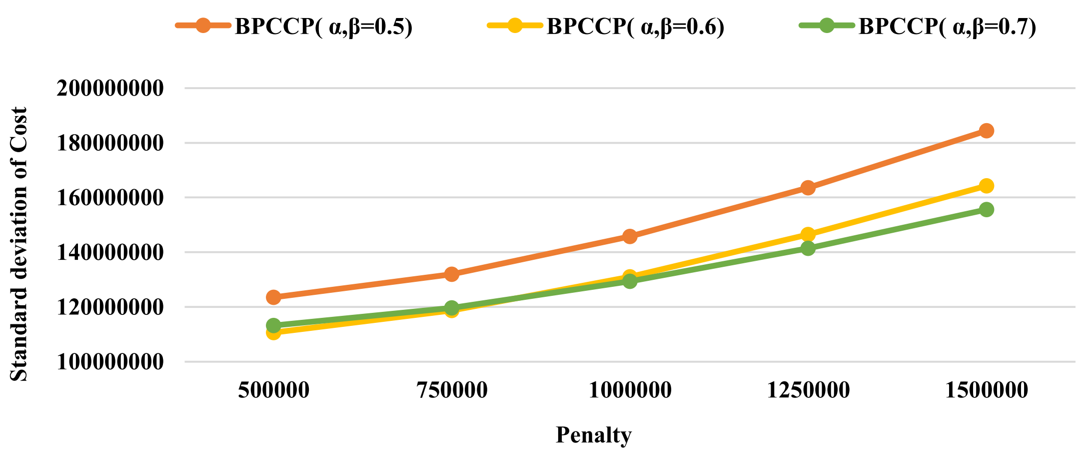

FIGURE 8. Standard deviation variations against the penalty values for the PP model with ten instances.

Among the four RPP models, SWRPP generally has a lower average cost and higher standard deviation for different penalties than the other models. Therefore, compared to the three other models, this model is better in terms of average cost and shows a weak performance regarding the standard deviation metric. As a result, the SWRPP model is preferred if the decision-making criterion is based on the average cost indicator. RPP-II and RPP-III have preferred paradigms if the decision-making criterion is based on the standard deviation indicator.

5. Conclusions and further research ideas

This research studied a two-level CGMCLP under the uncertainty of cost factors, demand, and capacity parameters. The uncertain parameters were deemed as fuzzy triangular numbers. The proposed model was a bi-objective programming model whose first objective function seeks to minimize total logistical costs while the second objective function maximized demand coverage of damaged points. According to the findings, the RPP models perform better than the PP model from the robustness perspective. Regarding the RPP models, the

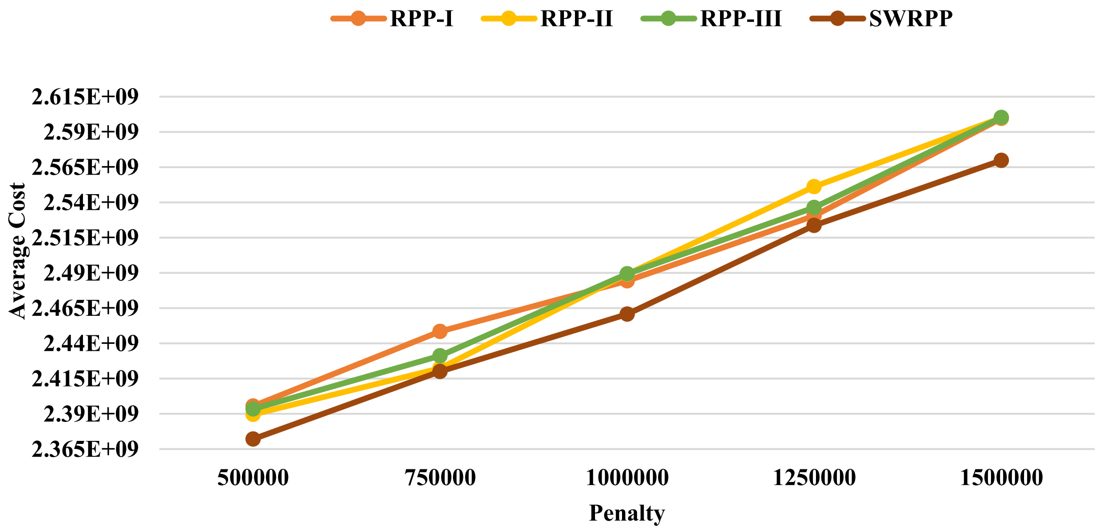

FIGURE 9. Average cost variations against the penalty values for the RPP models with ten instances.

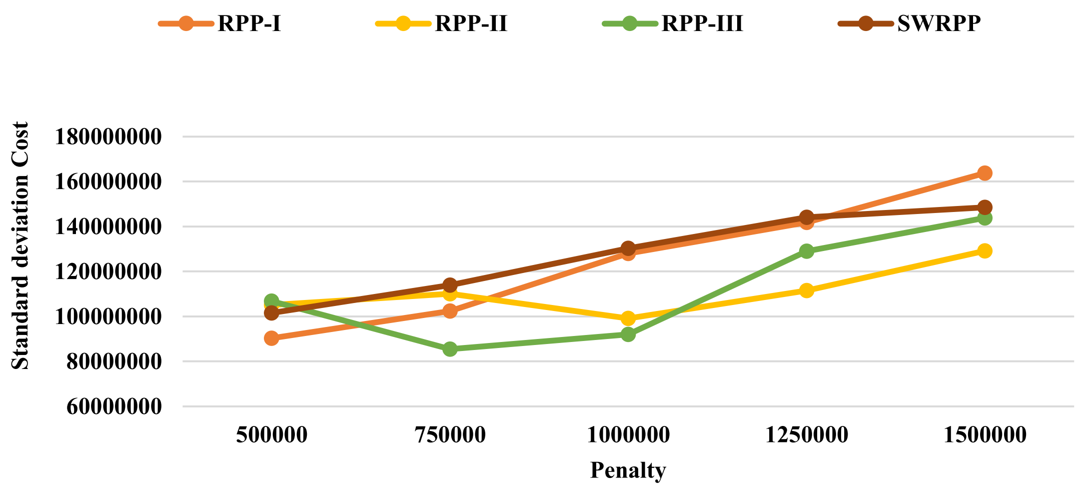

FIGURE 10. Standard deviation variations against the penalty values for the RPP models with ten instances.

SWRPP model is preferred if the decision-making criterion is based on the average cost indicator. RPP-II and RPP-III have preferred models if the decision-making criterion is based on the standard deviation indicator.

Future extensions

The major limitations of this study can be considered for the further extension of this model. Those are explained below.

For any RSCM, the main issue is the logistic problem. During the disaster, due to breakdown of road transportation, vehicles cannot be used to help the people immediately. Thus, only air-mode is utilized for that case due to road blockage, or disconnected road situations. Thus, the best way to get connected immediately to the affected people is by air. That is going on as present extension of this research study by the existing authors.

Another big issue is that the temporary relief centers are being made after immediate effect of any disaster. But if those temporary RCs are suddenly broken due to the immediate effect of disasters, then the whole relief situations are in trouble and the safety of human being are in trouble. Then, the technological development is immediate support to tackle the situation [15]. That will be another big area of research extension. The application of drones, helicopters for the transportation of food service for disaster-affected people can be another big research extension of the model. Though the way of optimization should not be then by the ε-constraints method. Then any multi-criteria optimization technique can be used rather than any single optimization technique. After any disaster, the debris management is really a tough job to maintain. How the non-governmental organization and government can control the debris management, that is a major research area for this field [11,50].

Abbreviations

RSC	Relief supply chain
RC	Relief center
SCM	Supply chain management
RSCM	Relief supply chain management
MCLP	Maximum covering location problem
CMCLP	Cooperative maximal covering location problem
GMCLP	Gradual maximal covering location problem
CGMCLP	Cooperative gradual maximal covering location problem
SCLP	Set covering location problem
DM	Decision-maker
GCLP	Gradual covering location problem
CCLP	Cooperative covering location problem
CGMCM	Cooperative gradual maximal covering model
RG	Relief goods
MIP	Mixed integer programming
SBSPM	Scenario-based stochastic programming model
CMCM	Cooperative maximal covering model
RPP-I	Robust possibilistic programming 1
RPP-II	Robust possibilistic programming 2
RPP-III	Robust possibilistic programming 3
MRPP	Modified robust possibilistic programming
HWRPP	Hard worst-case robust possibilistic programming
SWRPP	Soft worst-case robust possibilistic programming
PP	Possibilistic programming
RPP	Robust possibilistic programming
DC	Relief distribution center

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